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The 24th International Spin Symposium

Simulation of the polarized quark fragmentation within the String+$^3P_0$ model

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in collaboration with Xavier Artru and Anna Martin

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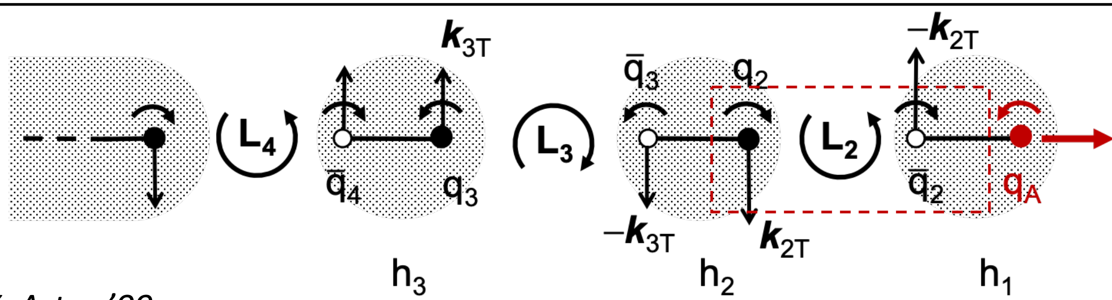
The Collins effect in the String+$^3P_0$ model

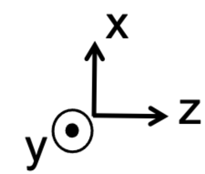
Collins FF $H_1^{\perp}$ → fragmentation of a transversely polarised quark into unpolarized hadrons
→ phenomenon interesting in its own right
→ gives access to chiral odd TMDs of the nucleon

	nucleon @ leading twist		
quark	**U**	**L**	**T**
U	$f_1(x,k_T^2)$ *(unpolarized)*		$f_{1T}^{\perp}(x,k_T^2)$ *(Sivers)*
L		$g_1(x,k_T^2)$ *(helicity)*	$g_{1T}(x,k_T^2)$ *(worm-gear)*
T	$h_1^{\perp}(x,k_T^2)$ *(Boer-Mulders)*	$h_{1L}^{\perp}(x,k_T^2)$ *(worm-gear)*	$h_1(x,k_T^2)$ *(transversity)* $h_{1T}^{\perp}(x,k_T^2)$ *(pretzelosity)*

non perturbative process → need of models
The String+$^3P_0$ model with pseudoscalar (PS) meson production
→ model for the Collins effect



Fragmenting quark q_A polarised along $\hat{y}$



X. Artru '09, arXiv:1001.1061

$q\bar{q}$ pair at string break-up in $^3P_0$ state (L=1, S=1, J=0)

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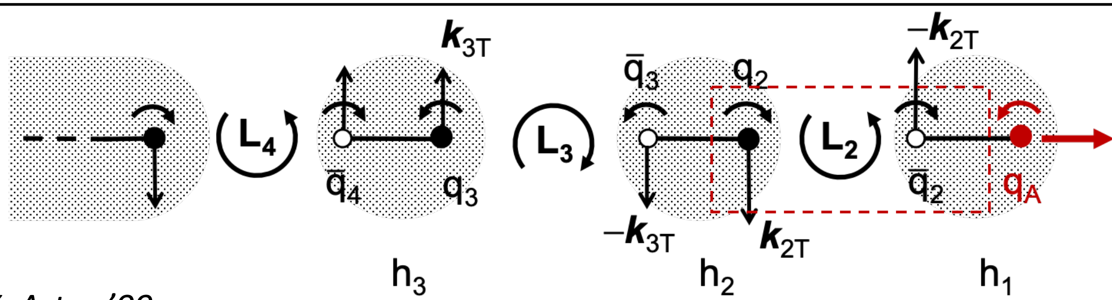

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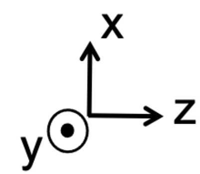
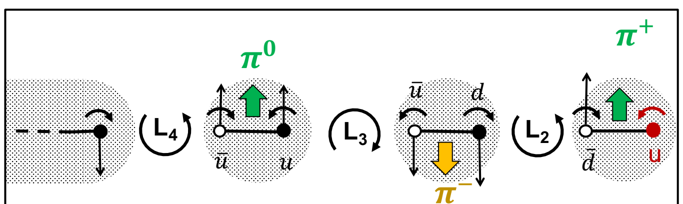
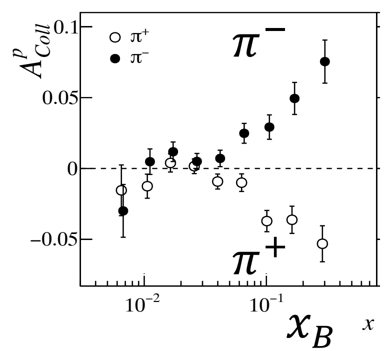


Assuming e.g. $q_A = u$



π^+ and π^- are emitted on opposite sides



Collins asymmetry
COMPASS PLB 717 (2012) 376

qualitative description of data
But for MC simulations
→quantum mechanical model

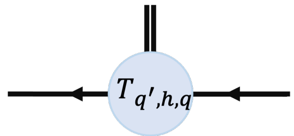
Stand alone MC implementation of (M18/M19)

String decay ↔ recursive repetition of the elementary splitting $q \to h + q'$

$p^+ = Zk^+, p^- = \varepsilon_h^2/p^+$
$\boldsymbol{p}_T = \boldsymbol{k}_T - \boldsymbol{k}'_T$

hadron = {h, $p = k - k'$}

quark' = {q', k', $\rho(q')$}



quark = {q, flavour
k, 4-momentum
$\rho(q)$} spin density matrix
(2x2, Pauli spinors)

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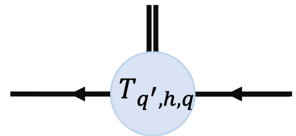

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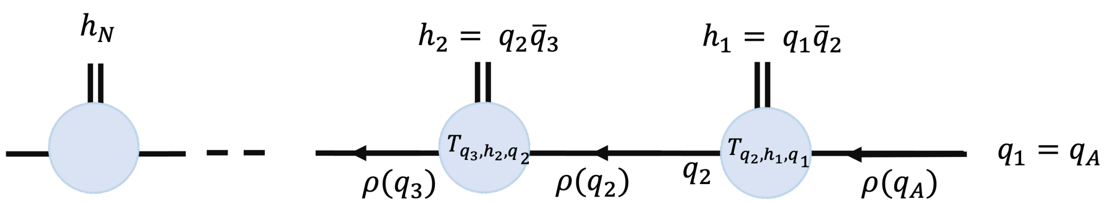
- Iterate points 1-4 until the exit condition (enough renamining mass to produce at least one baryonic resonance)



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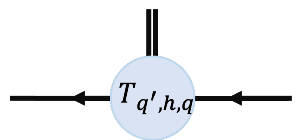

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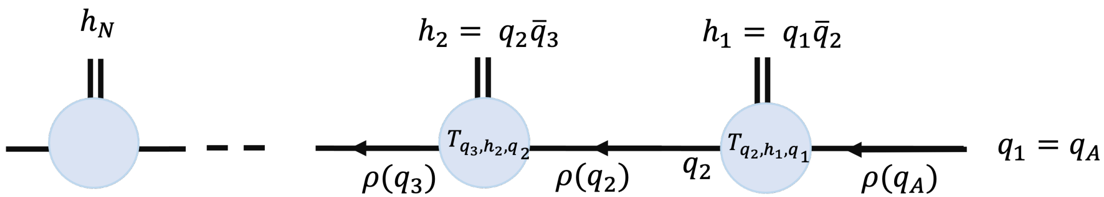


Few free parameters

$a, b_L, \mathrm{b_T}$ → string fragmentation dynamics (Lund Model, e.g. PYTHIA)

μ **complex mass** from $^3P_0$ mechanism → **responible for spin effects** $(\mathrm{Im}(\mu)$ → transverse)

Input function $\breve{g}$ → governs spin-independent $\mathbf{k}_\mathrm{T}$ - $\mathbf{k}'_\mathrm{T}$ correlations

