# **Small-***x* **Helicity Evolution and the Proton Spin Puzzle**



Daniel Pitonyak Lebanon Valley College, Annville, PA, USA



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# Outline

- Background and motivation
- > Theoretical results small-x helicity evolution
- Phenomenological results fit of the world polarized DIS data
- ➤ A possible path to resolving the spin puzzle
- Summary and outlook





# **Background and Motivation**













$$S_q(Q^2 = 10 \,\mathrm{GeV}^2) \approx 0.15 \div 0.20$$
  
 $S_g(Q^2 = 10 \,\mathrm{GeV}^2) \approx 0.13 \div 0.26$   
 $\mathcal{L}_q + \mathcal{L}_g = \mathrm{whatever} \mathrm{is} \mathrm{left}$ 

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$$\begin{split} S_q(Q^2 &= 10\,{
m GeV}^2) &pprox 0.15 \div 0.20 \\ S_g(Q^2 &= 10\,{
m GeV}^2) &pprox 0.13 \div 0.26 \\ \mathcal{L}_q + \mathcal{L}_g &= {
m whatever is left} \qquad \mbox{...but there are a few (significant) caveats} \end{split}$$

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1) The calculations of  $S_q$  and  $S_g$  are based on *truncated* integrals – can *never* measure down to x = 0!

$$S_q = \frac{1}{2} \int_{0.001}^1 dx \,\Delta\Sigma(x, Q^2) \qquad S_g = \int_{0.05}^1 dx \,\Delta g(x, Q^2)$$

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2) The extractions for  $\Delta \Sigma(x)$  and  $\Delta g(x)$  have large uncertainties at small *x* even *including EIC data*!



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The main reason for these issues is that "standard" extractions of helicity PDFs use DGLAP evolution and *parametrize* the x dependence at some initial scale  $Q_0$  and then evolve to higher Q





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- Such equations were derived by Y. Kovchegov, DP, and M. Sievert (KPS evolution) in a series of papers from 2015-2018 similar to the BK/JIMWLK evolution for unpolarized PDFs



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- Such equations were derived by Y. Kovchegov, DP, and M. Sievert (KPS evolution) in a series of papers from 2015-2018 similar to the BK/JIMWLK evolution for unpolarized PDFs

If we want to resolve the spin crisis, and accurately calculate values for  $S_q$  and  $S_g$  (integrating down to x = 0), we need to incorporate KPS evolution into phenomenological extractions of helicity PDFs





# **Theoretical Results – Small-***x* **Helicity Evolution**



# At small x, a process like DIS is dominated by the virtual photon splitting into a dipole:



$$x_{ij}\equiv |\underline{x}_i-\underline{x}_j|$$
 is the dipole size

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► All flavor singlet small-*x* helicity quantities (e.g.,  $\Delta \Sigma(x)$ ) depend on the "polarized dipole amplitude"  $G_q(x_{10}, zs)$ 

 $s = Q^{2}(1-x)/x$  is the invariant mass squared of the  $\gamma^{*}N$  system



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The evolution of the polarized dipole amplitude diagrammatically takes on the following form (Kovchegov, DP, Sievert: JHEP 1601 (2016), PRL 118 (2017), PRD 95 (2017), PLB 772 (2017), JHEP 1710 (2017); Kovchegov & Sievert PRD 99 (2019))



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> The evolution equations close in the large- $N_c$  (and large- $N_c$  & - $N_f$ ) limit:

$$G_{q}(s_{10},\eta) = G_{q}^{(0)}(s_{10},\eta) + \int_{s_{10}}^{\eta} d\eta' \int_{s_{10}}^{\eta'} ds_{21} \left[ \Gamma_{q}(s_{10},s_{21},\eta') + 3G_{q}(s_{21},\eta') \right]$$
polarized dipole flavor dependent initial condition
flavor independent evolution

$$\label{eq:eq:expansion} \boxed{ \begin{split} \eta \equiv \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{zs}{\Lambda^2} \\ s_{ij} \equiv \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{1}{x_{ij}^2 \Lambda^2} \quad \mathbf{7} \end{split} }$$

