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THE STRING+$^3P_0$ MODEL OF HADRONIZATION

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GOALS:

- Generalize the String Fragmentation Model (used in PYTHIA) by including the **spin degree of freedom** and implement it in a **Monte Carlo code**.
- We assume the **$^3P_0$ mechanism** of quark pair creation in the string breaking.
- We generate *pseudo-scalar* mesons and *vector* mesons with their polarizations.
- We generate the VM (anisotropic) decays.
- We respect **LR symmetry** (LR for ``**L**eft-**R**ight" or ``Quark ***L**ine **R**eversal*").

METHODS:

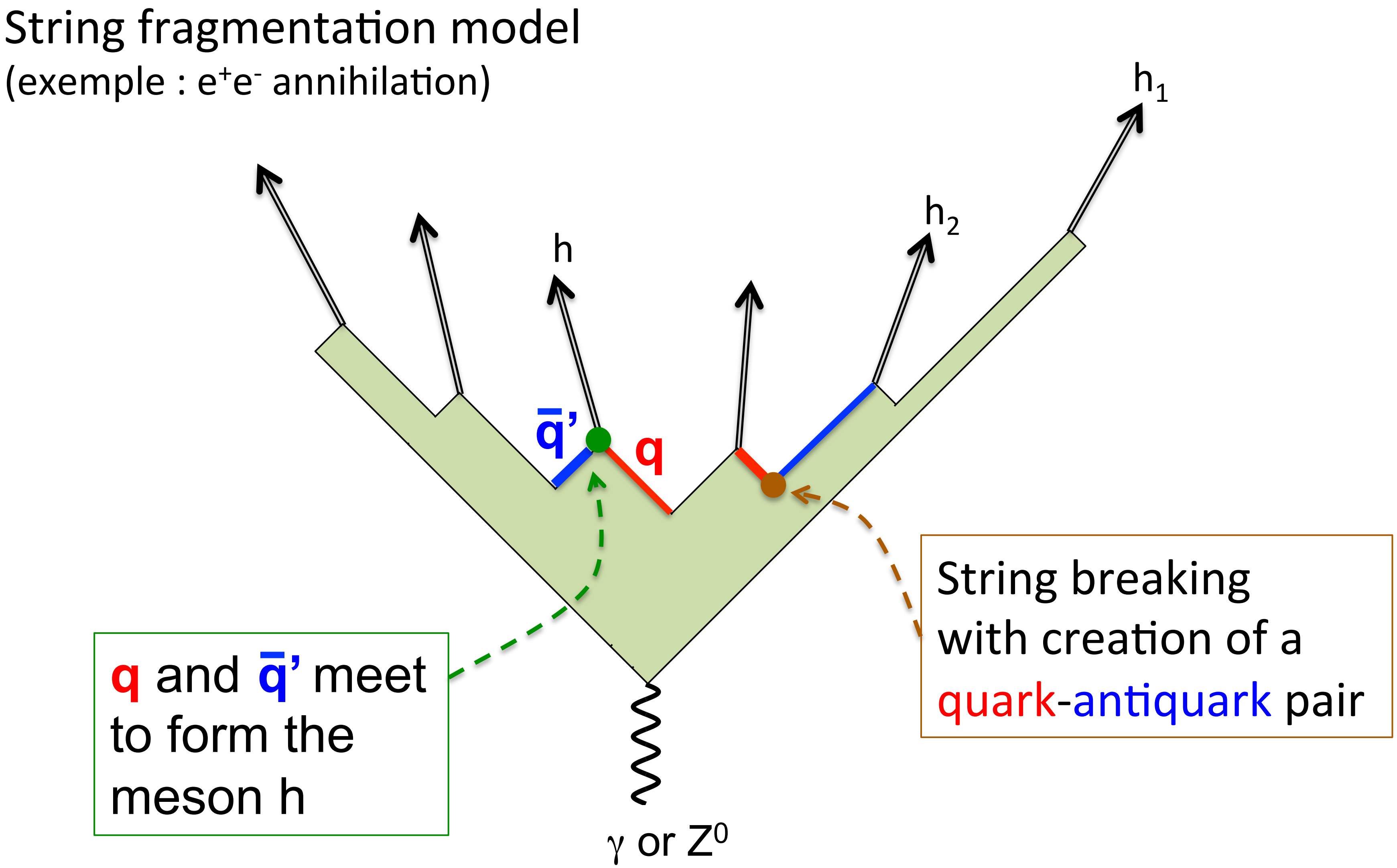
- We re-formulate the String Fragmentation Model as a *multiperipheral* [*] *model*, but with *quark* (instead of meson) exchanges.
- The basic objects are **quark propagators Δ**(k) and **quark-hadron vertices Γ**(k',p,k). Quark spinors are reduced to **Pauli** spinors. Δ(k) and Γ(k',p,k) are 2×2 matrices.
- To the recursive splitting process q(k) → h(p) + q'(k') we associate the *amplitude*

