The Evolution of Primordial Neutrino Helicities in Cosmic Gravitational and Magnetic Fields

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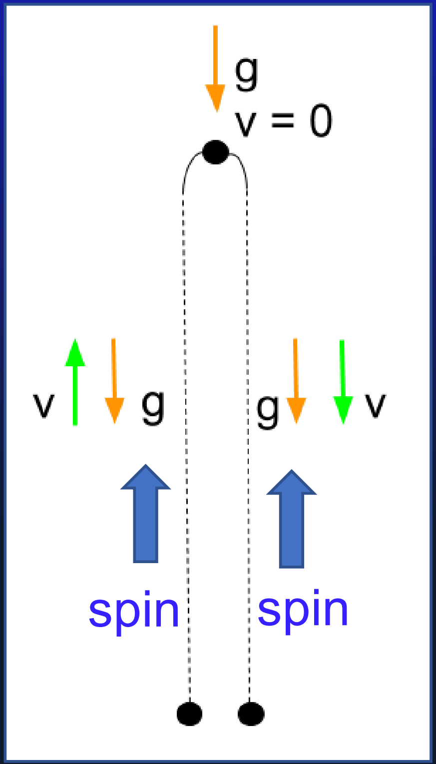
with **Jen-Chieh Peng** (following talk)

PRL 126, 191803 (2021) (magnetic)
PRD 103, 123019 (2021)(gravitational)



SPIN2021
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Relic neutrinos

Density of neutrinos left over from the big bang is about 338 /cm^3
(100 X solar neutrino density)
Some 20,000,000 our bodies now – only unprocessed relic of big bang

At least two of the three neutrino mass states are non-relativistic now:
$v < 1/4\ c$

But they all have a relativistic fermion distribution $f(p) = \frac{1}{e^{p/T} + 1}$

Could be Dirac or Majorana (own antiparticle)

Although they start out essentially with left handed helicity, gravitational inhomogeneities can change their helicity --

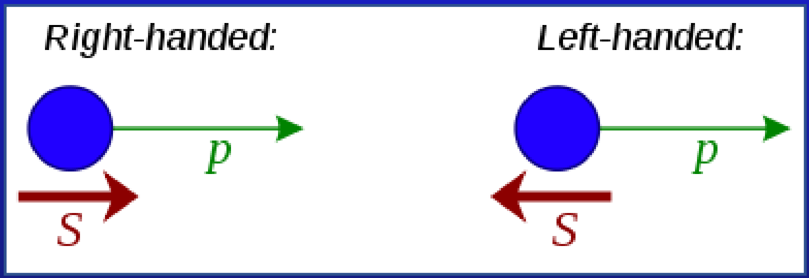
-- cosmic and galactic magnetic fields can also change helicity of Dirac relic neutrinos.

-- Never detected

What happens to neutrinos between 1 sec and now, 13.8 billion years later?

Neutrinos have negative helicity: L handed

& antineutrinos positive helicity: R handed



A property of the weak interaction processes, not an intrinsic property of neutrinos

Both cosmic, and later galactic, magnetic fields as well as gravitational inhomogeneities can rotate the spins with respect to the momentum, and thus give neutrinos an amplitude to be right handed, and antineutrinos left handed!

The helicities of relic neutrinos are a new probe of cosmic gravitational and magnetic fields.

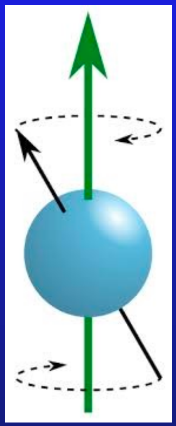
Magnetic field B rotates spins, but not momenta:

Since ν have non-zero mass they have magnetic moment

$$\frac{d\vec{S}_\perp}{dt} = 2\mu_\nu \left(\vec{S}_\parallel \times \vec{B}_\perp + \frac{1}{\gamma} \vec{S}_\perp \times \vec{B}_\parallel \right)$$

Bargmann-Michel-Telegdi equation

μ_ν = magnetic moment and $\gamma = 1/\sqrt{1-v^2}$ of neutrino



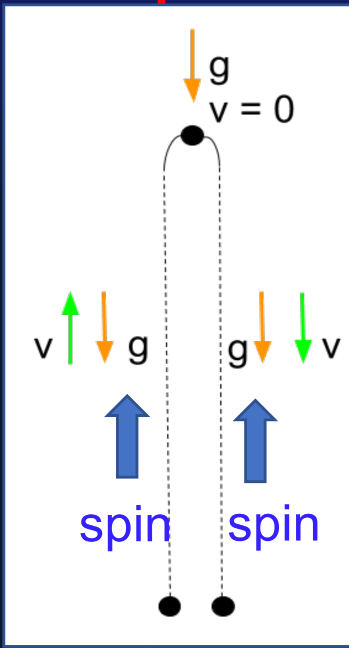
Gravitational potential Φ rotates momentum and spin:

$$\left.\frac{d\hat{p}}{dt}\right|_\perp = -\left(v + \frac{1}{v}\right) \vec{\nabla}_\perp \Phi \quad , \qquad \left.\frac{d\vec{S}}{dt}\right|_\perp = -\frac{2\gamma+1}{\gamma+1} \vec{S}\cdot\vec{v}\, \vec{\nabla}_\perp \Phi$$

relativistic effect

Spin bending lags momentum bending
(helicity = $h = \hat{p} \cdot \hat{S}$)

$$\left(h \frac{d\hat{S}}{dt} - \frac{d\hat{p}}{dt} \right)_\perp = \frac{m}{p} \vec{\nabla}_\perp \Phi$$



