Accessing pion GPDs through the Sullivan process: is it feasible?

Based on: hep-ph/2110.06052
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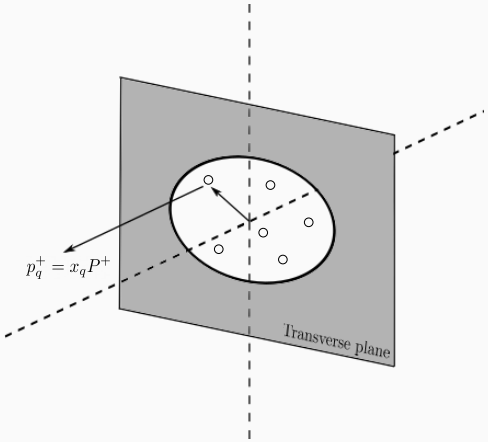
Introduction

Introduction

Question: *How can we gain insights into hadron's structure?*

1. Generalised parton distributions:
   - Probabilistic interpretation: "3D picture"
     [M.Burkardt-PRD:071503(62)2020]
   - Parametrise DVCS through CFFs.
     [X.Ji-PRL:610(78)1997]
   - PDFs as forward limit.
   - EFF and GFFs as Mellin moments.
2. Pions:
   - DCSB Nambu-Goldstone bosons.
   - Simpler than baryons.



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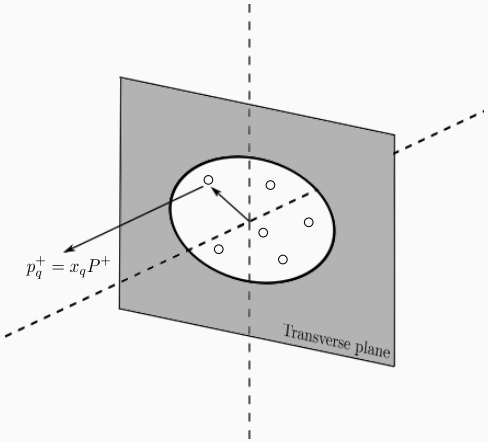


2. Pions:
   - DCSB Nambu-Goldstone bosons.
   - Simpler than baryons.

Two main questions guide this talk:

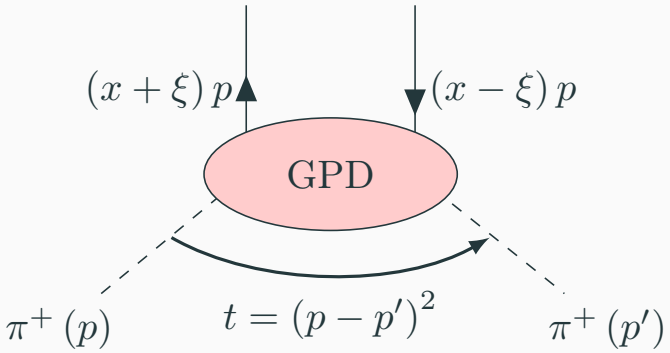
- Can we build "theoretically-complete" pion GPD models?
- Can we probe them in experiment?

  Pion GPDs through Sullivan process.
  [D.Amrath et al.-EPJC:179(58)2008]

Generalised parton distributions

Definition and properties

GPDs: definition and properties



x: Momentum fraction of p.

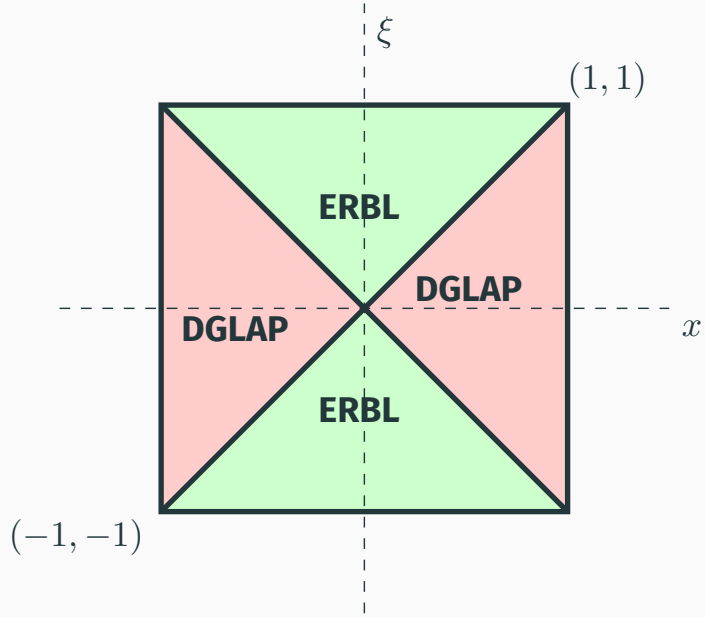
ξ: Fraction of momentum longitudinally tranfered.

t: Momentum transfer.

Kinematics:

[M.Diehl-Phys.Rept:41(388)2003]

- **DGLAP** ($|x| > |\xi|$):
  Emits/takes a quark ($x > 0$) or antiquark ($x < 0$).
- **ERBL:** ($|x| < |\xi|$):
  Emits pair quark-antiquark.



GPDs: definition and properties

- **Support:**
  [M.Diehl et al.-PLB:359(428)1998]

$$(x,\xi) \in [-1,1] \otimes [-1,1]$$

- **Polynomiality:** Order-m Mellin moments are degree-$(m+1)$ polynomials in ξ.
  [X.Ji-JPG:1181(24)1998, A.Radyushkin-PLB:81(449)1999]

$$\int_{-1}^{1} dx x^m H(x,\xi,t) = \sum_{\substack{k=0 \\ k \text{ even}}}^{m+1} c_k^{(m)}(t)\, \xi^k$$

Lorentz invariance

- **Positivity:**
  [P.V.Pobylitsa-PRD:114015(65)2002, B.Pire et al.-EPJC:103(8)1999]

$$|H^q(x,\xi,t=0)| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)} \qquad , \qquad |x| \geq \xi$$

Positivity of Hilbert space norm

- **Low energy soft-pion theorem**
  [M.V.Polyakov-NPB:231(555)1999, C.Mezrag et al.-PLB:190(741)2015]

PCAC/Axial-Vector WTI

GPD modelling

Can we build “theoretically-complete” pion GPD models?