$$\mathbf{T}(p,k) = \Delta(k')\,\Gamma(k',p,k)\;;$$

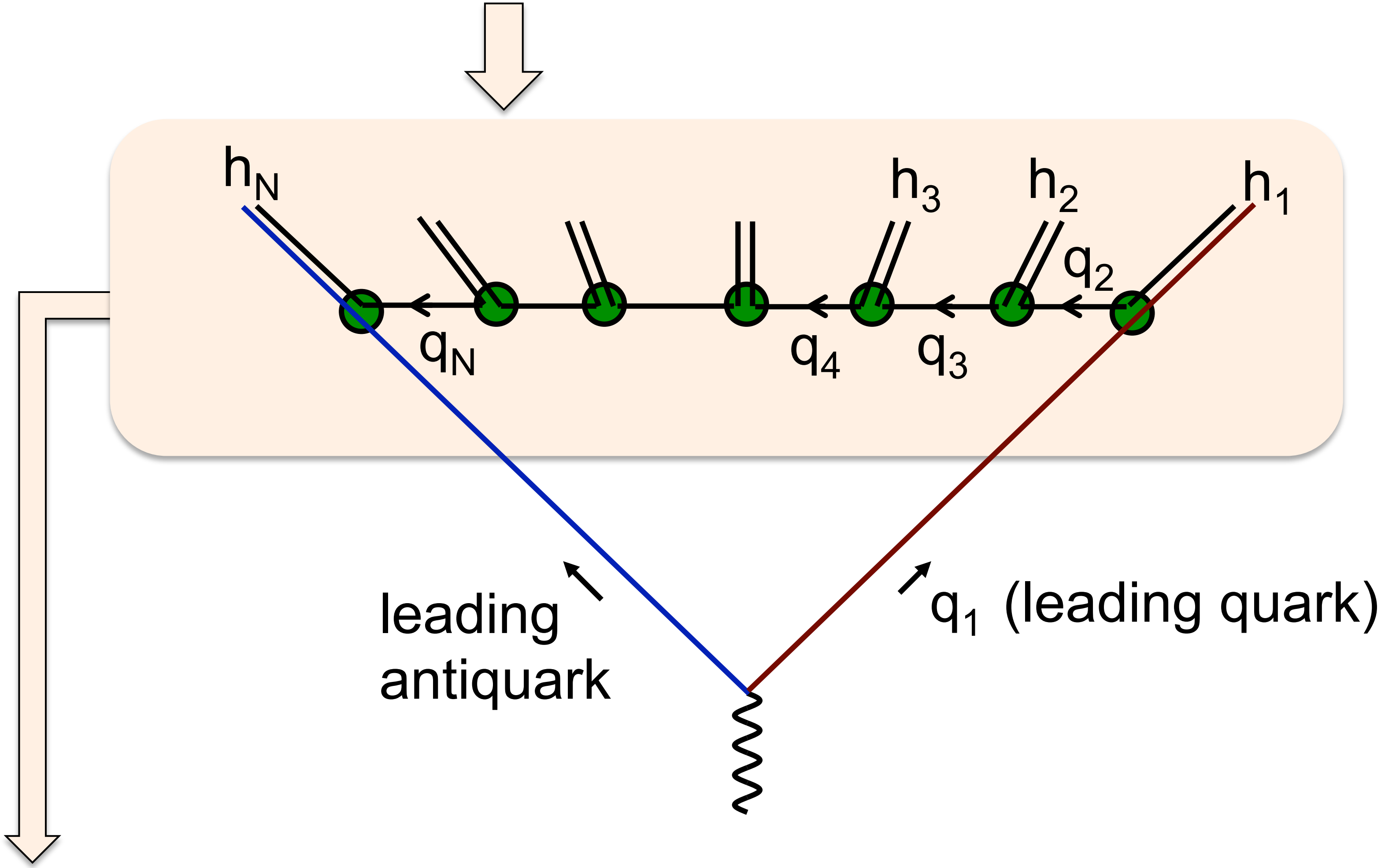
The *splitting function* is F(p,k) = trace {**T** ρ(q) **T**†},
where ρ(q) = ½ [1+ **σ.S**(q)] is the density matrix of q.

- The fact that we start with *amplitudes*, not probabilities, guaranties the quantum **positivity** and the **entanglement** between quark spin and vector meson spin.

