Helicity quasi-parton distributions in a large Nc nucleon

with
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Introduction

Parton distribution functions (PDFs)

How partons (quarks and gluons) are distributed inside a hadron

Probability density on the light-cone

Factorizations & universality

Non-perturbative QCD matrix elements

Theoretical understanding of PDFs

Effective models at low energy (low renormalization scale)

- providing initial conditions of the QCD evolution
- to understand the detailed mechanism
- Chiral quark-soliton model [D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

Parton distribution functions (PDFs)

How partons (quarks and gluons) are distributed inside a hadron

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Factorizations & universality

Non-perturbative QCD matrix elements

Theoretical understanding of PDFs

Lattice QCD

- fundamental difficulties being Euclidean: no direct computation is possible

- Mostly studied using the Mellin moments of the PDFs

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu,P^z) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P|\bar{\psi}(0)\gamma^z \exp\left[-ig\int_0^z dz' A^z(z')\right]\psi(z)|P\rangle + \mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2})$$

Large **M**omentum **E**ffective **T**heory

Spacelike separation→ can be calculated on Lattice!

No unique definition → **$\Gamma=\gamma^3$** or **$\Gamma=\gamma^0$**

Approaches to PDFs in the limit Pz →∞, or v → 1.

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu_R,P^z) = \int_{-1}^{1} \frac{dy}{|y|} \underline{C(\frac{x}{y},\frac{\mu_R}{\mu},\frac{\mu}{p^z})} q(y,\mu) + \mathcal{O}(\frac{\Lambda^2_{\mathrm{QCD}}}{(P^z)^2},\frac{M_N^2}{(P^z)^2})$$

Perturbative matching coefficients

Extensively studied for the Lattice computations

Market results P_z ~ 2-3 GeV

Enough accuracy and uncertainty for actual application?

Reliable model computations on quasi-PDFs?

Plenary talk by C. Alexandrou on 14:20 Thus 21/10

(Quasi-)PDFs in the chiral quark-soliton model

A successful description of the PDFs at low renormalization scale

[D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, Nuclear Physics B 480, 341 (1996)]

Nucleon matrix element in Euclidean separation

Lorentz boost → PDFs ~ quasi-PDF

Properties of qPDFs for quarks and antiquarks in the nucleon:

Sum-rules, positivity, evolution in P_z

Gravitational form factors are related to the momentum sum-rule:

$\bar{c}^q$ **is accessible only through the quasi-PDF**

In this talk,

Description of the chiral quark-soliton model

quasi-PDFs within the chiral quark-soliton model

Properties: quasi sum-rules as their Mellin moments

Numerical results for the unpolarized and longitudinally polarized quasi-quark distributions

Comparison of the Dirac structure defining the quasi-PDFs

The polarized antiquark asymmetry

Method

Chiral quark-soliton model

$$Z = \int \mathcal{D}\pi^a d\psi^\dagger d\psi \ \exp \int d^4x \psi^\dagger(x)(i\partial\!\!\!/ + iMU^{\gamma_5})\psi(x)$$

$$U^{\gamma_5}(x) = U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2} \qquad U(x) = \exp\left[\frac{i}{F_\pi}\pi^a(x)\tau^a\right]$$

From QCD to the low energy effective theory via the instantons

Parameters: $\bar{\rho} \sim 1/3 \text{ fm}$ & $\bar{R} \sim 1 \text{ fm}$

Intrinsic renormalisation scale $\Lambda \sim 1/\bar{\rho} \approx 600 \text{ MeV}$

Dynamically generated quark mass $M = 350 \text{ MeV}$

Interplays the quark-model and (topological) soliton picture of the baryons

Fully field theoretic: successively describes a wide class of baryon properties

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

Properties of quasi-PDFs in the large Nc

Quark distribution functions: large components

In the large Nc limit,

Isosinglet unpolarised	$u(x) + d(x)$	$\sim N_c^2 \, \rho(N_c x)$
Isovector polarised	$\Delta u(x) - \Delta d(x)$	

