The Ohio State University

Center for Frontiers in Nuclear Science

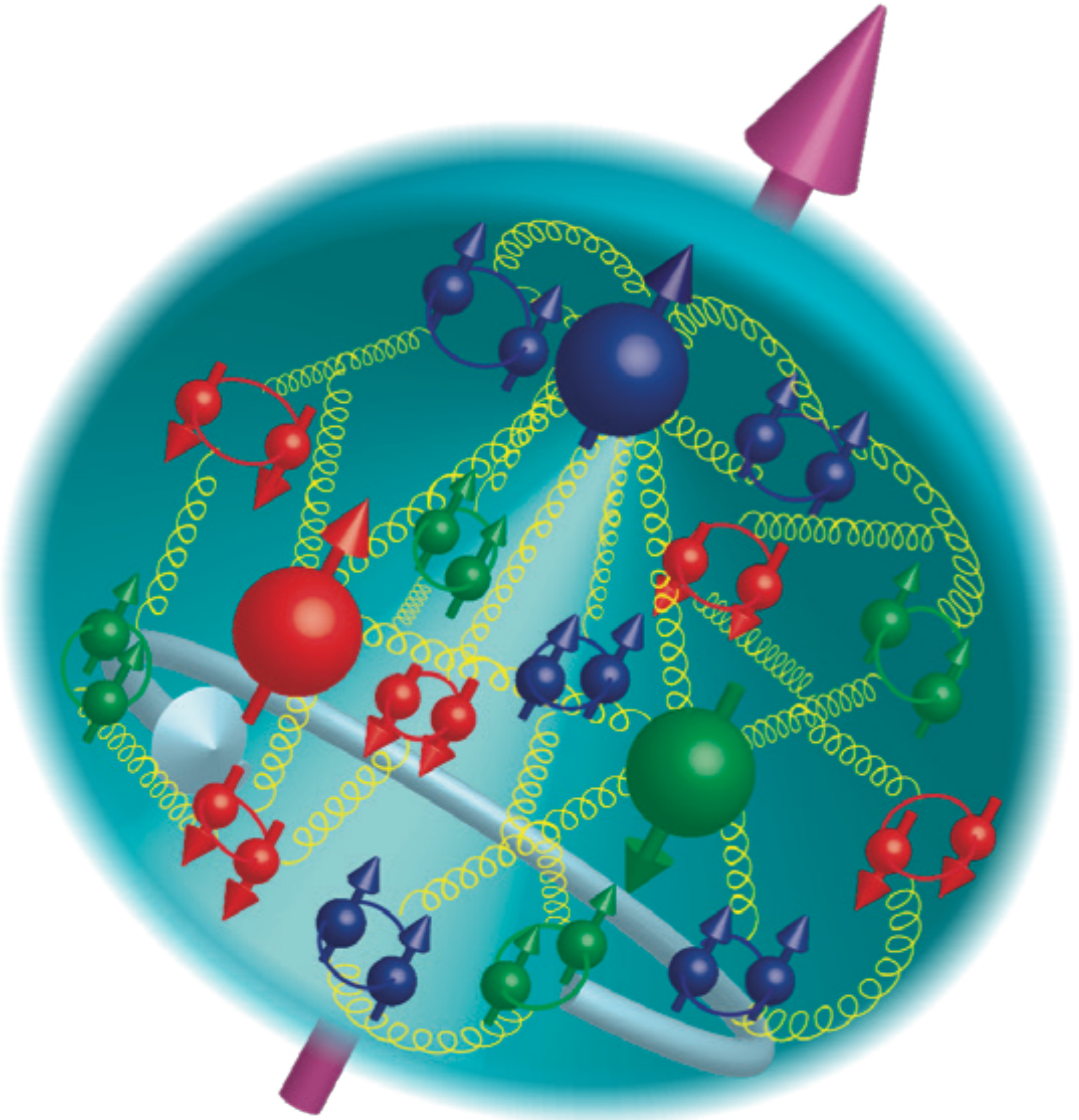
The role of the chiral anomaly in polarized deeply inelastic scattering: Topological screening and emergent axion-like dynamics

Andrey Tarasov

Based on Andrey Tarasov and Raju Venugopalan
Phys. Rev. D 102 (2020) 11, 114022 (arXiv:2008.08104), and arXiv:2109.10370

SPIN 2021

The proton's spin puzzle



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Spin

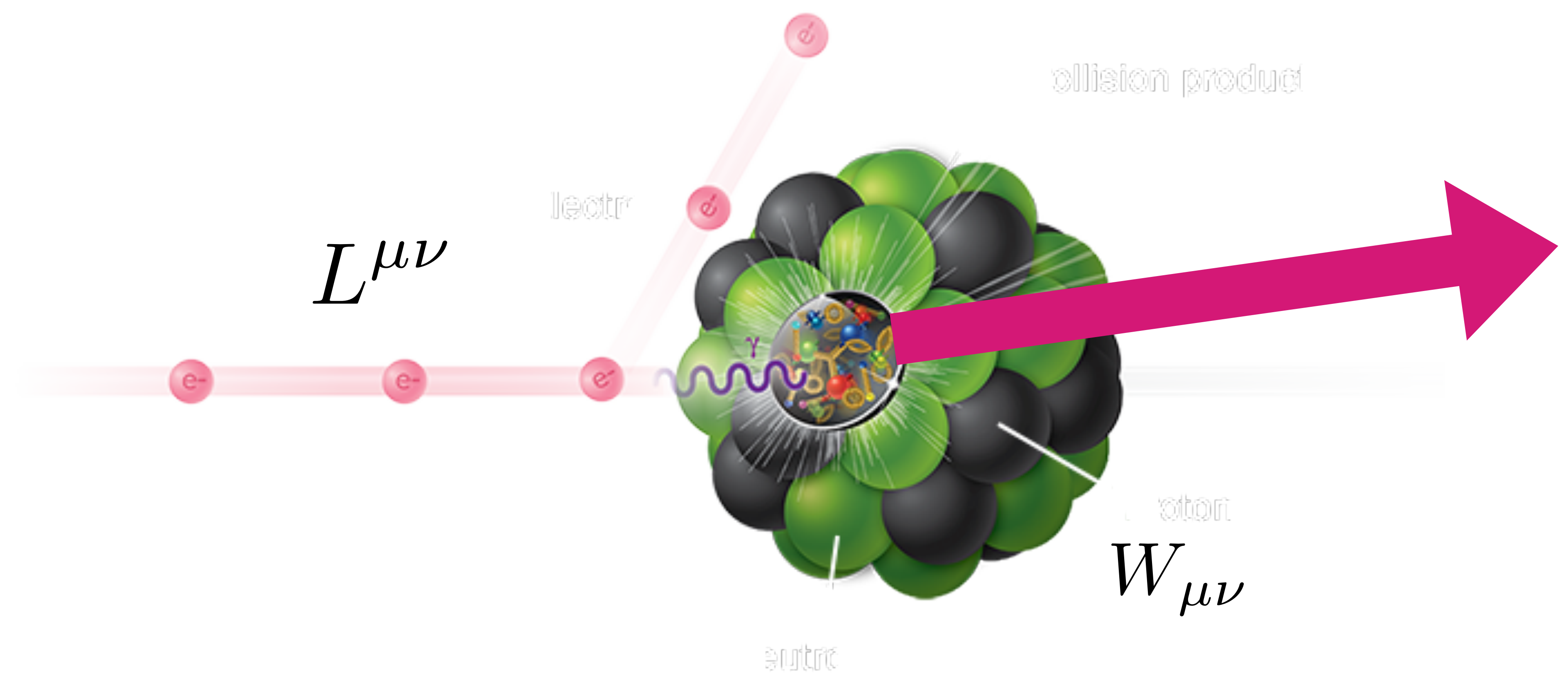
Quark helicity

Gluon helicity

Orbital angular momentum

- DIS experiments showed that quarks carry only about 30% of the proton's spin $\Delta\Sigma = 0.25 \sim 0.3$
- Failure of the constituent quark model to explain spin of the proton - *spin crisis*

First moment of g_1



$$\int_0^1 dx_B\, g_1(x_B, Q^2) = \frac{1}{18}\left(3F + D + 2\,\Sigma(Q^2)\right)$$

Combination of F and D can be measured in β-decay and hyperon decay experiments

Quark spin contribution is defined by iso-singlet axial vector current

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N}\sum_f \langle P, S|\bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f|P, S\rangle \equiv \frac{1}{M_N}\langle P, S|J_5^\mu(0)|P, S\rangle$$

The current is not conserved due to presence of the chiral anomaly

$$\partial^\mu J_\mu^5(x) = \frac{n_f \alpha_s}{2\pi}\,\mathrm{Tr}\left(F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)\right)$$

Perturbative and non-perturbative interplay

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N} \langle P, S | J_5^\mu(0) | P, S \rangle$$

R. L. Jaffe

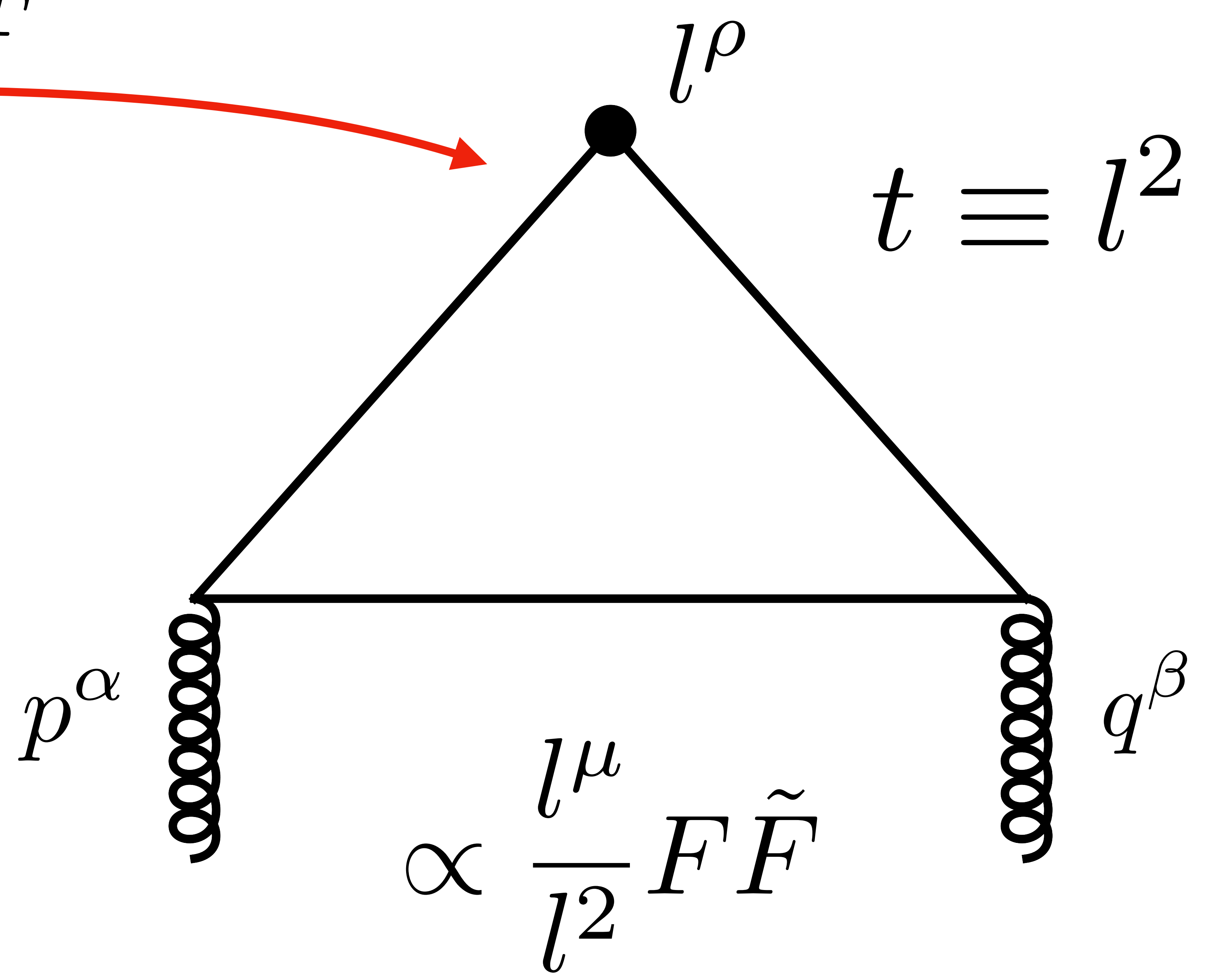
A. Manohar

The key role of the anomaly is seen from the structure of the triangle graph in the off-forward limit. The current is not conserved due to presence of the chiral anomaly.

The triangle diagram has an infrared pole and $\kappa(t) \propto F\tilde{F}$

$$\frac{1}{M_N} \langle P', S | J_5^\mu(0) | P, S \rangle = \boxed{\frac{l \cdot S\, l^\mu}{l^2} \kappa(Q^2, t)}$$

Exact result!



R. L. Jaffe, A. Manohar
Nucl. Phys., B337:509–546, 1990

Shore, Veneziano (1990)
Narison, Shore, Veneziano, hep-ph/9812333
G. M. Shore, hep-ph/0701171
K.-F. Liu (1992)

The existence of the infrared pole of the triangle diagram has not been addressed in pQCD calculations.