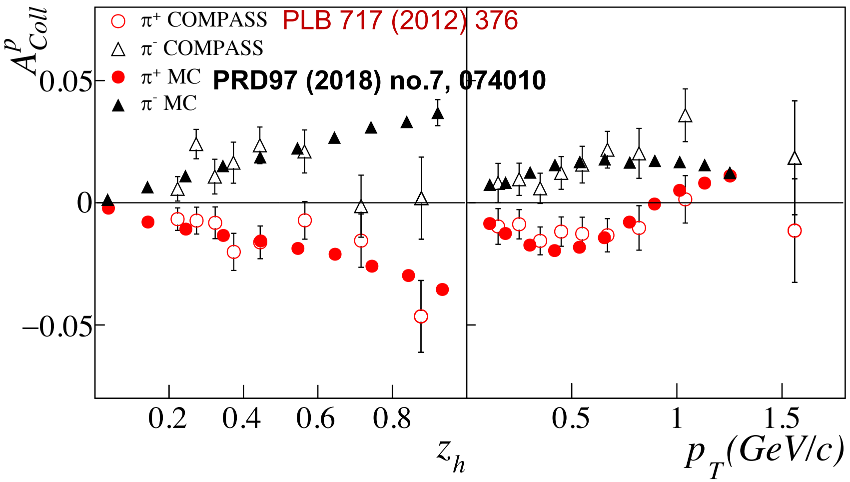
correlations → Model **M18** *AK, X. Artru, Z. Belghobsi, F. Bradamante, A. Martin, PRD 97 (**2018**) 7, 074010*

NO correlations → Model **M19** (much simpler) *AK, X. Artru, Z. Belghobsi, A. Martin ,PRD100 (**2019**) no.1, 014003*

Stand alone simulations (M19)

M18 and M19 have been implemented in stand alone MC programs, similar results
→ below results from fragmentations of **fully transversely polarized u quarks**

Collins asymmetry



$$A^p_{Coll}(z_h,p_T) \simeq \langle h_1^u/f_1^u\rangle\, a^{u\uparrow\to\pi+X}(z_h,p_T)$$

MC: Collins analysing power $a^{u\uparrow\to\pi+X}$ scaled by a constant factor

Comparison with HERMES asymmetries also OK

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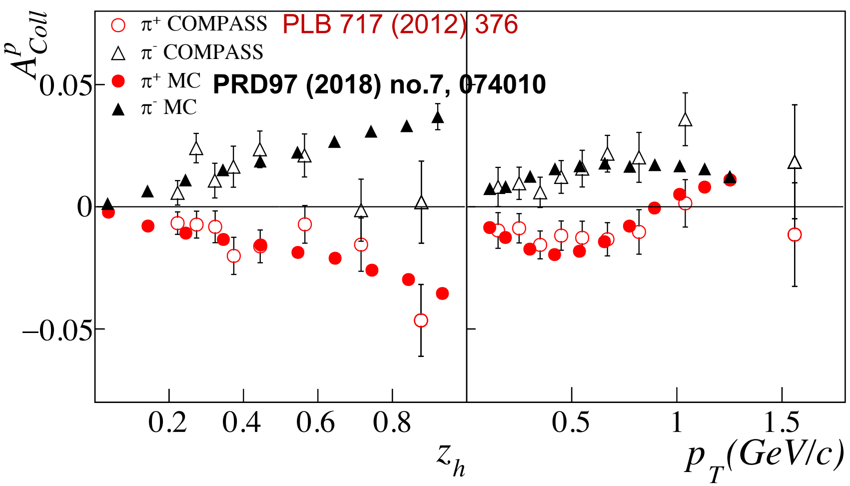
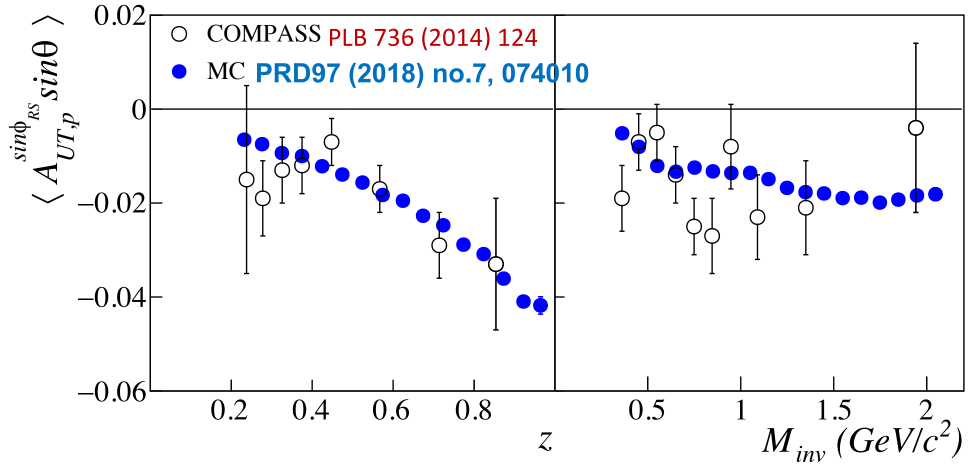


Dihadron asymmetry



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$$\langle A^{\sin\phi_{RS}}_{UT,p} \sin\theta \rangle \simeq \langle h_1^u / f_1^u \rangle \, a^{u\uparrow\to h^+h^-+X}(z, M_{inv})$$

MC: dihadron analysing power for h^+h^- pairs scaled by same factor

Comparison with HERMES asymmetries also OK

The model describes the main properties of data
few parameters, same mechanism for Collins and dihadron asymmetries, and jet handedness

Further developments

a) Interface of M19 with PYTHIA 8.2 for SIDIS → **StringSpinner** package

in collaboration with L. Lönnblad

→ more complete simulation of the collision event
→ spin effects in string fragmentation to PS mesons
→ parametrizations of transversity PDFs introduced (for u^v and d^v, can be changed)
→ ISR/FSR switched OFF
→ **first step towards a systematic implementation of spin effects in PYTHIA**

AK, L. Lönnblad, arXiv:2105.09730

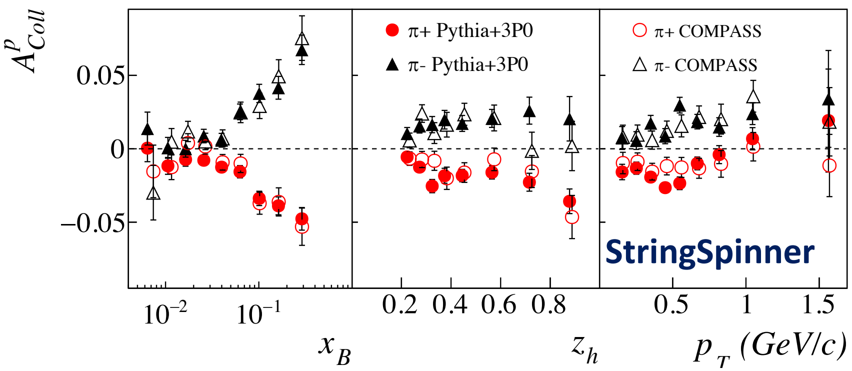
b) Extend M19 by introducing vector mesons (VM) → NEW model **M20**
→ decays products of VMs give a large contribution to the sample of final observed hadrons
→ improved description of the polarized fragmentation process
→ more complicated model

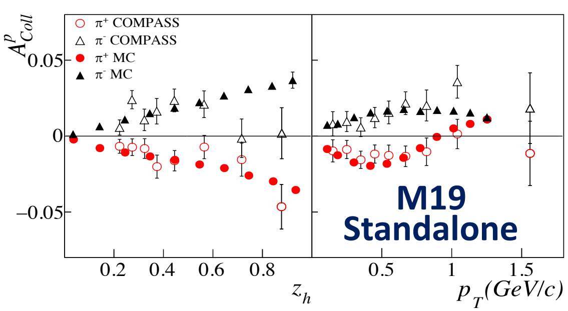
StringSpinner

https://gitlab.com/albikerbizi/stringspinner.git

Simulations of SIDIS off protons in the COMPASS kinematics [no intrinsic $\boldsymbol{k}_\perp$, $\mu = (0.78 + i0.38)\ \mathrm{GeV/c^2}$]

