The pelletron experiment -$^{12}C(p,\gamma)^{13}N$/$^{10}B(p,\alpha)^{7}Be$-

Juzo Zenihiro, RNC (2014)
revised by T.M. in 2016, 2017, 2018,
by H.I. in 2019

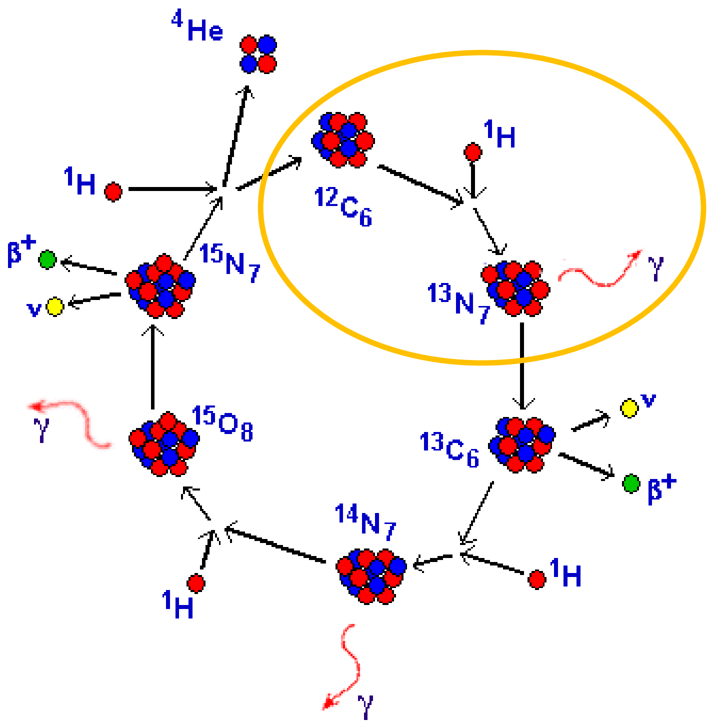
$^{12}C(p,\gamma)^{13}N$ $p + {}^{12}C \rightarrow \gamma + {}^{13}N$

- A reaction involved in the CNO cycle nuclear burning in **massive*** stars, …

- At low (astrophyiscal) energies, two dominant resonances are important.

$$p + {}^{12}C \rightarrow {}^{13}N^{*} \rightarrow \gamma + {}^{13}N$$

CNO cycle



* more massive than the sun

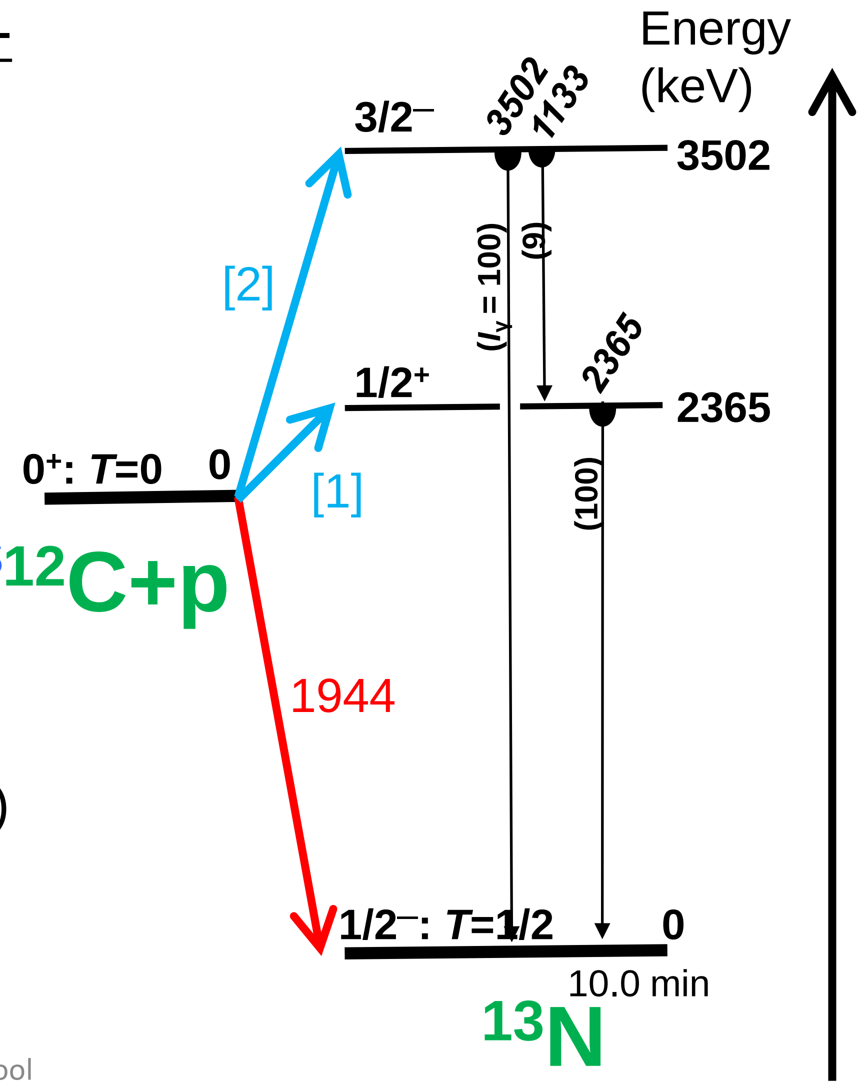
Appropriate energies of proton beams
from the accelerator (pelletron)?

Question 1
energy (mass*) difference between $^{12}C+p$ & ^{13}N

- Energy levels of at the <u>center-of-mass (CM) system</u>
- mass difference between $^{12}C+p$ and ^{13}N is 1944 keV
- Calculate the energy differences for [1] and [2].

Note: The level at 2365 keV (or 3502 keV) is a **resonance**.

Note: $E=mc^2$



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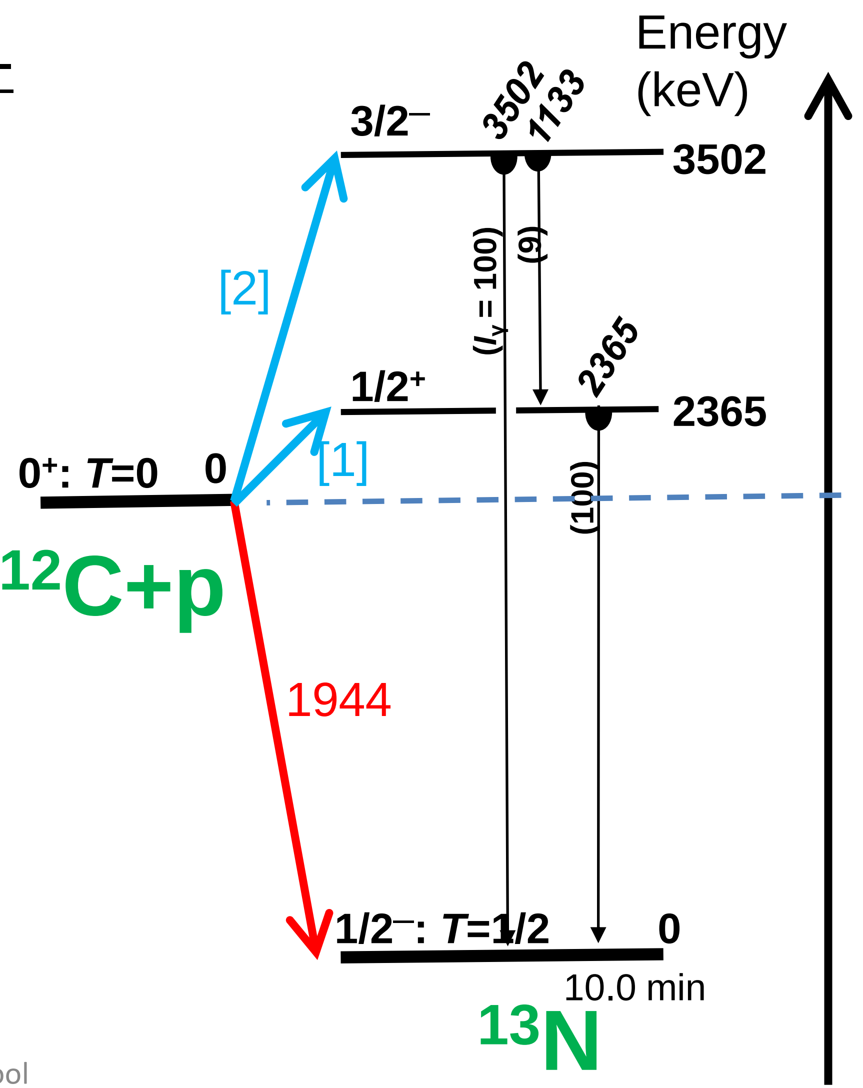

