Useful relations and sum rules for PDFs and multiparton distribution functions of spin-1 hadrons

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Reference: S. Kumano and Qin-Tao Song, JHEP 09 (2021) 141

Contents

What will be discussed in this talk

- Decomposition of f_{LT} in spin-1 hadrons
- Quark-gluon-quark correlation functions for spin-1 hadrons
- Wandzura-Wilczek (WW) type relation for spin-1 hadrons
- Burkhardt-Cottingham (BC) type sum rule for spin-1 hadrons

Spin parameters for spin-1 hadrons

Spin parameters for spin-1 hadrons

- Vector polarization S (3 parameters): same as the case of proton
- Tensor polarization T (5 Parameters): unique, investigated in this talk

$$T^{\mu\nu} = \frac{1}{2}\left[\frac{4}{3}S_{LL}\frac{(P^+)^2}{M^2}\bar{n}^\mu\bar{n}^\nu - \frac{2}{3}S_{LL}(\bar{n}^{\{\mu}n^{\nu\}} - g_T^{\mu\nu}) + \frac{1}{3}S_{LL}\frac{M^2}{(P^+)^2}n^\mu n^\nu + \frac{P^+}{M}\bar{n}^{\{\mu}S_{LT}^{\nu\}} - \frac{M}{2P^+}n^{\{\mu}S_{LT}^{\nu\}} + S_{TT}^{\mu\nu}\right],$$

$$n^\mu = \frac{1}{\sqrt{2}}(1,\,0,\,0,\,-1),\quad \bar{n}^\mu = \frac{1}{\sqrt{2}}(1,\,0,\,0,\,1)$$

A. Bacchetta and P. Mulders, PRD 62, 114004 (2000)

Tensor-polarized PDFs of spin-1 hadrons

$$\Phi(x,P,T)=\frac{1}{2}\Big[S_{LL}\not{\bar{n}}\, f_{1LL}(x)+\frac{M}{P^+}S_{LL}\,e_{LL}(x)+\frac{M}{P^+}\not{S}_{LT}\,f_{LT}(x)+\frac{M^2}{(P^+)^2}S_{LL}\not{n}\, f_{3LL}(x)\Big]$$

Twist-2 (f_{1LL}) → simliar as g_1 in proton

Twist-3 (e_{LL}, f_{LT}) → simliar as g_T in proton (f_{LT})

Twist-4 (f_{3LL})

Can we derive similar relations as the proton case?

1) Decomposition of g_T

2) Wandzura-Wilczek (WW) relation

3) Burkhardt-Cottingham (BC) sum rule

P. Hoodbhoy, R. L. Jaffe, and A. Manohar, NP B312 (1989) 571

L. L. Frankfurt and M. I. Strikman, NP A405 (1983) 557.

S. Kumano and Qin-Tao Song, PRD 103 (2021) 014025.

J. P. Ma, C. Wang and G. P. Zhang, arXiv:1306.6693 [hep-ph].

QCD operator expansion

Fock-Schwinger gauge is used for gluon field.

$$x_\mu A^\mu(x) = 0$$

Gluon field can be expressed as

$$A^\nu(\xi) = \int_0^1 dt\, t\, \xi_\mu G^{\mu\nu}(t\xi)$$

$$G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig\,[A^\mu, A^\nu]$$

If move to the light-cone limit of x, light-cone gauge is obtained.

$$A^+ = 0$$

QCD operator expansion

S. Kumano and Qin-Tao Song, JHEP 09 (2021) 141

Try to reexpress the twist-3 operator:

$$\xi_\mu\,\bar\psi(0)\,(\overrightarrow{\partial}^\mu\gamma^\alpha - \overrightarrow{\partial}^\alpha\gamma^\mu)\,\psi(\xi) = g\int_0^1 dt\,\bar\psi(0)\left[i(t-\frac{1}{2})G^{\alpha\mu}(t\xi) - \frac{1}{2}\gamma_5\widetilde{G}^{\alpha\mu}(t\xi)\right]\xi_\mu \not\xi\psi(\xi)$$

→ quark-gluon-quark terms

$$+\frac{g}{2}\int_0^1 dt\,\bar\psi(0)\left[\xi_\mu\xi^\alpha\widetilde{G}^{\mu\sigma}(t\xi) - \xi^2\widetilde{G}^{\alpha\sigma}(t\xi)\right]\gamma_\sigma\gamma_5\psi(\xi)$$

