The deconvolution problem of deeply virtual Compton scattering

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Overview

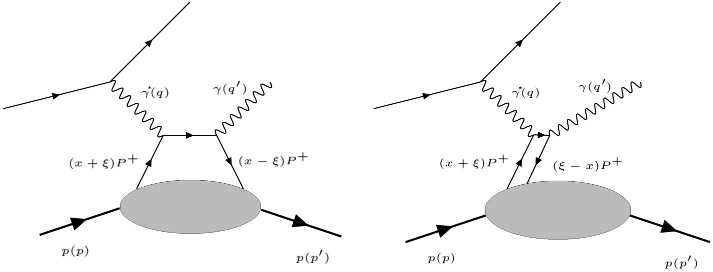
1. Deeply virtual Compton scattering and generalized parton distributions
2. Warming-up: extraction of gravitational form factors
3. Deconvoluting a Compton form factor: Shadow GPDs
4. Perspectives

1. Deeply virtual Compton scattering and generalized parton distributions

DVCS and generalized parton distributions

DVCS is the scattering of a lepton on a hadron via a photon of large virtuality, producing a real photon in the final state. It is an **exclusive process** with an intact recoil proton.

- x is the average light-front plus-momentum (longitudinal momentum in a fast moving hadron) fraction of the struck parton
- ξ describes the light-front plus-momentum transfer, linked to Björken's variable x_B
- $t = \Delta^2$ is the total four-momentum transfer squared



GPDs were introduced more than two decades ago in [Müller *et al*, 1994], [Radyushkin, 1996] and [Ji, 1997].

Tree-level depiction of DVCS for $x > |\xi|$ (left) and $\xi > |x|$ (right)

DVCS and generalized parton distributions

- For a large photon virtuality $Q^2 = -q^2$, finite x_B and small total four-momentum transfer squared t, **factorisation theorems** describe the amplitude of DVCS, parametrised by **Compton form factors** (CFFs) $\mathcal{F}$, as convolutions of perturbative **coefficient functions** T^a and non-perturbative **generalised parton distributions (GPDs)** F^a:

CFF convolution (leading twist) [Radyushkin, 1997], [Ji, Osborne, 1998], [Collins, Freund, 1999]

$$\mathcal{F}(\xi, t, Q^2) = \sum_{\text{parton type } a} \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^a\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) F^a(x, \xi, t, \mu^2) \quad (1)$$

$F^a(x, \xi, t, \mu^2) \to F^g(x, \xi, t, \mu^2)/x$ for the usual definition of gluon GPD

μ is the factorisation / renormalisation scale, α_s the strong coupling.

DVCS and generalized parton distributions

- The **forward limit** gives back the PDF:

$$H^q(x, \xi = 0, t = 0, \mu^2) = f^q(x, \mu^2) \tag{2}$$

- **Polynomiality property**: [Ji, 1998], [Radyushkin, 1999] due to Lorentz covariance,

$$\int_{-1}^{1} \mathrm{d}x \, x^n H^q(x, \xi, t, \mu^2) = \sum_{k=0 \text{ even}}^{n+1} H^q_{n,k}(t, \mu^2) \xi^k \tag{3}$$

This property implies that the GPD is the Radon transform of a **double distribution** F^q (DD) with an added D-term on the support $\Omega = \{(\beta, \alpha) \mid |\beta| + |\alpha| \leq 1\}$:

Double distribution formalism [Radyushkin, 1997], [Polyakov, Weiss, 1999]

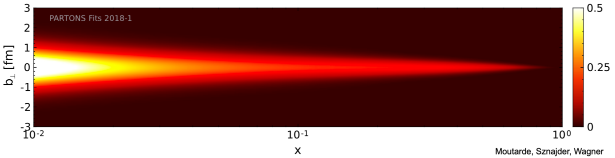
$$H^q(x, \xi, t, \mu^2) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \, \delta(x - \beta - \alpha\xi) \left[F^q(\beta, \alpha, t, \mu^2) + \xi \delta(\beta) D^q(\alpha, t, \mu^2)\right] \tag{4}$$