Answers:
421 keV for [1]
1558 keV for [2]

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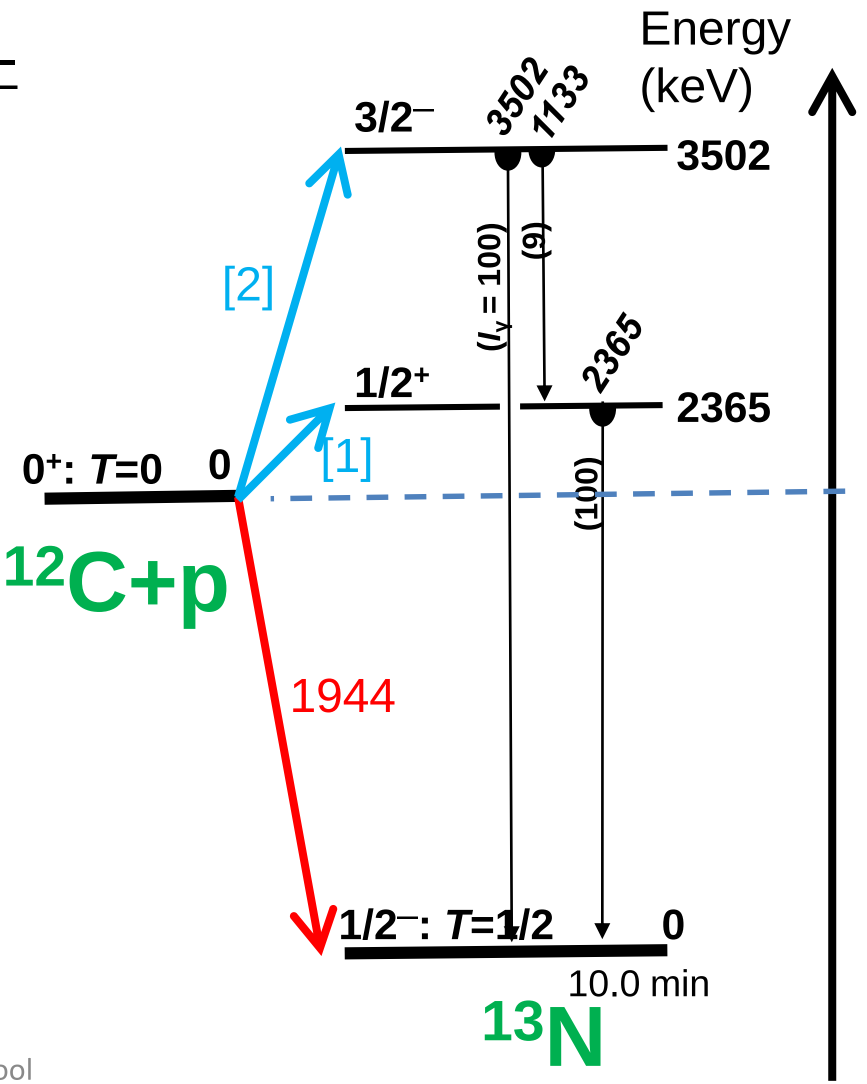
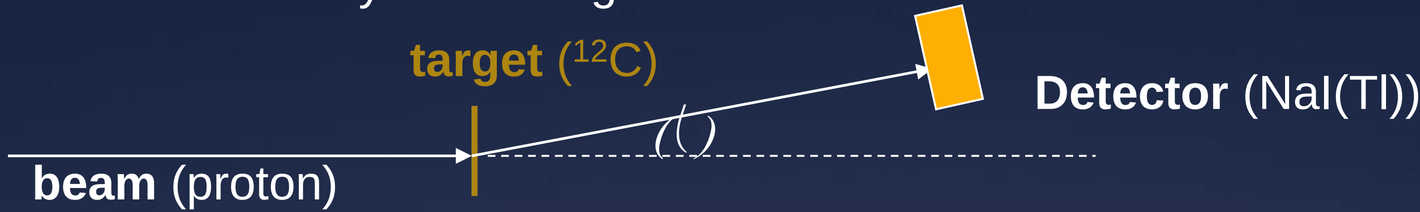


Note:
The level at 2365 keV (or 3502 keV) is a **resonance**.

In the "laboratory" frame

nuclear reaction study with energetic beams



Question 2

proton kinetic energy necessary to populate a resonance?

Is it

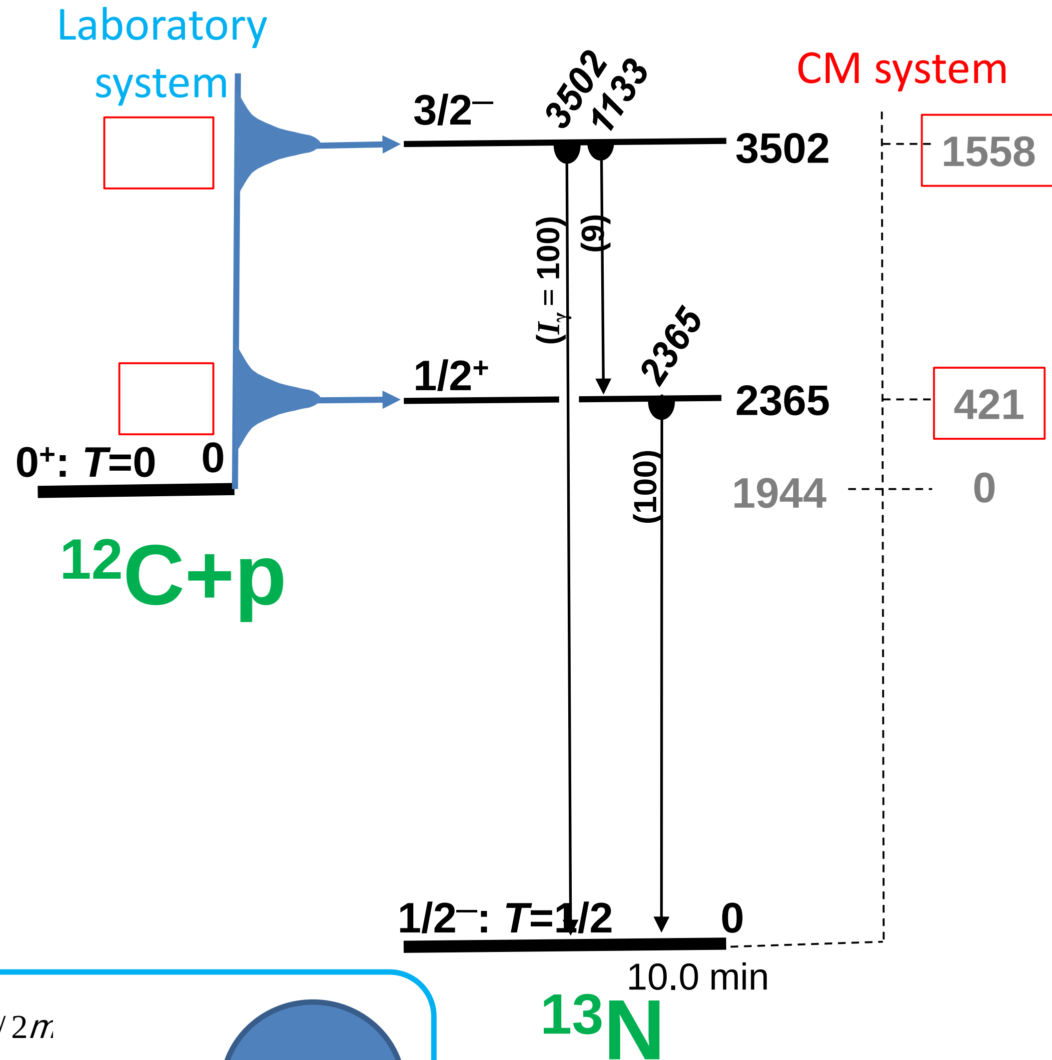
Laboratory <-> CM system :