Collins asymmetry





Nice description of Collins

→ trend of Collins asymmetry vs z_h modified by PYTHIA

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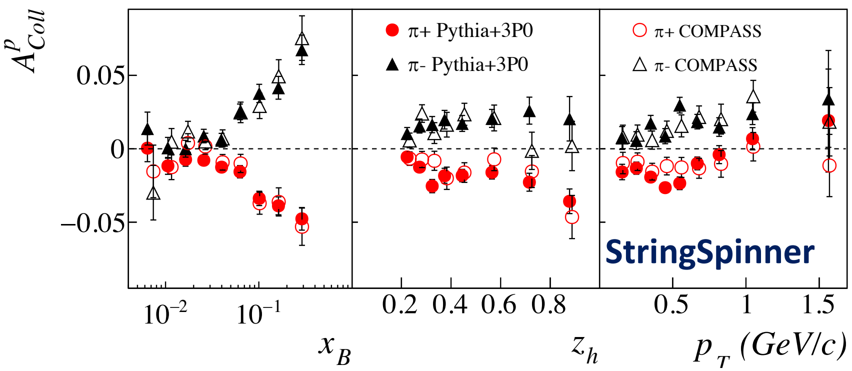

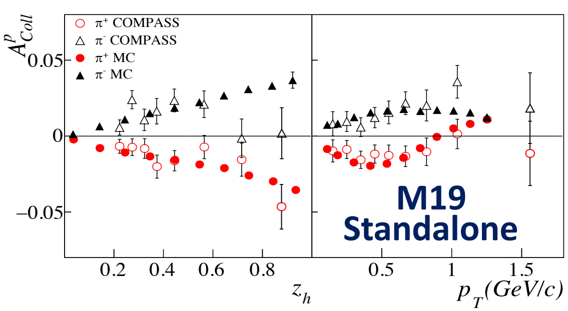

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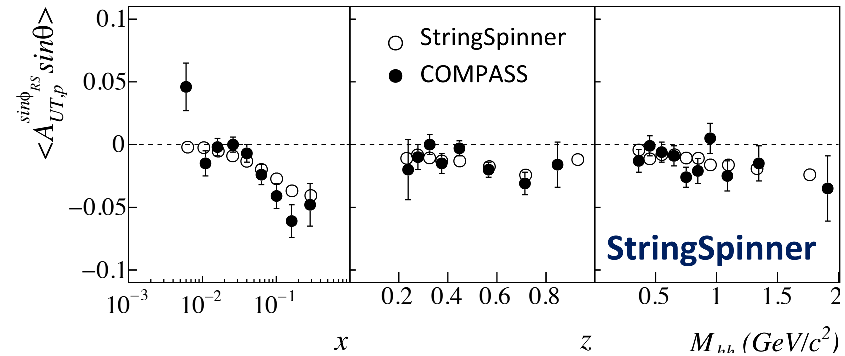

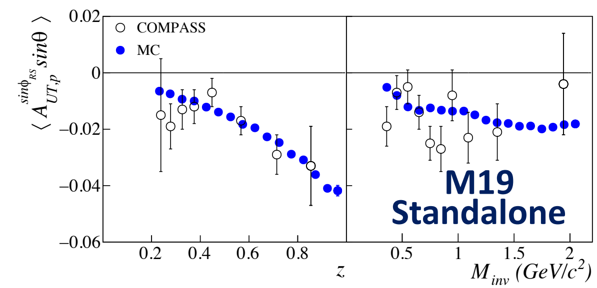

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AK, X. Artru, A. Martin, arXiv:2109.06124

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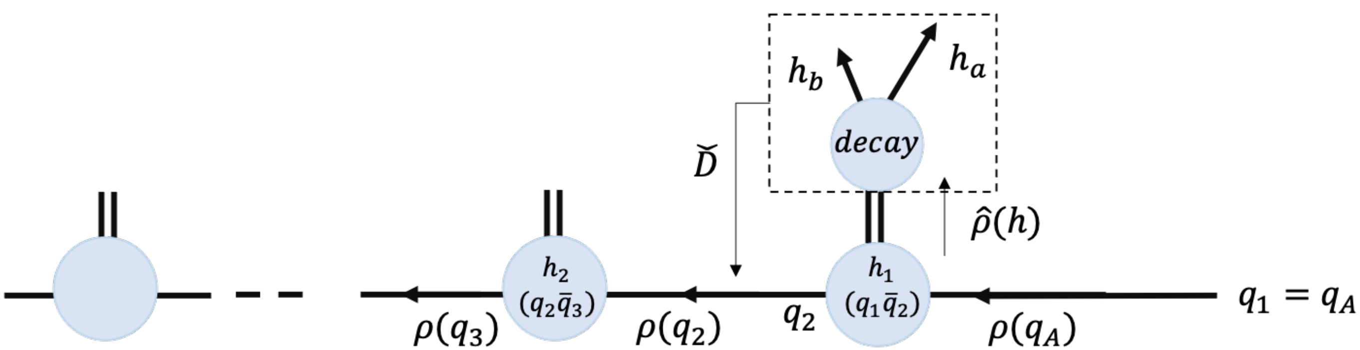
2. Choose h to be VM with prob. $\frac{f_{VM}}{f_{VM}+f_{PS}}$
3. Generate 4-momentum of h_1, using the splitting function $F = \mathrm{Tr}\, T\rho(q)T^\dagger$
4. If $h_1 = \mathrm{PS}$ go to 5
   If $h_1 =$ VM → calculate $\hat{\rho}(h)$
   a) generate decay hadrons in VM rest frame and boost to string frame
   b) construct the decay matrix $\breve{D}$
5. Calculate the spin density matrix of q_2

Collins '88
Knowles '88

- Iterate points 1-5 until the exit condition (enough renamining mass to produce at least one baryonic resonance)



Stand alone simulations with M20

Coupling of quarks to VM → complex free parameters

G_L → coupling to VM with linear L pol. along the string axis

G_T → coupling to VM with linear T pol. w.r.t the string axis

New free parameters for VMs

$|G_L|^2 + 2|G_T|^2$ → governs the to fraction f_{VM}/f_{PS} [also in PYTHIA]

$|G_L/G_T|$ → governs spin alignment, influences the Collins effect of the VM

$\theta_{LT} = \arg(\mathrm{G_L}/\mathrm{G_T})$ → governs oblique polarisation (LT)

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Next slides, simulations with

u quarks fully transversely polarized along $\hat{y}$

Energy calculated from a $\{x_B, Q^2\}$ sample of SIDIS events

no primordial KT

Values of the free parameters

all mesons

$a = 0.9$

$b_L = 0.5\ (\mathrm{GeV}/c^2)^{-2}$

$b_T = 8.43(\mathrm{GeV}/c)^{-2}$

$\mu = (0.42 + \mathrm{i}0.76)\ \mathrm{GeV/c^2}$

} *as in M19*

VM production

$$f_{VM}/f_{PS} = \begin{cases} 0.62 & \text{light mesons} \\ 0.725 & \text{strange mesons} \end{cases}$$ *as in PYTHIA 8*

$|G_L/G_T| = 1$

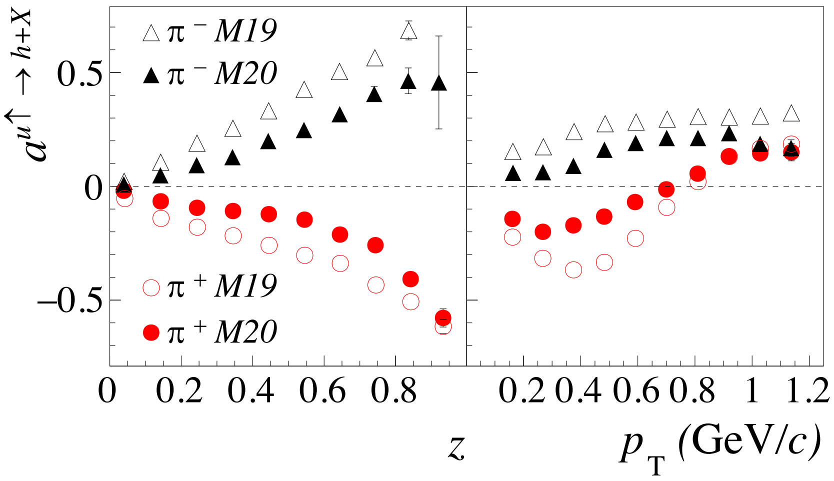
$\theta_{LT} = 0$

Czyzewski '96 (quark model)

