SPIN2021

Matsue, Japan

24th International Spin Symposium

October 18 -22, 2021

Collinear twist-3 approach to hyperon polarization in SIDIS

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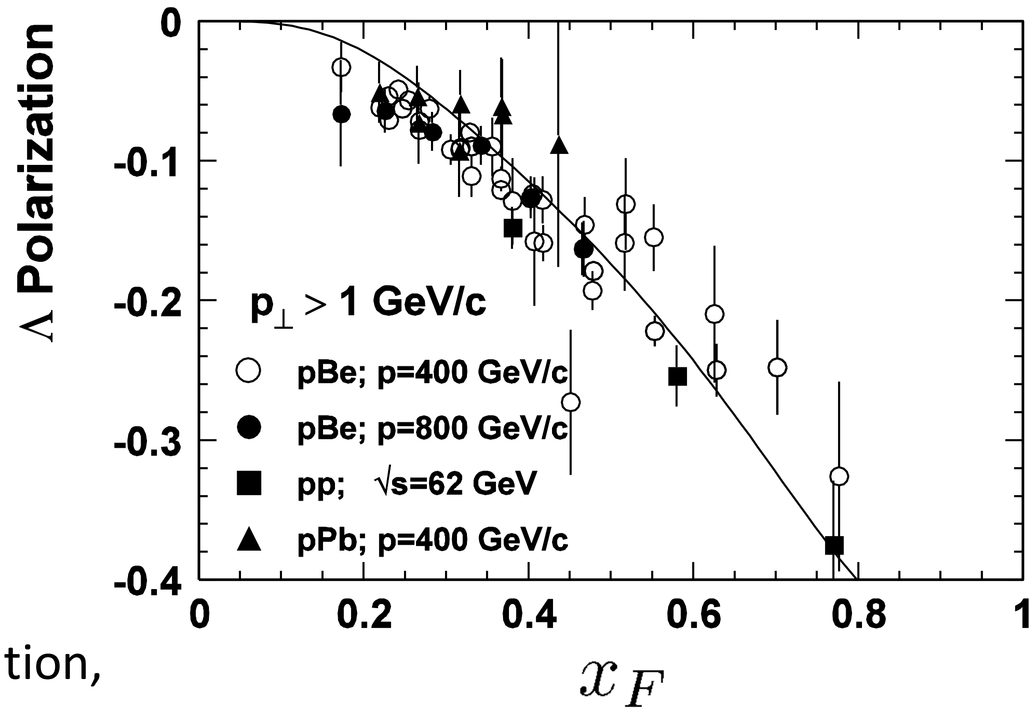
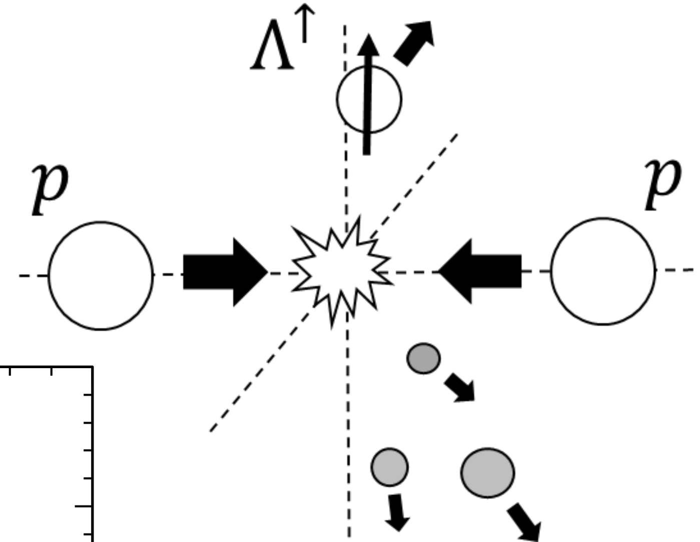
Niigata univ [A], SCNU [B]

Introduction

・Single Spin Asymmetry(SSA)

・ $pp \to \Lambda^{\uparrow} X$





In addition,

- Semi-inclusive deep inelastic scattering(SIDIS) $ep \to e\Lambda^{\uparrow} X$
- Drell-Yan process $p^{\uparrow} p \to l^{+} l^{-} X$
- Direct photon production $p^{\uparrow} p \to \gamma X$, etc.

Introduction

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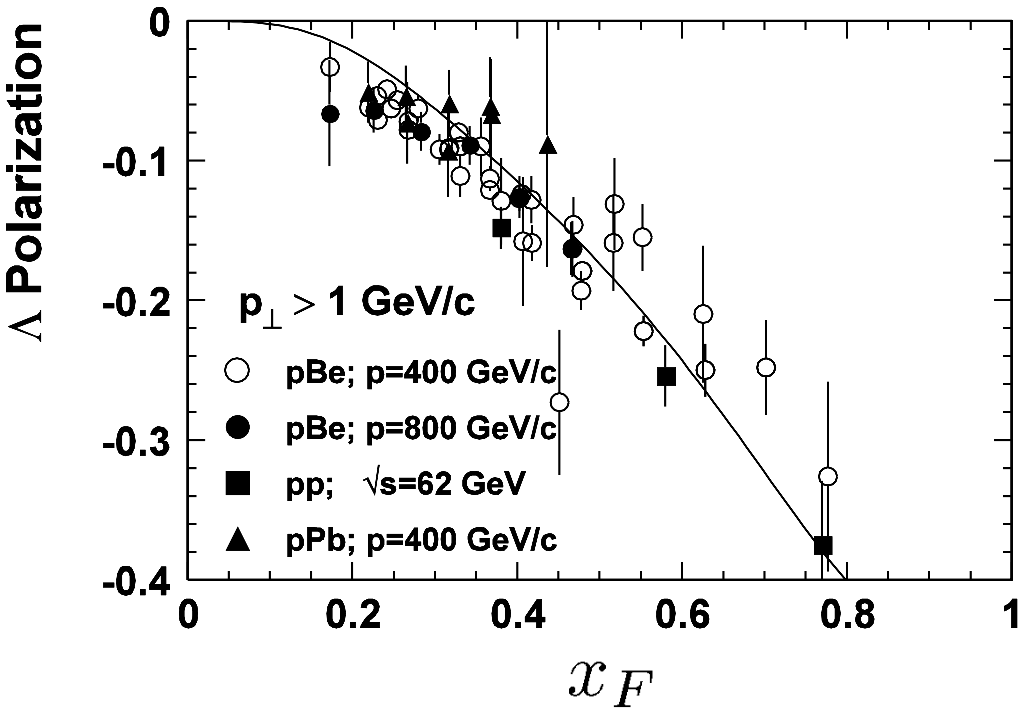
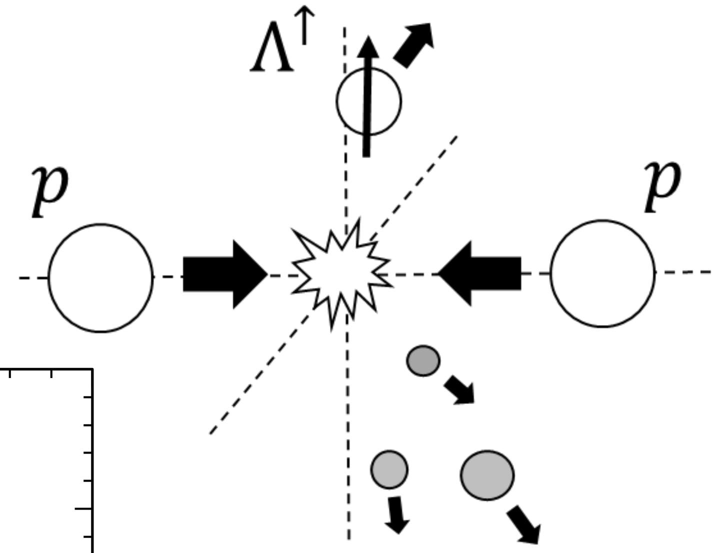
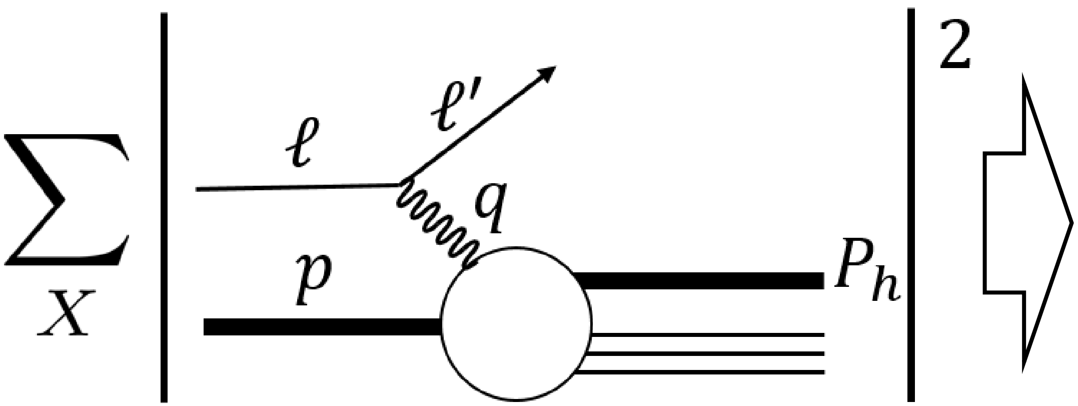