[*] multiperipheral model: Amati, Fubini, Stanghelini (1962)

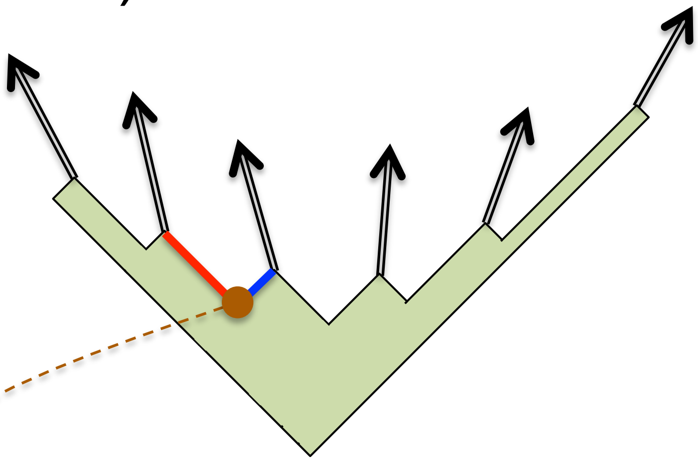

String fragmentation model
(exemple : e+e- annihilation)
h1
h2
h
q̄'
q
String breaking with creation of a quark-antiquark pair
q and q̄' meet to form the meson h
γ or Z0

Associated multiperipheral model (with quark exchanges)



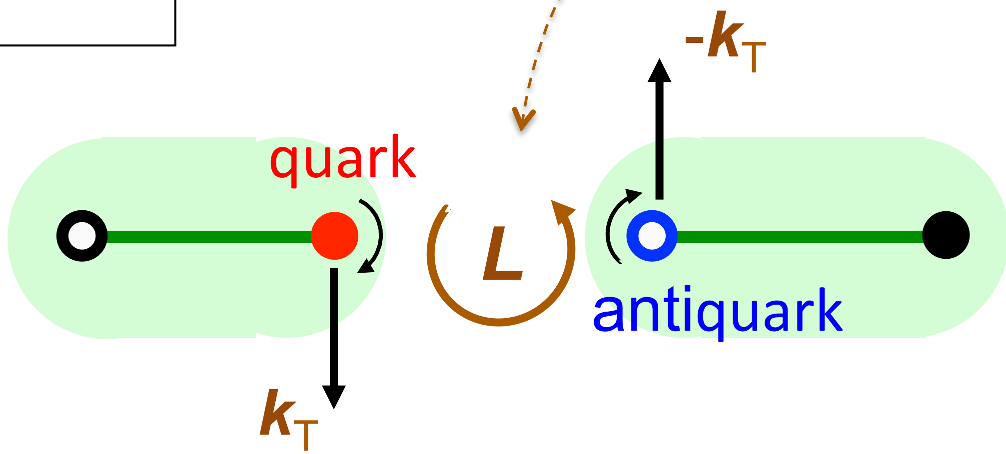
- considered as a *recursive quark cascade* [Lund, Bowler]
- LR symmetry is a non-trivial constraint (not satisfied by Feynman-Fields)

The $^3P_0$ mechanism (classical approach)



Hypothesis : the quark-antiquark pair is created in the $0^{++} = {}^3P_0$ state and zero total 4-momentum (vacuum quantum numbers)

$$\Rightarrow \quad \begin{array}{l} L + s_{\bar{q}} + s_q = 0 \\ s_{\bar{q}} = s_q \end{array}$$

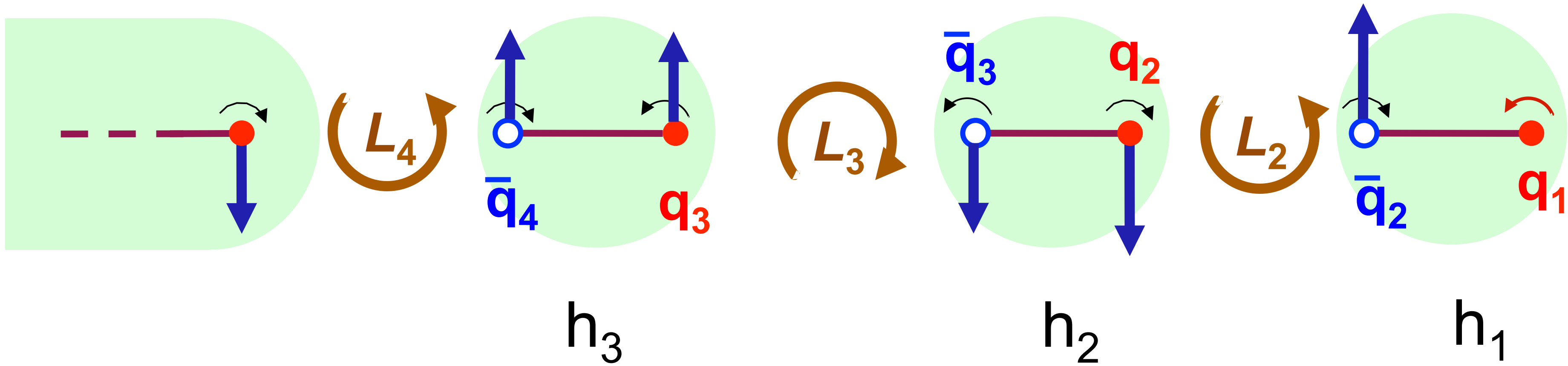


→ Local Compensation of Transverse Momentum

→ azimuthal correlation between k_T and **S**(quark)

= source of Collins effect

Iteration of the $^3P_0$ mechanism :
exemple with only *pseudoscalar* mesons (π, K, η^0)