Isovector unpolarised	$u(x) - d(x)$	$\sim N_c \, \rho(N_c x)$
Isosingletr polarised	$\Delta u(x) + \Delta d(x)$	

quasi-PDFs acquire overall factor of v

-> the Nc ordering is preserved and dominant
if v not too small

Quasi-PDFs in the χQSM

Quasi- quark and antiquark number densities

$$D_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x-\frac{k^3}{P_N}\right)\int d^3x\; e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\bar{\psi}_f\left(-\frac{\mathbf{x}}{2},t\right)\Gamma\psi_f\left(\frac{\mathbf{x}}{2},t\right)|N_v\rangle$$

$$\bar{D}_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x-\frac{k^3}{P_N}\right)\int d^3x\; e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\mathrm{Tr}\left[\Gamma\psi_f\left(-\frac{\mathbf{x}}{2},t\right)\bar{\psi}_f\left(\frac{\mathbf{x}}{2},t\right)\right]|N_v\rangle$$

Isoscalar unpolarized $x \in (-\infty, \infty)$

$$\sum_f q_f(x,v) = N_c M_N v \sum_{n,occ}\int \frac{d^3k}{(2\pi)^3}\delta(k^3 + vE_n - vM_N x)\left[\Phi_n^\dagger(\vec{k})(1+v\gamma^0\gamma^3)\gamma_0\Gamma\Phi_n(\vec{k})\right]$$

Isovector polarized (helicity)

$$\Delta u(x,v) - \Delta d(x,v) = -\frac{1}{3}(2T^3)\frac{N_c M_N v}{2\pi}\sum_{n,occ}\int \frac{d^2k_\perp}{(2\pi)^2}\delta(k^3 + vE_n - vM_N x)\left[\Phi_n^\dagger(\vec{k})(1+v\gamma^0\gamma^3)\gamma_0\Gamma\tau^3\gamma^5\Phi_n(\vec{k})\right]$$

Sum-rules

Baryon number

$$\int_{-\infty}^{\infty} dx\ q(x,v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases}$$

Momentum

$$\int_{-\infty}^{\infty} dx\ xq(x,v) = \begin{cases} 1, & \Gamma = \gamma^0 \\ v, & \Gamma = \gamma^3 \end{cases}$$

Bjorken

$$\int_{-\infty}^{\infty} dx\ (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

Interpretation for QCD quark symmetry currents

Sum-rules

Baryon number

$$\int_{-\infty}^{\infty} dx\ q(x,v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases}$$

Momentum

$$\int_{-\infty}^{\infty} dx\ x q(x,v) = \begin{cases} 1, & \Gamma = \gamma^0 \\ v, & \Gamma = \gamma^3 \end{cases}$$

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$$\int_{-\infty}^{\infty} dx\ (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

U(1): charge density (γ^0) vs flux (γ^3)

Sum-rules

Baryon number

$$\int_{-\infty}^{\infty} dx\; q(x,v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases}$$

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Momentum sum-rule is satisfied only by quarks

Energy-momentum tensor: momentum flux ($T^{30} \sim \partial_3 \gamma^0$) vs pressure ($T^{33} \sim \partial_3 \gamma^3$)

In general, $M_2^q(\Gamma = \gamma^3) = v\left(A^q(0) - \frac{1-v^2}{v^2}\bar{c}^q(0)\right)$

[Maxim Polyakov and HDS, JHEP 09 (2018) 156]