GPD modelling: general strategy

Question: *Can we build pion GPDs fulfilling all these constraints?*

1. **Overlap representation**
   [M.Diehl et al.-NPB:33(569)2001]

   Based on LFWFs, $\Psi^q\left(x, k_\perp^2\right)$

   Polynomiality ?
   Positivity ✓

---

2. **Double Distribution representation**
   [D.Müller et al.-Fort.Phys:2(42)1994, JLAB-THY-00-33]

   Relying on Radon transform, $\mathcal{R}$

   Polynomiality ✓
   Positivity ?

Problem: Different modelling strategies and different problems

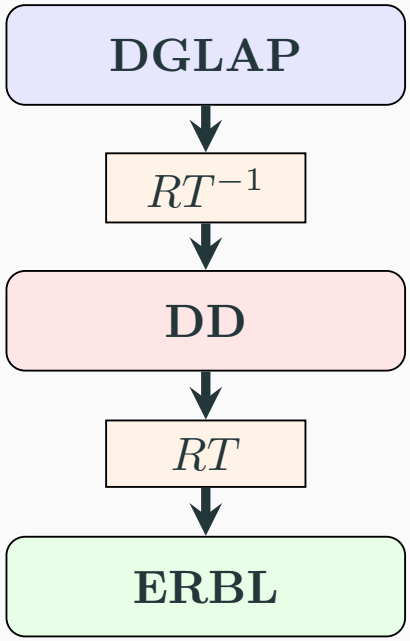
Solution:

Covariant extension: given a DGLAP-GPD, the covariant extension allows for computing the corresponding ERBL-GPD such that polynomiality is satisfied.[N.Chouika et al.-EPJC:906(77)2017]

GPD modelling: covariant extension

Covariant extension: given a DGLAP-GPD, the covariant extension allows for computing the corresponding ERBL-GPD such that polynomiality is satisfied. [N.Chouika et al.-EPJC:906(77)2017]

$$H(x,\xi,t) = \mathcal{R}\left[h(\beta,\alpha,t)\right] + \frac{1}{|\xi|} D^{+}\left(\frac{x}{\xi},t\right) + sign(\xi)\, D^{-}\left(\frac{x}{\xi},t\right)$$

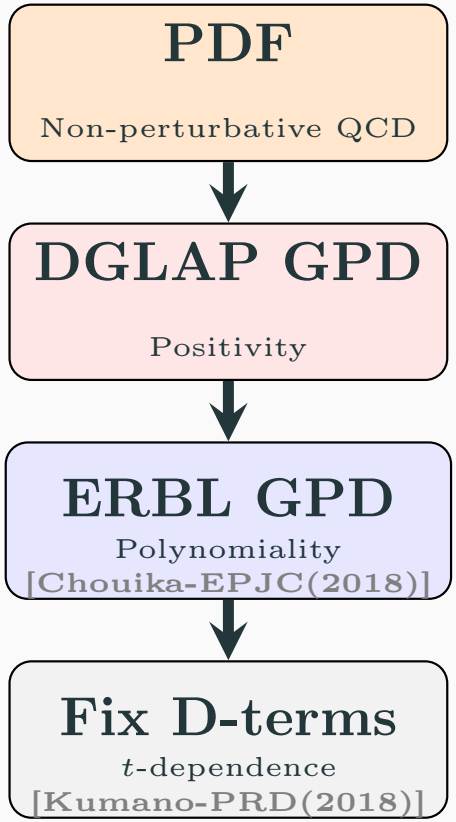


1. Build positive DGLAP GPD $\Rightarrow$ **How?**
2. Covariant extension: ERBL GPD
3. Soft pion theorem: fix $D^{\pm}(\alpha, 0)$

GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Poby.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓

Pion GPDs: from separable LFWFs to postive DGLAP GPDs

Question: *How can we build a positive DGLAP GPD?*



1. Overlap representation [M.Diehl-NPB:33(569)2001]

$$H^q\left(x,\xi,t\right)\big|_{|x|\geq\xi} = \int \frac{d^2k_\perp}{16\pi^3}\Psi^{q*}\left(x_-,k^2_{\perp,-}\right)\Psi^q\left(x_+,k^2_{\perp,+}\right)$$

2. Assume factorisation of the LFWF
[J.-L.Zhang et al.-PLB:136158(815)2021]

$$\Psi^q\left(x,k^2_\perp\right) \propto \varphi\left(x\right)\phi\left(k^2_\perp\right)$$

$$\downarrow \text{(Overalp rep.)}$$

$$H^q\left(x,\xi,t\right)\big|_{|x|\geq\xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right)q\left(\frac{x+\xi}{1+\xi}\right)}\,\Phi\left(x,\xi,t\right)$$

$$\downarrow (t=0)$$

$$H^q\left(x,\xi,0\right)\big|_{|x|\geq\xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right)q\left(\frac{x+\xi}{1+\xi}\right)}$$

Positivity saturated

Pion DGLAP GPDs

Pion GPDs: Positive DGLAP GPDs

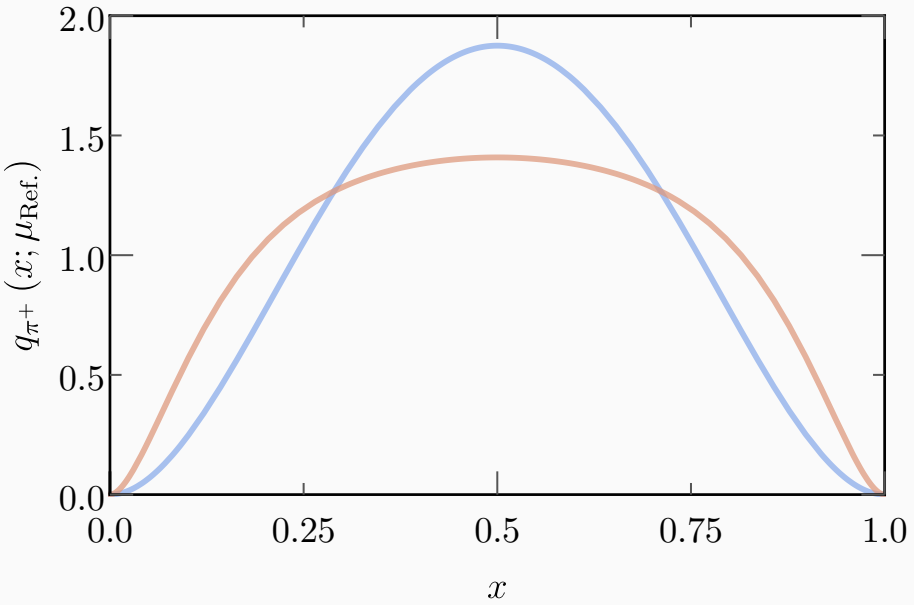
1. Under certain PTIR, chiral symmetry allows to factorize LFWF:
[J.-L.Zhang et al.-PLB:136158(815)2021]

$$\Psi_{\pi}^{\lambda_1\lambda_2}\left(x,k_{\perp}^{2}\right)=\sqrt{q_{\pi}\left(x\right)}\frac{i^{\lambda_1\lambda_2}M^{2}}{\left(k_{\perp}^{2}+M^{2}\right)^{2}}$$