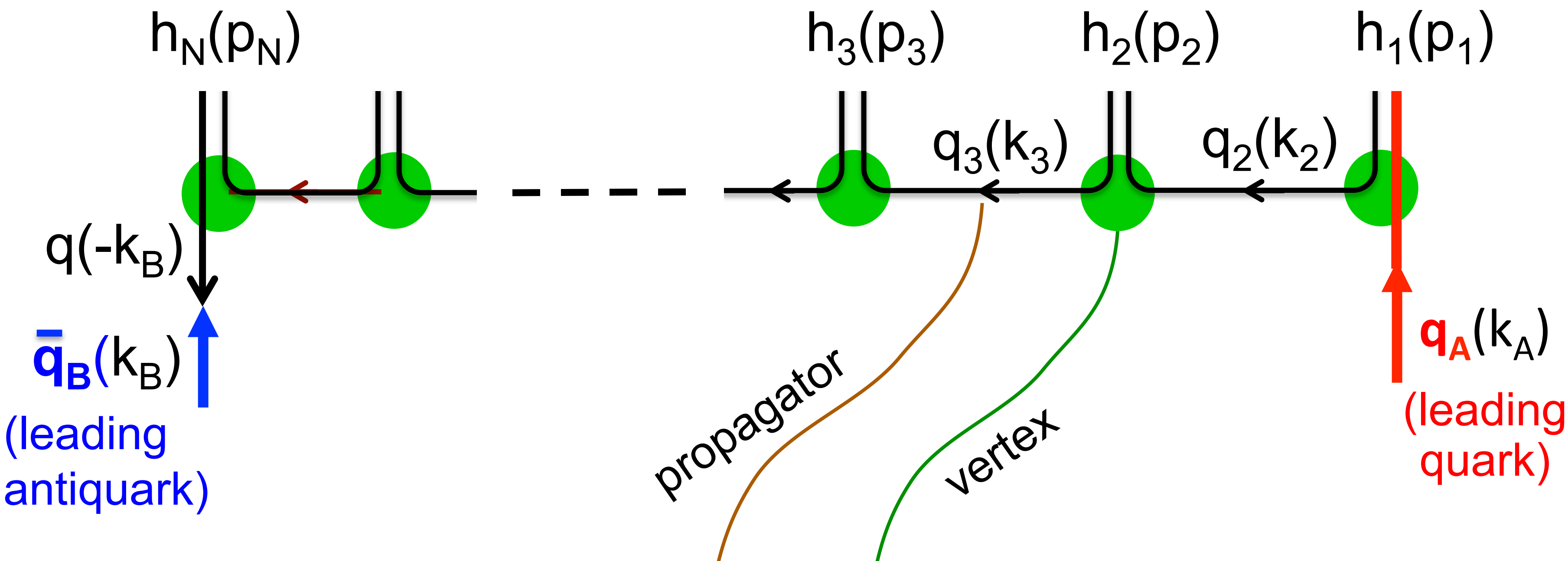
- quark spins are correlated two-by-two

  → propagates the spin information along the quark chain
- Collins effects are alternate in rank
- large Collins effect for the 2nd-rank (*unfavored*) meson,
- large *dihadron asymmetry* or *Relative* Collins Effect

} OK with exper^t

> *This description is classical ! Let us look for a quantum one*

Quantum approach : the multiperipheral model



Amplitude :

