Transverse spin sum rules

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Outline

- Introduction: good and bad news about the longitudinal spin sum rules
- Transverse-spin sum rules
  - Twist-2 case: a unique simple partonic spin sum rule
  - Twist-3 case: a rotation of Jaffe-Manohar sum rule
- Conclusions

Ref: Ji, Zhao, and Yuan, What we know and what we don't know about the proton spin after 30 years

Nature Reviews Physics 3, 65 (2021)

Introduction: good news news about longitudinal spin sum rules

Longitudinal spin sum rules

- The first longitudinal spin sum rule in QCD was put forward by Jaffe and Manohar (1990)

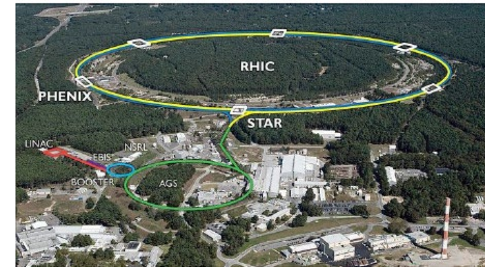
$$\frac{1}{2} = \frac{1}{2}\ \Delta\Sigma + \Delta G + \ell_q^z + \ell_g^z$$

- It is a sum rule in the infinite momentum frame $P^z = \infty$ or partonic sum rule.
- Good news: it involves "simplest" experimentally accessible quantities:

$\Delta\Sigma$: total quark helicity

ΔG: total gluon helicity

Progress in measuring ΔG



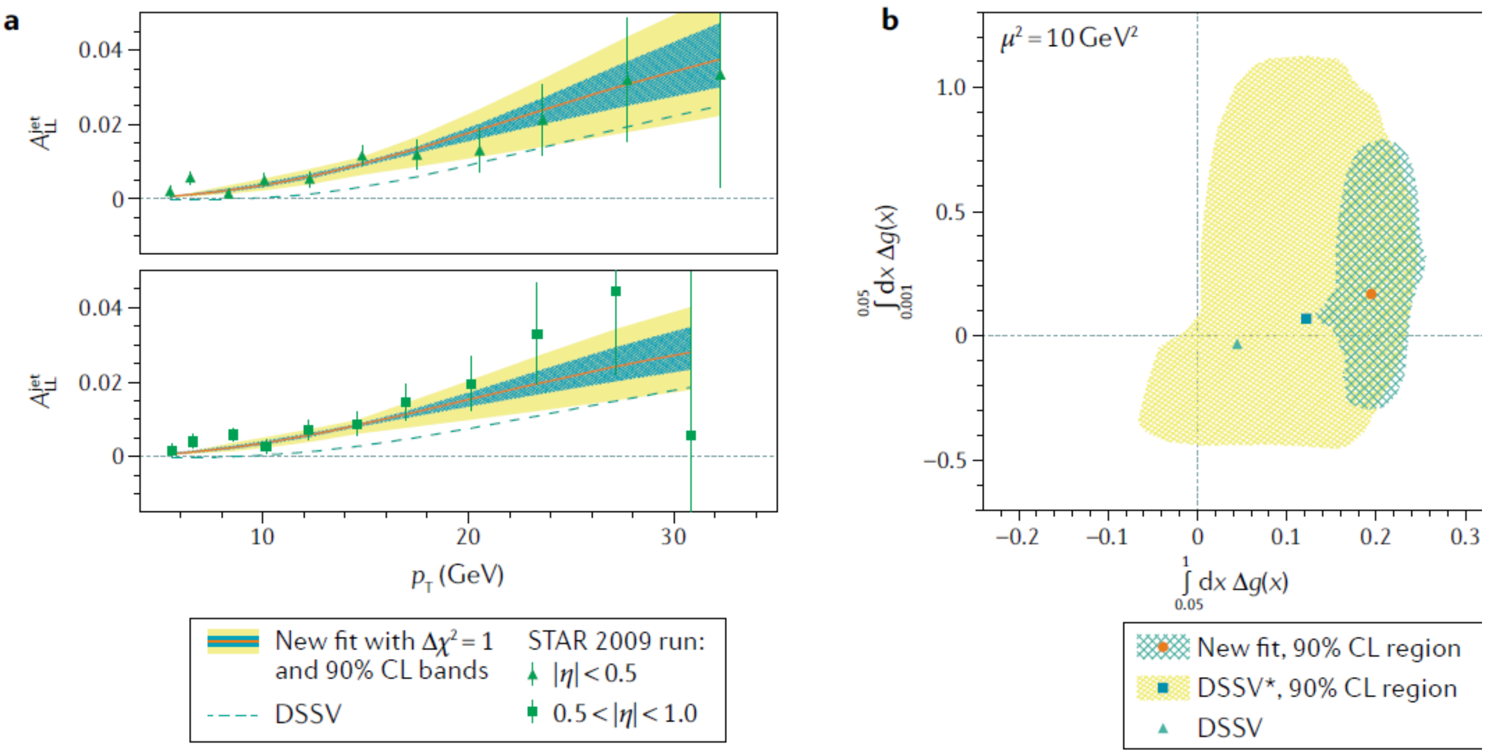


Fig. 2 | **The Relativistic Heavy Ion Collider at Brookhaven National Laboratory provides strong evidence for the gluon helicity contribution to the proton spin. a** | Double spin asymmetry A_{LL}^{jet} in inclusive jet production measured

Progress in understanding ΔG

- There is no gauge-invariant local axial vector corresponding to the gluon spin.
- There is a gauge-dependent candidate for ΔG

$$\vec{S}_g = \overrightarrow{E_a} \times \vec{A}_a$$

- In the infinite-momentum limit, the gauge-dependent part of the above operator vanishes

X. Ji, J. Zhang & Y. Zhao Phys.Rev.Lett. 111 (2013)

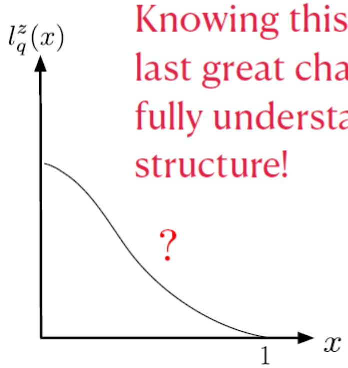
- This provides a recipe for lattice QCD calculation

Bad news about Jaffe-Manohar sum rules

- The helicity of the nucleon, ½, does not grow at the nucleon's momentum gets large. It stays as constant. Thus it is a twist-3 quantity.
- But it so happens that $\Delta\Sigma$ & ΔG are twist-2 quantities,
- OAM l_q and l_g are twist-3!
- The only way we know how to access them experimentally is through twist-three GPDs

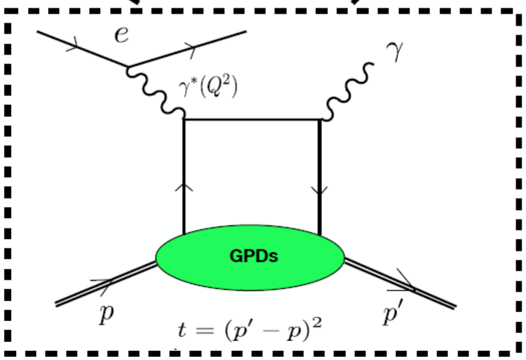
Parton OAM & twist-3 GPD

$$l_q^z = \int dx\, l_q^z(x) \longrightarrow l_q^z(x) = \int dy \left[G_{q,D,3}(x,y) + \mathcal{P}\frac{1}{y-x} G_{q,F,3}(x,y) \right]$$



OAM density

Knowing this curve will be the last great challenge in fully understanding proton spin structure!



Twist-3 GPDs extractable from DVCS & DVMP

Very very challenging! It will be a while before we learn how to measure twist-3 GPDs

A frame-independent sum rule

- Frame-independent longitudinal spin sum rule (ji, 1996)

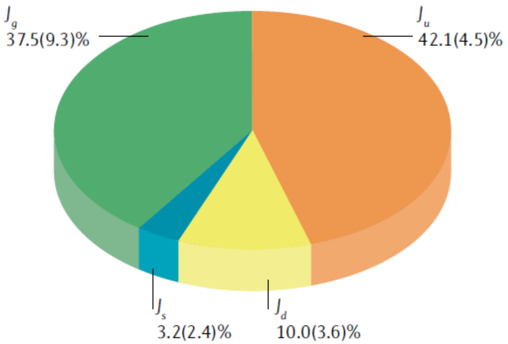
$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Delta\Sigma + L_q^z + \mathrm{J_g}$$