DVCS and generalized parton distributions

Impact parameter distribution (IPD) [Burkardt, 2000]

$$I_a(x, \mathbf{b}_\perp, \mu^2) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} F^a(x, 0, t = -\Delta_\perp^2, \mu^2) \quad (5)$$

is the density of partons with plus-momentum x and transverse position $\mathbf{b}_\perp$ from the center of plus momentum in a hadron $\rightarrow$ **hadron tomography**



Density of up quarks (valence GPD) in an unpolarized proton from a parametric fit to DVCS data in the PARTONS framework [Moutarde *et al*, 2018].

2. Warming-up: extraction of gravitational form factors from experimental data

Extraction of GFFs

- The energy-momentum tensor (EMT) is parametrised in terms of **gravitational form factors (GFFs)**, which can remarkably be accessed from GPDs. We focus on the GFF $C_q(t, \mu^2)$ since it only depends on the D-term thanks to the **polynomiality property** via

$$\int_{-1}^{1} \mathrm{d}z\, z D^q(z, t, \mu^2) = 4 C_q(t, \mu^2) \tag{6}$$

- The experimental data is sensitive to the D-term through the **subtraction constant** defined by the **dispersion relation** (see *e.g.* [Diehl, Ivanov, 2007])

Dispersion relation

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\,\mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi' \,\mathrm{Im}\,\mathcal{H}(\xi', t, Q^2)\left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right) \tag{7}$$

$$= \frac{2}{\pi} \sum_{\text{parton type } a} \int_1^{\infty} \mathrm{d}\omega \,\mathrm{Im}\, T^a\left(\omega, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \int_{-1}^{1} \mathrm{d}\alpha\, \frac{D^a(\alpha, t, \mu^2)}{\omega - \alpha} \tag{8}$$

Extraction of GFFs

- For instance, in a LO study, the quark contribution to the subtraction constant reads

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} \quad \text{but we are interested in} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) \tag{9}$$

- This is a prototype of the more complicated GPD extraction problem we will face later on. The known solution is through evolution.
- Let's expand the D-term on a basis of Gegenbauer polynomials

$$D^q(z,t,\mu^2) = (1-z^2) \sum_{\text{odd } n} d_n^q(t,\mu^2) \, C_n^{3/2}(z) \tag{10}$$

Then

GFF C_a extraction

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \quad \text{and} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) = \frac{4}{5} \, d_1(t,\mu^2) \tag{11}$$

Extraction of GFFs

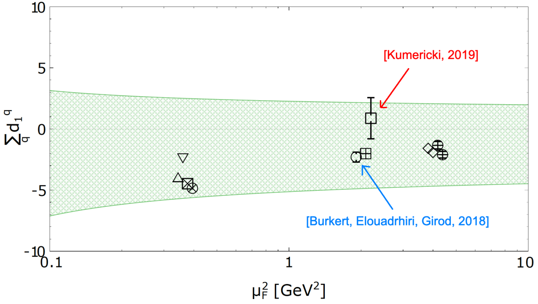
- Because Gegenbauer polynomials diagonalize the LO ERBL [Lepage, Brodsky, 1979], [Efremov, Radyushkin, 1979] evolution kernel, each term $d_n^q(t,\mu^2)$ actually $d_n^{\pm}$ but that does not change the argument evolves multiplicatively with a different anomalous dimension. Since exponentials are a free family on any non-vanishing interval, the decomposition

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \tag{12}$$

  is **unique, non-ambiguous and theoretically allows to entirely retrieve the** D**-term from the knowledge of the subtraction constant on any non-vanishing interval in** $Q^2 = \mu^2$.
- All is well on paper, but what about in real life?

