Artificial neural network techniques in modelling of GPDs

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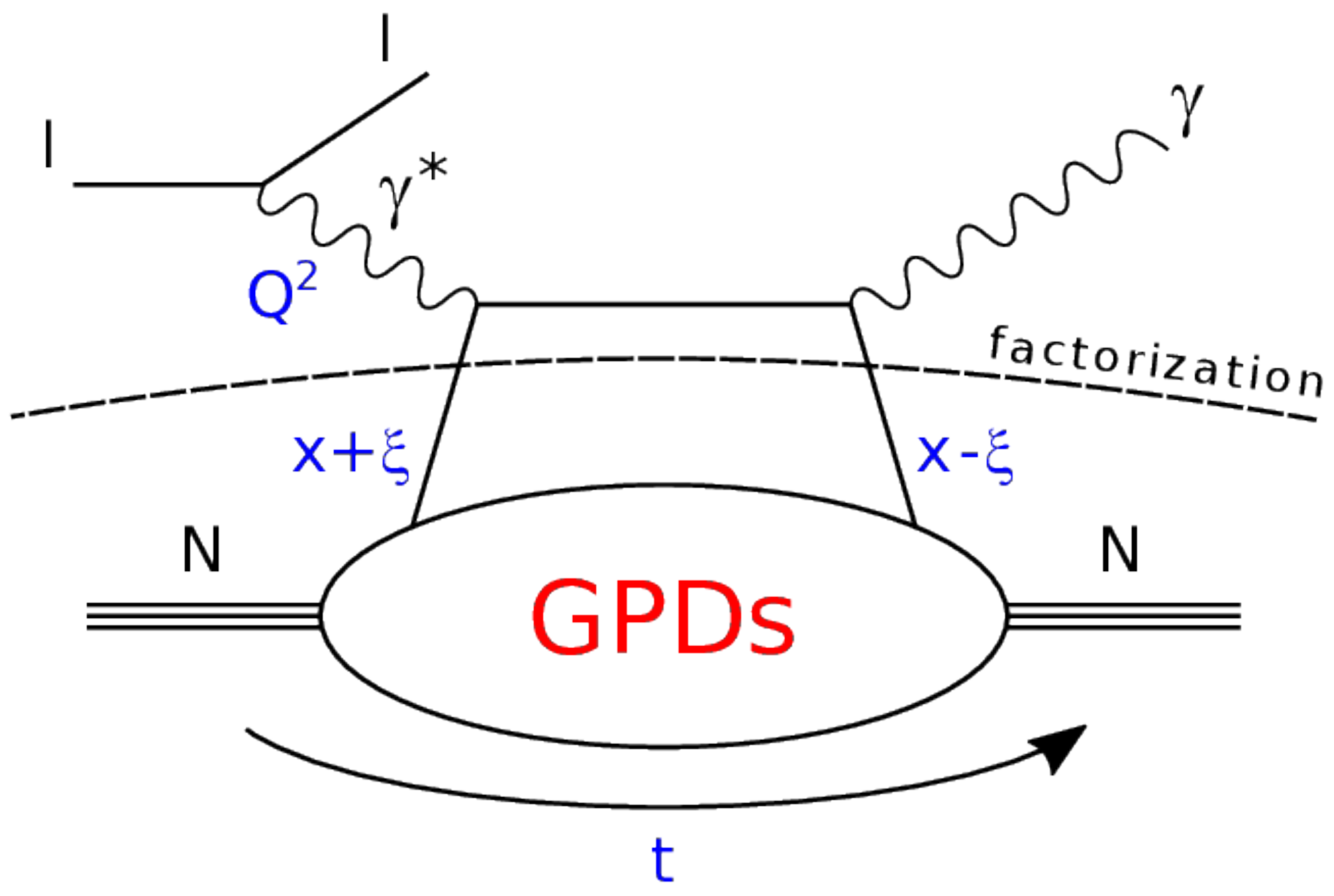
Deeply Virtual Compton Scattering (DVCS)



factorisation for |t|/Q^2 ≪ 1

Chiral-even GPDs:
(helicity of parton conserved)

$H^{q,g}(x,\xi,t)$	$E^{q,g}(x,\xi,t)$	*for sum over parton helicities*
$\widetilde{H}^{q,g}(x,\xi,t)$	$\widetilde{E}^{q,g}(x,\xi,t)$	*for difference over parton helicities*
nucleon helicity conserved	*nucleon helicity changed*	

Reduction to PDF:

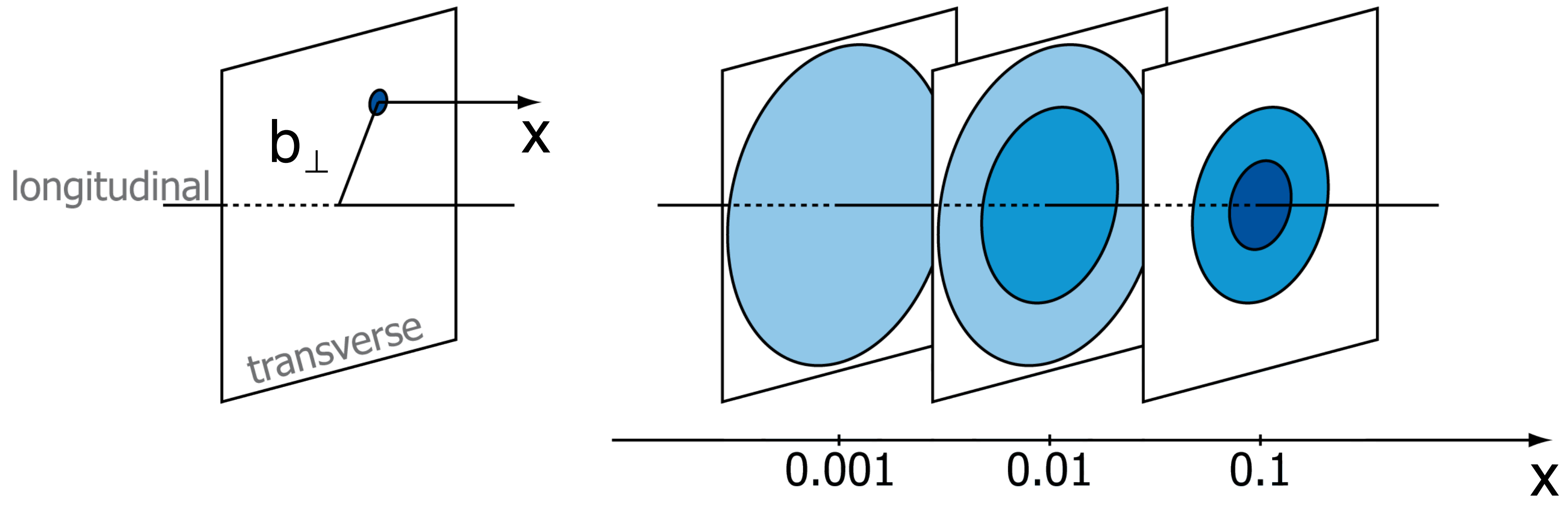
$$H(x, \xi = 0, t = 0) \equiv q(x)$$

Polynomiality - non-trivial consequence of Lorentz invariance:

$$\mathcal{A}_n(\xi, t) = \int_{-1}^{1} \mathrm{d}x x^n H(x, \xi, t) = \sum_{\substack{j=0 \\ \text{even}}}^{n} \xi^j A_{n,j}(t) + \mathrm{mod}(n, 2)\xi^{n+1} A_{n,n+1}(t)$$