Sum-rules

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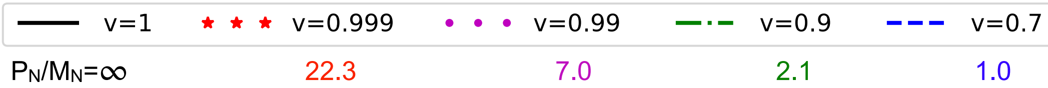
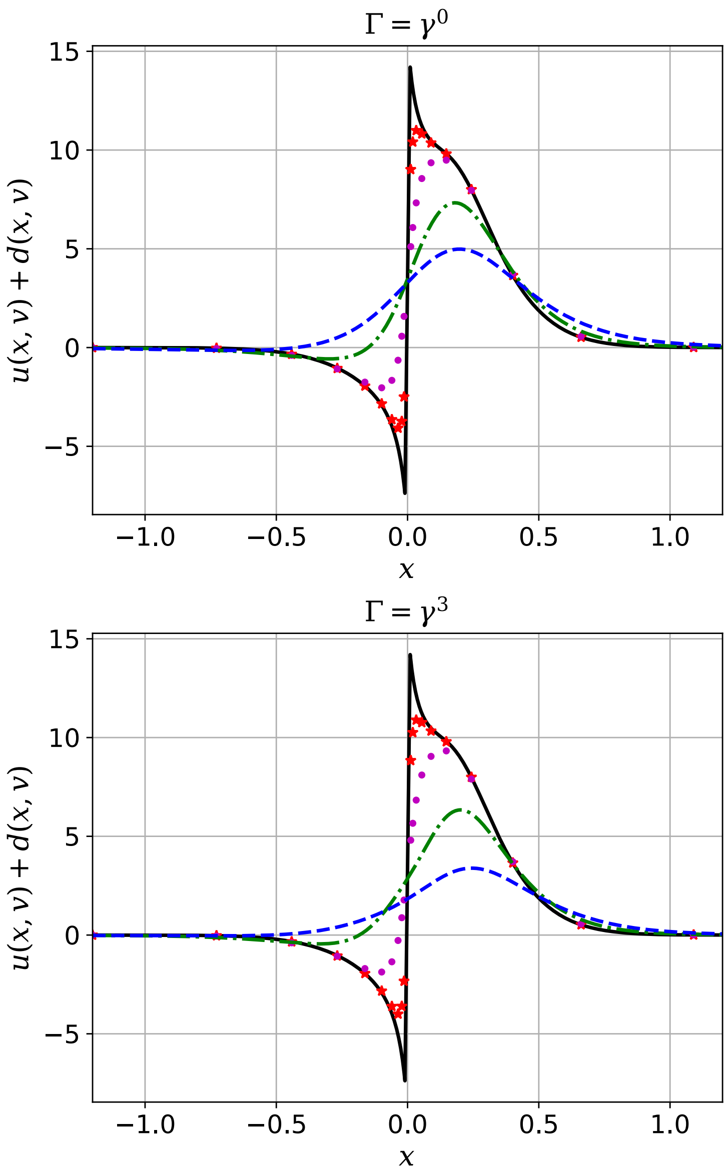
$$\int_{-\infty}^{\infty} dx\; (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

Axial current: $\gamma^3 \sim S^3 g_A^{(3)}$ vs $\gamma^0 \sim \overrightarrow{S} \cdot \overrightarrow{v}\, g_A^{(3)}$

Numerical results

Isoscalar unpolarized





$$\bar{q}(x) = -q(-x)$$

Strong v dependence at small x: due to smearing of the quark and antiquark parts

Antiquark part (negative x) breaks the positivity

Baryon number and Momentum sum-rules are satisfied in good accuracy

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808,135665 (2020)]

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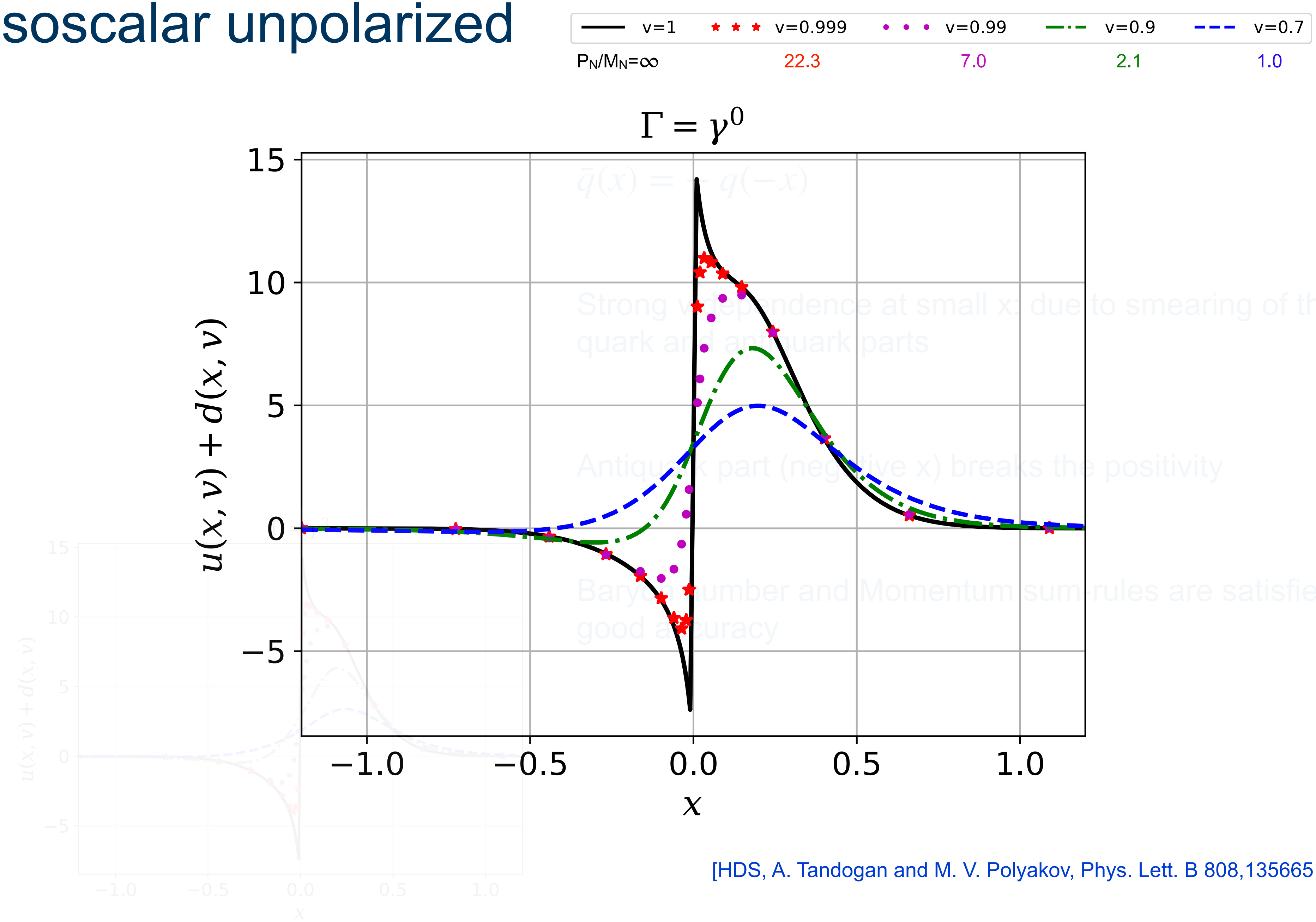
Isoscalar unpolarized