→ twist-4 terms

$$-\frac{i}{2}\xi_\mu\bar\psi(0)\sigma^{\alpha\mu}(\overrightarrow{\not D} - m_q)\psi(\xi) - \frac{i}{2}\xi_\mu\bar\psi(0)(\overleftarrow{\not D} + m_q)\sigma^{\alpha\mu}\psi(\xi) + \frac{i}{2}\xi_\mu\bar\partial_\rho\{\bar\psi(0)\gamma^\rho\sigma^{\alpha\mu}\psi(\xi)\}$$

Equation of motion term

Total derivative term

The coordinate is not necessay light cone

Compare with the case for g_1 and g_T in proton: $\xi_\mu\,\bar\psi(0)\,(\partial^\mu\gamma^\sigma - \partial^\sigma\gamma^\mu)\,\gamma_5\,\psi(\xi)$

$$\xi_\mu\,\bar\psi(0)\,(\partial^\mu\gamma^\sigma - \partial^\sigma\gamma^\mu)\,\gamma_5\,\psi(\xi) = g\int_0^1 dt\,\bar\psi(0)\left\{i\gamma_5\left(t-\frac{1}{2}\right)G^{\sigma\rho}(t\xi) - \frac{1}{2}\tilde{G}^{\sigma\rho}(t\xi)\right\}\xi_\rho\not\xi\psi(\xi)$$

$$+\,2m_q\bar\psi(0)\gamma_5\sigma^{\sigma\rho}\xi_\rho\psi(\xi)$$

$$+\,\bar\psi(0)\gamma_5\sigma^{\sigma\rho}\xi_\rho(i\not D - m_q)\psi(\xi) - \bar\psi(0)(i\overleftarrow{\not D} + m_q)\gamma_5\sigma^{\sigma\rho}\xi_\rho\psi(\xi)$$

We shall have a similar expression, one more term which is proportional to quark mass m_q

See the papers

I. I. Balitsky and V. M. Braun, NPB 311 (1989), 541-584
J. Kodaira and K. Tanaka, Prog. Theor. Phys. 101 (1999), 191-242.

Twist-3 level operator relation

$$\xi_\mu \bar{\psi}(0)(\partial^\mu \gamma^\alpha - \partial^\alpha \gamma^\mu)\psi(\xi) = g\int_0^1 dt\, \bar{\psi}(0)\left[i\left(t - \frac{1}{2}\right) G^{\alpha\mu}(t\xi) - \frac{1}{2}\gamma_5 \widetilde{G}^{\alpha\mu}(t\xi)\right]\xi_\mu \not{\xi}\, \psi(\xi)$$

Try to calulate the matrix elements of the above relation, express them with parton distributions.

Left side: $\xi_\mu \bar{\psi}(0)(\partial^\mu \gamma^\alpha - \partial^\alpha \gamma^\mu)\psi(\xi)$

First caculate the derivatives, then take the light-cone limit.

PDFs definitions in spin-1 hadrons

$$\langle P, T \,|\, \bar{\psi}(0)\, \gamma^\mu\, \psi(\xi) \,|\, P, T\rangle_{\xi^+=0,\, \vec{\xi}_T=0}$$
$$= \int_{-1}^{1} dx e^{-ixP^+\xi^-} 2P^+ \left[S_{LL}\, \bar{n}^\mu\, f_{1LL}(x) + \frac{M}{P^+} S_{LT}^\mu\, f_{LT}(x) + \frac{M^2}{(P^+)^2} S_{LL}\, n^\mu\, f_{3LL}(x)\right]$$

PDFs definitions at twist 3 with an abitary vector of ξ

$$\langle P,T \left| \bar{\psi}(0)\gamma^{\mu}\psi(\xi) \right| P,T \rangle = \int_{-1}^{1} dx\, e^{-ixP\cdot\xi} \left[\xi\cdot T\cdot\xi \left\{ A(x)\, P^{\mu} + B(x)\, \xi^{\mu} \right\} + C(x)\, T^{\mu\nu}\xi_{\nu} \right]$$

$$A(x) = \frac{3\,M^2}{(P\cdot\xi)^2} \left[f_{1LL}(x) - \frac{4}{3} f_{LT}(x) \right], \quad B(x) = \frac{3\,M^4}{2(P\cdot\xi)^3} \left[-f_{1LL}(x) + \frac{8}{3} f_{LT}(x) \right]$$