Spin and momentum bent equally for massless particle (photon); no spin bending of non-relativistic particle

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Evolution of primordial neutrinos from freezeout

Neutrinos produced in flavor eigenstates, linear superpositions of mass eigenstates 1,2,3,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Pontecorvo–Maki–Nakagawa–Sakata
PMNS mixing matrix

and in wave packets of size
~ electron mean free path $1/\alpha^2 T \sim 10^6 - 10^7 \mathrm{fm}$
and velocity $v = p/\sqrt{p^2 + m^2}$

Velocity dispersion of mass components $\delta v = (\Delta m/m) m^2/p^2$
>> velocity dispersion $(\delta p/p) m^2/p^2$ of given mass component

Flavor eigenstate (a) arrives at Earth in three well separated mass packets with relativistic thermal distributions:

$$f_a(p) = \sum_i \frac{|U_{ai}|^2}{e^{p/T_e} + 1}$$

Neutrino propagation in an expanding universe

Metric of expanding universe with weak gravitational inhomogeneities:

$$ds^2 = a(u)^2[-(1+2\Phi)du^2 + (1-2\Phi)d\vec{x}^{\,2}]$$

a = scale factor grows from ~10^{-10} at T =1 MeV to a=1 now
u = conformal time, dt = a du
x = comoving spatial coordinates

Φ = weak potential, driven by density and pressure fluctuations

$$\nabla_x^2\Phi = 4\pi G\left(\delta\rho(\vec{x}\,) + 3\delta P(\vec{x}\,)\right)a(u)^2$$

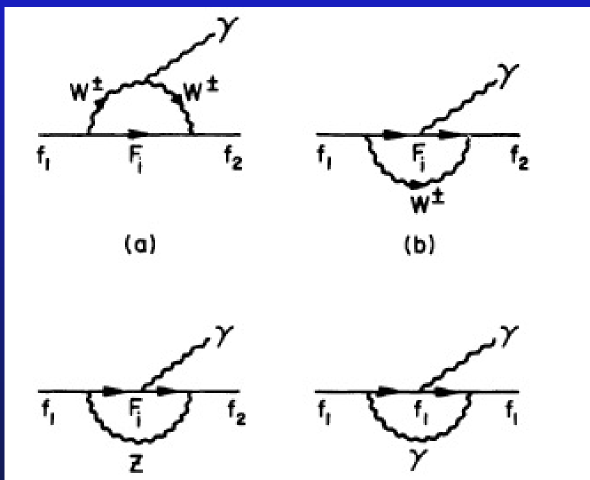
Φ(x) independent of *a*, at long wavelengths $\delta\rho \;\; a^2 \propto a^0$

Radiation dominated era ($P = \rho/3$) , down to redshift ~ 10^4 :

$$\delta\rho/\bar{\rho} \sim a^2, \quad \delta\rho \sim 1/a^2$$

Matter dominated era, from redshift ~ 10^4 to now, $\delta\rho/\bar{\rho} \sim a, \quad \delta\rho \sim 1/a^2$

Rotation of neutrino spins in magnetic fields via neutrino magnetic moment



Standard model processes lead to a non-zero neutrino magnetic moment

$$\mu_\nu^{\rm SM} \simeq \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu \simeq 3 \times 10^{-21} m_{-2}\, \mu_B$$

Fujikawa-Schrock PRL 1980

μ_B = Bohr magneton $= e/2m_e$

$m_{-2} = m_\nu / 10^{-2}{\rm eV}$

But the magnetic moment could be much larger (BSM physics!)

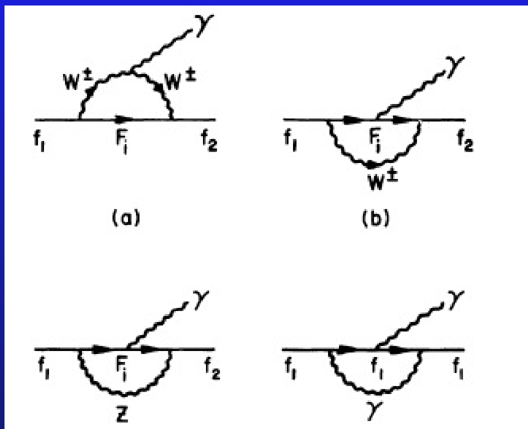
Upper bounds: $\mu_\nu < 2.9 \times 10^{-11} \mu_B$ GEMMA Kalinin reactor expt (2010) $\bar{\nu} + e^-$

$\mu_{\nu_e} < 2.8 \times 10^{-11} \mu_B$ Borexino (2017, solar $\nu + e^-$)

Theoretical "naturalness" bound: $\mu_\nu \lesssim 10^{-16} m_{-2} \mu_B$

Bell et al. PRL 2005

Diagonal vs. transition magnetic moments



Diagonal: interaction with magnetic field between equal mass states (neutrino $m_1 = m_2$)

Transition: interaction only between different mass states ($m_1 \neq m_2$)

Are neutrinos Dirac or Majorana fermions?

Dirac neutrinos can have both diagonal and transition moments.