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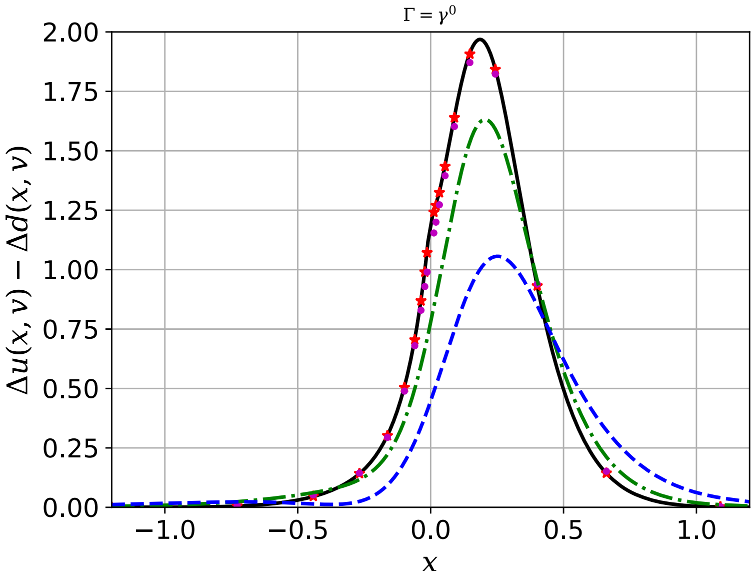
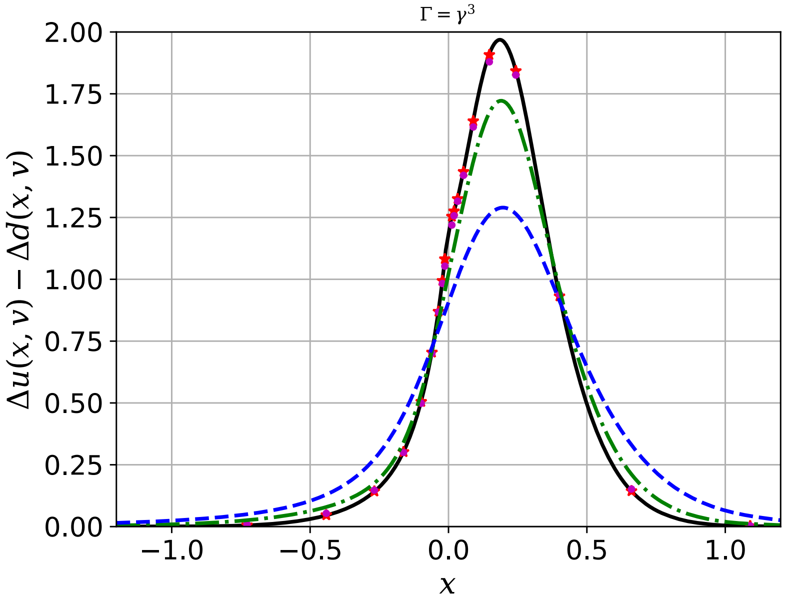
Isovector polarized