Other combinations of $|\mathrm{G_L/G_T}|$ and θ_{LT} explored

Effect of VM production on TSA

Collins analysing power



Different trends as compared to M19
Dilution of about 50%

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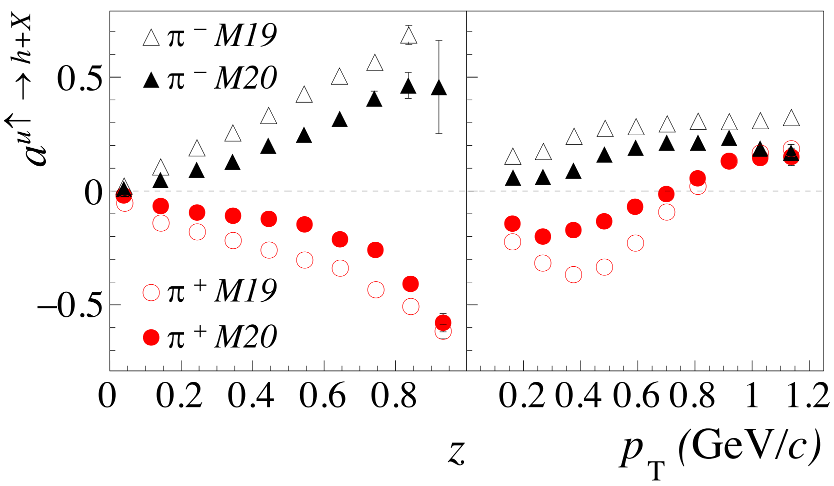

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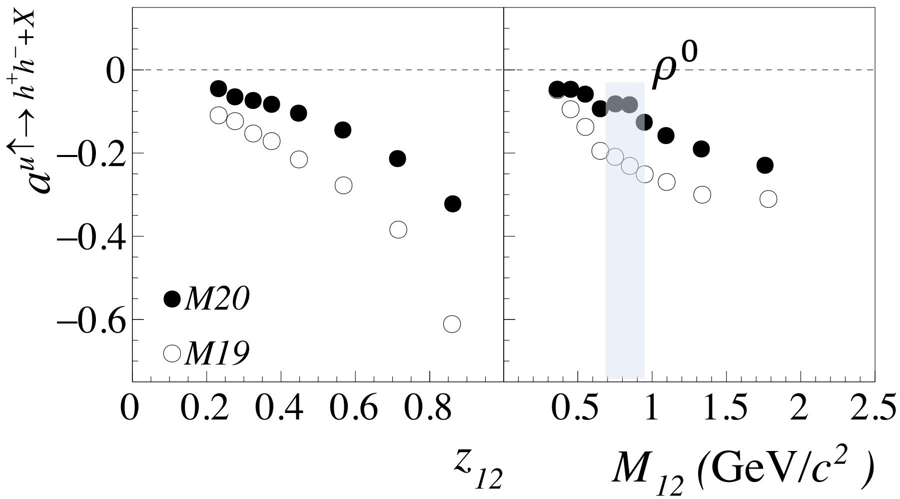
Di-hadron analysing power



VM decays do not contribute to the 2h asymmetry
(decay symmetric w.r.t $\boldsymbol{R} \leftrightarrow -\boldsymbol{R}$)
50% dilution w.r.t M19
→ stronger dilution around ρ^0

Large effect on Collins and dihadron analysing powers
→ important to understand how different contributions lead to the observed TSA

Comparison with SIDIS data

To investigate the possible values of $|G_L G_T|$ and θ_{LT} we tried different combinations of the parameters

oblique

L		T		
	$\theta_{LT} = -\pi/2$		$\theta_{LT} = 0$	$\theta_{LT} = +\pi/2$
	$\lvert G_L/G_T \rvert = 5$		$\lvert G_L/G_T \rvert = 5$	$\lvert G_L/G_T \rvert = 5$
	$\lvert G_L/G_T \rvert = 1$		$\lvert G_L/G_T \rvert = 1$	$\lvert G_L/G_T \rvert = 1$
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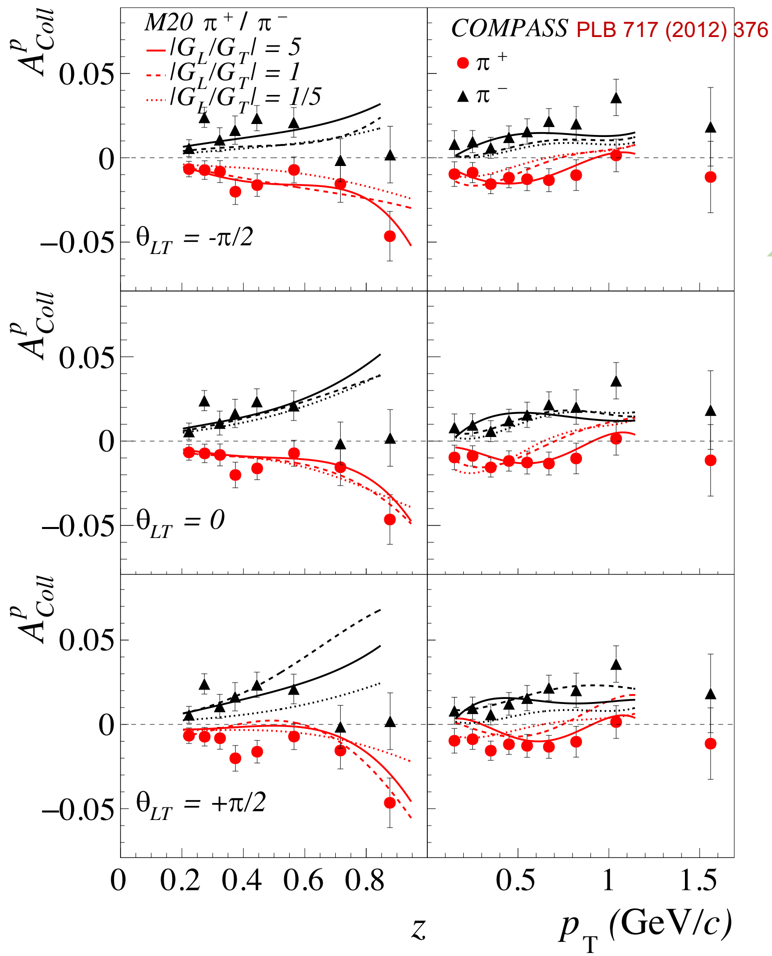

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oblique

L | T

$\theta_{LT} = 0$	$\theta_{LT} = +\pi/2$
$\theta_{LT} = -\pi/2$	$\theta_{LT} = 0$	$\theta_{LT} = +\pi/2$
$|G_L/G_T| = 5$	$|G_L/G_T| = 5$	$|G_L/G_T| = 5$
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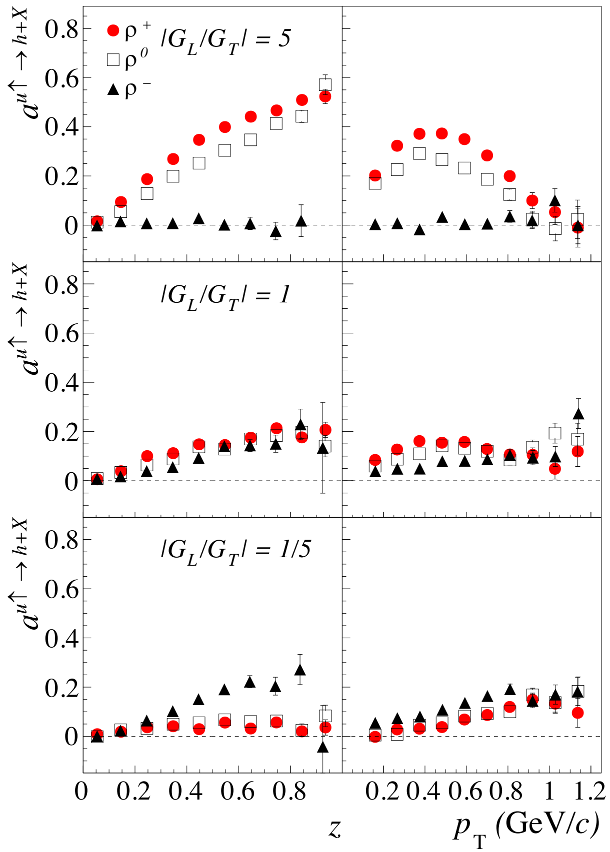
$|G_L/G_T| = 1/5$	$|G_L/G_T| = 1/5$	$|G_L/G_T| = 1/5$