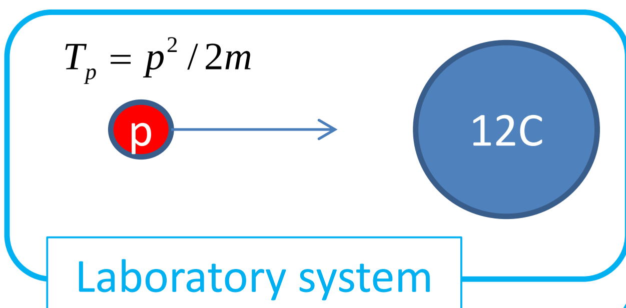
$$T_{\mathrm{CM}} = \frac{p_{\mathrm{rel}}^2}{2m}, m = \frac{m_1 m_2}{m_1 + m_2}, p_{rel} = \frac{m_2 p_1 - m_1 p_2}{m_1 + m_2}$$

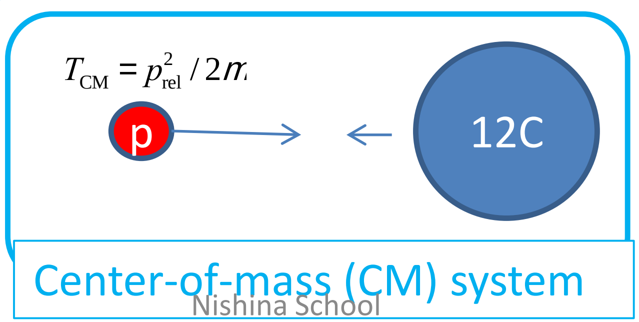
$$m_1 = m, m_2 \,@\, 12m, p_1 = p, p_2 = 0$$

$$\backslash \quad T_{\mathrm{CM}} = \frac{p^2}{2m} \times \frac{12}{13} = T_p \times \frac{12}{13}$$

μ : reduced mass







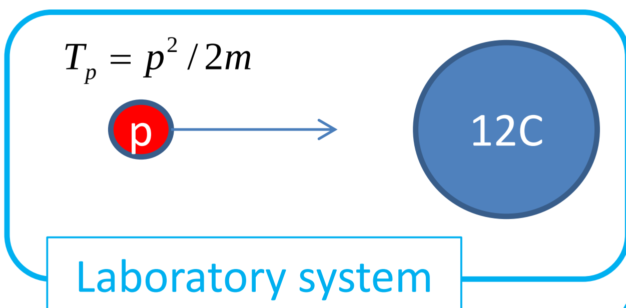
Laboratory <-> CM system :

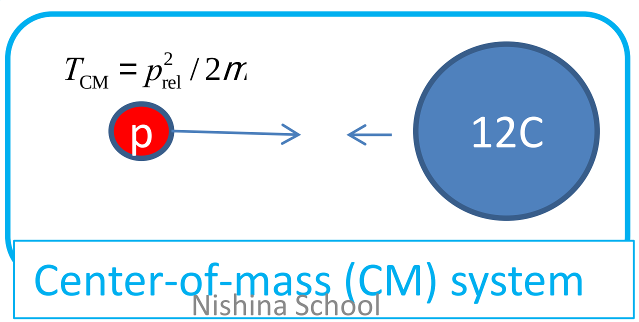
$$T_{CM} = \frac{p_{rel}^2}{2m}, m = \frac{m_1 m_2}{m_1 + m_2}, p_{rel} = \frac{m_2 p_1 - m_1 p_2}{m_1 + m_2}$$

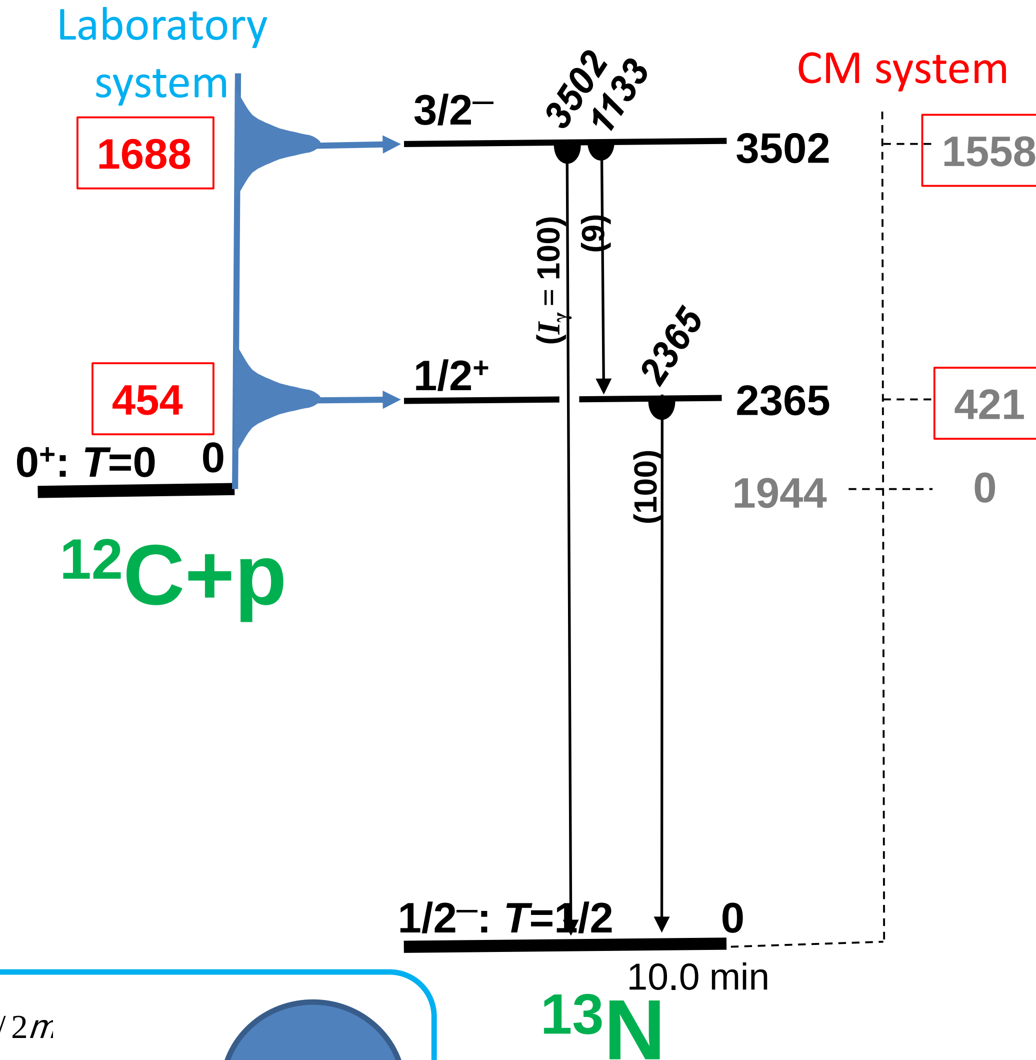
$$m_1 = m, m_2 \approx 12m, p_1 = p, p_2 = 0$$

$$\therefore \quad T_{CM} = \frac{p^2}{2m} \times \frac{12}{13} = T_p \times \frac{12}{13}$$