	v=1	v=0.999	v=0.99	v=0.9	v=0.7
P_N/M_N	∞	22.3	7.0	2.1	1.0





$$\Delta\bar{q}(x) = \Delta q(-x)$$

At v=0.9 (P ~ 2 GeV), qPDF ~ PDF

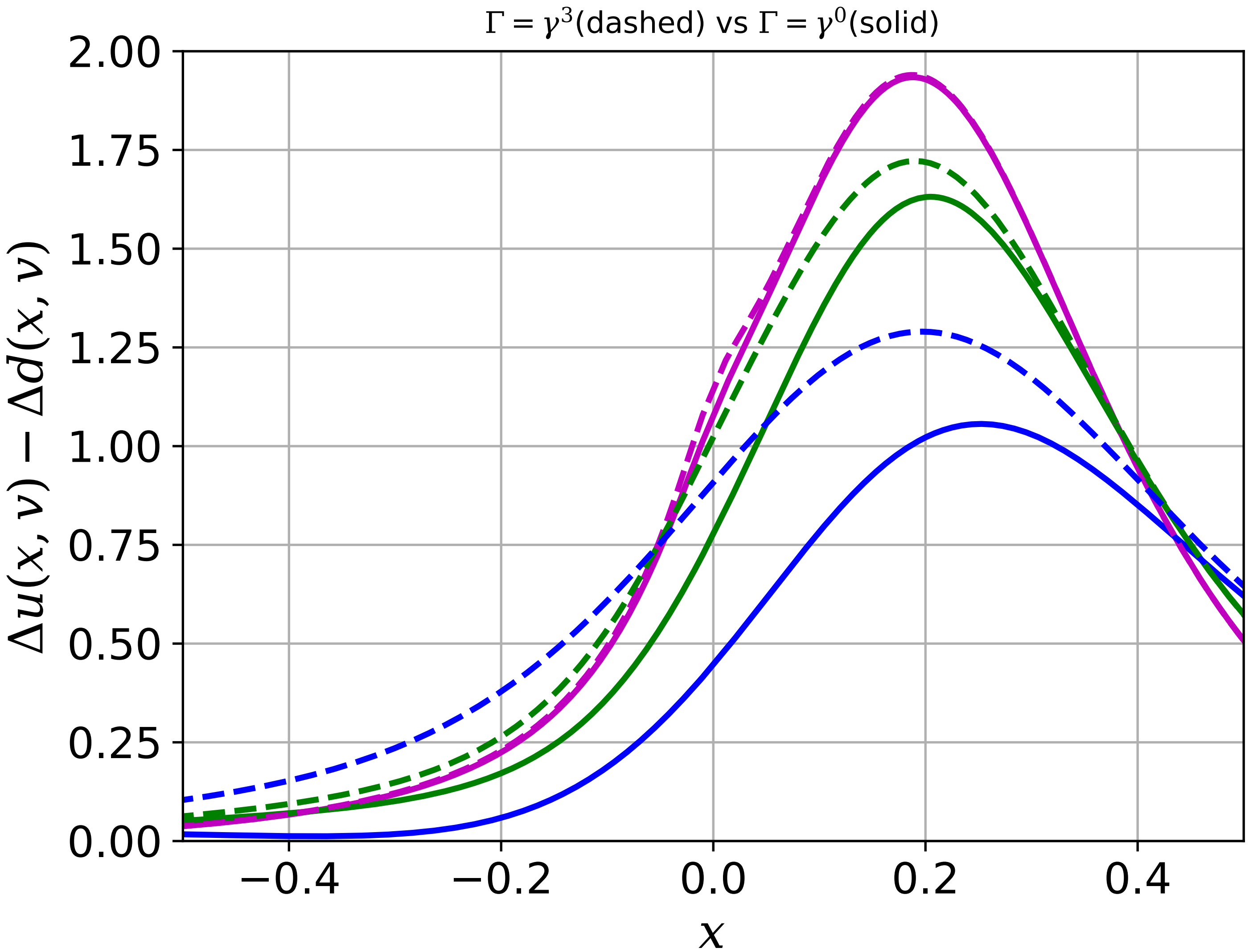
Sum-rules are satisfied in good accuracy