Extraction of GFFs

Complete details found in **[Dutrieux et al, Eur.Phys.J.C 81 (2021) 4, 300]** **see talk by H. Moutarde on Wednesday 16:30 - Joint GPD - Future session**. We perform a neural network fit of CFFs over world DVCS data, which gives a **subtraction constant compatible with 0** $\rightarrow$ also found in [Kumericki, 2019]. Then fixing the t-dependence with an Ansatz and assuming all d_n for $n > 1$ to be 0 gives



In green, 68% confidence interval found for $\sum_q d_1^q(t = 0, \mu^2)$. Results obtained by the two other data-driven extractions highlighted. **But uncertainty here is driven by the experimental uncertainty on the subtraction constant. There is another source of uncertainty.**

Extraction of GFFs

- Since the LO subtraction constant reads

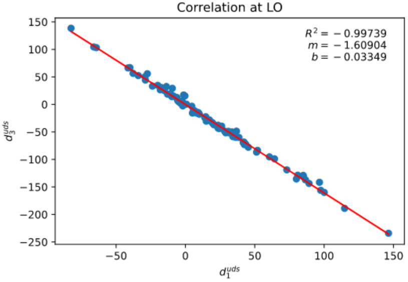
$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \tag{13}$$

if we allow d_3^q to be non-zero, at some scale μ_0^2, we can have $d_1^q(\mu_0^2) = -d_3^q(\mu_0^2)$, so a **vanishing subtraction constant, but non-zero GFF** $C_q(\mu_0^2)$. If the effect of evolution is not significant enough, these configurations are not ruled out and add a considerable uncertainty.

$d_1^{uds}(\mu_F^2)$	-0.5 ± 1.2

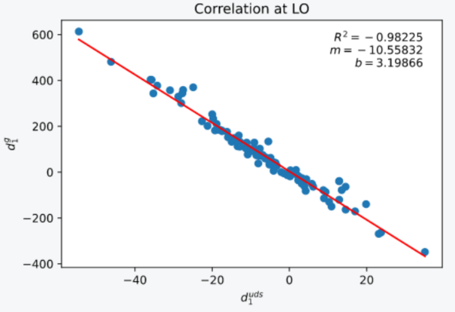
$\longrightarrow$

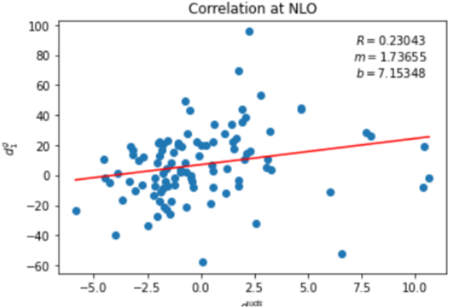
$d_1^{uds}(\mu_F^2)$	11 ± 25
$d_3^{uds}(\mu_F^2)$	-11 ± 26



Extraction of GFFs

- **Exclusive preliminary result** We reanalysed our work in a full NLO formalism **[Dutrieux et al, in preparation]**. We tried different fitting scenarios and conclude that NLO effects are generally rather small. A noticeable effect is obtained when allowing an intrinsic gluon contribution not purely generated by evolution.





3. Deconvoluting a Compton form factor: shadow GPDs

Deconvoluting a Compton form factor

Position of the problem
Assuming a CFF has been extracted from experimental data with excellent precision – and the different gluon and flavour contributions have been separated –, we are left with the convolution:

$$\int_{-1}^{1} \frac{\mathrm{d}x}{\xi}\, T^q\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) H^q(x, \xi, t, \mu^2) = T^q(Q^2, \mu^2) \otimes H^q(\mu^2) \tag{14}$$

where T^q is a coefficient function computed in pQCD. **Can we then "de-convolute" eq. (14) to recover $H^q(x, \xi, t, \mu^2)$ from $T^q(Q^2, \mu^2) \otimes H^q(\mu^2)$?**

Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was proposed as a crucial element in [Freund, 1999], but the question has remained essentially open.
- We show that GPDs exist which bring contributions to the LO and NLO CFF of only subleading order even under evolution. We call them **LO and NLO shadow GPDs**.