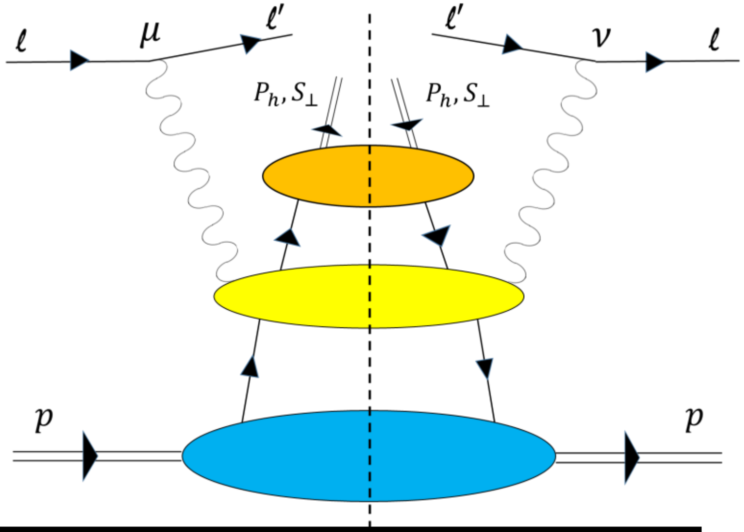


The origin of SSA ➡ **Multi parton correlation functions (quark-gluon, purely gluonic)**

Introduction

- $ep \to e\Lambda^{\uparrow}X$ (Semi–inclusive deep inelastic scattering(SIDIS))





$$e(l) + p(p) \to e(l') + \Lambda^{\uparrow}(P_h, S_\perp) + X$$

$$d\Delta\sigma = \frac{1}{2S_{ep}} \frac{d^3\vec{P_h}}{(2\pi)^3 2P_h^0} \frac{d^3\vec{l'}}{(2\pi)^3 2l'^0} \frac{e^4}{q^4} L^{\mu\nu}(l, l') W_{\mu\nu}(p, q, P_h)$$

Leptonic tensor

Hadronic tensor

Introduction

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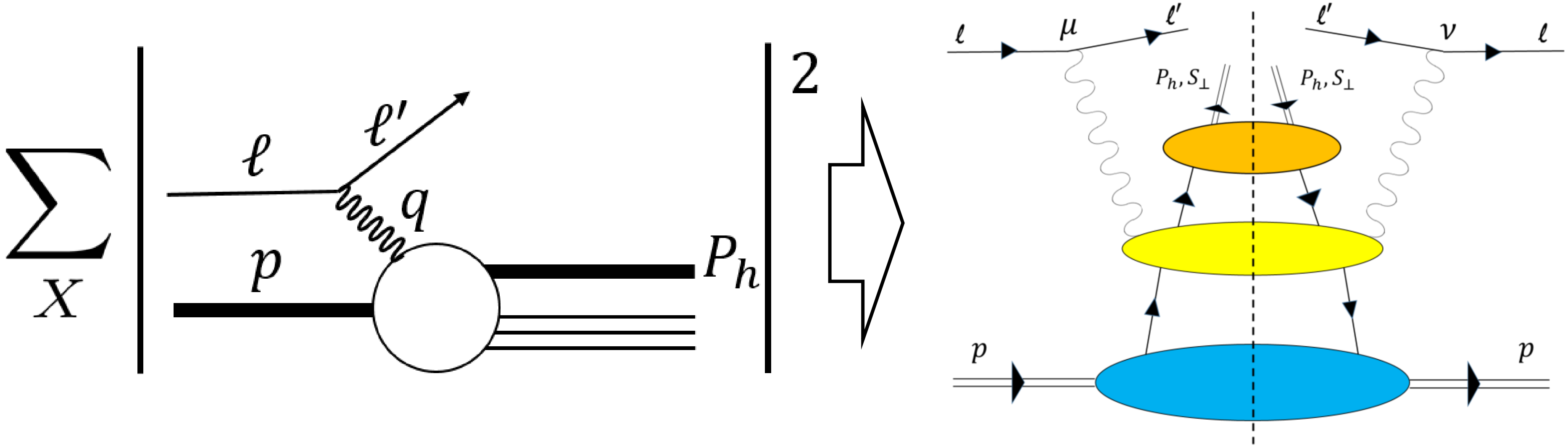

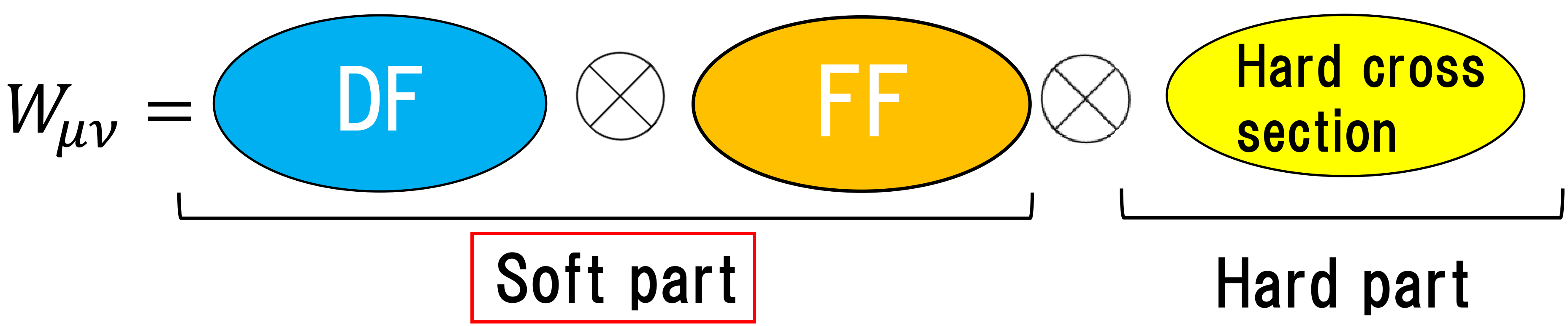

Quark-gluon correlation function

Introduction

$\boldsymbol{ep \to e\Lambda^{\uparrow}X}$ based on

the collinear twist-3 factorization formalism.

Large-p_T $\quad (\Lambda_{QCD} \ll Q \sim P_T)$

Introduction

$ep \to e\Lambda^{\uparrow}X$ based on
the collinear twist-3 factorization formalism.

