Theory of muon g-2

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Based on: 2006.04822(WP Theory Initiative) and references in the executive summary therein

SPIN 2021

18/10/2021

Theory initiative: prepare the Standard Model prediction for $(g\text{-}2)_{\mu}$

PAST WORKSHOPS

Fourth Plenary Workshop of the Muon g-2 Theory Initiative
Fourth Plenary Workshop of the Muon g-2 Theory Initiative
A virtual workshop hosted by KEK (Tsukuba, Japan), held on 28 June - 02 July 2021.

The hadronic vacuum polarization from lattice QCD at high precision
https://indico.cern.ch/event/956699/
A virtual topical workshop of the Muon g-2 Theory Initiative, 16-20 Nov 2020.

Hadronic contributions to (g-2) **μ**
https://indico.fnal.gov/event/21626/
held at the Institute for Nuclear Theory, University of Washington, Seattle, WA, 9-13 September 2019

Second workshop of the Muon g-2 Theory Initiative
https://wwwth.kph.uni-mainz.de/g-2/
held at the Helmholtz Institute Mainz, University of Mainz, Mainz, Germany, 18-22 June 2018

Muon g-2 Theory Initiative Hadronic Light-by-Light working group workshop
https://indico.phys.uconn.edu/event/1/
held at the University of Connecticut, Storrs, CT, 12-14 March 2018

Workshop on Hadronic Vacuum Polarization Contributions to Muon g-2
https://www-conf.kek.jp/muonHVPws/
held at KEK, Tsukuba, Japan, 12-14 Feb 2018

First workshop of the Muon g-2 Theory Initiative
https://indico.fnal.gov/event/13795/
held in St. Charles, IL, USA, 3-6 June 2017

Put together in a *coherent & conservative* way the results of various groups, *before the Fermilab result*

https://muon-gm2-theory.illinois.edu

White Paper: arXiv:2006.04822 (Phys. Rept. 887 (2020) 1-166)

Next workshop: Higgscentre (Edinburgh), 5-9 September 2022

Lepton Magnetic Anomaly: from Dirac to QED

- Magnetic dipole moment of a charged lepton: $\vec{\mu} = g \frac{e}{2m} \vec{s}$

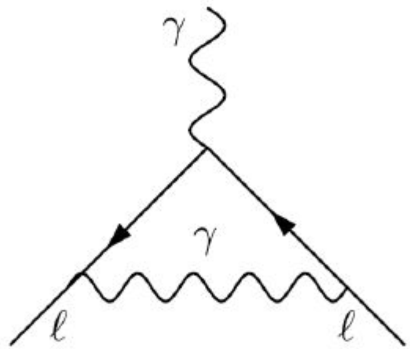
  Dirac (1928) $g_e=2$ $a_e=0$

- "anomaly" = deviation w.r.t. Dirac's prediction: $a = \frac{g-2}{2}$

anomaly discovered:
Kusch-Foley (1948) $a_e = (1.19 \pm 0.05)\ 10^{-3}$

and explained by $O(\alpha)$ QED contribution:
Schwinger (1948) $a_e = \alpha/2\pi = 1.16\ 10^{-3}$