$\Gamma = \gamma^3$ qPDF converges faster to the lightcone PDF

$$\int_{-\infty}^{\infty} dx\, (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

[HDS, manuscript in preparation]

Isovector polarized



[HDS, manuscript in preparation]

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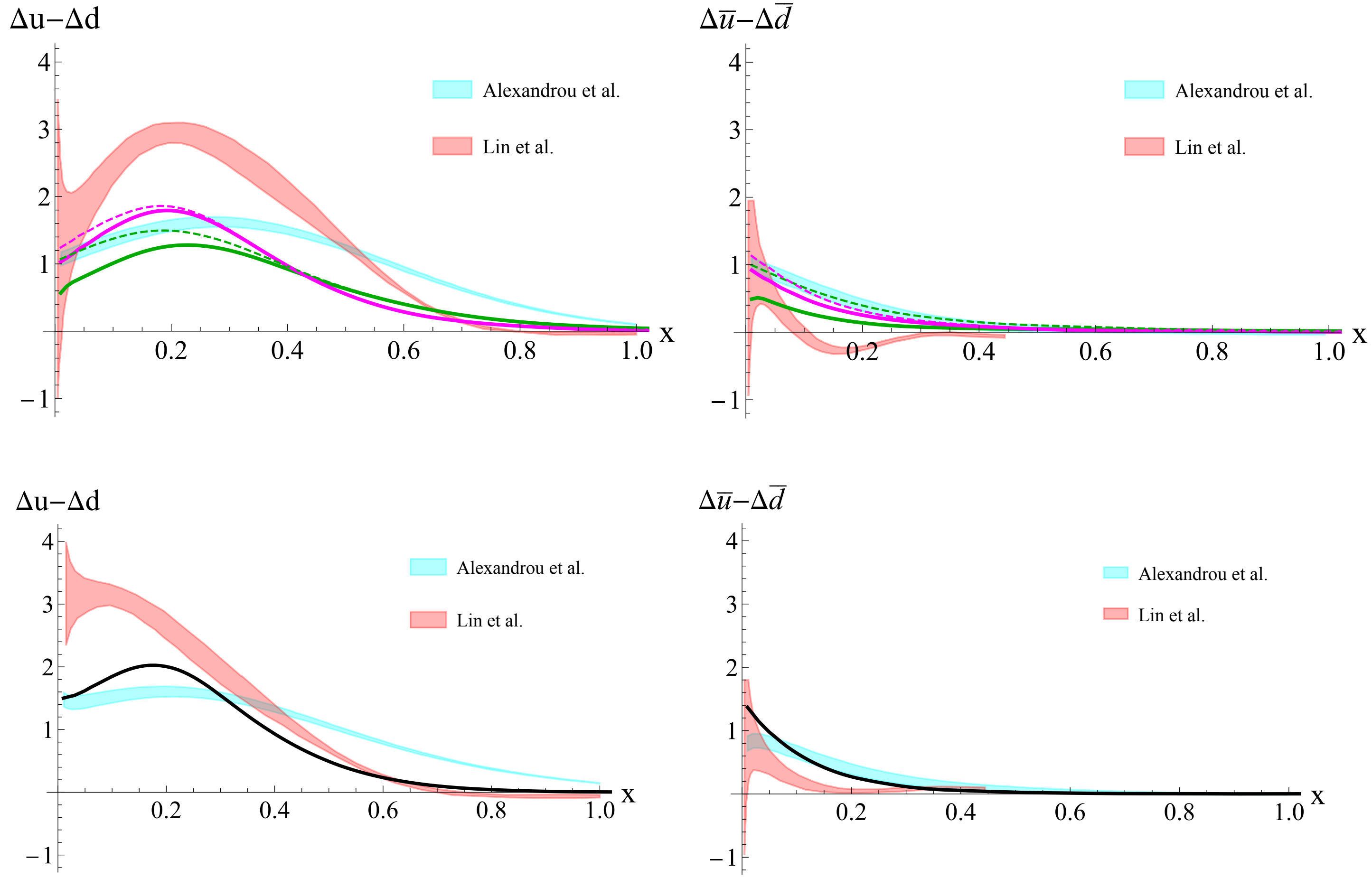
vs. Lattice results

v = 1 | [v = 0.93, Γ = γ^0] | [v = 0.93, Γ = γ^3] | [v = 0.77, Γ = γ^0] | [v = 0.77, Γ = γ^3]