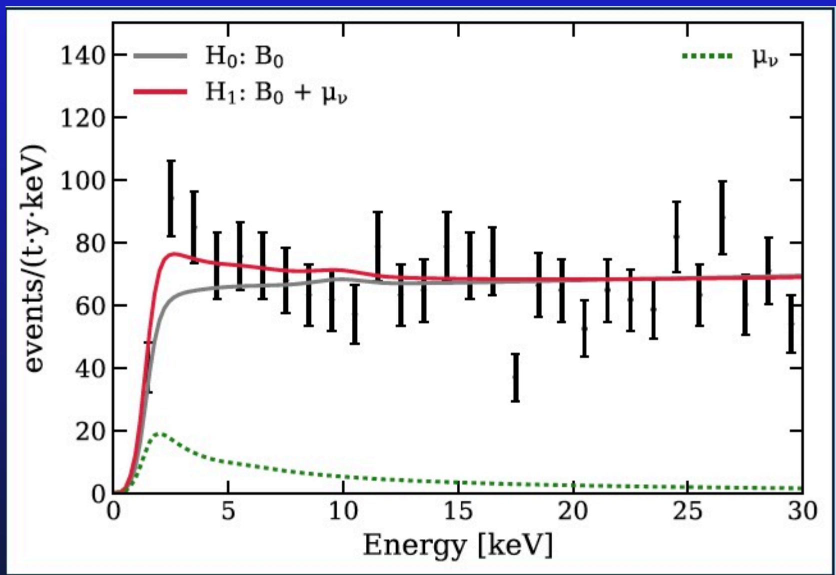
Diagonal moments of Majorana neutrinos identically zero; only transition moments: $\mathrm{CPT} => \langle i|\mu_\nu \vec{S}|j\rangle = -\langle \bar{j}\,|\mu_\nu \vec{S}|\bar{i}\,\rangle$

Propagation through cosmic and galactic magnetic fields cannot change neutrino mass state.

Only Dirac neutrinos can have helicities changed by magnetic fields.

XENON1T low energy (1-7 keV) electron event excess

Aprile et al. PR D 102, 072004 (2020)

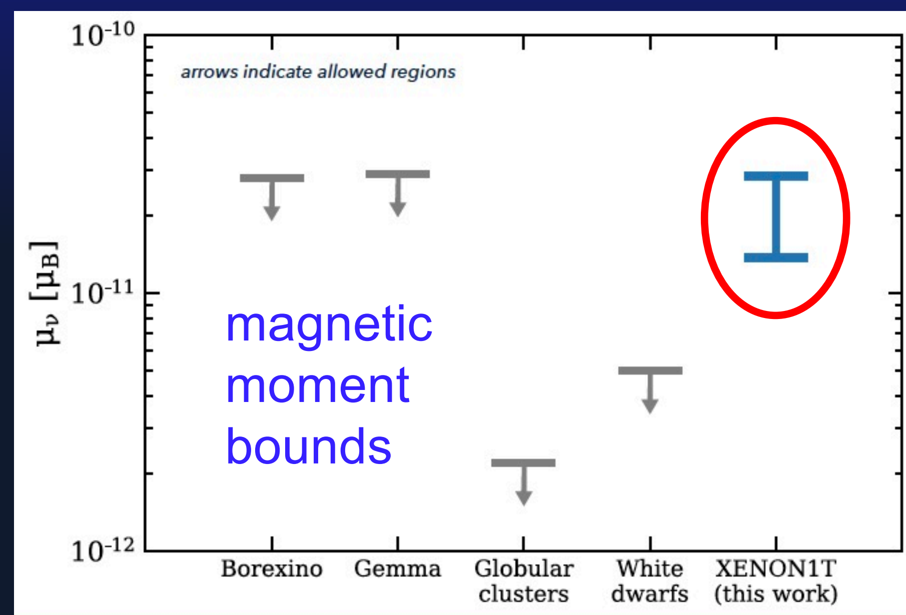


Possible explanations:

Large neutrino mag. moment (3.2σ)
Solar axions (3.5σ)
Tritium (in Xe) beta decays

Excess consistent with neutrino magnetic moment

$$\mu_{\nu,1T} \sim 1.4 - 2.9 \times 10^{-11} \mu_B$$



Beyond Standard Model physics??

No information on whether diagonal or transition moment

Spin precesses in magnetic field, but momentum does not

(neutrinos are electrically neutral)

Magnetic fields change neutrino helicity: $h = \hat{S} \cdot \hat{p}$

Spin rotation by angle θ => helicity reversal probability $\sin^2(\theta/2)$

Define spin in rest frame of neutrino.

Precession in terms of magnetic field in "lab" frame for small angles

BMT: $$\frac{d\vec{S}_\perp}{dt} = 2\mu_\nu \vec{S}_\parallel \times \vec{B}_\perp$$ where ⊥ is transverse to momentum

Cumulative spin rotation along ν trajectory:

$$\frac{\vec{S}_\perp}{|\vec{S}\,|} = \pm 2\mu_\nu \int dt\, \hat{v} \times \vec{B}(t)$$