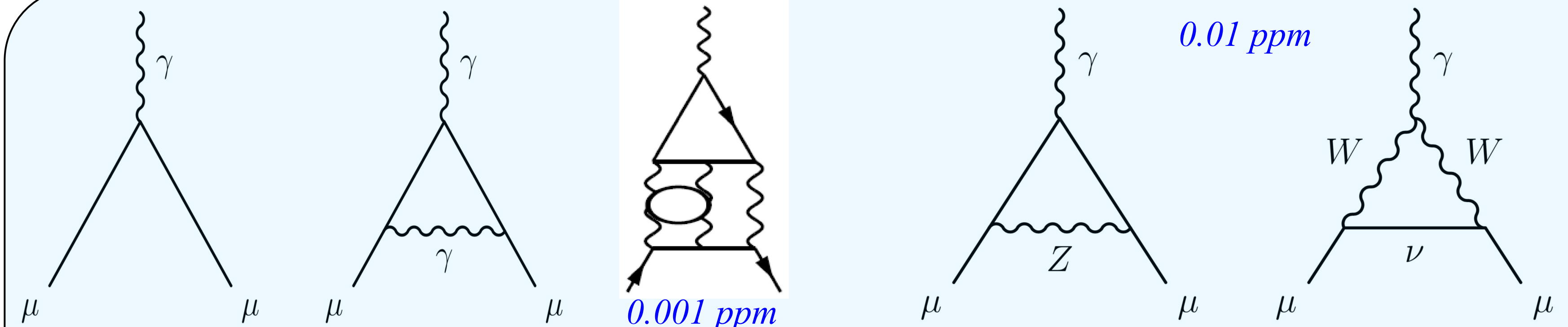
first triumph of QED



$\Rightarrow$ a_e sensitive to quantum fluctuations of fields

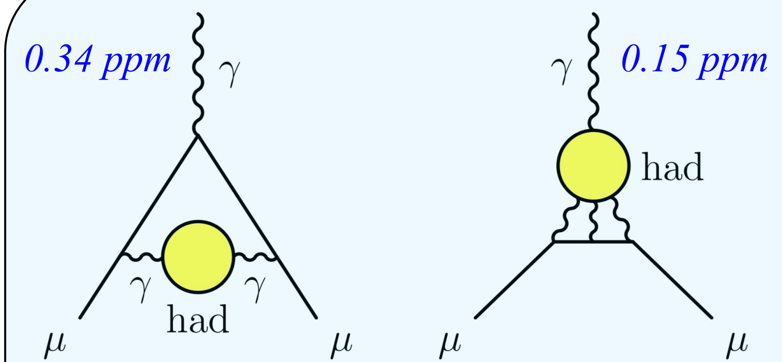
More Quantum Fluctuations

Why is it (so) complicated to compute one number ? (*very precisely*)

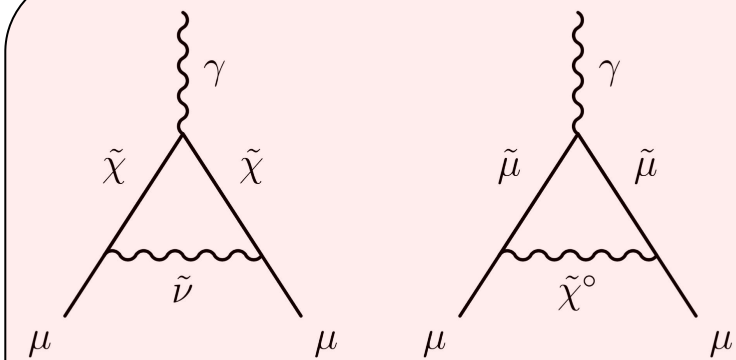


+ Many other diagrams at higher orders…

QED up to O(α^5) (Kinoshita et al.)



Dominant uncertainties: non-perturbative...



? ? ?

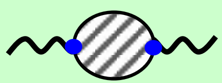
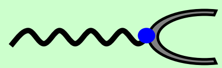
Hadronic Vacuum Polarization and Muon $(g\text{-}2)_\mu$

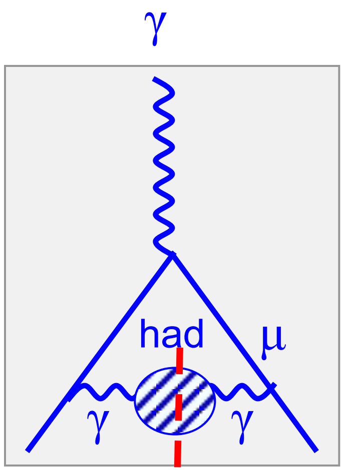
Dominant uncertainty for the theoretical prediction: from lowest-order HVP piece

Cannot be calculated from QCD (low mass scale), but one can use experimental data on $e^+e^- \to$ hadrons cross section

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

$$12\pi\ \mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0\ [e^+e^- \to \text{hadrons}\ (\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$