Definition of an NLO shadow GPD

For a given scale μ_0^2,

$$\forall \xi, \forall t, \; T^q_{NLO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x, \xi = 0, t = 0, \mu_0^2) = 0 \tag{15}$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2, \; T^q_{NLO}(Q^2, \mu^2) \otimes H^q(\mu^2) = \mathcal{O}(\alpha_s^2(\mu^2)) \tag{16}$$

- Let H^q be an NLO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same NLO CFF up to a numerically small and theoretically subleading contribution.

Shadow GPDs at leading order

- Complete details in **[Bertone et al, Phys.Rev.D 103 (2021) 11, 114019]**
- We search for our shadow GPDs as simple **double distributions (DD)** $F(\beta, \alpha, \mu^2)$ to respect polynomiality, with a zero D-term. Then, thanks to dispersion relations, we can restrict ourselves to the imaginary part only $\mathrm{Im}\ T^q(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0$.
- We search our DD as a polynomial of order N in (β, α), characterised by $\sim N^2$ coefficients c_{mn}:

$$F(\beta, \alpha, \mu_0^2) = \sum_{m+n \leq N} c_{mn}\, \alpha^m \beta^n \tag{17}$$

Shadow GPDs at next-to-leading order

- **First study beyond leading order:** Apart from the **LO** part, the NLO CFF is composed of a **collinear part** (compensating the α_s^1 term resulting from the convolution of the LO coefficient function and the evoluted GPD) and a genuine **1-loop NLO** part.

$$\mathcal{H}^q(\xi, Q^2) = C_0^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_1^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_{coll}^q \otimes H^{q(+)}(\mu_0^2) \log\left(\frac{\mu^2}{Q^2}\right) \tag{18}$$

An explicit calculation of each term for our polynomial double distribution gives that

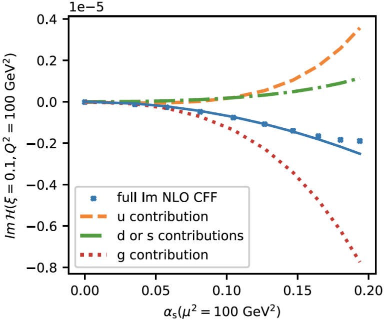
$$\mathrm{Im}\ T_{coll}^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto$$
$$\alpha_s(\mu^2) \log\left(\frac{\mu^2}{Q^2}\right) \left[\left(\frac{3}{2} + \log\left(\frac{1-\xi}{2\xi}\right)\right) \mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N+1} \frac{k_w^{(coll)}}{(1+\xi)^w} \right] \tag{19}$$

and assuming $\mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) = 0$,

$$\mathrm{Im}\ T_1^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto \alpha_s(\mu^2) \left[\log\left(\frac{1-\xi}{2\xi}\right) \mathrm{Im}\ T_{coll}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N-1} \frac{k_w^{(1)}}{(1+\xi)^w} \right]$$

Shadow GPDs at next-to-leading order

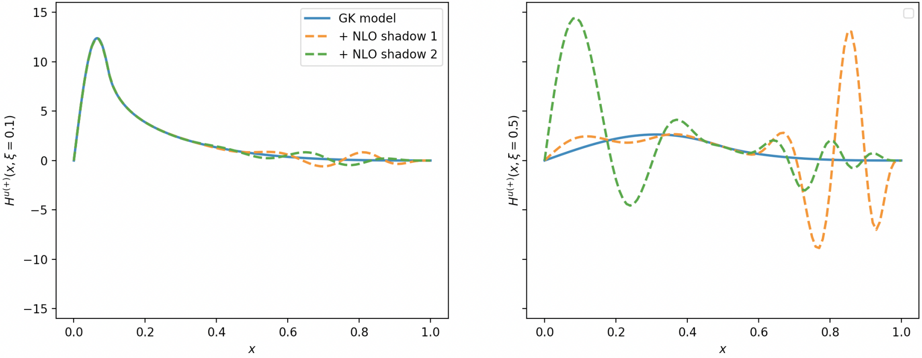
- By linearity of both the CFF convolution and the evolution equation, we can evaluate separately the contribution to the CFF of a quark shadow NLO GPD under evolution.
- We probe the prediction of evolution as $\mathcal{O}(\alpha_s^2(\mu^2))$ with our previous NLO shadow GPD on a lever-arm in Q^2 of $[1, 100]$ GeV^2 (typical collider kinematics) using APFEL++ code.