Apply to galaxies, and
to cosmic magnetic fields

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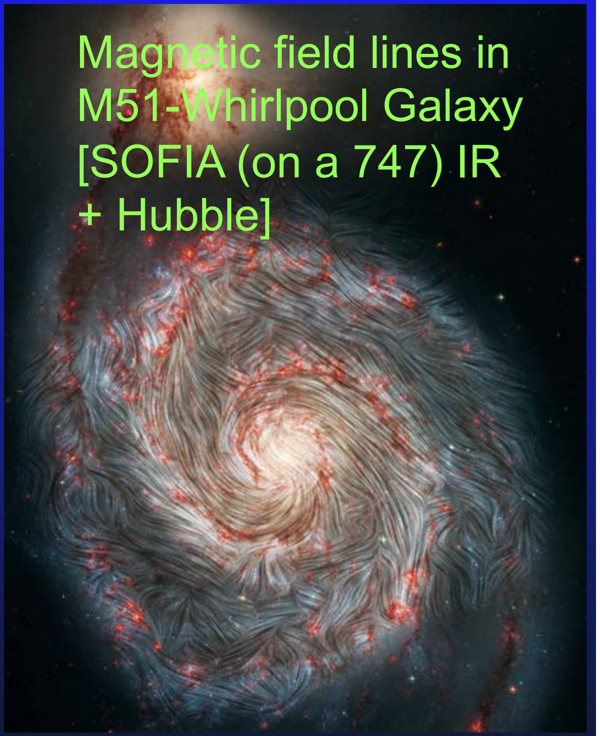
Neutrino spin rotation by galactic magnetic field

For uniform galactic magnetic field: $\theta_g \sim 2\mu_\nu B_g \frac{\ell_g}{v}$

ℓ_g = mean crossing distance of the galaxy

Galactic fields uniform only over coherence length Λ_g ~ kpc. Spin random walks in magnetic field.

$$\langle\theta^2\rangle_g \simeq \left(2\mu_\nu B_g \frac{\Lambda_g}{v}\right)^2 \frac{\ell_g}{\Lambda_g}$$



ex., Milky Way with characteristic parameters (spherical cow approx):

$$B_g \sim 10\,\mu\mathrm{G}, \ell_g \sim 16\mathrm{kpc}, \Lambda_g \sim \mathrm{kpc}$$

$$\langle\theta^2\rangle_{\mathrm{MW}} \sim 4 \times 10^{29} m_{-2}^2 \left(\frac{\Lambda_g}{1\,\mathrm{kpc}}\right)\left(\frac{B_g}{10\mu\mathrm{G}}\right)^2\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

$\mu_\nu \sim 1.5 \times 10^{-15}\mu_B \sim 10^{-4}\mu_{1T} => \sqrt{\langle\theta^2\rangle} \sim 1$: helicity randomizes

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Neutrino spin rotation by cosmic magnetic fields

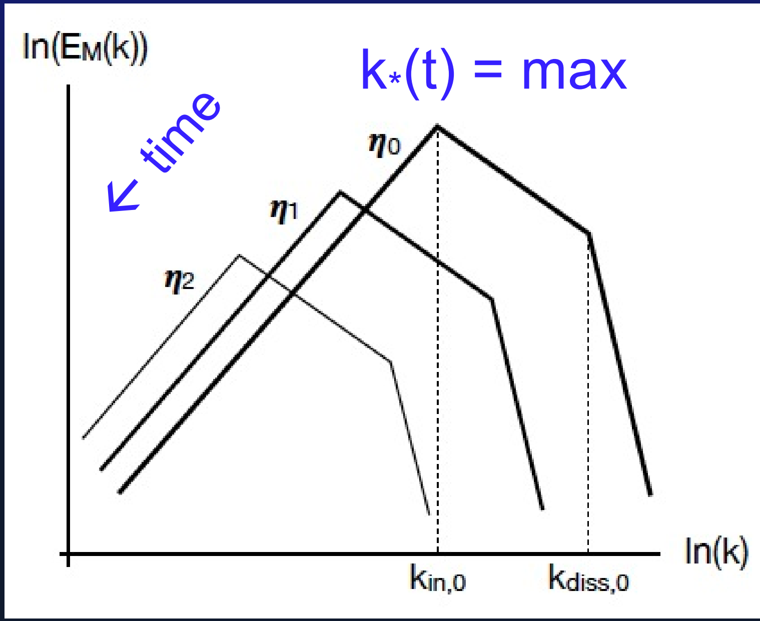
$$\frac{\vec{S}_\perp}{|\vec{S}\,|} = \pm 2\mu_\nu \int dt\, \hat{v} \times \vec{B}(t) \quad \Rightarrow \quad \langle \theta^2 \rangle_c = 4\mu_\nu^2 \Big\langle \Big(\int dt \vec{B}_\perp(t) \Big)^2 \Big\rangle$$

perp to v

Magnetic field correlation function:

$$\langle B_i(\vec{x}) B_j(\vec{x}\,') \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{2} P_B(k) e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}$$

+ helical part (no role here)



Schematic of $P_B(k)$ with increasing conformal time (η =u): $\eta_0 < \eta_1 < \eta_2$

T. Vachaspati, Rep. Prog. Phys. **84** 074901 *(2021)*

$$k < k_* : P_B \sim k^s, s \sim 2$$

$$k > k_* : P_B \sim k^{-q}, q \sim 2 + 5/3$$

sum rule: $$\int \frac{d^3k}{(2\pi)^3} P_B(k) = \langle \vec{B}^{\,2} \rangle$$

$$P_B(k) = (2\pi)^2 E_M(k)/k^2$$

$$\langle\theta^2\rangle_c \simeq \mu_\nu^2 \pi \int_{u_d}^{u_0} du\, a(u)^2 \frac{\langle\vec{B}^2\rangle(u)}{k_*(u)}$$