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"neighbor dipole"

$$\Gamma_q(s_{10}, s_{21}, \eta') = G_q^{(0)}(s_{10}, \eta') + \int_{s_{10}}^{\eta'} d\eta'' \int_{s_{21} - \eta' + \eta''}^{\eta''} ds_{32} \left[ \Gamma_q(s_{10}, s_{32}, \eta'') + 3G_q(s_{32}, \eta'') \right]$$

$$\eta \equiv \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{zs}{\Lambda^2}$$
$$s_{ij} \equiv \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{1}{x_{ij}^2 \Lambda^2} \quad \mathbf{7}$$





➤ In the asymptotic high-energy regime (Kovchegov, DP, Sievert PRL 118 (2017))

$$\Delta \Sigma(x) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 2.31 \sqrt{\frac{\alpha_s N_c}{2\pi}} \stackrel{\alpha_s = 0.3}{\approx} 0.874$$





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➤ Using a different operator,

$$G_{10}^{i}(z) \equiv \frac{1}{4N_{c}} \int_{-\infty}^{\infty} dx^{-} \left\langle \operatorname{tr} \left[ V_{\underline{0}}[\infty, -\infty] V_{\underline{1}}[-\infty, x^{-}] \left(-ig\right) \tilde{A}^{i}(x^{-}, \underline{x}) V_{\underline{1}}[x^{-}, \infty] \right] + \operatorname{c.c.} \right\rangle(z)$$

one can derive the small-*x* evolution for  $\Delta g(x)$  and finds the following high-energy asymptotics (Kovchegov, DP, Sievert JHEP **118** (2017)):

$$\Delta g(x) \sim \left(\frac{1}{x}\right)^{\alpha_h^G} \quad \text{with} \quad \alpha_h^G = \frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 1.88 \sqrt{\frac{\alpha_s N_c}{2\pi}} \stackrel{\alpha_s = 0.3}{\approx} 0.712$$





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 $\implies S_q \gg S_g$  at small x





# Phenomenological Results – Fit of the World Polarized DIS Data

D. Adamiak, Y. Kovchegov, W. Melnitchouk, DP, N. Sato and M. Sievert, PRD 104, L031501 (2021) [arXiv:2102.06159 [hep-ph]]

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# NSE

➢ Using the polarized dipole amplitude, we can calculate

 $\Delta q^+(x,Q^2) \equiv \Delta q(x,Q^2) + \Delta \bar{q}(x,Q^2)$ 

$$=\frac{1}{\alpha_s \pi^2} \int_{0}^{\ln \frac{Q^2}{x \Lambda^2}} d\eta \int_{\max\left\{\eta - \ln \frac{1}{x}, 0\right\}}^{\eta} ds_{10} G_q(s_{10}, \eta)$$

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and at LO

$$g_1(x,Q^2) = \frac{1}{2} \sum_q e_q^2 \Delta q^+(x,Q^2)$$

This allows us to carry out a fit of the world polarized DIS data on  $A_1$  and  $A_{||}$ 

$$A_1 \sim A_{\parallel} = \frac{\sigma_{+-} - \sigma_{++}}{\sigma_{+-} + \sigma_{++}} \sim \frac{g_1}{F_1}$$





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N.B.: Only using DIS data does *not* allow for a flavor separation of the helicity PDFs unless one assumes SU(3) symmetry of the sea and uses constraints from the octet axial charge  $a_8$ . We do not utilize this constraint and instead extract only  $g_1^p$  and  $g_1^n$ .

$$G_p^{(0)}(s_{10},\eta) = a_p \,\eta + b_p \, s_{10} + c_p$$

$$G_n^{(0)}(s_{10},\eta) = a_n \eta + b_n s_{10} + c_n$$





The KPS evolution equations must start at some low value of  $x = x_0$ . We fit the world polarized DIS data (proton, deuteron, and <sup>3</sup>He targets) using a cut of  $x < x_0$  on the data



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The results on the next slides are for  $x_0 = 0.1$  (122 data points). The fits were preformed within the Jefferson Lab Angular Momentum (JAM) Collaboration Monte Carlo framework – refer to as JAMsmallx







$$G_p^{(0)}(s_{10},\eta) = a_p \eta + b_p s_{10} + c_p$$
$$G_n^{(0)}(s_{10},\eta) = a_n \eta + b_n s_{10} + c_n$$

$$a_p = -1.33 \pm 0.30$$
  $a_n = -2.47 \pm 0.65$   
 $b_p = 0.49 \pm 0.44$   $b_n = 3.03 \pm 1.01$   
 $c_p = 2.24 \pm 0.16$   $c_n = 0.30 \pm 0.36$ 