μ : reduced mass

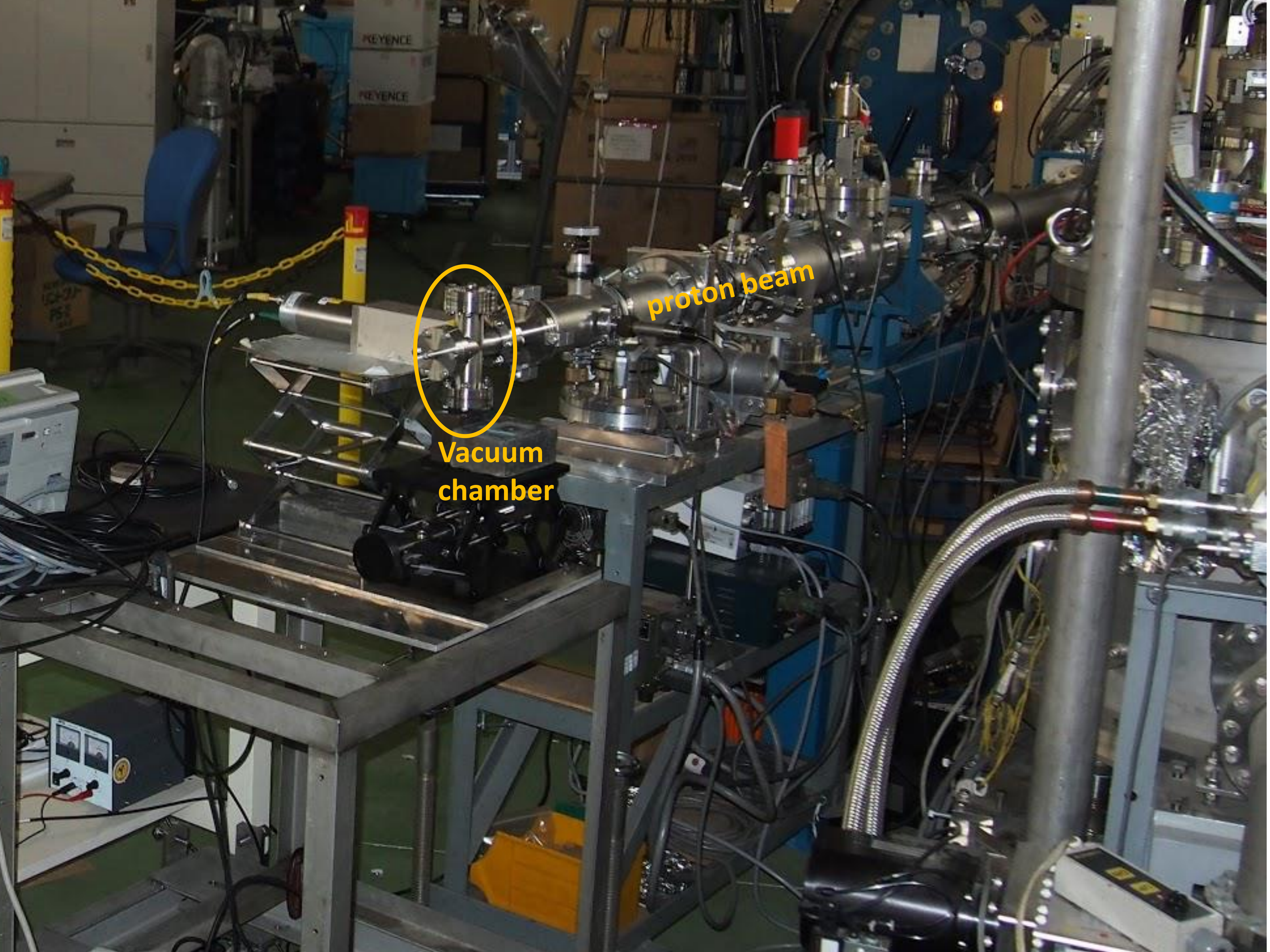


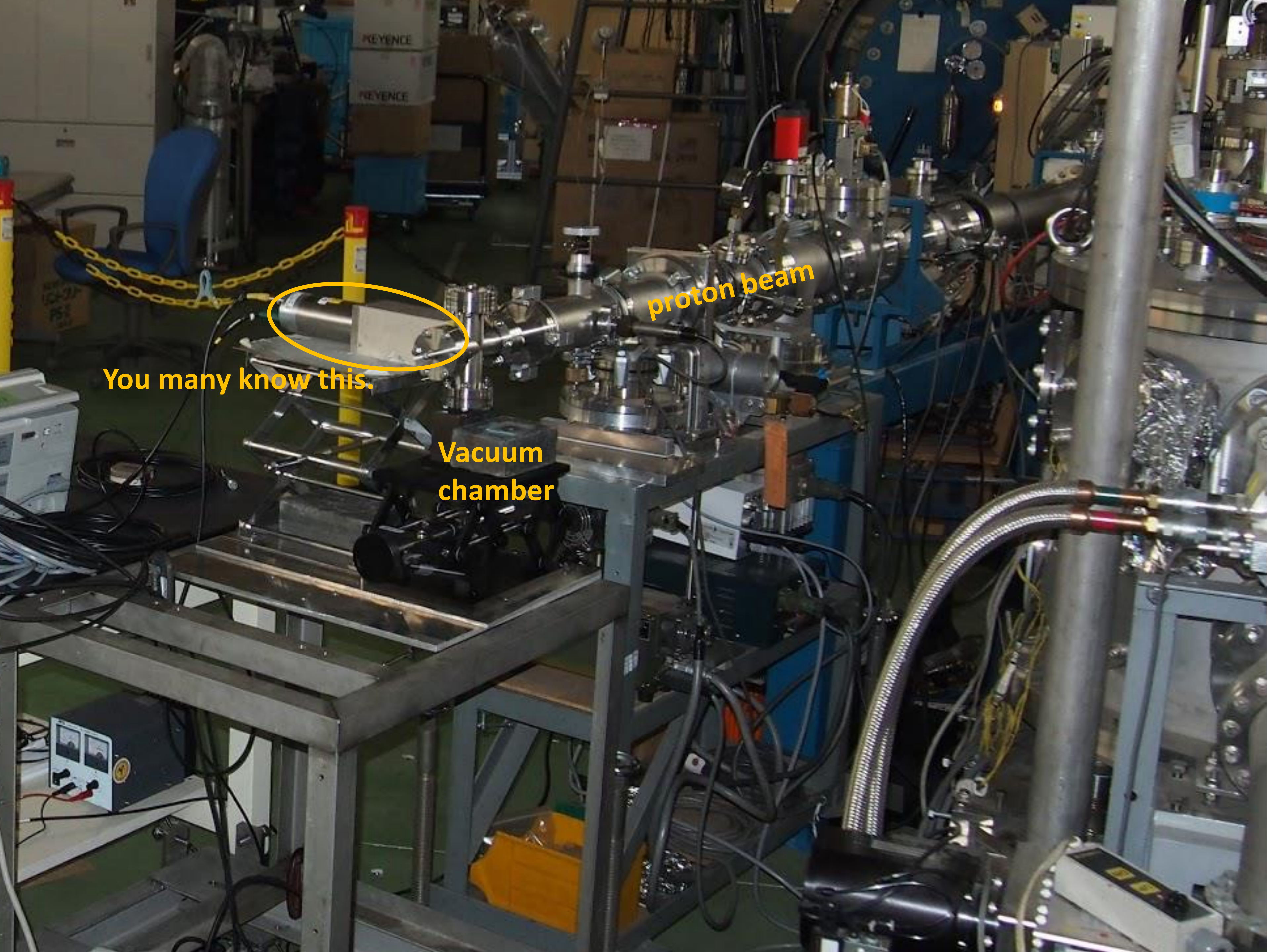




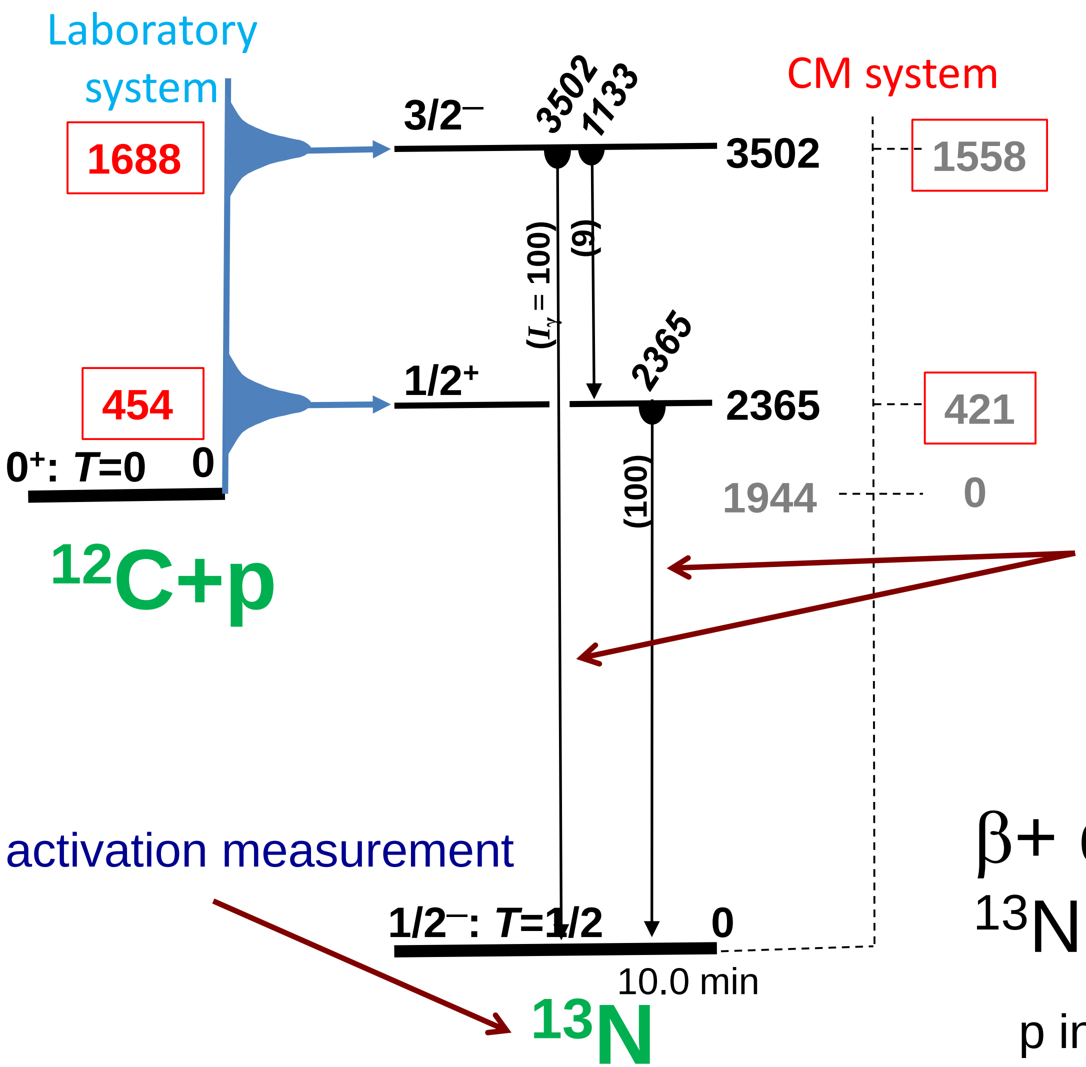
How do we measure the **cross sections**?