Perturbative and non-perturbative interplay

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N}\langle P, S | J_5^\mu(0) | P, S\rangle$$

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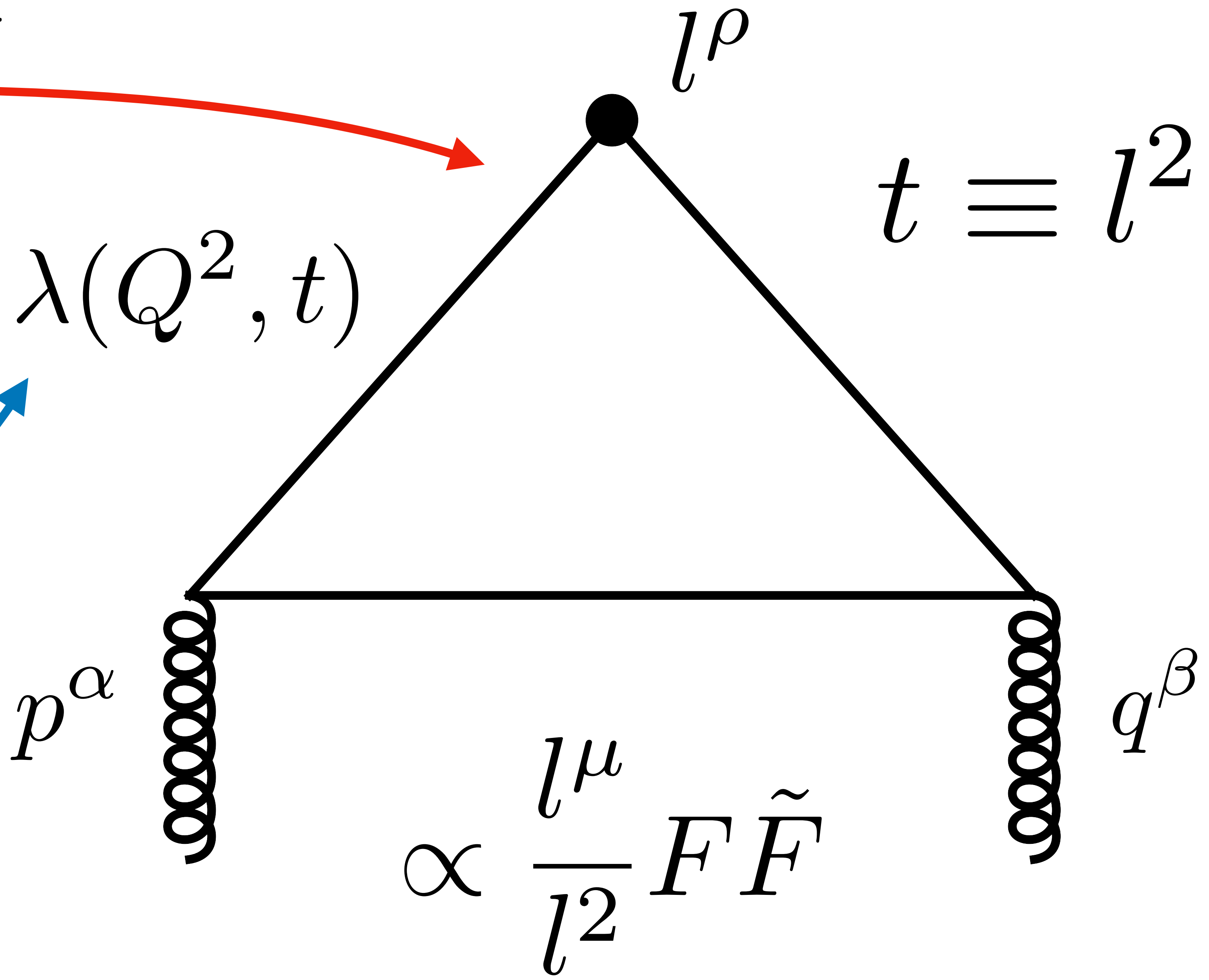
$$\frac{1}{M_N}\langle P', S | J_5^\mu(0) | P, S\rangle = \boxed{\frac{l\cdot S\, l^\mu}{l^2}\kappa(Q^2, t)} + \left(S^\mu - \frac{l\cdot S\, l^\mu}{l^2}\right)\lambda(Q^2, t)$$

Pseudoscalar contribution



R. L. Jaffe, A. Manohar
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Perturbative and non-perturbative interplay

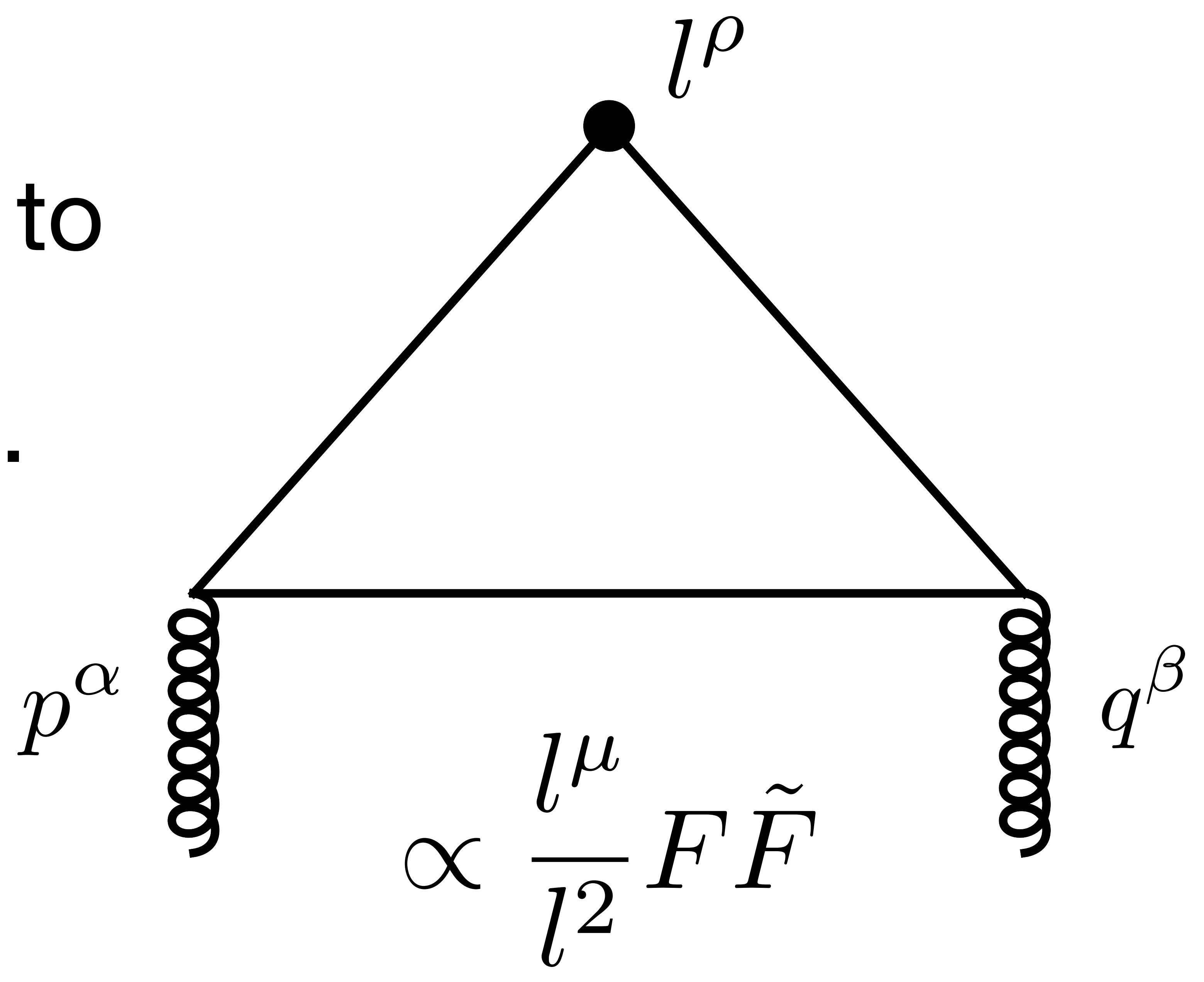
The infrared pole of the triangle diagram must be cancelled by a pole in the pseudo-scalar sector of the theory:

$$\Sigma(Q^2) \propto \frac{n_f\,\alpha_s}{2\pi\,M_N} \lim_{l_\mu \to 0} \langle P', S|\mathrm{Tr}\big(F\tilde{F}\big)(0)|P, S\rangle$$

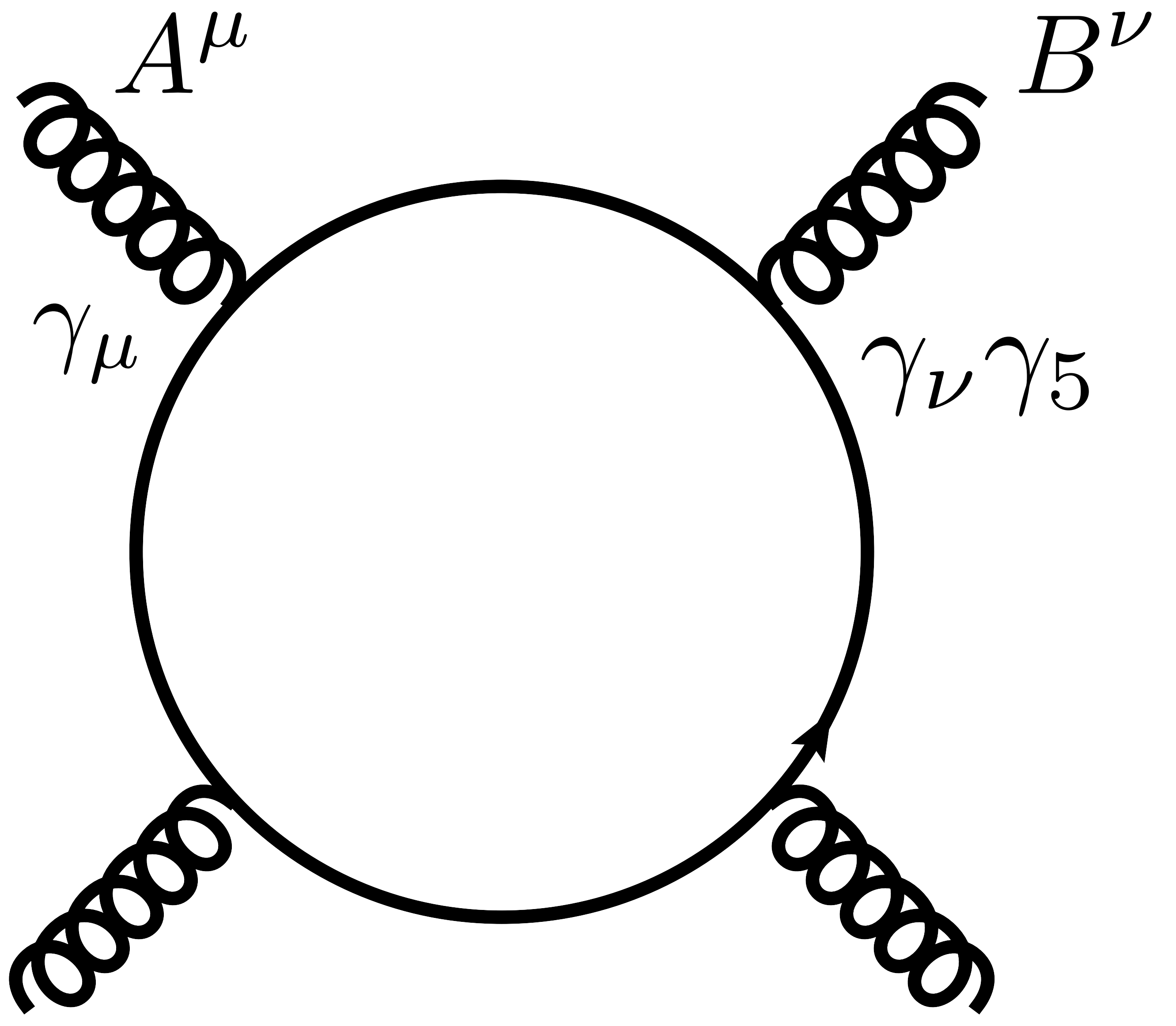
The result is manifestly gauge invariant!

The presence of the pole in the triangle diagram is related to topological properties of QCD (measure of the QCD path integral), which are described by the chiral Ward identities. The triangle diagram is not local!

Generalization of this result to $g_1(x, Q^2)$, and interplay with non-perturbative physics can be explored efficiently in a worldline framework



The worldline formalism



We consider a functional integral representation (worldline) of the imaginary part (anomaly!) of the Dirac determinant

$$-\mathcal{W}[A,B] = \ln \mathrm{Det}\ \left[\not{p} - \not{A} - \gamma_5 \not{B}\right]$$

Heat-kernel regularization:

$$\mathcal{W}[A,B] = -\frac{1}{2}\mathrm{Tr}\ \int_0^\infty \frac{dT}{T} \exp\left\{ -T\Big[\not{p} - \not{A} - \gamma_5 \not{B}\Big]^2 \right\}$$

Bern,Kosower, NPB 379 (1992) 145
Strassler, NPB 385 (1992) 145
Schubert, Phys. Rep. (2001)
D'Hoker, Gagne (1996)
Berezin, Marinov (1976)
Barducci (1977)

One can introduce bosonic and fermionic coherent states and construct a functional integral representation effective action W:

Grassmann numbers

$$\int d^4x\, |x\rangle\langle x| = \int d^4p\, |p\rangle\langle p| = 1; \quad \langle x|p\rangle = e^{ipx}$$

$$i\int |\xi\rangle\langle\xi| d^2\xi = -i\int |\bar\xi\rangle\langle\bar\xi| d^2\bar\xi = 1\,; \quad \langle\xi|\bar\xi\rangle = e^{\sum_{i=1}^2 \xi_i\bar\xi_i}\,; \quad \langle\bar\xi|\xi\rangle = e^{\sum_{i=1}^2 \bar\xi_i\xi_i}$$

The triangle anomaly in the worldline formalism

$$\mathcal{W}[A,B] = -\frac{1}{2}\mathrm{Tr}_c \int_0^\infty \frac{dT}{T} \int \mathcal{D}x \int_{AP} \mathcal{D}\psi$$

Functional integrals over trajectories of a point-like particle

$$\times \exp\Big\{ -\int_0^T d\tau \Big(\underbrace{\frac{1}{4}\dot{x}^2 + \frac{1}{2}\psi_\mu \dot{\psi}^\mu}_{\text{Free ``propagation''}} + \underbrace{ig\dot{x}^\mu A_\mu - ig\psi^\mu\psi^\nu F_{\mu\nu}}_{\text{Vector coupling}}$$

$$\underbrace{-2i\psi_5 \dot{x}^\mu \psi_\mu \psi_\nu B^\nu + i\psi_5 \partial_\mu B^\mu + (D-2)B^2}_{\text{Axial coupling}} \Big) \Big\}$$