- $ep \to e\Lambda^{\uparrow}X$

Contribution of the

1. twist-3 quark distribution function [1]

2. twist-3 quark fragmentation function [2]

3. twist-3 gluon fragmentation function

[1] H. Eguchi, Y. Koike, K. Tanaka, Nucl. Phys. B763 (2007) 198-227,
[2] K. Kanazawa, Y. Koike, Phys. Rev. D88 (2013) 074022,

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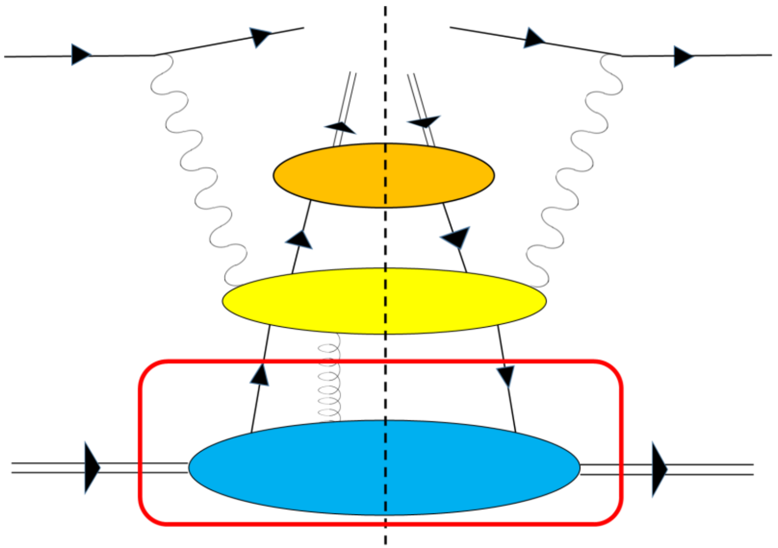

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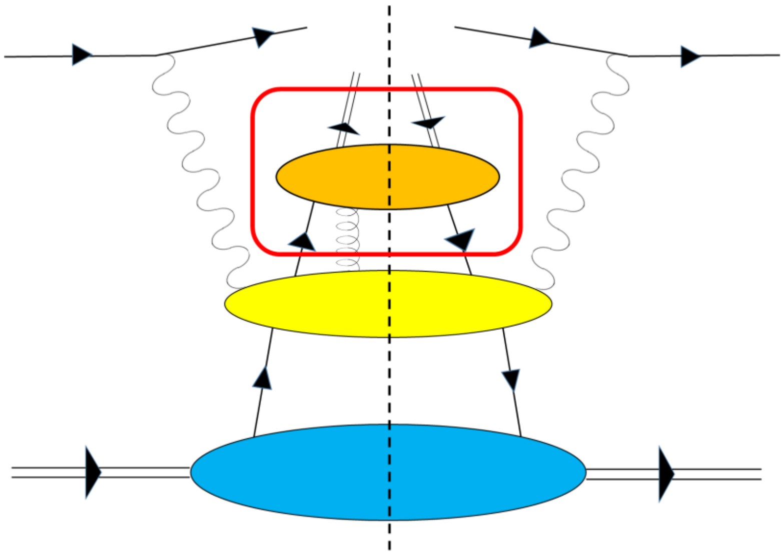

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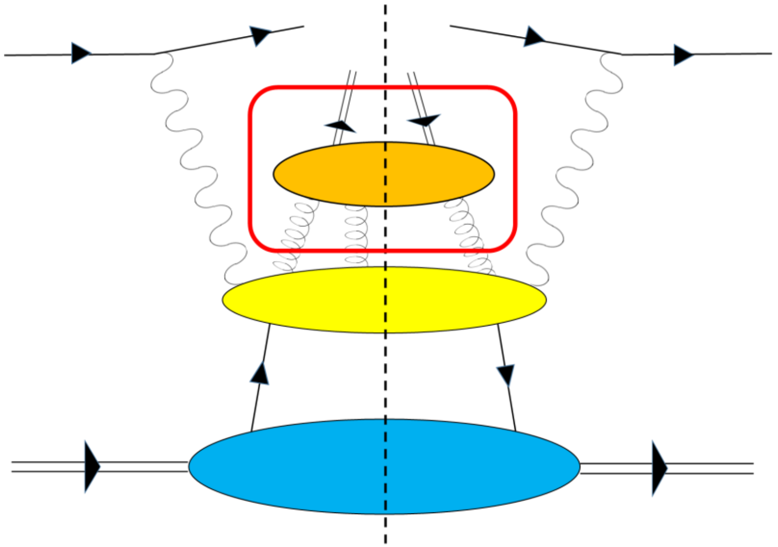

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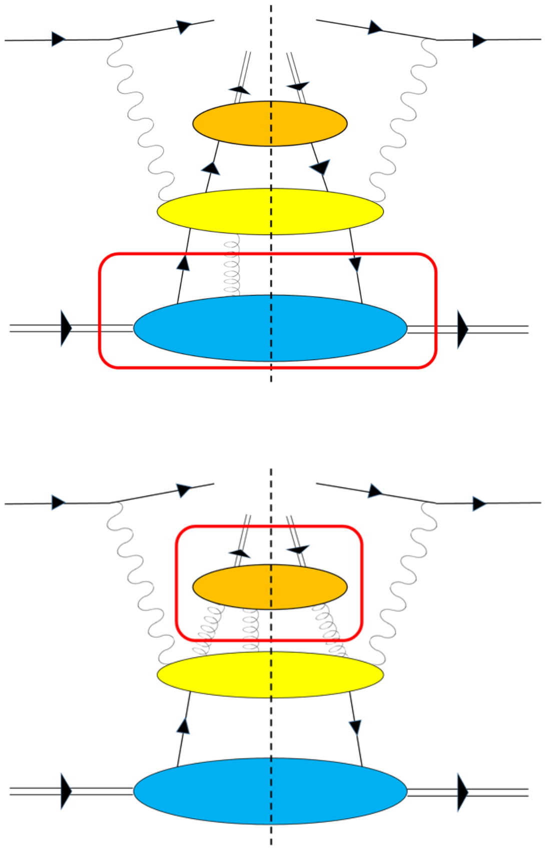
→ Talk by R. Ikarash(TMD session)



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“Hadron frame” and spin vector

- Five Lorentz invariants.

$$S_{ep} = (p+l)^2$$

$$x_{bj} = \frac{Q^2}{2p\cdot q}$$

$$Q^2 = -q^2 = -(l-l')^2$$

$$z_f = \frac{p\cdot P_h}{p\cdot q}$$

$$q_T = \sqrt{-q_t^2}$$

$$q_t^\mu = q^\mu - \frac{P_h\cdot q}{p\cdot P_h}p^\mu - \frac{p\cdot q}{p\cdot P_h}P_h^\mu$$

$$p^\mu = \left(\frac{Q}{2x_{bj}}, 0, 0, \frac{Q}{2x_{bj}}\right)$$

$$q^\mu = (0,0,0,-Q)$$

$$P_h^\mu = \frac{z_f Q}{2}\left(1+\frac{q_T^2}{Q^2}, \frac{2q_T}{Q}\cos\chi, \frac{2q_T}{Q}\sin\chi, -1+\frac{q_T^2}{Q^2}\right)$$

$$T^\mu = \frac{1}{Q}(q^\mu + 2x_{bj}p^\mu) = (1,0,0,0)$$

$$X^\mu = \frac{1}{q_T}\left\{\frac{P_h^\mu}{z_f} - q^\mu - (1+\frac{q_T^2}{Q^2})x_{bj}p^\mu\right\} = (0,\cos\chi,\sin\chi,0)$$

$$Y^\mu = \epsilon^{\mu\nu\rho\sigma}Z_\nu X_\rho T_\sigma = (0,-\sin\chi,\cos\chi,0)$$

$$Z^\mu = -\frac{q_T^\mu}{Q} = (0,0,0,1)$$

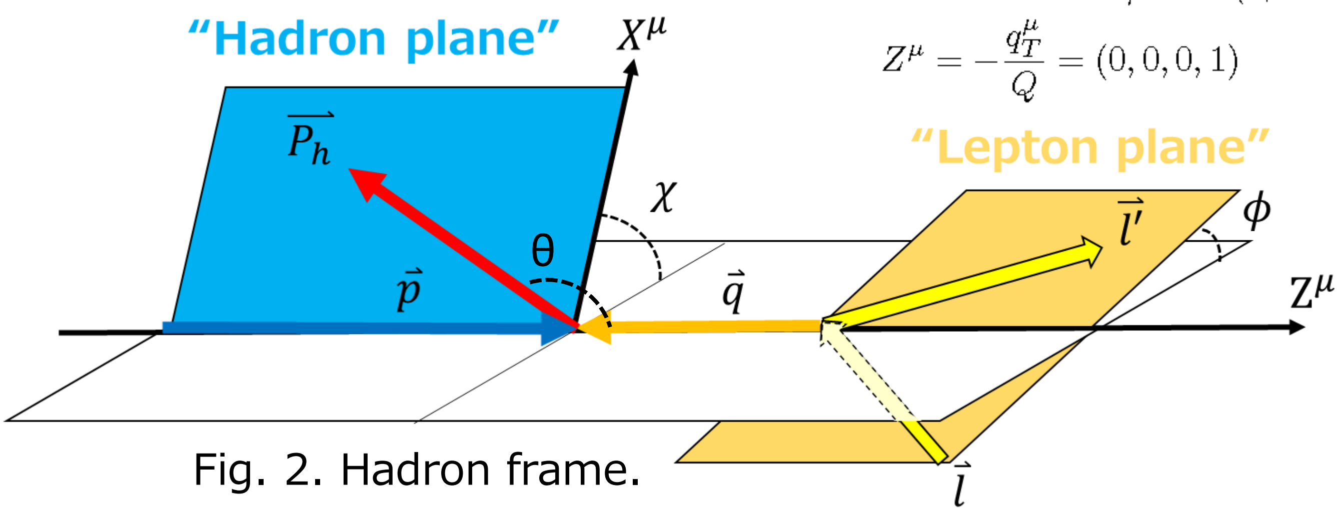

Fig. 2. Hadron frame.