last year


proton beam
Vacuum chamber


proton beam
You many know this.
Vacuum
chamber

“in-beam” and “activation”



$e^+ + e^- \rightarrow 2\gamma$ (511 keV*)

* $m_e c^2$

β+ decay

$^{13}N \rightarrow {}^{13}C + e^+ + \nu$

p in ^{13}N decays! (to n)

Remember a question of Sakurai san

Extraction of the (resonant capture) **cross section** of $^{12}C(p,\gamma)^{13}N$

Important parameters

1. The number of protons : N_0
2. The number of ^{12}C : N_t
3. The number of the 1st or 2nd resonance populated : N_r
4. The number of emitted γ: N_γ

Extraction of the (resonant capture) cross section of $^{12}C(p,\gamma)^{13}N$ -- more in details --

I : electric current of the beam
e : electron charge
t : measurement time
σ : cross section of p+12C -> 13N*(2nd :3502 keV) reaction
ρ : density of target carbon foil
T : thickness of target carbon foil
N_A : Avogadro number
P_γ : decay branching ratio of the 3502 keV resonance
Ω : solid angle of NaI(Tl)
ε : photo peak efficiency of 3502 keV

$$N_0 = I \cdot t / e$$

$$N_t = N_A \cdot T \cdot \rho / 12$$

$$N_r = N_0 \cdot N_t \cdot \sigma$$

$$N_\gamma = N_r \cdot P_\gamma \cdot (\Omega / 4\pi) \cdot \varepsilon$$

[Parameters]

I : electric current of the beam

e : electron charge

t : measurement time

σ : cross section of p+12C -> 13N*(2[nd] :3502 keV) reaction

ρ : density of target carbon foil

T : thickness of target carbon foil

N_A : Avogadro number

P_γ : decay branching ration of 3502 keV

Ω : solid angle of NaI(Tl)

ε : photo peak efficiency of 3502 keV

$$\sigma = N_\gamma \cdot 12 \cdot 4\pi / (I / e \cdot t \cdot N_A \cdot T \cdot \rho \cdot P_\gamma \cdot \Omega \cdot \varepsilon)$$

$$d\sigma / d\Omega = N_\gamma \cdot 12 / (I / e \cdot t \cdot N_A \cdot T \cdot \rho \cdot P_\gamma \cdot \Omega \cdot \varepsilon)$$

O : should be measured during the experiment

Design of the experiments
Yield estimation

- the yield of the measurement to check the feasibility

$$d\sigma / d\Omega = N_\gamma \cdot 12 / \left(I / e \cdot t \cdot N_A \cdot T \cdot \rho \cdot P_\gamma \cdot \Omega \cdot \varepsilon \right)$$ **[μbarn/sr]**

- Let’s estimate the yield of N_γ this afternoon!

Useful information for yield estimation

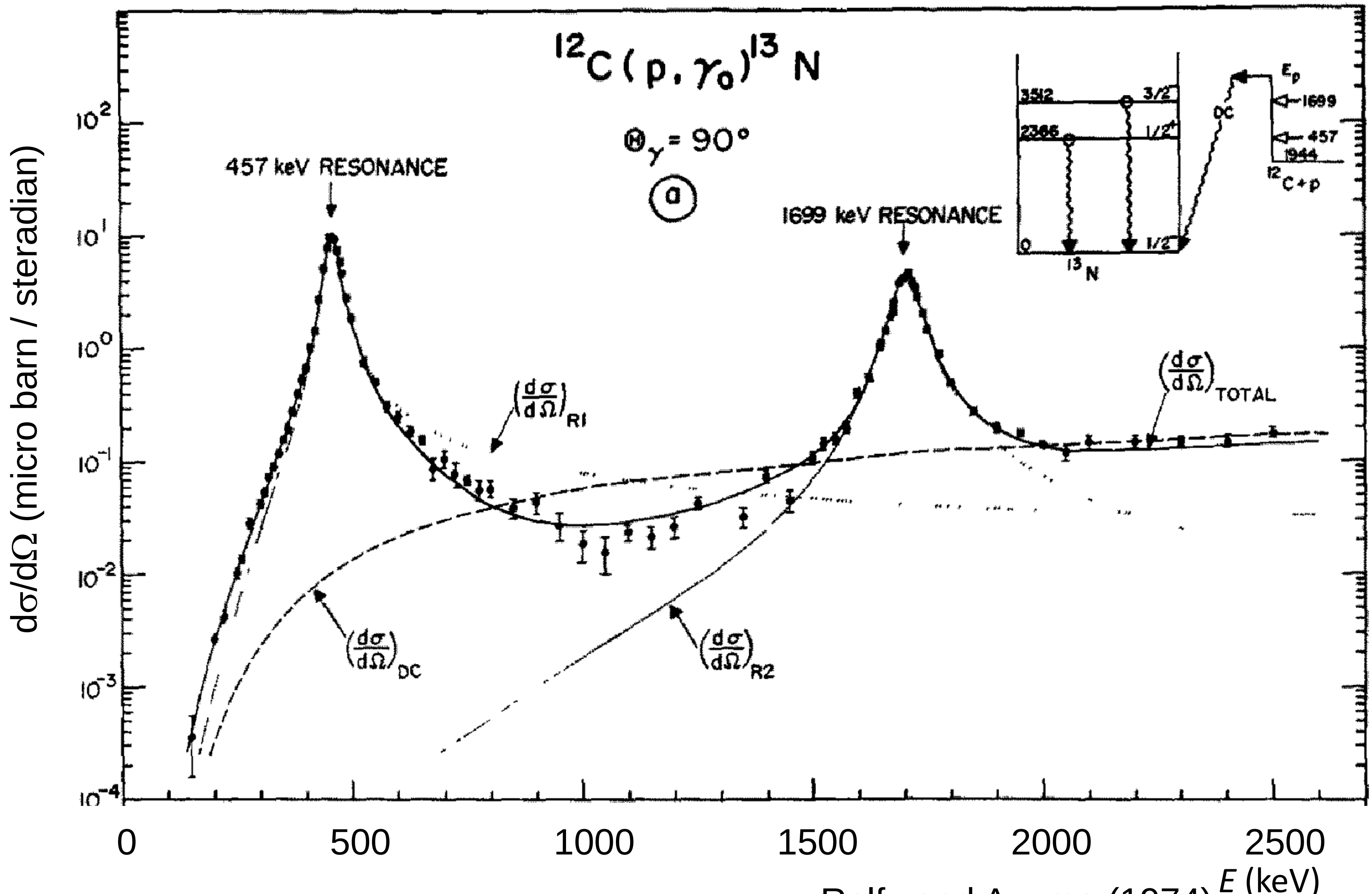
- Information
  - Branching ratio of γ(3502keV) decay from 2nd excited state of ^{13}N : **P_γ = 100/(100+9)**

- Assumption
  - Typical beam current : **I = 100 [nA]**
  - Typical measurement time of each beam energy setting : **30 [min.]**

- Please consider and check whether **the measurement time (t)** and **the distance (l)** of NaI(Tl) to be set against the target are realistic or not, and also how precisely the cross section are determined.