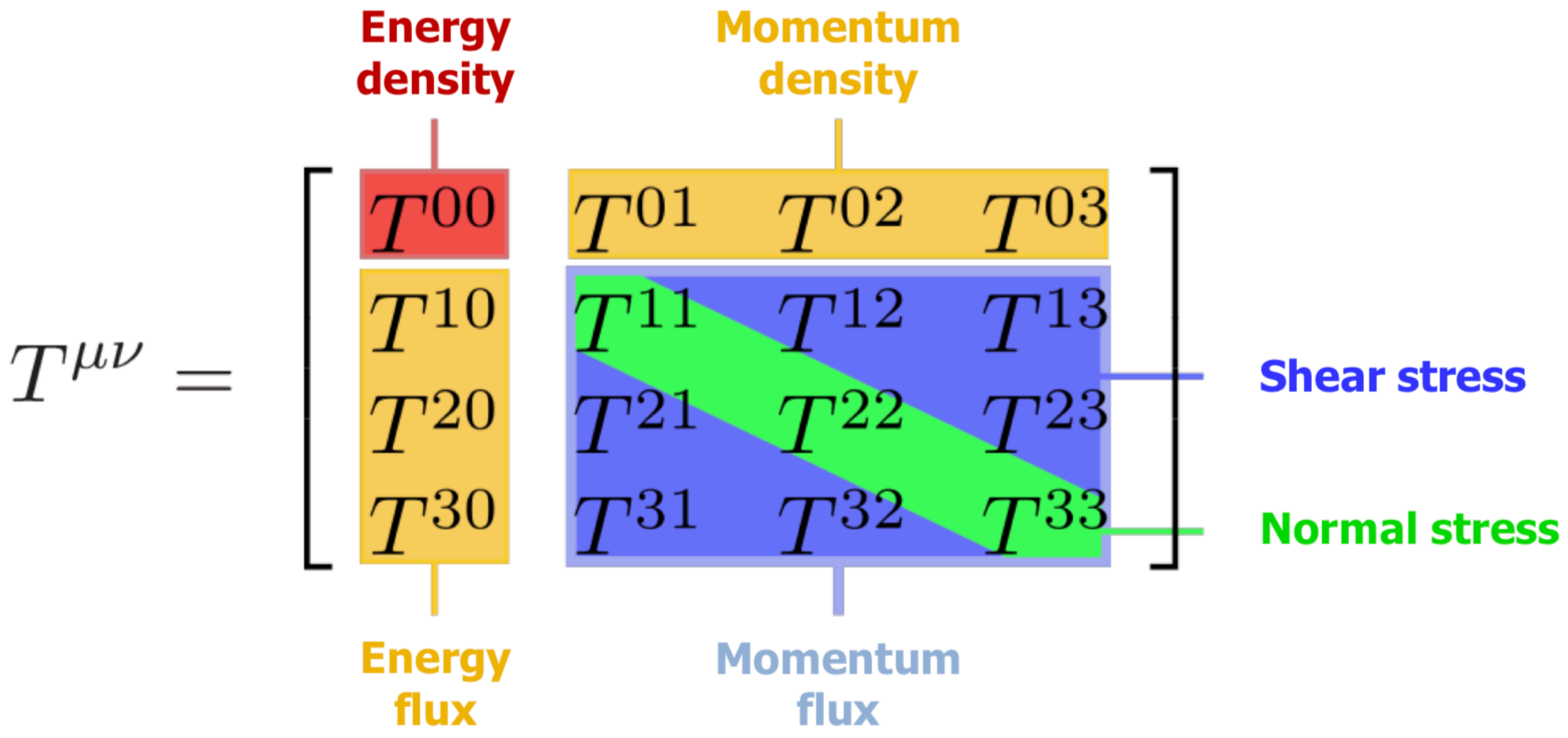
Positivity bounds - positivity of norm in Hilbert space, e.g.:

$$|H(x, \xi, t)| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right) \frac{1}{1-\xi^2}}$$



Nucleon tomography

$$q(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\mathbf{\Delta}}{4\pi^2} e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}} H^q(x, 0, t = -\mathbf{\Delta}^2)$$



Energy momentum tensor in terms of form factors (OAM and mechanical forces)

$$\langle p', s' | \widehat{T}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \Bigg[\frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C(t) + M\eta^{\mu\nu}\bar{C}(t) + \frac{P^\mu i\sigma^{\nu\lambda}\Delta_\lambda}{4M}\left[A(t) + B(t) + D(t)\right] + \frac{P^\nu i\sigma^{\mu\lambda}\Delta_\lambda}{4M}\left[A(t) + B(t) - D(t)\right] \Bigg] u(p, s)$$

Motivation

- we witness a substantial progress in:
  - measurement and description of exclusive processes
  - understanding of fundamental problems like the deconvolution of GPDs from DVCS data (see H. Dutrieux talk)
  - lattice-QCD (see A. Scapellato and C. Alexandrou talks)
- however, problem of the model dependency of GPDs is still poorly addressed, with the exception of the extraction of D-term (see H. Moutarde talk)
- no GPD models that could be considered non-parametric → no tools to study model dependency of the extraction of GPDs, nucleon tomography and orbital angular momentum

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modelling in (x, ξ)-space

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Polynomiality:

$$A_n(\xi) = \int_{-1}^{1} \mathrm{d}x x^n H(x,\xi) = \sum_{\substack{j=0 \\ \text{even}}}^{n} \xi^j A_{n,j} + \mathrm{mod}(n,2)\xi^{n+1} A_{n,n+1}$$