[4] R. b. Meng, F. I. Olness and D. E. Soper, Nucl. Phys. B **371**, 79 (1992).

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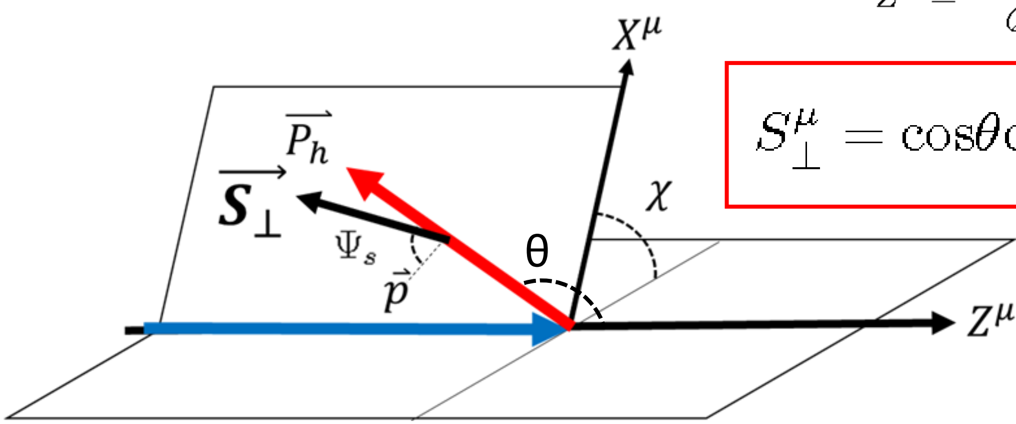

Fig.3. Hadron plane and spin vector.

$$S_\perp^\mu = \cos\theta\cos\Psi_s X^\mu + \sin\Psi_s Y^\mu - \sin\theta\cos\Psi_s Z^\mu$$

"Spin vector of final state hadron"

[4] R. b. Meng, F. I. Olness and D. E. Soper, Nucl. Phys. B **371**, 79 (1992).

“Hadron frame” and spin vector

$$\frac{d^6\Delta\sigma^{q-frag}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2}{128\pi^4 x_{bj}^2 S_{ep}^2 Q^2} z_f \int \frac{dx}{x} f(x) L^{\mu\nu}(l,l') W_{\mu\nu}(p,q,P_h)$$

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Expansion coefficients

$$L^{\mu\nu}W_{\mu\nu} = \sum_{k=1,2,\cdots 9} [L_{\mu\nu}\mathscr{V}_k^{\mu\nu}]\underline{[W_{\rho\sigma}\widetilde{\mathscr{V}}_k^{\rho\sigma}]}$$

$$L^{\mu\nu}(l,l') = 2(l^\mu l'^\nu + l^\nu l'^\mu) - Q^2 g^{\mu\nu}$$

・Symmetric tensors

$$\begin{aligned}
\mathscr{V}_1^{\mu\nu} &= X^\mu X^\nu + Y^\mu Y^\nu \\
\mathscr{V}_2^{\mu\nu} &= g^{\mu\nu} + Z^\mu Z^\nu \\
\mathscr{V}_3^{\mu\nu} &= T^\mu X^\nu + X^\mu T^\nu \\
\mathscr{V}_4^{\mu\nu} &= X^\mu X^\nu - Y^\mu Y^\nu \\
\mathscr{V}_8^{\mu\nu} &= T^\mu Y^\nu + Y^\mu T^\nu \\
\mathscr{V}_9^{\mu\nu} &= X^\mu Y^\nu + Y^\mu X^\nu
\end{aligned}$$

$\Longleftrightarrow$

$$\begin{aligned}
\widetilde{\mathscr{V}}_1^{\mu\nu} &= \frac{1}{2}(2T^\mu T^\nu + X^\mu X^\nu + Y^\mu Y^\nu) \\
\widetilde{\mathscr{V}}_2^{\mu\nu} &= T^\mu T^\nu \\
\widetilde{\mathscr{V}}_3^{\mu\nu} &= -\frac{1}{2}(T^\mu X^\nu + X^\mu T^\nu) \\
\widetilde{\mathscr{V}}_4^{\mu\nu} &= \frac{1}{2}(X^\mu X^\nu - Y^\mu Y^\nu) \\
\widetilde{\mathscr{V}}_8^{\mu\nu} &= -\frac{1}{2}(T^\mu Y^\nu + Y^\mu T^\nu) \\
\widetilde{\mathscr{V}}_9^{\mu\nu} &= \frac{1}{2}(X^\mu Y^\nu + Y^\mu X^\nu)
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$$L^{\mu\nu}W_{\mu\nu} = \sum_{k=1,2,\cdots 9} [L_{\mu\nu}\mathcal{V}_k^{\mu\nu}][W_{\rho\sigma}\widehat{\mathcal{V}}_k^{\rho\sigma}] = Q^2 \sum_{k=1,2,\cdots 9} \mathcal{A}_k(\phi-\chi)[W_{\rho\sigma}\widetilde{\mathcal{V}}_k^{\rho\sigma}]$$

$$L^{\mu\nu}(l,l') = 2(l^\mu l'^\nu + l^\nu l'^\mu) - Q^2 g^{\mu\nu}$$

$$\mathcal{A}_k(\phi-\chi) = L_{\mu\nu}\mathcal{V}_k^{\mu\nu}/Q^2$$

$$l^\mu = \frac{Q}{2}(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, -1)$$

$$l'^\mu = \frac{Q}{2}(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, 1)$$

$$\mathcal{A}_1(\varphi) = 1+\cosh^2(\psi)$$
$$\mathcal{A}_2(\varphi) = -2$$
$$\mathcal{A}_3(\varphi) = -\cos(\varphi)\sinh(2\psi)$$
$$\mathcal{A}_4(\varphi) = \cos(2\varphi)\sinh^2(\psi)$$
$$\mathcal{A}_8(\varphi) = -\sin(\varphi)\sinh(2\psi)$$
$$\mathcal{A}_9(\varphi) = \sin(2\varphi)\sinh^2(\psi)$$

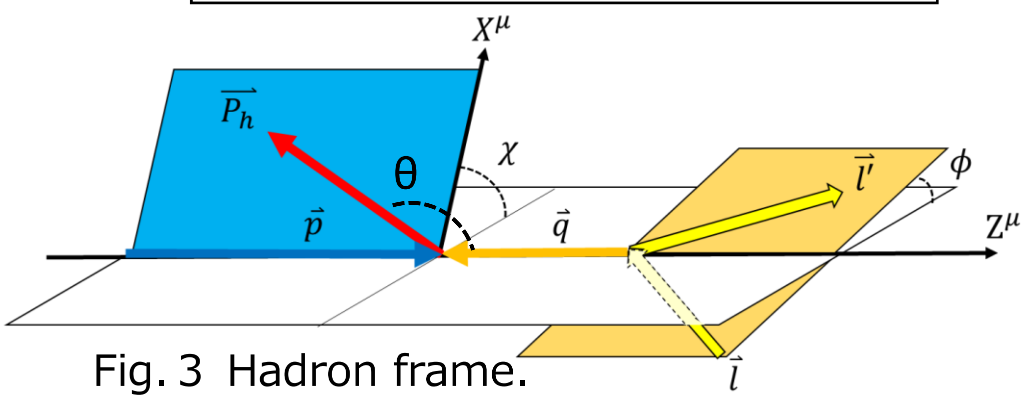


Fig. 3 Hadron frame.

$$\cosh\psi = \frac{2x_{bj}S_{ep}}{Q^2} - 1$$

Contribution of the twist-3 distribution function

・Hadronic tensor[1]

