Hadron spin structure in lattice QCD

Constantia Alexandrou

University of Cyprus

The Cyprus Institute

Extended Twisted Mass Collaboration

STIMULATE
European Joint Doctorates

SPIN2021
Matsue, Japan

The 24th International Spin Symposium, Matsue, Japan,
18-22 Oct. 2021

Extended Twisted Mass Collaboration (ETMC)

❋ **Gauge ensembles generated by ETMC**

These are now generated with 2+1+1 flavours
at physical values of the light, strange and charm quark masses

❋ **Analysis of these ensembles for various observables**

❋ Main collaborators for spin structure

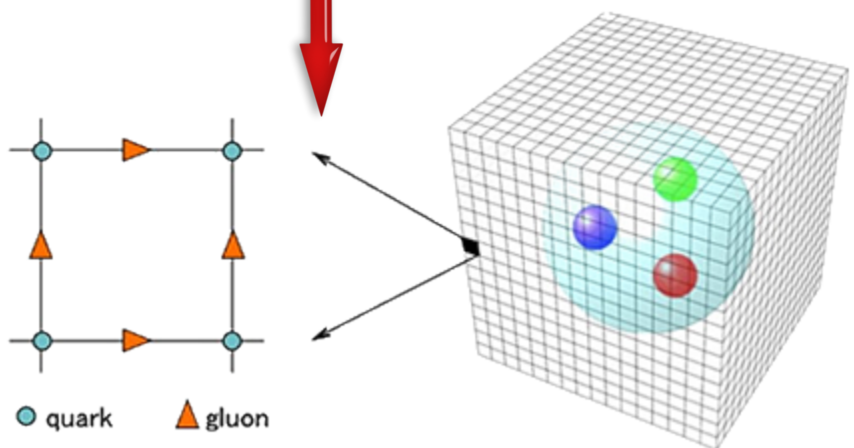
- *S. Bacchio, The Cyprus Institute*
- *M. Constantinou, Temple University*
- *J. Finkenrath, The Cyprus Institute*
- *K. Hadjiyiannakou, University of Cyprus & The Cyprus Institute*
- *K. Jansen, DESY-Zeuthen*
- *G. Koutsou, The Cyprus Institute*
- *C. Lauer, Temple University*
- *H. Panagopoulos, University of Cyprus*
- *M. Petschlies, University of Bonn*
- *G. Spanoudes, The Cyprus Institute*
- *C. Urbach, University of Bonn*

Outline

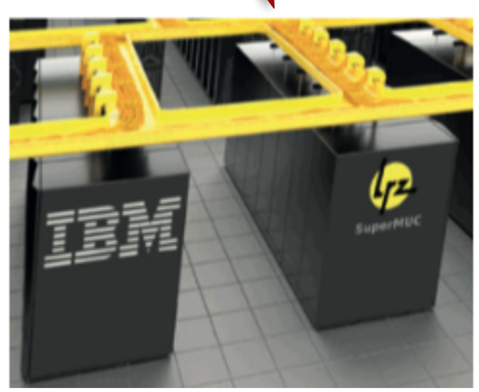
- ❋Introduction
- ❋Spin structure of the nucleon (and momentum fraction of pion)
- ❋Quasi-parton distributions in large momentum effective theory (LaMET)
- ❋Results and open issues
- ❋ Conclusions

Extraction of matrix elements in lattice QCD

$$\langle \mathcal{O}\rangle = \frac{1}{Z}\int \mathcal{D}[U]\mathcal{O}(D^{-1}[U],U)\left(\prod_{f=u,d,s,c}\text{Det}(D_f[U])\right)e^{-S_{\text{QCD}}[U]}$$

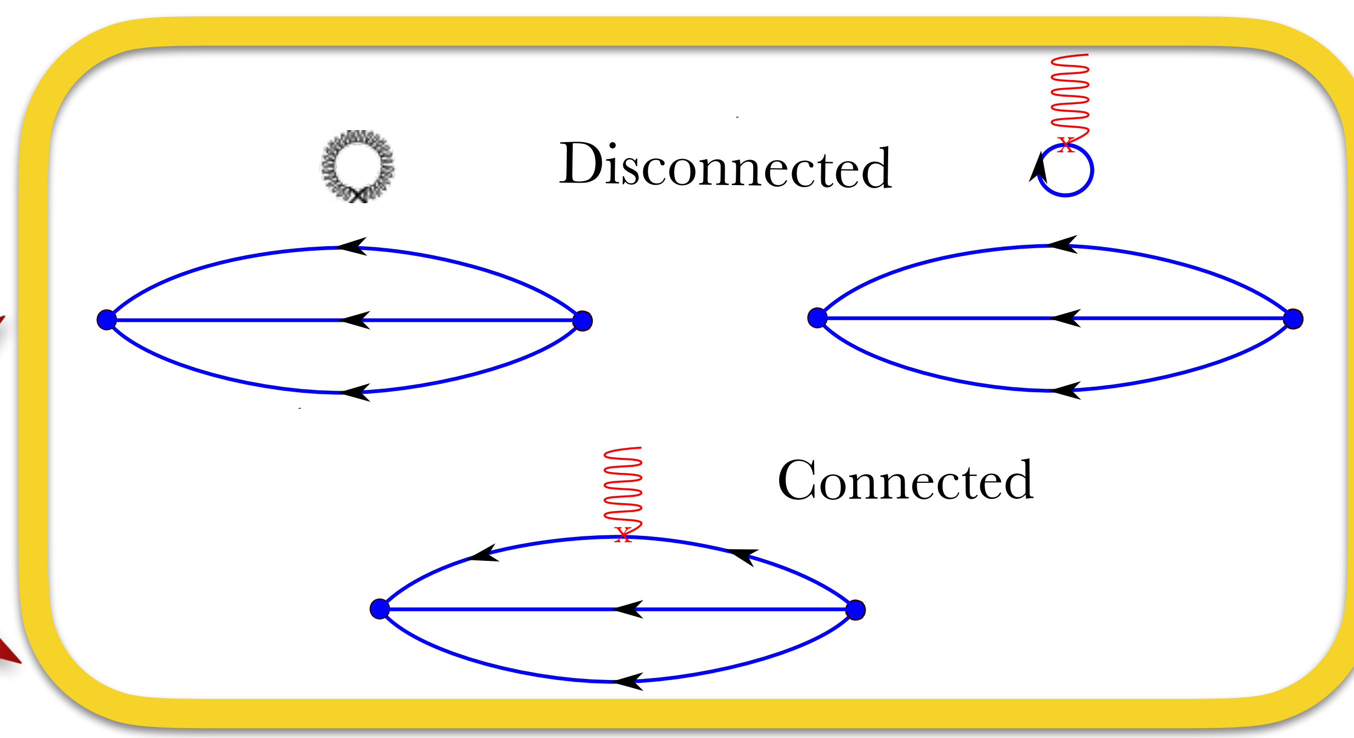


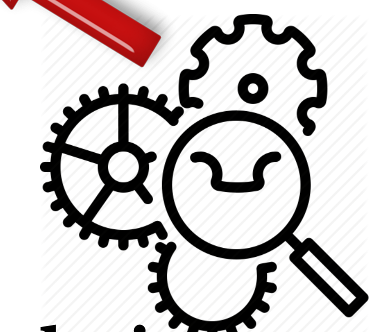
Simulation of gauge configurations U



Quark propagators

contractions



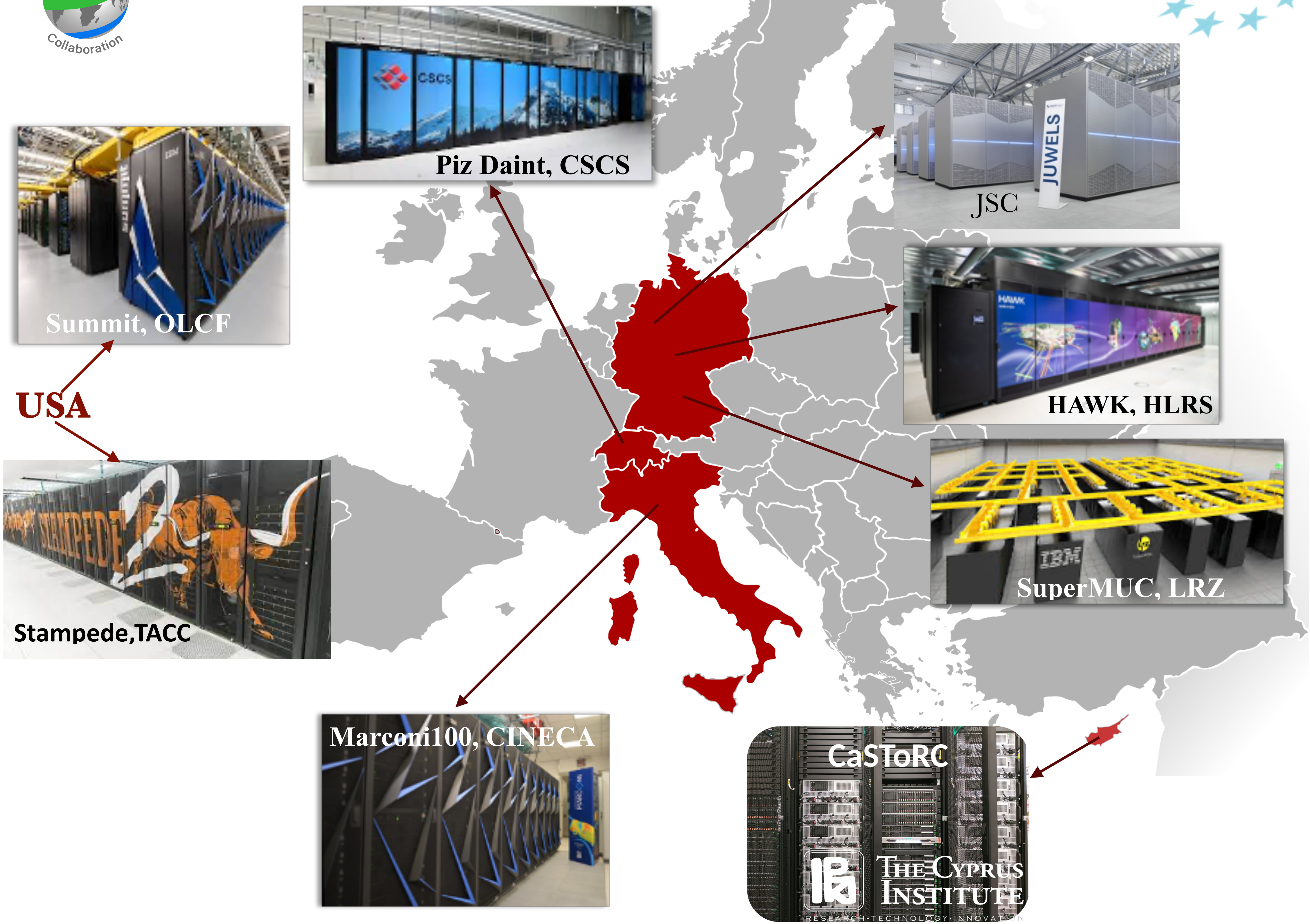


Data Analysis

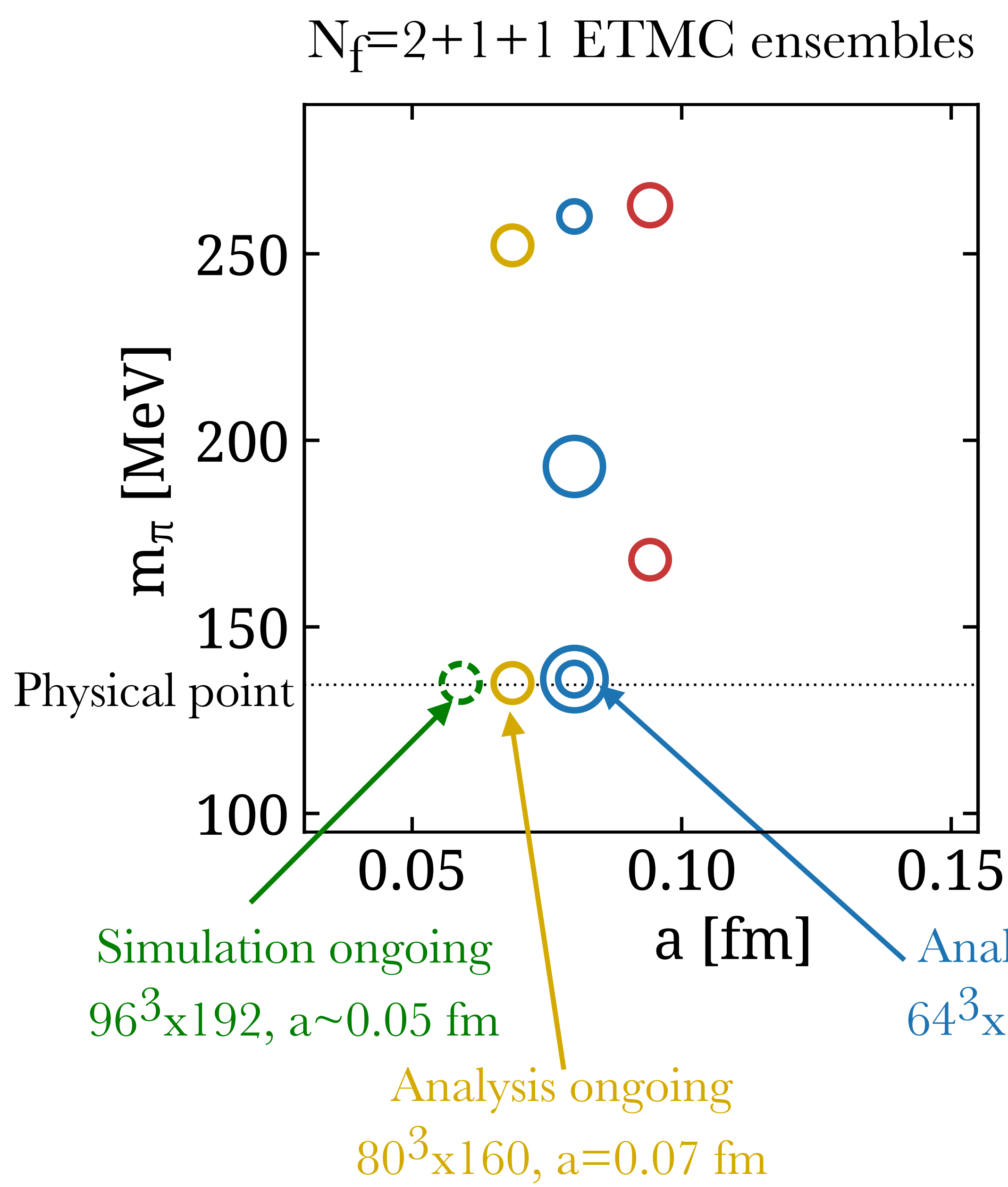
Extended Twisted Mass Collaboration

Computational resources

PRACE



Gauge ensembles generated by ETMC

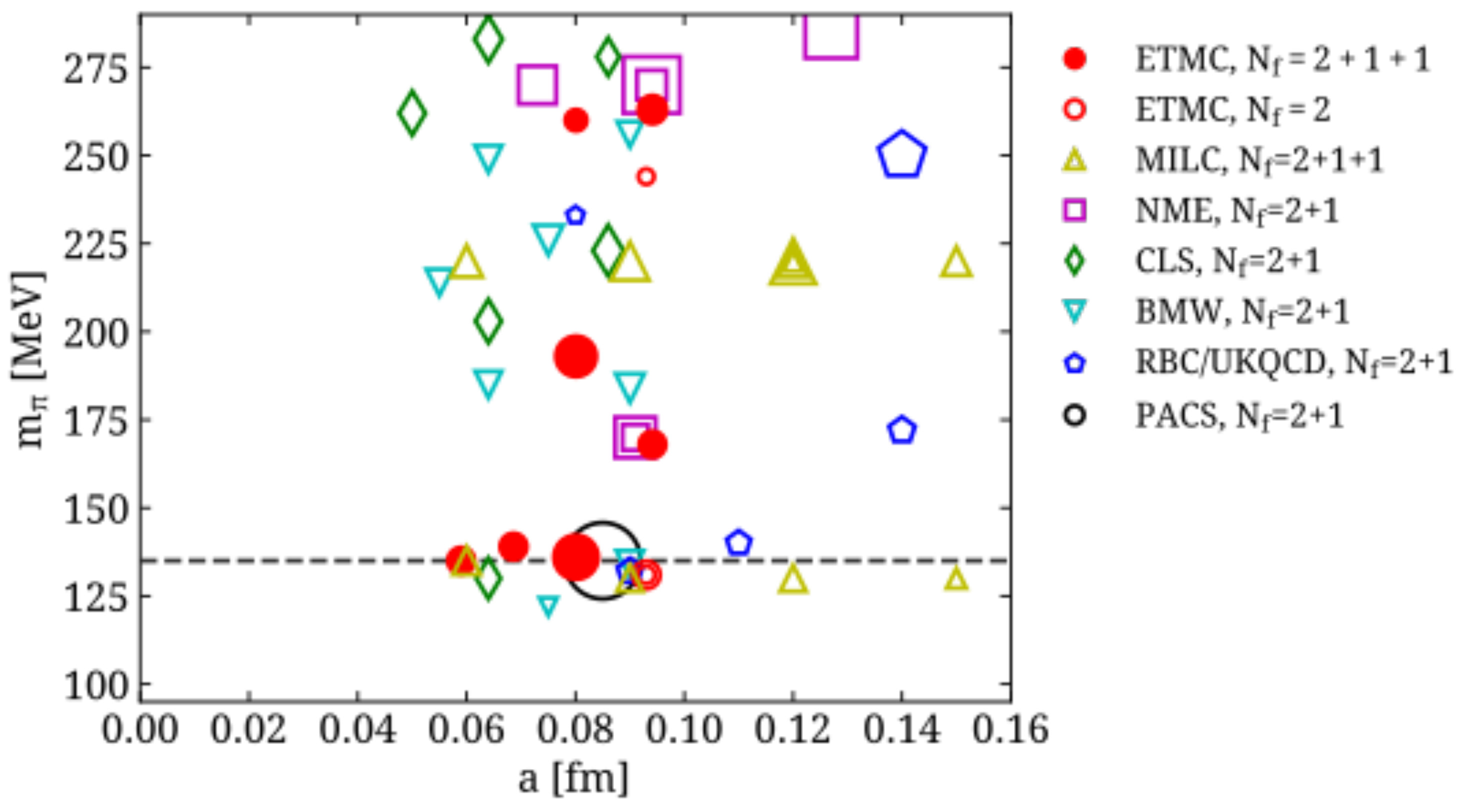


- 4 ensembles at physical pion mass
  - 3 lattice spacings 0.05<a<0.1 fm —> take continuum limit **directly at the physical point** avoiding chiral extrapolation removing a major systematic error in the baryon sector
  - Two volumes at a=0.08 fm of Lm_π=3.6 (5.1 fm) and Lm_π=5.4 (7.7 fm)
- Algorithmic improvements needed to go to a<0.05 fm due to critical slow down in HMC (long autocorrelations) —> new approaches e.g. Machine learning approaches using equivariant flows