D.G.C. McKeon, C. Schubert, Phys. Lett. B 440 (1998) 101

The triangle anomaly:

$$\langle P', S | J_5^\kappa | P, S \rangle = \int d^4y \, \frac{\partial \mathcal{W}_I[A,B]}{\partial B_\kappa(y)} \Big|_{B_\kappa = 0} e^{ily} \equiv \Gamma_5^\kappa[l]$$

The triangle anomaly in the worldline formalism

$$\Gamma_5^\kappa[l] = \frac{1}{4\pi^2}\boxed{\frac{l^\kappa}{l^2}}\int \frac{d^4k_2}{(2\pi)^4}\int \frac{d^4k_4}{(2\pi)^4}\ \mathrm{Tr_c} F_{\alpha\beta}(k_2)\tilde{F}^{\alpha\beta}(k_4)\ (2\pi)^4\delta^4(l+k_2+k_4)$$

Steven Adler

John S. Bell

We reproduce famous infrared pole of the anomaly

$$S^\mu \int_0^1 dx_B\, g_1(x_B, Q^2)\Big|_{Q^2\to\infty}$$

$$= \sum_f e_f^2 \frac{\alpha_S}{2i\pi M_N} \lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P', S|\mathrm{Tr_c} F_{\alpha\beta}(0)\tilde{F}^{\alpha\beta}(0)|P, S\rangle$$

Roman Jackiw

The infrared pole must be cancelled by a pseudoscalar contribution

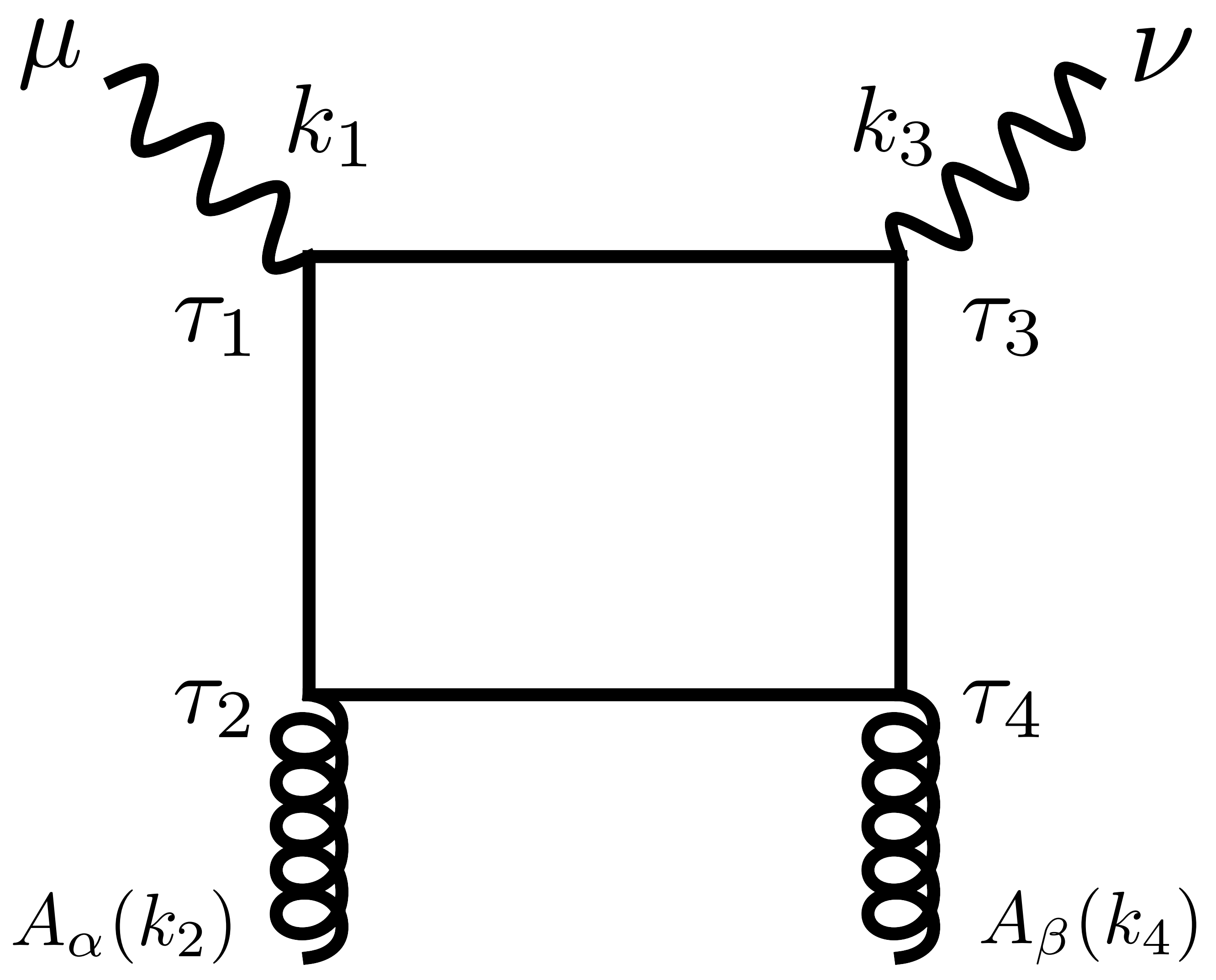
Finding the axial anomaly in the box diagram

Tarasov, Venugopalan (2019)

$$\Gamma_A^{\mu\nu}[k_1,k_3] = \int \frac{d^4k_2}{(2\pi)^4}\int \frac{d^4k_4}{(2\pi)^4}\ \Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4]\ \mathrm{Tr_c}(\tilde{A}_\alpha(k_2)\tilde{A}_\beta(k_4))$$

Hadronic tensor

Box diagram



Worldline representation of the box diagram (one loop exact!):

$$\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4] \equiv -\frac{g^2e^2e_f^2}{2}\int_0^\infty \frac{dT}{T}\ \int \mathcal{D}x \int \mathcal{D}\psi\ \exp\Big\{-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi\cdot\dot{\psi}\Big)\Big\}$$
$$\times\Big[V_1^\mu(k_1)V_3^\nu(k_3)V_2^\alpha(k_2)V_4^\beta(k_4)-(\mu\leftrightarrow\nu)\Big]$$

where the vector current $$V_i^\mu(k_i) \equiv \int_0^T d\tau_i(\dot{x}_i^\mu + 2i\psi_i^\mu k_j\cdot\psi_j)e^{ik_i\cdot x_i}$$

The box diagram in the Bjorken limit

In the Bjorken limit $Q^2 \to \infty$ and x_B is fixed

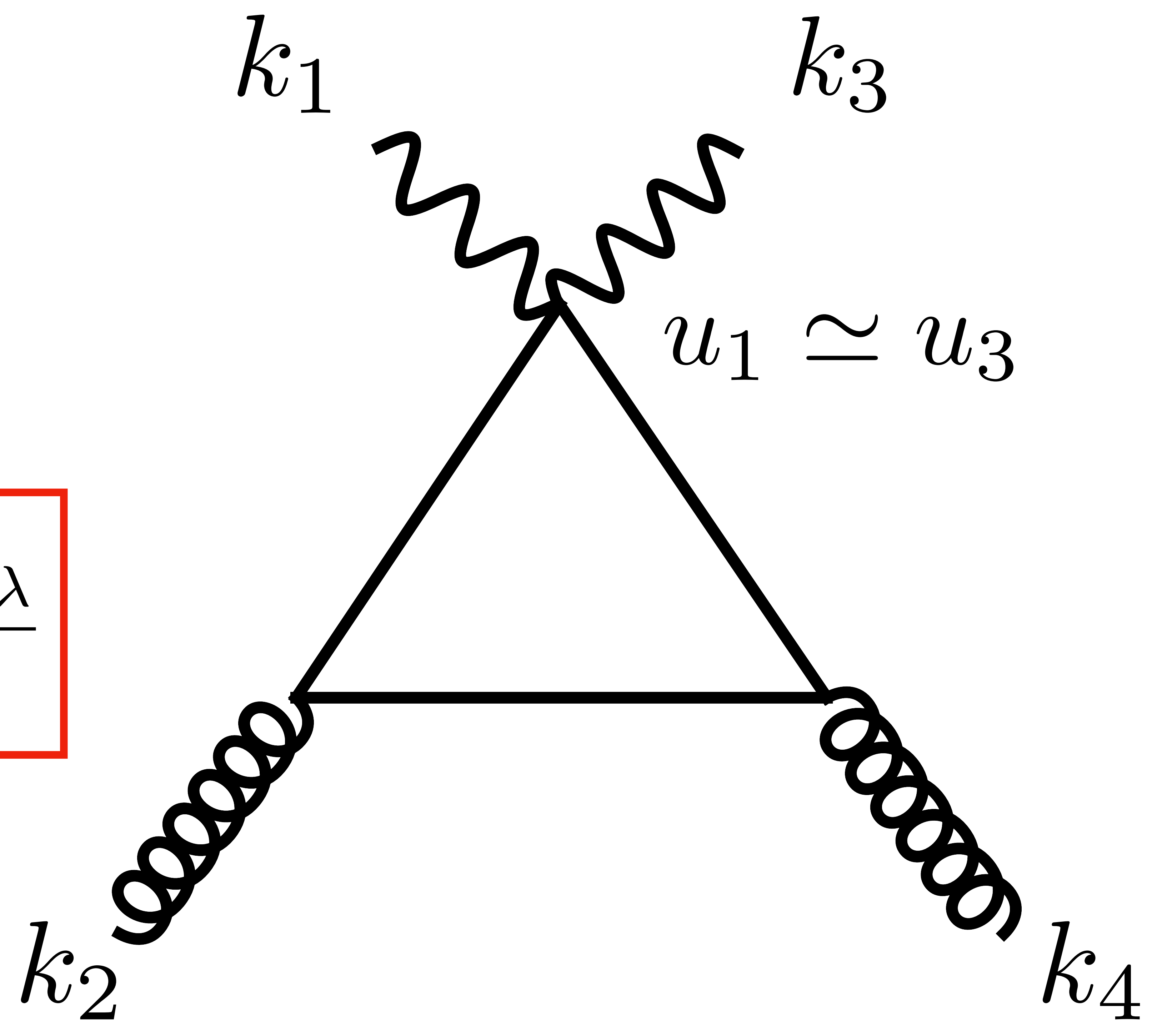
$$\Gamma_A^{\mu\nu\alpha\beta}[k_1, k_3, k_2, k_4]\Big|_{Q^2\to\infty}$$

$$= -2\frac{g^2 e^2 e_f^2}{\pi^2}\epsilon^{\mu\nu\eta}{}_{\kappa}(k_{1\eta} - k_{3\eta})(2\pi)^4\delta^{(4)}(\sum_{i=1}^{4} k_i)\boxed{\frac{(k_2^\kappa + k_4^\kappa)\epsilon^{\alpha\beta\sigma\lambda}k_{2\sigma}k_{4\lambda}}{(k_2+k_4)^2}}$$

$$\times \prod_{k=2}^{4}\int_0^1 d\beta_k\ \delta(1 - \sum_{k=2,3,4}\beta_k)\frac{1}{(k_1+k_2)^2\beta_3 + k_1^2\beta_4 + k_3^2\beta_2}$$

triangle diagram

Kinematic factor, dependence on x_B and Q^2



$$u_1 - u_3 \sim \Lambda_{\rm QCD}^2/Q^2$$

The structure function g_1

infrared pole

non-local operator

non-pole contribution

$$S^\mu g_1(x_B, Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P', S|\mathrm{Tr_c} F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P, S\rangle + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$

The structure function g_1 is dominated by the triangle anomaly, hence g_1 is topological. The result is formally identical in both Bjorken and Regge limits. $g_1(x_B, Q^2)$ is topological in both asymptotic limits of QCD

Tarasov, Venugopalan (arXiv:2008.08104)

First moment of g_1:

$$S^\mu \int_0^1 dx_B\, g_1(x_B, Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_S}{2i\pi M_N}\lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P', S|\mathrm{Tr_c} F_{\alpha\beta}(0)\tilde{F}^{\alpha\beta}(0)|P, S\rangle$$

matches calculation of the triangle diagram

The pole must be removed by contribution of the pseudo-scalar sector

Pseudoscalar contribution

To take into account contribution of the pseudoscalar sector we use the most general form of the imaginary part of the effective action

$$-\mathcal{W}[A,B,\Phi,\Pi] = \ln \mathrm{Det}\,\left[\not{p} - i\Phi - \gamma_5\,\Pi - \not{A} - \gamma_5\not{B}\right]$$

scalar field pseudoscalar field

Functional integral representation of the effective action:

D'Hoker, Gagne,hep-th/9508131

$$\mathcal{W}_{\mathcal{I}} = -\frac{i}{32}\int_{-1}^{1} d\alpha \int_0^\infty dT\,\mathcal{N}\int_{\mathrm{PBC}} \mathcal{D}x\,\mathcal{D}\psi\;\mathrm{tr}\;\chi\,\bar{\omega}(0)\,\exp\left[-\int_0^T d\tau \mathcal{L}_{(\alpha)}(\tau)\right]$$

It generates the iso-singlet Wess-Zumino-Witten coupling $\propto \eta_0 F\tilde{F}$

Tarasov, Venugopalan arXiv:2109.10370

$$\mathcal{W}_{\mathcal{I}}[\Pi A^2] = \frac{ig^2\,2n_f}{16\pi^2}\frac{1}{\Phi}\,\mathrm{tr}_c\int d^4x\,\Pi(x)\,F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)$$

in agreement with the corresponding term in $\mathscr{L}_{\mathrm{WZW}}$ which was derived from chiral perturbation theory Leutwyler (1996); Herrara-Sikody et al (1997); Leutwyler-Kaiser (2000)