- The fit by $\alpha_s^2(\mu^2)$ is very good up to values of α_s of the order of its $\overline{MS}$ values. For larger values, large logs and higher orders slightly change the picture.
- The numerical effect of evolution remains very small. For a GPD of order 1, the NLO CFF is only of order 10^{-5}.

Shadow GPDs at next-to-leading order

The orange and brown models are **Goloskokov-Kroll model + NLO shadow GPDs**. For ξ close to 0 and x close to ξ, by design, they are very close, but vastly different otherwise. They give rise to NLO CFFs which are exactly identical at this scale, and different by a negligible amount for expected Q^2 lever arm.



$\xi = 0.1$ (left) and $\xi = 0.5$ (right)

4. Perspectives

Perspectives

- Modelling shadow GPDs allows to quantify the uncertainty involved in GPD extractions from DVCS data. **Ongoing effort to produce such models to propagate uncertainties in GPD studies, especially taking into account positivity constraints.** For $x \geq |\xi|$

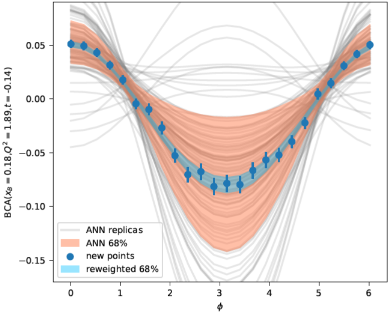
$$\left| H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E^q(x,\xi,t) \right| \leq \sqrt{\frac{1}{1-\xi^2} f^q\left(\frac{x+\xi}{1+\xi}\right) f^q\left(\frac{x-\xi}{1-\xi}\right)} \qquad (21)$$

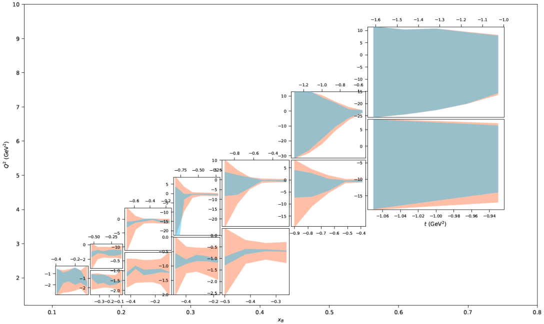
see talk by P. Sznajder on Wednesday 17:20 - Joint GPD - Future session.

- Other exclusive processes can be expressed in terms of GPDs. Close parent to DVCS is **time-like Compton scattering** (TCS) [Berger *et al*, 2002]. Although its measurement will reduce the uncertainty, especially on $\mathrm{Re}\,\mathcal{H}$ [Jlab proposal PR12-12-001], and produce a valuable check of the universality of the GPD formalism, the similar nature of its convolution (see [Müller *et al*, 2012]) makes it subject to the same shadow GPDs.

Perspectives

- Reducing uncertainties on CFFs itself is a very useful task. *e.g.* proton pressure anisotropy is compatible with 0 largely because of the uncertainty on $\mathrm{Re}\,\mathcal{H}$ in **[Dutrieux et al, Eur.Phys.J.C 81 (2021) 4, 300]**.
- The proposal to install a positron beam at JLab [Afanasev *et al*, 2019] can help on this task. We have performed in **[Dutrieux et al, Eur.Phys.J.A 57 (2021) 8, 250]** a reweighting of our neural network replicas of CFFs against simulated new experimental points.