  G. Kanwar, et al., Phys. Rev. Lett. 125 (2020) no.12, 121601 2003.06413; D. Boyda, et al., 2008.05456

C. A. *et al.* (ETMC), "Simulating twisted mass fermions at physical light, strange and charm quark masses" Phys. Rev.,D98(5):054518, 2018

Status of current simulations



✷ A number of collaborations has physical point ensembles:

- Wilson-type: ETMC, BMW, CLS, PACS
    - Most have 1-2 lattice spacings 0.05<a<0.1 fm
    - PACS has a large volume ensemble

- Staggered at physical point: MILC with 3 lattice spacings 0.06<a<0.15 fm

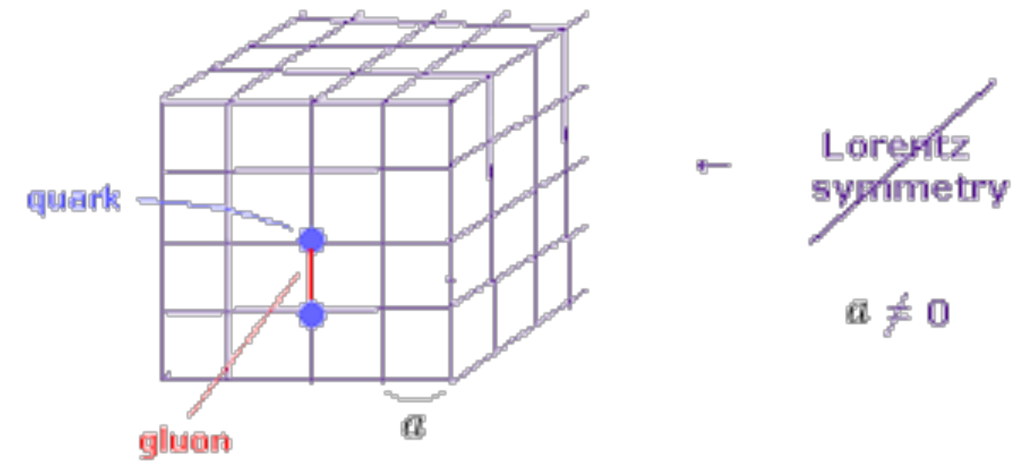
- Domain wall at physical point RBC/UKQCD with 2 lattice spacings

Systematics & Challenges

- **Discretisation effect:** Continuum limit
  —> need simulations for at least 3 lattice spacings

Typically done using simulations for heavier than physical values of the pion mass

- **Finite volume effects:** Infinite volume limit
  —> need simulations for at least 3 volumes



- **Simulations directly at the physical point**
  Systematic effects from chiral extrapolation are eliminated

- **Ground-state identification**
  Cross-check (one-, two- and three-state fits, summation)

- **Renormalisation**
  Non-perturbatively with improvements e.g using perturbative subtraction of lattice artefacts

- **Large momentum limit - important for direct evaluation of GPDs**
  Challenging since statistical errors grow exponentially

- In what follows we assume **isospin symmetry** i.e. up and down quarks have equal mass, and **neglect EM effects**

Nucleon spin structure

Energy and momentum tensor (EMT)

Energy and momentum tensor taken to be symmetric

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu}$$

X. Ji, PRD 52, 271,1995, hep-ph/9502213

Traceless and trace parts

- **Physical matrix elements of the traceless part can be written in terms of two gauge invariant terms**

$$\bar{T}^{\mu\nu} = \bar{T}_q^{\mu\nu} + \bar{T}_g^{\mu\nu}$$

$$\bar{T}_q^{\mu\nu} = \bar{\psi} i \gamma^{\{\mu} \overleftrightarrow{D}^{\nu\}} \psi \,, \qquad \bar{T}_g^{\mu\nu} = F^{\{\mu\rho} F^{\nu\}}{}_{\rho}$$

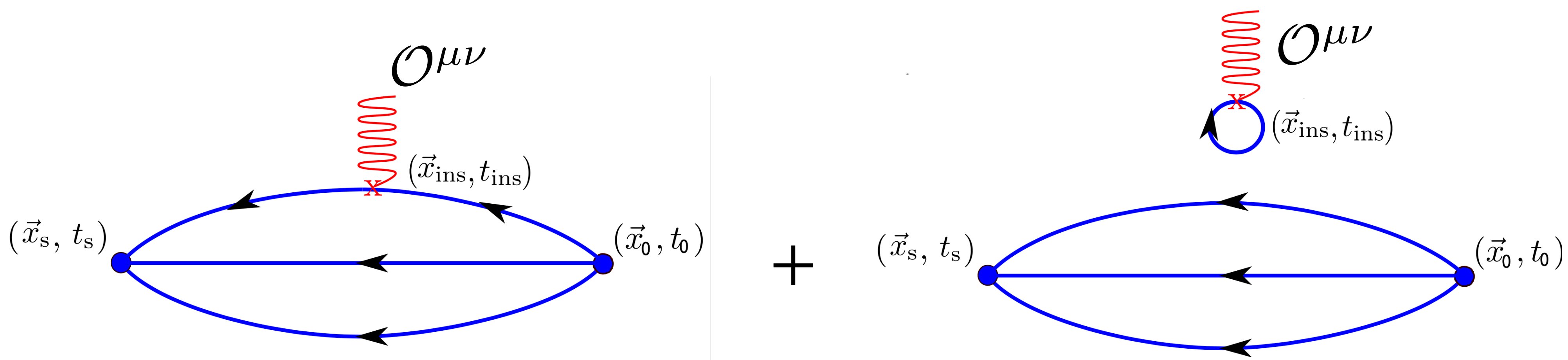
Symmetrisation over indices and subtraction of trace

Scheme and scale dependent

Related to quark and gluon total angular momentum

Nucleon matrix elements

$$C_{3\mathrm{pt}}^{\mu\nu}(\Gamma;\vec{q}=0,t_s,t_{\mathrm{ins}})=\sum_{\vec{x}_{\mathrm{ins}},\vec{x}_s}\mathrm{Tr}\left[\langle \Gamma J_N(t_s,\vec{x}_s)\mathcal{O}^{\mu\nu}(t_{\mathrm{ins}},\vec{x}_{\mathrm{ins}})\bar{J}_N(t_0,\vec{x}_0)\rangle\right]$$



connected   disconnected

✱ Identification of nucleon matrix element $\mathcal{M}$ ($t_0=0$)

Plateau and two-state fit:

$$R^{\mu\nu}(\Gamma;\vec{q}=\vec{0},t_s,t_{\mathrm{ins}})=\frac{C_{3\mathrm{pt}}^{\mu\nu}(t_s,t_{\mathrm{ins}})}{C_{2\mathrm{pt}}(\Gamma_0,t_s)}\longrightarrow \boxed{\mathcal{M}}+\mathcal{O}(e^{-\Delta E(t_s-t_{\mathrm{ins}})})+\mathcal{O}(e^{-\Delta E t_{\mathrm{ins}}})$$

Included in the two-state fit

Summation:

$$\sum_{t_{\mathrm{ins}}=a}^{t_s-a}R^{\mu\nu}(\Gamma;\vec{q}=\vec{0},t_s,t_{\mathrm{ins}})\longrightarrow c+\boxed{\mathcal{M}}t_s+\mathcal{O}(e^{-\Delta E t_s})$$

L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys., B293: 420, 1987

Nucleon matrix elements of EMT

$$\langle N(p',s')|\bar{T}^{\mu\nu}_{\rm q}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]u_N(p,s)$$

D-term

$$\langle x\rangle = A_{20}(0) \quad \text{and} \quad J_q = \frac{1}{2}\left[A^q_{20}(0) + B^q_{20}(0)\right]$$

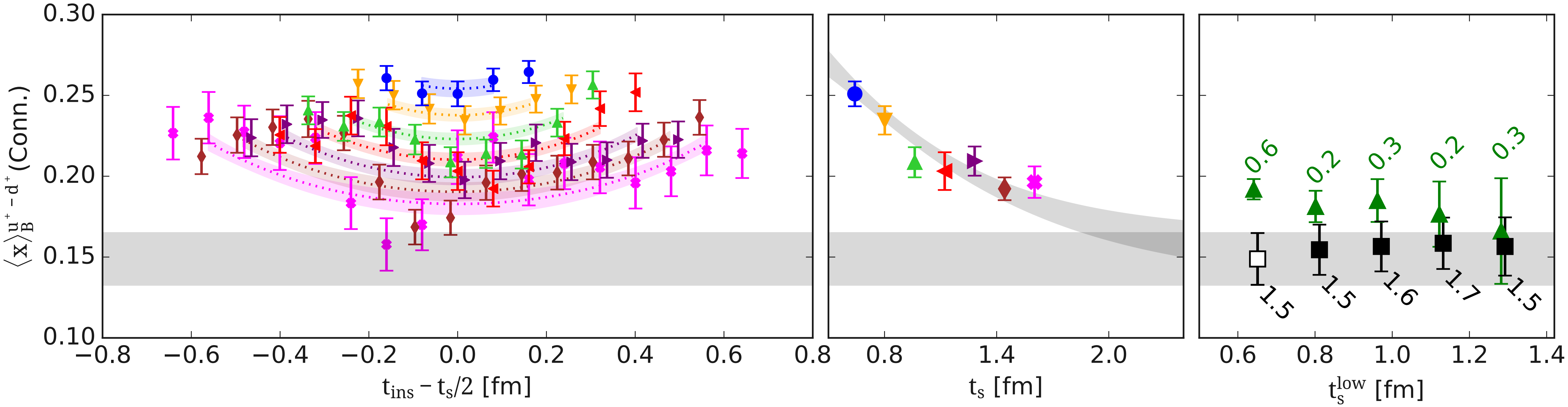
Ground-state dominance

- Plateau - one state
- Two- state fits
- Summation

$$\frac{C_{3pt}}{C_{2pt}}$$

Isovector

Plateau values

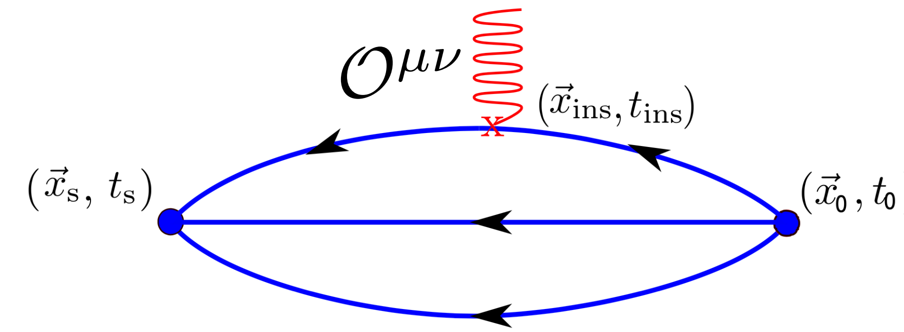


C. Alexandrou et al., PRD 101, 094513 (2020), arXiv: 2003.08486

Momentum fraction for each quark

N_f=2+1+1 twisted mass fermions with a clover term

- Lattice size 64^3 x 128
- a=0.08 fm determined from the nucleon mass
- m_π=139 MeV
- Lm_π=3.6



Statistics for connected contribution

t_s/a	$N_{\rm cnfs}$	$N_{\rm srcs}$	$N_{\rm meas}$
8	750	1	750
10	750	2	1500
12	750	4	3000
14	750	6	4500
16	750	16	12000
18	750	48	36000
20	750	64	48000
2-pt	750	264	198000

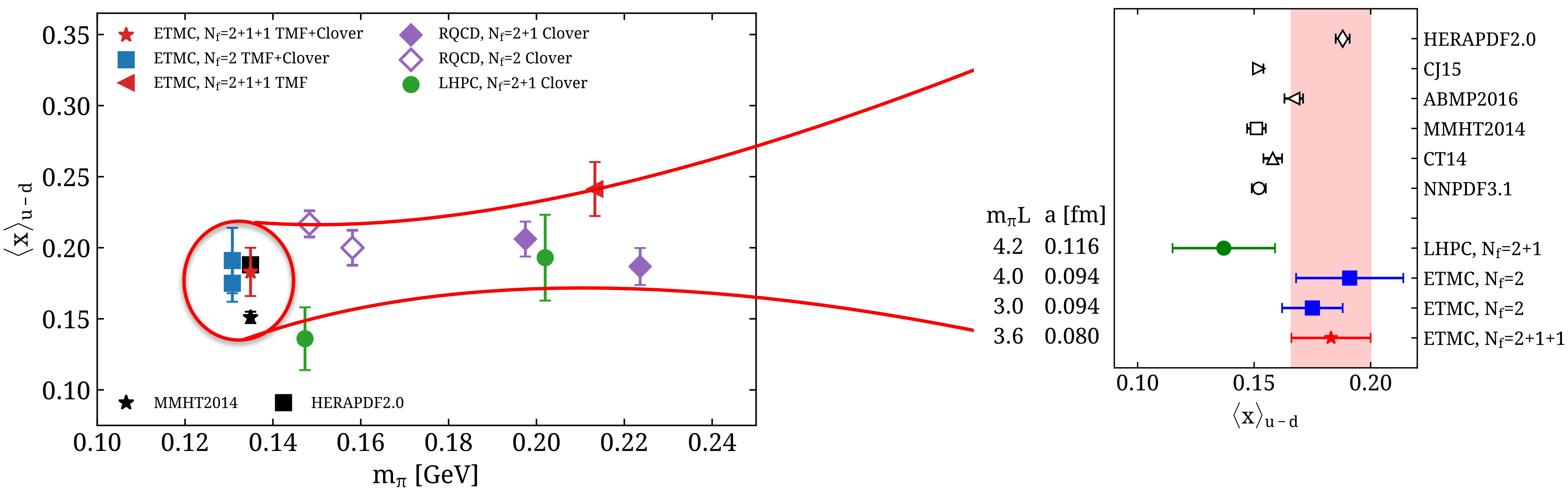
0.64 fm

Needed for studying excited states

1.6 fm

Increase statistics to keep approx. constant error

Nucleon isovector momentum fraction results



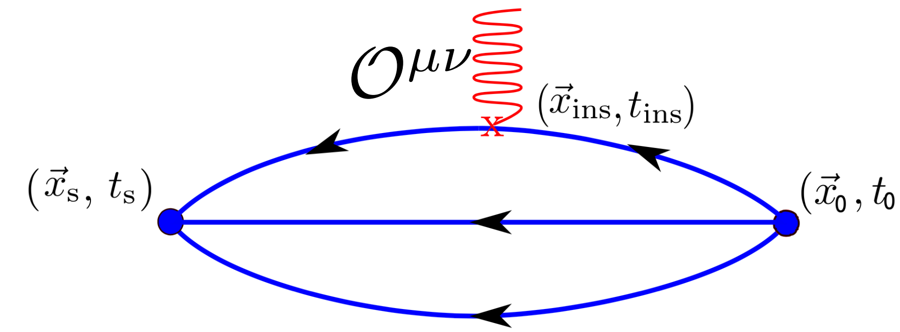
Comparison among lattice collaborations

- A number of calculations at the physical point
- Phenomenological determinations yield different values with spread compatible with the statistical error of lattice QCD

Momentum fraction for each quark

N_f=2+1+1 twisted mass fermions with a clover term

- Lattice size 64^3 x 128
- a=0.08 fm determined from the nucleon mass
- m_π=139 MeV
- Lm_π=3.6



Statistics for connected contribution

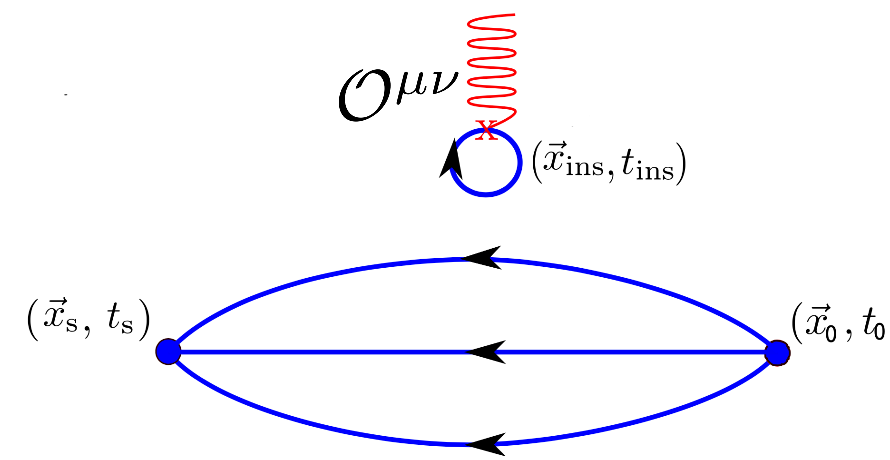
t_s/a	$N_{\rm cnfs}$	$N_{\rm srcs}$	$N_{\rm meas}$
8	750	1	750
10	750	2	1500
12	750	4	3000
14	750	6	4500
16	750	16	12000
18	750	48	36000
20	750	64	48000
2-pt	750	264	198000

0.64 fm

Needed for studying excited states

1.6 fm

Increase statistics to keep approx. constant error



Use hierarchical probing

no. of Hadamard vectors

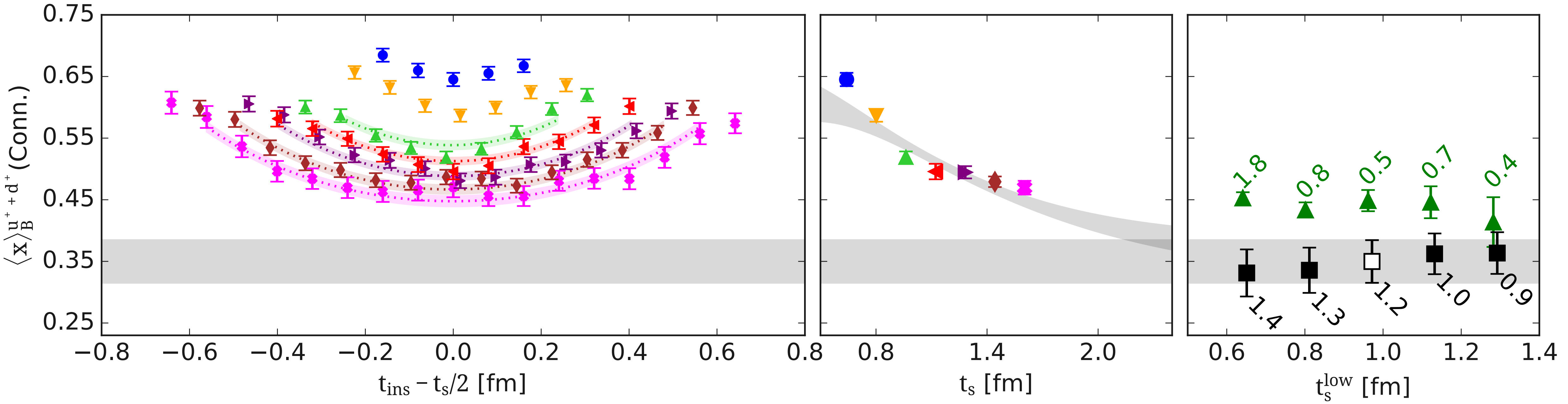
Statistics for disconnected contribution

