18-22 octobre 2021

THE STRING+$^3P_0$ MODEL OF HADRONIZATION

Xavier Artru

Institut de ***P***hysique des **2** ***I***nfinis de Lyon

Université de Lyon and CNRS (France)

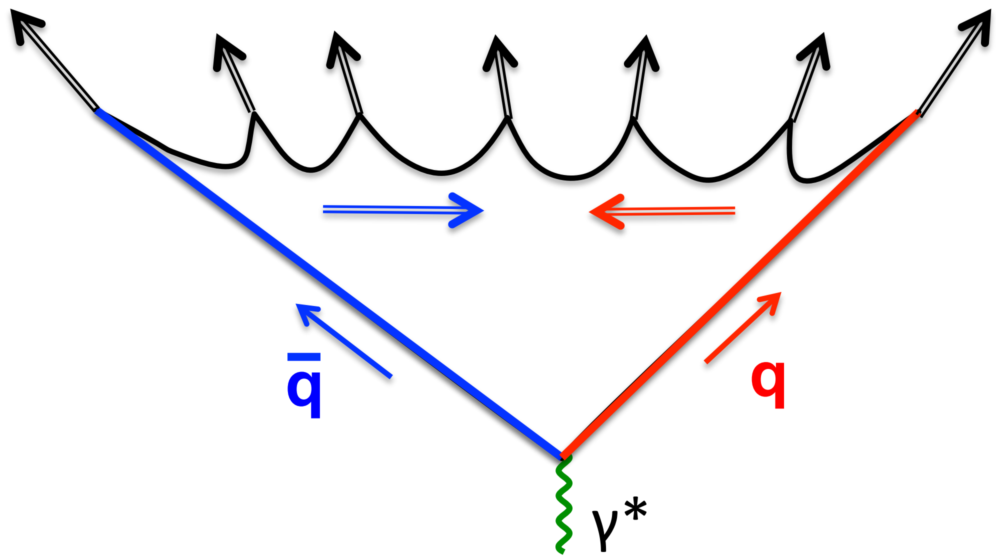
In collaboration with Albi kerbizi and Anna Martin

GOALS (reached)

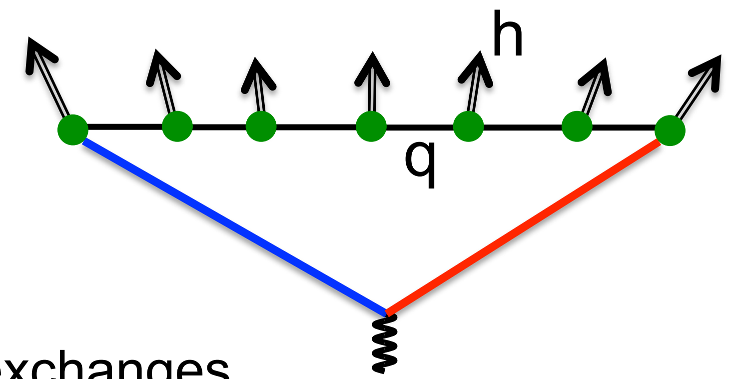
- Include the **spin degree of freedom** in the String Fragmentation Model and implement it in a **Monte Carlo code** for polarized quark jets. It can serve as a guide in **quark polarimetry** [X.A. and Z. Belghobsi 2013]
- Quantize the **$^3P_0$ mechanism** of quark pair creation in the string breaking.
- Generate *pseudo-scalar* mesons and *polarized vector* mesons.
- Generate the *anisotropic* decays of the VM.
- Respect the so-called "**L**eft-**R**ight" **symmetry**

(I prefer : "symmetry of Quark **L**ine **R**eversal")

[works by X.A. and Z. Belghobsi, Dubna-spin'11 and '13]

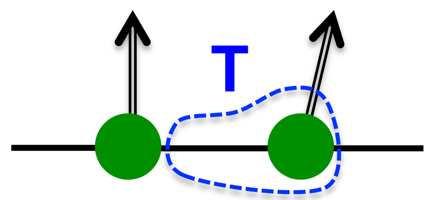


METHODS



- The String Fragmentation Model is reformulated as a *multiperipheral model* [*] , but with *quark exchanges* instead of meson exchanges.
- Quark spinors are reduced to **Pauli** spinors (α_z = +1 subspace)
  Quark polarization ↔ density matrix $\rho(q) = \frac{1}{2}\,(1+ \boldsymbol{\sigma}.\mathbf{S}_q)$;
- The basic objects are **quark propagators Δ**(k)
  and **quark-hadron vertices Γ**(k',p,k) , which are 2×2 matrices.
- *Splitting amplitude* of the *recursive process* $q \to h + q'$:

$$\mathbf{T}(p,k) = \Delta(k')\, \Gamma(k',p,k)$$

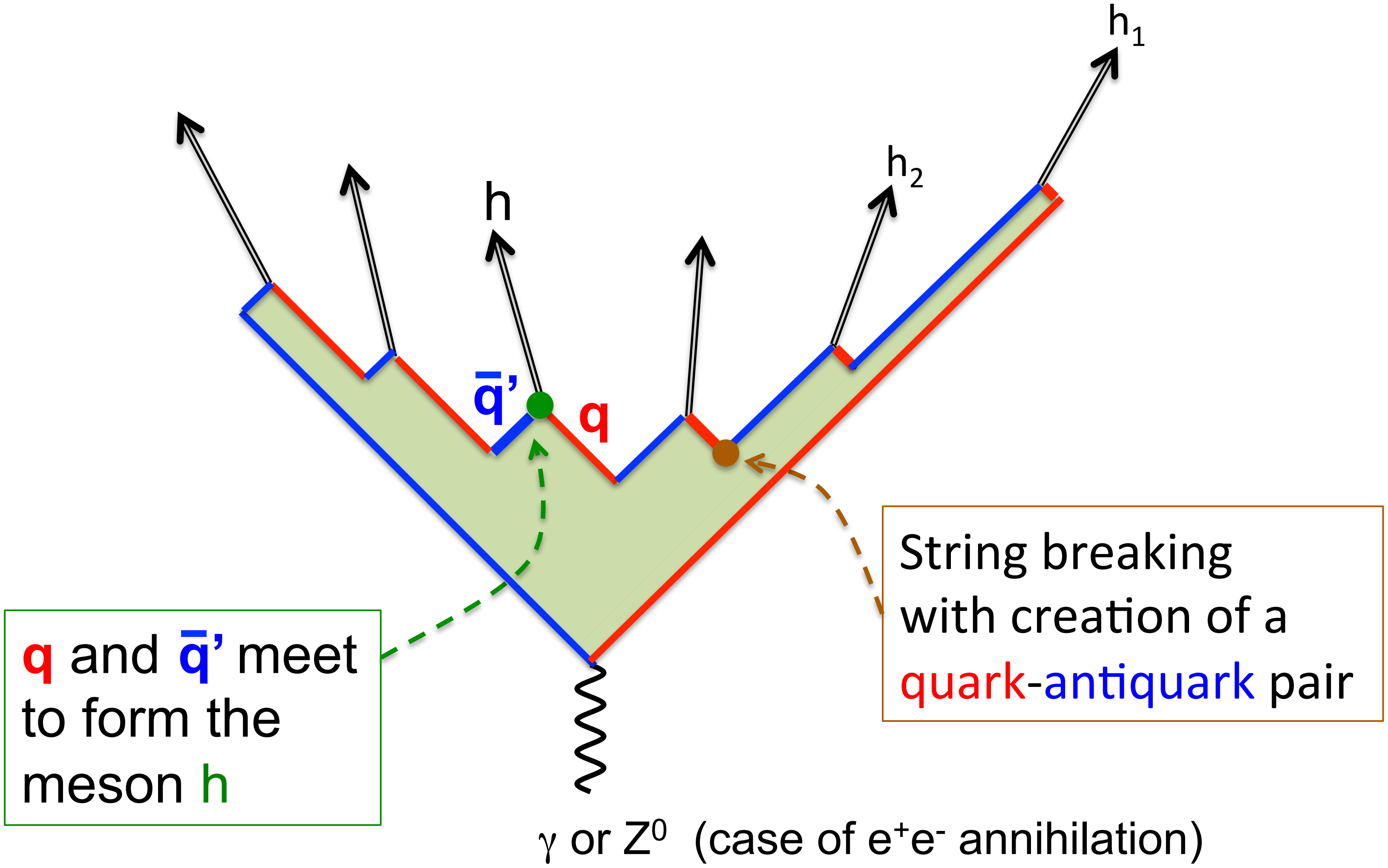


Splitting *function* : $F(p,k) = \text{trace}\,\{\mathbf{T}\, \rho(q)\, \mathbf{T}^\dagger\}$