- J_q & J_g are related to the EMT form factor

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right]$$

- They can be calculated using the standard lattice QCD approach

ETMC collaboration

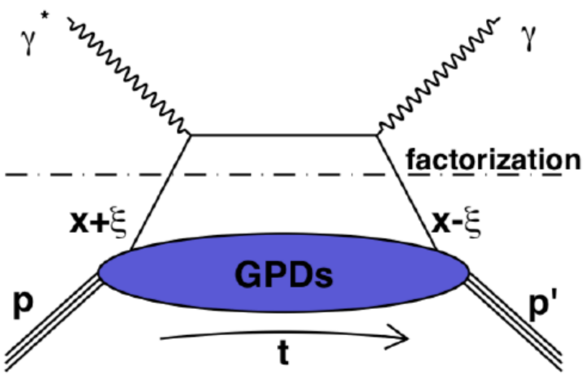


Twist-2 GPD sum rule and DVCS etc

- GPDs were introduced to extract the form factors of EMT through the sum rules

$$\int_{-1}^{1} dx\ xH(x,\xi,t) = A(t) + \xi^2\ C(t)\ ,$$
$$\int_{-1}^{1} dx\ xE(x,\xi,t) = B(t) - \xi^2\ C(t)\ .$$

- To extract H & E from DVCS and similar processes model-independently are somewhat challenging



Problem with frame-indepedent sum rule

- It does not have simple parton interpretation when considered in IMF.
- Even though H and E come from twist-two operators, but in the longitudinal polarized state, its parton structure is complicated!
- Consider a twist-2 operator

$$\bar{\psi}\gamma^{(\mu_1} iD^{\mu_2} \dots iD^{\mu_n)}\psi \quad \text{(all } \mu \text{ arbitray)}$$

Its matrix element has only simple partonic interpretation when all indices +.

Transverse spin sum rules: twist-2 case

Transverse spin is a twist-2 quantity!

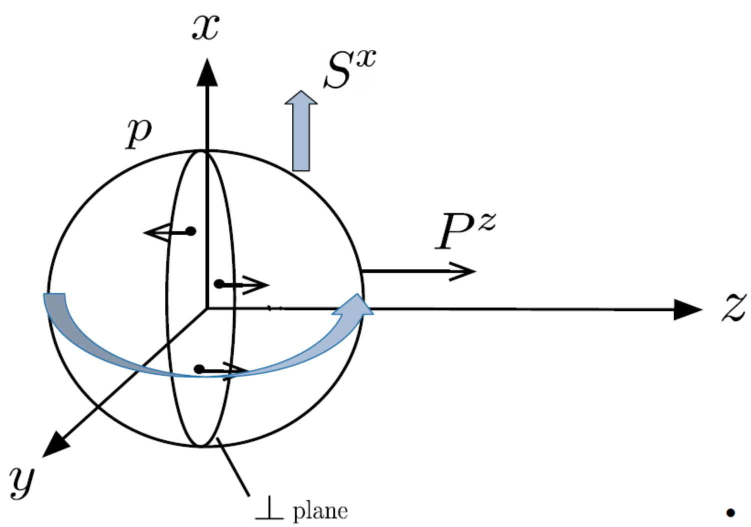
- Transverse AM grows with the external momentum of the hadron
- Consider the second order tensor $J^{\mu\nu} = (\vec{J}, \vec{K})$
- It transforms under Lorentz trans. Like $\left(\vec{B}, \vec{E}\right)$

Under Lorentz trans., $B_\perp$ increase by factor of γ, thus behave like a twist-2 quantity.

transverse spin: $\frac{\hbar}{2} \rightarrow \frac{\gamma\hbar}{2}$

ji & Yuan *Phys.Lett.B* 810 (2020) 135786

Transversely Polarized Proton:



$$J^x = \sum_i r_i^y p_i^z - r_i^z p_i^y$$

Kinematics: $\sim \gamma$ (Tw-2) $\quad \sim \gamma^{-1}$ (Tw-4)

$$\gamma = (1-\beta^2)^{-1/2}$$

$$[J^{x,y}, K^z] \neq 0$$

- This means that J^x is P^z (frame)-dependent ☹

- Less studied than Longitudinal case because:
  1. It is frame-dependent with non-trivial boost properties
  2. A key issue is separating intrinsic contributions from CM ones
- This has led to ~~some~~ a lot of controversy in previous works

Frame dependence

Transverse Polarization Sum Rules:

- Let's look again at transverse AM, but **split in terms of its 2 contributions**:

$\xi^y T^{0z}(\xi)$ $\xi^z T^{0y}(\xi)$

Note: equal in rest frame by rotational symmetry

$$J^x = \left(\gamma - \frac{1}{2\gamma}\right) J^{x(2)} + \frac{1}{2\gamma} J^{x(3)} = \gamma \frac{\hbar}{2}$$

$J^{x(2)}$ & $J^{x(3)}$ ~ proton matrix elements of q and g fields

- Solution: $J^{x(2)} = J^{x(3)} = \frac{\hbar}{2}$

Twist-2 Ji Sum Rule

Novel Twist-3 Sum Rule

Spin sum rules are γ-independent!