2pt	(u+d)-quark loop	s-quark loop	c-quark loop
600000	750×512 + deflation of 200 modes	750×512	9000×32

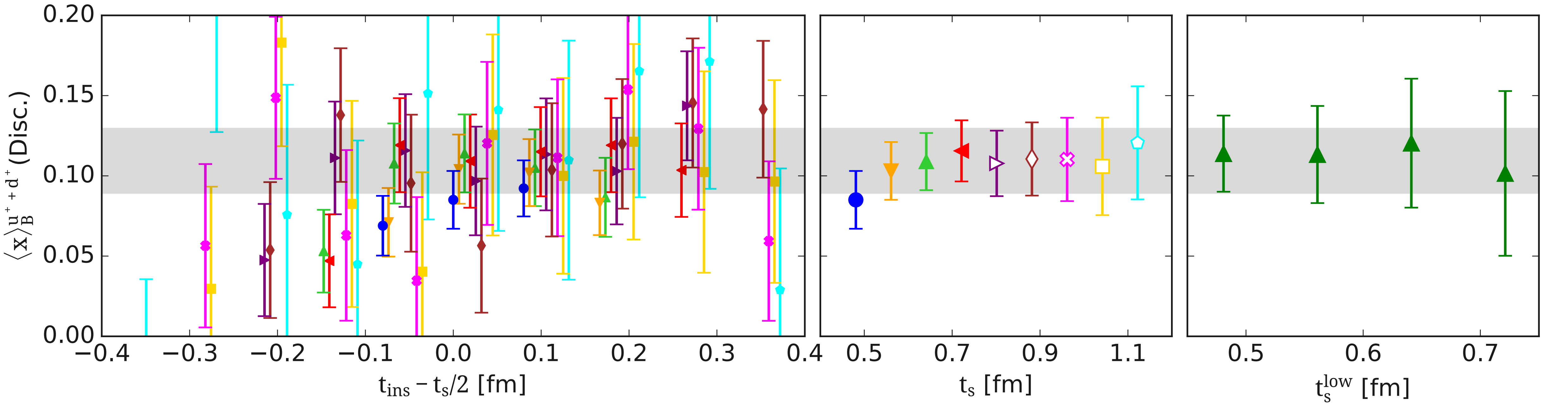
no. of stochastic vectors

Nucleon isoscalar momentum fraction

Connected



Disconnected

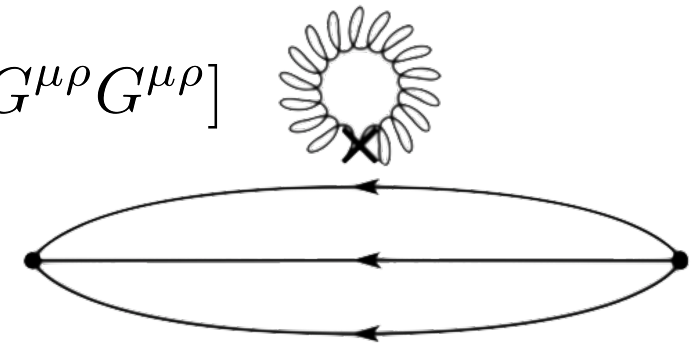


Gluon momentum fraction

$\langle x \rangle_g = A^g_{20}(0)$

$\mathcal{O}^{\mu\nu}_g = 2\mathrm{Tr}\left[G^{\mu\rho}G^{\mu\rho}\right]$

Field strength tensor



Use stout smearing to reduce UV noise: n_s=10

- In the rest frame of the nucleon: $\dfrac{\langle N|\mathcal{O}^{44}_g - \frac{1}{3}\mathcal{O}^{jj}_g|N\rangle}{\langle N|N\rangle} = -2m_N < x >_g$
- In a moving frame: $\dfrac{\langle N|\mathcal{O}^{4i}_g|N\rangle}{\langle N|N\rangle} = p_i < x >_g$
- Quark isoscalar & gluon momentum fractions mix —> need 2x2 matrix:

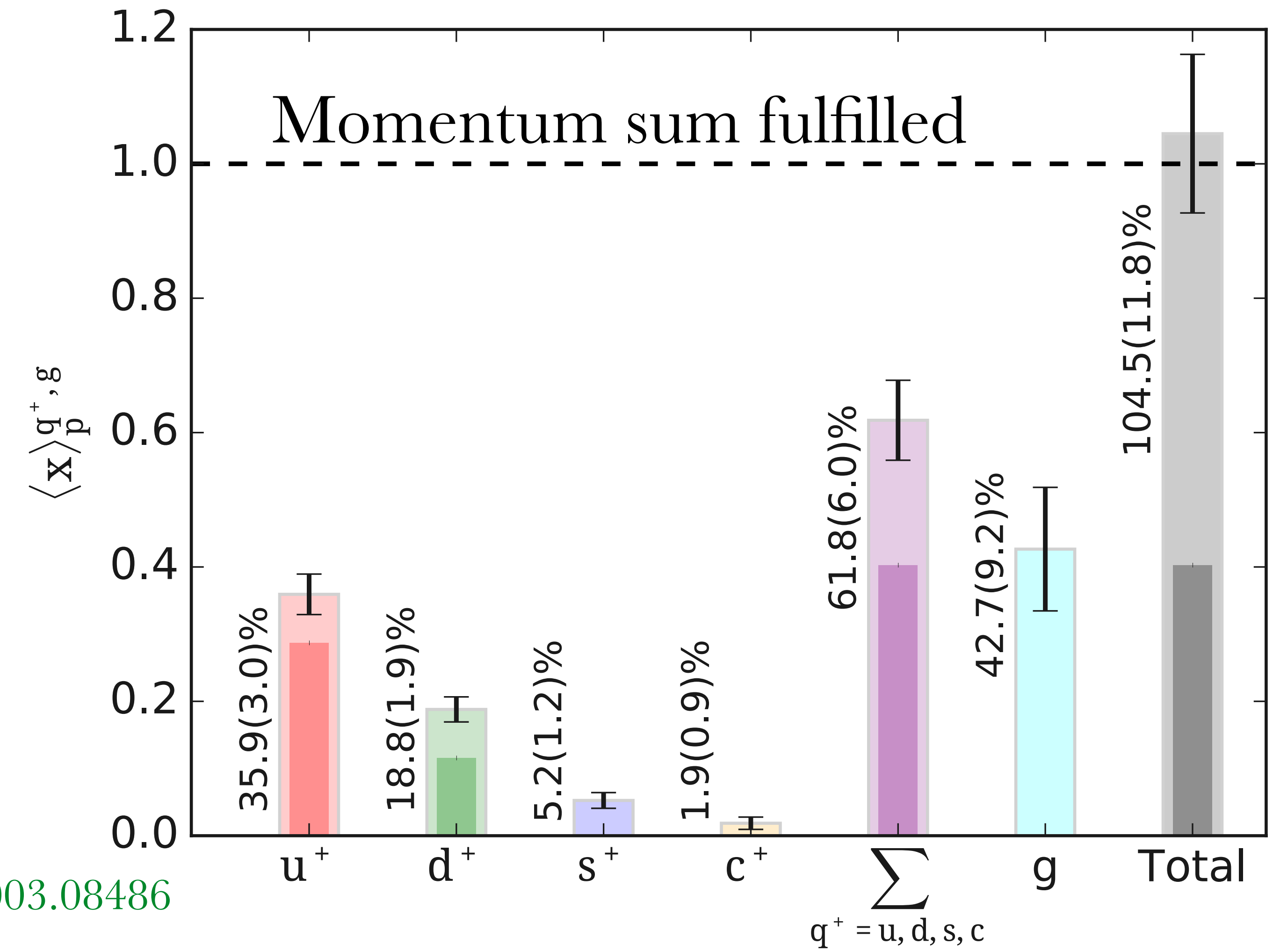
$$\langle x \rangle^R_g = Z_{gg}\langle x \rangle^B_g + Z_{gq}\sum_q \langle x \rangle^B_q$$

- Z_{qq} and Z_{gg} computed non-perturbatively

$$Z_{gq} = \frac{g^2 C_f}{16\pi^2}\left(0.8114 + 0.4434 c_{SW} - 0.2074 c^2_{SW} + \frac{4}{3}\log(a^2\bar{\mu}^2)\right)$$

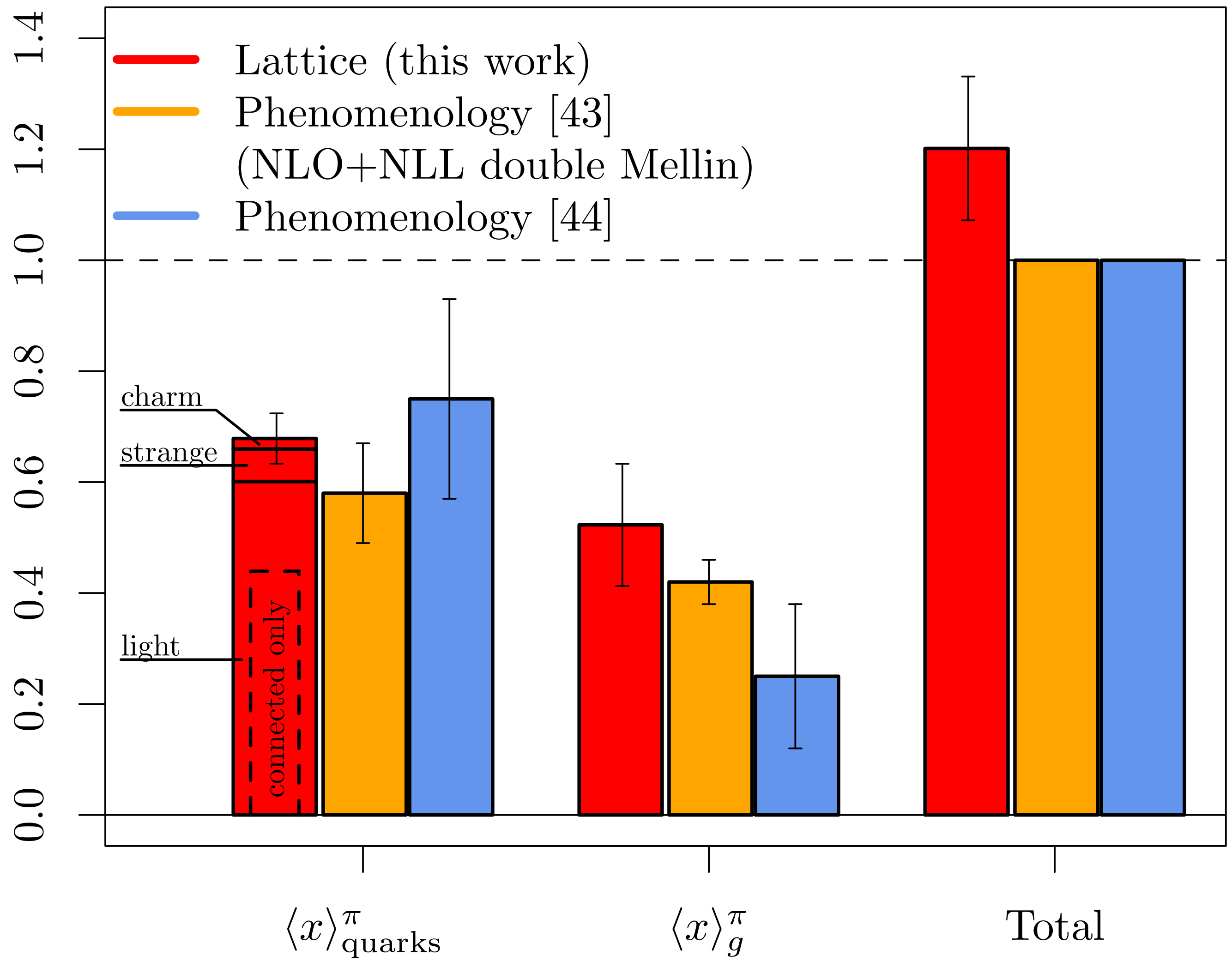
$$Z_{qg} = \frac{g^2 C_f}{16\pi^2}\left(0.2164 + 0.4511 c_{SW} + 1.4917 c^2_{SW} - \frac{4}{3}\log(a^2\bar{\mu}^2)\right)$$

computed perturbatively



C. Alexandrou et al., PRD 101, 094513 (2020), arXiv: 2003.08486

Momentum fraction of pion



N_f=2+1+1 twisted mass fermions

- Lattice size 64^3 x 128
- a=0.08 fm determined from the nucleon mass
- m_π=139 MeV
- Lm_π=3.6

$\overline{\text{MS}}$ scheme at 2 GeV

$$x_{u+d} = 0.601(28),\ x_s = 0.059(13),\ x_c = 0.019(05),\ \text{and}\ x_g = 0.52(11)$$

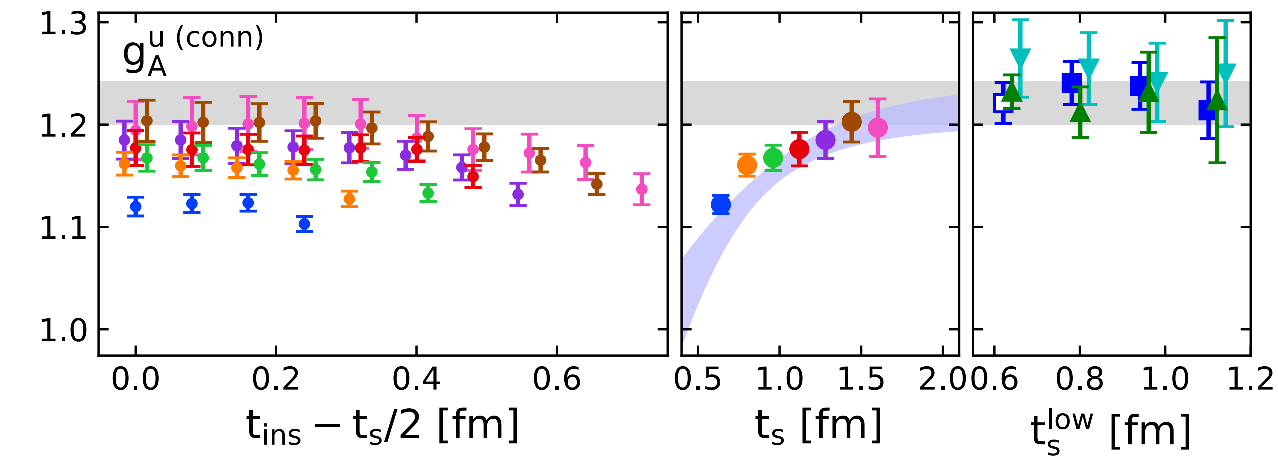
C. Alexandrou *et al.* (ETMC), arXiv: 2109.10692

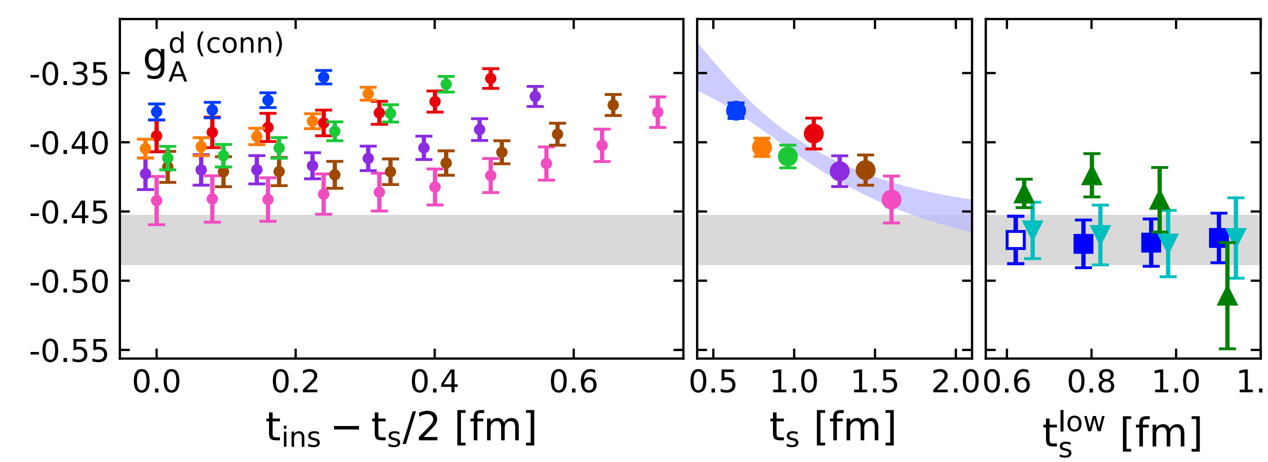
Flavor decomposition of axial charge

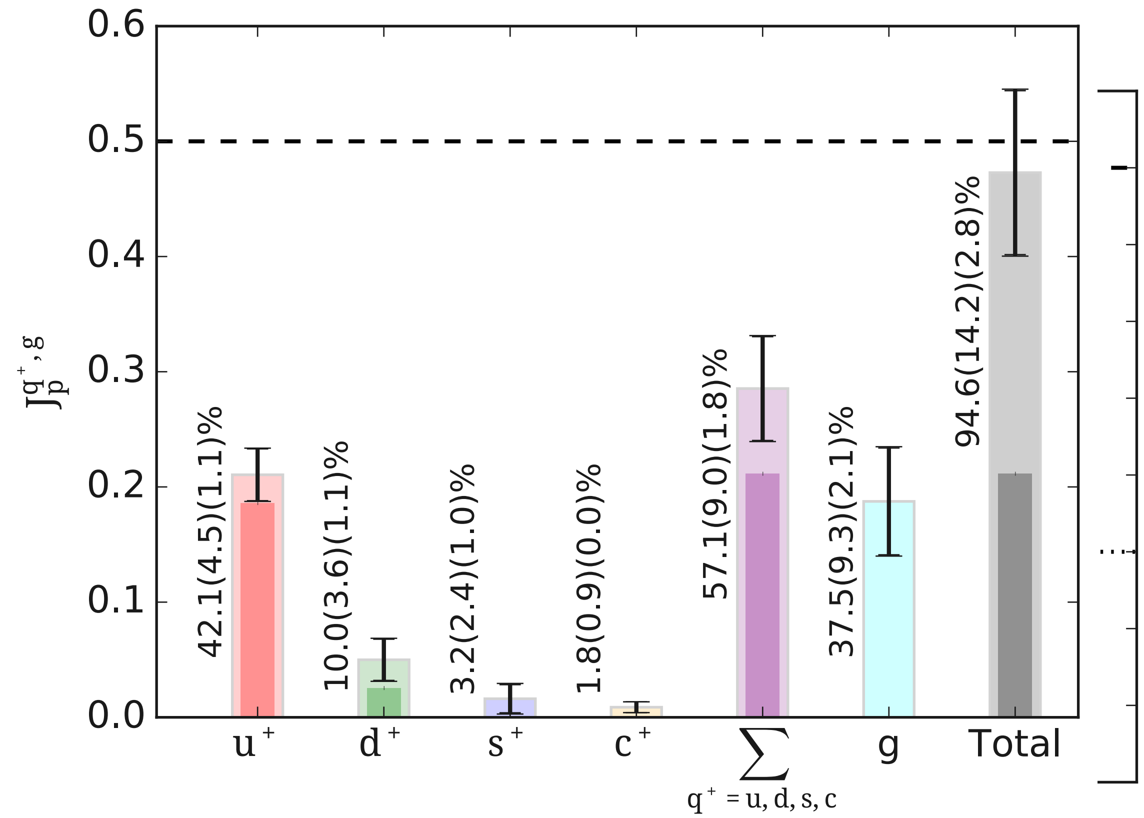
- Determines intrinsic spin carried by each quark

$$\Delta\Sigma_{q_+}(\mu^2) = \int_0^1 dx\, \left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] = g_A^q$$