Let us express GPD by:

$$H^N(x,\xi) = \sum_{\substack{j=0 \\ \text{even}}}^{N} f_j(x)\xi^j$$

only even j as there is no odd power of ξ in polynomiality expansion

Support:

$$f_j(-1) = f_j(1) = 0$$

we want GPDs to vanish at |x| = 1

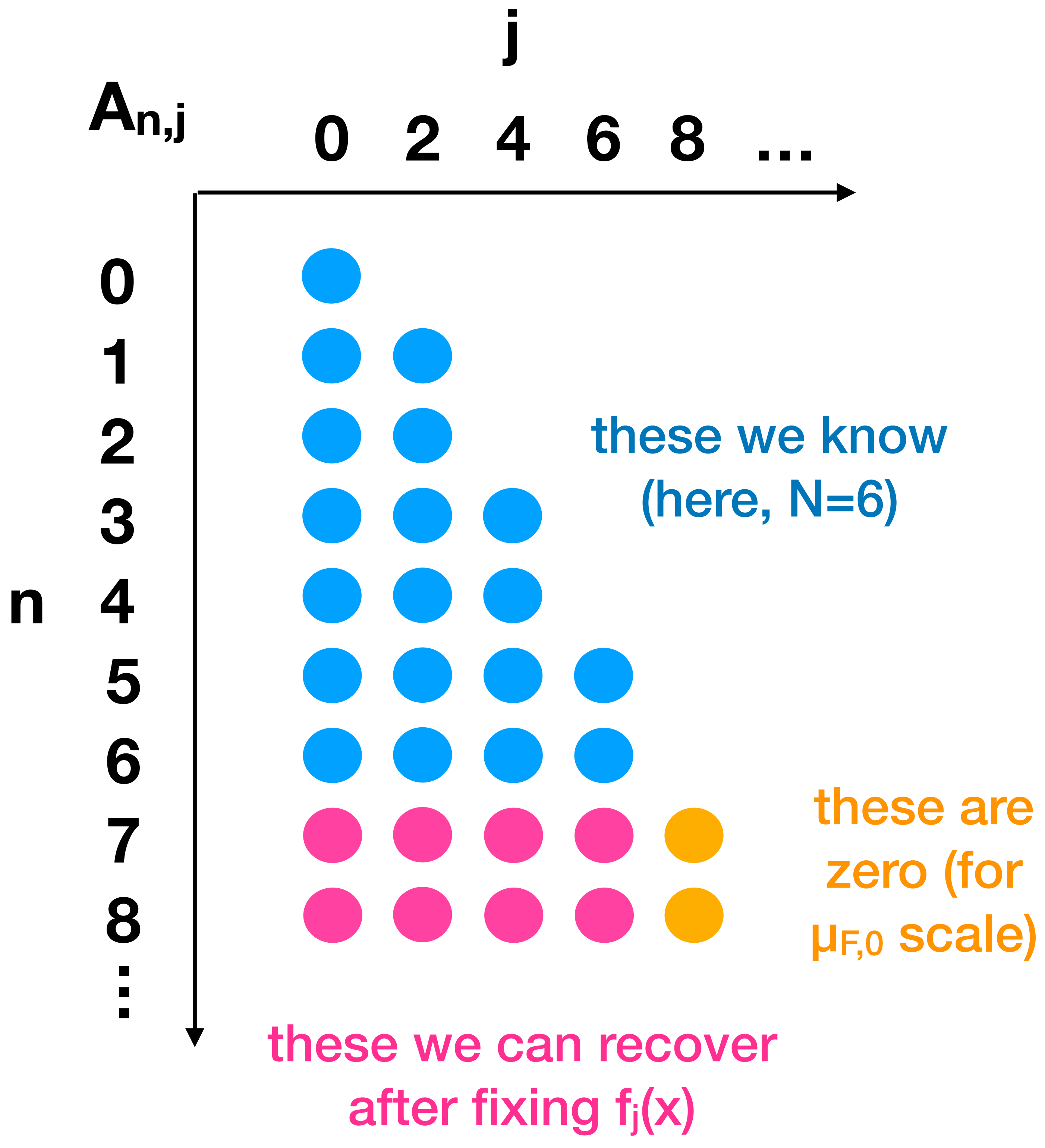
Mellin coefficients:

$$A_{n,j} = \int_{-1}^{1} \mathrm{d}x x^n f_j(x)$$

choice of $f_j(x)$ functional form is arbitrary

where e.g.:

$$A_{1,2} = \int_{-1}^{1} \mathrm{d}x\, x\, f_2(x) = 0$$



Principles of modelling

Polynomial basis:

This basis leads to Dual Parameterisation → M. Polyakov, A. Shuvaev, hep-ph/0207153

Any attempt of describing GPDs by orthogonal polynomials will lead to this basis

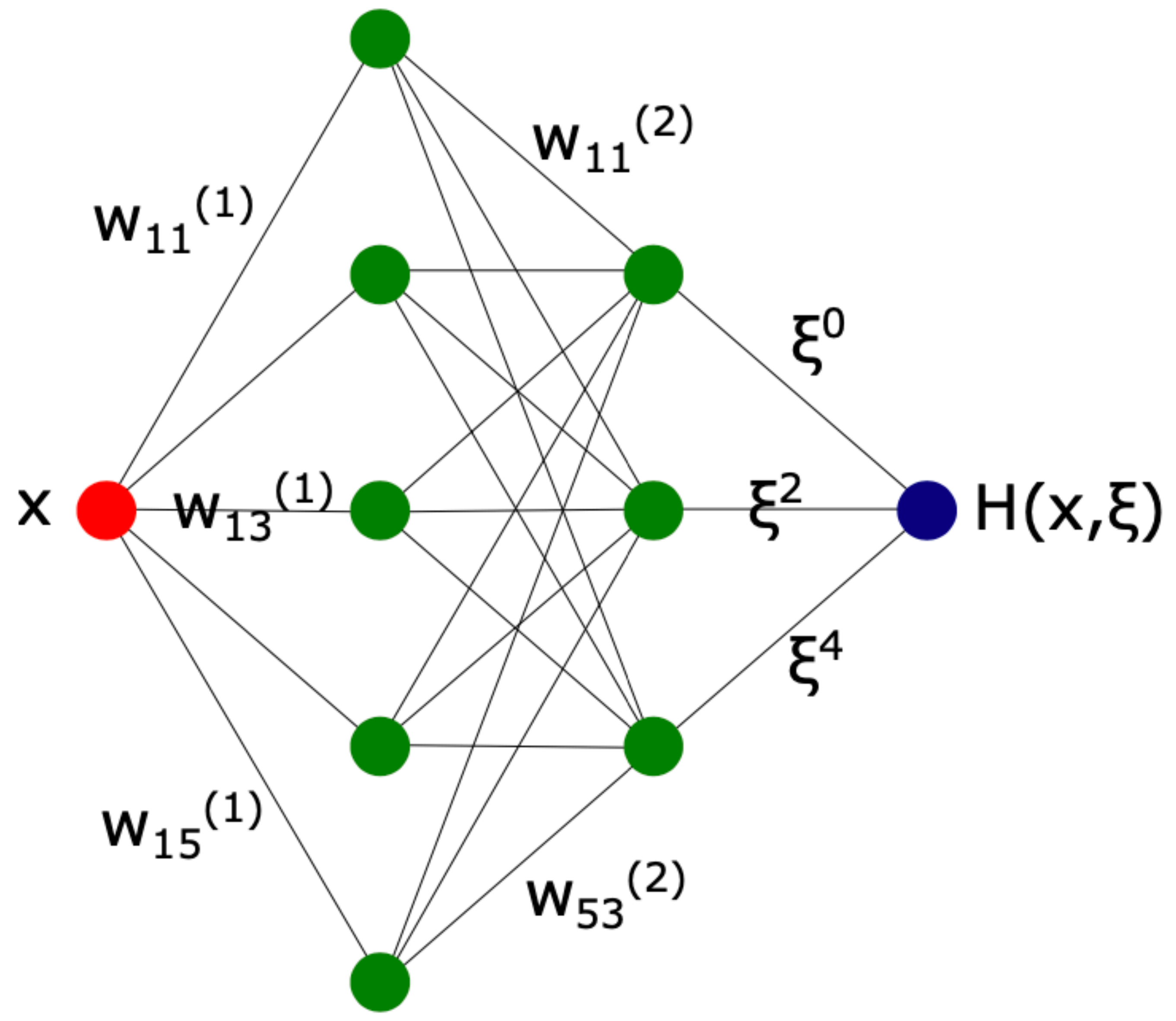
$$f_j(x) = \sum_{i=0}^{N+2} w_{i,j} x^i$$

GPD will be expressed by sum of monomials $x^i\xi^j$

ANN basis:

New!