- [barn] = 10^{-28} [m^2]

$^{12}C(p,\gamma)^{13}N$ cross section in literatures

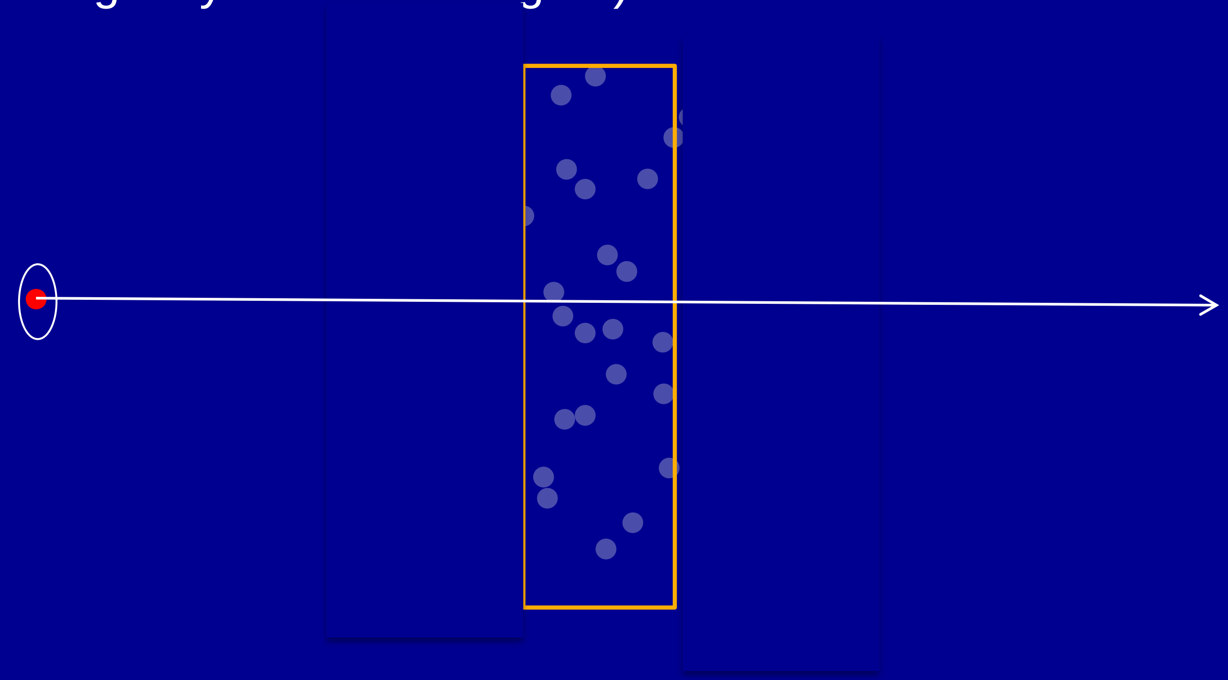


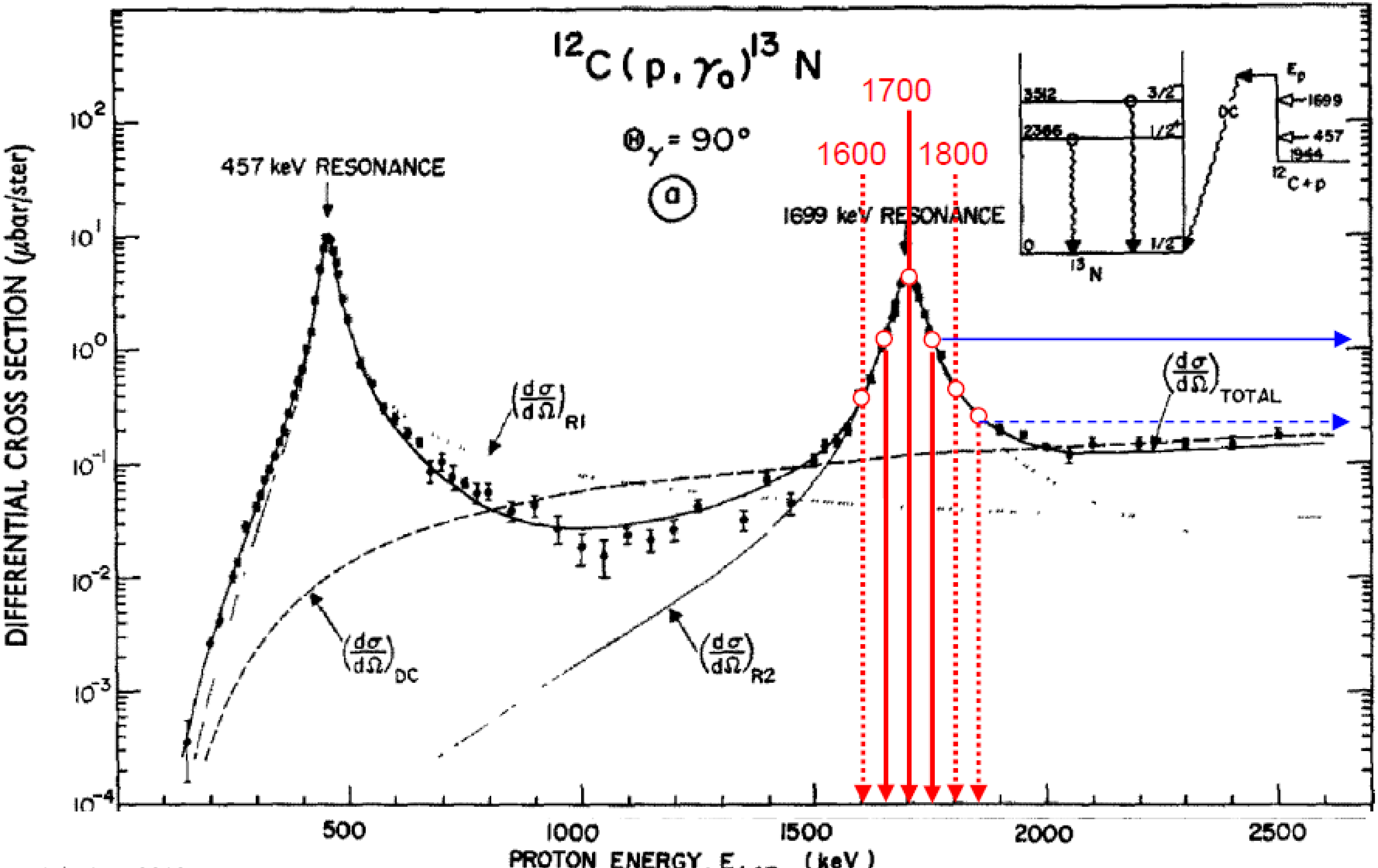
Rolfs and Azuma (1974)

The usual method for a certain energy interval

Target: thin so that the p energy-change (loss) is small.

σ: cross section (area of the imaginary circle in the figure)