Connected up & down - like isovector

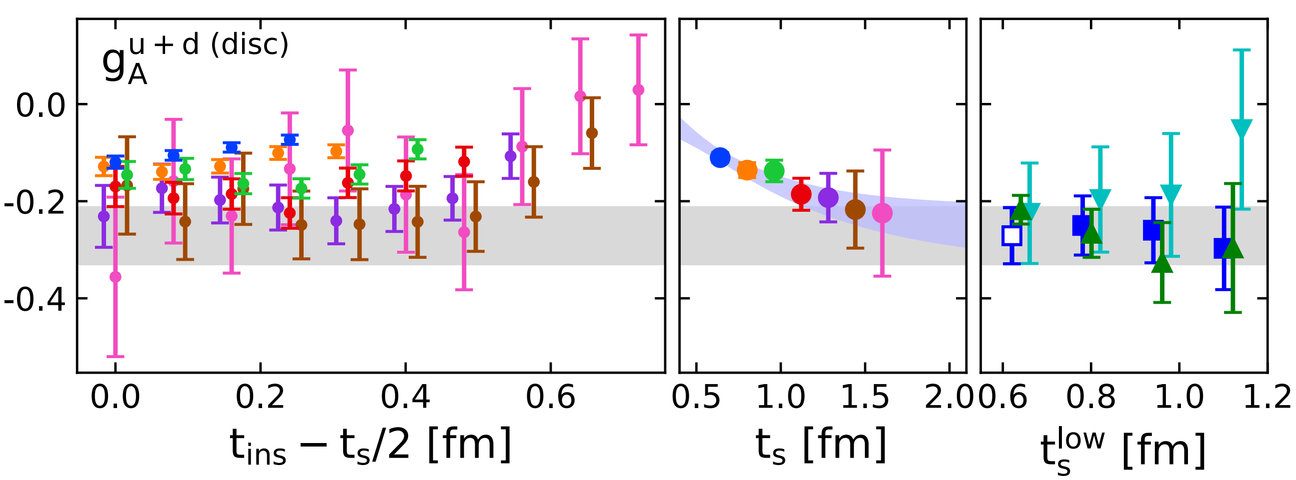


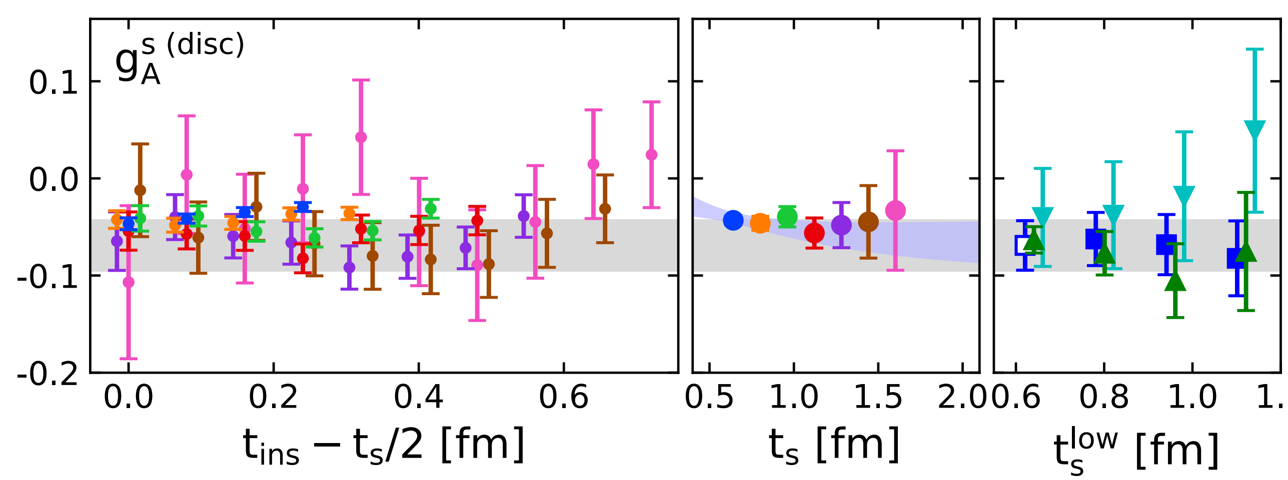




C. A. *et al.* (ETMC) Phys.Rev.D 102 (2020) 5, 054517, 1909.00485
C. A. *et al.* (ETMC) Phys.Rev.D 101 (2020) 9, 094513, 2003.08486

Disconnected





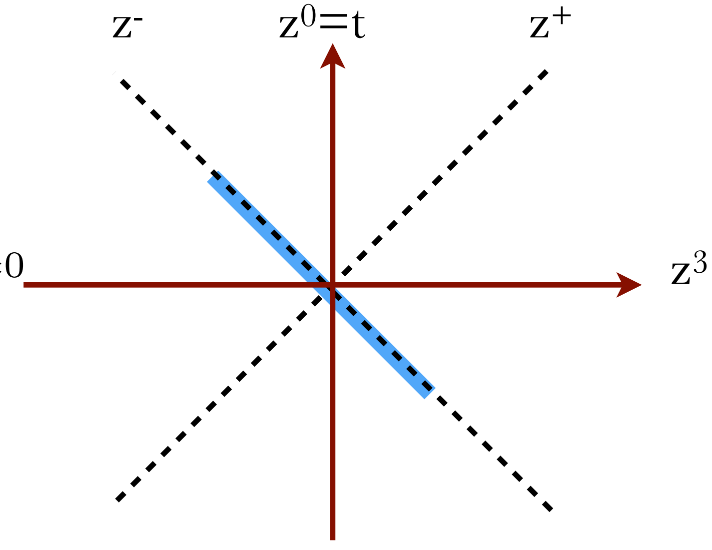
New era of direct computation of x-dependencne of parton distributions

Quasi-parton distributions

X. Ji, Phys. Rev. Lett. 110 (2013) 262002 [arXiv:1305.1539]

- PDFs light-cone correlation matrix elements

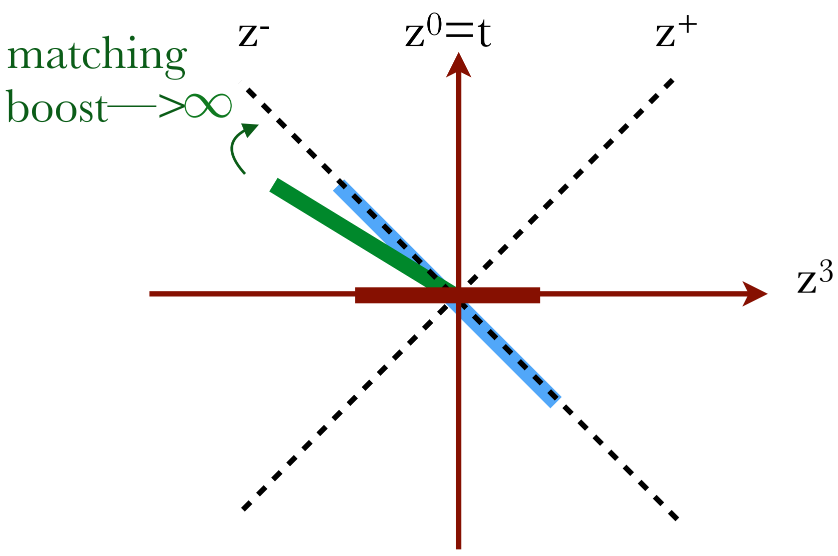
$$F_\Gamma(x) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-}\langle N(p)|\bar\psi(-z/2)\Gamma W(-z/2,z/2)\psi(z/2)|N(p)\rangle|_{z^+=0,\vec{z}=0}$$



- Define spatial correlators e.g. along z^3 and boost nucleon state to large momentum

- Renormalise quasi-PDFs and increase nucleon boost

- Use LaMET to match to the infinite momentum frame using the matching kernel computed in perturbation theory



Parton distributions on the lattice

❋Quasi-PDFs:

- Formulate an effective field theory, Large Momentum Effective Theory (LaMET) with large momentum scale to match lattice results to the infinite momentum frame (IMF)
- Quasi-PDFs and IMF PDFs have the same infrared physics and thus the matching can be done in perturbation theory i.e. UV regularization is necessarily taken first, before the infinite momentum limit

X. Ji, Phys. Rev. Lett. 110 (2013) 262002 [arXiv:1305.1539]

❋Other approaches:

- Pseudo-distributions

A. Radyushkin, Phys. Lett. B767, 314 (2017), 1612.05170

- Current-current correlators

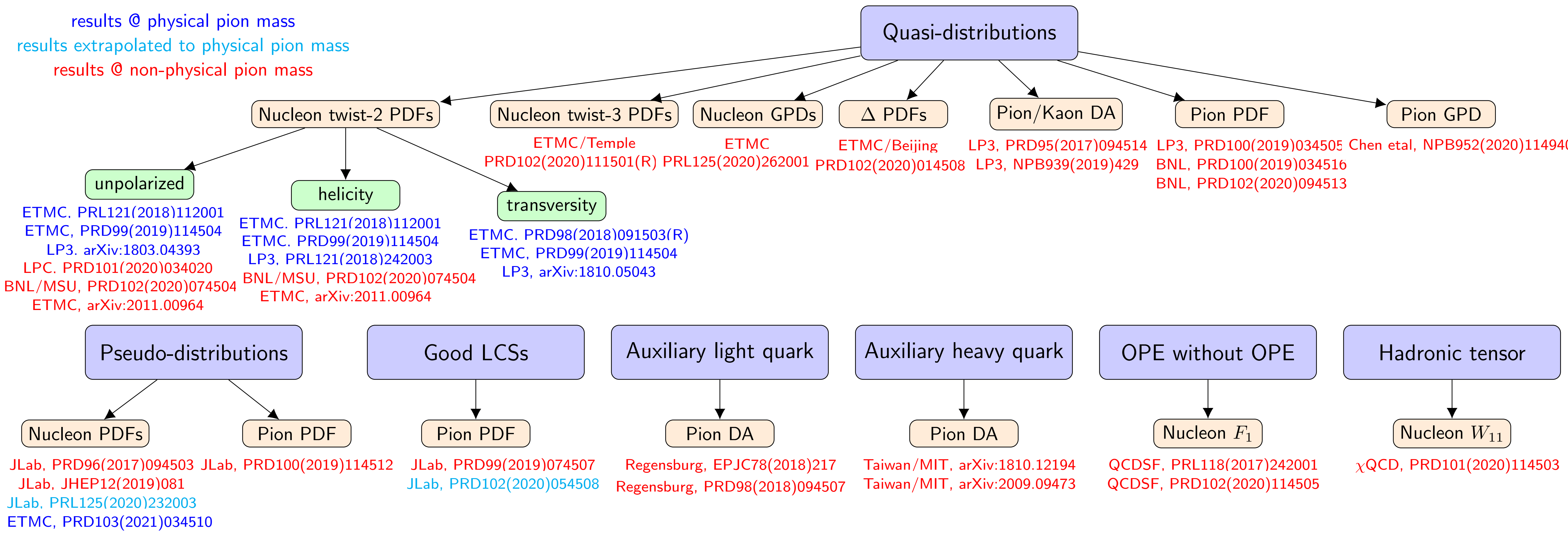
X. Ji and C. Jung Phys. Rev. Lett. 86 (2001) 208, hep-lat/0101014

V. Braun and D. Müller, EPJC 55 (2008) 349

A. J. Chambers et al. (QCDSF) Phys. Rev. Lett. 118 (2017) 2420 , arXiv:1703.01153

- For more see recent reviews
  - M. Constantinou *et al.* (2020) 2006.08636
  - H.-W. Lin *et al.* Prog. Part. Nucl. Phys. 100, 107, 1711.07916
  - X. Ji, Y. Liu, J.-H. Zhang, 2004.03543

Overview of results from different approaches



Courtesy K. Cichy

Reviews

- M. Constantinou *et al.* (2020) 2006.08636
- H.-W. Lin *et al.* Prog. Part. Nucl. Phys. 100, 107, 1711.07916
- X. Ji, Y. Liu, J.-H. Zhang, 2004.03543
- K. Cichy and M. Constantinou, *Adv.High Energy Phys.* (2019) 3036904,1811.07248

Computation of quasi-PDFs

X. Ji, Phys. Rev. Lett. 110 (2013) 262002 [arXiv:1305.1539]

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle |_\mu$$

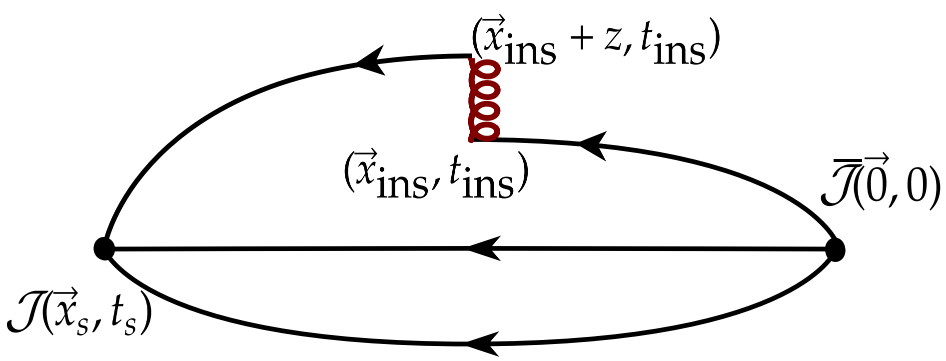
Renormalise non-perturbatively, $Z(z,\mu)$
Eliminates both UV and exponential divergences

- Match using LaMET

Perturbative kernel

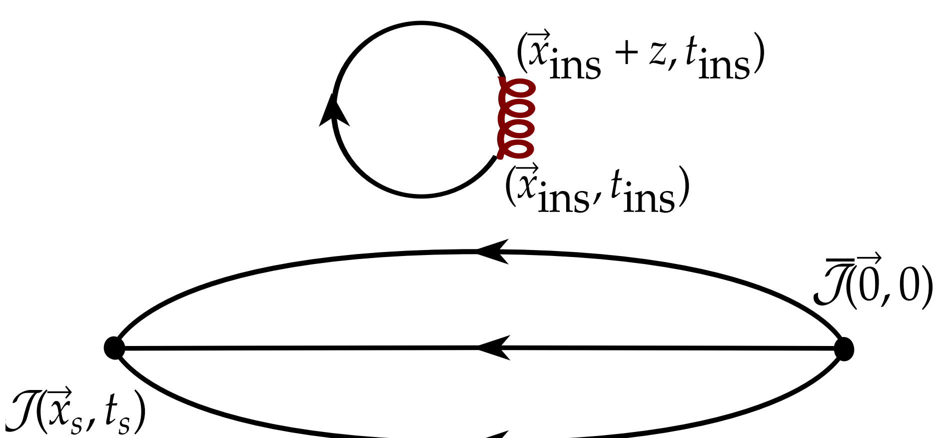
$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y,\mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{\rm QCD}^2}{P_3^2}\right)$$

Isovector (u-d) and isoscalar (u+d) connected

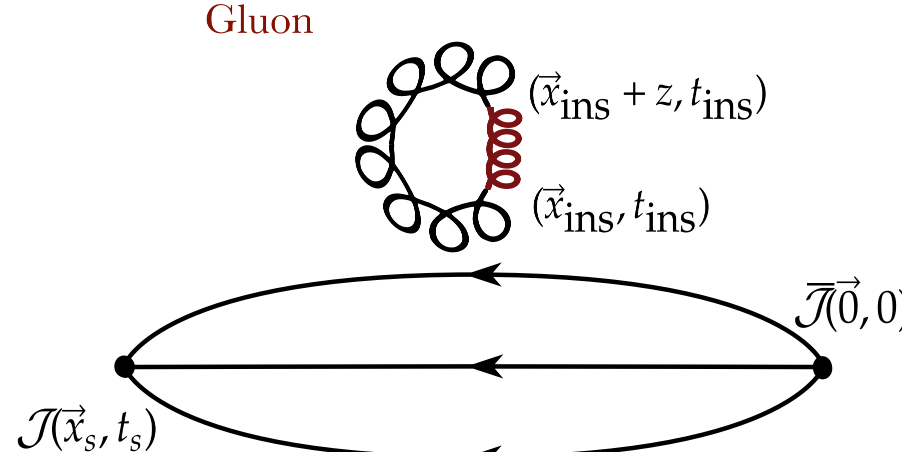


$$\Gamma = \begin{cases} \gamma_0 & \text{unpolarised} \\ \gamma_5\gamma_3 & \text{helicity} \\ \sigma_{3i},\, i=1,2 & \text{transversity} \end{cases}$$