Matrix element of the current

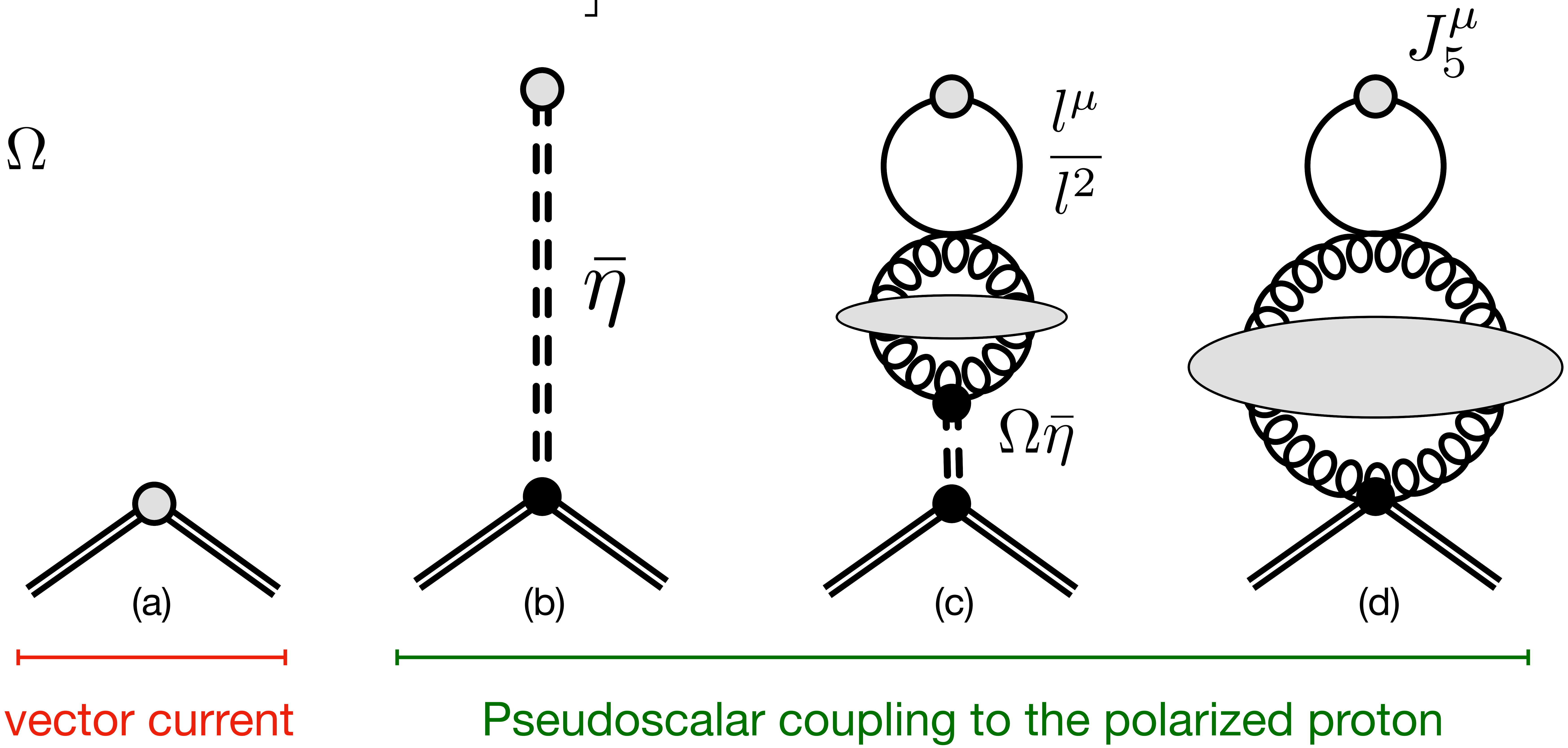
With this form of the effective action, we see explicitly how the pole of the anomaly is canceled by a massless $\bar{\eta}$ exchange ("primordial" ninth Goldstone boson)

$$\langle P', S|J_5^\mu|P, S\rangle = \bar{u}(P', S)\Big[\gamma^\mu\gamma_5 G_A(l^2) + l^\mu\gamma_5 G_P(l^2)\Big]u(P, S)$$

$$S^{\bar{\eta}}_{\mathrm{WZW}} = i\frac{\sqrt{2\,n_f}}{F_{\bar{\eta}}}\int d^4x\,\bar{\eta}\,\Omega$$

$$\Omega = \frac{\alpha_s}{4\pi}\mathrm{Tr}\left(F\tilde{F}\right)$$

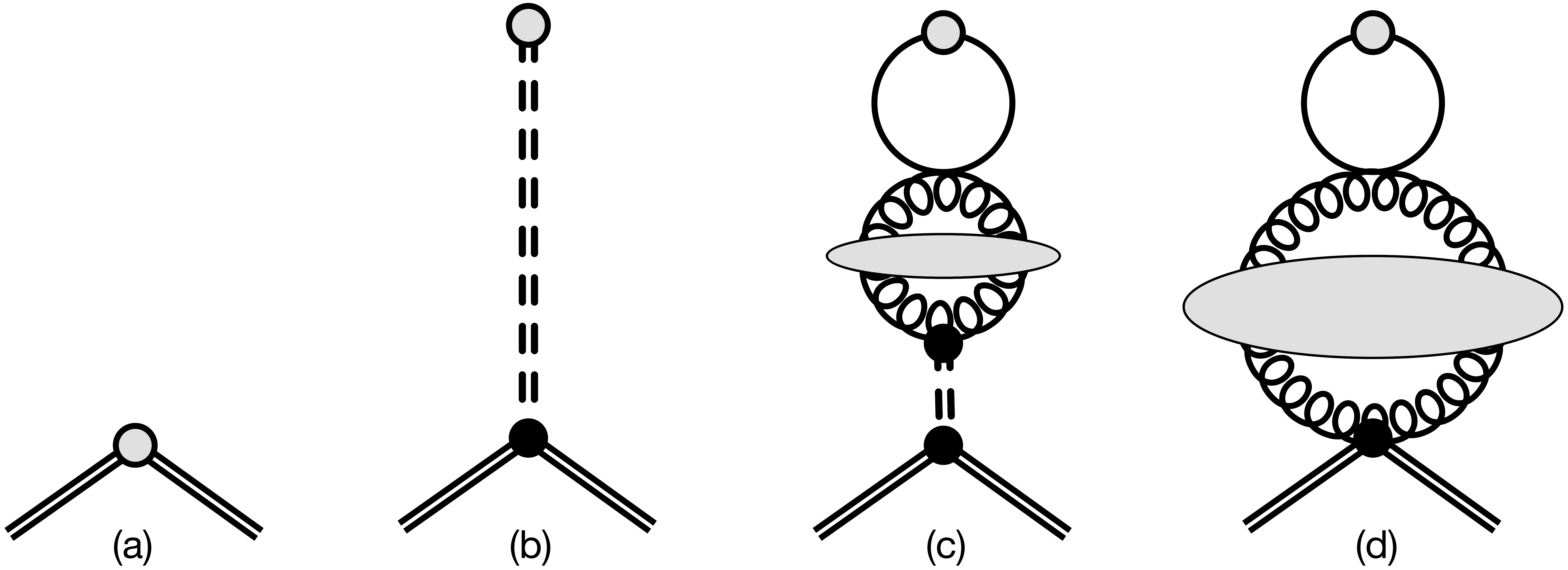
We'll establish relations between diagrams



Matrix element of the current

$$\langle P', S|J_5^\mu|P,S\rangle = \bar{u}(P',S)\Big[\gamma^\mu\gamma_5 G_A(l^2) + l^\mu\gamma_5 G_P(l^2)\Big]u(P,S)$$

To establish relations between diagrams we use two conditions: anomaly equation for the current and absence of the infrared pole in $G_P(l^2)$



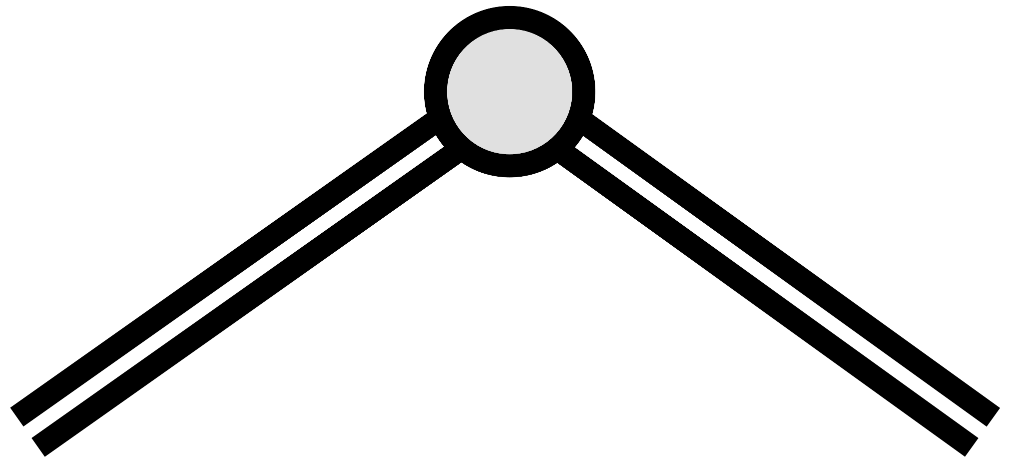
Direct axial vector current

Pseudoscalar coupling to the polarized proton

$G_P(l^2)$ form factor doesn't have a pole: $\lim_{l\to 0}\Big[\langle P',S|J_5^\mu|P,S\rangle|_{\text{Fig. b}} + \langle P',S|J_5^\mu|P,S\rangle|_{\text{Figs. c+d}}\Big] = 0$

Imaginary part of the effective action

The diagram represents the direct coupling of the axial vector currents to the nucleon target. It is fundamentally different from other diagrams since it is the only diagram which contributes to the axial form factor $G_A(l^2)$



(a)

$$\langle P', S| J_5^\mu |P, S\rangle\Big|_{Fig.\ a} = G_A(l^2)\,\bar{u}(P', S)\,\gamma^\mu\gamma_5\,u(P, S)$$

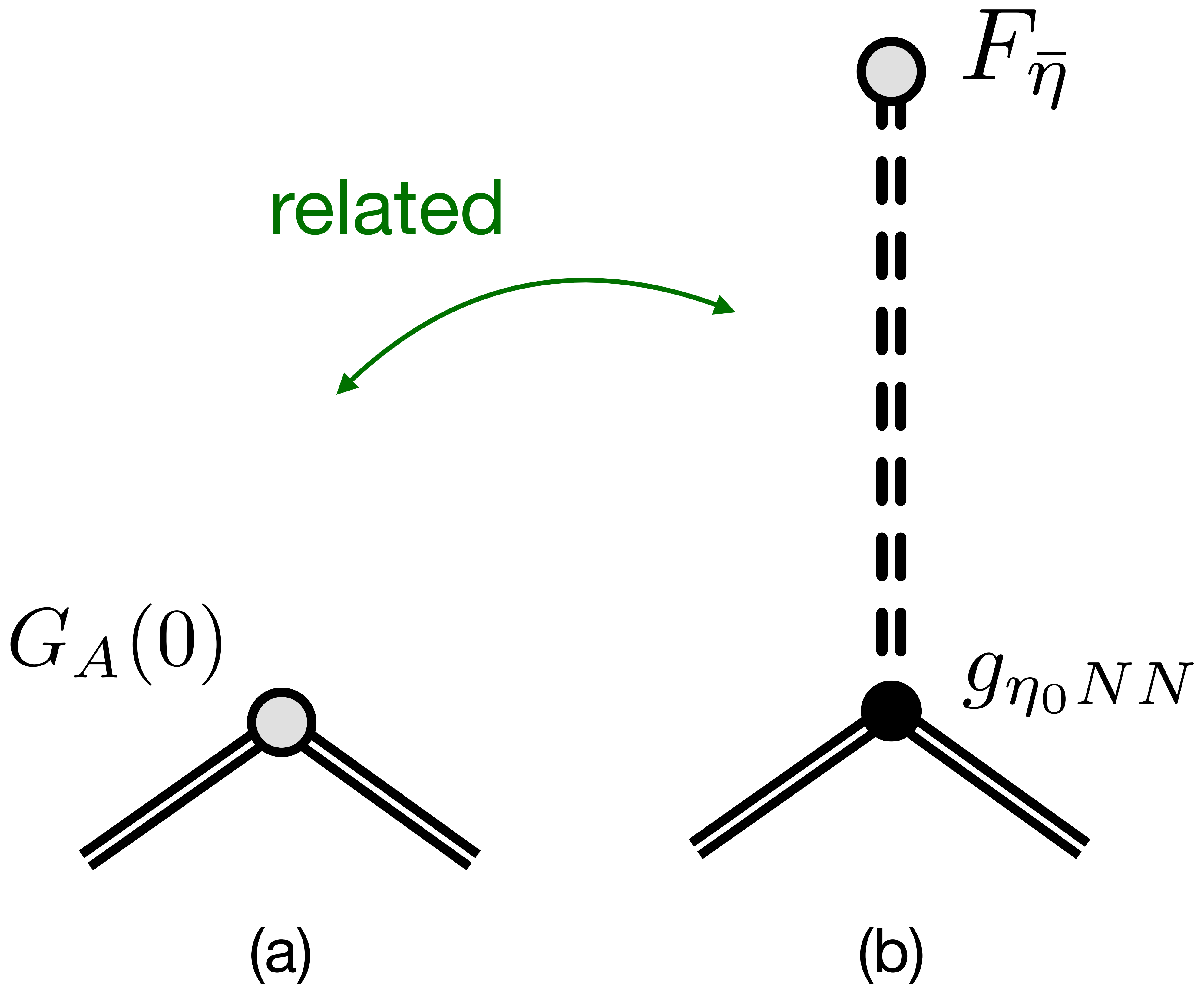
$$\lim_{l\to 0}\langle P', S| J_5^\mu |P, S\rangle = 2M_N\, G_A(0)\, S^\mu$$

The anomaly equation for the divergence of the singlet axial current and the requirement of the infrared pole cancellation impose relation of this result to the contribution of other diagrams

Goldberger-Treiman relation

$$\langle P', S|\partial_\mu J_5^\mu|P,S\rangle = \langle P', S|2\, n_f\, \Omega|P,S\rangle$$

Using the anomaly equation we derive the generalization of the well-known Goldberger-Treiman relation to the isosinglet axial vector current