2. Pion GPD saturating positivity

$$H_{\pi}^{q}\left(x,\xi,t\right)|_{\mathrm{DGLAP}}=\frac{\sqrt{q_{\pi}\left(x_{-}\right)q_{\pi}\left(x_{+}\right)}}{\left(1+z^{2}\right)^{2}}\left[3+\frac{1-2z}{1+z}\frac{\operatorname{arctanh}\left(\sqrt{\frac{z}{1+z}}\right)}{\sqrt{\frac{z}{1+z}}}\right]$$

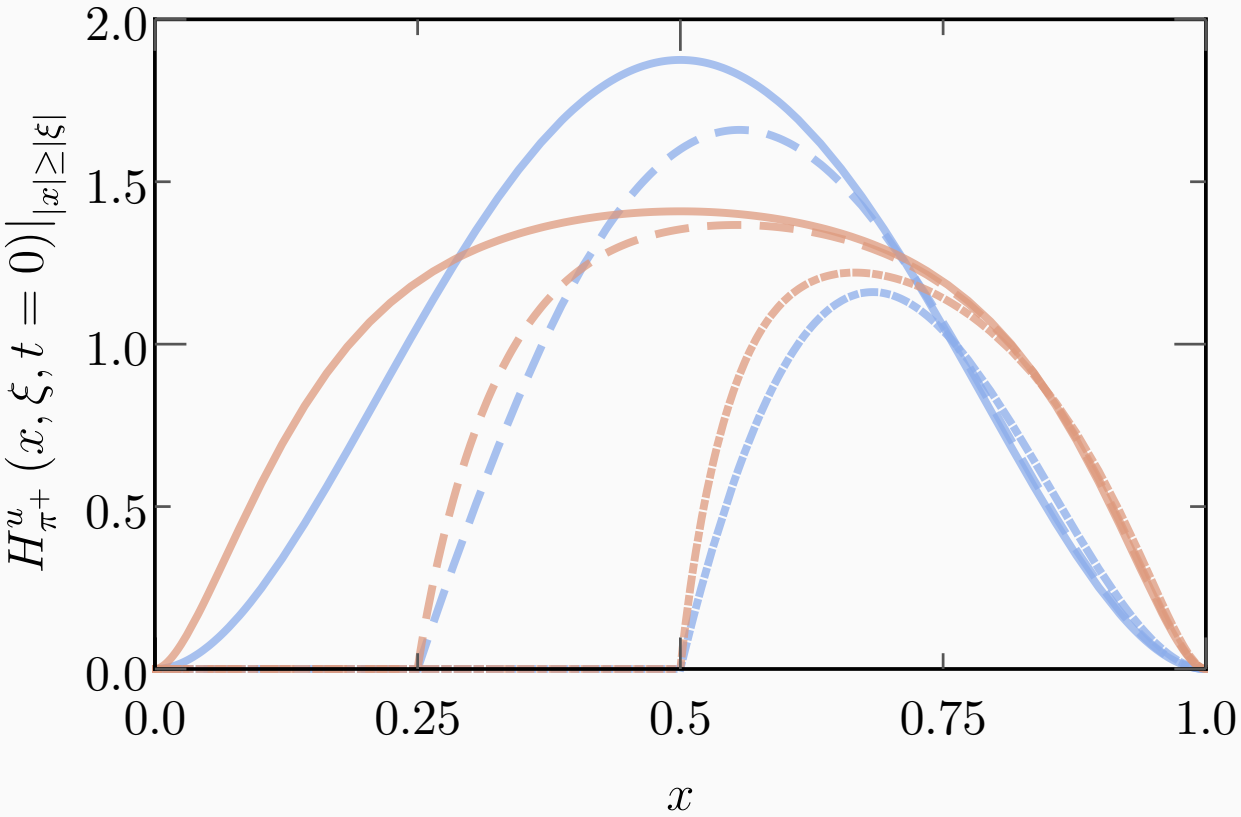
$$z=-t\left(1-x\right)^{2}/4M^{2}\left(1-\xi^{2}\right)$$



Two models:

- Algebraic model
  $q_{\pi}\left(x\right)=30x^{2}\left(1-x\right)^{2}$
- Realistic model (DSE)
  [M.Ding et al.-PRD:054014(101)2020]
  $q_{\pi}\left(x\right)=\mathcal{N}_{q}x^{2}\left(1-x\right)^{2}$
  $\times\left[1+\gamma x\left(1-x\right)+\rho\sqrt{x\left(1-x\right)}\right]$

Pion GPDs: Positive DGLAP GPDs



Pion ERBL GPDs: covariant extension

Pion GPDs: covariant extension

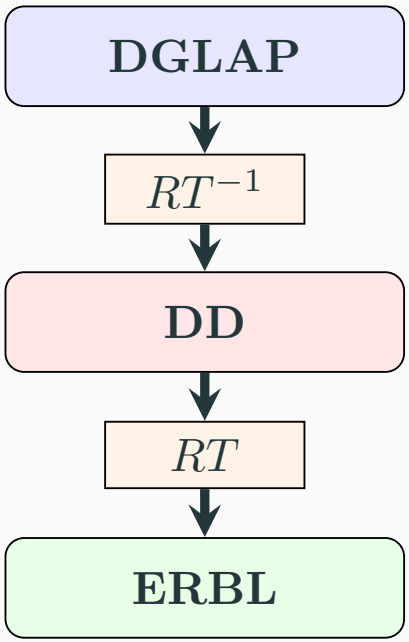
Covariant extension:

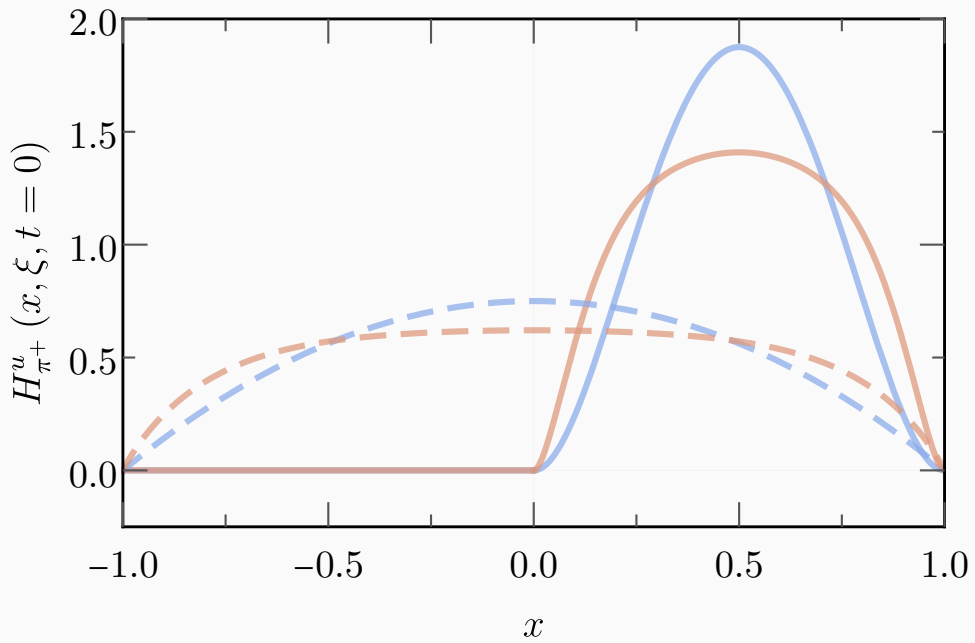
$$H^q(x,\xi,t) = \mathcal{R}\left[h(\beta,\alpha,t)\right] + \frac{1}{|\xi|} D^+\left(\frac{x}{\xi},t\right) + sgn(\xi)\, D^-\left(\frac{x}{\xi},t\right)$$

Fix D-terms with soft pion theorem:

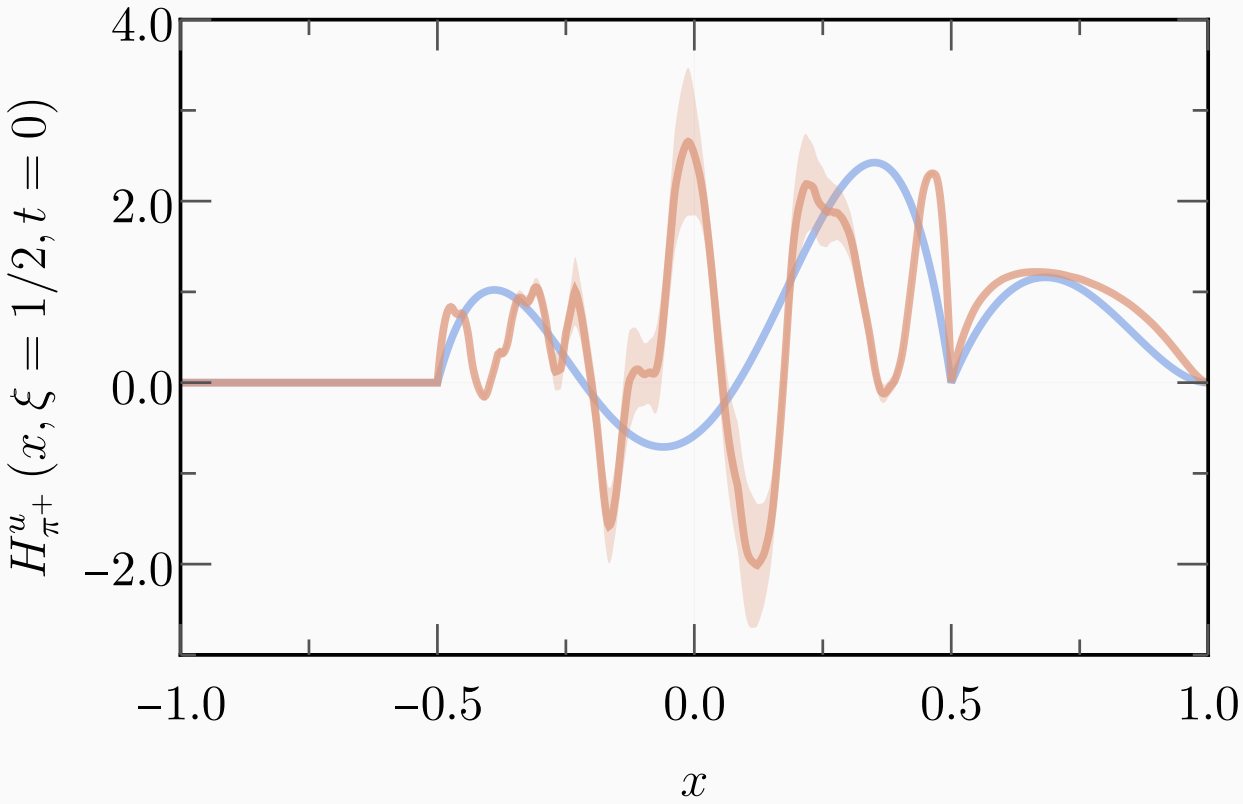
[M.V.Polyakov-NPB:231(555)1999, C.Mezrag at al.-PLB:190(741)2015]

$$H^{I=0}_{\pi^+}(x,\xi,t)\big|_{\xi=1,t=0} = H_{\pi^+}(x,\xi,t) - H_{\pi^+}(-x,\xi,t)\big|_{\xi=1,t=0} = 0$$
$$H^{I=1}_{\pi^+}(x,\xi,t)\big|_{\xi=1,t=0} = H_{\pi^+}(x,\xi,t) + H_{\pi^+}(-x,\xi,t)\big|_{\xi=1,t=0} = \varphi\left(\tfrac{1+x}{2}\right)$$





Pion GPDs: covariant extension



Phenomenology of pion GPDs

Can we probe them in experiment?

Phenomenology of pion GPDs: Sullivan process

Question: *Can we probe those pion GPDs through experiment?*
[D.Amrath at al. EPJC:179(58)2008]

Sullivan process [J.D.Sullivan-PRD:1732(5)1972]

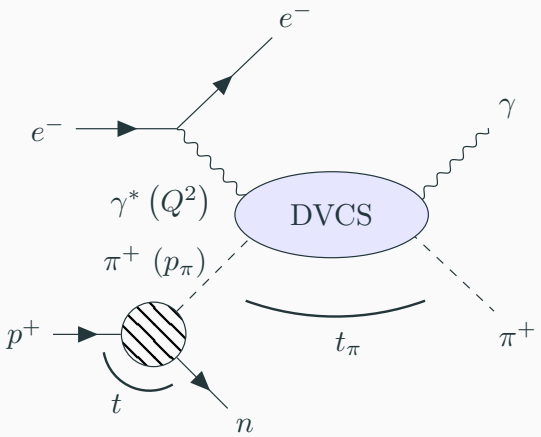
Deep inelastic electron-proton scattering with πn fixed final states.