Isoscalar (u+d) disconnected, s and c



Gluon

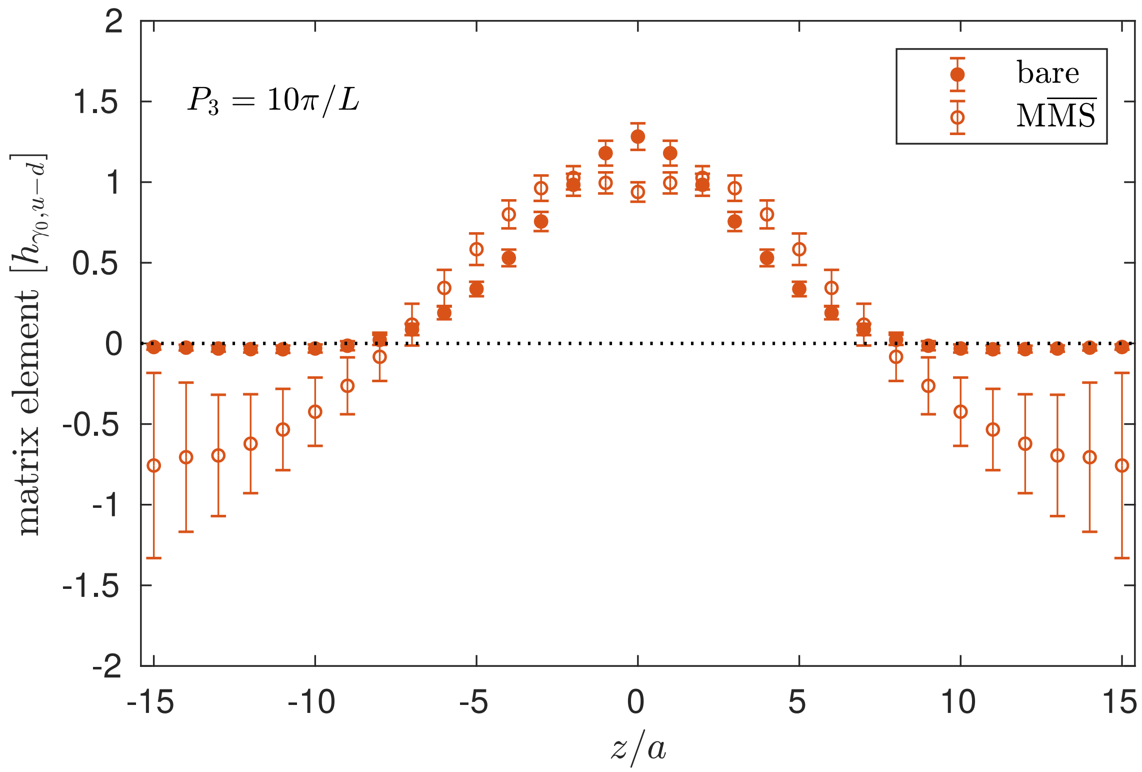


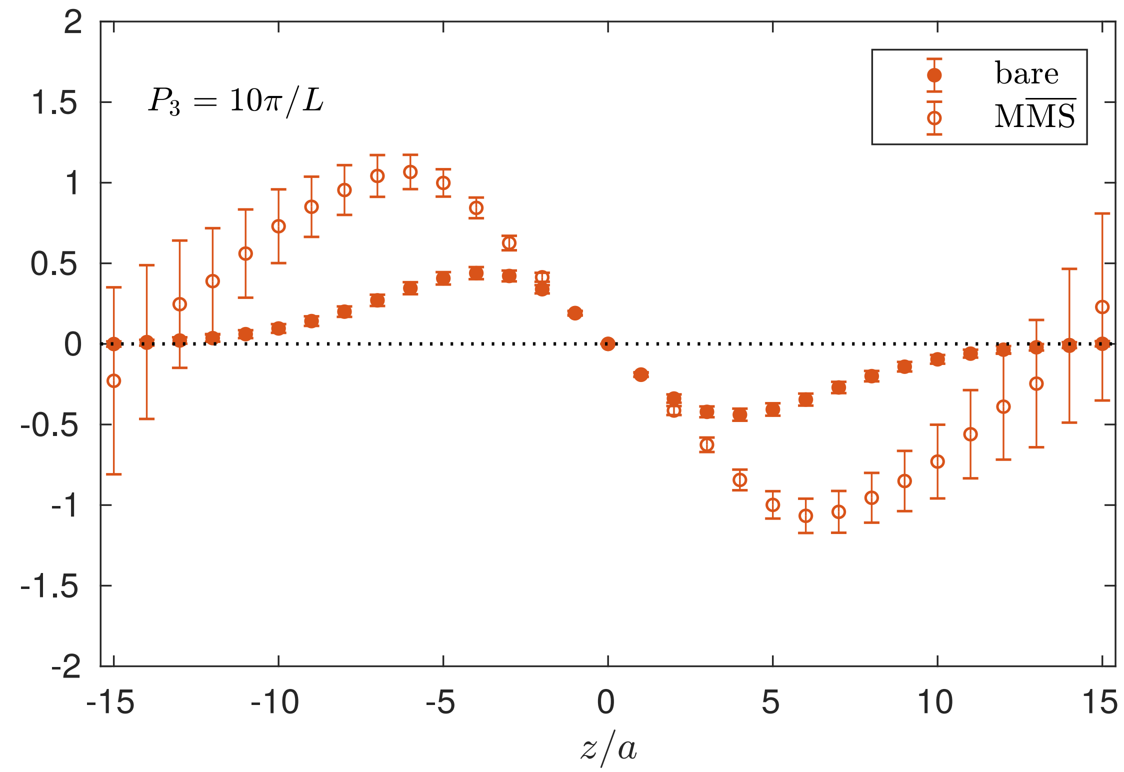
Quasi-PDFs: Challenges (I)

- Non-perturbative evaluation of the renormalization functions $\mathcal{Z}$(z,μ) to eliminate power and logarithmic divergences - Renormalizability proven to all orders in PT

C. A. *et al.*, Nucl. Phys. B 923, 394 (2017). 1706.00265;
J.-W. Chen *et al.*, Phys. Rev. D 95, 094514 (2017), 1702.00008
T. Ishikawa, Y. Q. Ma, J. W. Qiu, S. Yoshida, Phys. Rev. D 96 (2017) 094019, 1707.03107
X. Ji, J.-H. Zhang, Y. Zhao, *Phys.Rev.Lett.* 120 (2018)11, 11200, 1706.08962;
J. Green, K. Janse, F. Steffens, Phys. Rev. Lett. 121, 022004 (2018), 1707.07152

- Follow the same procedure as for local operators i.e. use RI-MOM and convert to $\overline{MS}$



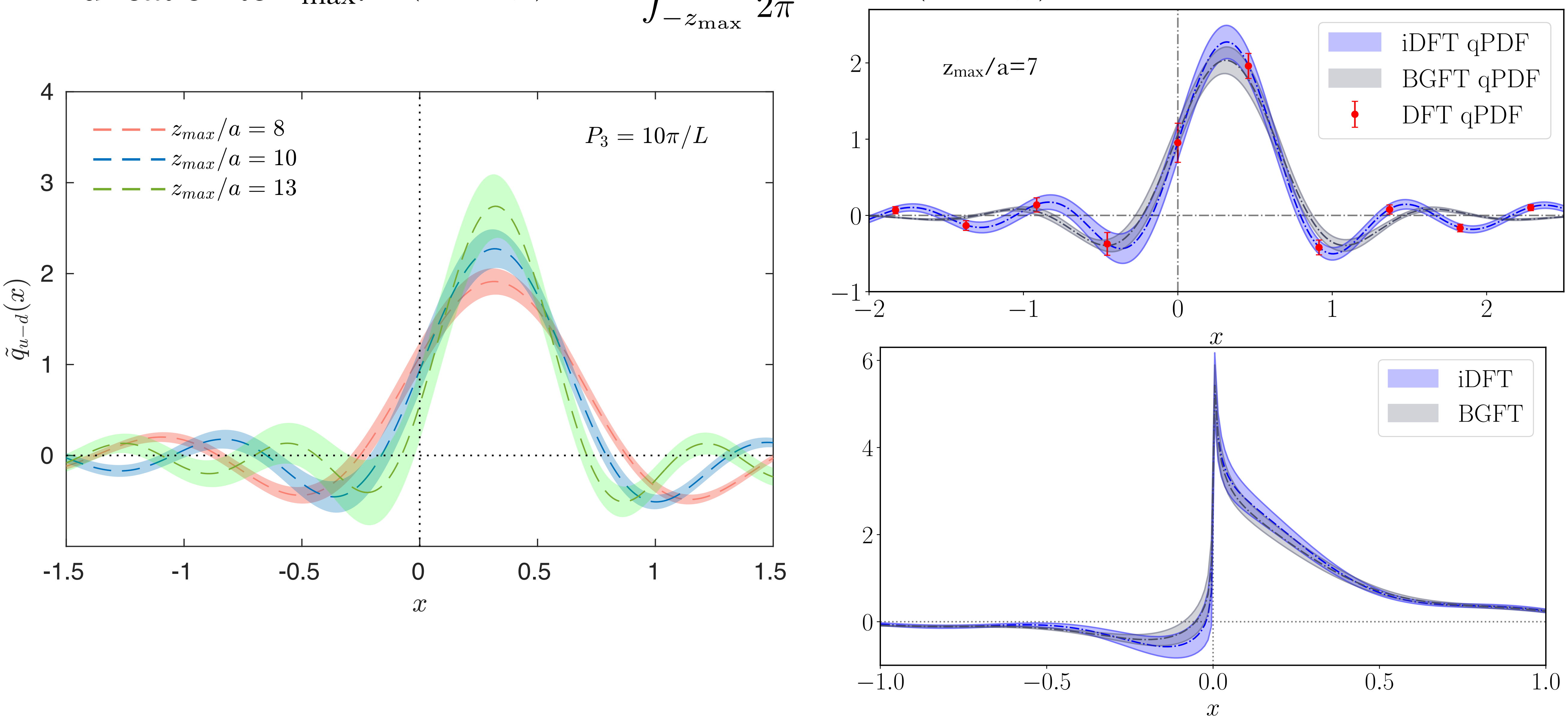


Quasi-PDFs: Challenges (II)

- Non-perturbative evaluation of the renormalization functions $\mathcal{Z}(z,\mu)$ to eliminate power and logarithmic divergences - Renormalizability proven to all orders in PT

- Discrete Fourier transform: - ill-posed problem

Truncation to z_{max}: $\tilde{q}(P_3, x, \mu) = P_3 \int_{-z_{\max}}^{z_{\max}} \frac{dz}{2\pi} e^{ixzP_3} h(P_3, z, \mu)$ ⟵ Renormalised matrix element



- Apply Gaussian Progress Regression, a nonparametric Bayesian approach, instead of discrete Fourier transform

L. Ambrogioni, E. Maris, 1704.02828

—> reduces oscillations but small effect on PDF

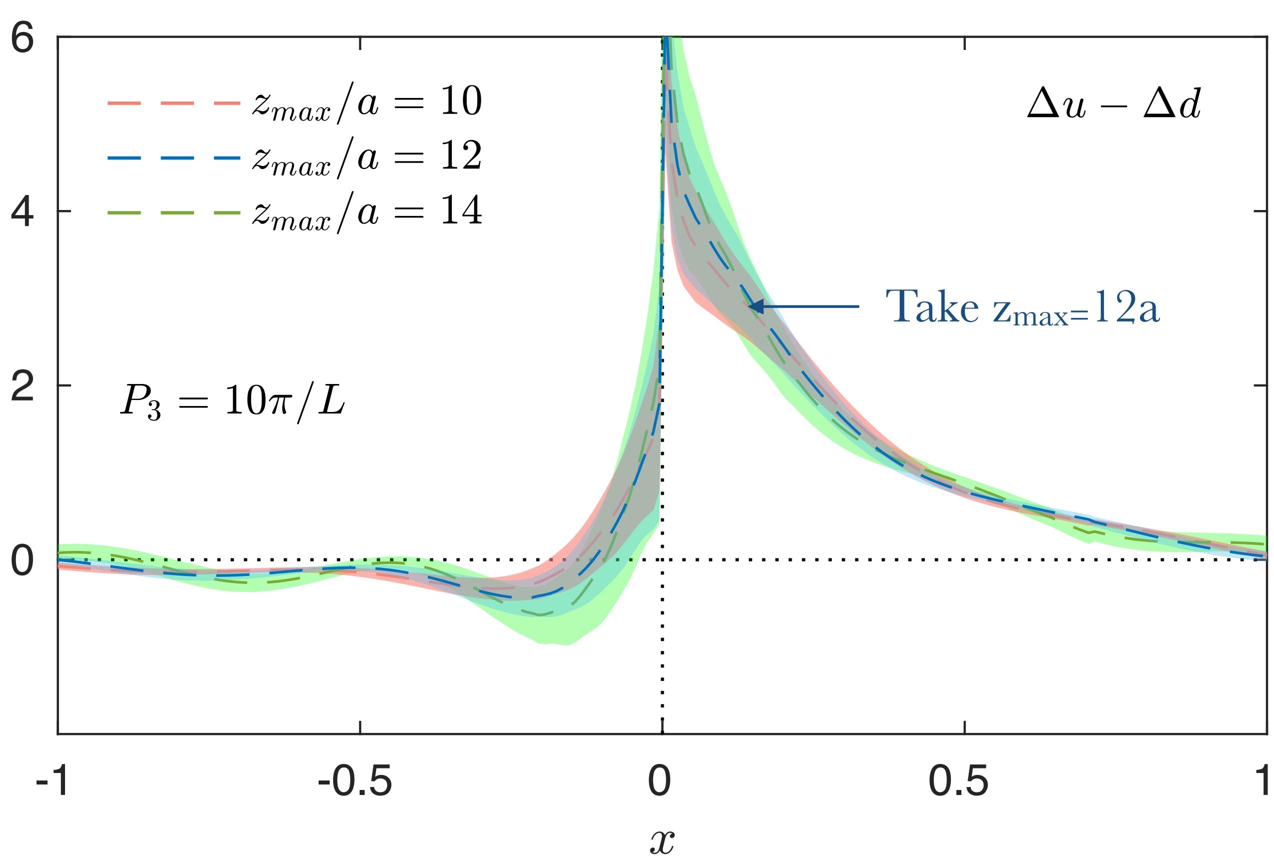
C.A., G. Iannelli, K. Jansen, F. Manigrasso, et al. Phys.Rev.D 102 (2020) 9, 094508, arXiv:2007.13800

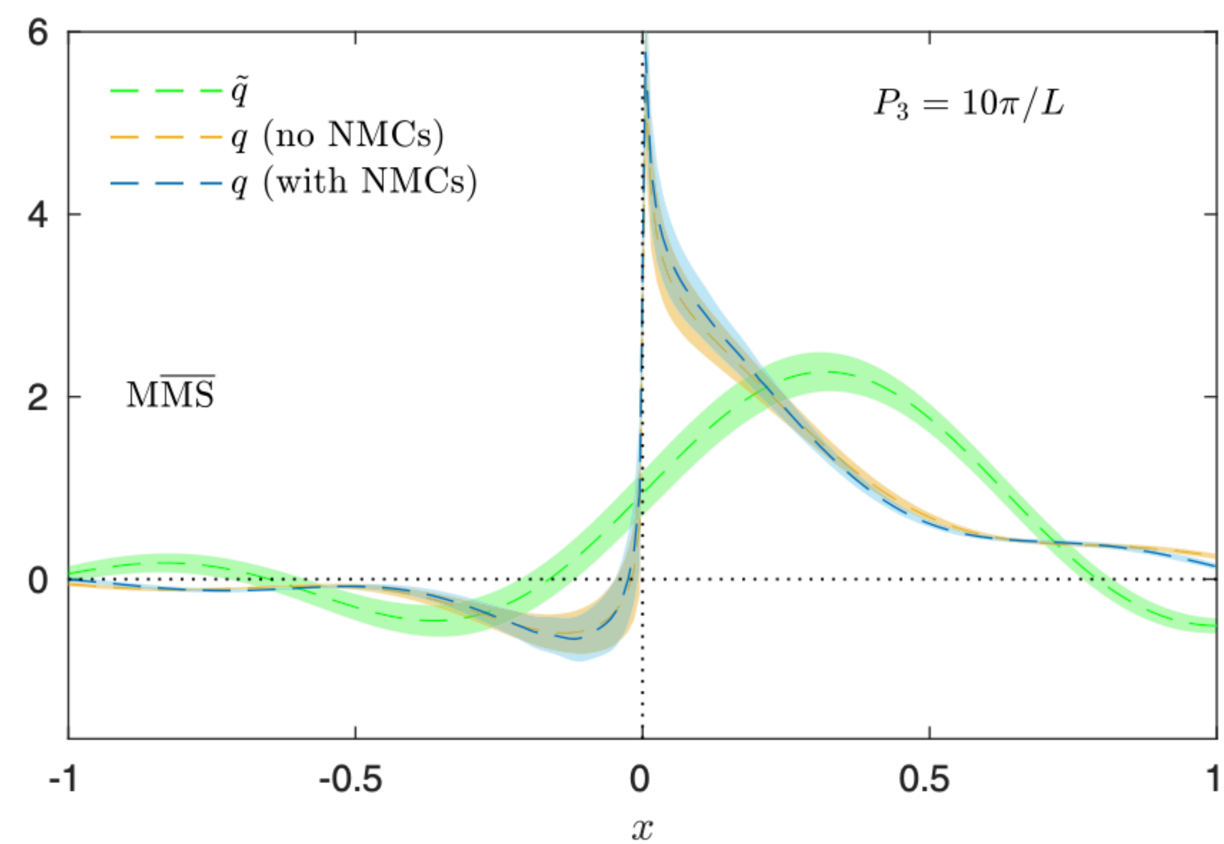
Quasi-PDFs: Mass corrections

- Non-perturbative evaluation of the renormalization functions $\mathcal{Z}(z,\mu)$ to eliminate power and logarithmic divergences - Renormalizability proven to all orders in PT
- Discrete Fourier transform
- Large boost needed for perturbative matching

$$\tilde{q}(x, P_3\mu) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) q(y,\mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{\rm QCD}^2}{P_3^2}\right)$$

matching function computed perturbatively





- Nucleon Mass Corrections, J.W. Chen et al., Nucl.Phys. B911 (2016) 246, 1603.06664

Quasi-PDFs: Challenges (III)

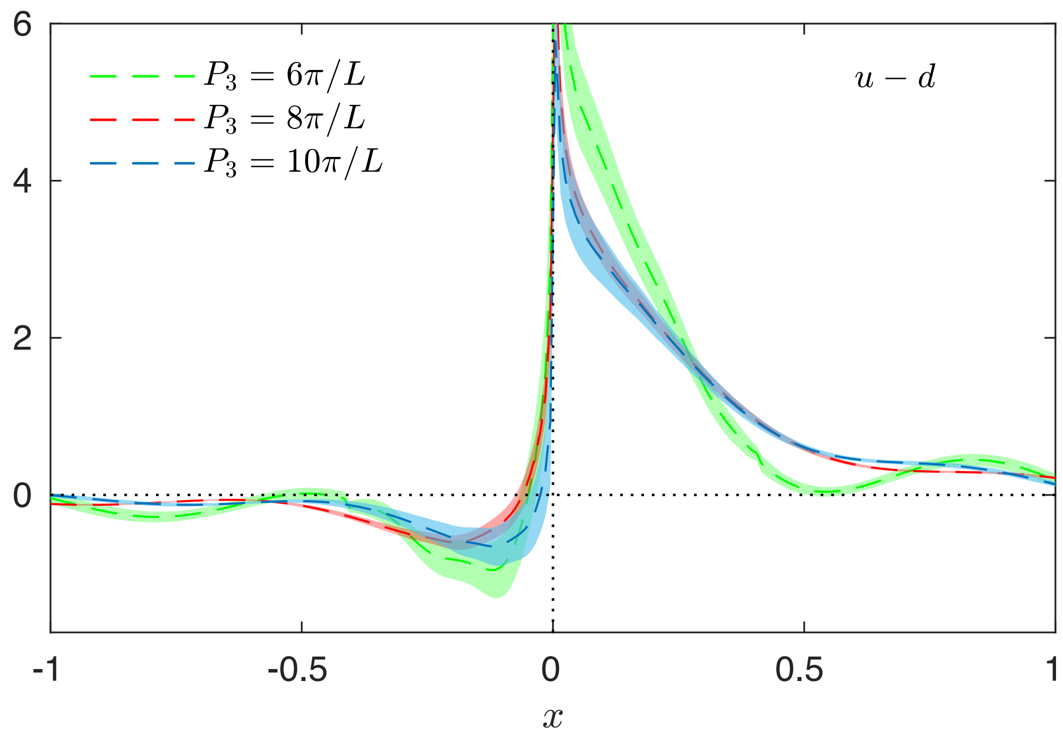
- Non-perturbative evaluation of the renormalization functions $\mathcal{Z}(z,\mu)$ to eliminate power and logarithmic divergences - Renormalizability proven to all orders in PT
- Discrete Fourier transform
- Large boost needed for perturbative matching - how large?