Conservation of flux: $a^2B\sim$ const. => $\langle\vec{B}^2(u)\rangle \simeq B_0^2/a(u)^4$ (0= now)

$$k_*(u) \sim \frac{2\pi}{\Lambda_0 a(u)^{1/2}}$$

(Λ_0 = coherence length of cosmic B field)

$$\langle\theta^2\rangle_c = \frac{1}{2}\mu_\nu^2 B_0^2 \Lambda_0 \int_{u_d}^{u_0} \frac{du}{a(u)^{3/2}}$$

Main contribution is from radiation-dominated era (a ~ u):
from neutrino decoupling, u_d ($a_d \sim 10^{-10}$)
to matter-radiation equality, u_{eq} ($a_{eq} \sim 0.8 \times 10^{-4}$)

$$\langle\theta^2\rangle_c \simeq 9\left(\frac{\Lambda_0}{R_u}\right)\frac{(\mu_\nu t_0 B_0)^2}{(a_{eq}a_d)^{1/2}}$$

$R_u = cu_0$ = radius of universe
$u_0 = 3t_0$

$$\simeq 2\times 10^{27}\left(\frac{\Lambda_0}{1\,\mathrm{Mpc}}\right)\left(\frac{B_0}{10^{-12}\mathrm{G}}\right)^2\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

Cosmic magnetic field rotation of neutrino spin

$$\langle \theta^2 \rangle_c \simeq 2 \times 10^{27} \left(\frac{\Lambda_0}{1\,\mathrm{Mpc}} \right) \left(\frac{B_0}{10^{-12}\mathrm{G}} \right)^2 \left(\frac{\mu_\nu}{\mu_B} \right)^2$$

A magnetic moment $\mu_\nu \sim 10^{-3} \mu_{1T} \sim 10^{-14} \mu_B$
(naturalness upper bound) would be experimentally significant :
~1% of neutrinos could flip helicity

Effects of standard model magnetic moment insignificant.

If the neutrino is Majorana, no helicity changes from magnetic fields.

To within uncertainties in magnetic fields, correlation lengths, and neutrino masses, spin rotation in cosmic magnetic fields ~ galactic

Rotation of neutrino spins by gravitational inhomogeneities

Gravitational potential Φ rotates momentum and spin:

$$\frac{d\hat{p}}{dt}\Big|_\perp = -\left(v + \frac{1}{v}\right)\vec{\nabla}_\perp \Phi \qquad , \qquad \frac{d\vec{S}}{dt}\Big|_\perp = -\frac{2\gamma+1}{\gamma+1}\vec{S}\cdot\vec{v}\,\vec{\nabla}_\perp\Phi$$

Spin bending lags momentum bending $\left(h\frac{d\hat{S}}{dt} - \frac{d\hat{p}}{dt}\right)_\perp = \frac{m}{p}\vec{\nabla}_\perp\Phi$

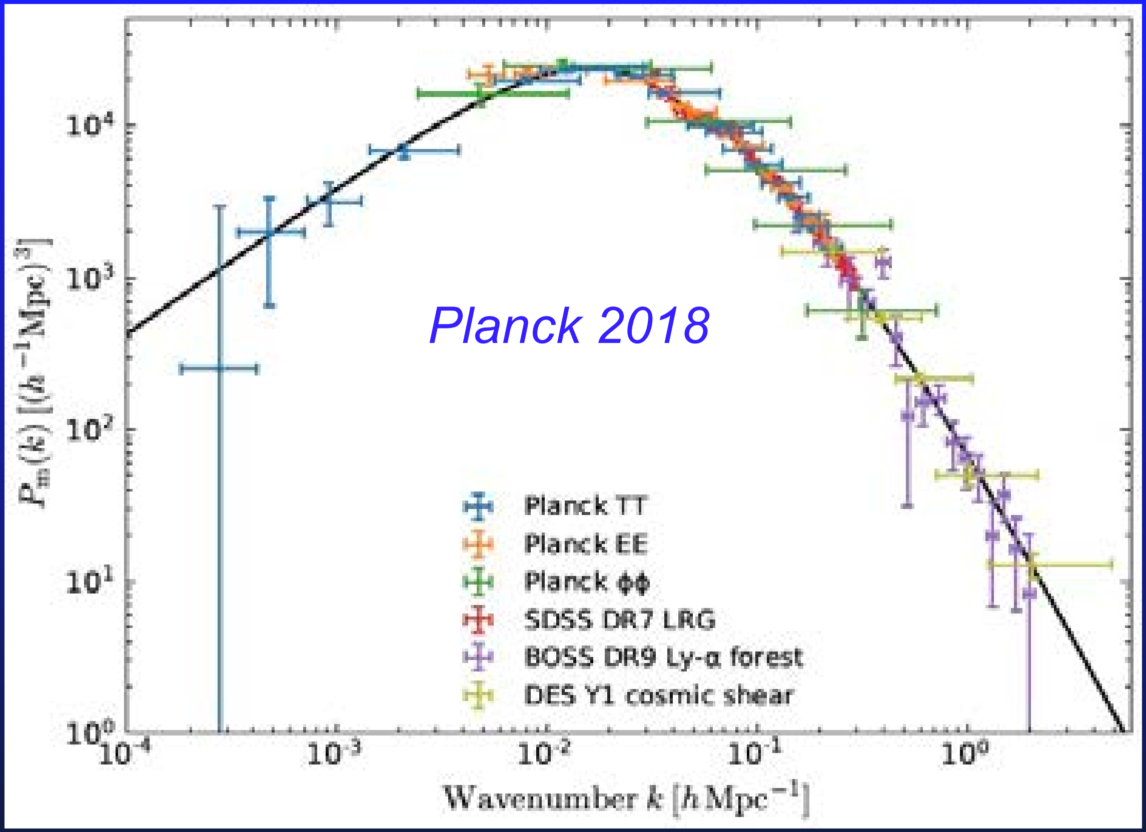
Again, neutrino undergoes a random walk through the inhomogeneities.