$P_N/M_N=\infty$ | 3.0 GeV | 1.4 GeV



$(m_\pi, P_z, \mu) =$ **(0.37, 1.4, 2.0) [Alexandrou et al. Phys. Rev. D, vol. 96, no. 1, p. 014513, 2017],**

(0.135, 3.0, 3.0) [Lin et al.Phys. Rev. Lett., vol. 121, no. 24, p. 242003, 2018]

[HDS, manuscript in preparation]

Antiquark asymmetries in the nucleon

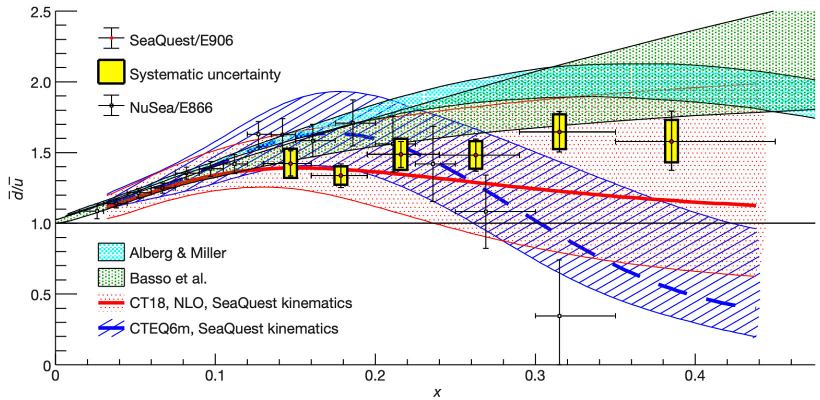


Fig. 2 | Ratios $\bar{d}(x)/\bar{u}(x)$. Ratios $\bar{d}(x)/\bar{u}(x)$ in the proton (red filled circles) with their statistical (vertical bars) and systematic (yellow boxes) uncertainties extracted from the present data based on NLO calculations of the Drell–Yan cross-sections. Also shown are the results obtained by the NuSea experiment (open black squares) with statistical and systematic uncertainties added in quadrature[4]. The cyan band shows the predictions of the meson–baryon model of Alberg & Miller[25] and the green band shows the predictions of the statistical parton distributions of Basso et al.[21]. The red solid (blue dashed) curves show the ratios $\bar{d}(x)/\bar{u}(x)$ calculated with CT18[29] (CTEQ6[35]) parton distributions at the scales of the SeaQuest results. The horizontal bars on the data points indicate the width of the bins.

[SeaQuest, Nature 590 (2021) 7847, 561-565]