Shore and Veneziano (89-92)

$$G_A(0) = \frac{\sqrt{2\tilde{n}_f}}{2M_N}\, F_{\bar\eta}\, g_{\eta_0 NN}$$

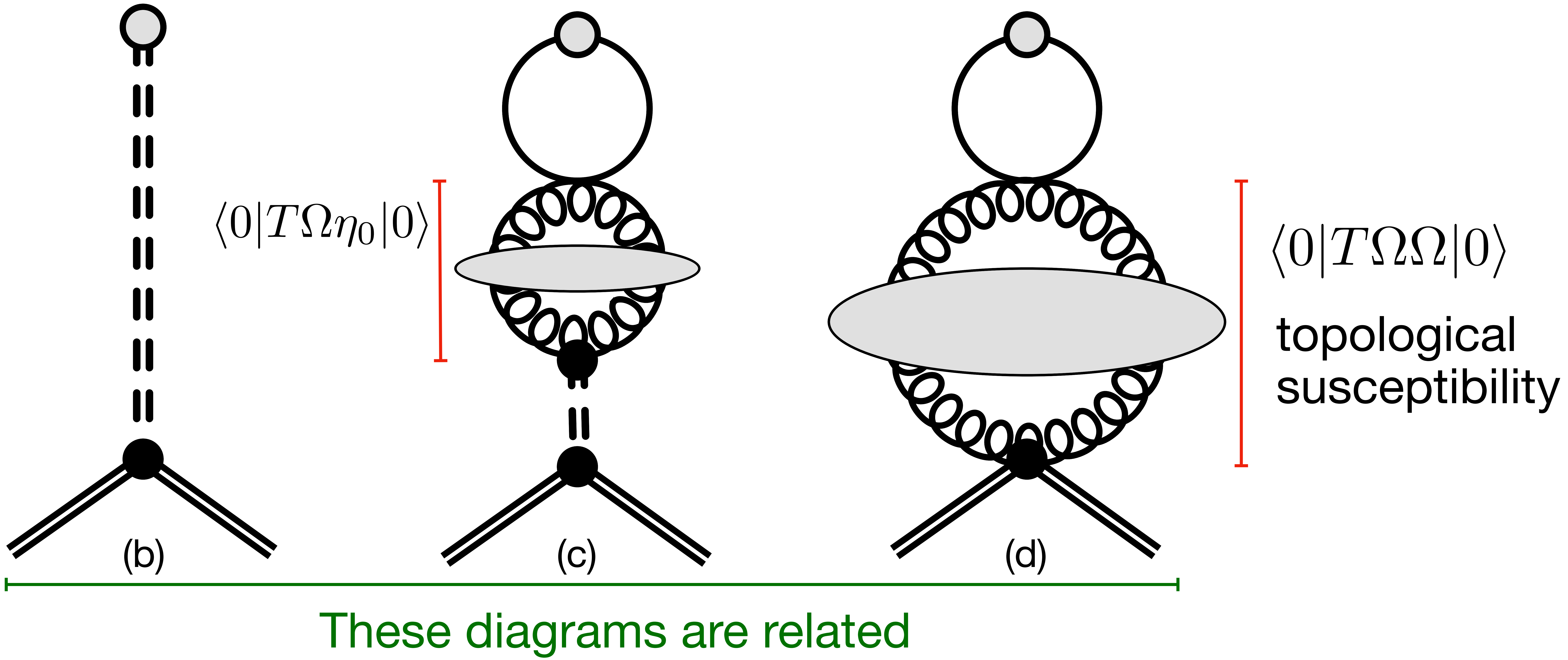
$$\lim_{l\to 0}\langle P', S|J_5^\mu|P,S\rangle = \sqrt{2\tilde{n}_f}\, F_{\bar\eta}\, g_{\eta_0 NN} S^\mu$$

decay constant

coupling to the nucleon

Cancellation of the infrared pole

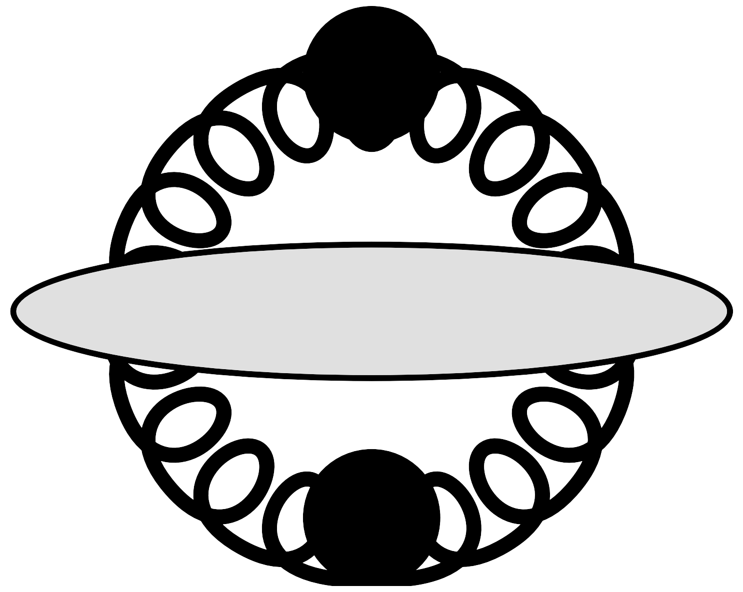
From requirement of the infrared pole cancellation we find a relation between the product of the η_0 decay constant and its coupling to the nucleon to the matrix element of the topological charge density in the forward limit



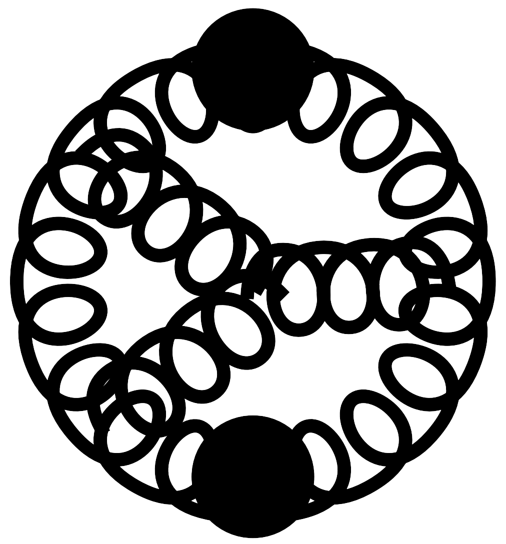
Topological susceptibility

We study the effect of the WZW coupling in the two-point Green functions and demonstrate how it generates a nonzero mass of the η'

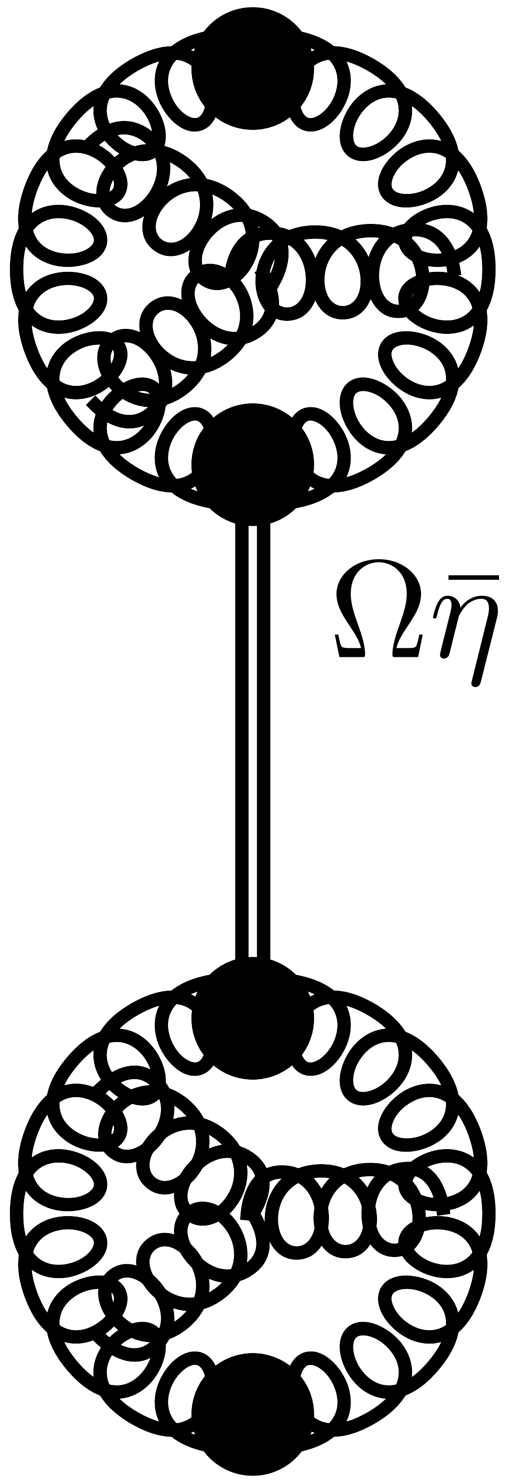
$$\chi(l^2) = i \int d^4x \, e^{ilx} \langle 0 | T\, \Omega(x) \Omega(0) | 0 \rangle$$



$=$



$+$



$+ \quad \cdots$

topological susceptibility $\chi(l^2)$

YM topological susceptibility $\chi_{YM}(l^2)$

Witten-Veneziano formula for $m^2_{\eta'}$

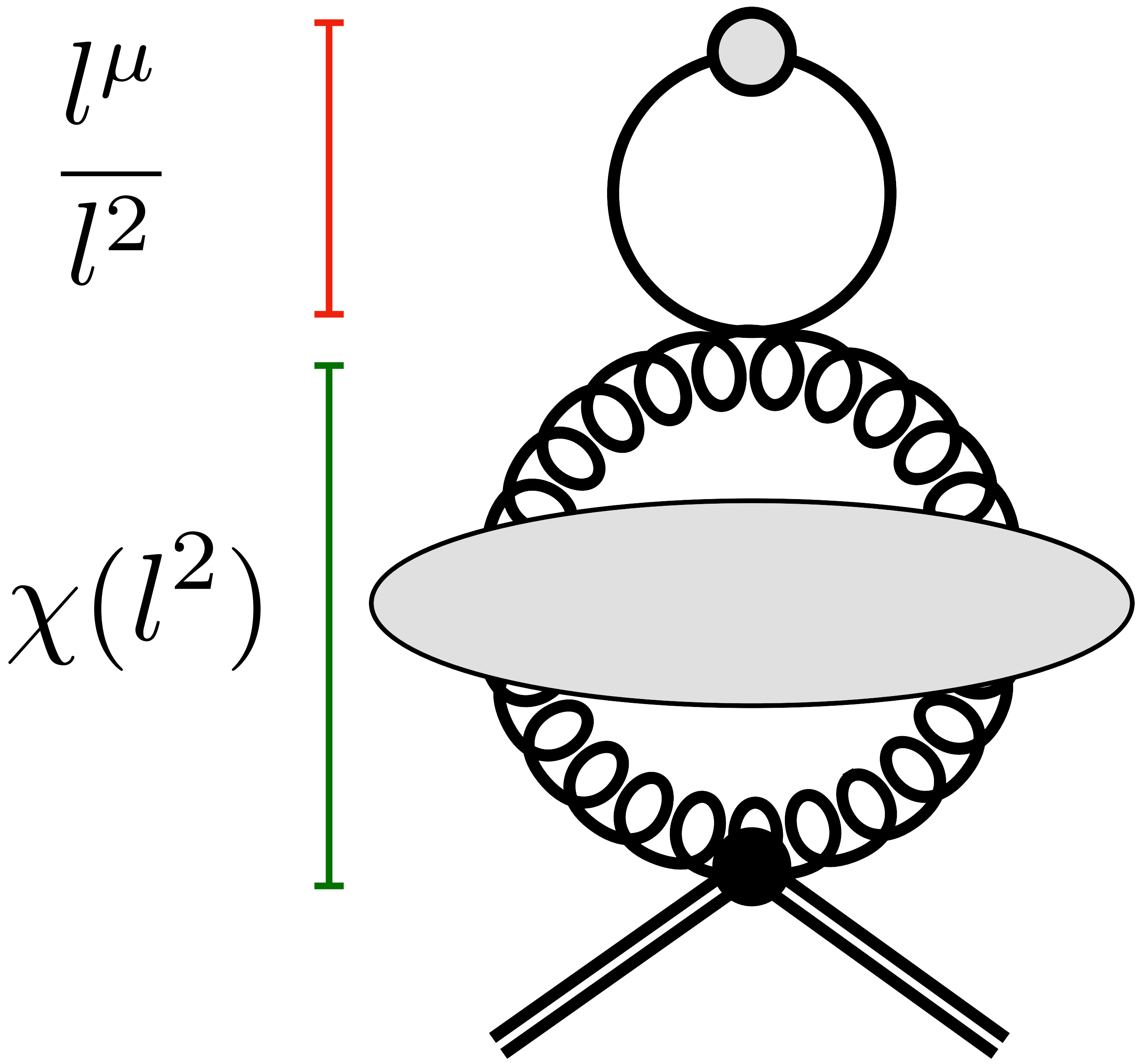
Taking into account the coupling between $\bar{\eta}$ and Ω, as specified by the WZW action, it is easy to obtain

$$\chi(l^2) = l^2 \frac{1}{l^2 - m^2_{\eta'}} \chi_{\mathrm{YM}}(l^2) \qquad m^2_{\eta'} \equiv -\frac{2\, n_f}{F^2_{\bar{\eta}}} \chi_{\mathrm{YM}}(0)$$

The WZW-$\bar{\eta}$ term and the topological susceptiblity

The topological susceptibility $\chi(l^2)$ vanishes in the forward limit because of topological mass generation

$$\chi(l^2) = l^2 \frac{1}{l^2 - m_{\eta'}^2} \chi_{\mathrm{YM}}(l^2) \qquad m_{\eta'}^2 \equiv -\frac{2\,n_f}{F_{\bar{\eta}}^2} \chi_{\mathrm{YM}}(0)$$



(d)

For this diagram we have:

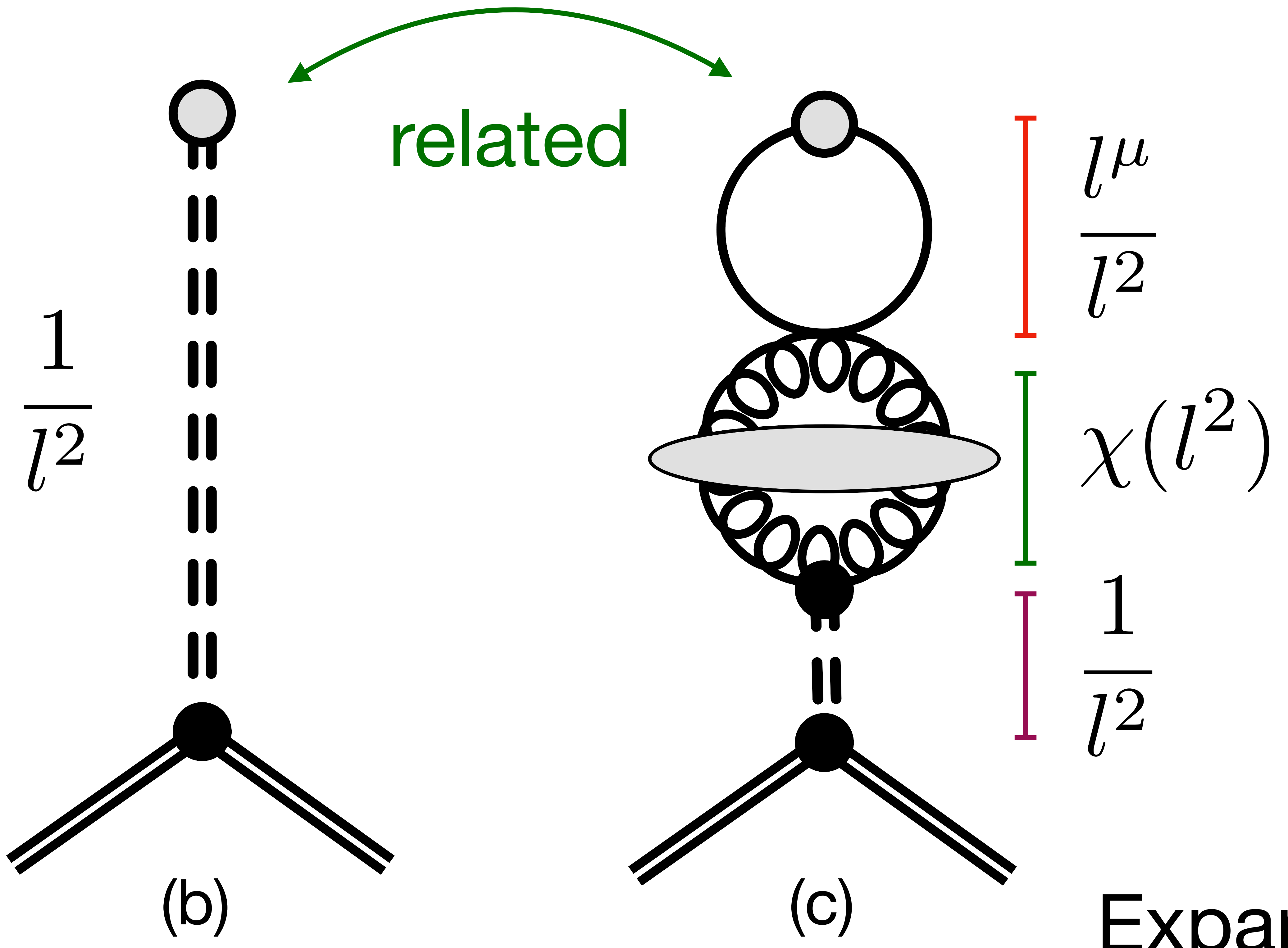
$$\langle P', S | J_5^\mu | P, S\rangle \Big|_{Fig.d} = 2n_f \frac{l^\mu}{l^2 - m_{\eta'}^2} \chi_{\mathrm{YM}}(l^2) \cdot g_{\Omega NN}\, \bar{u}(P',S)\, \gamma_5\, u(P,S)$$

In the forward limit the contribution of this diagram:

$$\lim_{l \to 0} \langle P', S | J_5^\mu | P, S\rangle \Big|_{Fig.d} = 0$$

The WZW-$\bar{\eta}$ term and the topological susceptiblity

Going back to the requirement of the infrared pole cancellation we obtain a relation between the decay constant and the topological susceptibility



$$\sqrt{2\tilde{n}_f}\, F_{\bar{\eta}} = 2n_f \lim_{l\to 0} i \langle 0|T\,\Omega\eta_0|0\rangle$$

Contribution of the diagram (c) is not zero even in the forward limit (note a pseudoscalar pole in the expression for the diagram)

$$\langle 0|T\Omega\eta_0|0\rangle = -i\frac{1}{l^2}\frac{\sqrt{2\tilde{n}_f}}{F_{\bar{\eta}}}\chi(l^2)$$

Expanding $\chi(l^2)$ in powers of l^2 and taking into account that $\chi(0) = 0$:

$$F_{\bar{\eta}}^2 = 2n_f\chi'(0)$$

The WZW-$\bar{\eta}$ term and the topological susceptiblity

Using relation $F_{\bar{\eta}}^2 = 2n_f \chi'(0)$ we can rewrite the matrix element of the axial vector

$$\langle P', S | J_5^\mu | P, S \rangle = \sqrt{\frac{2}{3}}\, 2n_f\, g_{\eta_0 NN} \sqrt{\chi'(0)}\, S^\mu$$

$$\Sigma(Q^2) = \sqrt{\frac{2}{3}}\, \frac{2n_f}{M_N}\, g_{\eta_0 NN} \sqrt{\chi'(0)}$$

Magnitude of the OZI violation:

$$\frac{a^0}{a^8} \simeq \frac{\sqrt{6}}{f_\pi} \sqrt{\chi'(0)}$$

$$\boxed{a^0|_{Q^2=10GeV^2} = 0.33 \pm 0.05}$$

Shore, Veneziano (1992)
Narison, Shore, Veneziano (1998)

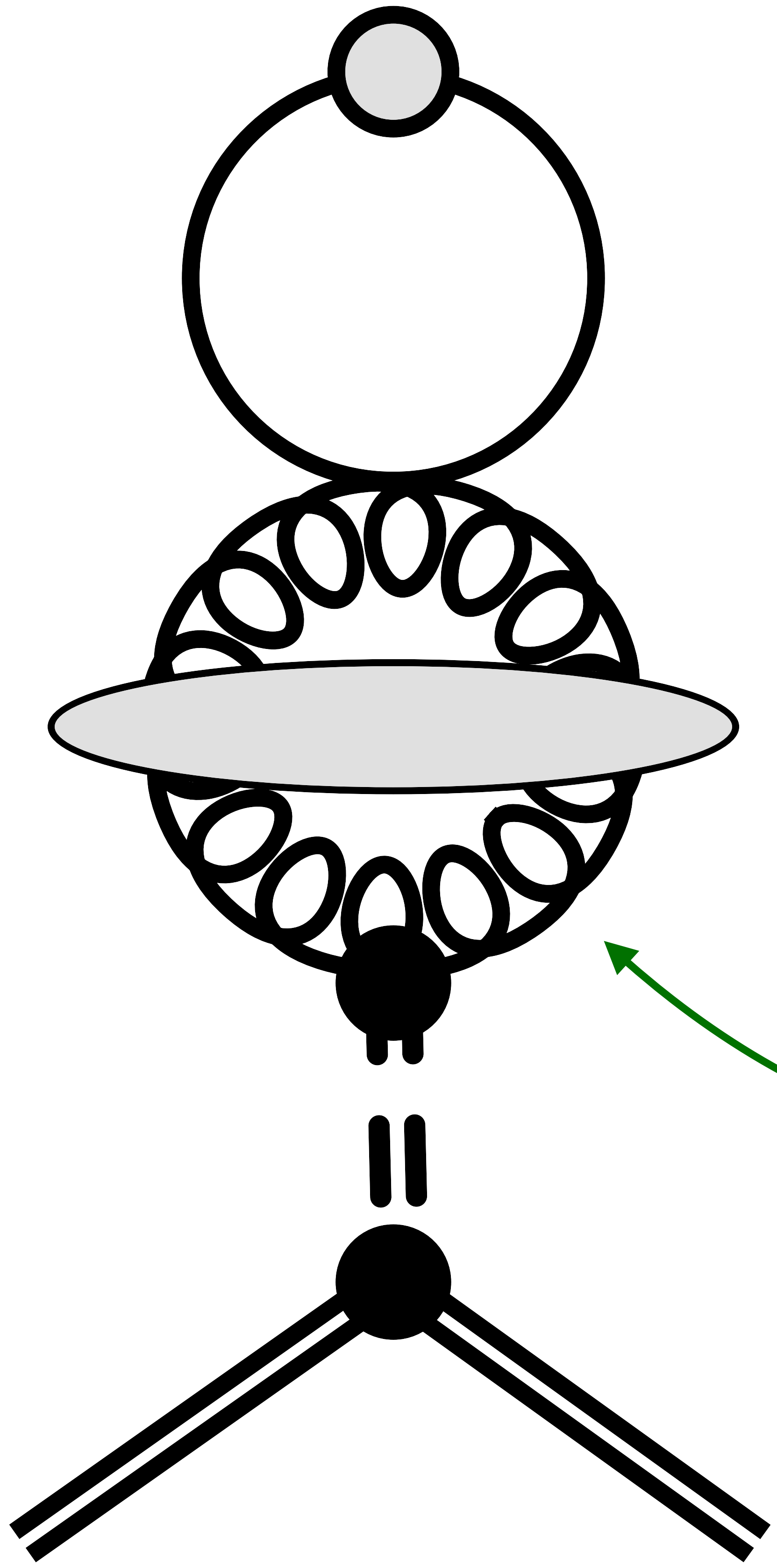

Shore


Veneziano

In agreement with COMPASS ($a^0|_{Q^2=3GeV^2} = 0.35 \pm 0.08$)
and HERMES data ($a^0|_{Q^2=5GeV^2} = 0.330 \pm 0.064$)

Axion-like effective action

One can construct an effective action which is based on the chiral Ward identities and incorporates cancelation of the infrared pole



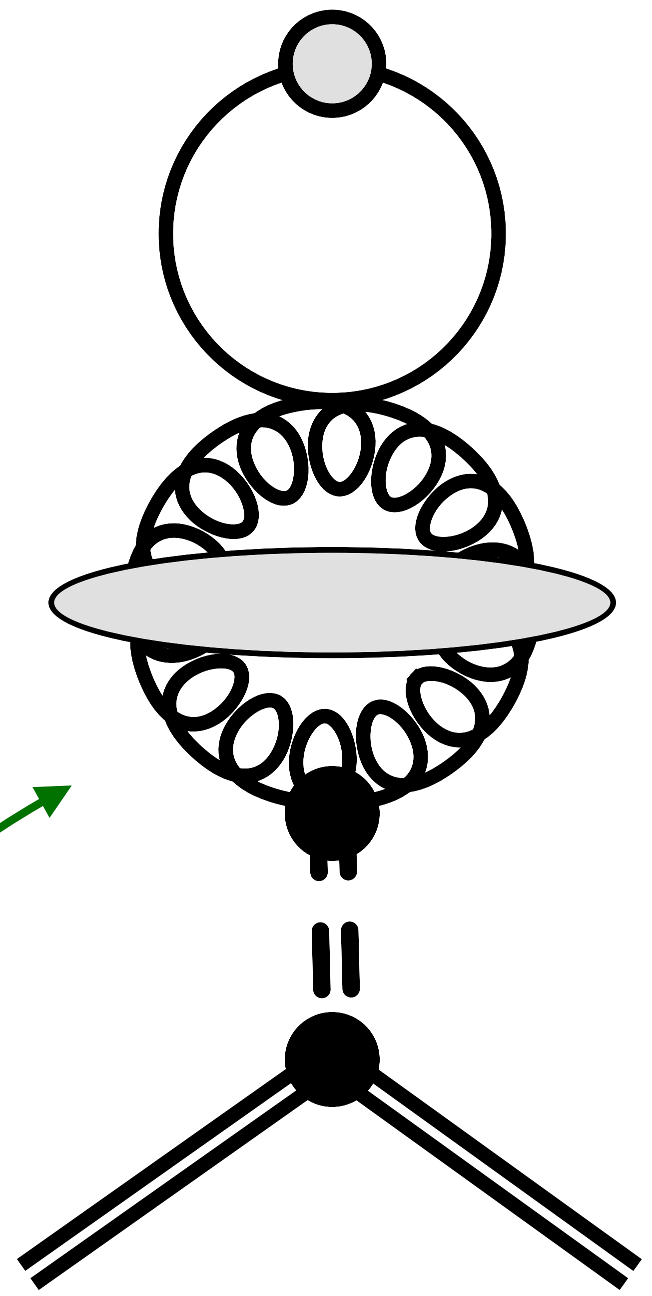
$$S_{\bar{\eta}} = \int d^4x \left[\frac{1}{2} \left(\partial_\mu \bar{\eta}\right) \left(\partial^\mu \bar{\eta}\right) - \frac{\sqrt{2n_f}}{F_{\bar{\eta}}} \bar{\eta}\, \Omega - \frac{\Omega^2}{2\, \chi_{\mathrm{YM}}} \right]$$