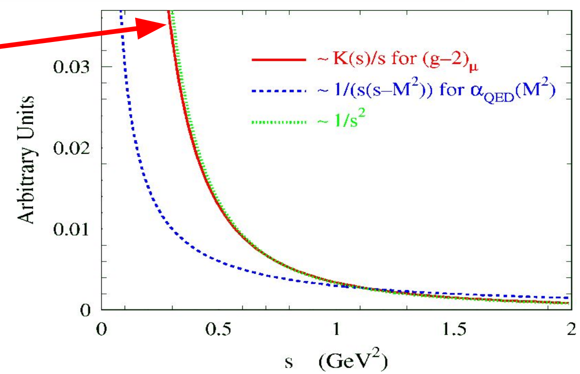
Im[] ∝ |  hadrons $|^2$



$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

Dispersion relation

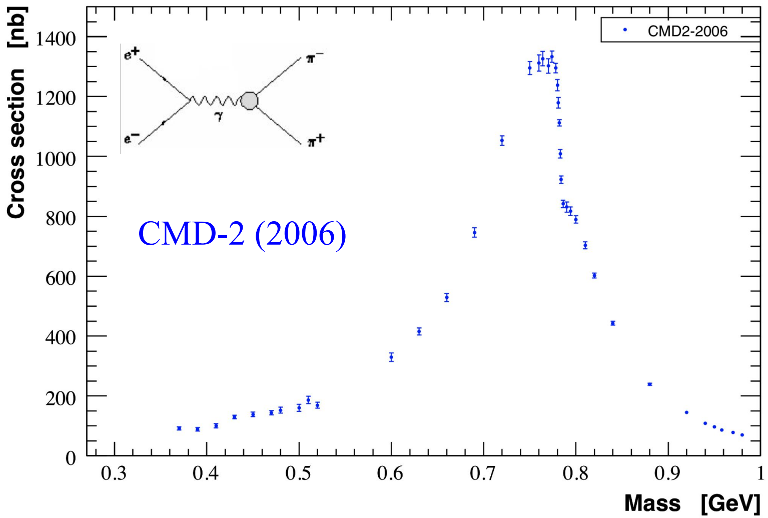
Bouchiat and Michel, 1961

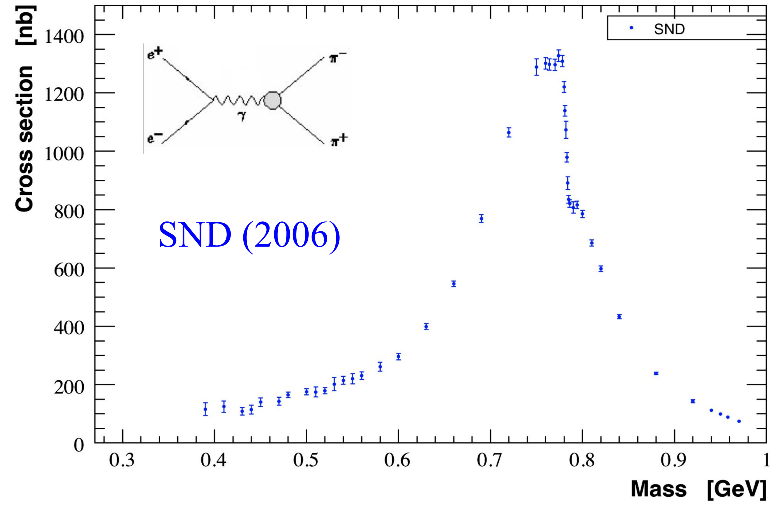


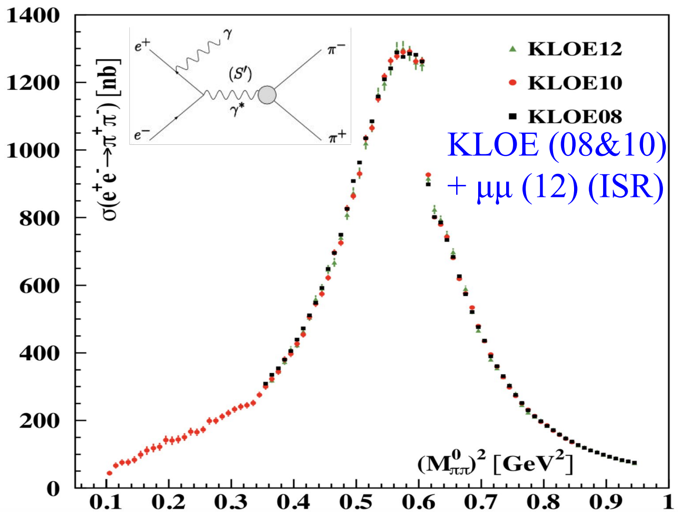
→ Precise $\sigma(e^+e^- \to \text{hadrons})$ measurements at low energy are very important