For massless neutrino, momentum bending angle is:

$$\langle(\Delta\theta_p)^2\rangle = 4\int dx_3 dx'_3 \nabla_{x\perp}\cdot\nabla_{x'\perp}\langle\Phi(x_3)\Phi(x'_3)\rangle$$

Relate gravitational fluctuation power spectrum, $\langle\Phi(x)\Phi(x')\rangle$ to primordial density fluctuation spectrum ($\nabla_x^2\Phi = 4\pi G\left(\delta\rho(\vec{x}\,) + 3\delta P(\vec{x}\,)\right)a(u)^2$)

$$\langle\delta\rho(\vec{x})\,\delta\rho(\vec{x}\,')\rangle = (\bar{\rho})^2\int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}P(k)$$



Density fluctuation spectrum

$P(k) \sim k$ for $k < k_{max}$

(Harrison-Zel'dovich)

$P(k) \sim k^{-\nu}$ for $k > k_{max}$

Scales as

a^2 in matter dom. era

a^4 in rad. dom. era, $k < k_{max}$

$$\langle (\Delta\theta_p)^2 \rangle = \frac{2}{\pi}\zeta \int du (4\pi G \bar{\rho} a^2)^2 \int dk \frac{P(k)}{k}$$

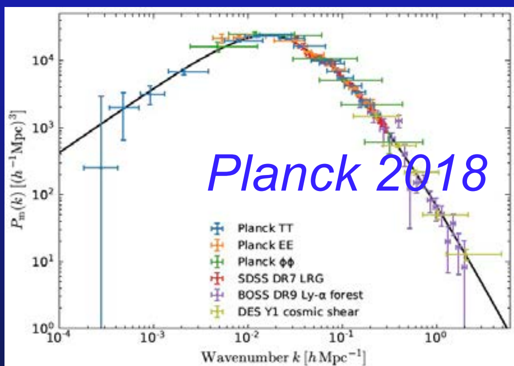
Radiation dom. ζ=4
Matter dom ζ=1

At present $$\int (dk/k) P_0(k) \simeq 7.25 \times 10^4 (\mathrm{Mpc/h})^3 \equiv \mathcal{P}$$

h = Hubble parameter ~ 0.7

Gravitational lensing of cosmic neutrino background

$$\langle(\Delta\theta_p)^2\rangle = \frac{2}{\pi}\zeta\int du(4\pi G\bar{\rho}a^2)^2\int dk\frac{P(k)}{k}$$



$$\langle(\Delta\theta_p)^2\rangle = \frac{9}{2\pi}\mathcal{P}H_0^3\int_0^1\frac{da}{a^2}\left(\Omega_M a+\Omega_V a^4\right)^{3/2}$$

$$\simeq 2.2\times 10^{-6}$$

Ω_M = matter fraction, Ω_V = dark energy fraction

RMS momentum bending = lensing of cosmic neutrino background ~ 5.1 arcmin

Lensing of CMB ~ 2.7 arcmin. Most efficient at smaller z (≲10).
Reionization of intergalactic H => photon-e scattering.

(Weak electron-neutrino scattering after reionization insignificant)

Gravitational spin rotation with respect to momentum, Θ

Main effect in matter dominated era from redshift ~ 10^4 to now.
Slower neutrinos have greater rotation of momentum vs spin

Momentum rotation with finite neutrino mass:

Neutrino velocity $v(a) = p_0/\sqrt{p_0^2 + (m_\nu a)^2}$. p_0 = present momentum

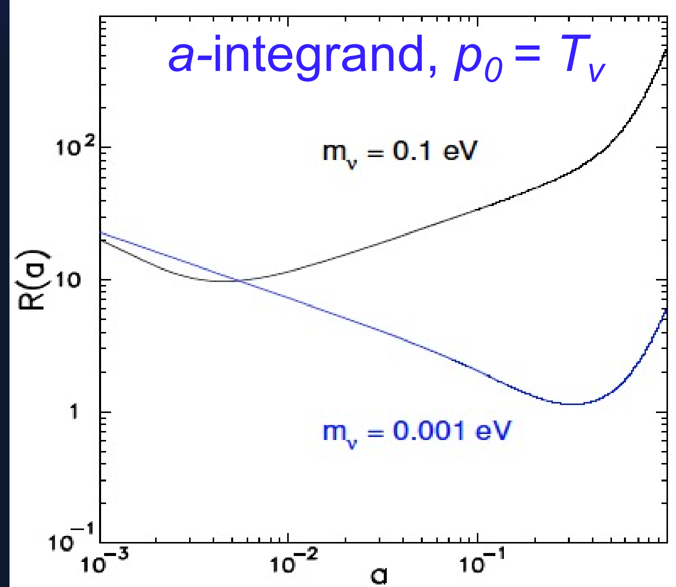
$$\langle(\Delta\theta_p)^2\rangle = \frac{9}{8\pi}\mathcal{P}H_0^3\int_{a_{eq}}^1 \frac{da}{a^2}\left(\Omega_M a + \Omega_V a^4\right)^{3/2} v(a)\left(v(a) + \frac{1}{v(a)}\right)^2$$