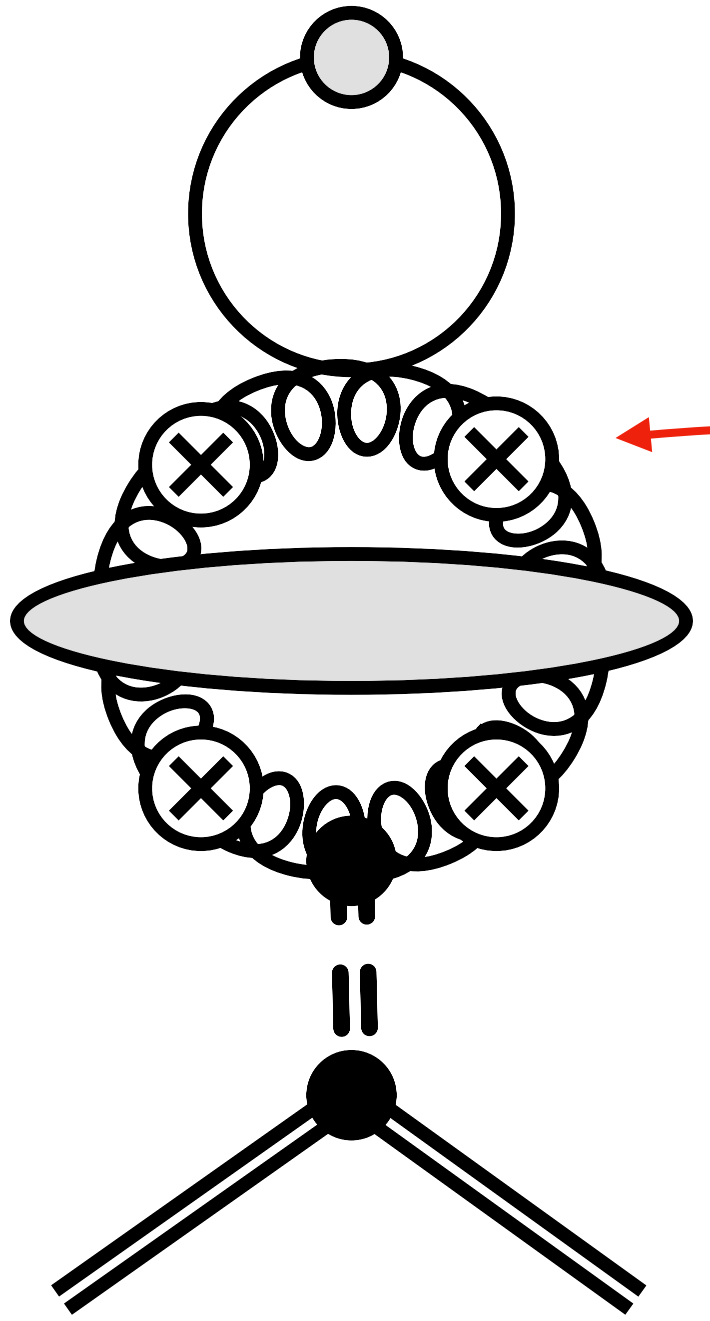
Shore,Veneziano (1990); Hatsuda (1990); Dvali, Jackiw, Pi (1995); t'Hooft (1976); Schafer, Shuryak (1996); Forte, Shuryak (1990); Qian, Zahed (2016)

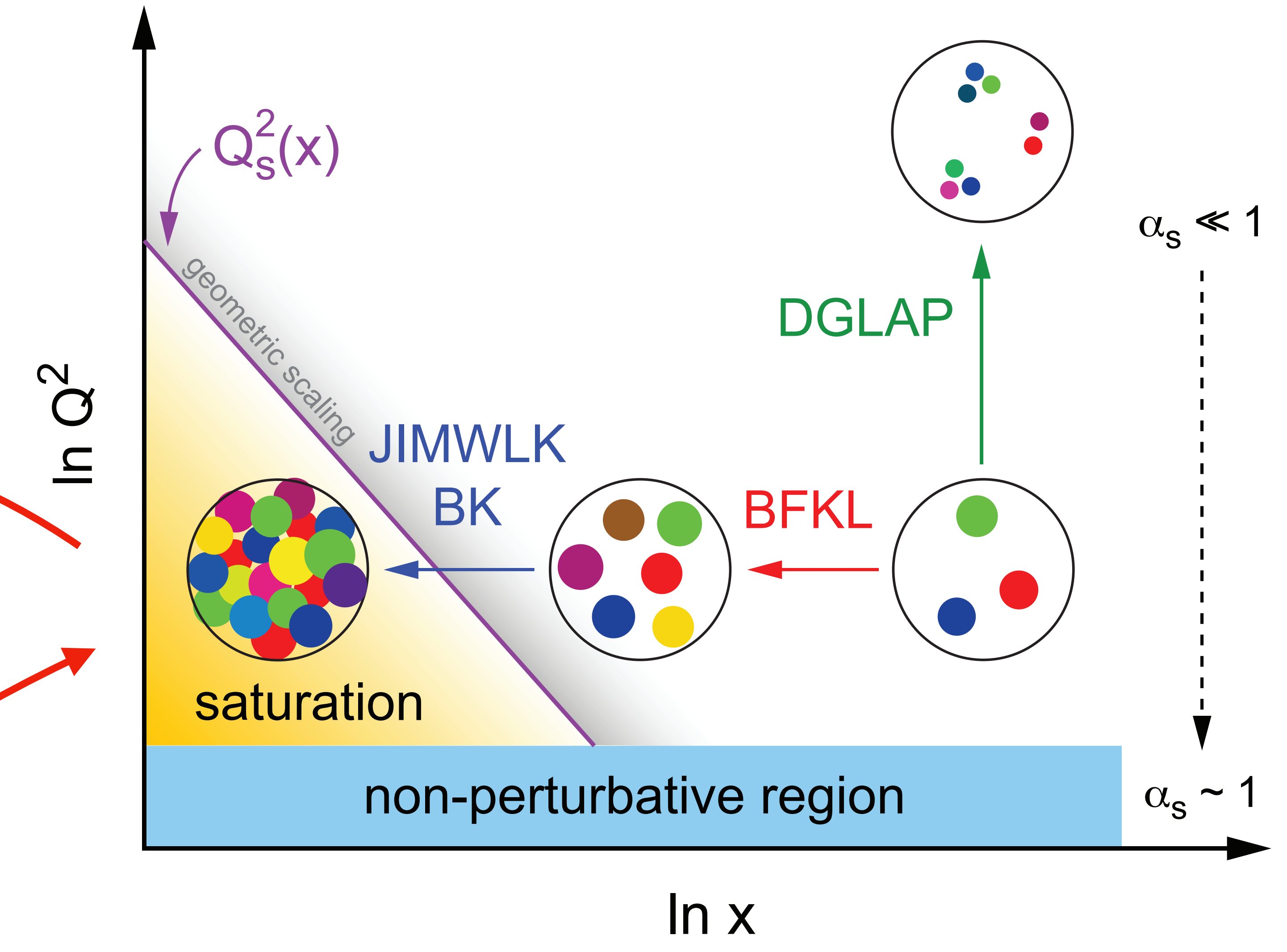
The gluon field, i.e. the YM topological susceptiblity χ_{YM} , is dominated by the instanton configurations. The typical scale is $m^2_{\eta'}$.

Effective action at small-x

Since the anomaly dominates the box diagram, the anomaly effects should be visible in analysis of the x_B dependence of $g_1(x_B, Q^2)$. In particular, novel features emerge at small x_B.





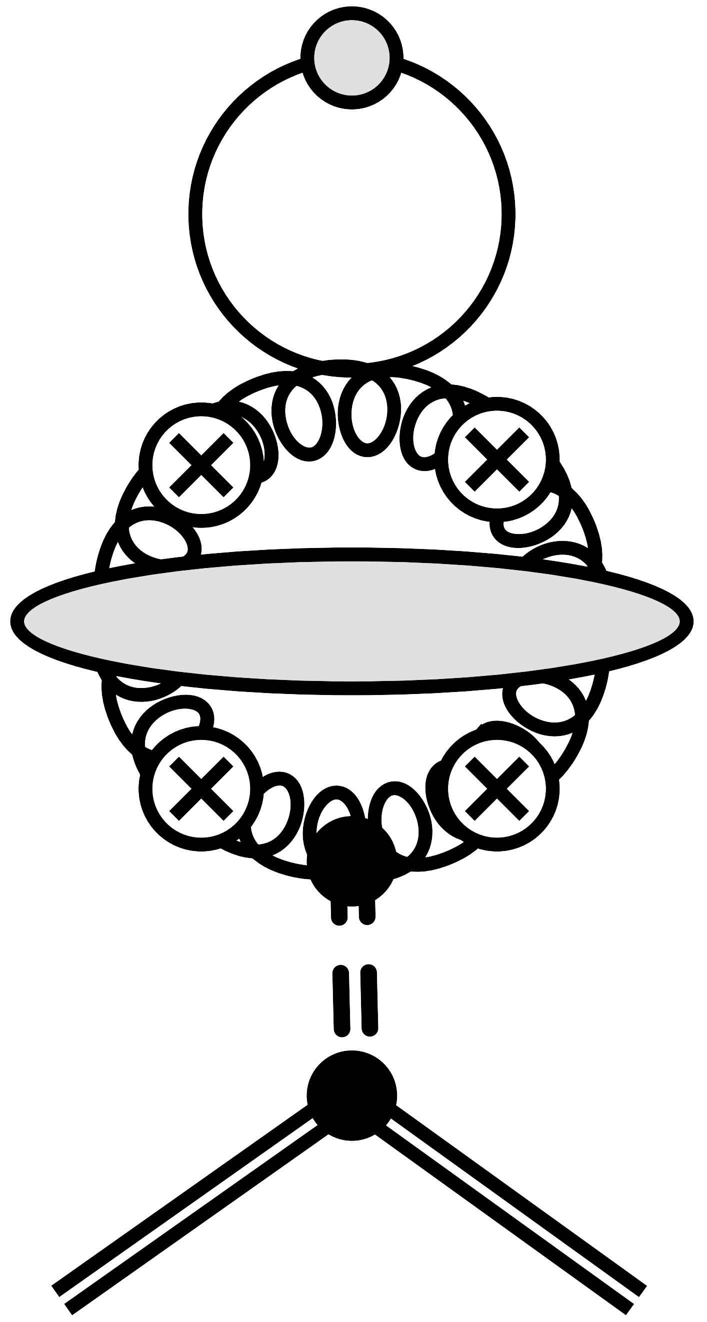


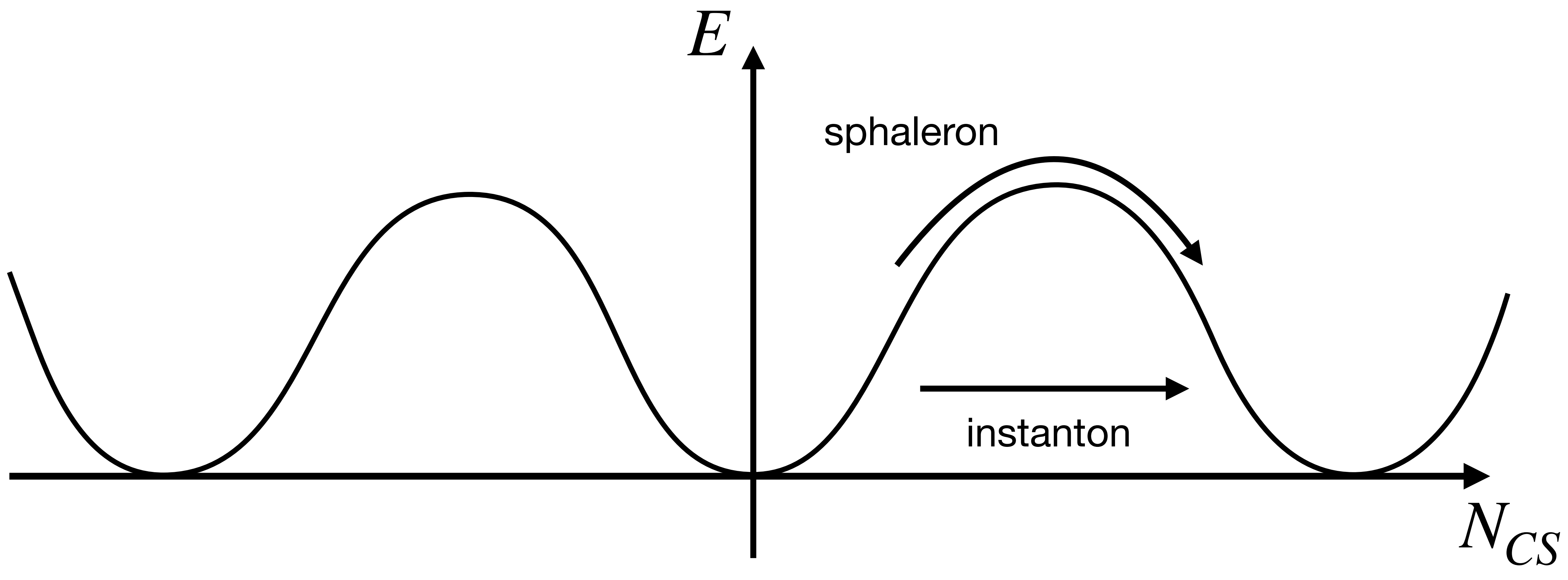
At large-x the gluon field is dominated by the instanton configurations.

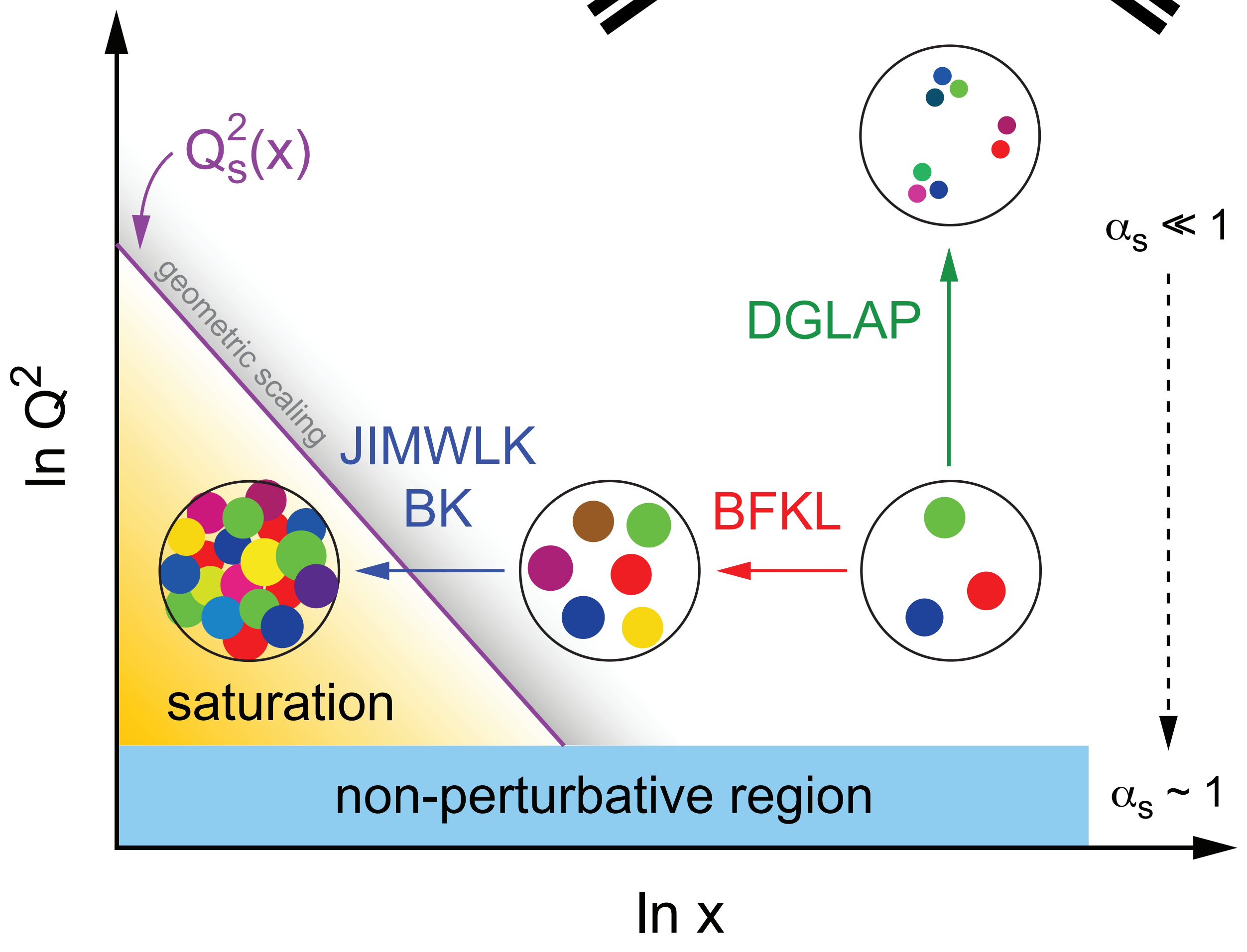
At small-x there is a background of small-x gluons characterized by the saturation scale Q_s^2.