We can describe GPD by a single ANN



$$f_j(x) = \mathrm{ANN}_j(x)$$

GPD will be expressed by sum of ANNs multiplied by ξ^j

Demonstration of results

Test model (see e.g.: hep-ph/2110.06052):

$$H_\pi(x,\xi) = \Theta(x-|\xi|)\,\frac{30(1-x)^2(x^2-\xi^2)}{(1-\xi^2)^2} + \Theta(|\xi|-|x|)\,\frac{15(1-x)(\xi^2-x^2)(x+2x\xi+\xi^2)}{2\xi^3(1+\xi)^2}$$

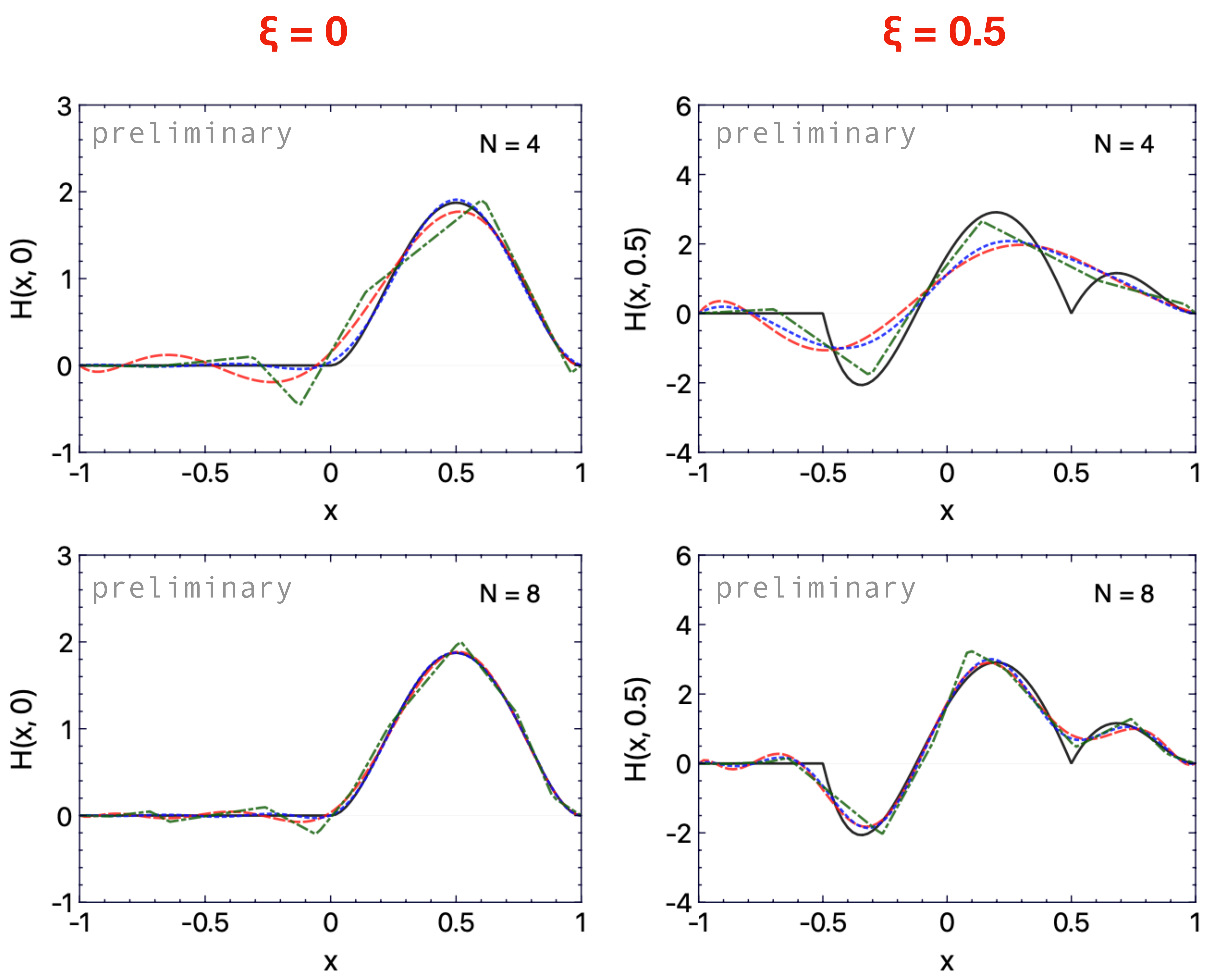
Polynomial basis

ANN basis - sigmoid

$$\varphi_k^{(2)}(\cdot) = \frac{1}{1+\exp(-(\cdot))}$$

ANN basis - ReLU

$$\varphi_k^{(2)}(\cdot) = (\cdot)\,\Theta(\cdot)$$



Note: positivity not addressed here

Possible modifications

Basic:

$$H(x,\xi) = \sum_{\substack{j=0 \\ \text{even}}}^{N} f_j(x)\xi^j$$

With explicit PDF:

$$H(x,\xi) = q(x) + \sum_{\substack{j=2 \\ \text{even}}}^{N} f_j(x)\xi^j$$

Vanishing at x=xi:

$$H(x,\xi) = (x^2 - \xi^2) \sum_{\substack{j=0 \\ \text{even}}}^{N} f_j(x)\xi^j$$

With D-term:

$$H(x,\xi) = D_{\text{term}}(x/\xi) + \sum_{\substack{j=0 \\ \text{even}}}^{N} f_j(x)\xi^j$$

modelling in (β,α)-space

Double distribution:

$$H(x,\xi,t) = g(x,\xi)\int \mathrm{d}\Omega\, F(\beta,\alpha,t)$$

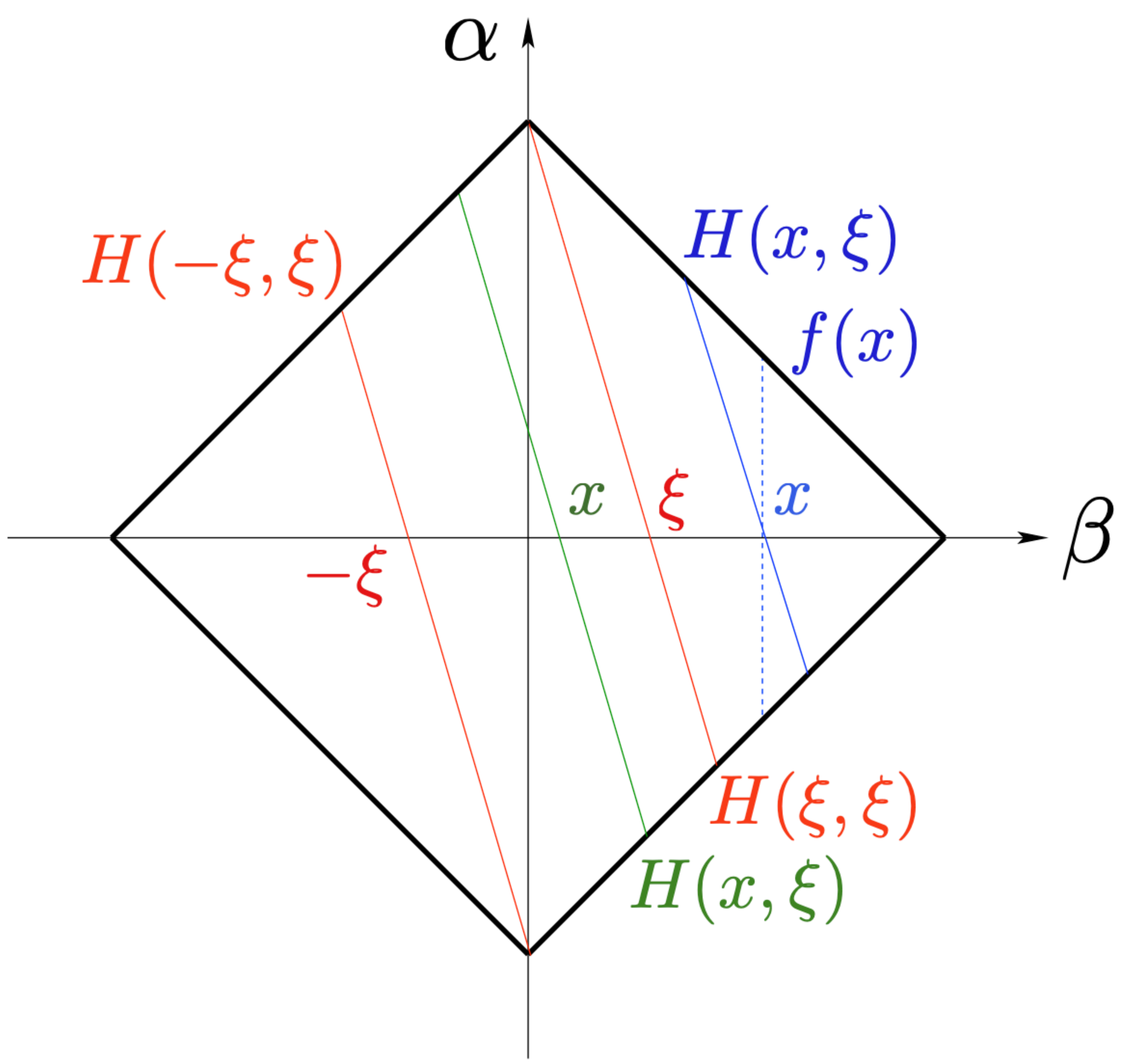
where:

$$\mathrm{d}\Omega = \mathrm{d}\beta\,\mathrm{d}\alpha\,\delta(x-\beta-\alpha\xi)$$

$$|\alpha| + |\beta| \leq 1$$

The gauge function:

$$g(x,\xi) = 1 \qquad \text{for Polyakov-Weiss gauge}$$
$$g(x,\xi) = 1-|x| \qquad \text{for Pobylitsa gauge}$$
$$g(x,\xi) = x^2-\xi^2 \qquad \text{for ``shadow'' gauge}$$



from PRD83, 076006, 2011

Principles of modelling

Double distribution:

$$(1-|x|)F_U(\beta,\alpha) + (x^2-\xi^2)F_S(\beta,\alpha) + \xi F_D(\beta,\alpha)$$

Usual term:

$$F_U(\beta,\alpha) = f(\beta)h_U(\beta,\alpha)\frac{1}{1-|\beta|}$$

$$f(\beta) = \mathrm{sgn}(\beta)q(|\beta|)$$

$$h_U(\beta,\alpha) = \frac{\mathrm{ANN}_U(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_U(|\beta|,\alpha)}$$

Shadow term:

$$F_S(\beta,\alpha) = f(\beta)h_S(\beta,\alpha)$$

$$f(\beta) = \mathrm{sgn}(\beta)q(|\beta|)$$

$$h_S(\beta,\alpha)/N_S = \frac{\mathrm{ANN}_S(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_S(|\beta|,\alpha)} - \frac{\mathrm{ANN}_{S'}(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_{S'}(|\beta|,\alpha)} .$$

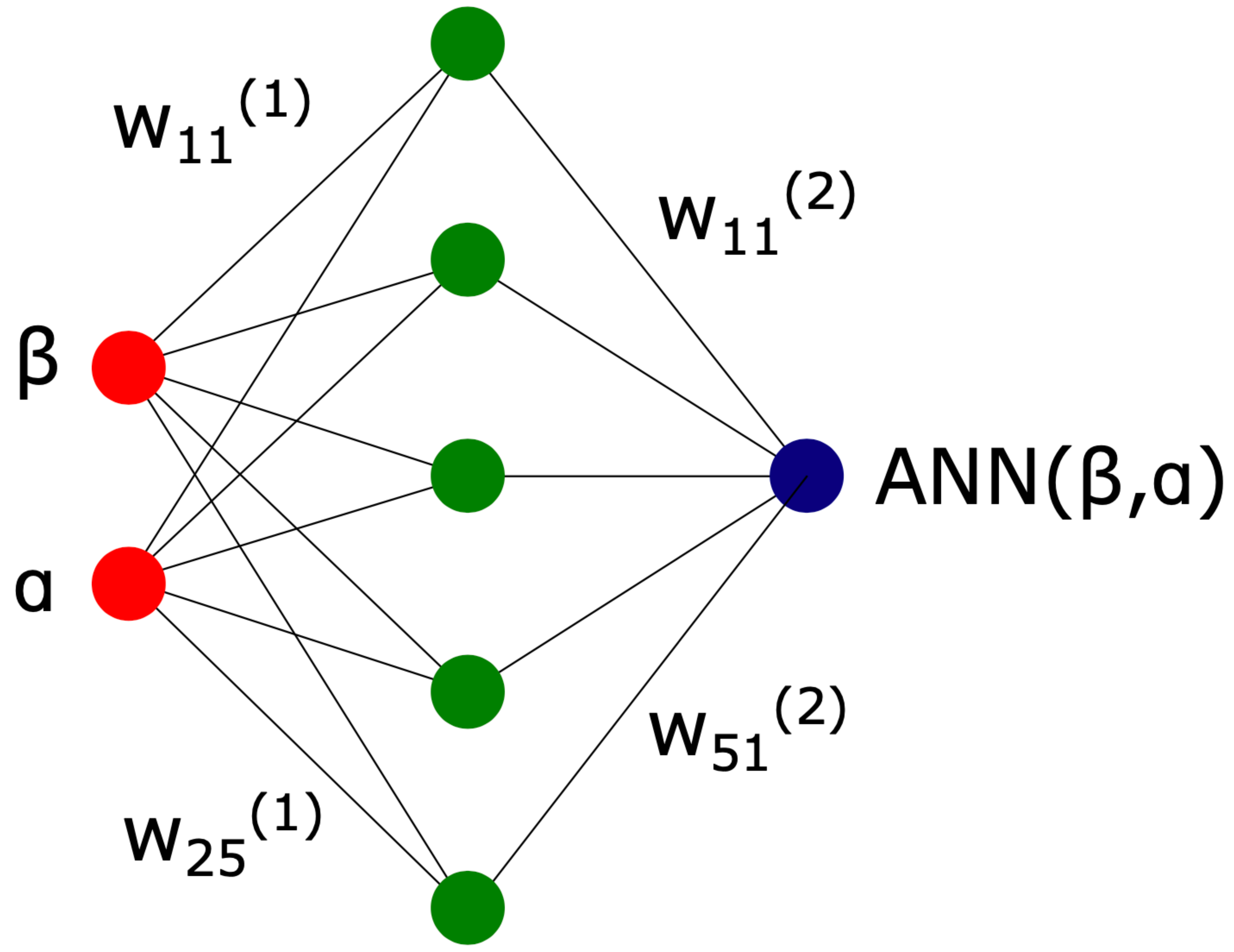
$$\mathrm{ANN}_{S'}(\beta,\alpha) \equiv \mathrm{ANN}_U(\beta,\alpha)$$