Conclusion

- Explicit demonstration of NLO shadow GPDs of considerable size with a very small and subleading contribution to CFFs. **Such shadow GPDs will be hidden in typical statistical and systematic uncertainties of DVCS**. TCS or LO DVMP face similar issues. We foresee that our discussion can be extended to higher order DVCS. Other exclusive processes will help discriminate the DVCS shadow GPDs. Especially DDVCS or Lattice QCD for instance should escape the dimensionality of data problem.
- Potential impact on **hadron tomography** due to the $\xi \to 0$ extrapolation, determination of **OAM** and mechanical properties to study.
- An extraction of GPDs with lesser systematic uncertainty requires a **multi-channel analysis**, and the development of integrated analysis tools, like **PARTONS**
- More precise data over a much larger Q^2 range promised by future colliders will be very welcome here and for the extraction of mechanical properties as well.
- More theoretical constraints, like **positivity** could play a significant role in reducing the uncertainty.

Conclusion

Thank you for your attention !

Shadow GPDs at leading order

- For our LO shadow GPD, we first want $H^{q(+)}(\xi,\xi,\mu_0^2) = 0$, so we notice that

$$H^{q(+)}(\xi,\xi,\mu_0^2) = \sum_{w=1}^{N+1} \frac{k_w}{(1+\xi)^w} \quad \text{where} \quad k_w = \sum_{u,v} C_w^{uv}\, q_{uv}\,, \quad C_w^{uv} = (-1)^{u+v+w} \binom{v}{u-w}$$

Cancelling the LO CFF

$$H^{q(+)}(\xi,\xi,\mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{22}$$

Shadow GPDs at leading order

Cancelling the LO CFF

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- We then want $H^{q(+)}(x,\xi=0,\mu_0^2)=0$, so we notice that

$$H^{q(+)}(x,0,\mu_0^2) = \sum_{w=0}^{N+1} q_w x^w \quad \text{where} \quad q_w = \sum_{u,v} Q_w^{uv}\, q_{uv}\,, \quad Q_w^{uv} = 2\delta_w^v$$

Cancelling the forward limit

$$H^{q(+)}(x,\xi=0,\mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \tag{23}$$

Shadow GPDs at leading order

Cancelling the LO CFF

$$H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{22}$$

Cancelling the forward limit

$$H^{q(+)}(x, \xi = 0, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \tag{23}$$

- Both linear systems $C.R$ and $Q.R$ are systems of $\sim N$ equations for $\sim N^2$ variables, so the number of solutions grows quadratically with N, order of the polynomial DD.

Shadow GPDs at leading order

Cancelling the LO CFF

$$H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{22}$$

Cancelling the forward limit

$$H^{q(+)}(x, \xi = 0, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \tag{23}$$

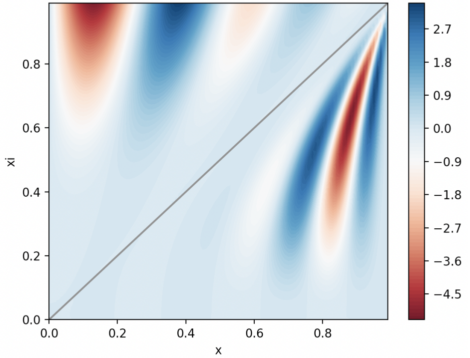
LO shadow GPDs

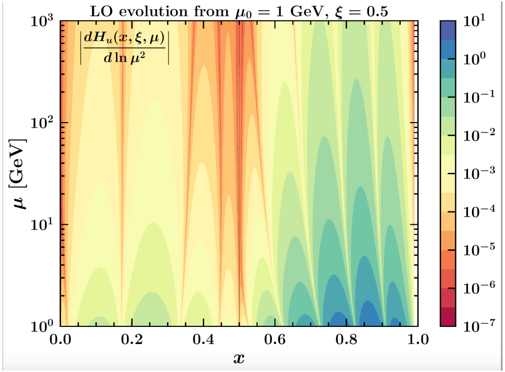
Here is an example of an infinite family of LO shadow DDs, each being of degree $N \geq 9$ odd