MC scaled by a factor λ for each combination
Large variations due to different values of $|G_L|/|G_T|$ and θ_{LT}
→ both parameters are important

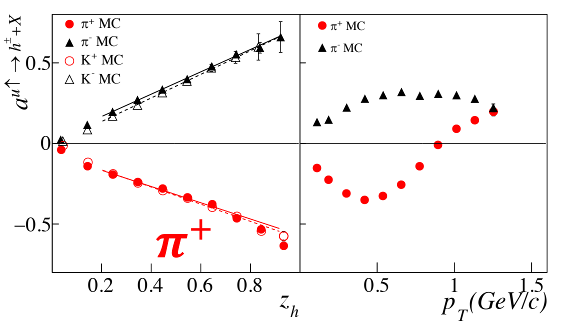
hint for $|G_L/G_T| = 5, \theta_{LT} = -\pi/2$ or $|G_L/G_T| = 1, \theta_{LT} = 0$
→ more precise data would help to fix the free parameters

comparison with 2h asymmetry OK, less sensitive to $|G_L G_T|$ and θ_{LT}

Collins effect for ρ mesons



→ Opposite sign w.r.t π^+ (Czyzewski prediction)
→ Strong dependence on $|G_L/G_T|$
→ both size and shapes change



model M19 → only PS

PRD 100 (2019) 1, 014003,

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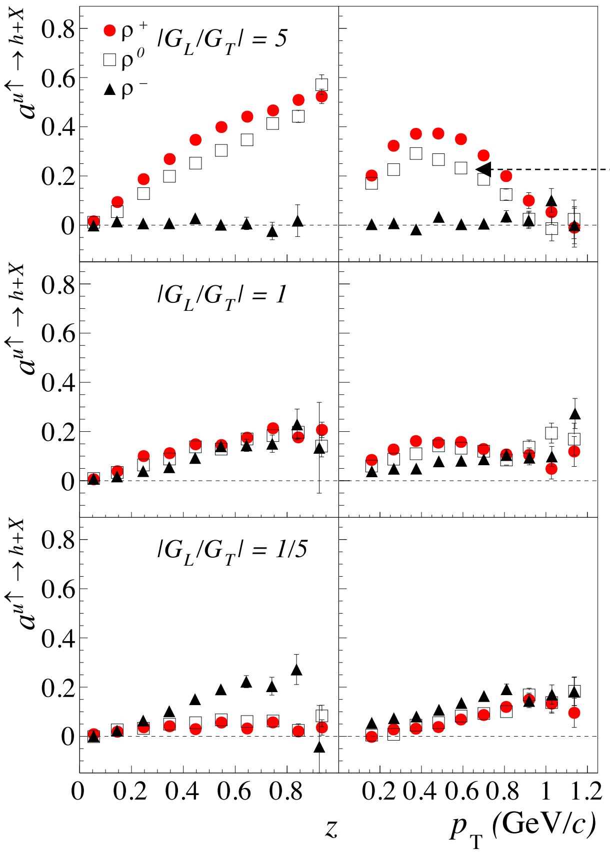
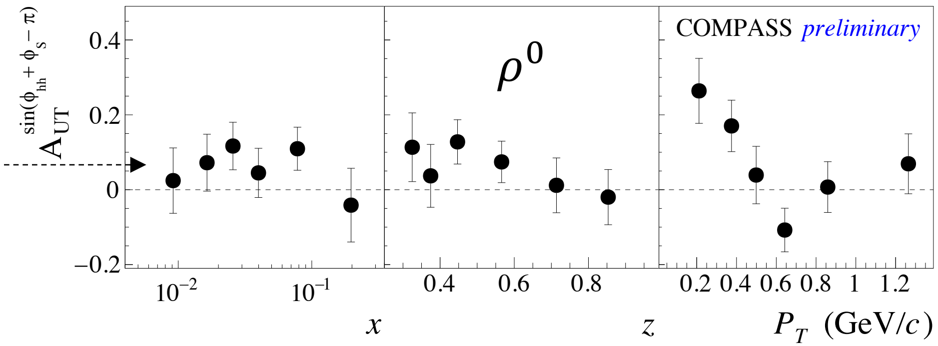




NEW COMPASS results *see talk by A. Moretti*

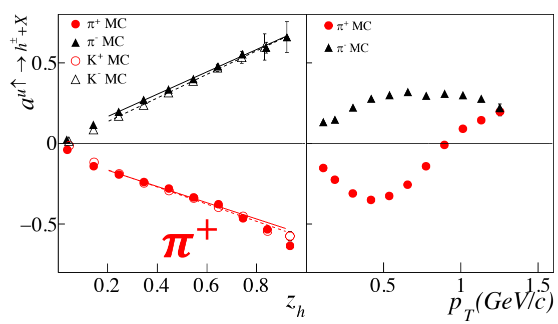
→ Measurements feasible, could be used to fix the parameters

→ Hint for $\frac{|G_L|}{|G_T|} > 1$ (in particular from p_T)

→ Opposite sign w.r.t π^+ (Czyzewski prediction)

→ Strong dependence on $|G_L/G_T|$

→ both size and shapes change



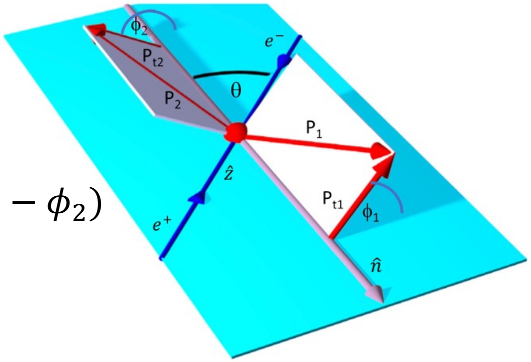
model M19 → only PS

PRD 100 (2019) 1, 014003,

Comparison with e⁺e⁻ data

Collins asymmetries for back-to-back pions measured recently by BELLE *PRD 100, 092008 (2019)*

see also talk by G. Schnell



$$R_{12}^{UL} = \frac{N_{\pi^+\pi^-} + N_{\pi^-\pi^+}}{N_{\pi^+\pi^+} + N_{\pi^-\pi^-}} \approx 1 + A_{12}^{UL}\cos(\phi_1 - \phi_2)$$

The $A_{12}^{UL}(z, p_T)$ asymmetry can be written as

$$A_{12}^{UL}(z, p_T) = \left\langle \frac{\sin^2\theta}{1+\cos^2\theta} \right\rangle \times \left|a^{fav}(z, p_T)\right|^2 \times \left(\frac{5 + 5\alpha^2 + 2\alpha'^2}{5 + 5\beta^2 + 2\beta'^2} - \frac{5\alpha + \alpha'^2}{5\beta + \beta'^2}\right)$$