$$C(x) = \frac{4\,M^2}{P\cdot\xi} f_{LT}(x),$$

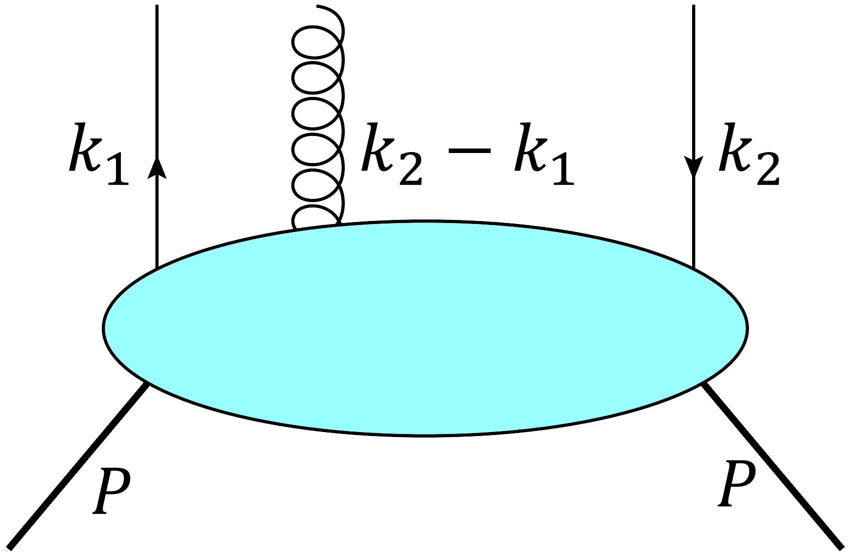
Minus x indicates the antiquark distribution.

Substitute the above relation into the matrix element of left side:

$$\xi_{\mu} \langle P,T \left| \bar{\psi}(0)(\partial^{\mu}\gamma^{\alpha} - \partial^{\alpha}\gamma^{\mu})\psi(\xi) \right| P,T \rangle = 2MS_{LT}^{\alpha} \int_{-1}^{1} dx\, e^{-ixP^{+}\xi^{-}} \left[-\frac{3}{2} f_{1LL}(x) + f_{LT}(x) - \frac{d}{dx} \{ x f_{LT}(x) \} \right]$$

$\dashrightarrow$ S_{LT} polarization

The matrix element of right side at light-cone limit



Quark-gluon-quark distributions

Quark-gluon-quark distributions is needed for spin-1 hadrons

$$\Phi_G^\alpha(x_1,x_2) = \frac{M}{2}\Big[iS_{LT}^\alpha F_{G,LT}(x_1,x_2) - \epsilon_T^{\alpha\mu} S_{LT\mu}\gamma_5 G_{G,LT}(x_1,x_2) + iS_{LL}\gamma^\alpha H_{G,LL}^\perp(x_1,x_2) + iS_{TT}^{\alpha\mu}\gamma_\mu H_{G,TT}(x_1,x_2)\Big]\not{n}$$

$\dashrightarrow$ S_{LT} polarization

There are four quark-gluon-quark distributions at twist-3 level for tensor-polarized spin-1 hadrons

S. Kumano and Qin-Tao Song, JHEP 09 (2021) 141
J. P. Ma, C. Wang and G. P. Zhang, arXiv:1306.6693 [hep-ph].

Quark-gluon-quark distributions

$$S_{LT}^{\nu} F_{G,LT}(x_1,x_2) = -\frac{i}{2M}\, g \int \frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi}\, e^{ix_1 P^+\xi_1^-}\, e^{i(x_2-x_1)P^+\xi_2^-} \times \langle P,T \,|\, \bar{\psi}(0)\, \not{n}\, n_\mu G^{\mu\nu}(\xi_2^-)\psi(\xi_1^-) \,|\, P,T \rangle$$

$$S_{LT}^{\nu} G_{G,LT}(x_1,x_2) = \frac{i}{2M}\, g \int \frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi}\, e^{ix_1 P^+\xi_1^-}\, e^{i(x_2-x_1)P^+\xi_2^-} \times \langle P,T \,|\, \bar{\psi}(0)\, i\gamma_5 \not{n}\, n_\mu \tilde{G}^{\mu\nu}(\xi_2^-)\psi(\xi_1^-) \,|\, P,T \rangle$$