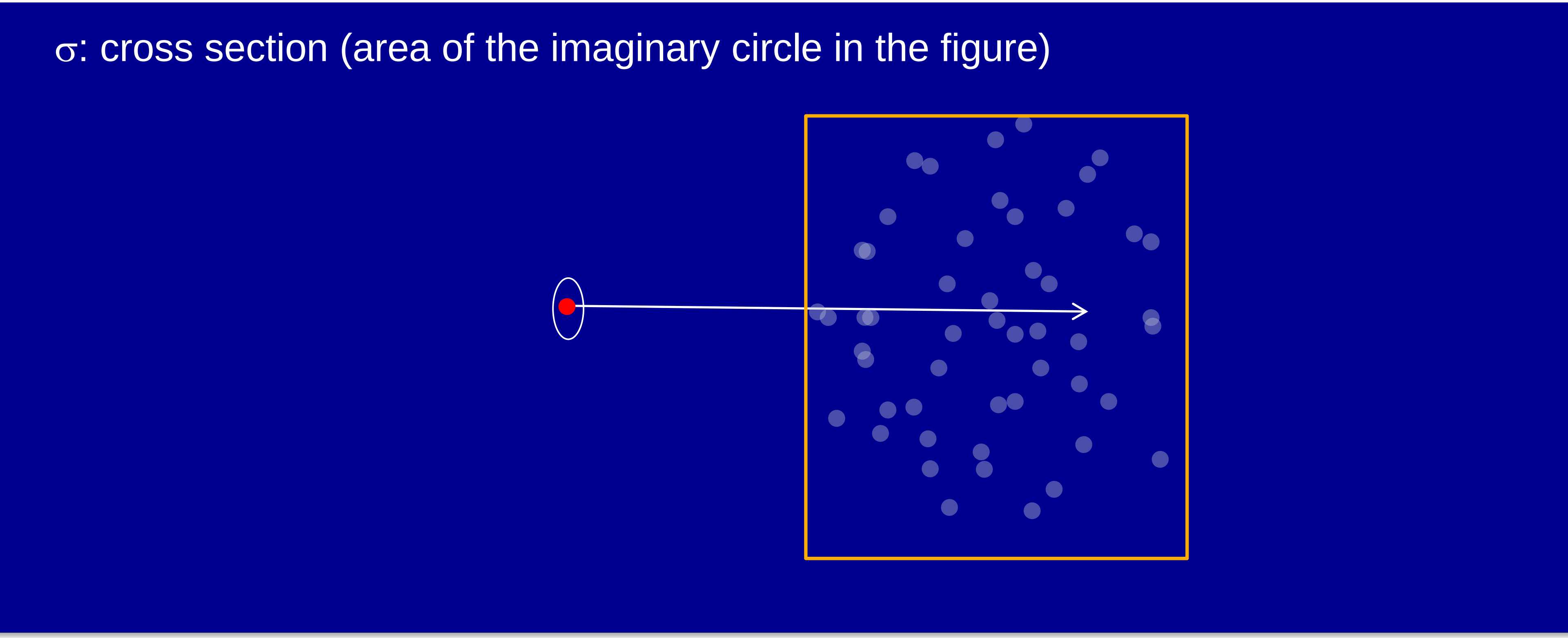




$^{12}C(p,\gamma_0)^{13}N$
$\Theta_\gamma = 90°$
(a)
457 keV RESONANCE
1699 keV RESONANCE
1600
1700
1800
$\left(\frac{d\sigma}{d\Omega}\right)_{R1}$
$\left(\frac{d\sigma}{d\Omega}\right)_{DC}$
$\left(\frac{d\sigma}{d\Omega}\right)_{R2}$
$\left(\frac{d\sigma}{d\Omega}\right)_{TOTAL}$
DIFFERENTIAL CROSS SECTION (μbar/ster)
PROTON ENERGY E_{LAB} (keV)
500
1000
1500
2000
2500
3512
3/2
2366
1/2
0
1/2
^{13}N
DC
E_p
1699
457
1944
$^{12}C+p$

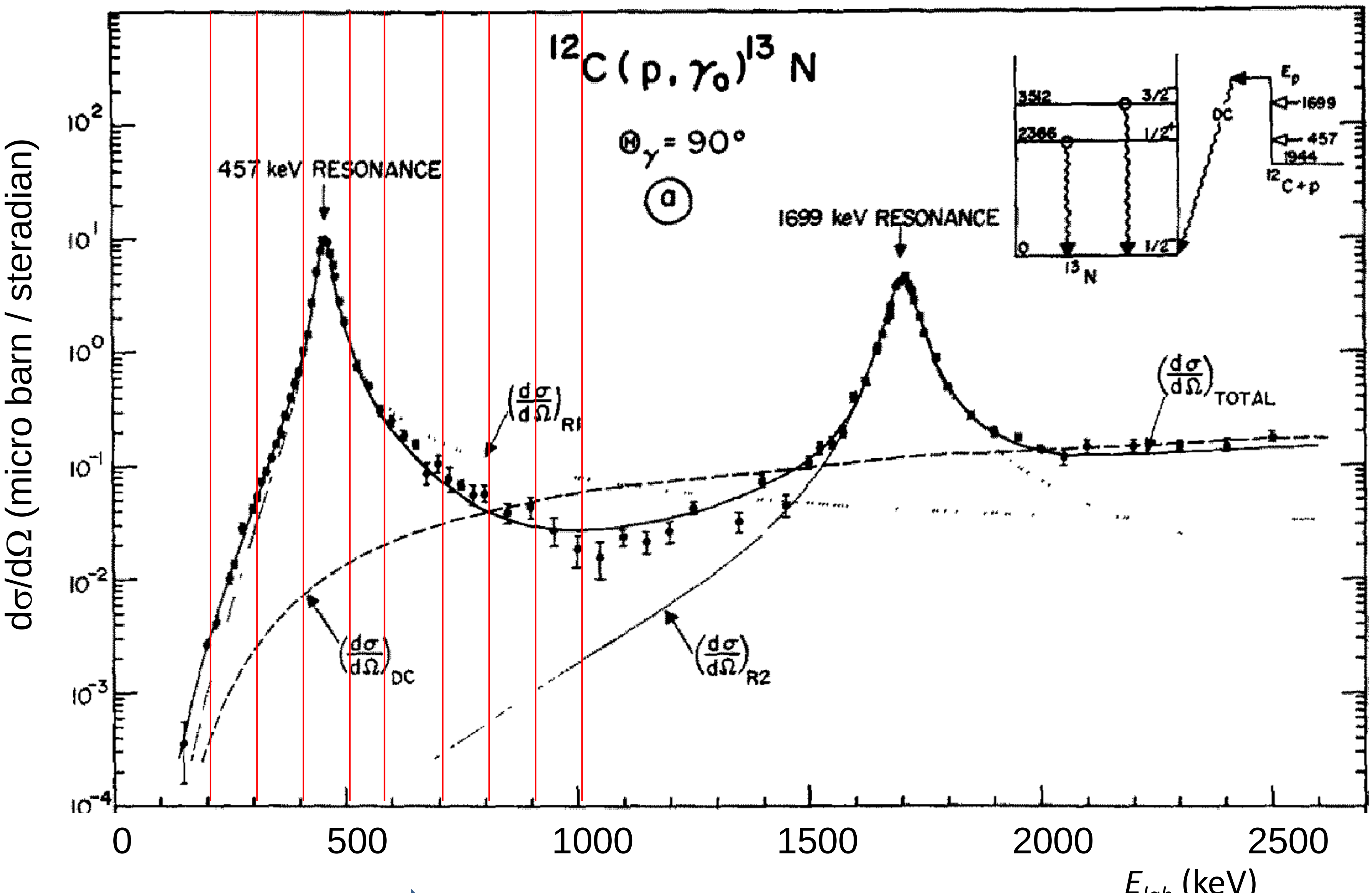
The thick target method

Target: thick enough to stop the beam



γ rays can be emitted in various different proton energies.

→ cross section integrated over a certain energy range.



Covert to target thickness

Rolfs and Azuma (1974)

Calculation for Range → SRIM code (free soft)

http://www.srim.org/

======= Target Composition ========

Atom Name	Atom Numb	Atomic Percent	Mass Percent
C	6	100.00	100.00

=============================

Proton beam in Carbon

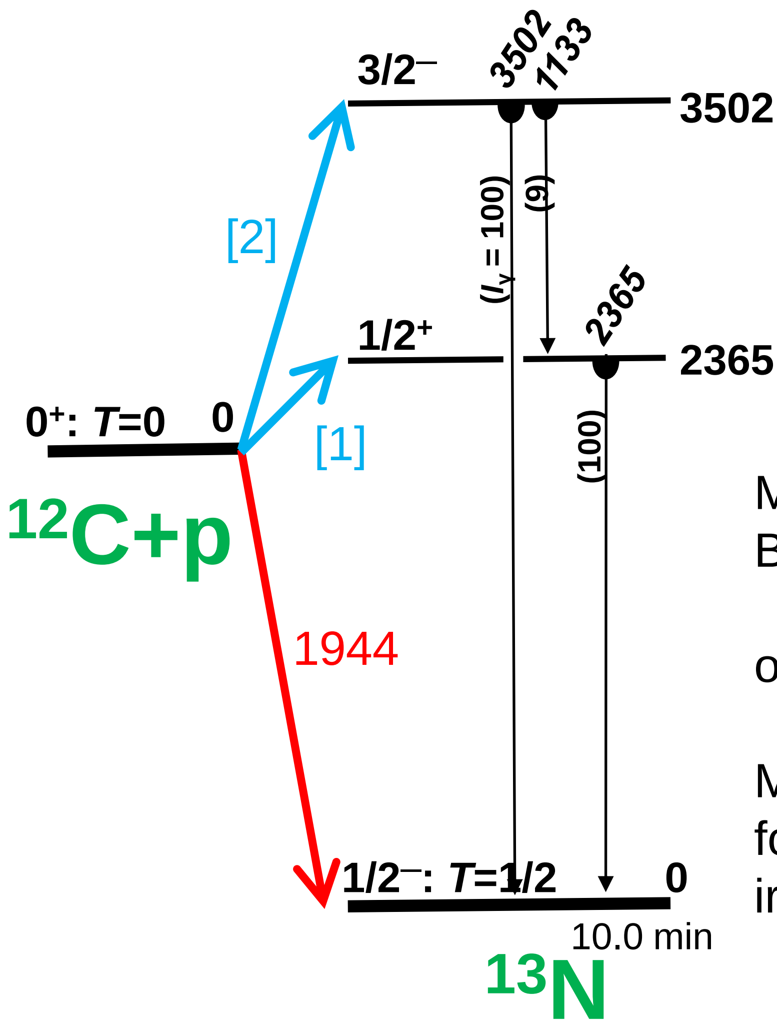
Bragg Correction = 0.00%

Stopping Units = MeV / (mg/cm2)