$z_1 = z_2 \equiv z$
$p_{1T} = p_{2T} \equiv p_T$

0.91

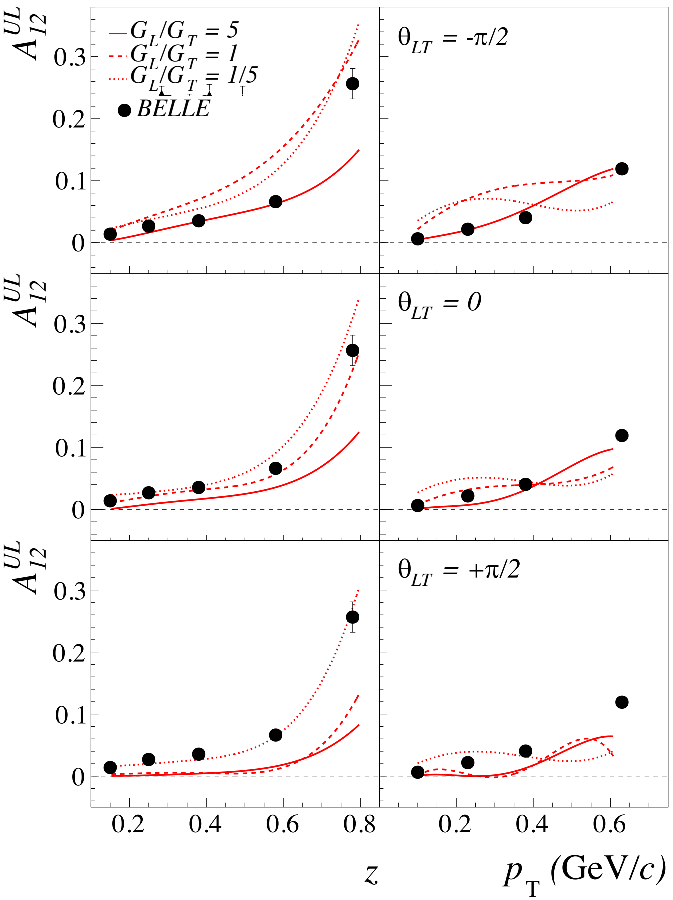
Collins analyzing power for fav. fragmentation

isospin + charge congj.

$\alpha = H_1^{fav}/H_1^{unfav}$ $\alpha' = H_{1,s}^{unfav}/H_1^{unfav}$

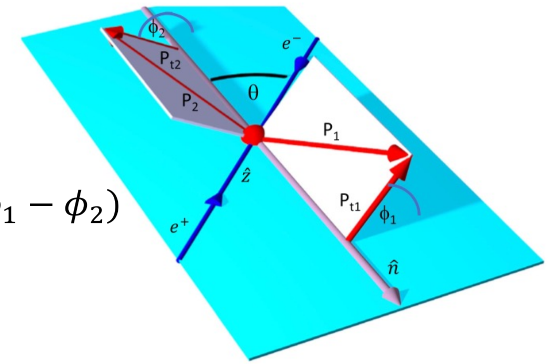
$\beta = D_1^{fav}/D_1^{unfav}$ $\beta' = D_{1,s}^{unfav}/D_1^{unfav}$

Comparison with e⁺e⁻ data



Collins asymmetries for back-to-back pions measured recently by BELLE *PRD 100, 092008 (2019)*

see also talk by G. Schnell



$$R_{12}^{UL} = \frac{N_{\pi^+\pi^-} + N_{\pi^-\pi^+}}{N_{\pi^+\pi^+} + N_{\pi^-\pi^-}} \approx 1 + A_{12}^{UL}\cos(\phi_1 - \phi_2)$$

The $A_{12}^{UL}(z, p_T)$ asymmetry can be written as

$$A_{12}^{UL}(z,p_T) = \left\langle \frac{\sin^2\theta}{1+\cos^2\theta} \right\rangle \times \left|a^{fav}(z,p_T)\right|^2 \times \left(\frac{5+5\alpha^2+2\alpha'^2}{5+5\beta^2+2\beta'^2} - \frac{5\alpha+\alpha'^2}{5\beta+\beta'^2}\right)$$

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Belle data corrected for charm production
A_{12}^{UL} has been **evaluated** using M20 (cuts as in data, MC not rescaled)
→ good description for $\left|\frac{G_L}{G_T}\right| = 5, \theta_{LT} = -\frac{\pi}{2}$ and $\left|\frac{G_L}{G_T}\right| = 1, \theta_{LT} = 0$ as for SIDIS data

A new di-hadron asymmetry

VMs do not contribute to the standard dihadron asymmetry but contribute to the z-ordered di-hadron asymmetry

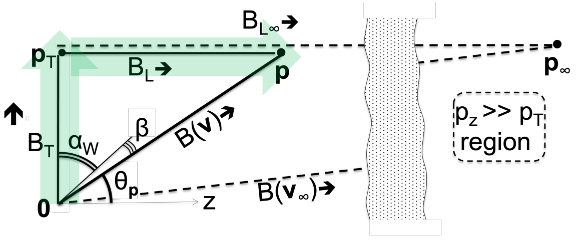
$$a^{u\uparrow\to h_1h_2+X} = 2\langle\sin(\phi_R-\phi_{S_u})\rangle\,,\ \boldsymbol{R}_T = z_2\boldsymbol{p}_{1T}/z - z_1\boldsymbol{p}_{2T}/z,$$

z-ordering: $z_1 > z_2$

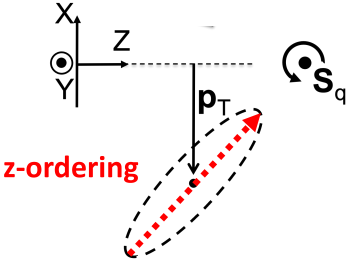
→ **Due to the oblique polarization ρ_{XZ} which depends on θ_{LT}**

Ex : for a rank 1 VM
$\rho^{LR}_{XZ} \propto S_Y \sin\theta_{LT}$

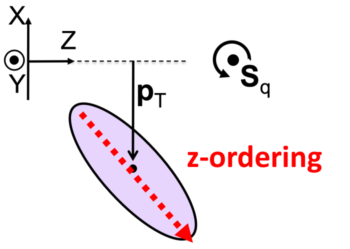
Physics → Left-Right symmetric rest frame



$\theta_{LT} = -\pi/2$



$\theta_{LT} = +\pi/2$



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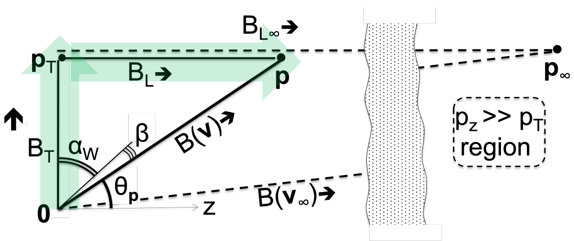
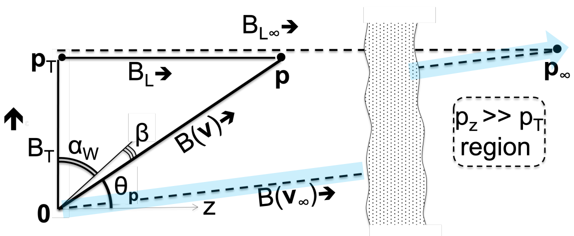
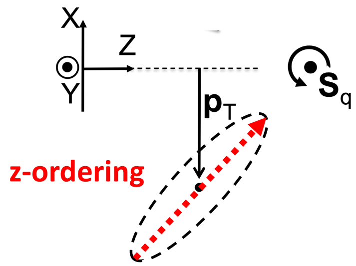


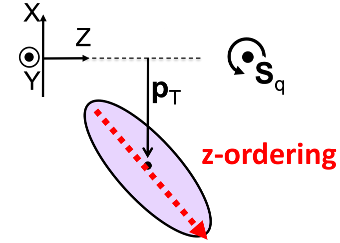
Measurement → probe the VM density matrix in the Infinite momentum frame



$\theta_{LT} = -\pi/2$



$\theta_{LT} = +\pi/2$



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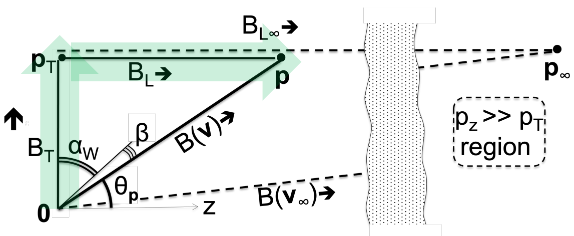