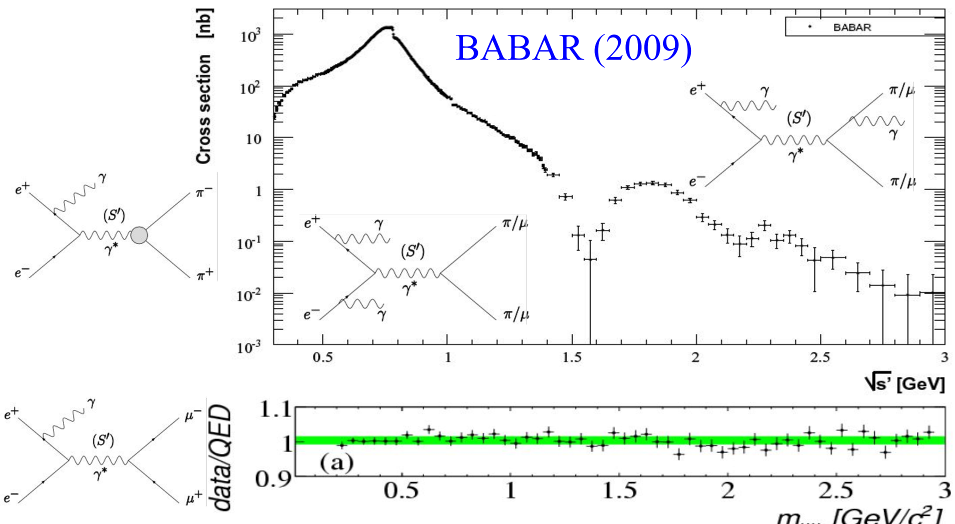
→ Do not use hadronic τ decays data anymore (less precise + theory uncertainties)