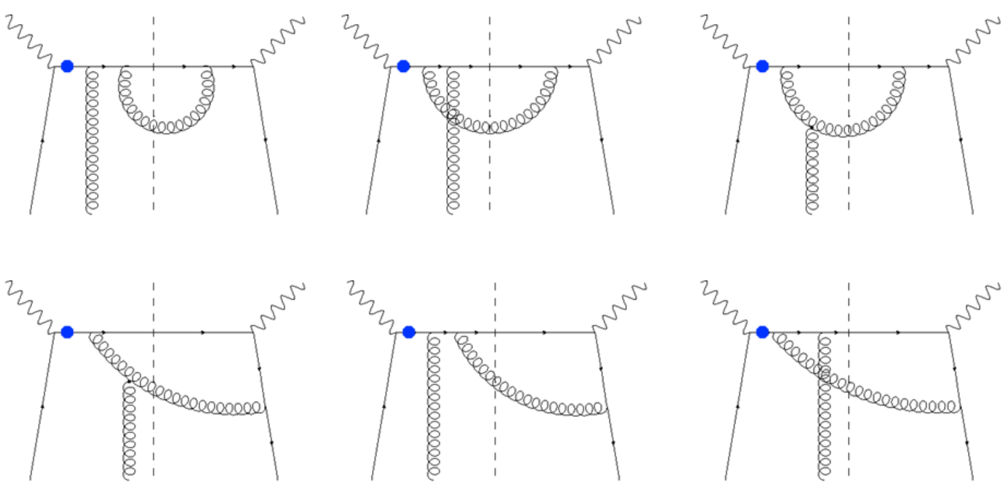
$$w_{\mu\nu}(p,q,P_h/z) = \int dx_1 \int dx_2 \mathrm{Tr}\left[i\omega^{\alpha}{}_{\beta} M_F^{\beta}(x_1,x_2) \frac{\partial S^{\mathrm{HP/SGP}}_{\sigma,\mu\nu}(k_1,k_2,q,P_h/z)p^{\sigma}}{\partial k_2^{\alpha}}\bigg|_{k_i=x_ip} \right]$$

$$= \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p|\bar{\psi}_j(0) g F^{\alpha n}(\mu n)\psi_i(\lambda n)|p\rangle$$

$$= \frac{M_n}{4}\epsilon^{\alpha\beta n p}(\gamma_5\gamma_\beta \not{p})_{ij} E_F(x_1,x_2) + \ldots$$

Twisi-3 quark DF for initial proton

・ **Hard pole**



$$\frac{1}{x_1 - x_{bj} \pm i\epsilon} = P\frac{1}{x_1 - x_{bj}} \mp i\pi\delta(x_1 - x_{bj})$$

$$\Rightarrow E_F(x_{bj}, x)$$

[1] H. Eguchi, Y. Koike, K. Tanaka, Nucl. Phys. B763 (2007) 198-227

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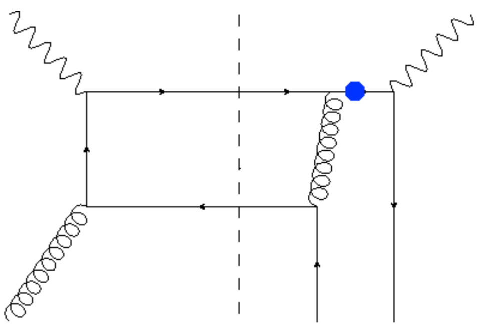

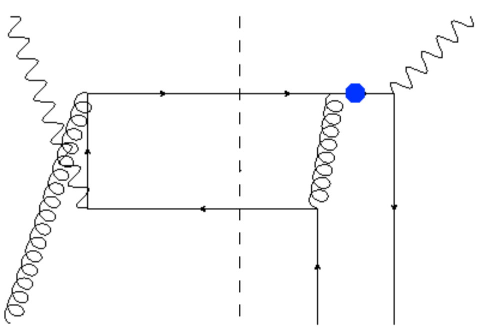

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$$\Rightarrow E_F(x_{bj}, x_{bj} - x)$$

[4] Y. Koike, K. Tanaka, (2009) arXiv:0907.2797v1

Contribution of the twist-3 distribution function

- **Hard pole**

Twist-2 FF for transversely polarized hyperon

$$\frac{d^6\Delta\sigma^{\rm HP}}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi}=\frac{\alpha_{em}^2\alpha_s}{16\pi^2S_{ep}^2x_{bj}^2Q^2}\left(\frac{\pi M_N}{4}\right)\int\frac{dz}{z}H_1(z)\int\frac{dx}{x}$$

$$\times\Bigg[\left(\frac{4}{Nq_T}-\frac{4NQ^2(\hat{x}-1)}{q_T^3\hat{x}}\right)\sinh^2\psi\sin\{\Phi_s+2(\phi-\chi)\}\times\frac{2}{1-\hat{x}}E_F(x_{bj},x)$$

$$+\frac{8\hat{x}}{Nq_T\hat{z}}(1+\cosh^2\psi)\sin\Phi_s\times E_F(x_{bj},x_{bj}-x)$$

$$+\left\{-\frac{8Q(\hat{x}-1)}{Nq_T{}^2\hat{z}}\right\}\sinh 2\psi\sin(\Phi_s+\phi-\chi)\times E_F(x_{bj},x_{bj}-x)$$

$$+\left\{\frac{8Q^2(\hat{x}-1)^2}{Nq_T{}^3\hat{x}\hat{z}}\right\}\sinh^2\psi\sin\{\Phi_s+2(\phi-\chi)\}\times E_F(x_{bj},x_{bj}-x)\Bigg]$$

$$\times\,\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$

Twist-3 DF for initial proton

Contribution of the twist-3 distribution function

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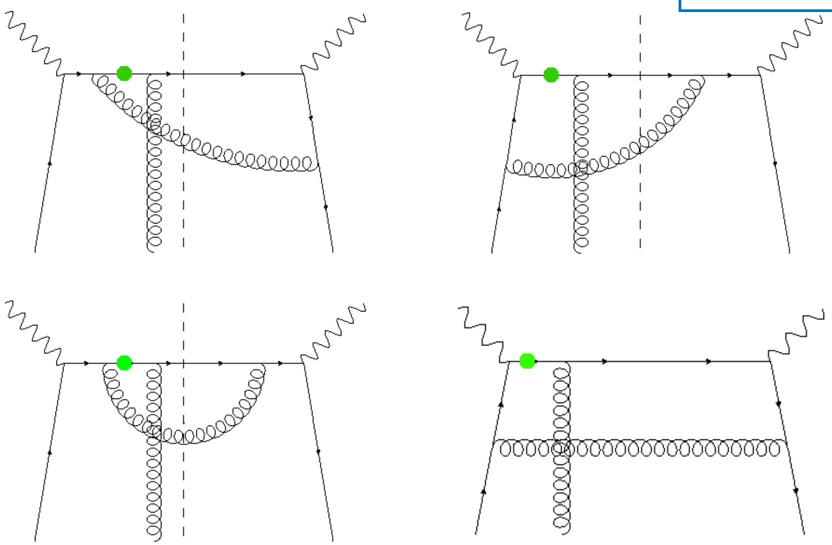
Twisi-3 quark DF for initial proton

- **Soft gluon pole**



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$$\Rightarrow E_F(x,x), \frac{dE(x,x)}{dx}$$

[1] H. Eguchi, Y. Koike, K. Tanaka, Nucl. Phys. B763 (2007) 198-227

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- **Soft-gluon pole**

Twist-2 FF for transversely polarized hyperon

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$$\times\left[8\left\{\frac{\hat{x}}{q_T^2(\hat{x}-1)}(-2Q^2(\hat{x}-1)+q_T^2(1+2\hat{x}))E_F(x,x)-\frac{2\hat{x}}{1-\hat{z}}x\frac{dE(x,x)}{dx}\right\}\sin\Phi_S(1+\cosh^2\psi)\right.$$

$$-\frac{4Q}{q_T}\left\{\frac{1}{q_T^2(\hat{x}-1)}(3Q^2(\hat{x}-1)-q_T^2(1+\hat{x}))E_F(x,x)+\frac{4\hat{x}}{1-\hat{z}}x\frac{dE_F(x,x)}{dx}\right\}\sinh 2\psi\sin\left(\Phi_S+\phi-\chi\right)$$

$$\left.+\frac{8Q^2}{q_T^2}\left\{\frac{1}{q_T^2(\hat{x}-1)}(-q_T^2+Q^2(\hat{x}-1))E_F(x,x)-\frac{2\hat{x}}{1-\hat{z}}x\frac{dE_F(x,x)}{dx}\right\}\sinh^2\psi\sin\left\{\Phi_S+2(\phi-\chi)\right\}\right]$$