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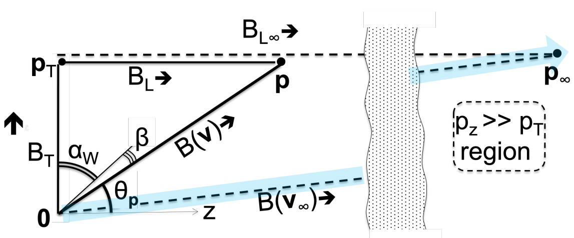
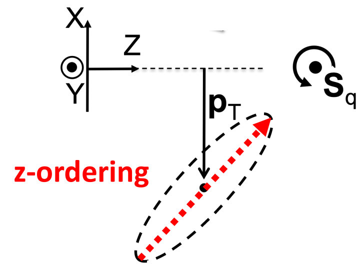
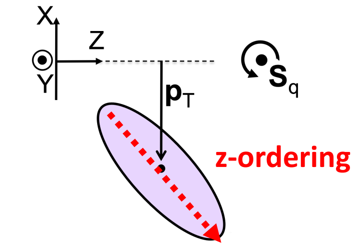


Relevant to study the oblique polarization of VMs
→ asymmetry still large below the ρ^0 region
→ interesting to be measured in SIDIS and e^+e^-

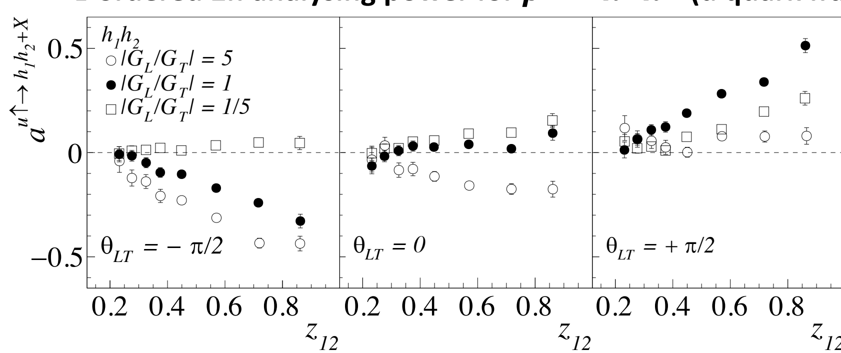
$\theta_{LT} = -\pi/2$



$\theta_{LT} = +\pi/2$



z-ordered 2h analysing power for $\rho^0 \to \pi^+\pi^-$ (u quark fragm.)



large and negative — **Small effect of Wigner rot.** — **large and positive**

Conclusions

The String+$^3P_0$ model with PS meson emission studied in detail (M18, M19)
→ describes the main features of Collins and di-hadron asymmetries!

M19 has been interfaced to PYTHIA 8.2 → StringSpinner
→ polarized SIDIS
→ first step to introduce spin effects in PYTHIA hadronization done

VM production introduced in the String+$^3P_0$ model (M20)
→ detailed study of Collins/dihadron effects for PS and VM, new 2h asymmetry
→ current data provide hints for values of the new free parameters
→ more precise data would help (COMPASS 2021-2022 d run, JLab12 ..)
→ a more complete simulation of the e^+e^- event is needed

All considered, the results are promising, the model has few free parameters and currently it is a **good candidate to be used for the preparation of a polarized MC event generator**

backup

Elementary splitting with PS meson emission

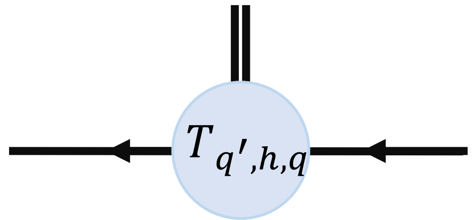
string decay = recursive repetition of the elementary splitting $q \rightarrow h + q'$

X. Artru, Z. Belghobsi DSPIN-2011, 2013

hadron = {h, $p = k - k'$}

$p^+ = Zk^+, p^- = \varepsilon_h^2/p^+$
$\boldsymbol{p}_T = \boldsymbol{k}_T - \boldsymbol{k}'_T$

quark' = {q', k', $\rho(q')$}



quark = {q, — flavour
k, — 4-momentum
$\rho(q)$} — spin density matrix (2x2, Pauli spinors)

Basic ingredients for the MC simulation — **Lund string Model** — **$^3P_0$ mechanism**

a) Transition Amplitude $\quad T_{q',h,q}(Z, \boldsymbol{p}_T|\boldsymbol{k}_T) \rightarrow$ Splitting in flavour $\otimes$ momentum $\otimes$ spin space

b) Splitting Function $\quad F_{q'hq}(Z, \boldsymbol{p}_T|\boldsymbol{k}_T, \boldsymbol{S}_q) = tr\, T_{q',h,q}\, \rho(\boldsymbol{S}_q)\, T^{\dagger}_{q',h,q} \rightarrow$ Generation of h

c) Density matrix of q' $\quad \rho(\boldsymbol{S}_{q'}) \rightarrow T_{q',h,q}\, \rho(\boldsymbol{S}_q)\, T^{\dagger}_{q',h,q}/Tr[\text{idem}]$

Few free parameters

$a, b_L, \mathrm{b_T} \rightarrow$ string fragmentation dynamics (Lund Model, e.g. PYTHIA)

μ **complex mass** from $^3P_0$ mechanism → **responible for spin effects** ($\mathrm{Im}(\mu) \rightarrow$ transverse)

Input function $\check{g} \rightarrow$ governs spin-independent $\mathbf{k}_\mathrm{T}$ - $\mathbf{k'}_\mathrm{T}$ correlations

correlations → Model **M18** *PRD 97 (**2018**) 7, 074010*

NO correlations → Model **M19** (much simpler) *PRD100 (**2019**) no.1, 014003*

Elementary splitting with VM emission (M20)

AK, X. Artru, A. Martin
arXiv: 2109.06124 [hep-ph]

see talk by X. Artru

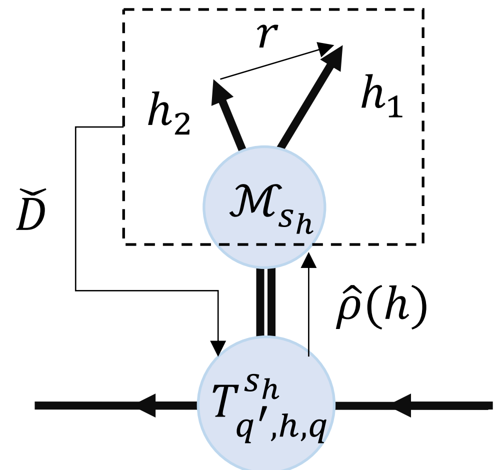
ρ, K^*, ω, ϕ

inv. mass

spin matrix

hadron {h, M_h, Z, $\boldsymbol{p}_T$, $\hat{\rho}(h)_{3\times 3}$ }

quark' $\{q', k', \rho(\boldsymbol{S}_{q'})\}$



quark $\{q, k, \rho(\boldsymbol{S}_q)\}$

NEW recipe (more complicated than PS) *more details also in backup*

Introduce **NEW Transition Amplitude** $T^{s_h}_{q',h,q}(M_h, Z, \boldsymbol{p}_T, s_h|\boldsymbol{k}_T)$

→ Generate h using the NEW Splitting Function

→ Transfer spin information to h [density matrix $\hat{\rho}(h)$]

→ Decay of h (e.g. VM→ $h_1 h_2$, VM→ $h_1 h_2 h_3$) [use $\hat{\rho}(h)$]

→ Bring decay information back to q' [decay matrix $\breve{D}$]

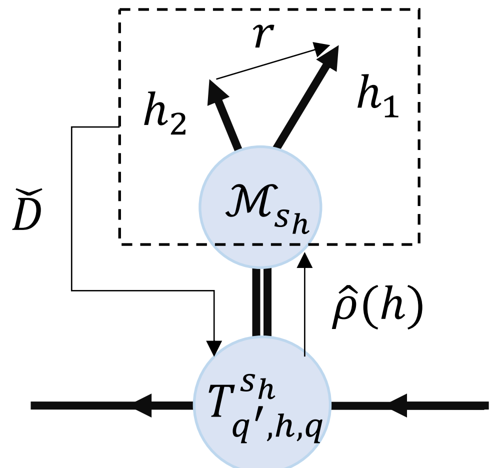
→ Transfer spin information to q' [density matrix $\rho(\boldsymbol{S}_{q'})$]