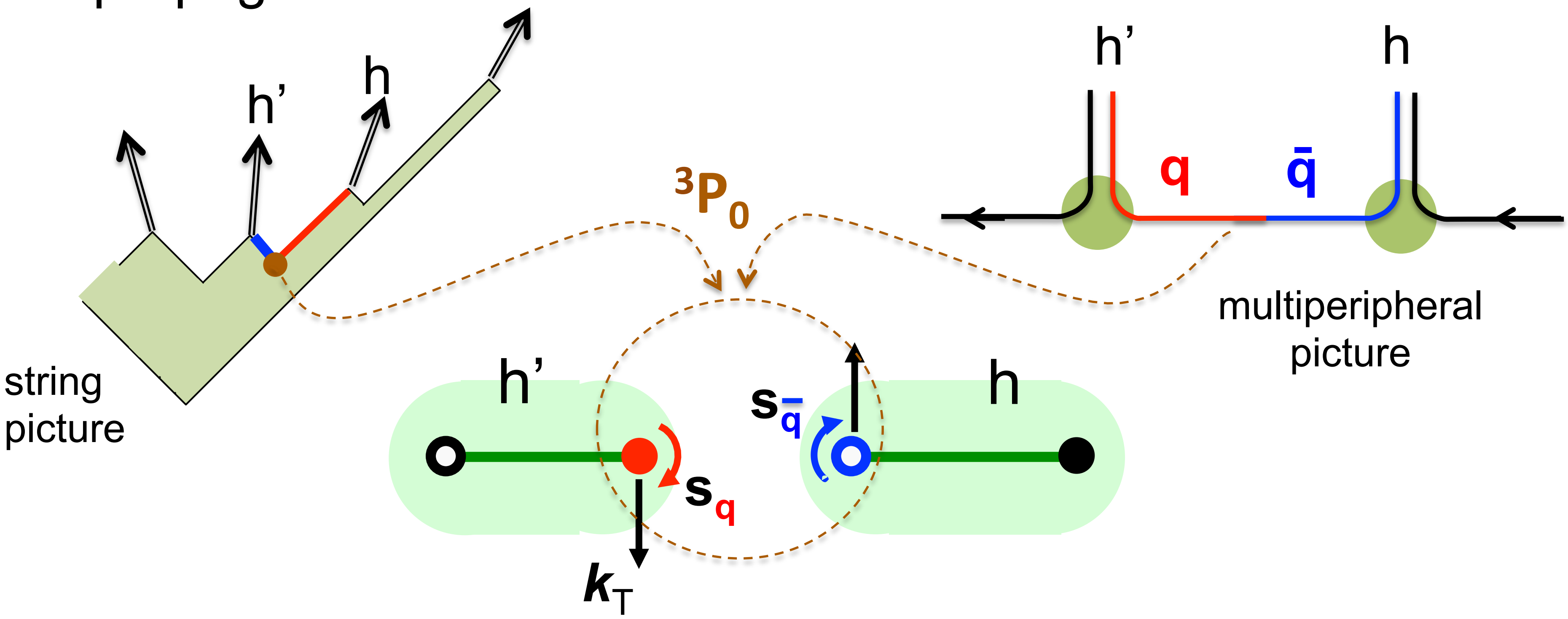
$$\Gamma(k_{N+1},p_N,k_N)\ \Delta(k_N)\ \ldots\ldots\ \Delta(k_3)\ \Gamma(k_3,p_2,k_2)\ \Delta(k_2)\ \Gamma(k_2,p_1,k_1)$$

sandwiched between **Pauli** spinors $\chi^\dagger(-\mathbf{S}_B)\,\sigma_z$ and $\chi(\mathbf{S}_A)$

analogue of the Dirac spinor $v^\dagger(-k,\mathbf{S}')$

Spin part of the propagator.

It correlates $\mathbf{s}_{\bar{q}}$ in h and $\mathbf{s}_q$ in h'



$^3P_0$ wave function : $\chi^\dagger(-\mathbf{S}_q)\,\sigma_z\ \ \boldsymbol{\sigma}.\mathbf{k}\ \ \chi(\mathbf{S}_{\bar{q}})$

During tunneling k_z is not fixed, but imaginary: $k_z \sim \mathbf{i}\,(m_q^2 + \mathbf{k}_T^2)^{1/2}$.

We replace it, phenomenologically, by a *complex mass parameter* μ, with $\mathrm{Im}(\mu) > 0$

$\rightarrow$ The spin propagator is $\sigma_z\,\boldsymbol{\sigma}.\mathbf{k} = \boxed{\mu + \sigma_z\,\boldsymbol{\sigma}.\mathbf{k}_T}$

(analogue of $m+\gamma.p$)

The vertex matrix.

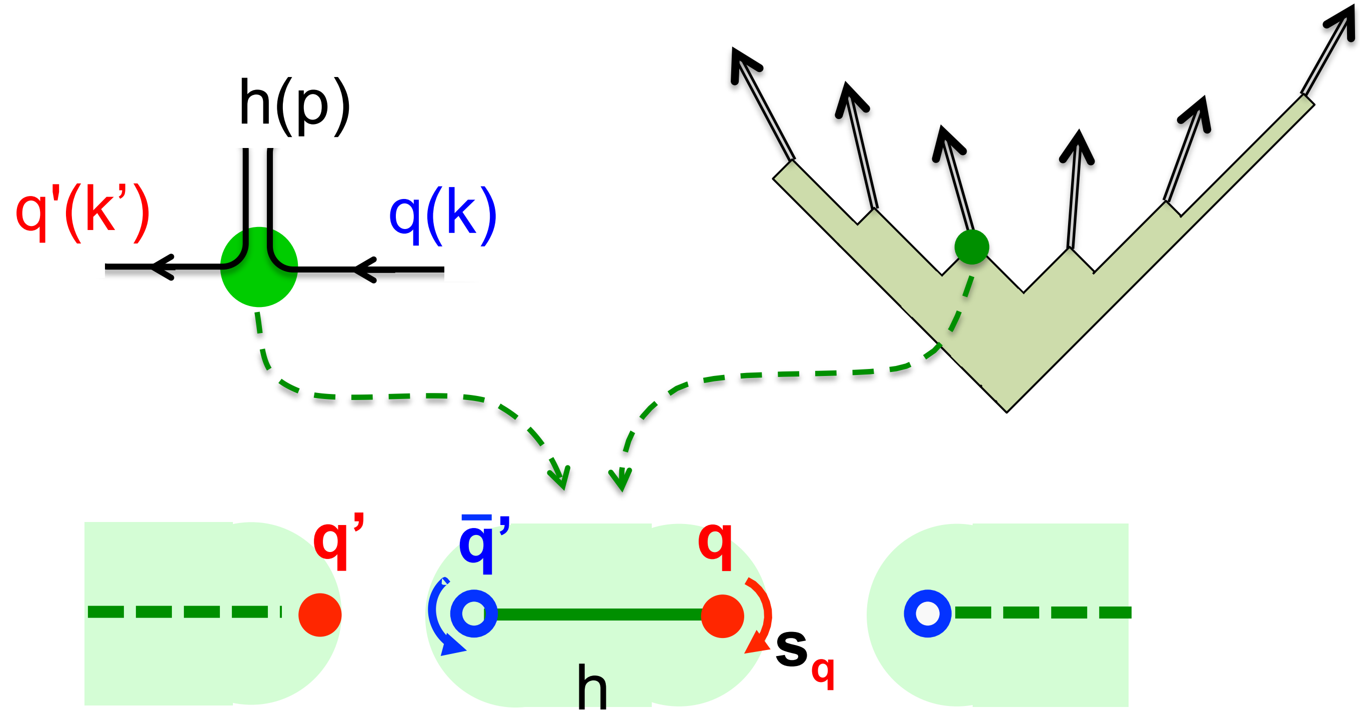
It correlates $\mathbf{s}_q$ and $\mathbf{s}_{\bar{q}'}$ in the same hadron