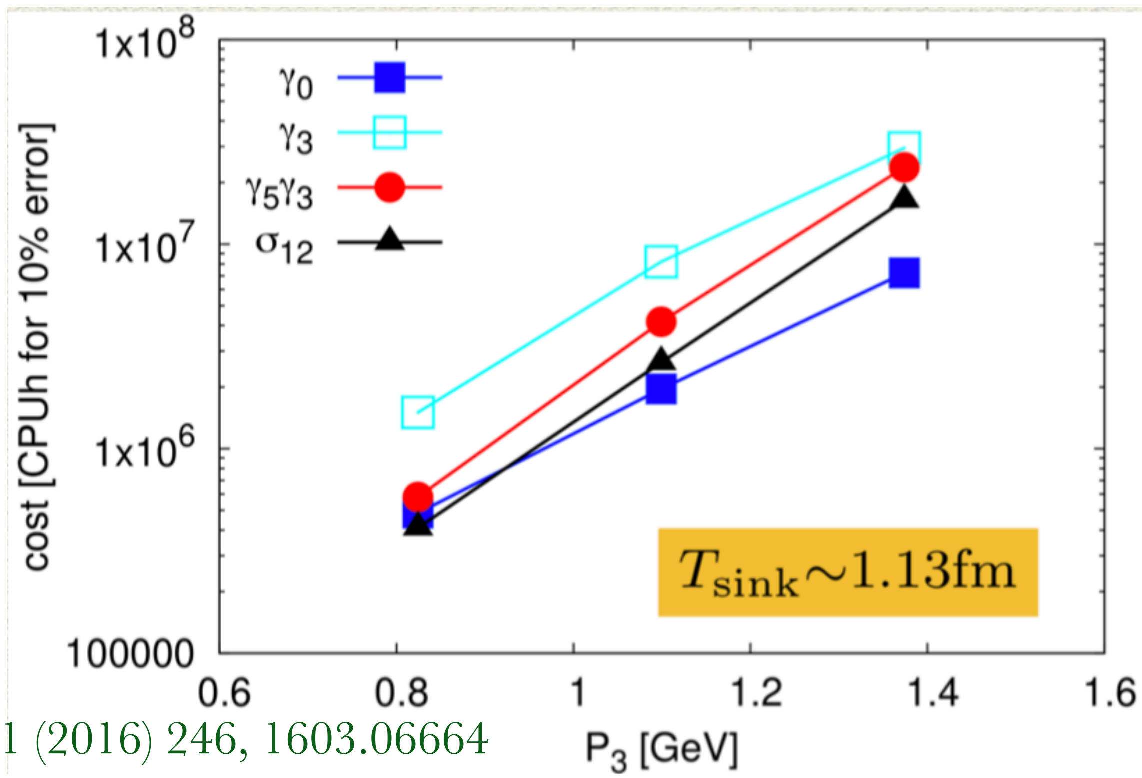
$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y,\mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{\rm QCD}^2}{P_3^2}\right)$$

matching function computed perturbatively

Going to large boost is difficult:

- errors increase rapidly even with momentum smearing which reduces errors by more than 2-order of magnitude
- larger finite a effects
- more severe excited states effects
- see e.g. K. Cichy, Lattice 2021arXiv:2110.07440



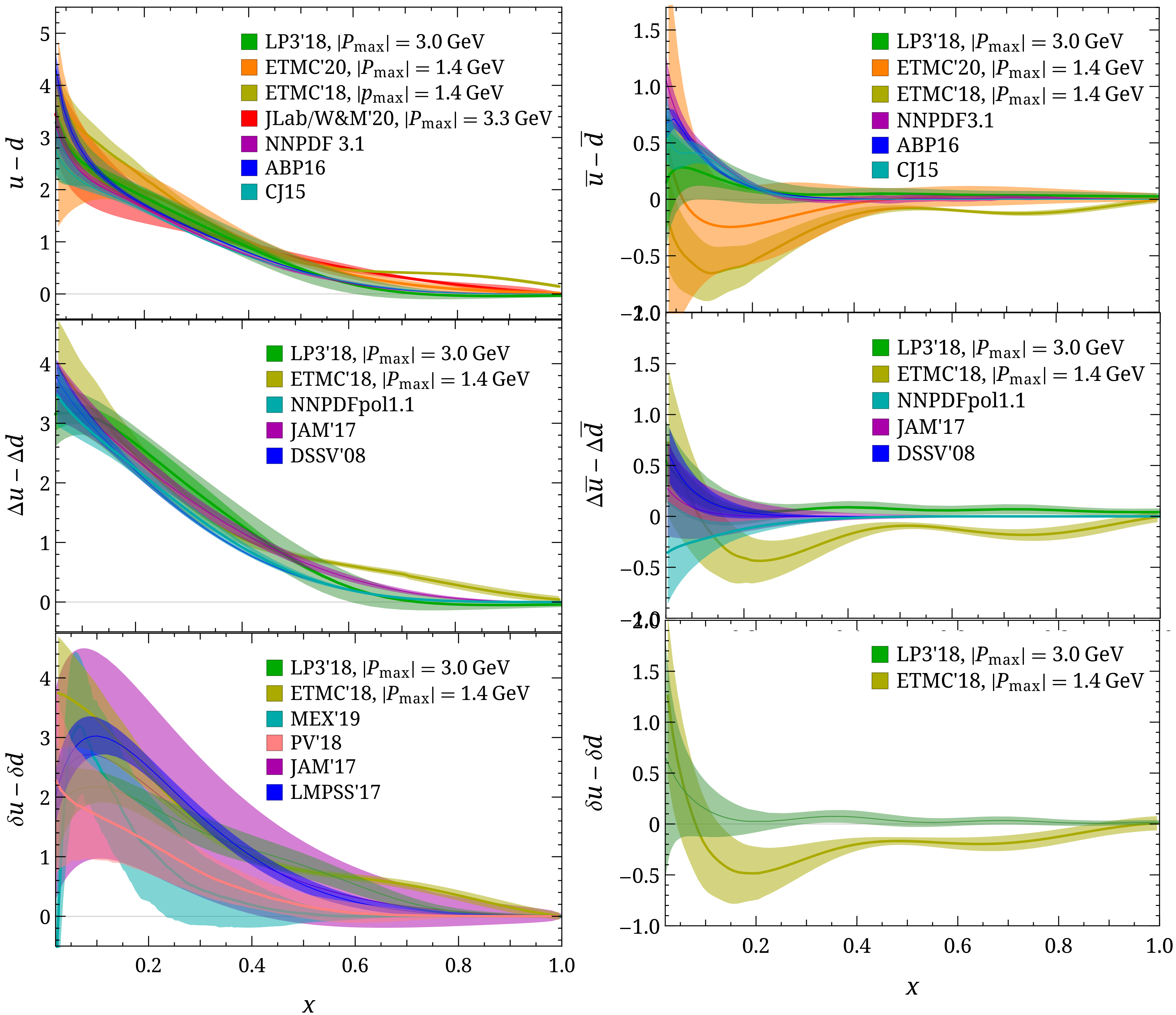


- Nucleon Mass Corrections, J.W. Chen et al., Nucl.Phys. B911 (2016) 246, 1603.06664
- Continuum limit: three studies at heavier than physical pion mass

C. A. *et al* (ETMC), (2020), arXiv:2011.00964
H.-W. Lin, J.-W. Chen, and R. Zhang, (2020), arXiv:2011.14971
K. Zhang *et al.*, (χQCD), (2020) arXiv:2012.05448

Isovector PDFs

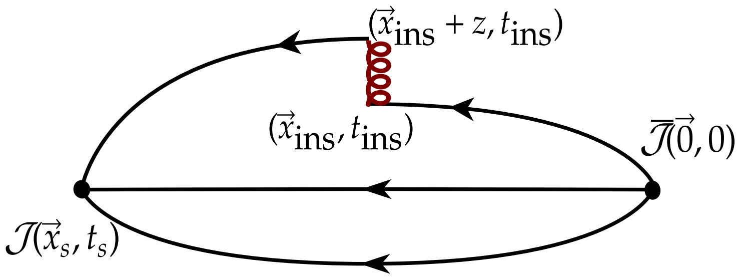
State-of-the-art results



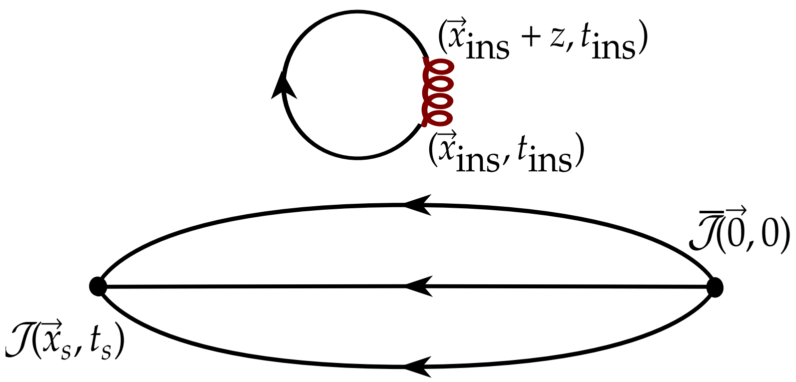
No continuum extrapolation yet

M. Constantinou *et al.* (2020) 2007.08636

Isoscalar and strange PDFs



Connected isoscalar: compute like isovector



$$\mathcal{L}(t_{\rm ins}, z) = \sum_{\vec{x}_{\rm ins}} {\rm Tr}\left[D_q^{-1}(x_{\rm ins}; x_{\rm ins}+z)\gamma^3\gamma^5 W(x_{\rm ins}, x_{\rm ins}+z)\right]$$

Two studies on disconnected with heavier than physical pion mass:

- Mixed action - clover valence on staggered sea, m_π=310 and m_π=690 MeV, only strange

R. Zhang, H.W. Lin, B. Yoon (2020), 2005.011

- Twisted mass fermions

$32^3 \times 64$	a=0.0938(3)(2) fm	$m_N = 1.050(8)$ GeV
$L = 3.0$ fm	$m_\pi \approx 260$ MeV	$m_\pi L \approx 4.0$

C. A., M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Manigrasso (2020), 2009.13061

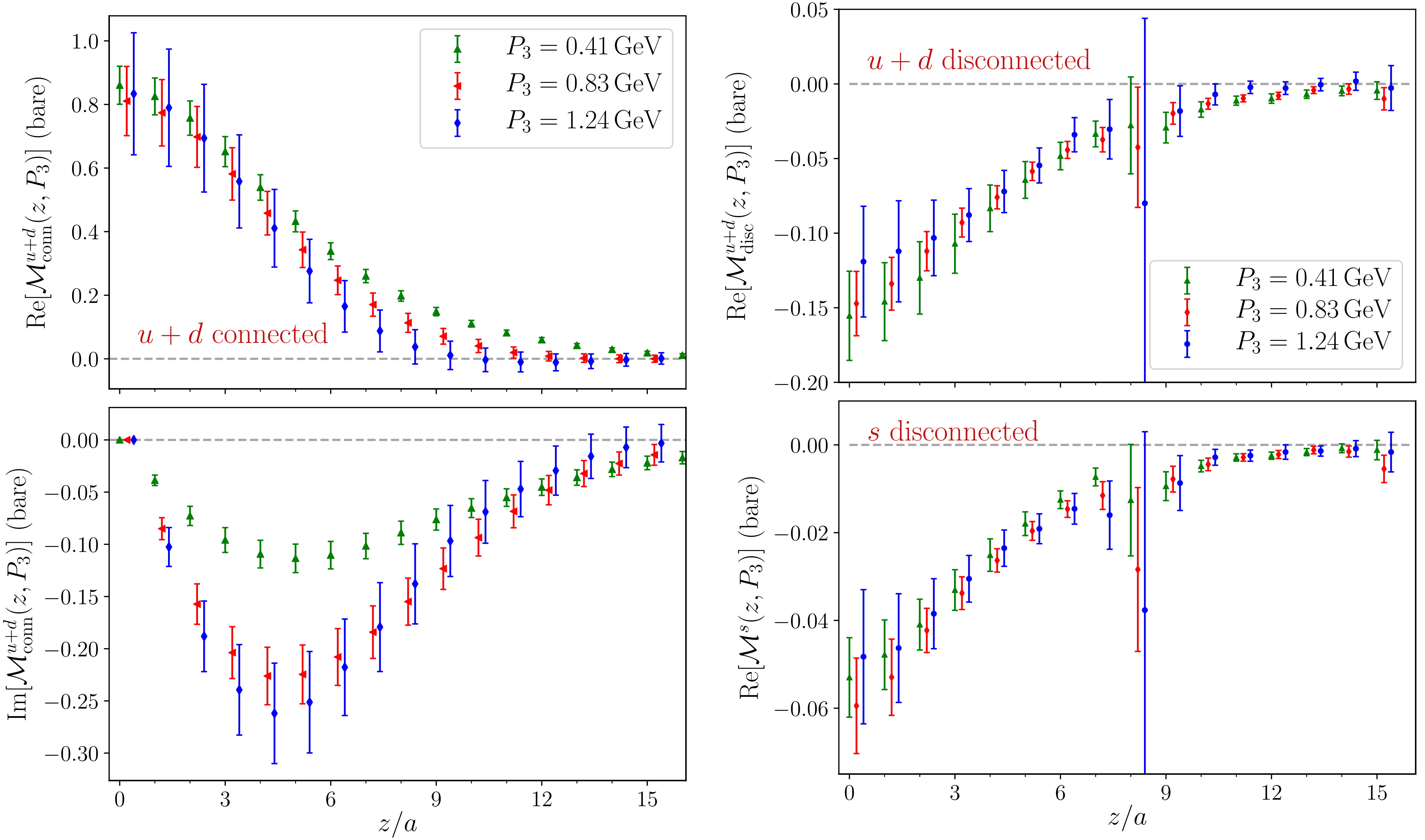
Quark helicity PDFs

$$\mathcal{O}(z) = \overline{\psi}(z)\,\gamma^3\gamma^5 W(0,z)\psi(0)$$

$$\frac{C_{3pt}(t_s, t_{\text{ins}}, P_3)}{C_{2pt}(t_s, P_3)} \xrightarrow{t_{\text{ins}}\gg 1; t_s \gg 1} \mathcal{M}(z, P_3)$$

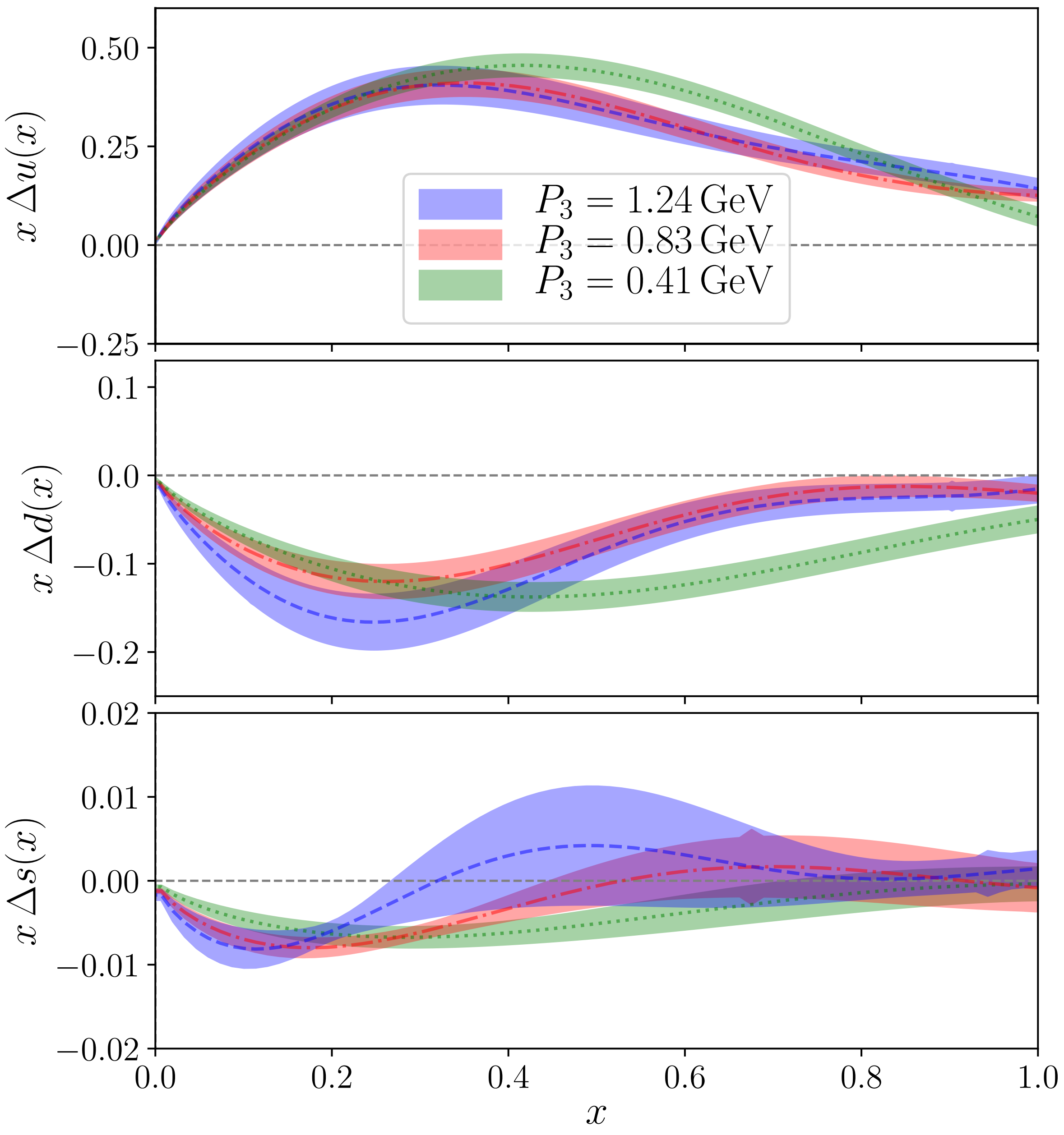
Using distance 8 hierarchal probing (HP)
—> develop asymmetric HP in-direction