2 Sum Rules:

$$J_q^{x(2)} + J_g^{x(2)} = \frac{\hbar}{2}$$ Twist-2

$$J_q^{x(3)} + J_g^{x(3)} = \frac{\hbar}{2}$$ Twist-3

Simple parton sum rule for transverse spin

- Spin sum rule as an infinite momentum frame simple parton sum rule for transverse spin

Twist-2 Sum Rule:

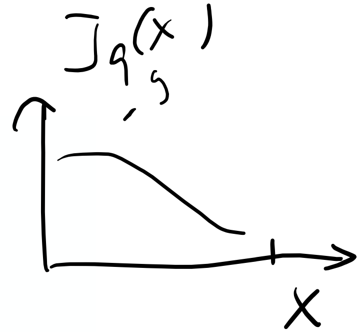
$$J_q^{x(2)} + J_g^{x(2)} = \frac{\hbar}{2}$$

X. Ji, PRL 78 (1997) 610

X. Ji, X. Xiong, and F. Yuan, PLB 717, 214 (2012)

$$J_q^{x(2)}(x) = \frac{x}{2}\left(H_q(x) + E_q(x)\right)$$

$$J_g^{x(2)}(x) = \frac{x}{2}\left(H_g(x) + E_g(x)\right)$$



$$J_{q,g} = \int_0^1 J_{q,g}(x)\,dx$$

- Complete twist-2 partonic sum rule!

Transverse spin sum rules: twist-3 case

Twist-3 transverse spin sum rule

- Rotated version of the Jaffe Manohar Longitudinal spin sum rule

X. Ji, Y. Guo & K. Shiells, Nuc. Phys. B 969, 115440 (2021)

Transverse Polarization: $$\frac{1}{2}\Delta q_T + \Delta G_T + l_q^{x(3)} + l_g^{x(3)} = \frac{\hbar}{2}$$

Δq_T , ΔG_T Involve measurable PDFs in DIS and correspond to spin

$l_q^{x(3)}$, $l_g^{x(3)}$ Involve twist-3 GPDs and correspond to canonical OAM

- Δq_T is related to g_2 structure function
- ΔG_T is a twist-three, spin-dependent gluon distribution
- Twist-3 OAM are a challenge to measure

g_2 structure function

- Can be measured with a transversely polarized target.
- Jlab 6 GeV, 12 GeV
- Moments have been calculated on lattice in the past
- X distribution can be calculated using large momentum effective theory (talks in the previous session)

ΔG_T: twist-3 gluon polarization

- When the target is transversely polarized, there is a twsit-3 gluon density.
- This mixes with g_2 in factorization formula.
- No experimental data.
- Can be calculated on lattice.

$$\langle PS | G^{+i}(0)\, \tilde{G}^{\perp i}(\lambda n) | P \rangle$$

$\int l$,

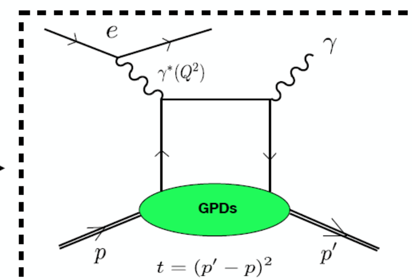
Experimental Roadmap for spin sums

	Longitudinal	Transverse
Frame-independent sum rule:		$\frac{1}{2}\Delta q + L_q^i + J_g^i = \frac{\hbar}{2}$ → $J_q^x + J_g^x = \frac{\hbar}{2}$ $\sim \int dxx$ moments of: $H_{q,g}(x), E_{q,g}(x)$ — 4 twist-2 GPDs extractable from DVCS & DVMP!
IMF sum rule:	$\frac{1}{2}\Delta q + l_q^z + \Delta G + l_g^z = \frac{\hbar}{2}$	$\frac{1}{2}\Delta q_T + l_q^x + \Delta G_T + l_g^x = \frac{\hbar}{2}$

$\int dx g_1(x)$
Quark helicity distributions probed in SIDIS

$\int dx \Delta G(x)$
Gluon helicity distribution probed in p-p collisions at RHIC

$\int dx dy \mathcal{F}^{(3)}_{\mathrm{GPD}}(x,y)$
Twist-3 OAM quark and Gluon GPDs extractable From DVCS & DVMP



Less experimental constraints

$g_1, \; g_T = g_1 + g_2$

Conclusion

- The simplest spin sum rule is twist-2 transverse spin sum rule
- For long. pol., Jaffe-Manohar sum rule is a twist-3 one.
- For long. Pol, the frame independent sum rule does not have a simple parton interpretation,
- There is a frame-rotated version of transverse spin sum rule which involves g_2 and ΔG_T
- All spin sum rules involve GPDs