$$F_N(\beta,\alpha,\mu_0^2)=\beta^{N-8}\Big[\alpha^8-\tfrac{28}{9}\alpha^6\Big(\tfrac{N^2-3N+20}{(N+1)N}+\beta^2\Big)+\tfrac{10}{3}\alpha^4\Big(\tfrac{N^2-7N+40}{(N+1)N}+\tfrac{2(N^2-3N+44)}{3(N+1)N}\beta^2+\beta^4\Big)$$
$$-\tfrac{4}{3}\alpha^2\Big(\tfrac{N^2-11N+60}{(N+1)N}-\tfrac{N-8}{N}\beta^2-\tfrac{N^2-3N-28}{(N+1)N}\beta^4+\beta^6\Big)+\tfrac{1}{9}(1-\beta^2)^2\Big(\tfrac{N^2-15N+80}{(N+1)N}-\tfrac{2(N-8)}{N}\beta^2+\beta^4\Big)\Big] \tag{24}$$

Shadow GPDs at next-to-leading order

- Cancelling both terms gives rise to two additional systems with a linear number of equations. The first NLO shadow GPD is found for $N = 21$, and adding the condition that the DD vanishes at the edges of its support gives a first solution for $N = 25$ (see below).





Color plot of an NLO shadow GPD at initial scale 1 GeV^2, and its evolution for $\xi = 0.5$ up to 10^6 GeV^2 via APFEL++ and PARTONS [Bertone].

Perspectives

- **Deeply virtual meson production** (DVMP) [Collins *et al*, 1997] is also an important source of knowledge on GPDs, with currently a larger lever arm in Q^2. The process involves form factors of the general form

$$\mathcal{F}(\xi, t) = \int_0^1 \mathrm{d}u \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} \phi(u) T\left(\frac{x}{\xi}, u\right) F(x, \xi, t) \tag{25}$$

  with $\phi(u)$ is the leading-twist meson distribution amplitude (DA).
- At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the form factor.
- Situation at NLO remains to be clarified, it is foreseeable new shadow GPDs (dependent on the DA) could be generated also for this process.

Perspectives

- **New experimental channels**: more experimentally challenging processes offer a richer access to GPDs thanks to more handles with kinematic variables.
  - Double deeply virtual Compton scattering (DDVCS) – proposed at JLab with SOLID (LOI12-15-005) and CLAS12 (LOI12-16-004) – which gives access directly to the (x, ξ) value of GPDs in the ERBL region at LO.
  - Multiparticle production: diphoton [Pedrak *et al*, 2017], photon-rho [Boussarie *et al*, 2017]
- **Lattice QCD**: low order Mellin moments of GPDs will not change significantly the previously exposed picture. Where a new order of DVCS put N constraints on a DD of polynomial order N, a new Mellin moment only brings a finite number of constraints.
- Extractions of the x-dependence of parton distributions are an interesting prospects, which we start to consider.

Perspectives

Positivity constraints [Radyushkin, 1999], [Pire *et al*, 1999], [Diehl *et al*, 2001], [Pobylitsa, 2002]

- Stemming from the representation of GPDs as overlap between light-front wave functions, positivity constraints are a Cauchy-Schwart like inequality relating GPDs to the PDFs, *e.g.* for $x \geq |\xi|$

$$\left| H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E^q(x,\xi,t) \right| \leq \sqrt{\frac{1}{1-\xi^2} f^q\left(\frac{x+\xi}{1+\xi}\right) f^q\left(\frac{x-\xi}{1-\xi}\right)} \tag{26}$$

- This inequality puts a maximal bound on the size of shadow GPDs in the DGLAP region, and is especially constraining for large x.
- Since shadow GPDs are maximally violating positivity (their forward limit is 0), they are a tool to correct a model giving satisfactory experimental agreement, but violating positivity. (Work in progress)