Based on the Collins-Knowles recipe *(Collins '88, Knowles '88)*

→ accounts for correlations between polarization of q' ↔ momenta of decay hadrons

elementary splitting: emission of a VM

AK, X. Artru, A. Martin
arXiv: 2109.06124 [hep-ph]



ρ, K^*, ω, ϕ

inv. mass

spin matrix

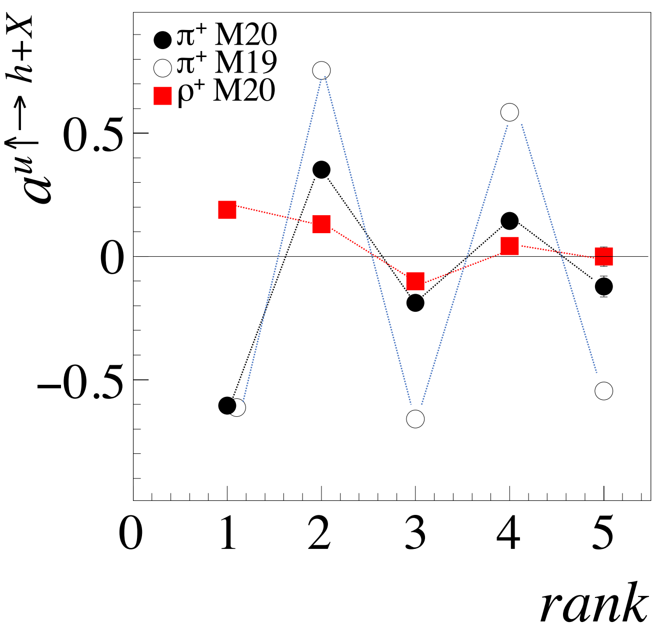
hadron {h, M_h, Z, $\boldsymbol{p}_T$, $\hat{\rho}(h)_{3\times 3}$ }

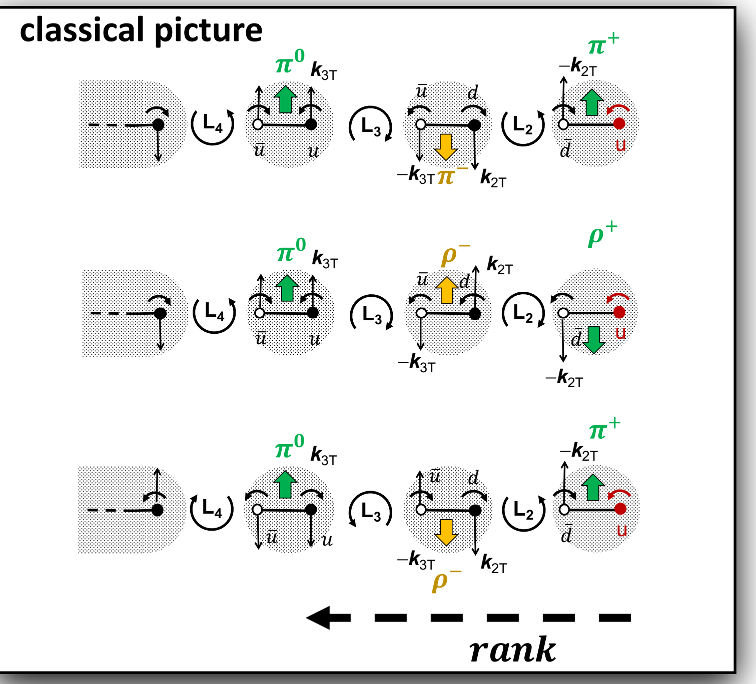
quark' {q', k', $\rho(\boldsymbol{S}_{q'})$}

quark {q, k, $\rho(\boldsymbol{S}_q)$}

NEW Transition Amplitude	$T^{s_h}_{q',h,q}(M, Z, \boldsymbol{p}_T, s_h \vert \boldsymbol{k}_T)$
Splitting Probability	
→ **NEW Splitting Function**	$F_{q'hq}(M, Z, \boldsymbol{p}_T \,\vert \boldsymbol{k}_T, \boldsymbol{S}_q) = tr\, T^{s_h}_{q',h,q}\rho(\boldsymbol{S}_q)T^{s_h\,\dagger}_{q',h,q}$
Spin transfer to h	
→ spin density matrix of h	$\hat{\rho}_{s_h s_h'}(h) \propto tr\, T^{s_h}_{q',h,q}\,\rho(\boldsymbol{S}_q)\,\mathrm{T}^{s_h'\dagger}_{q',h,q}$
Decay (e.g. VM→ $h_1 h_2$)	
→ angular distribution	$dN/d\Omega \propto \mathcal{M}_{s_h}\,\hat{\rho}_{s_h s_h'}\mathcal{M}^{\dagger}_{s_h'}$
Bring decay information back to q'	
→ decay matrix	$\breve{D}_{s_h' s_h} = \mathcal{M}^{\dagger}_{s_h'}\mathcal{M}_{s_h}$
Transfer spin information to q'	
→ spin density matrix of q'	$\rho(\boldsymbol{S}_{q'}) = \breve{D}_{s_h s_h'}\,T^{s_h'}_{q',h,q}\rho(\boldsymbol{S}_q)\mathrm{T}^{s_h\dagger}_{q',h,q}$

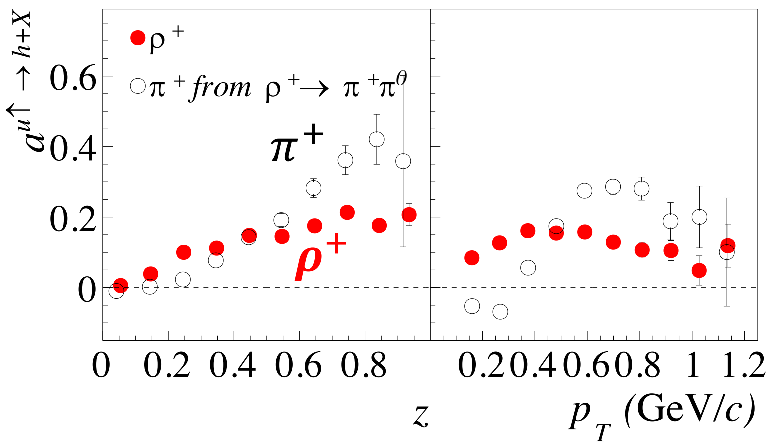
Collins analysing power as function of rank

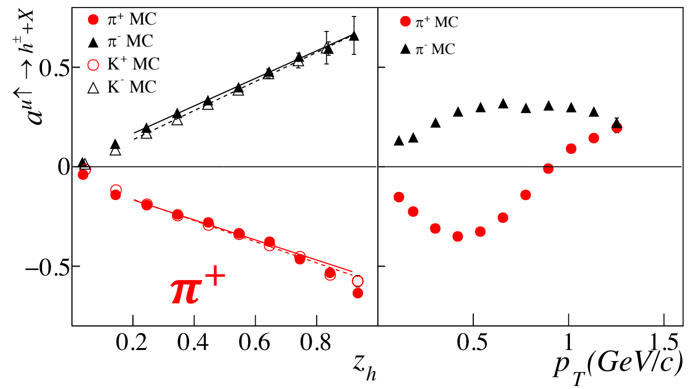




- classical picture reproduced
  ρ^+ have opposite effect w.r.t π^+
- quark spin information decays along the chain
  faster decay in M20

Collins analysing power for ρ and decay π





PRD97 (2019) no.7, 074010

ρ^+ analysing power
- opposite to π^+
- ~ 3 times smaller than π^+

as expected from the M20 prediction

model M19 → only PS

$$\left.\frac{a^{u\uparrow\to\rho+X}}{a^{u\uparrow\to\pi+X}}\right|_{rank=1} = -\frac{|G_L|^2}{2|G_T|^2+|G_L|^2}$$

decay π^+ → larger analysing power w.r.t ρ^+ at large z_h
different trend as function of p_T