1) pseudoscalar meson

↓

spin singlet

$\mathbf{s}_q = - \mathbf{s}_{\bar{q}'}$



The spin part of the vertex is $\Gamma(k',p,k) = \sigma_z$

⇩

analogue of γ_5

The vertex. 2) vector meson (ρ, ω, ϕ)

- it depends on the vector amplitude **V** of the VM
- Parity and LR symmetry

$$\Gamma(k',p,k) = G_L V^*_z + G_T \mathbf{V^*.\sigma}_T \sigma_z$$

(analogues of $\gamma_\mu V^\mu$ and $\sigma_{\mu\nu} V^\mu p^\nu$ couplings)

G_L and G_T can be complex. New relevant parameters are :

$|G_L|^2 + 2 |G_T|^2$ → abundance of vector mesons

$|G_L/G_T|^2$ = relative abundance longitudinal/transverse VM

$\mathrm{Arg}(G_L/G_T)$ → **oblique** polarization of the VM

= source of dihadron transverse spin asymmetry

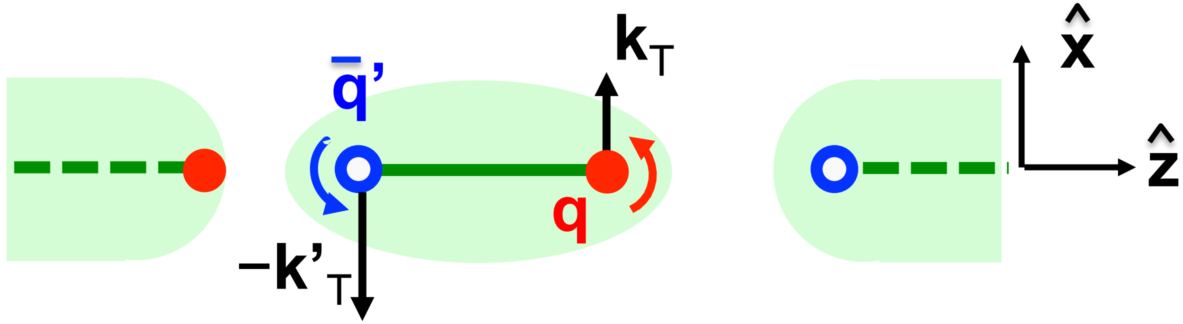
Effects linked to **V**

Take **V** real (linear polarization) and $\mathbf{S}_q = \hat{y}$ (spining anticlockwise)

a) **V** = $\hat{x}$ or $\hat{z}$

$s_y(VM) = \pm 1$

$s_y(\bar{q}') = s_y(q) = \pm ½$



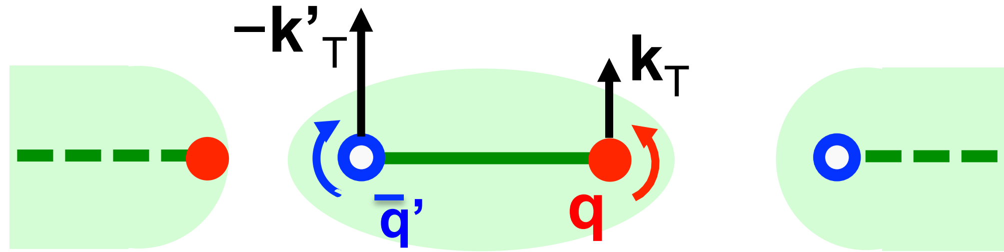
➡ VM of rank > 1: **smaller** $\langle p_T^2 \rangle$ compared to pseudoscalars.