Twist-3 DF for initial proton

Contribution from the twist-3 quark FFs

・Hadronic tensor[2]

$$W_{\mu\nu} = \int \frac{dz}{z^2} \mathrm{Tr}\left[\Delta(z) S_{\mu\nu}(z)\right]$$

→ Intrinsic quark FF $D_T(z)$

$$+ \Omega^\alpha_\beta \int \frac{dz}{z^2} Im \mathrm{Tr}\left[\Delta^\beta_\partial(z) \frac{\partial S_{\mu\nu}(z)}{\partial k^\alpha}\Big|_{c.l.}\right]$$

→ Kinematical quark FF $D_{1T}^{\perp(1)}(z)$

$$+ \Omega^\alpha_\beta \int \frac{dz}{z^2} \int \frac{dz'}{z'^2} \mathrm{P}\left(\frac{1}{1/z' - 1/z}\right) Im \mathrm{Tr}\left[\Delta^\beta_F(z,z') S^L_{\mu\nu,\alpha}(z',z) + (\rho \leftrightarrow \sigma)\right]$$

→ Dynamical quark FF $\hat{D}_{FT}(z,z'), \hat{G}_{FT}(z,z')$

・Equation of motion relation, Lorentz invariance relation[5]

EOM

$$\int_z^\infty \frac{dz'}{z'^2} \frac{1}{1/z - 1/z'} \left(Im\, \widehat{D}_{FT}(z,z') - Im\, \widehat{G}_{FT}(z,z') \right) = \frac{D_T(z)}{z} + D_{1T}^{\perp(1)}(z)$$

LIR

$$-\frac{2}{z} \int_z^\infty \frac{dz'}{z'^2} \frac{Im \widehat{D}_{FT}(z,z')}{(1/z' - 1/z)^2} = \frac{D_T(z)}{z} + \frac{d\left(D_{1T}^{\perp(1)}(z)/z\right)}{d(1/z)}.$$

[2] K. Kanazawa, Y. Koike, Phys. Rev. D88 (2013) 074022

[5] K .Kanazawa, Y. Koike, A. Metz, D. Pitonyak and M. Schlegel, Phys. Rev. D93 (2016) 054024

The twist-3 quark fragmentation functions

FF

- Intrinsic quark fragmentation functions.

$$\Delta_{ij}(z) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|\psi_i(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|\bar{\psi}_j(\lambda w)|0\rangle$$

$$= M_h \epsilon^{\alpha S_\perp w P_h}(\gamma_\alpha)_{ij}\frac{D_T(z)}{z} + \cdots$$

- Kinematical quark fragmentation functions.

$$\Delta^\alpha_{\partial ij}(z) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|[\infty w, 0]\psi_i(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|\bar{\psi}_j(\lambda w)[\lambda w, \infty w]|0\rangle \overleftarrow{\partial}^\alpha$$

$$= -iM_h \epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij}\frac{D_{1T}^{\perp(1)}(z)}{z} + \cdots$$

- Dynamical quark fragmentation functions

$$\Delta^\alpha_{Fij}(z, z') = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z'}} e^{-i\mu(\frac{1}{z}-\frac{1}{z'})} \langle 0|\psi_i(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|\bar{\psi}_j(\lambda w) gF^{\alpha w}(\mu w)|0\rangle$$

$$= M_h \epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij}\frac{\widehat{D}^*_{FT}(z, z')}{z} - iM_h S^\alpha_\perp(\gamma_5 \not{P}_h)_{ij}\frac{\widehat{G}^*_{FT}(z, z')}{z} + \cdots$$

Contribution from the twist-3 quark FFs

・Hadronic tensor[2]

$$W_{\mu\nu} = \int \frac{dz}{z^2} \mathrm{Tr}\left[\Delta(z) S_{\mu\nu}(z)\right]$$

$$+ \Omega^\alpha_\beta \int \frac{dz}{z^2} Im\mathrm{Tr}\left[\Delta^\beta_\partial(z) \frac{\partial S_{\mu\nu}(z)}{\partial k^\alpha}\Big|_{c.l.}\right]$$

$$+ \Omega^\alpha_\beta \int \frac{dz}{z^2} \int \frac{dz'}{z'^2} \mathrm{P}\left(\frac{1}{1/z' - 1/z}\right) Im\mathrm{Tr}\left[\Delta^\beta_F(z,z') S^L_{\mu\nu,\alpha}(z',z) + (\rho \leftrightarrow \sigma)\right]$$

→ Intrinsic quark FF $D_T(z)$

→ Kinematical quark FF $D^{\perp(1)}_{1T}(z)$

→ Dynamical quark FF $\hat{D}_{FT}(z,z'), \hat{G}_{FT}(z,z')$

・Equation of motion relation, Lorentz invariance relation[5]

EOM
$$\int_z^\infty \frac{dz'}{z'^2} \frac{1}{1/z - 1/z'} \left(Im\,\widehat{D}_{FT}(z,z') - Im\,\widehat{G}_{FT}(z,z') \right) = \frac{D_T(z)}{z} + D^{\perp(1)}_{1T}(z)$$

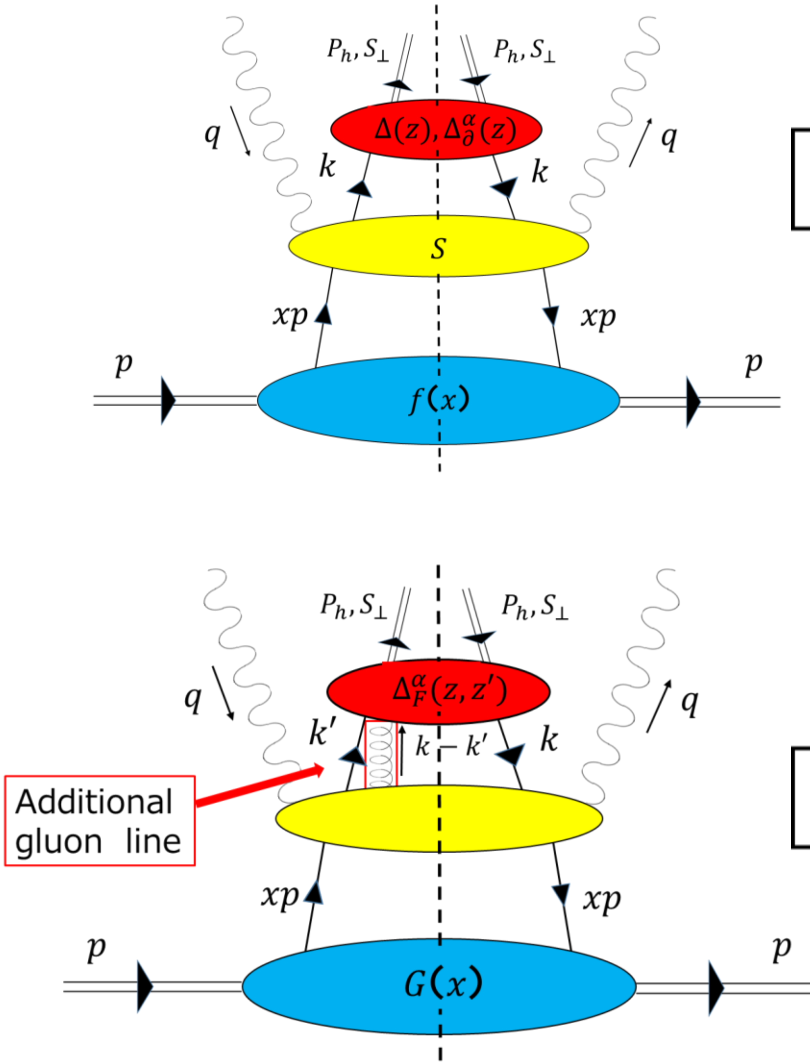
LIR
$$-\frac{2}{z} \int_z^\infty \frac{dz'}{z'^2} \frac{Im\widehat{D}_{FT}(z,z')}{(1/z' - 1/z)^2} = \frac{D_T(z)}{z} + \frac{d\left(D^{\perp(1)}_{1T}(z)/z\right)}{d(1/z)}.$$