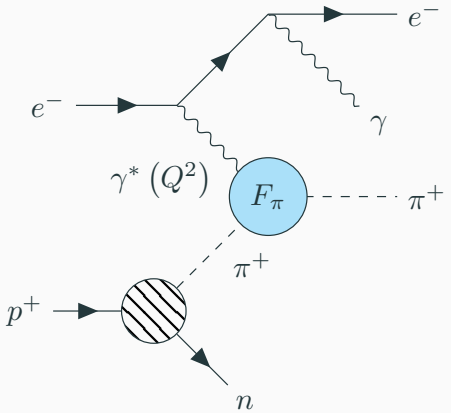
One pion exchange approximation: [D.Amrath et al.-EPJC:179(58)2008]

- $|t|^{\text{Max.}} = 0.6\,\text{GeV}^2$
- $\sigma_L >> \sigma_\perp$

} Met at EIC [EICYR:phys.ins-det/2103.05419]

DVCS amplitudes are parametrized by hadron GPDs.
[X.Ji-PRD:7114(55)1997]





Employed for EFFs.
[G.M.Huber at al. PRC:045203(78)2008]

Can we probe DVCS contribution through experiment?
[D.Amrath at al. EPJC:179(58)2008]

Phenomenology of pion GPDs: Sullivan process

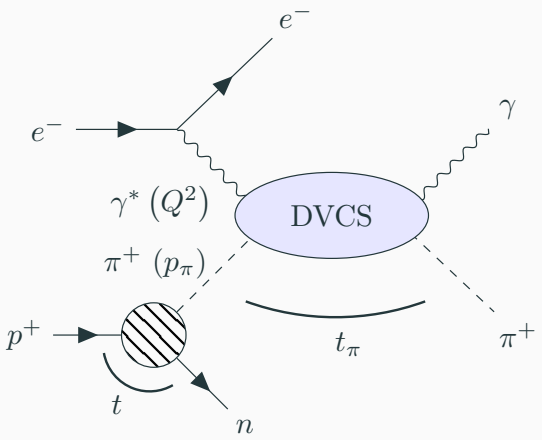
In fact... this has already been advocated in the EIC-Yellow report
[EICYR:phys.ins-det/2103.05419]

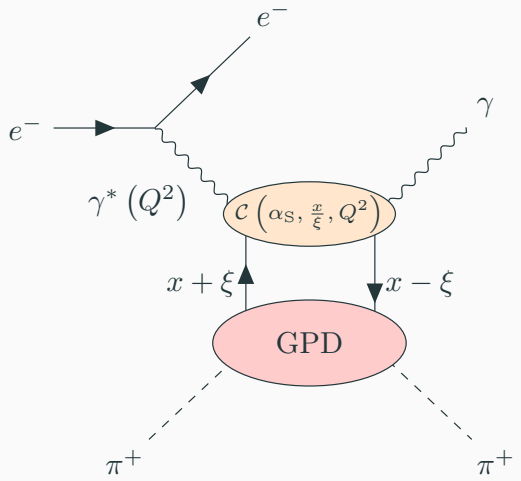
Science Question	Key Measurement	Key Requirements
What are the quark and gluon energy contributions to the pion mass?	Pion structure function data over a range of x and Q^2.	• Need to uniquely determine $e+p \to e'+X+n$ (low $-t$) • CM energy range ~10-100 GeV • Charged and neutral currents desirable
Is the pion full or empty of gluons as viewed at large Q^2?	Pion structure function data at large Q^2.	• CM energy ~100 GeV • Inclusive and open-charm detection
What are the quark and gluon energy contributions to the kaon mass?	Kaon structure function data over a range of x and Q^2.	• Need to uniquely determine $e+p \to e'+X+\Lambda/\Sigma^0$ (low $-t$) • CM energy range ~10-100 GeV
Are there more or less gluons in kaons than in pions as viewed at large Q^2?	Kaon structure function data at large Q^2.	• CM energy ~100 GeV • Inclusive and open-charm detection
Can we get quantitative guidance on the emergent pion mass mechanism?	Pion form factor data for Q^2 = 10-40 $(\mathrm{GeV}/c)^2$.	• Need to uniquely determine exclusive process $e+p \to e'+\pi^++n$ (low $-t$) • $e+p$ and $e+D$ at similar energies • CM energy ~10-75 GeV
What is the size and range of interference between emergent-mass and the Higgs-mass mechanism?	Kaon form factor data for Q^2 = 10-20 $(\mathrm{GeV}/c)^2$.	• Need to uniquely determine exclusive process $e+p \to e'+K+\Lambda$ (low $-t$) • L/T separation at CM energy ~10-20 GeV • Λ/Σ^0 ratios at CM energy ~10-50 GeV
What is the difference between the impacts of emergent- and Higgs-mass mechanisms on light-quark behavior?	Behavior of (valence) up quarks in pion and kaon at large x.	• CM energy ~20 GeV (lowest CM energy to access large-x region) • Higher CM energy for range in Q^2 desirable
What is the relationship between dynamically chiral symmetry breaking and confinement?	Transverse-momentum dependent Fragmentation Functions of quarks into pions and kaons.	• Collider kinematics desirable (as compared to fixed-target kinematics) • CM energy range ~20-140 GeV
More speculative observables		
What is the trace anomaly contribution to the pion mass?	Elastic J/Ψ production at low W off the pion.	• Need to uniquely determine exclusive process $e+p \to e'+J/\Psi+\pi^++n$ (low $-t$) • High luminosity ($\geq 10^{34}\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$) • CM energy ~70 GeV
Can we obtain tomographic snapshots of the pion in the transverse plane? What is the pressure distribution in a pion?	Measurement of DVCS off pion target as defined with Sullivan process.	• Need to uniquely determine exclusive process $e+p \to e'+\gamma+\pi^++n$ (low $-t$) • High luminosity ($\geq 10^{34}\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$) • CM energy ~10-100 GeV
Are transverse momentum distributions universal in pions and protons?	Hadron multiplicities in SIDIS off a pion target as defined with Sullivan process.	• Need to uniquely determine SIDIS off pion $e+p \to e'+h+X+n$ (low $-t$) • High luminosity ($10^{34}\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$) • $e+p$ and $e+D$ at similar energies desirable • CM energy ~10-100 GeV

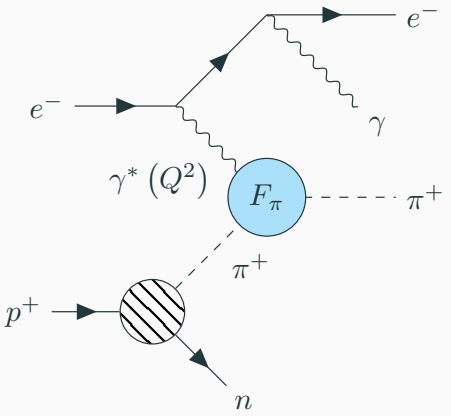
Let us see if that would be feasible in a future electron-ion collider.