HVP: Data on $e^+e^- \rightarrow$ hadrons

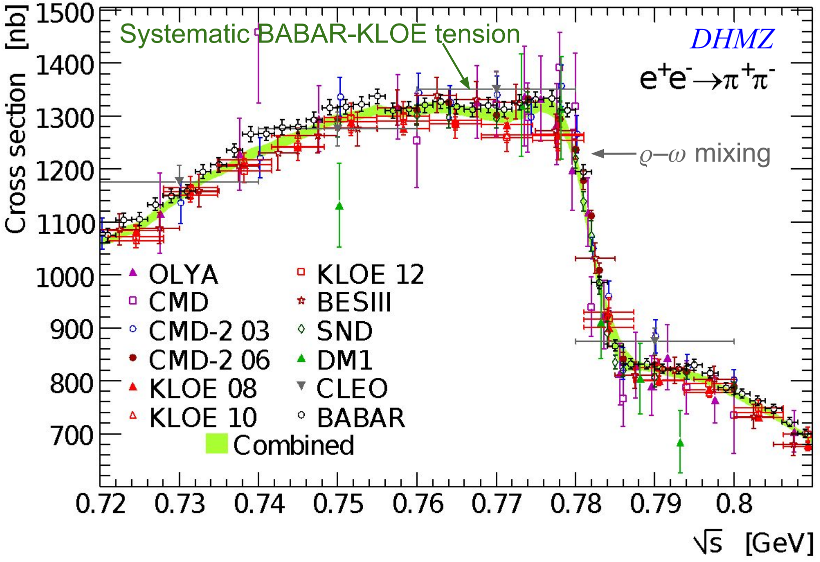


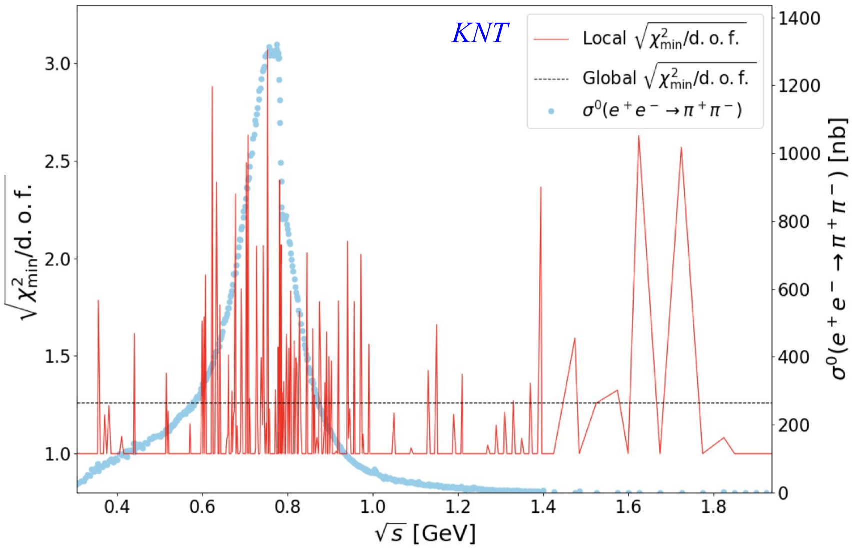


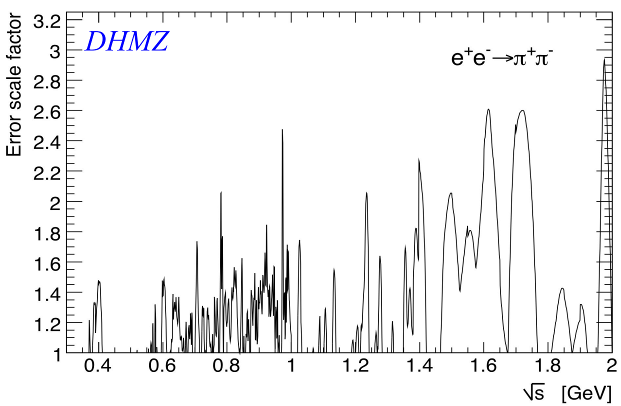




Combination (for the $e^+e^- \to \pi^+\pi^-$ channel, plus other 31 channels)





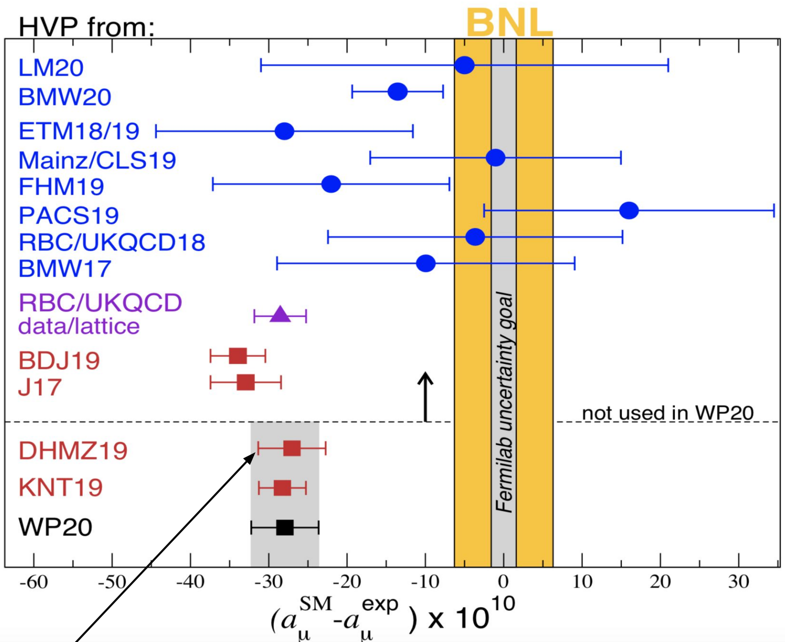


→ Local tensions taken into account by both DHMZ & KNT, but this is not sufficient

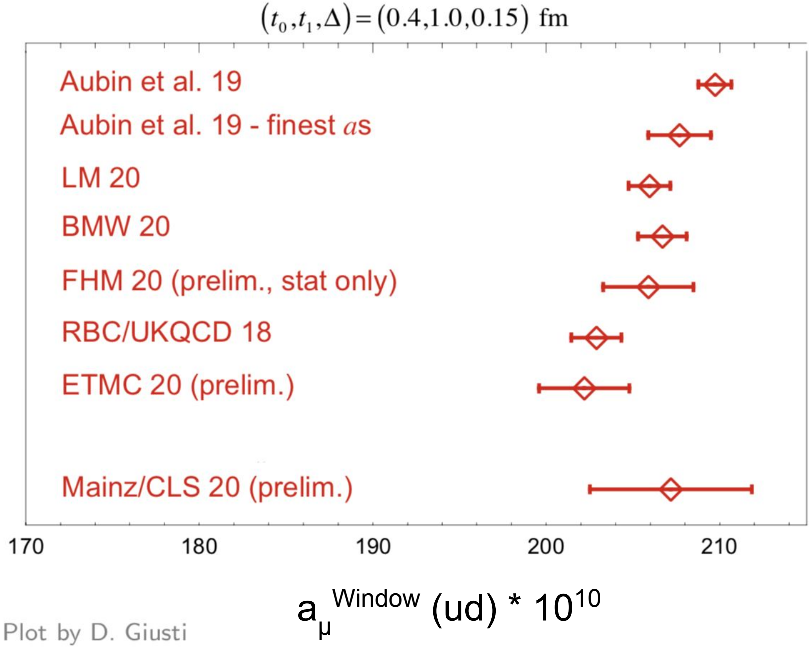
→ Systematic from (BABAR-KLOE) tension accounted for by DHMZ: 2.8×10^{-10} (dominant uncertainty - half of the difference between integrals w/o BABAR or KLOE)