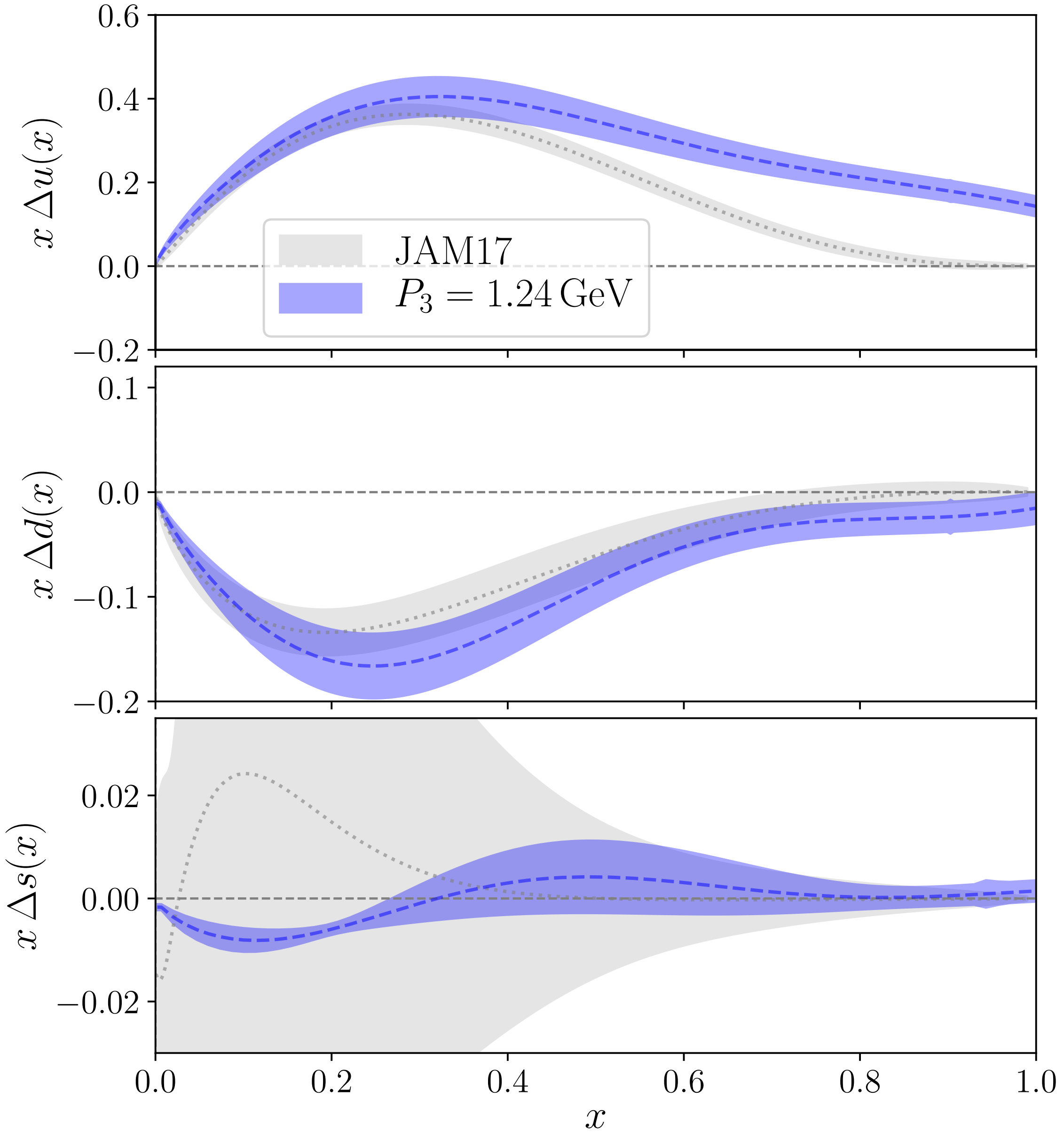
Bare matrix elements



small but clearly non-zero

Helicity distributions





C. A., M. Constantinou, K. Jansen, F. Manigrasso, Phys. Rev. Lett. 126 (2021) 10, 102003, arXiv:2009.13061

- Convergence of results seen as we increase the momentum from P_3=0.83 to P_3=1.24 GeV
- We find agreement between discrete Fourier transform and a Gaussian process regression method for the Fourier transform —> the systematic effect due to the discretization of the Fourier transform is negligible

C.A., G. Iannelli, K. Jansen, F. Manigrasso, Phys. Rev. D 102 (2020) 9, 094508, arXiv:2007.13800

- Computation at the physical point is currently on-going

Generalised parton distributions

- Compute space-like matrix element with different initial and final nucleon boosts

$$h_\Gamma(z,\tilde{\xi},Q^2,P_3) = \langle N(P_3\hat{e}_z + \vec{Q}/2)|\,\overline{\psi}(z)\,\Gamma W(z,0)\,\psi(0)|\,N(P_3\hat{e}_z - \vec{Q}/2)\rangle$$

$$\tilde{\xi} = -\frac{Q_3}{2P_3}\ :\ \mathrm{quasi-skewness} \quad \tilde{\xi} = \xi + \mathcal{O}(\frac{1}{P_3^2})$$

- Rest of the steps are the same as for quasi-PDFs: i.e. renormalise, take the Fourier transform and match

$$\tilde{F}_\Gamma(z,\tilde{\xi},Q^2,P_3,\mu^0,\mu_3^0) = \int_{-1}^{1} \frac{dy}{y} C_\Gamma\left(\frac{x}{y},\frac{\mu}{yP_3},\frac{\mu_3^0}{yP_3},\frac{(\mu^0)^2}{(\mu_3^0)^2}\right) F_\Gamma(y,Q^2,\xi,\mu) + \mathcal{O}\left(\frac{m^2}{P_3^2},\frac{Q^2}{P_3^2},\frac{\Lambda^2_{\mathrm{QCD}}}{x^2P_3^2}\right)$$

RI-scale

$\overline{\mathrm{MS}}$ − scale

Reduces to the matching kernel for ξ=0
Does not depend on Q^2

X.Ji *et al.*, Phys.Rev. D92 (2015) 014039
X.Xiong, J-H. Zhang, Phys.Rev. D92 (2015) 054037
Y-S. Liu *et al.*, Phys.Rev. D100 (2019), 034006

- Two first studies for pion and nucleon GPDs

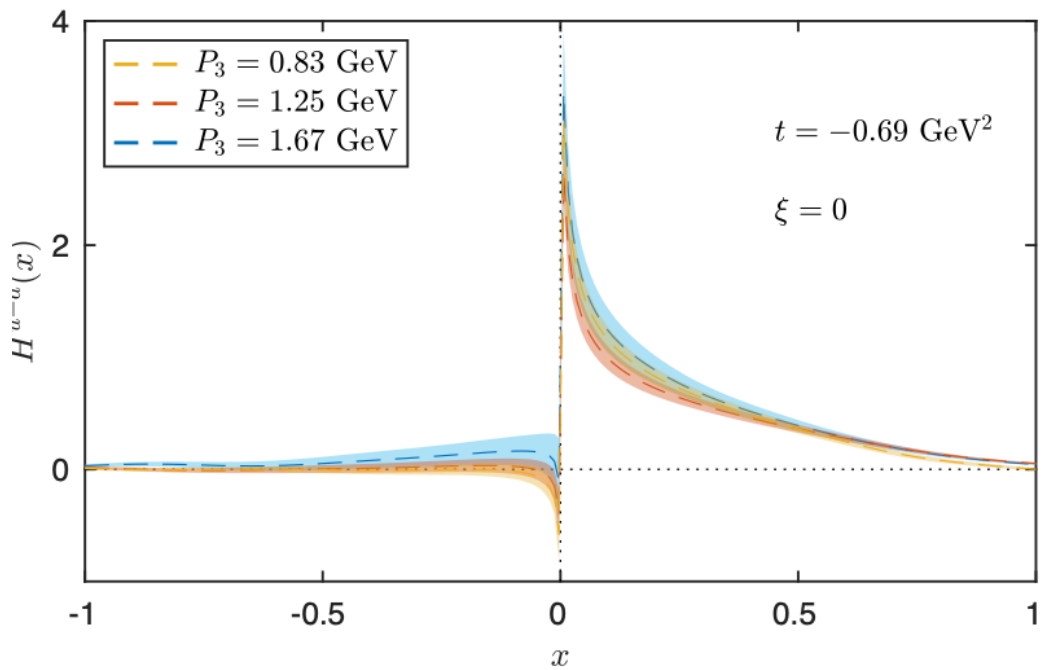
J.W. Chen, H.W. Lin, J.H. Zhang, Nucl. Phys. B 952, 114940 (2020), 1904.12376
C. A. *et al.*, Phys.Rev.Lett. 125 (2020) 26, 262001, 2008.10573

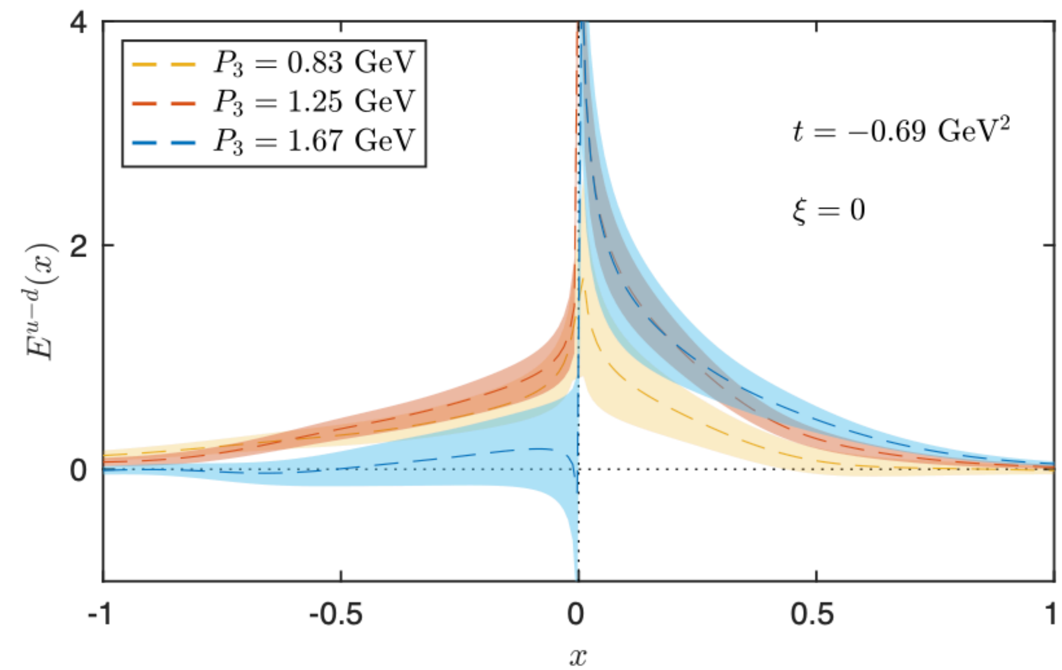
Results on unpolarised and helicity GPDs

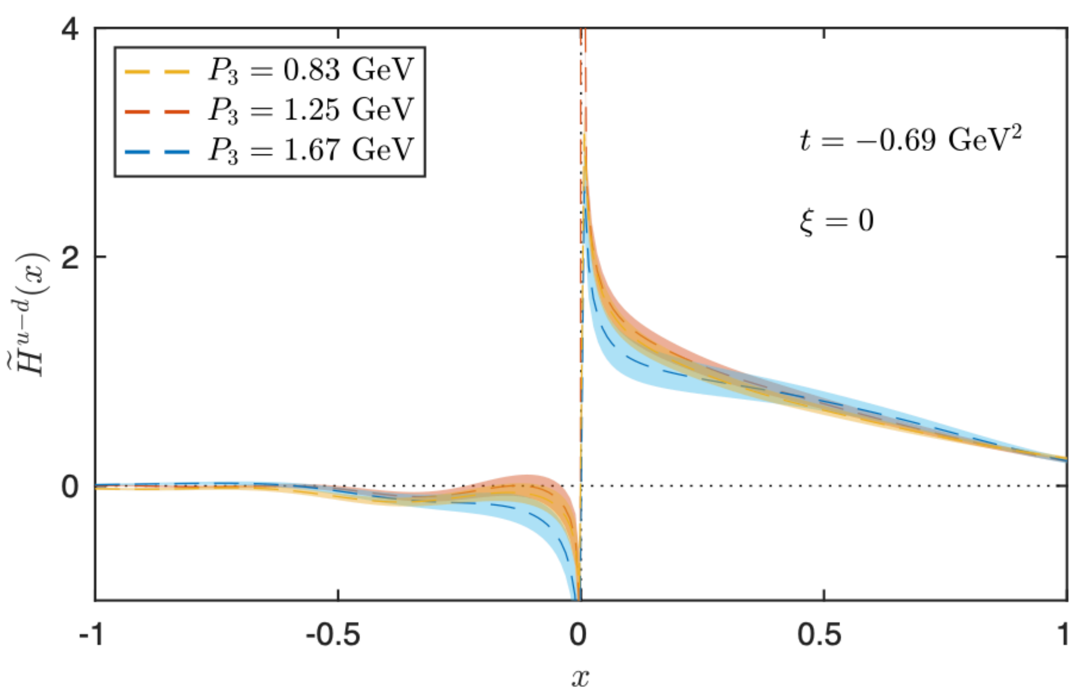
Convergence with momentum

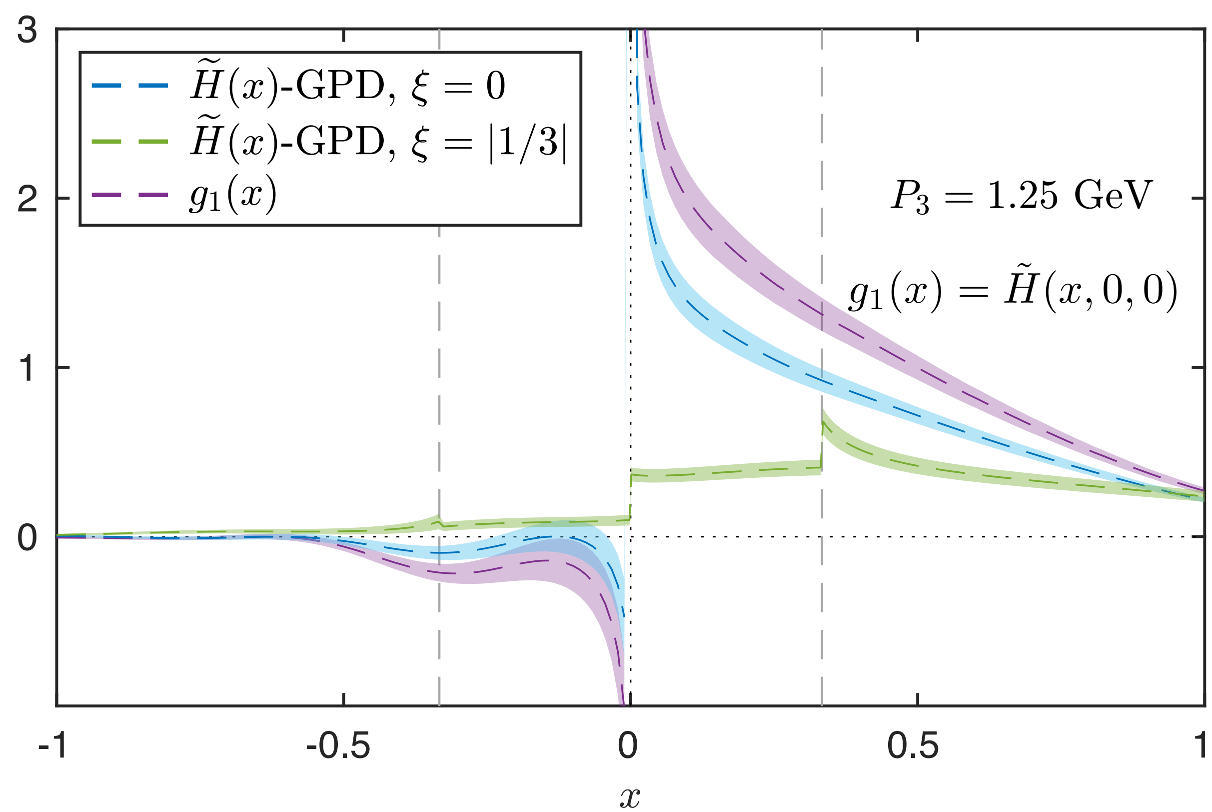
t=-Q²

$32^3 \times 64$	a=0.0938(3)(2) fm	$m_N = 1.050(8)$ GeV
$L = 3.0$ fm	$m_\pi \approx 260$ MeV	$m_\pi L \approx 4.0$









Conclusions

- Moments of PFDs can be extracted precisely (precision era of lattice QCD) - we can extract a lot of interesting physics and also reconstruct the PDFs
- Direct computation of PDFs very promising with calculations using simulations with physical pion mass being carried out for isovector PDFs using various approaches (**quasi-distributions**, pseudo-distributions, current-current correlates, etc)
- The calculation of sea quark contributions is feasible providing valuable input e.g. for the determination of strange helicity
- Exploratory studies of isovector GPDs
- Way forward:
  - ✦ continuum limit, larger boosts, study volume effects
  - ✦ twist-3, TMDs, other hadrons, …

Very much progress over the last five years!!