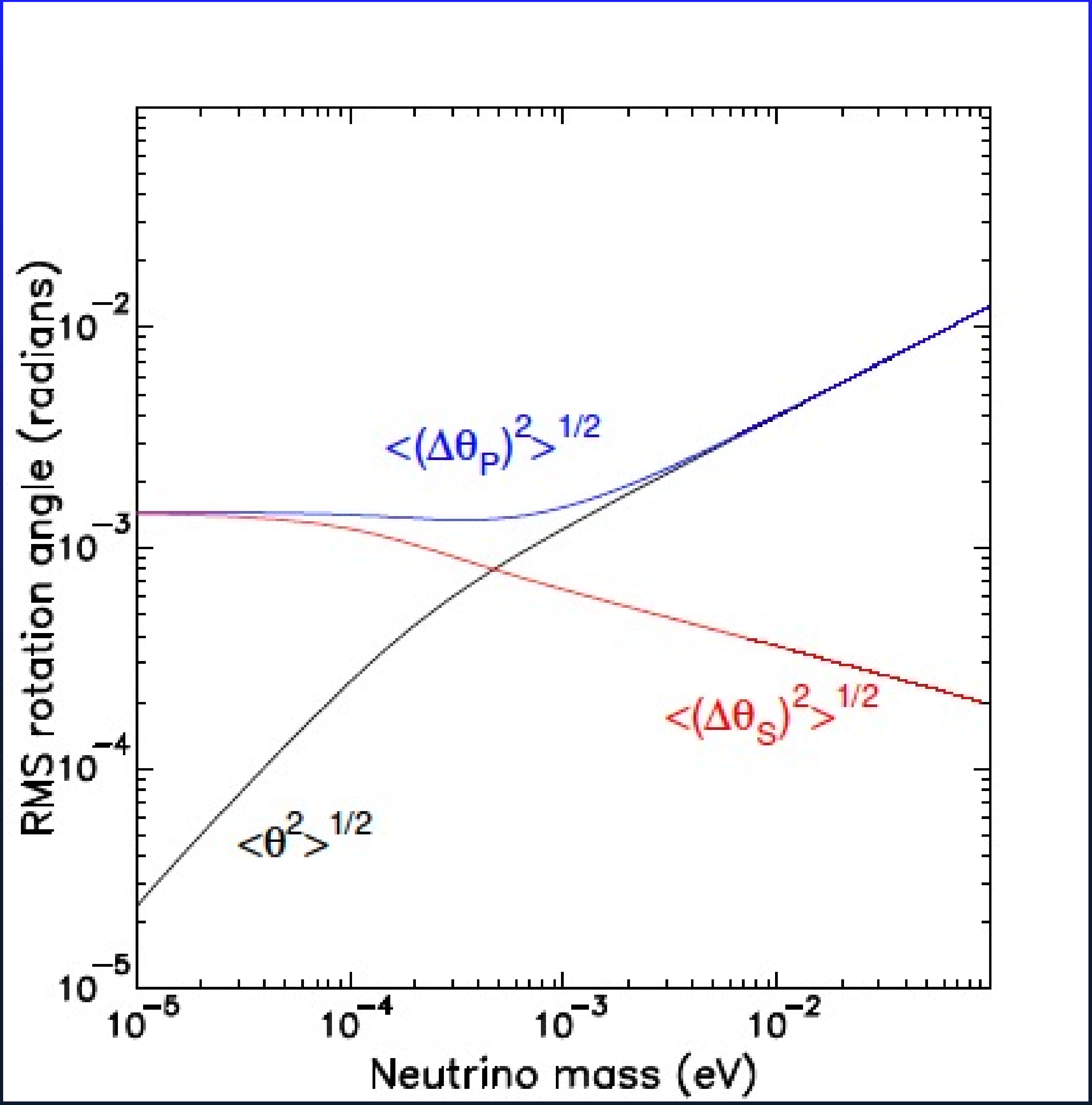
Ω_M = matter fraction, Ω_V = dark energy fraction

Spin rotation with respect to momentum.

$$\langle\theta^2\rangle = \frac{9}{8\pi}\mathcal{P}H_0^3\int_0^1 \frac{da}{a^2}\left(\Omega_M a + \Omega_V a^4\right)^{3/2}\left(\frac{1}{v} - v\right)$$