VM of rank 1: Collins effect **opposite** to that of pseudoscalar

b) **V** = $\hat{y}$

$S_y(VM) = 0$

$s_y(\bar{q}') = - s_y(q)$



➡ Same $\langle p_T^2 \rangle$ and Collins effect as pseudoscalars (for all ranks)

- The *average* Collins effect is opposite to that of pions ,[J. Czyzewski 1996]

The full splitting amplitude

$\mathbf{T}(p,k) = \boldsymbol{\Delta}(k') \, \boldsymbol{\Gamma}(k',p,k)$

$$= (F_{Lund})^{1/2} \times (\mu + \sigma_z \, \boldsymbol{\sigma}.\mathbf{k'}_T) \times \begin{cases} \sigma_z & \text{(PS meson)} \\ (G_L \, V_z^* + G_T \, \mathbf{V}_T^*.\boldsymbol{\sigma}_T \, \sigma_z) & \text{(V-meson)} \end{cases}$$

⇓

Lund fragmentation function (without spin):

$$F_{Lund}(Z, \mathbf{p}_T) = N \; (1-Z)^a \; \underbrace{\exp\{-b_L(m^2+\mathbf{p}_T^2)/Z\}}_{\text{string area factor}} \; \underbrace{\exp\{-b_T \, \mathbf{k'}_T^2\}}_{\text{tunneling factor}}$$

Z ↓ p^+/k^+

Splitting function and polarization of q'

- The probability of the splitting process q(k) → h(p) + q'(k')
  is the *splitting function*

$$F(Z, \mathbf{p}_T) = \text{trace}\,\{\mathbf{T}\,\rho(q)\,\mathbf{T}^\dagger\},$$

- If h is a *vector meson* $\mathbf{T} = \mathbf{T}_\alpha V_\alpha$ (α = x, y, or z)
  If the VM polarization is not analysed,

$$F(Z, \mathbf{p}_T) = \text{trace}\,\{\mathbf{T}_\alpha\,\rho(q)\,\mathbf{T}_\alpha^\dagger\}$$

- The spin density matrix of q' is

$$\rho(q') = \begin{cases} \mathbf{T}\,\rho(q)\,\mathbf{T}^\dagger\} \,/\, \text{trace}\,\{\mathbf{T}\,\rho(q)\,\mathbf{T}^\dagger\} & \text{if } h = \text{pseudo-scalar meson} \\ \mathbf{T}_\alpha\,\rho(q)\,\mathbf{T}_\alpha^\dagger\} \,/\, \text{trace}\,\{\mathbf{T}_\beta\,\rho(q)\,\mathbf{T}_\beta^\dagger\} & \text{if } h = \textit{not analysed}\ \text{VM} \end{cases}$$

  q' is polarized by spin transfer from q and by $\mathbf{k}'_T$ - $\mathbf{s}_{q'}$ correlations.

- a VM is *automatically analysed* by the directions of its decay products.
  We need to calculate ρ(q') in an other way (second slide below)

Polarized vector meson decay

- The VM is polarized. Its density matrix is, in Cartesian basis

$$\rho_{\alpha\beta}(h) = \mathrm{trace}\{ \mathbf{T}_\alpha\, \rho(q)\, \mathbf{T}^\dagger_\beta \} \qquad\qquad (\text{recall } \mathbf{T} = \mathbf{T}_\alpha V_\alpha)$$

$\downarrow$

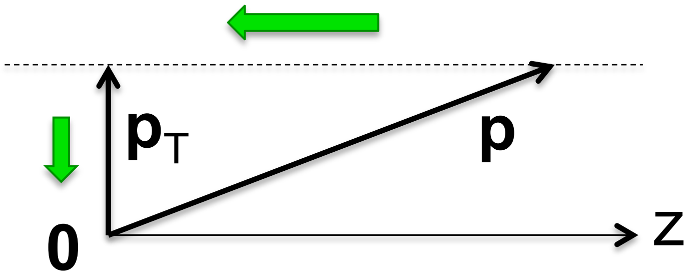
(summation over the spin of q')

$\rightarrow$ Its decay is anisotropic (unlike in PYTHIA).
Let **r** be the relative momentum of the decay products
and $\langle \mathbf{r} |\, M | \alpha \rangle$ the decay amplitude. The angular distribution of **r** is

$$\langle \mathbf{r} | M | \alpha \rangle\, \rho_{\alpha\beta}\, (h)\, \langle \beta | M^\dagger | \mathbf{r} \rangle$$

- **The question of frame :** **r** is measured in the VM rest frame. But different Lorentz transformations can bring the VM at rest, hence different "frames" (*helicity frame*, *Jackson frame*, etc), differing by Wigner rotations.

LR symmetry imposes the composition of two boosts :



Polarization of q' following a VM decay

- the spins of the VM and of q' are *entangled*.

→ The polarization of q' depends on the directions of the decay products

→ In the Monte Carlo code, we **cannot** treat ***separately*** the decay of the VM and the fragmentation of q'.