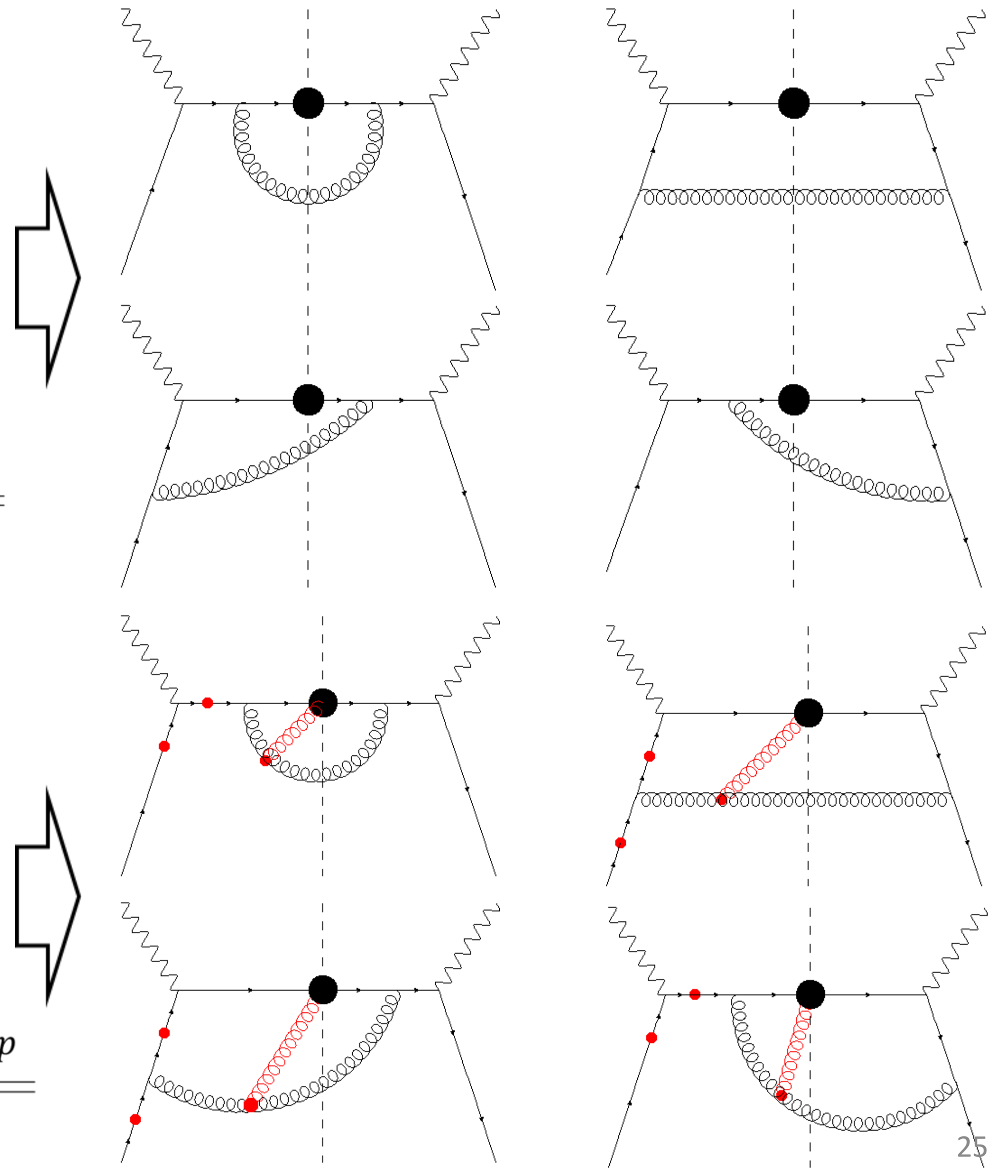
[2] K. Kanazawa, Y. Koike, Phys. Rev. D88 (2013) 074022

[5] K .Kanazawa, Y. Koike, A. Metz, D. Pitonyak and M. Schlegel, Phys. Rev. D93 (2016) 054024

Contribution from the twist-3 quark FFs

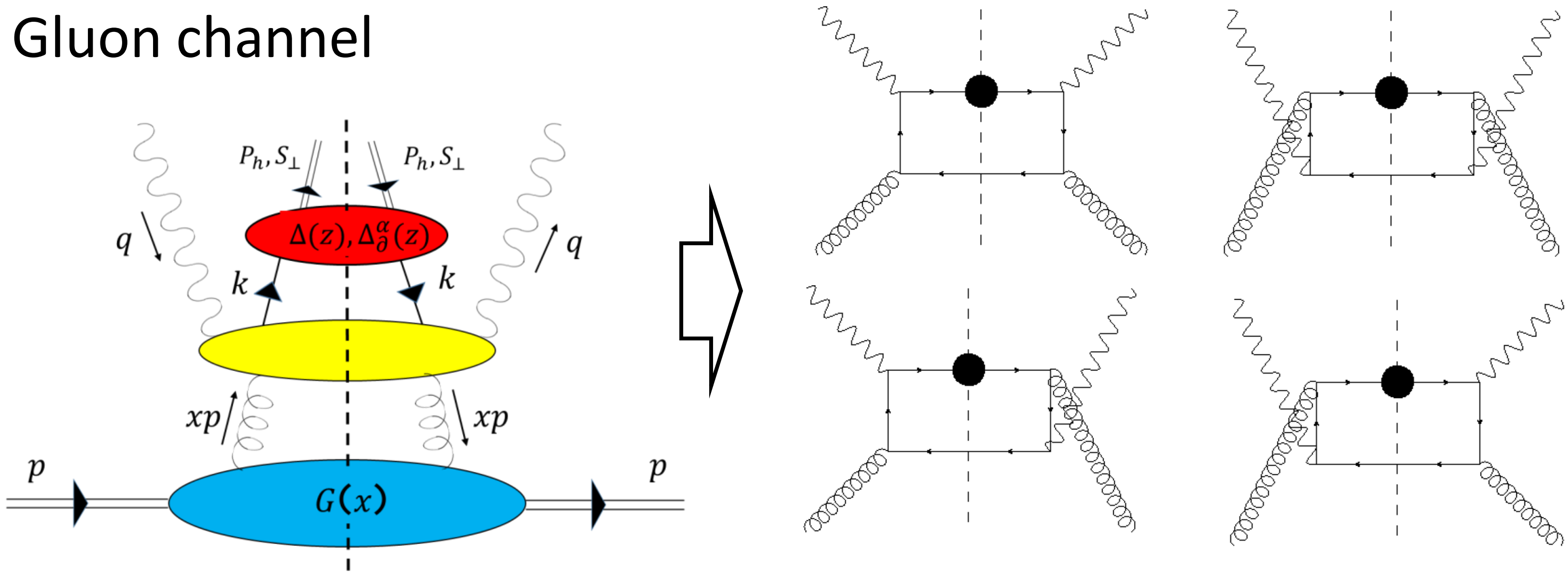
- Quark channel

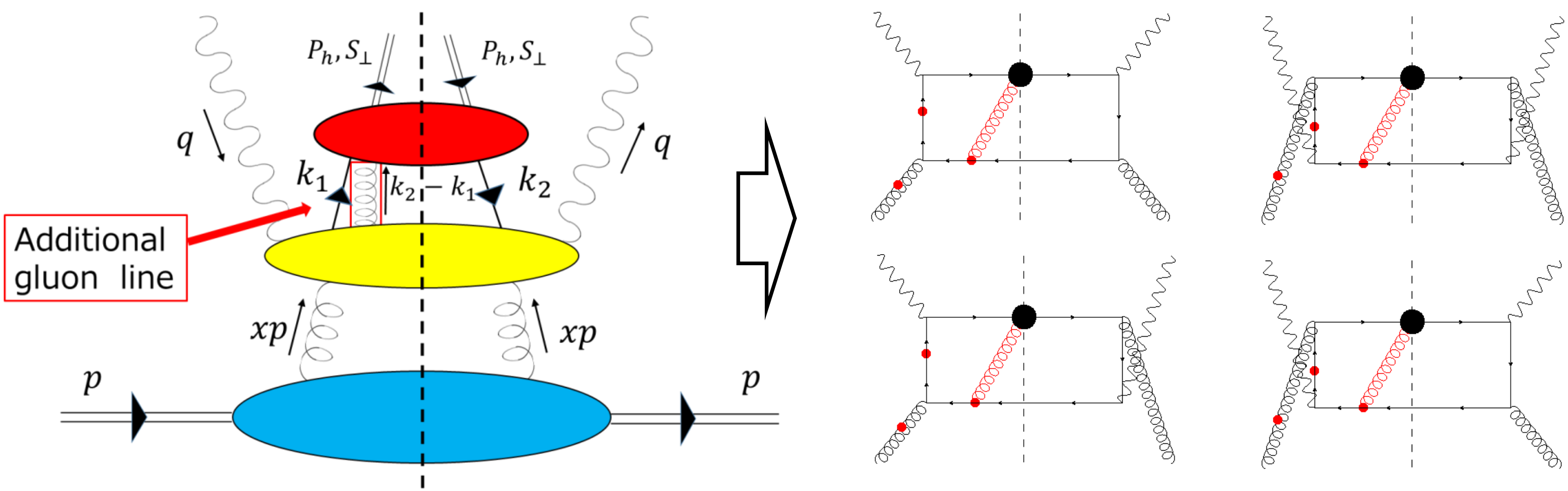




Contribution from the twist-3 quark FFs

- Gluon channel





Result

- Quark channel

$$\mathcal{S}_{1,2,3,4} = \sin\Psi_s, \mathcal{S}_{8,9} = \cos\Psi_s$$

$$\frac{d^6\Delta\sigma^{q-frag,q-dist}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2\alpha_s M_h}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2}\sum_k \mathcal{A}_k(\phi-\chi)\mathcal{S}_k \int\frac{dx}{x}f(x)\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$