Phenomenology of pion GPDs: DVCS

Goal: Employ our GPD models to describe the DVCS contribution to the Sullivan process in the one pion exchange approximation.







$$\mathcal{M}_{e\pi} = \mathcal{M}_{\mathrm{DVCS}} + \mathcal{M}_{\mathrm{BH}}$$

Phenomenology of pion GPDs: DVCS

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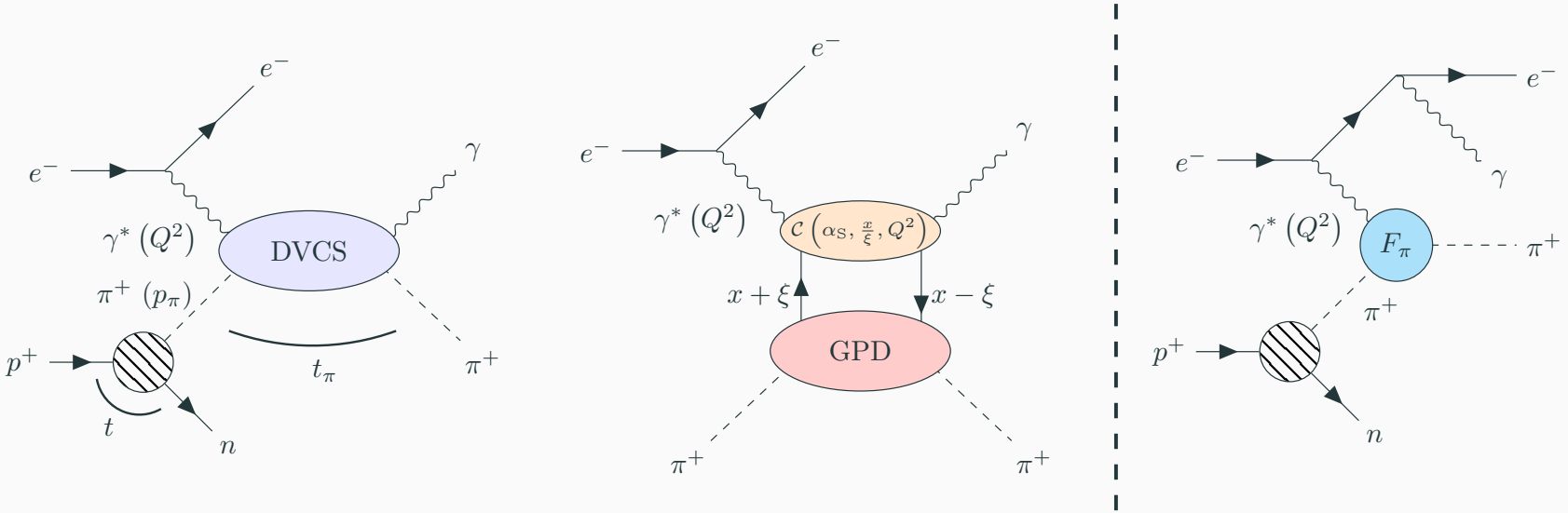


$$\mathcal{M}_{e\pi} = \mathcal{M}_{\text{DVCS}} + \mathcal{M}_{\text{BH}}$$

Phenomenology

1. σ-DVCS
   [D.Amrath et al.-EPJC(2008)]
2. Asymmetry
3. ...

$=$

QCD Evolution

APFEL++

[V.Bertone et al.] -CPComm(2014),

[V.Bertone et al.: hep-ph/1708.00911]