Substitute the above relation into the matrix element of right side:

$$\int \frac{d(P\cdot\xi)}{2\pi} e^{ix_1 P\cdot\xi} \left\langle P,T \middle| g\int_0^1 dt\, \bar{\psi}(0)\left[i\left(t-\frac{1}{2}\right) G^{\alpha\mu}(t\xi) - \frac{1}{2}\gamma_5\, \tilde{G}^{\alpha\mu}(t\xi)\right] \xi_\mu \not{\xi}\psi(\xi) \middle| P,T \right\rangle_{\xi^+=\vec{\xi}_T=0}$$

$$= -2M S_{LT}^{\nu}\, \mathscr{P}\int_{-1}^{1} dx_2 \frac{1}{x_1-x_2}\left[\frac{\partial}{\partial x_1}\{F_{G,LT}(x_1,x_2)+G_{G,LT}(x_1,x_2)\} + \frac{\partial}{\partial x_2}\{F_{G,LT}(x_2,x_1)+G_{G,LT}(x_2,x_1)\}\right]$$

Final expression

Combine the results

$$x\,\frac{df_{LT}(x)}{dx} = -\frac{3}{2} f_{1LL}(x) - f_{LT}^{(HT)}(x)$$

$$f_{LT}^{(HT)}(x) = -\mathscr{P}\int_{-1}^{1} dy \frac{1}{x-y}\left[\frac{\partial}{\partial x}\{F_{G,LT}(x,y)+G_{G,LT}(x,y)\} + \frac{\partial}{\partial y}\{F_{G,LT}(y,x)+G_{G,LT}(y,x)\}\right]$$

Twist-2 PDF

Twist-3 quark-gluon-quark distributions

Decomposition of f_{LT} at twist-3 level

$$f_{LT}(x) = \frac{3}{2}\int_x^{\epsilon(x)} \frac{dy}{y} f_{1LL}(y) + \int_x^{\epsilon(x)} \frac{dy}{y} f_{LT}^{(HT)}(y)$$

Twist-3 PDF

Quark mass term does not exist compared with g_T in proton

S. Kumano and Qin-Tao Song, JHEP 09 (2021) 141

WW type relation and BC type sum rule in spin-1 hadrons

Introduce the plus function

$$f^+(x) \equiv f(x) + \bar{f}(x) = f(x) - f(-x), \quad f = f_{1LL},\ f_{LT},\ f_{LT}^{(HT)}, \quad x > 0$$

Then

$$f_{LT}^+(x) = \frac{3}{2}\int_x^1 \frac{dy}{y} f_{1LL}^+(y) + \int_x^1 \frac{dy}{y} f_{LT}^{(HT)+}(y)$$

Define the PDF f_{2LT}

$$f_{2LT}(x) \equiv \frac{2}{3} f_{LT}(x) - f_{1LL}(x)$$

⟶ Similar as g_2 in proton

WW type relation and BC type sum rule in spin-1 hadron

$$f_{2LT}^{+}(x) = -f_{1LL}^{+}(x) + \int_x^1 \frac{dy}{y} f_{1LL}^{+}(y) + \frac{2}{3}\int_x^1 \frac{dy}{y} f_{LT}^{(HT)}(y)$$

Twist –2 PDF

Twist-3 quark-gluon-quark distributions

If the twist-3 term is neglected, we obtain a relation

$$f_{2LT}^{+}(x) = -f_{1LL}^{+}(x) + \int_x^1 \frac{dy}{y} f_{1LL}^{+}(y)$$

→ Wandzura-Wilczek (WW) type relation in spin-1 hadrons

$$\int_0^1 dx\, f_{2LT}^{+}(x) = 0$$

→ Burkhardt-Cottingham (BC) type sum rule

S. Kumano and Qin-Tao Song, JHEP 09 (2021) 141
S. Wandzura and F. Wilczek, Phys. Lett.B 72, 195 (1977).
H. Burkhardt and W. N. Cottingham, Annals Phys. 56, 453 (1970).

Summay

Relations for higher twist PDFs in spin-1 hadrons

- Decomposition of f_{LT} in spin-1 hadrons
- Quark-gluon-quark correlation functions
- Wandzura-Wilczek (WW) type relation
- Burkhardt-Cottingham (BC) type sum rule

Possible measurements of those quantities can be conducted at Fermilab, JLab and NICA.

Tensor-polarized deuteron target at Fermilab and JLab → D. Keller, D. Crabb, and D. Day, Nucl. Instrum. Meth. A 981, 164504 (2020).

NICA summary report → A. Arbuzov et al., Prog. Nucl. Part. Phys. 119, 103858 (2021) 1

Thank you