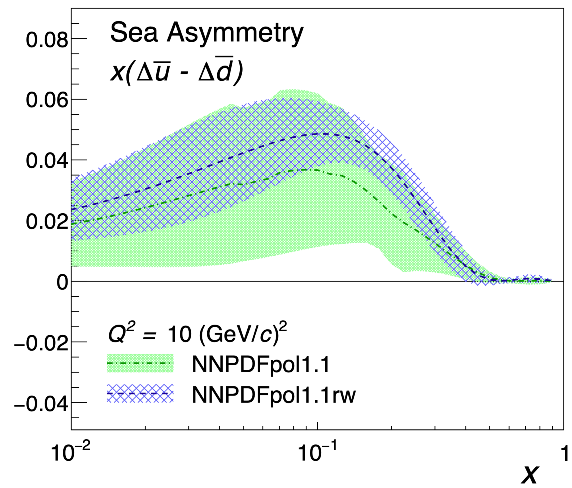
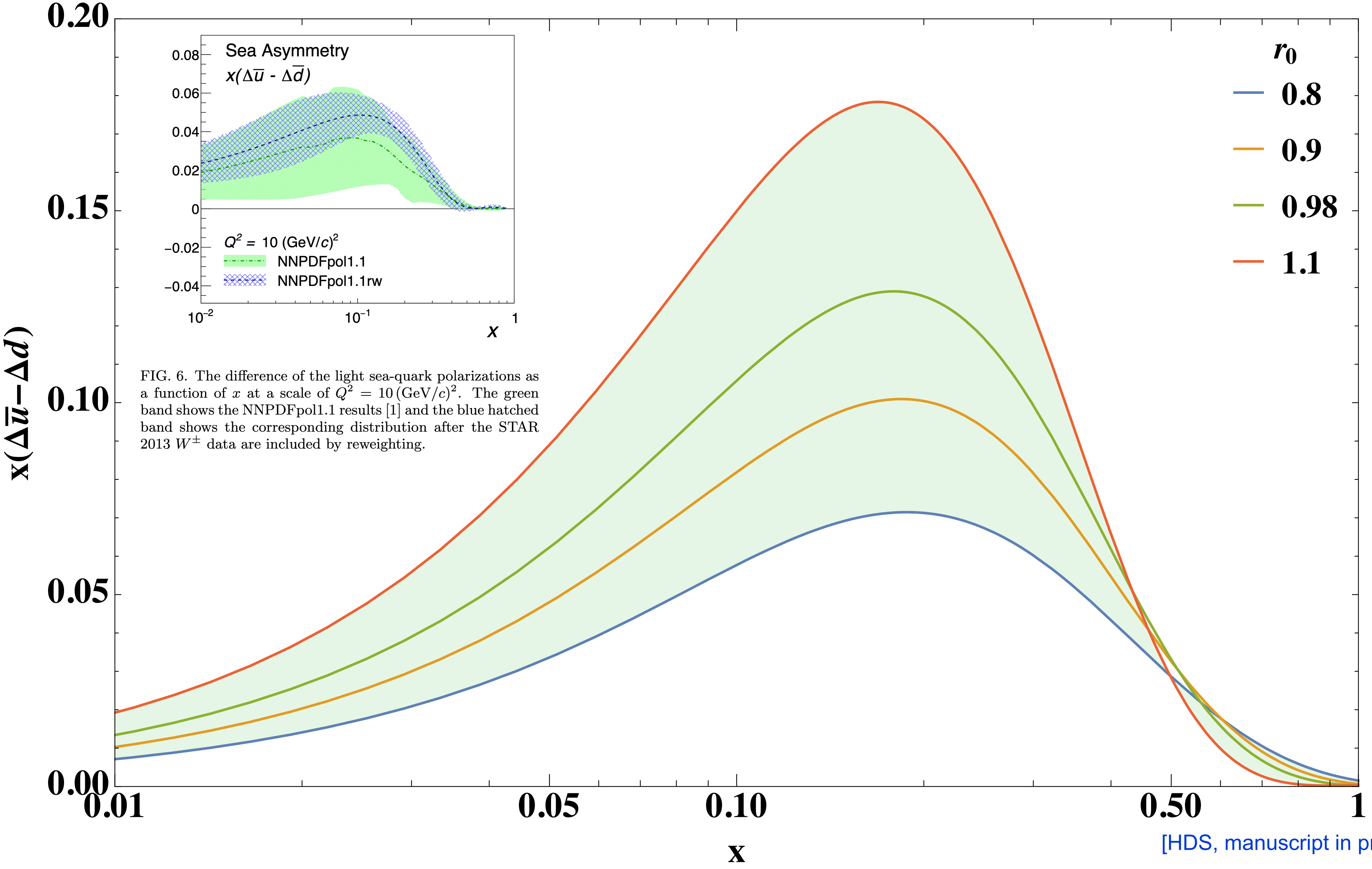


FIG. 6. The difference of the light sea-quark polarizations as a function of x at a scale of $Q^2 = 10\,(\mathrm{GeV}/c)^2$. The green band shows the NNPDFpol1.1 results [1] and the blue hatched band shows the corresponding distribution after the STAR 2013 $W^{\pm}$ data are included by reweighting.

[STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

Antiquark flavor asymmetry: model case



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[HDS, manuscript in preparation]

Closing remarks

Summary

χQSM is a working framework for understanding the (quasi-)PDFs

Generalized sum-rules: $\bar{c}^q$, 'better' Γ for the convergence to the PDFs,

Good convergence of the isovector polarized

vs. poor $\mathrm{P_z} \to \infty$ convergence for the isoscalar unpolarized quasi-PDF

Antiquark flavor asymmetry is predicted in the model calc.

Perspectives

Transversity PDF and other small components in the large Nc

Nontrivial large x behavior of quasi-PDFs near the end-point

gluon PDFs

Nucleon Quasi-DAs (in collaboration with J.-Y Kim, for light-cone DAs see ***2110.05889****)*

Thank you very much!