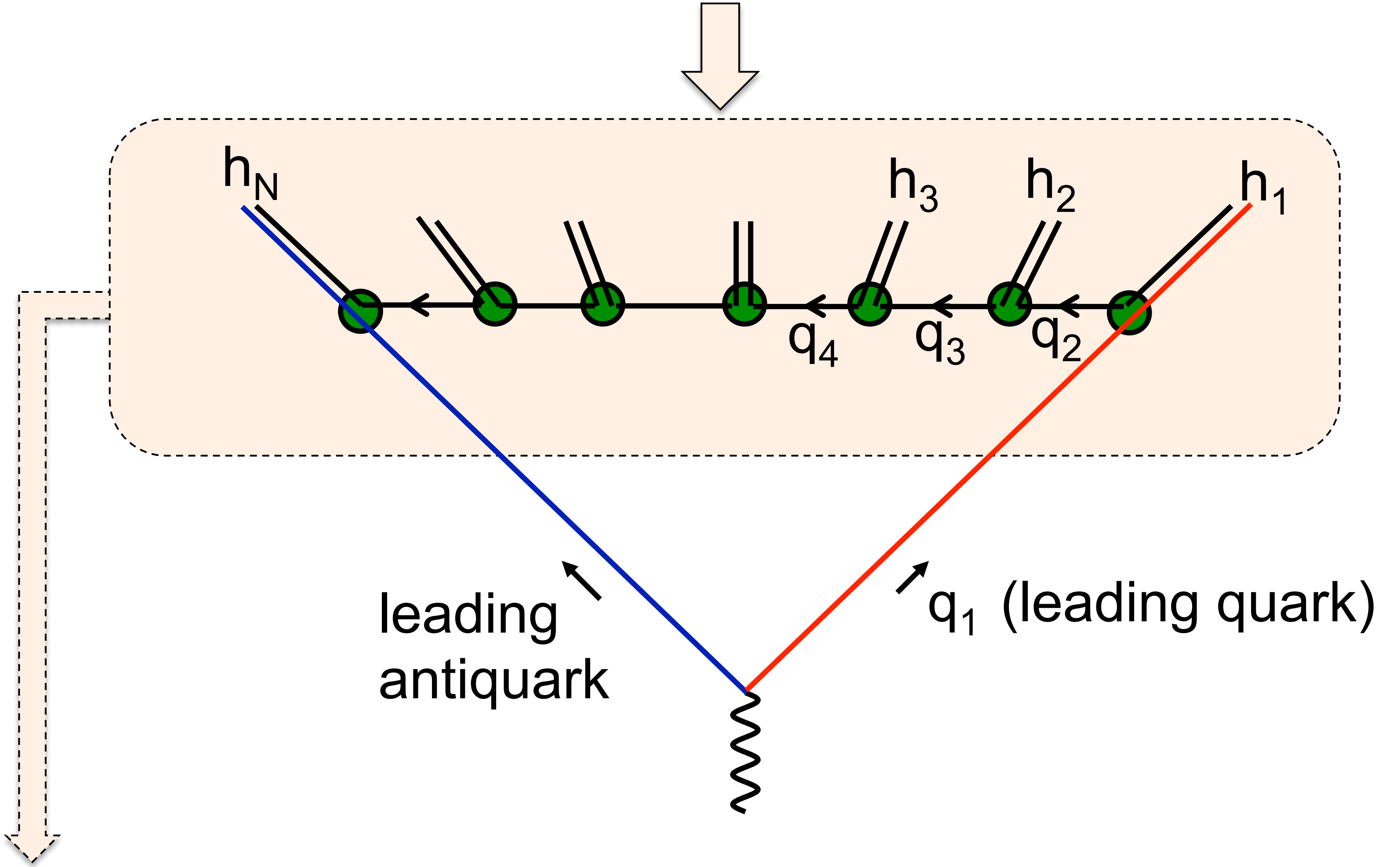
- The fact that we start with *amplitudes*, instead of probabilities guaranties the quantum properties of **positivity** and **entanglement**.

[*] *multiperipheral model* : Amati, Fubini, Stanghelini (1962)