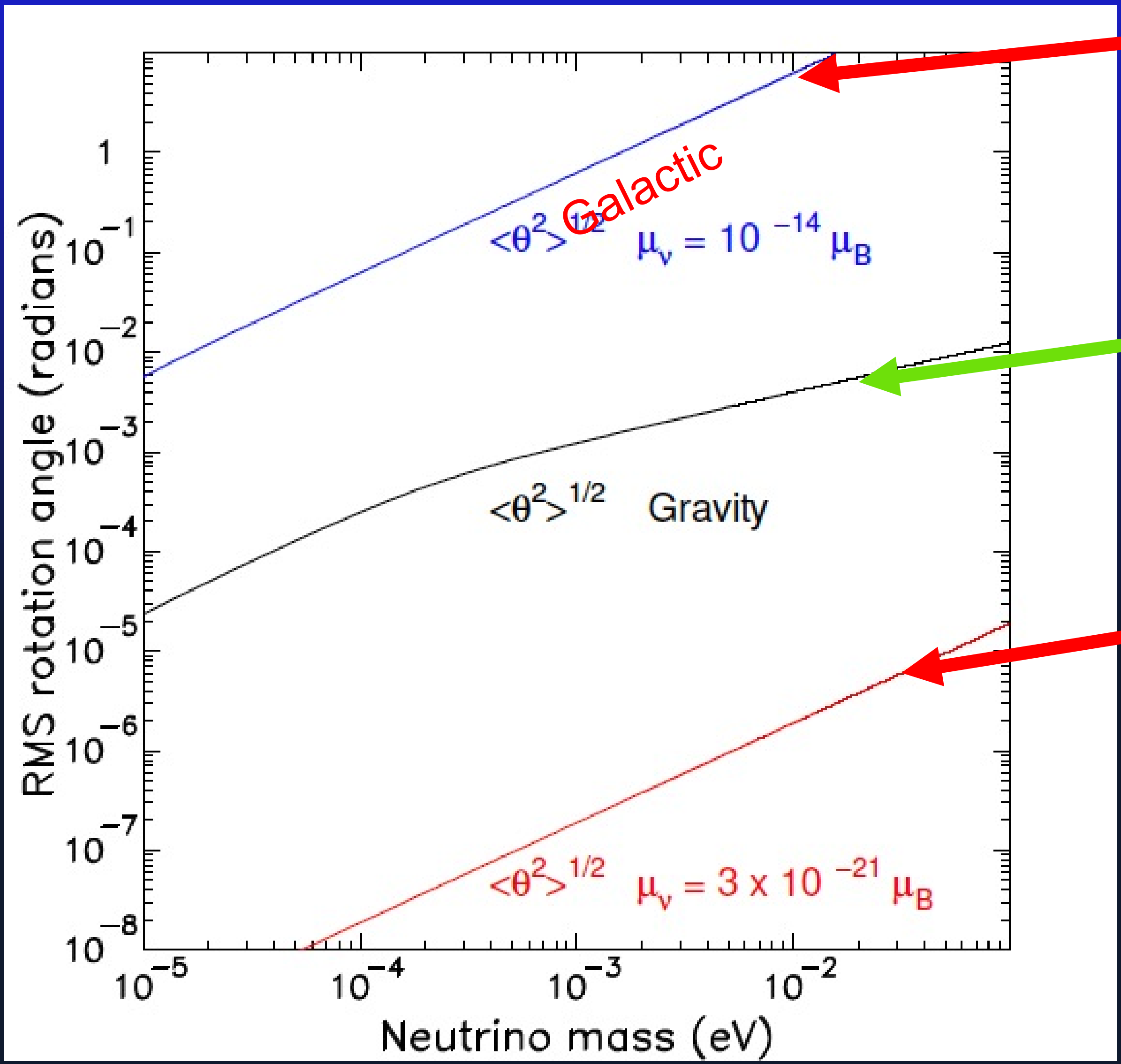
Measure of helicity changes





RMS rotation angle (radians)
Neutrino mass (eV)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
$<(\Delta\theta_P)^2>^{1/2}$
$<(\Delta\theta_S)^2>^{1/2}$
$<\theta^2>^{1/2}$

Spin rotation from gravitational vs. magnetic fields



Rotation in Milky Way

$$B_g = 10\,\mu\text{G},\ \Lambda_g = 1\,\text{kpc}$$

$$\mu_\nu = 10^{-14}\mu_B$$

Gravitational rotation

GB+JCP PRD

Rotation in Milky Way with standard model magnetic moment

$$\mu_\nu^{\rm SM} \simeq 3\times 10^{-19} m_{\rm eV}\,\mu_B$$

Probability of helicity flip

$$\sim \langle\theta^2\rangle/4$$

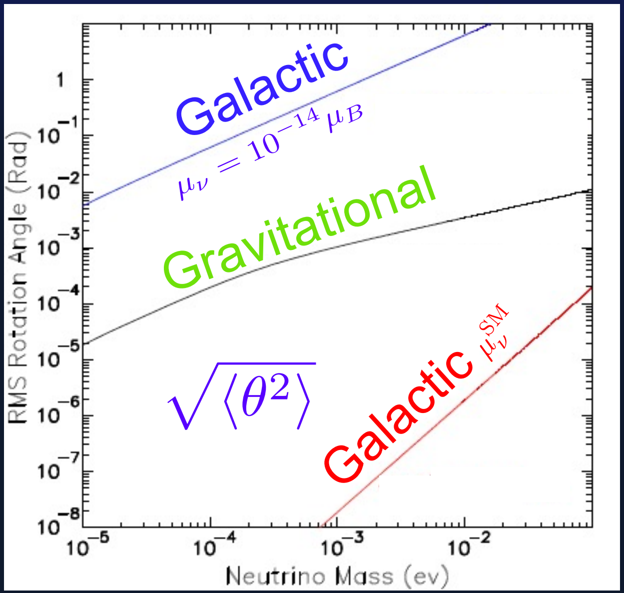
Conclusions

Relic neutrino helicities new probe of cosmic gravitational and magnetic fields

Significant helicity changes of relic neutrinos for neutrino magnetic moment μ_ν even three-four orders of magnitude smaller than suggested by XENON1T.

Gravitational helicity changes few orders of magnitude smaller cf. large μ_ν rotations, but much larger than for μ_ν in Standard Model

New processes for generation of sterile neutrinos

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