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$$G_p^{(0)}(s_{10}, \eta) = a_p \eta + b_p s_{10} + c_p$$
$$G_n^{(0)}(s_{10}, \eta) = a_n \eta + b_n s_{10} + c_n$$
$$p = -1.33 \pm 0.30 \quad a_n = -2.47 \pm 0.65$$

$$b_p = 0.49 \pm 0.44$$
  $b_n = 3.03 \pm 1.01$   
 $c_p = 2.24 \pm 0.16$   $c_n = 0.30 \pm 0.36$ 



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still "blow up" even including EIC data





An opportunity presented by the EIC that can allow us to obtain a flavor separation using only DIS data is a measurement of parity-violating DIS (PVDIS)

$$g_1^{\gamma Z}(x, Q^2) = \sum_q e_q \, g_V^q \, \Delta q^+(x, Q^2)$$

As a proof of principle that we can extract  $\Delta u^+$ ,  $\Delta d^+$ ,  $\Delta s^+$  with our JAMsmallx framework, we fit EIC pseudo-data for PVDIS (and DIS) along with the current experimental DIS data

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![](_page_37_Picture_2.jpeg)

![](_page_37_Figure_3.jpeg)

![](_page_38_Picture_2.jpeg)

![](_page_38_Figure_3.jpeg)

![](_page_39_Picture_0.jpeg)

![](_page_39_Picture_2.jpeg)

# A Possible Path to Resolving the Spin Puzzle

![](_page_40_Picture_2.jpeg)

$$\frac{1}{2} = S_q + \mathcal{L}_q + S_g + \mathcal{L}_g$$
$$S_q|_{\text{large } x} = \frac{1}{2} \int_{0.001}^1 dx \,\Delta\Sigma(x, Q^2) \approx 0.18$$
$$S_g|_{\text{large } x} = \int_{0.05}^1 dx \,\Delta g(x, Q^2) \approx 0.20$$

![](_page_41_Picture_2.jpeg)

$$\frac{1}{2} = S_q + \mathcal{L}_q + S_g + \mathcal{L}_g$$
$$S_q|_{\text{large } x} = \frac{1}{2} \int_{0.001}^1 dx \,\Delta\Sigma(x, Q^2) \approx 0.18$$
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$$J_{0.05}$$

$$\mathcal{L}_q + \mathcal{L}_g =$$
whatever is left ???

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![](_page_42_Figure_2.jpeg)

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![](_page_43_Picture_2.jpeg)

Using similar techniques as those used to derive small-x helicity evolution, one can show at small x (Hatta and Yang (2018); Kovchegov (2019))

$$\mathcal{L}_q(x,Q^2) = -\Delta\Sigma(x,Q^2)$$

small x contribution to  $S_q + \mathcal{L}_q$  is given by

$$(S_q + \mathcal{L}_q)\big|_{\text{small }x} = -\frac{1}{2} \int_0^{x_{max}} dx \,\Delta\Sigma(x, Q^2)$$

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![](_page_44_Picture_2.jpeg)

Using similar techniques as those used to derive small-x helicity evolution, one  $\geq$ can show at small x (Hatta and Yang (2018); Kovchegov (2019))

 $\mathcal{L}_q(x, Q^2) = -\Delta \Sigma(x, Q^2)$ 

small x contribution to  $S_q + \mathcal{L}_q$  is given by

 $\left(S_q + \mathcal{L}_q\right)\Big|_{\text{small }x} = -\frac{1}{2} \int_{10-5}^{x_{max}} dx \,\Delta\Sigma(x, Q^2)$  $\Delta \Sigma^{[x_{\max}]}$  at  $Q^2 = 10.00 \text{ GeV}^2$ Preliminar PRELIMINARY result 0.4from JAMsmallx fit of 0.2polarized DIS+SIDIS data 0.0-0.2-0.4 $10^{-4}$  $10^{-2}$  $10^{-3}$  $10^{-1}$  $10^{-5}$  $x_{
m max}$ 