D-term:

$$F_D(\beta,\alpha) = \delta(\beta)D(\alpha)$$

$$D(\alpha) = (1-\alpha^2)\sum_{\substack{i=1\\ \mathrm{odd}}} d_i C_i^{3/2}(\alpha)$$

Our ANNs:



Requirements:

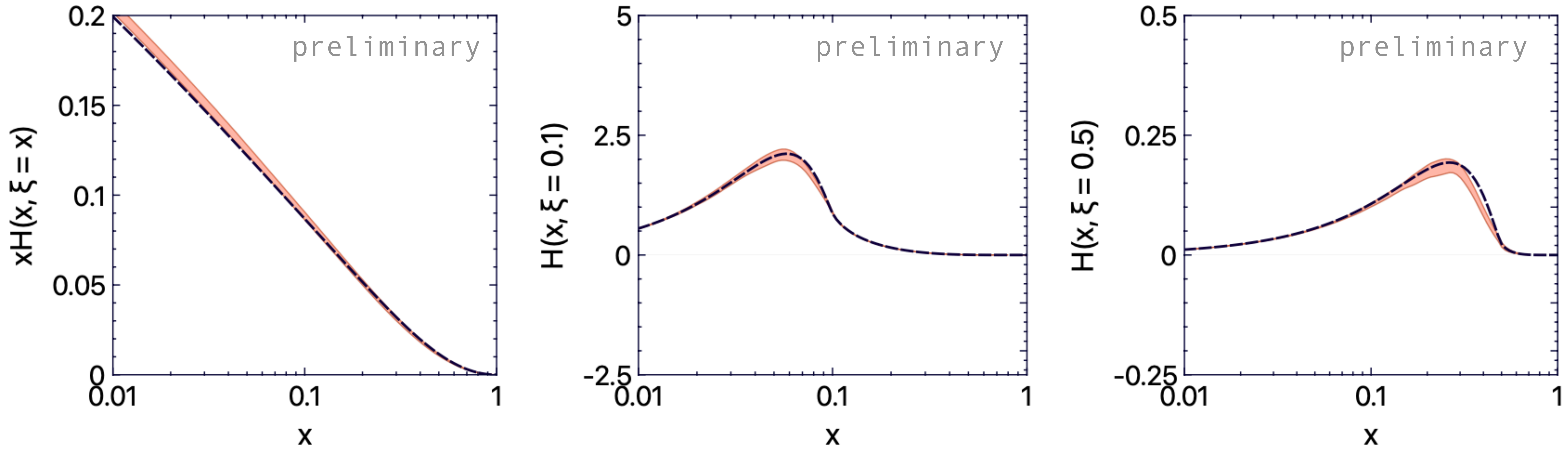
symmetric w.r.t. α

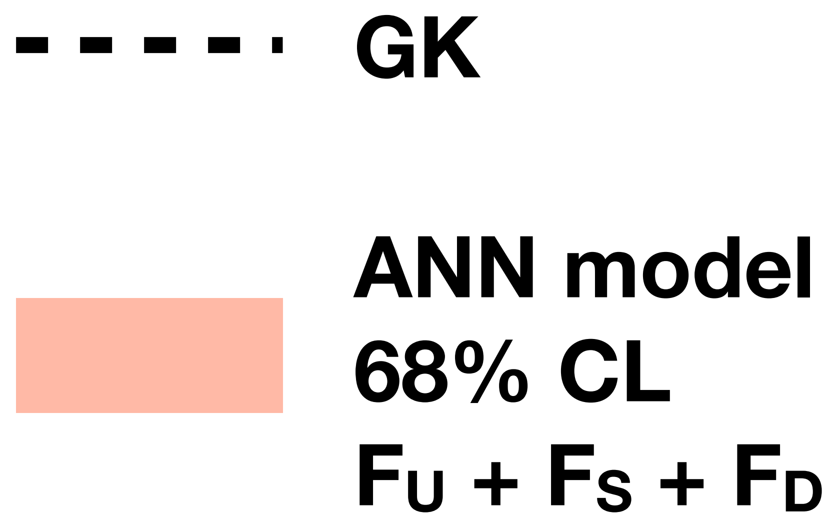
symmetric w.r.t. β

vanishes at | α | + | β | = 1

Activation function:

$$\left(\varphi_i\left(w_i^{\beta}|\beta| + w_i^{\alpha}\alpha/(1-|\beta|) + b_i\right) - \varphi_i\left(w_i^{\beta}|\beta| + w_i^{\alpha} + b_i\right)\right) + (w^{\alpha} \to -w^{\alpha})$$