→ DHMZ also accounts for correlations between different channels (in addition to correlations between bins/points and between experiments)

Status of a_μ (HVP)



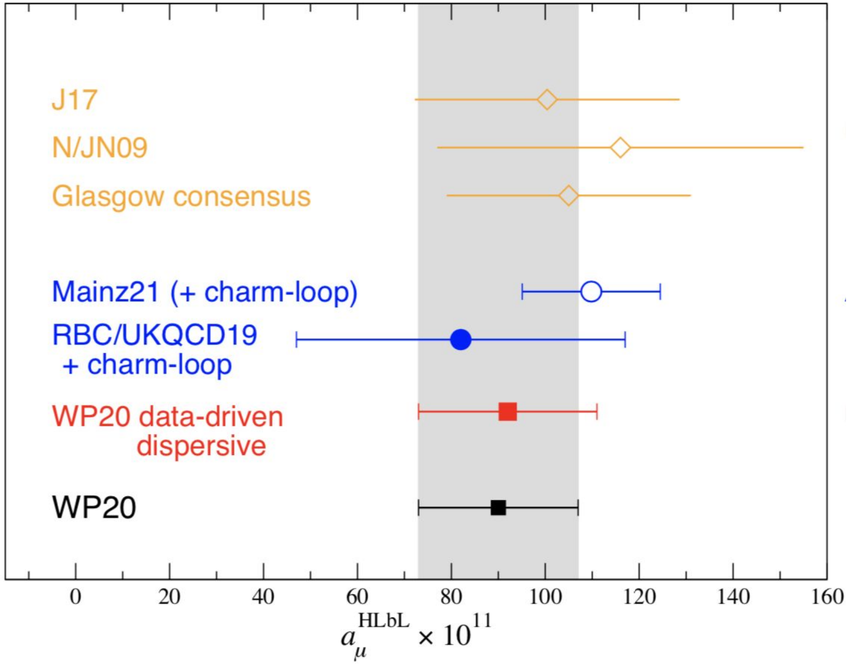
Important to account for BABAR-KLOE diff. & inter-channel correlations



→ *HVP(WP20)*: *Merging* of model independent results: DHMZ and KNT (and CHHKS for $\pi^+\pi^-$ & $\pi^+\pi^-\pi^0$) Central value from simple average; BABAR-KLOE tension & correlations between channels from DHMZ; Max(DHMZ & KNT uncertainties) in each channel

→ Excellent progress on the Lattice QCD (+QED) calculations; *Precision of BMW20* (*to be cross-checked* by other lattice groups) became *similar to the one of dispersive approaches*; *Ongoing cross-checks* using Euclidean time windows (related to HVP with suppression of very low and high energies) for which various groups achieved similar precision; *If BMW20 result is confirmed, the difference w.r.t. dispersive results* to be understood.