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![](_page_45_Picture_2.jpeg)

Using similar techniques as those used to derive small-x helicity evolution, one can show at small x (Hatta and Yang (2018); Kovchegov (2019))

 $\mathcal{L}_q(x,Q^2) = -\Delta\Sigma(x,Q^2)$ 

• small *x* contribution to  $S_q + \mathcal{L}_q$  is given by

$$(S_q + \mathcal{L}_q)\Big|_{\text{small }x} = -\frac{1}{2} \int_{10^{-5}}^{10^{-3}} dx \,\Delta\Sigma(x, Q^2) \approx 0.1$$

![](_page_45_Figure_7.jpeg)

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![](_page_46_Picture_2.jpeg)

$$\frac{1}{2} = S_q + \mathcal{L}_q + S_g + \mathcal{L}_g$$
$$S_q|_{\text{large } x} = \frac{1}{2} \int_{0.001}^1 dx \,\Delta\Sigma(x, Q^2) \approx 0.18$$
$$S_g|_{\text{large } x} = \int_{0.05}^1 dx \,\Delta g(x, Q^2) \approx 0.20$$
$$(S_q + \mathcal{L}_q)|_{\text{small } x} = -\frac{1}{2} \int_{10^{-5}}^{10^{-3}} dx \,\Delta\Sigma(x, Q^2) \approx 0.1$$

Recall  $S_q \gg S_g$  at small x

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Also,  $\mathcal{L}_g(x,Q^2) \ll \Delta g(x,Q^2)$  at small x (Kovchegov (2019))

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![](_page_47_Picture_2.jpeg)

$$\frac{1}{2} = S_q + \mathcal{L}_q + S_g + \mathcal{L}_g$$
$$S_q|_{\text{large } x} = \frac{1}{2} \int_{0.001}^1 dx \,\Delta\Sigma(x, Q^2) \approx 0.18$$
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$$(S_q + \mathcal{L}_q)|_{\text{small } x} = -\frac{1}{2} \int_{10^{-5}}^{10^{-3}} dx \,\Delta\Sigma(x, Q^2) \approx 0.1$$

Recall  $S_q \gg S_g$  at small x

Also,  $\mathcal{L}_g \approx 0$  at small x

 $(S_q)$ 

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![](_page_48_Picture_2.jpeg)

$$\frac{1}{2} = S_q + \mathcal{L}_q + S_g + \mathcal{L}_g$$
$$S_q|_{\text{large } x} = \frac{1}{2} \int_{0.001}^{1} dx \,\Delta\Sigma(x, Q^2) \approx 0.18$$
$$S_g|_{\text{large } x} = \int_{0.05}^{1} dx \,\Delta g(x, Q^2) \approx 0.20$$
$$+ \mathcal{L}_q)|_{\text{small } x} = -\frac{1}{2} \int_{10^{-5}}^{10^{-3}} dx \,\Delta\Sigma(x, Q^2) \approx 0.1$$

Recall  $S_q \gg S_g$  at small xAlso,  $\mathcal{L}_g \approx 0$  at small x

If quark and gluon (Jaffe-Manohar) OAM is negligible at large x, small-x partons may give the remainder of the proton spin

![](_page_49_Picture_0.jpeg)

![](_page_49_Picture_2.jpeg)

# **Summary and Outlook**

- We have shown the JAMsmallx framework, utilizing KPS evolution, can fit the world polarized DIS data at x < 0.1, with significantly reduced uncertainties (compared to "standard" DGLAP fits) as one extends into the unmeasured (small-x) region</p>
- → We are working on including polarized SIDIS data to perform a flavor separation and make a genuine prediction for the spin carried by quarks at small x. Also polarized proton-proton collisions will be explored (access to  $\Delta g(x)$ ).
- Additional future updates will also include single-log corrections (Kovchegov, Tarasov, Tawabutr (2021)) and using solutions for the large- $N_c \& -N_f KPS$  evolution equations (Kovchegov and Tawabutr (2020))

KPS evolution provides a controlled way to extend helicity PDFs down to very small x and will be a crucial ingredient to resolve the proton spin puzzle, especially once EIC data is available