$$\times\left[\frac{D_T(z)}{z}\hat{\sigma}_1^k+\left\{\frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\hat{\sigma}_2^k\right\}+D_{1T}^{\perp(1)}(z)\hat{\sigma}_3^k\right.$$

$$+\int\frac{dz'}{z'^2}\mathrm{P}\left(\frac{1}{1/z'-1/z}\right)\left\{Im\hat{D}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{DF3}^k-\frac{1}{Q^2}\frac{1}{1-(1-q_T^2/Q^2)\,z_f/z'}\hat{\sigma}_{DF4}^k\right]\right.$$

$$\left.\left.+Im\hat{G}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{GF3}^k-\frac{1}{Q^2}\frac{1}{1-(1-q_T^2/Q^2)\,z_f/z'}\hat{\sigma}_{GF4}^k\right]\right\}\right]$$

Result

$$\mathcal{S}_{1,2,3,4} = \sin\Psi_s, \mathcal{S}_{8,9} = \cos\Psi_s$$

- Quark channel

$$\frac{d^6\Delta\sigma^{q-frag,q-dist}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2\alpha_s M_h}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2}\sum_k \mathcal{A}_k(\phi-\chi)\mathcal{S}_k\int\frac{dx}{x}f(x)\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$

$$\times\left[\frac{D_T(z)}{z}\hat{\sigma}_1^k + \left\{\frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\hat{\sigma}_2^k\right\} + D_{1T}^{\perp(1)}(z)\hat{\sigma}_3^k\right.$$

$$\cdots \frac{z'}{z}\hat{\sigma}^k_{DF3} - \frac{1}{Q^2}\frac{1}{1-(1-q_T^2/Q^2)\,z_f/z'}\hat{\sigma}^k_{DF4}\Big]$$

$$\cdots \frac{\cdots}{\cdots/Q^2)\,z_f/z'}\hat{\sigma}^k_{GF4}\Big]\Big\}\Big]$$

Example. k=2

$$\hat{x} = \frac{x_{bj}}{x} \quad \hat{z} = \frac{z_f}{z}$$

$$\sigma_1^2 = -\frac{1}{N}\frac{8\hat{x}\hat{z}}{q_T} + N\frac{8(-1+2\hat{z})\hat{x}}{q_T}$$

Result

- Gluon channel

$\mathcal{S}_{1,2,3,4} = \sin\Psi_s, \mathcal{S}_{8,9} = \cos\Psi_s$

$$\frac{d^6\Delta\sigma^{q-frag,g-dist}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2\alpha_s M_h}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2}\sum_k \mathcal{A}_k(\phi-\chi)\mathcal{S}_k \int\frac{dx}{x}G(x)\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$

$$\times\left[\frac{D_T(z)}{z}\hat{\sigma}^k_{1,GC}+\left\{\frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\hat{\sigma}^k_{2,GC}\right\}+D_{1T}^{\perp(1)}(z)\hat{\sigma}^k_{3,GC}\right.$$

$$+\int\frac{dz'}{z'^2}\mathrm{P}\left(\frac{1}{1/z'-1/z}\right)\left\{Im\hat{D}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}^k_{DF3,GC}-\frac{1}{Q^2}\frac{1}{1-(1-q_T^2/Q^2)\,z_f/z'}\hat{\sigma}^k_{DF4,GC}\right]\right.$$

$$\left.\left.+Im\hat{G}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}^k_{GF3,GC}-\frac{1}{Q^2}\frac{1}{1-(1-q_T^2/Q^2)\,z_f/z'}\hat{\sigma}^k_{GF4,GC}\right]\right\}\right]$$

Result

$\mathcal{S}_{1,2,3,4} = \sin\Psi_s, \mathcal{S}_{8,9} = \cos\Psi_s$

- Gluon channel

$$\frac{d^6\Delta\sigma^{q-frag,g-dist}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2\alpha_s M_h}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2}\sum_k \mathcal{A}_k(\phi-\chi)\mathcal{S}_k \int\frac{dx}{x}G(x)\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$

$$\times\left[\frac{D_T(z)}{z}\hat{\sigma}^k_{1,GC} + \left\{\frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\hat{\sigma}^k_{2,GC}\right\} + D_{1T}^{\perp(1)}(z)\hat{\sigma}^k_{3,GC}\right.$$

$$\frac{1}{Q^2}\frac{1}{1-(1-q_T^2/Q^2)\,z_f/z'}\hat{\sigma}^k_{DF4,GC}\Big]$$

$$\frac{\;}{z'}\hat{\sigma}^k_{GF4,GC}\Big]\Big\}\Big]$$

Example. k=2 $\hat{x} = \frac{x_{bj}}{x} \quad \hat{z} = \frac{z_f}{z}$

$$\sigma^2_{1,GC} = -\frac{8(-1+\hat{x})\hat{x}(-1+\hat{x}+\hat{x}\hat{z})}{q_T\hat{z}(-1+\hat{x}+\hat{z})} + \frac{1}{N^2-1}\frac{16(-1+\hat{x})(-1+\hat{z})\hat{x}}{q_T\hat{z}}$$

Result

- Gluon channel

$$\mathcal{S}_{1,2,3,4} = \sin\Psi_s,\ \mathcal{S}_{8,9} = \cos\Psi_s$$

$$\frac{d^6\Delta\sigma^{q-frag,g-dist}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2\alpha_s M_h}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2}\sum_k \mathcal{A}_k(\phi-\chi)\mathcal{S}_k \int\frac{dx}{x}G(x)\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$

$$\times\left[\frac{D_T(z)}{z}\hat{\sigma}^k_{1,GC}+\left\{\frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\hat{\sigma}^k_{2,GC}\right\}+D_{1T}^{\perp(1)}(z)\hat{\sigma}^k_{3,GC}\right.$$

$$+\int\frac{dz'}{z'^2}\mathrm{P}\left(\frac{1}{1/z'-1/z}\right)\left\{Im\hat{D}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}^k_{DF3,GC}-\frac{1}{Q^2}\frac{1}{1-(1-q_T^2/Q^2)\,z_f/z'}\hat{\sigma}^k_{DF4,GC}\right]\right.$$

$$\left.\left.+Im\hat{G}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}^k_{GF3,GC}-\frac{1}{Q^2}\frac{1}{1-(1-q_T^2/Q^2)\,z_f/z'}\hat{\sigma}^k_{GF4,GC}\right]\right\}\right]$$

$$\frac{d^6\Delta\sigma^{q-frag}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{d^6\Delta\sigma^{q-frag,q-dist}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} + \frac{d^6\Delta\sigma^{q-frag,g-dist}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi}$$

Completed!!

Summary

・We have calculated the twist-3 distribution and the twist-3 fragmentation contribution to polarized hyperon production in SIDIS.

・ $ep \to e\Lambda^{\uparrow}X$

Contribution of the

1. twist-3 quark distribution function → **Completed**
2. twist-3 quark fragmentation function → **Completed**
3. twist-3 gluon fragmentation function → Talk by R. Ikarashi (TMD session)

⇒ Global analysis leads to understanding of nucleon spin structure.

・The process may be observed in Electron-Ion-Collider(EIC) experiment.

Next stage. NLO correction, ...