$+$

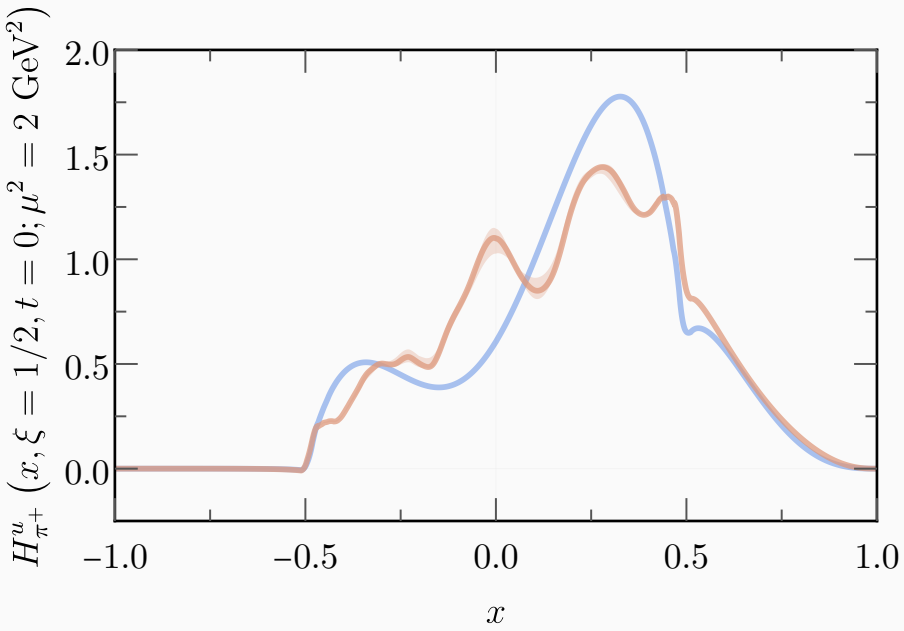
LO/NLO CFFs

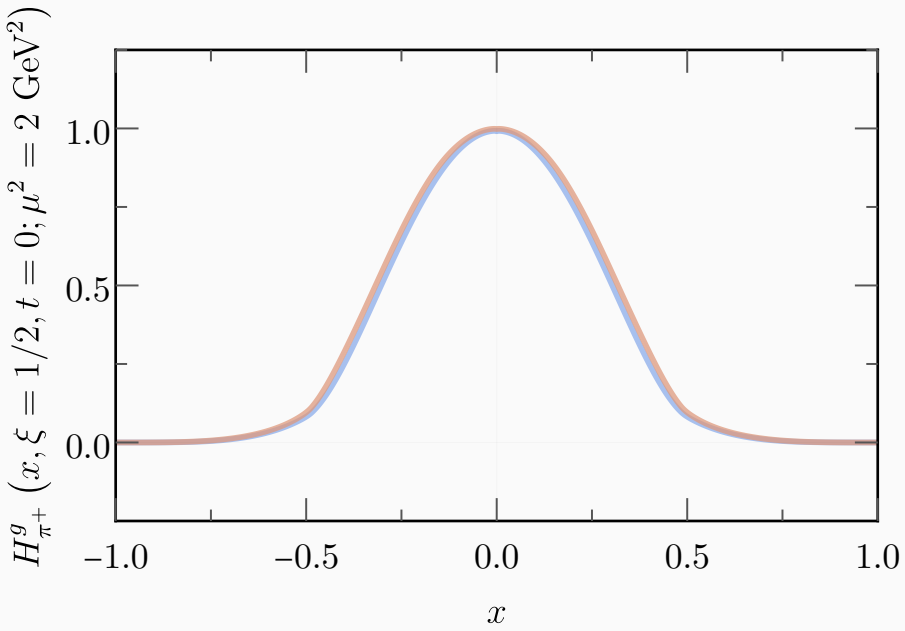
PARTONS

[B.Berthou et al. -EPJC(2018)]

QCD evolution

Phenomenology of pion GPDs: QCD evolution

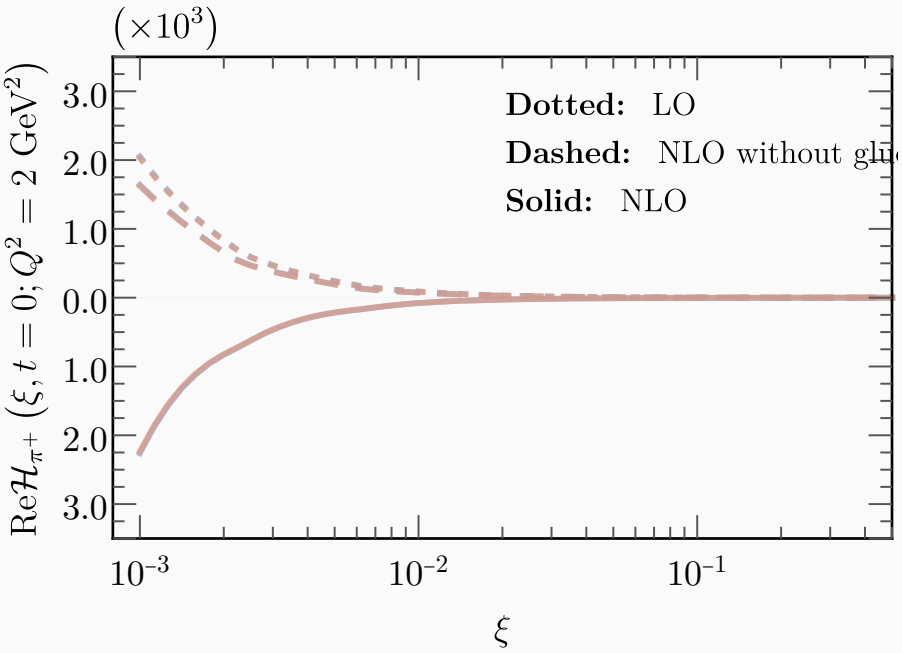


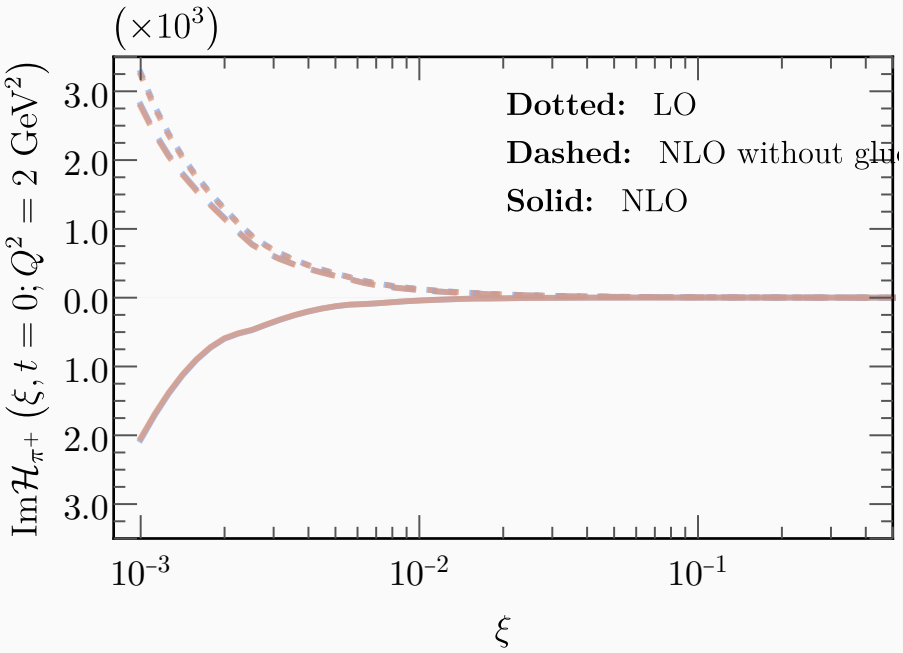


- Uncertainty band narrowed by scale evolution.
- Continuity along $x = \xi$ lines.
- Non-zero gluon distribution generated by scale evolution.

Compton form factors

Phenomenology of pion GPDs: Compton Form Factors





- Undistinguishable results for both models
- Dominant effect of NLO corrections (gluon distributions)

Gluon dominance makes essential at least NLO accuracy in any phenomenlogical analysis of DVCS at an EIC.