Effective action at small-x

Gluon saturation can induce over the barrier sphaleron-like transitions







While at large x_B the gluon field is dominated by the instanton configurations, at small x_B the CGC solution ($Q_s^2 \gg m_{\eta'}^2$) starts to dominate and we predict over-the-barrier sphaleron transitions between different topological sectors of QCD vacuum

Tarasov, Venugopalan
arXiv:2109.10370

Axion-like effective action with coupling to CGC

We construct an axion-like effective action at small x_B that describes the interplay between gluon saturation and the topology of the QCD vacuum. It contains the WZW coupling and a kinetic term for the $\bar{\eta}$ field. This dynamics is governed by the $m^2_{\eta'}$

$$\langle \mathcal{O} \rangle^{\text{Regge}}_{\text{pol.}} = \int \mathcal{D}\rho \, W_Y[\rho] \int D\bar{\eta} \, \tilde{W}_{P,S}[\bar{\eta}] \int \mathcal{D}A \; \mathcal{O}[A] \; e^{iS_{\text{pCGC}}[A,\rho,\bar{\eta}]}$$

$$S_{\text{pCGC}}[A, \rho, \bar{\eta}] = S_{\text{CGC}}[A, \rho] + \int d^4x \left[\frac{1}{2} \left(\partial_\mu \bar{\eta}\right) \left(\partial^\mu \bar{\eta}\right) - \frac{\sqrt{2n_f}}{F_{\bar{\eta}}} \bar{\eta} \, \Omega \right]$$

The background gauge configurations are static classical configurations and their dynamics is described by the Color Glass Condensate (CGC) Effective Field Theory and controlled by the saturation scale Q_s^2

$$S_{\text{CGC}}[A, \rho] = -\frac{1}{4} \int d^4x F^{\mu\nu}_a F^a_{\mu\nu} + \frac{i}{N_c} \int d^2x_\perp \; \text{tr}_c \left[\rho(x_\perp) \ln \left(U_{[\infty, -\infty]}(x_\perp) \right) \right]$$

McLerran, Venugopalan (1994)

The $g_1(x_B, Q^2)$ structure function

The dynamics of the structure function is governed by two scales $m^2_{\eta'}$ and Q^2_s

$$g_1^{\rm Regge}(x_B,Q^2) = \left(\sum_f e_f^2\right) \frac{n_f\alpha_s}{\pi M_N} i \int d^4y \int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int\frac{d\xi}{2\pi}e^{-i\xi x}\int \mathcal{D}\rho\, W_Y[\rho]\int D\bar\eta\,\tilde W_{P,S}[\bar\eta]\int[DA]$$

$$\times\,{\rm Tr_c} F_{\alpha\beta}(\xi n)\tilde F^{\alpha\beta}(0)\,\eta_0(y)\,\exp\left(iS_{\rm CGC}+i\int d^4x\left[\frac{1}{2}\left(\partial_\mu\bar\eta\right)\left(\partial^\mu\bar\eta\right)-\frac{\sqrt{2n_f}}{F_{\bar\eta}}\bar\eta\,\Omega\right]\right)$$

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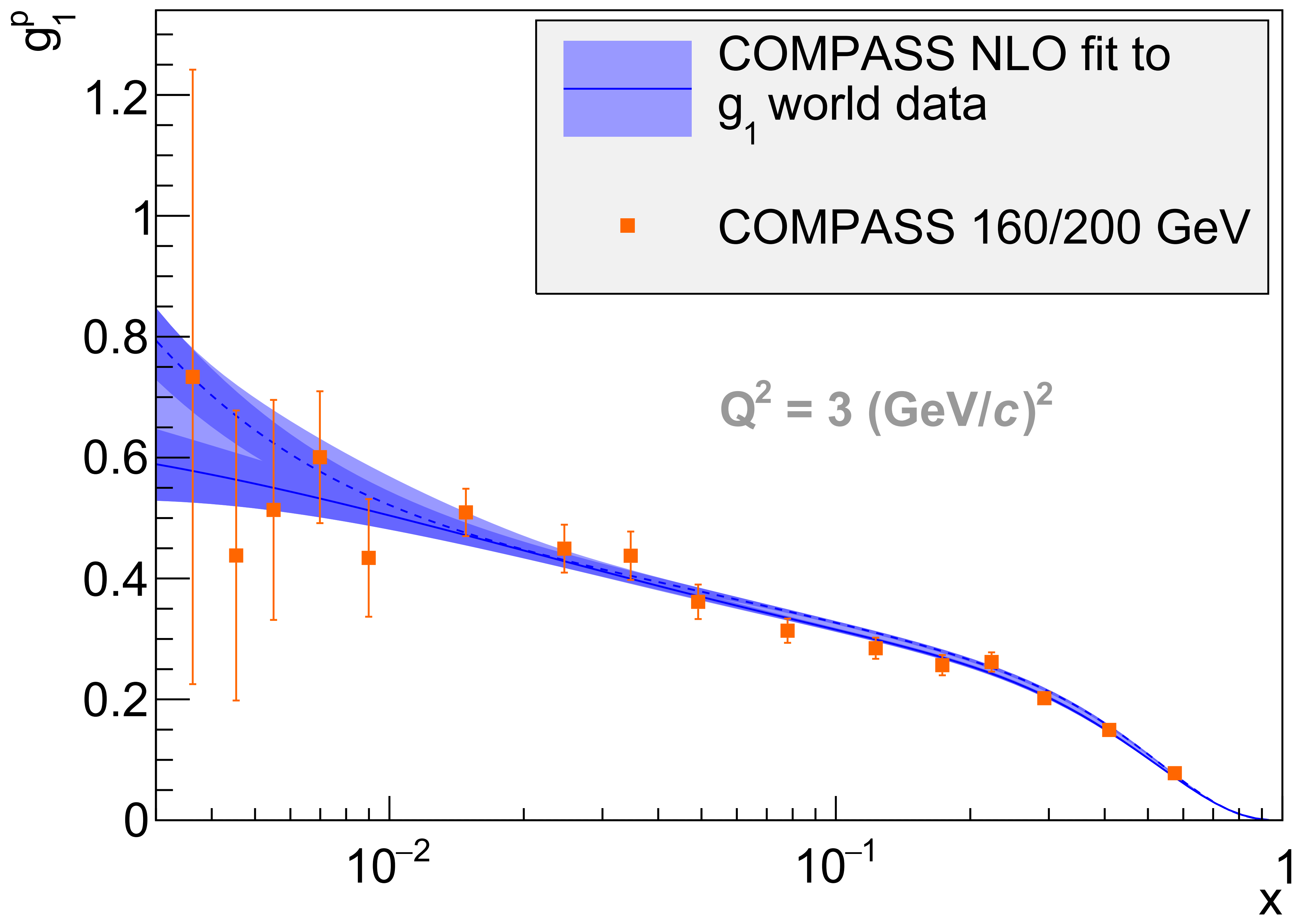
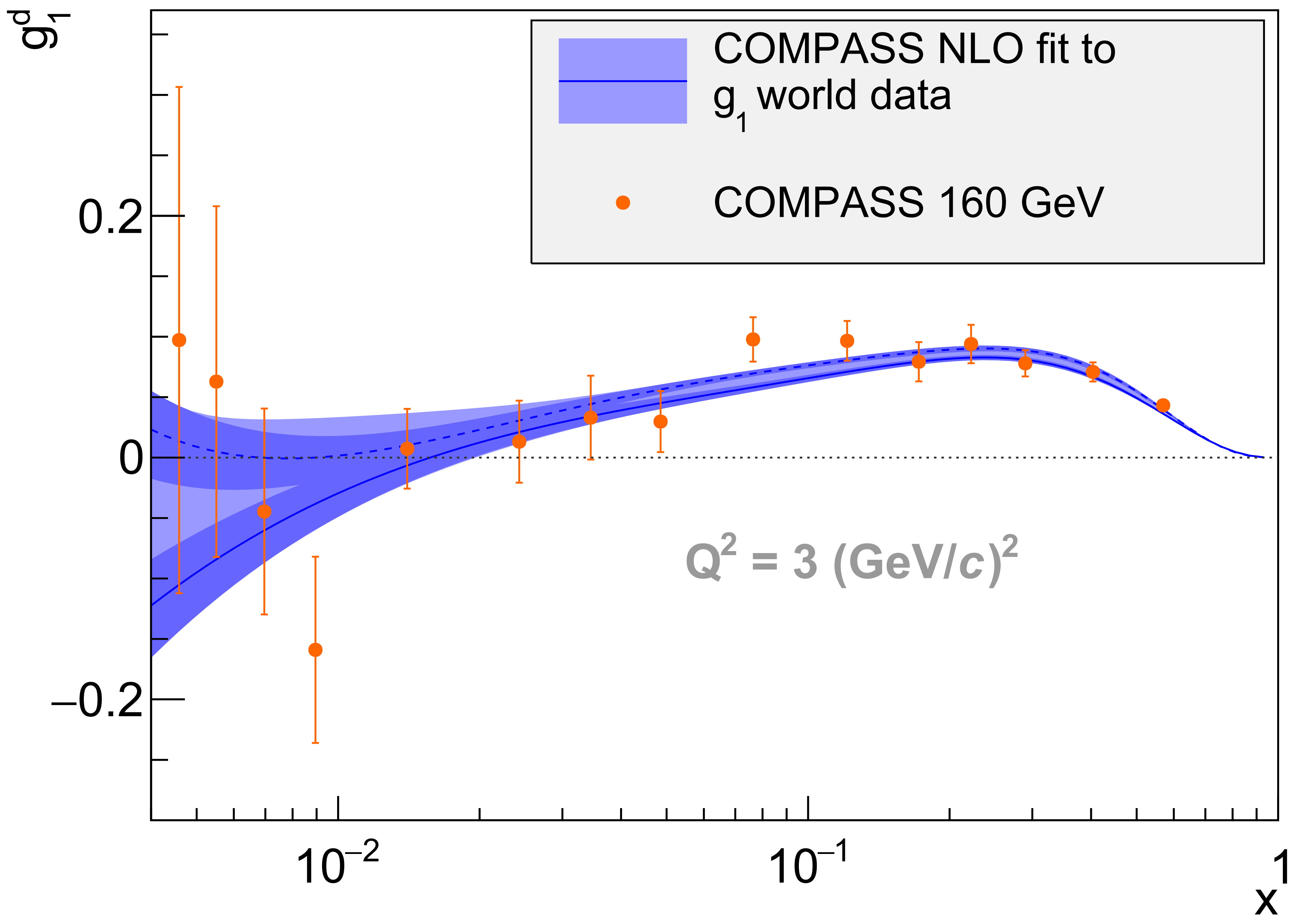
We estimate the structure function at $m^2_{\eta'} > Q^2_s$ employing analysis of the axion-like dynamics in a hot QCD plasma (see McLerran,Mottola,Shaposhnikov (1990)). We predict a rapid spin diffusion due to “drag force” on “axion” propagation in the shock wave background which is proportional to sphaleron transition rate.

$$g_1^{\rm Regge}(x_B,Q^2) \propto \frac{Q_S^2 m^2_{\eta'}}{F^3_{\bar\eta} M_N}\,\exp\left(-4\,n_f C\,\frac{Q_S^2}{F^2_{\bar\eta}}\right)$$

COMPASS data

$$g_1^{\rm Regge}(x_B, Q^2) \propto \frac{Q_S^2 m_{\eta'}^2}{F_{\bar\eta}^3 M_N} \exp\left(-4\, n_f C\, \frac{Q_S^2}{F_{\bar\eta}^2}\right)$$





The difference between g_1^p and g_1^n can be explained by a difference in density of color sources

arXiv:1503.08935; 1612.00620

Summary

- We show that the anomaly appears in both the Bjorken limit of large Q^2 and in the Regge limit of small x_B. We find that the infrared pole in the anomaly arises in both limits
- The cancellation of the pole involves a subtle interplay of perturbative and nonperturbative physics that is deeply related to the $U_A(1)$ problem in QCD
- We demonstrate the fundamental role of the WZW term both in topological mass generation of the η' and in the cancellation of the off-forward pole arising from the triangle anomaly in the proton's helicity. We recover the result by Shore and Veneziano that $\Sigma \propto \sqrt{\chi'(0)}$
- We introduce an axion-like effective action at small-x which describes the interplay between gluon saturation and the topology of the QCD vacuum
- We outline the role of "over-the-barrier" sphaleron-like transitions in spin diffusion at small x_B. Such topological transitions can be measured in polarized DIS at a future Electron-Ion Collider.