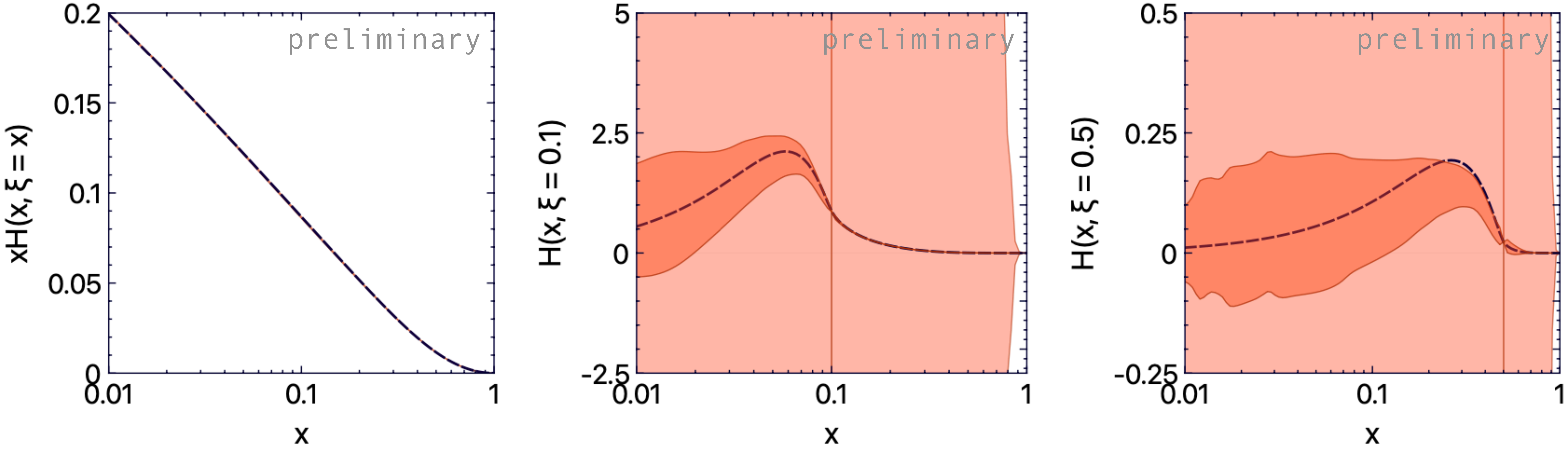
Conditions:

- Input: 400 x ≠ xi points generated with GK model
- Positivity not forced

Technical detail of the analysis:

- Minimisation with genetic algorithm
- Replication for estimation of model uncertainties
- “Global” detection of outliers
- Dropout algorithm for regularisation

Demonstration of results



GK

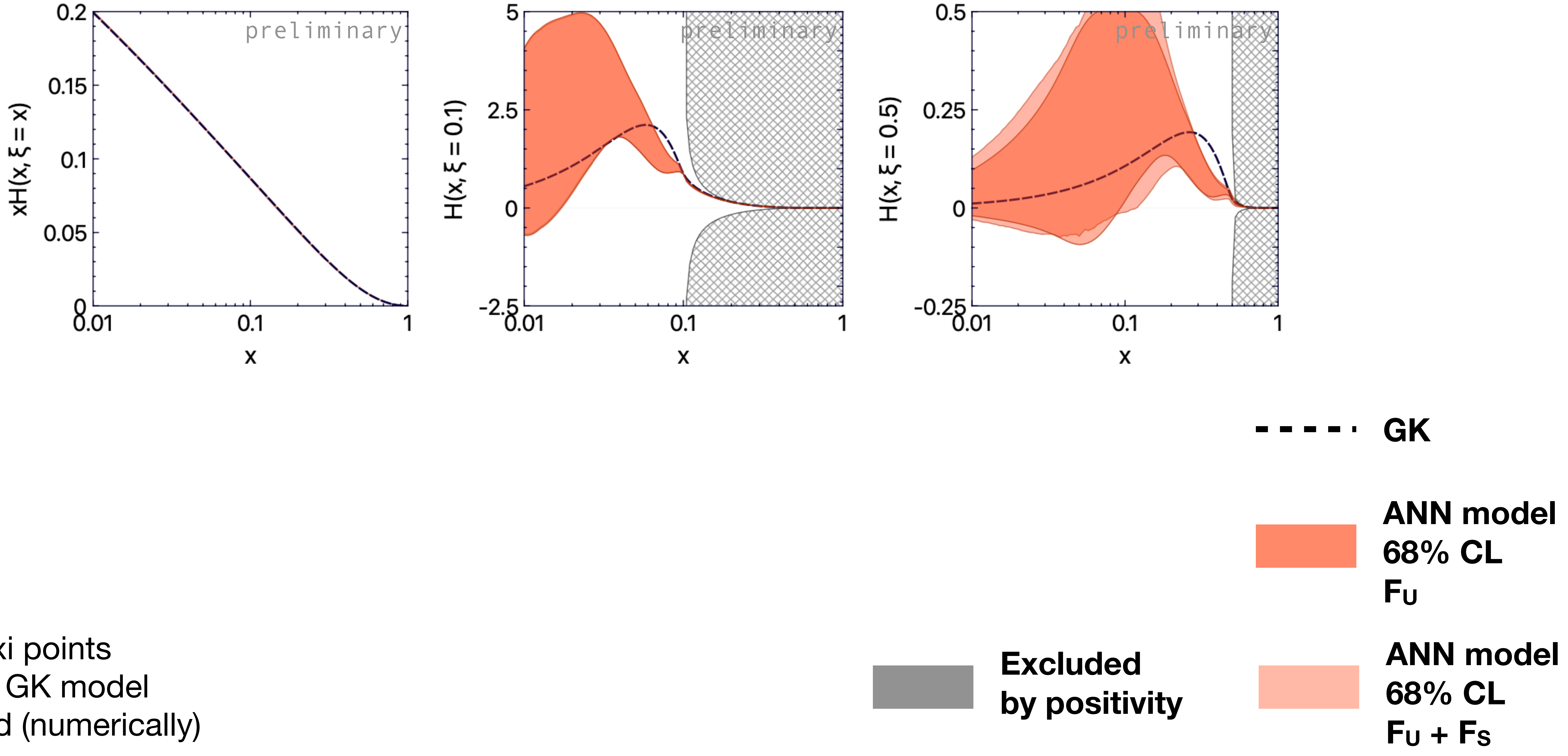
ANN model 68% CL F_U

ANN model 68% CL $F_U + F_S$

Conditions:

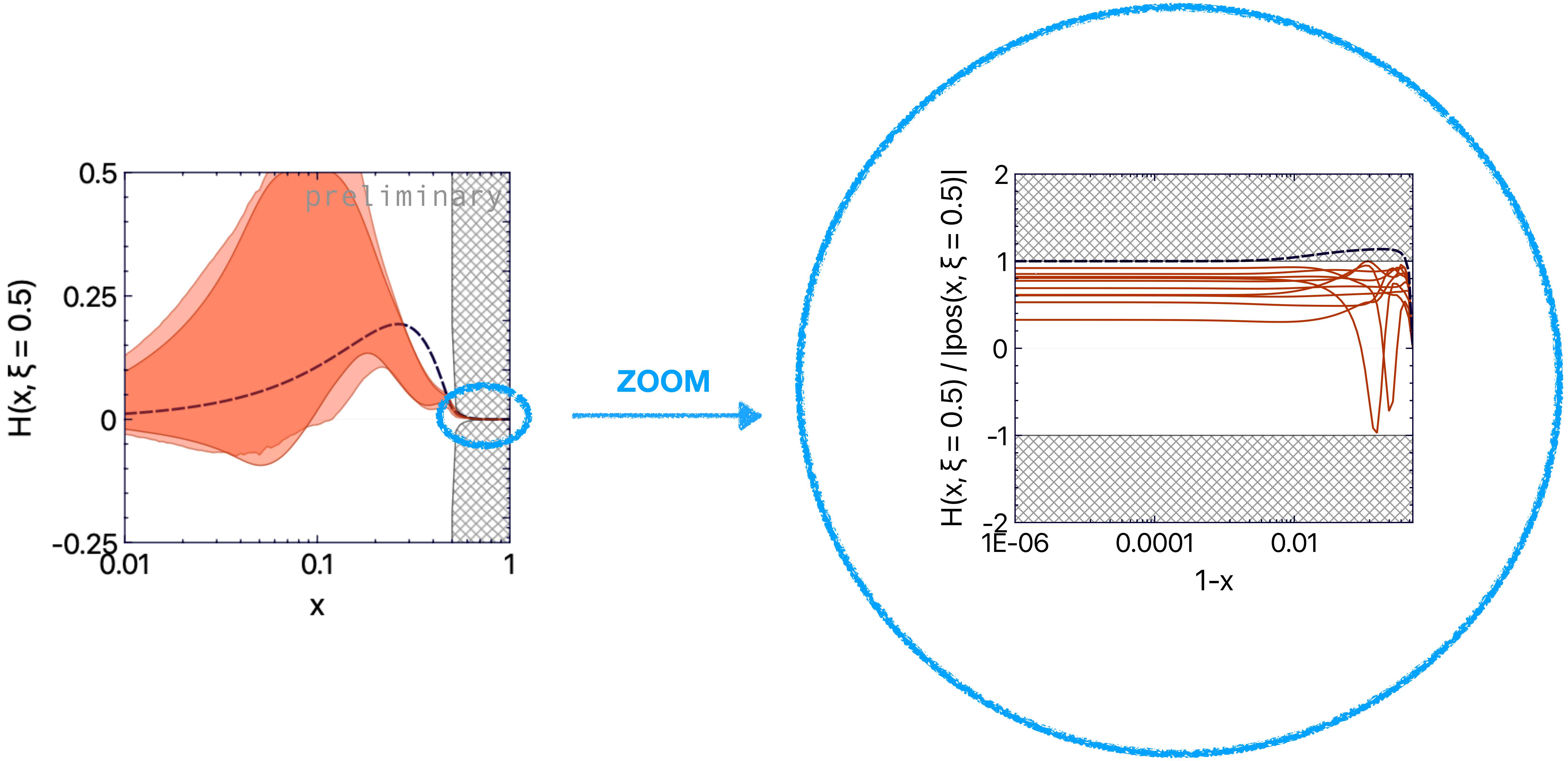
- Input: 200 x = xi points generated with GK model
- Positivity not forced

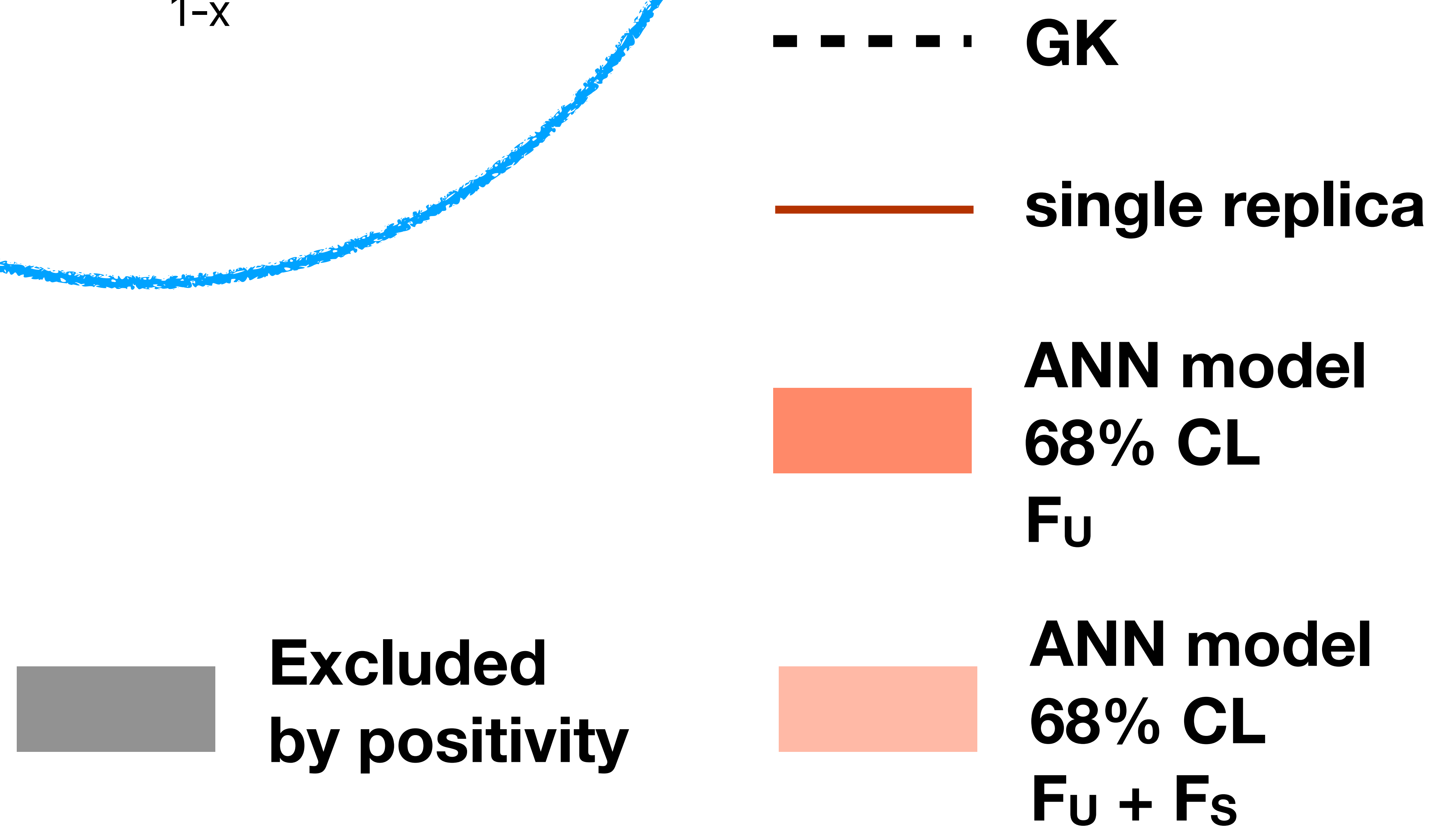
Demonstration of results



Conditions:

- Input: 200 x = xi points generated with GK model
- Positivity forced (numerically)





Conditions:

- Input: 200 x = xi points generated with GK model
- Positivity forced

Summary

- We propose new ways of modeling GPDs based on ANNs
- Our models fulfil all theory-driven constraints (including positivity)
- Can easily accommodate lattice-QCD results
- These are new tools to address the long-standing problem of model dependency of GPDs
- Easy extension to the t-dependent case
- Easy to plug in any x-space dependent GPD computing code
  (with evolution, coefficient functions, etc.) and thus ready for multi-channel phenomenology