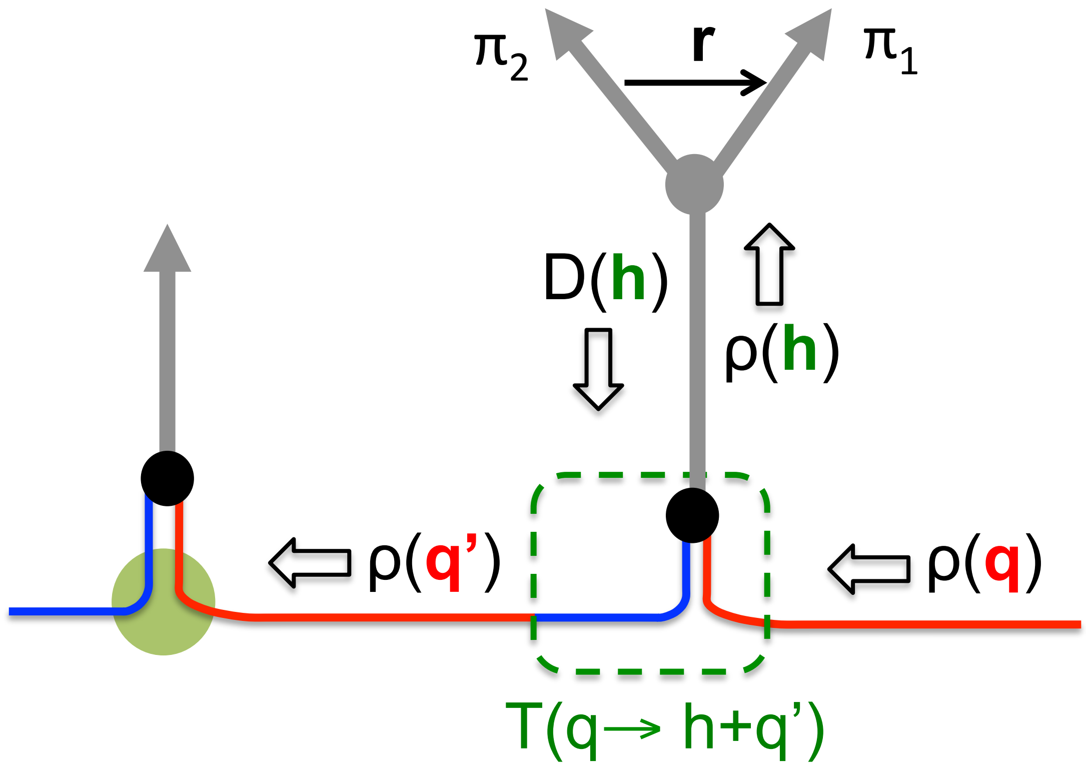
We must use the *algorithm of Collins and Knowles* (1988) :

1) calculate $\rho_{\alpha\beta}(h) = \text{trace}\{ \mathbf{T}_\alpha \, \rho(q) \, \mathbf{T}^\dagger_\beta \}$
2) generate the directions $\mathbf{r}$ of the decay products with the angular distribution $\langle \mathbf{r}|M|\alpha\rangle \, \rho_{\alpha\beta}(h) \, \langle\beta|M^\dagger|\mathbf{r}\rangle$

(done last slide) [steps 1–2]

3) calculate the *decay matrix* $D_{\beta\alpha}(h) = \langle\beta|M^\dagger|\mathbf{r}\rangle \, \langle \mathbf{r}|M|\alpha\rangle$
4) deduce $\rho(q') = D_{\beta\alpha}(h) \, \mathbf{T}_\alpha \, \rho(q) \, \mathbf{T}^\dagger_\beta$

Flow of the spin information in VM decay



- $D_{\beta\alpha}$ carries the information about the direction of **r** "**backward in time**", from the decay event to the splitting event (this cannot be understood with classical causality !)

Main conclusions

- We extended the Lund model of string fragmentation by including the spin degree of freedom in accordance with the rules of quantum mechanics.
- The $^3P_0$ mechanism, in agreement with experimental Collins asymmetry, is implemented by the factor $\mu + \sigma_z\, \boldsymbol{\sigma}.\mathbf{k}_T$ of the quark propagator.
- Vector mesons have been included. They have opposite Collins effect w.r.t. pions, in accordance with Czyzewski's prediction, and smaller $\langle \mathbf{p}_T^2 \rangle$ ("hidden spin" effect).
- A complex value of the parameter G_L/G_T leas to an *oblique* polarization of the vector mesons, resulting in a dihadron spin asymmetry.
- In the Monte Carlo code, the anisotropic decay of a polarized VM must be generated **before** continuing the recursive quark fragmentation (Collins and Knowles algorithm). Indeed, due to a quantum entanglement, the polarization of the leftover quark depends on the directions of the decay products.
- The translation of this model in a Monte Carlo code and the comparison with experimental data is presented by Albi Kerbizi.

Thank you !!!

Last reference : A. Kerbizi, X. Artru and A. Martin, arXiv:2109.06124, submitted to Phys. Rev. D