DVCS cross-section and beam spin asymmetries

Phenomenology of pion GPDs: Generating events

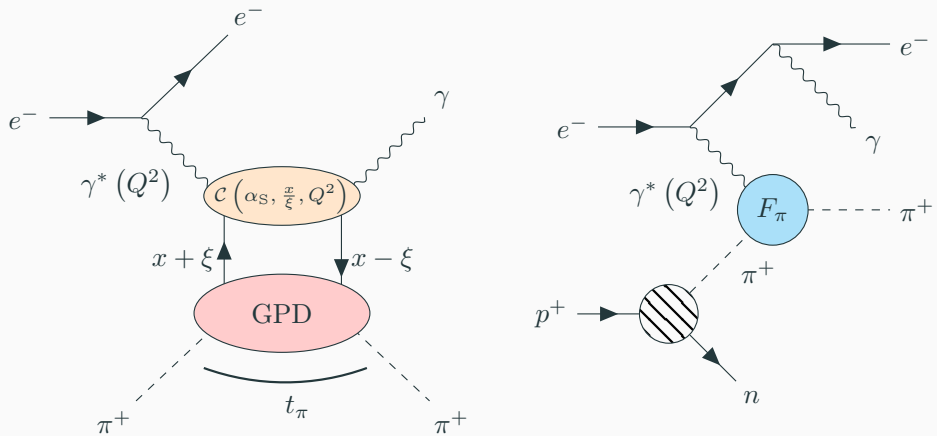
Question: *Can we measure DVCS?*

Sullivan process amplitude

1. One pion exchange approx.

$\left|t_\pi^{\text{Max}}\right| = 0.6\,\text{GeV}^2, \quad \sigma_L >> \sigma_\perp$

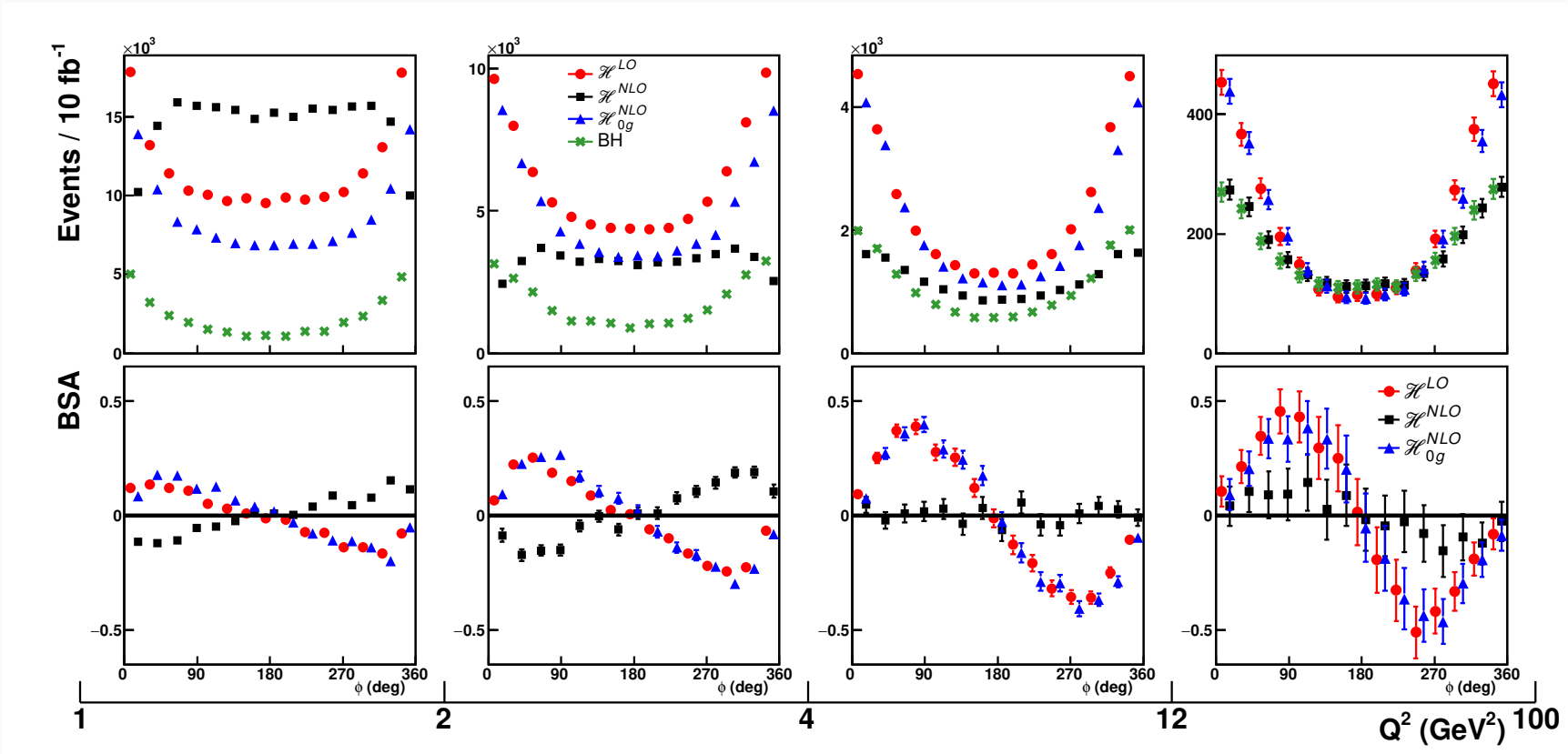
$\mathcal{M}_{e\pi} = \mathcal{M}_{\text{DVCS}} + \mathcal{M}_{\text{BH}}$



2. Monte Carlo event generation

- Select event compatible with detector geometry and performance: EIC. [EICYR:phys.ins-det/2103.05419]
- Add kinematical cuts: [D.Amrath et al. EPJC:179(58)2008]
  - DVCS kinematics and one pion exchange approximation:
    - $s_\pi^{\text{Min}} = 4\,\text{GeV}^2$
    - $|t_\pi|^{\text{Max.}} = 0.6\,\text{GeV}^2$
  - Reduce contamination of resonances (Δ): $W \gtrsim 2\,\text{GeV}^2$
- Integrated one-year luminosity

Phenomenology of pion GPDs: EIC



- **Visible signal at EIC**
- "Destructive interference" between quark and gluon content
- Sign change in beam spin asymmetry due to gluon contributions

Summary and perspectives

Summary and perspectives

- **GPD modelling: can we build "theoretically-complete" GPDs?**

  YES! Pion GPD models fulfilling every theoretical constraint
  - **Positivity: factorisation hypothesis**
  - **Polynomiality: covariant extension**
  - **Support and soft-pion theorem**
  - **Continuity along $x = \xi$**
  - **Low-t behaviour in agreement with data**

- **Phenomenology: can we probe pion GPDs in experiment?**

  YES!
  - **Signal expected at EIC kinematics**
  - **Gluon-quark "destructive interference"**
  - **Gluon dominance: NLO analyses**
  - **Gluon dominance: Beam spin asymmetry sign change**

Thank you!