See bottom of Table for other Stopping units

Ion Energy	dE/dx Elec.	dE/dx Nuclear	Projected Range	Longitudinal Straggling	Lateral Straggling
500.00 keV	3.534E-01	2.738E-04	4.40 um	1837 A	1974 A
550.00 keV	3.334E-01	2.524E-04	5.04 um	2092 A	2182 A
600.00 keV	3.160E-01	2.342E-04	5.72 um	2347 A	2402 A
650.00 keV	3.007E-01	2.186E-04	6.44 um	2601 A	2632 A
700.00 keV	2.872E-01	2.051E-04	7.19 um	2856 A	2873 A
800.00 keV	2.640E-01	1.828E-04	8.80 um	3732 A	3385 A
900.00 keV	2.450E-01	1.651E-04	10.54 um	4553 A	3934 A
1.00 MeV	2.291E-01	1.507E-04	12.41 um	5350 A	4520 A
1.10 MeV	2.163E-01	1.387E-04	14.40 um	6134 A	5138 A
1.20 MeV	2.035E-01	1.286E-04	16.51 um	6915 A	5788 A
1.30 MeV	1.923E-01	1.199E-04	18.74 um	7703 A	6473 A
1.40 MeV	1.825E-01	1.124E-04	21.11 um	8501 A	7191 A
1.50 MeV	1.737E-01	1.058E-04	23.59 um	9309 A	7943 A
1.60 MeV	1.659E-01	9.998E-05	26.20 um	1.01 um	8727 A
1.70 MeV	1.588E-01	9.481E-05	28.93 um	1.10 um	9543 A
1.80 MeV	1.524E-01	9.017E-05	31.77 um	1.18 um	1.04 um
2.00 MeV	1.412E-01	8.219E-05	37.81 um	1.48 um	1.22 um

In-beam or activation

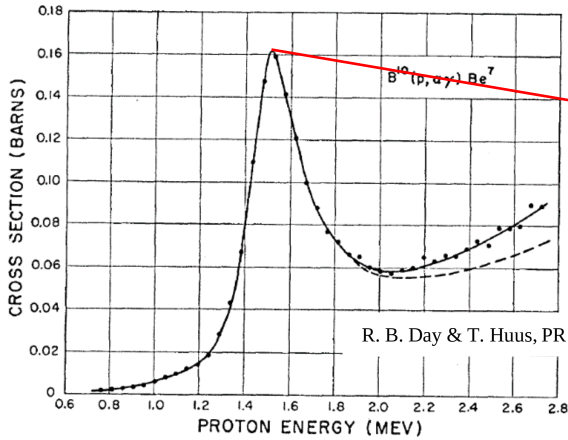


Measuring γ rays during
BOT (the proton beams on the target)

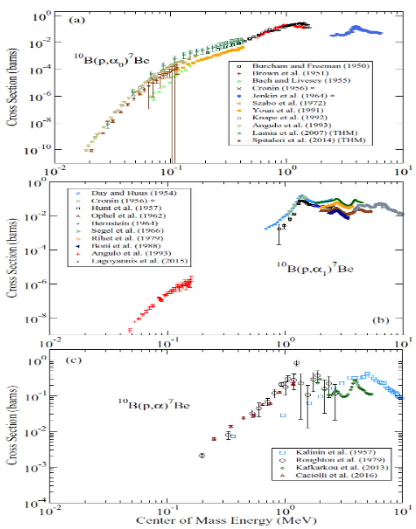
or

Measuring the ^{13}N activity (511 keV photons following the β^+ decay) after the beam irradiation

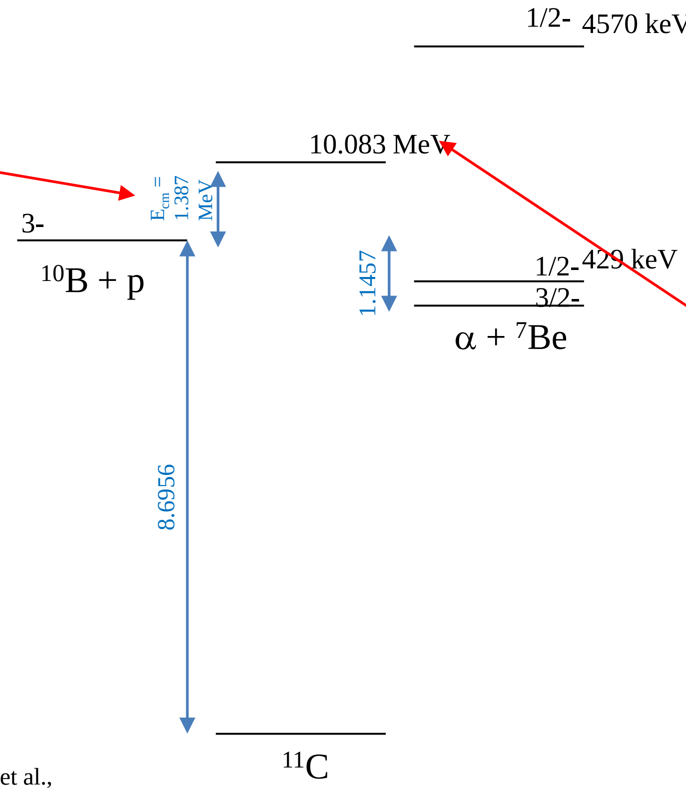
$^{10}B(p, \alpha)^{7}Be$

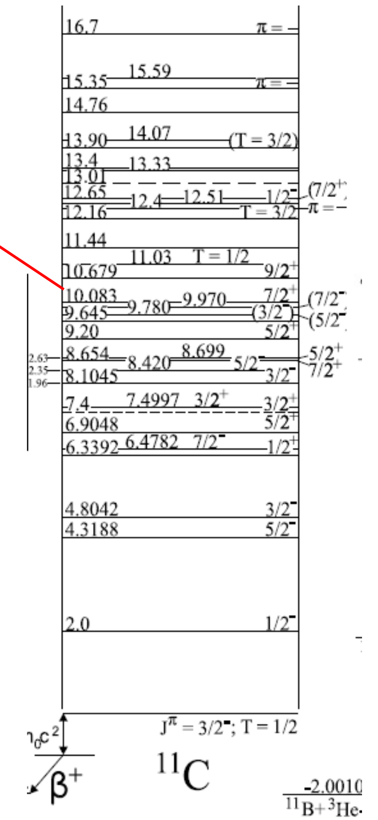


R. B. Day & T. Huus, PR 95(1954)1003



M. Wiescher, et al.,
PRC 95 (2017)044617.





Radioactive decay

$N(t) = N_0 \exp[-t/\tau]$ $(N_0 e^{-t/\tau})$

N: number of the initial nucleus (survived)

or number of decay per unit time

τ: mean life

$t_{1/2}$ = half life --- $N(t_{1/2}) = N_0/2$

Q: Estimate the counts after 30 min. proton irradiation.

Solid angle (unit: steradian)

Beam intensity: We may assume 100 nA. (elementary charge: 1.6×10^{-19} C; 1 A = 1 C/s)

Size of the NaI(Tl) crystal: 6x6x12 cm^2

"Target thickness"

divide into 10 or 20 slices.

Photo (full-energy-peak) efficiency of the NaI(Tl) ?

Which distance from the target to the center of NaI(Tl) scintillator should be?

on-site / off-site