Status of a_μ (Hadronic Light-by-Light)



Hadronic model + pQCD

Lattice QCD+QED

Data-driven

→ Good progress on systematically improvable approaches

→ The g-2 Theory Initiative provides an adequate environment for cross-checks

→ Uncertainty controlled at 0.15ppm

Theory initiative white paper executive summary & new results

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

→ Dominant uncertainty: HVP LO → Based on *merging of model-independent methods*

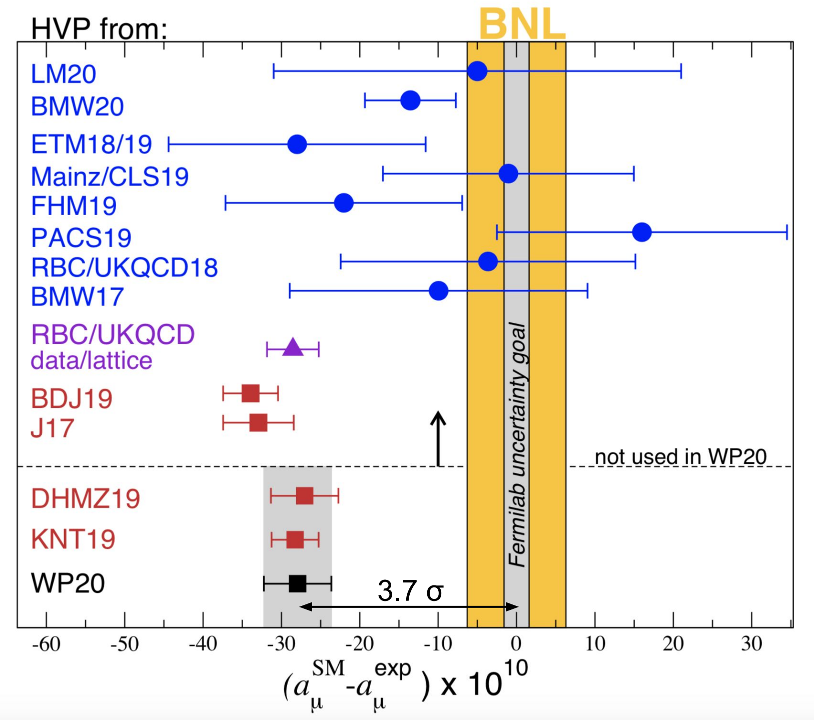
→ HLbL also has an important uncertainty

→ Lattice results become more and more interesting

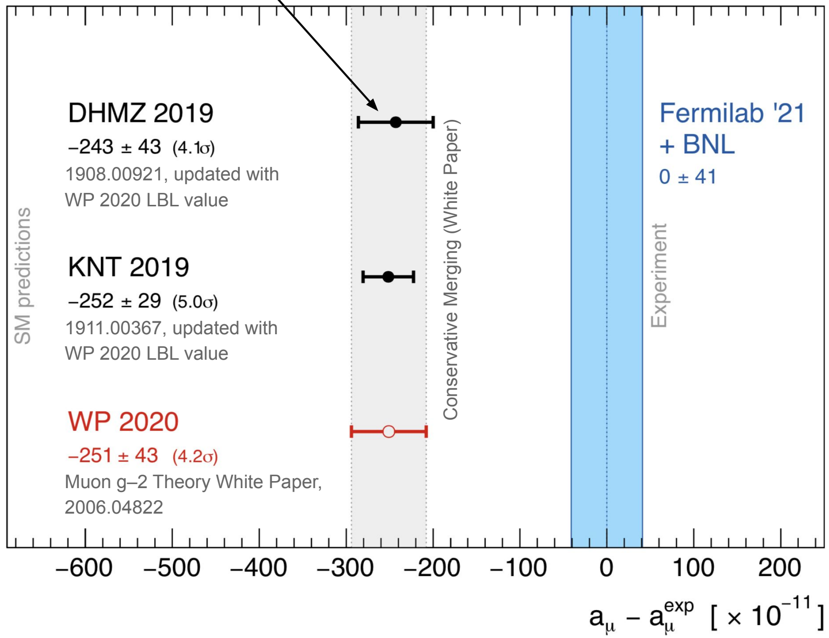
→ A tension between the BNL measurement and the reference SM prediction: ~ 3.7 σ (~ 4.2 σ including FNAL)

→ Tension significantly smaller when using BMW20 for the LO HVP (TBC by other lattice groups)

Status of a_μ before/with Fermilab result



Important to account for BABAR-KLOE diff. & inter-channel correlations



→ Caution about significance:

- statistics-dominated measurement
- prediction uncertainty limited by non-Gaussian systematic effects

→ Nevertheless, large discrepancy between measurement and reference SM prediction (to be significantly improved in view of the forthcoming updates of the Fermilab measurement)

We have an interesting, long standing, multifaceted problem to solve...

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From executive summary of White Paper: arXiv:2006.04822 (Phys. Rept. 887 (2020) 1-166)

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