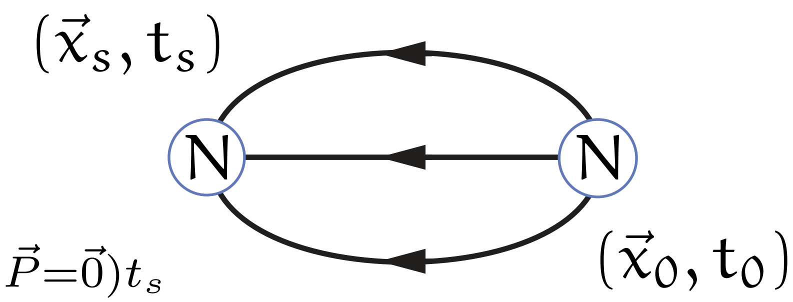
F. Manigrasso

C. Lauer

Extended Twisted Mass Collaboration

Backup slides

Nucleon propagator

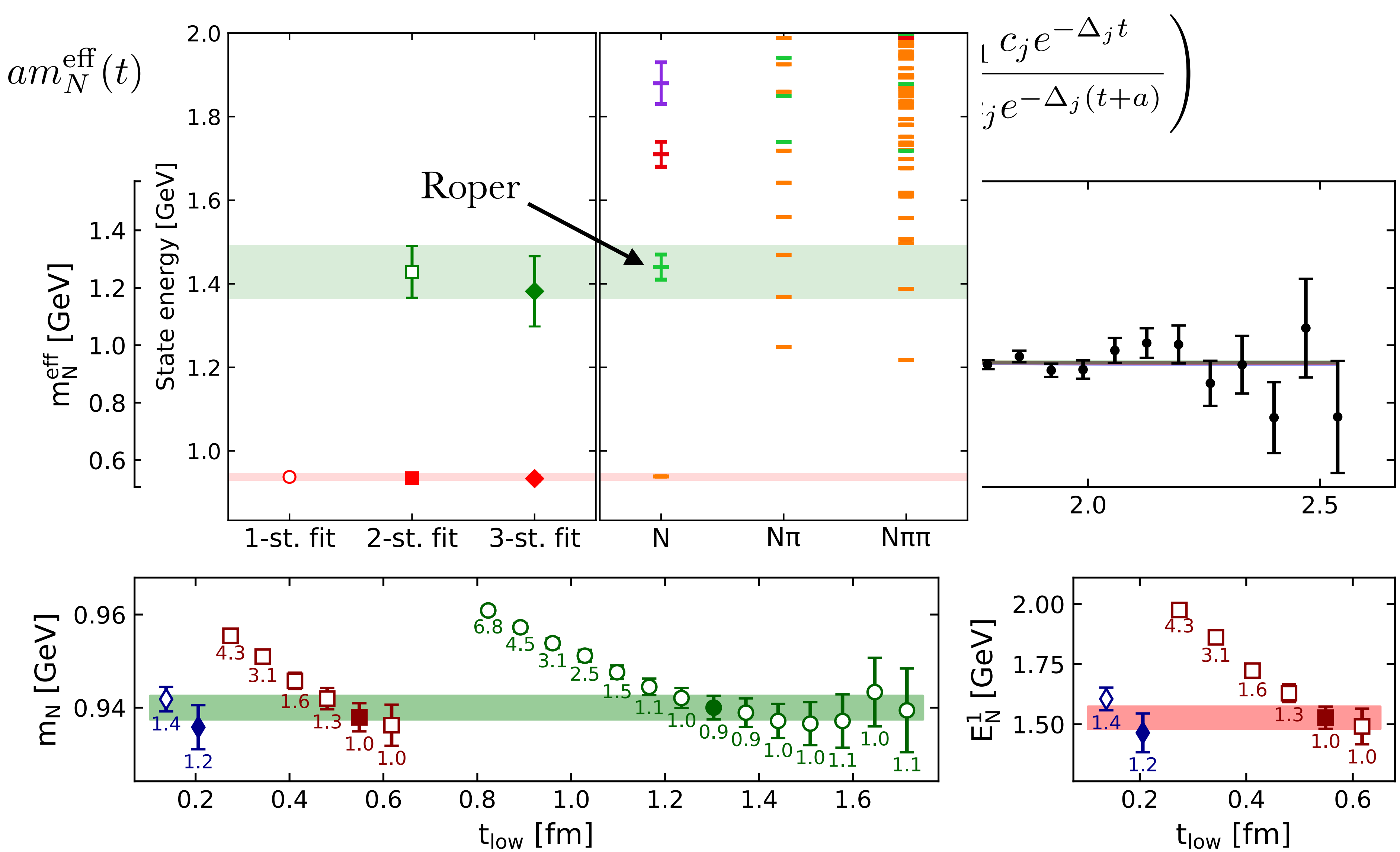


Analysis of two- and three-point functions C_{2pt} and C_{3pt}

$$C_{2\text{pt}}(\Gamma_0;\vec{P}=\vec{0},t_s)=\sum_{\vec{x}_s}\text{Tr}\left[\langle\Gamma_0 J_N(t_s,\vec{x}_s)\bar{J}_N(t_0,\vec{x}_0)\rangle\right]=\sum_{n=0}^{\infty}a_n e^{-E_n(\vec{P}=\vec{0})t_s}$$

$$\xrightarrow{t_s\to\infty} a_0 e^{-m_N t_s}+\mathcal{O}\left(e^{-E_1(\vec{P}=\vec{0})t_s}\right)$$

Fit the nucleon two-point function or effective mass keeping up to two excited states



Renormalisation

- Non-perturbative renormalisation employing the RI' -MOM scheme:

the forward amputated Green function computed in the chiral limit and at a given (large Euclidean) scale $p^2 = \mu^2$ is set equal to its tree-level value.

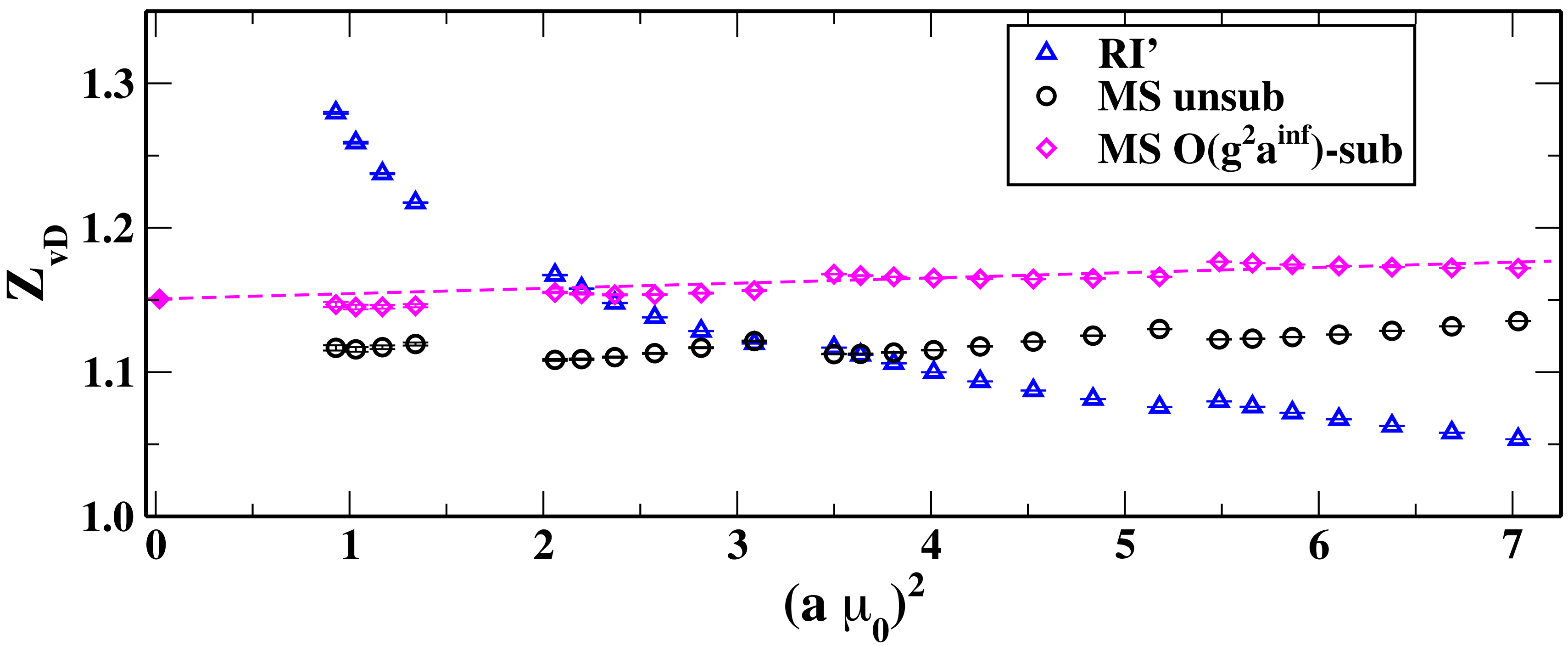
G. Martinelli, C. Pittori, C. T. Sachrajda, M. Testa and A. Vladikas, Nucl. Phys. B 445 (1995) 81, hep-lat/9411010

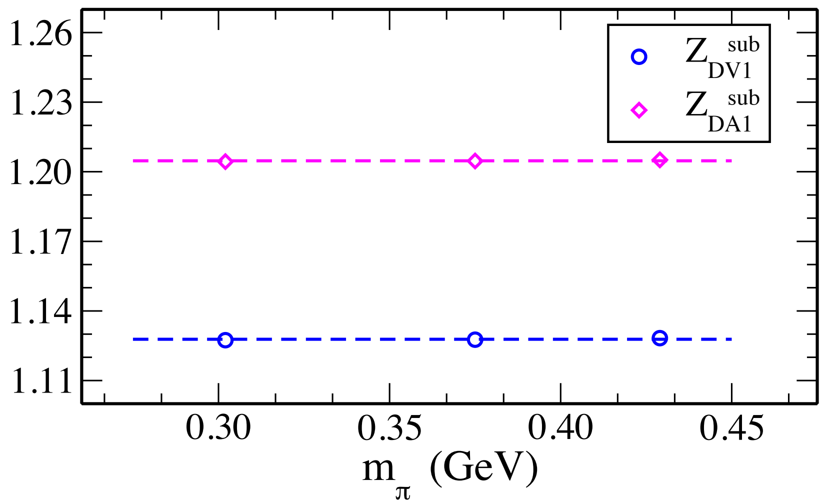
- Use N_f=4 ensembles to take chiral limit - very mild dependence
- Subtract lattice artefacts to $\mathcal{O}(g^2 a^\infty)$ perturbatively
- For scheme dependent operators translate them to the $\overline{\text{MS}}$ scheme at μ =2 GeV using a conversion factor computed in perturbation theory to three-loops

C.A, M. Constantinou, H. Panagopoulos, Phys. Rev. D95, 034505 (2017), 1509.00213

- Momentum source method leads to small statistical errors

M. Gockeler *et al.* (QCDSF) Nucl. Phys. B544, 699 (1999), hep-lat/9807044; Phys. Rev. D 82 (2010) 114511, 1003.5756

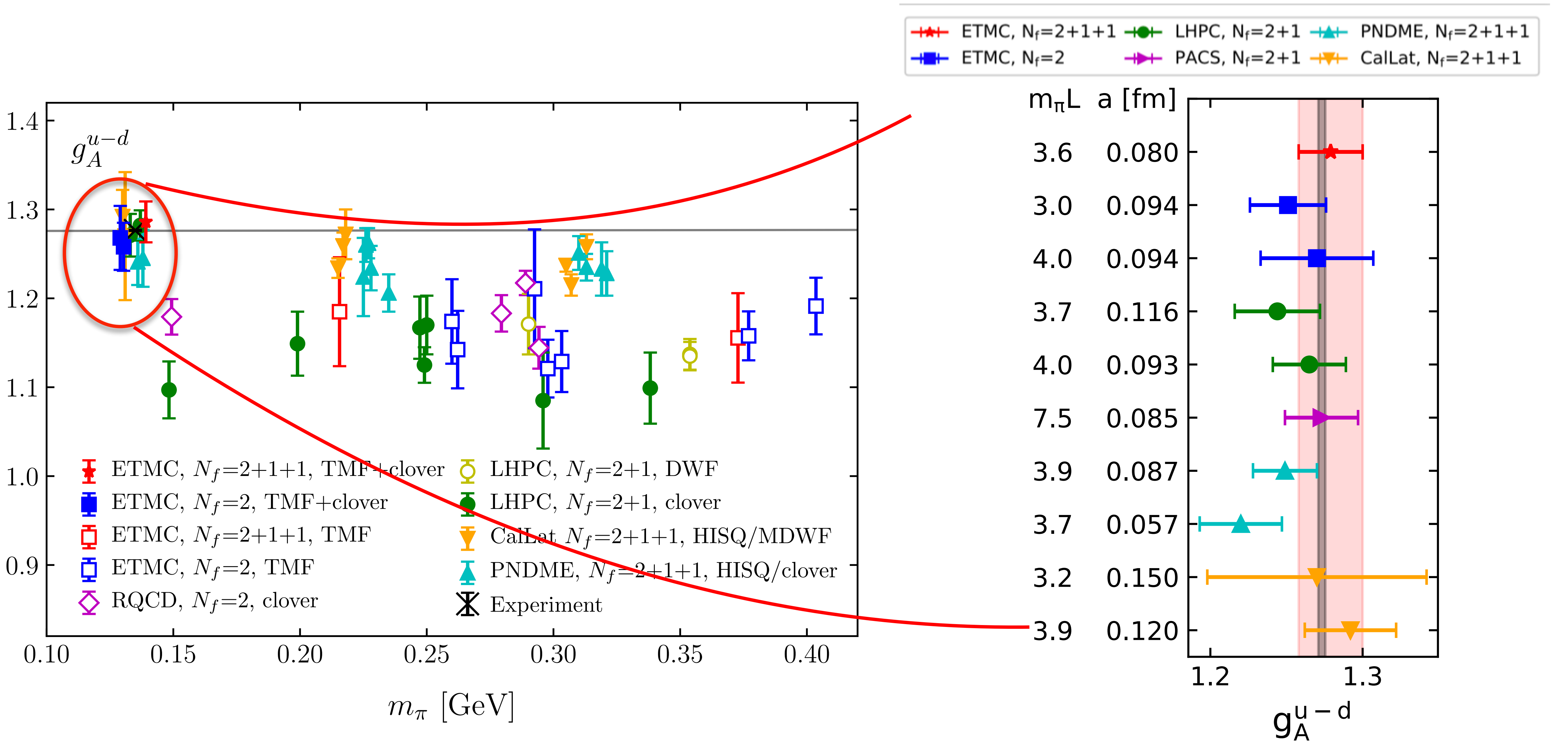




State-of-the-art results on nucleon axial charge

Comparison among lattice collaborations

- A number of calculations at the physical point
- Agreement with experimental value



Benchmark for lattice QCD computations of matrix elements

Other approaches

❋Pseudo-PDFs: use the same hadronic matrix element as quasi-PDFs

A. Radyushkin, Phys. Rev. D96 (2017) 034025
A. Radyushkin, Phys. Rev. D98 (2018) 014019

$$h(z, P_3) = \langle P_3 | \overline{\psi}(0)\, \Gamma W(0, z)\, \psi(z) | P_3 \rangle$$

- Express in terms of the Ioffe time $\nu = zP_3$ and z^2 : $h(z, P_3) \to \bar{h}(\nu, z^2)$

- Renormalzation cancels in the ratio: $\mathcal{M}(\nu, z^2) = \dfrac{\bar{h}(\nu, z^2)}{\bar{h}(0, z^2)}$

→ Related to PDFs via the Ioffe time PDF, $Q(\nu,\mu^2)$

$$\mathcal{M}(\nu, z^2) = \mathcal{Q}(\nu, \mu^2) + \mathcal{O}(z^2)$$

→ $q(x, \mu^2) = \int \frac{d\nu}{2\pi}\, e^{-ix\nu}\, \mathcal{Q}(\nu, \mu^2)$

Need large Ioffe times so large momentum

❋Current-current corelaltors

A. J. Chambers et al. (QCDSF) Phys. Rev. Lett. 118 (2017) 2420 , arXiv:1703.01153

- Compute Compton amplitude: $T_{\mu\nu}(p, q, s) = \int d^4 z\, e^{iqz} \langle p, s | \mathcal{T}\, J_\mu(z) J_\nu(0) | p, s \rangle$
- Use Feynman-Hellmann approach: $\mathcal{L} \to \mathcal{L} + \lambda J_3(x), \quad J_3 = Z_V cos(\vec{q}.\vec{x})\bar{\psi}\gamma_3\psi$
- Take the second derivative of the nucleon two-point function to obtain T_{33}

→ Extract unpolarize structure function F_1

Gaussian Process Regression (GPR)

- We construct a GPR with the following characteristics:
  - the interpolation is **non-parametric,** so is not restricted to a specific parametrized function
  - it is based on Bayesian inference so the information on the asymptotic behavior of the function can be incorporated into the **prior distribution**
  - **the uncertainties of the measurements** are incorporated into the interpolation through Bayes theorem
  - it is possible to **impose a chosen level of smoothness** to the interpolating function
  - the result of the interpolation is continuous, defined over whole domain of interest and its F**ourier transform is computable in closed form**

- Choice of regression function

$$\mu(z) = \mu_P(z) + \sum_{ij} k_P(z, z_i) \tilde{K}(z_i, z_j)^{-1} (h(z_j) - \mu_P(z_j))$$

$$\tilde{K}(z_i, z_j) = k_P(z_i, z_j) + \Delta h_i^2 \delta_{ij}$$

Encodes prior knowledge, if no prior $\mu_P(z)=0$

Correlation between h(z) and h(z'): $k_P(z, z') = \sigma^2 \exp\left(-\frac{(z-z')^2}{2\ell^2}\right)$

Hyperparameters determined via maximum likehood estimation

- The resulting Gaussian process posterior $\mu(z)$ is infinitely mean-square differentiable with our choices of $\mu_P(z)$ and $k_P(z,z')$