Recall about the String Fragmentation Model

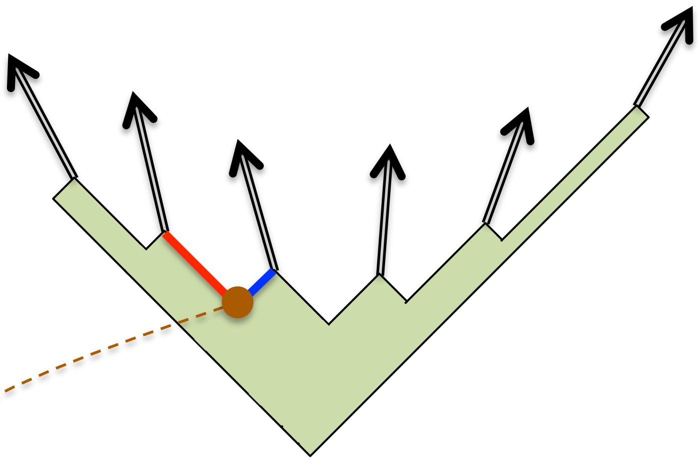


Associated multiperipheral model with quark exchanges



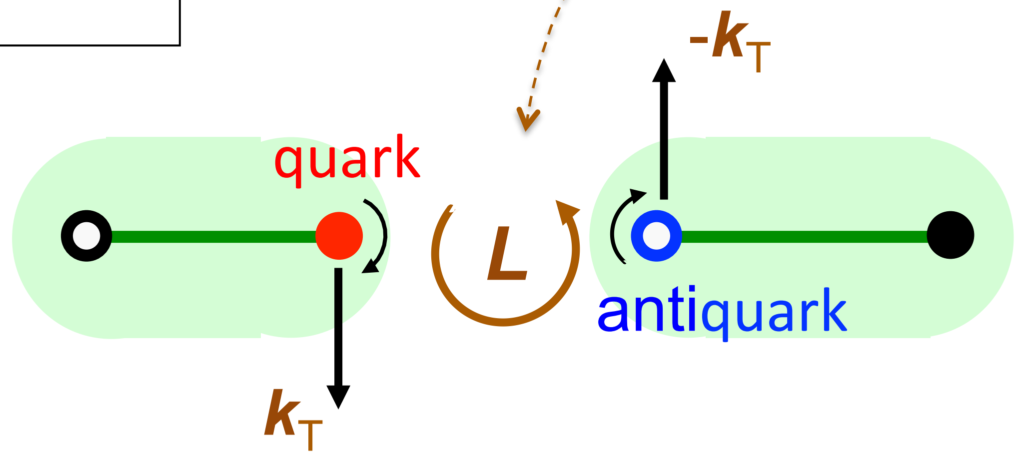
- viewed as a *recursive quark cascade* [Lund, Bowler]
- LR symmetry is a non-trivial constraint (not satisfied by Feynman-Fields)

The ${}^3P_0$ mechanism. 1) classical approach



Hypothesis : the quark-antiquark pair is created in the 0^{++} = ${}^3\mathbf{P}_0$ state and zero total 4-momentum (vacuum quantum numbers)

$$\Rightarrow \boxed{\begin{array}{l} L + \mathbf{s}_{\bar{q}} + \mathbf{s}_q = 0 \\ \mathbf{s}_{\bar{q}} = \mathbf{s}_q \end{array}}$$



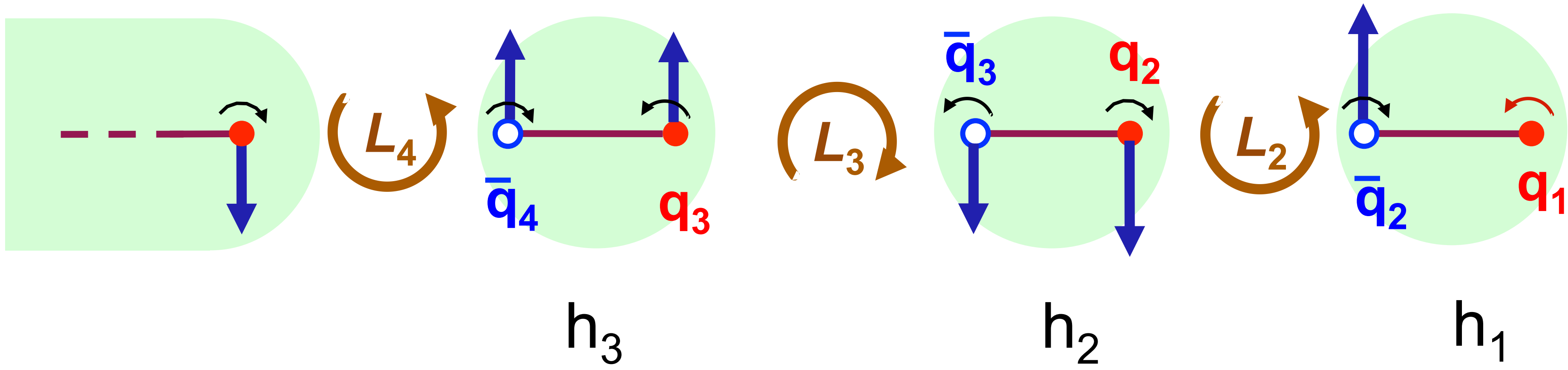
$\longrightarrow$ Local Compensation of Transverse Momentum

$\longrightarrow$ azimuthal correlation between k_T and $\mathbf{s}_q$ = source of Collins effect.

[Similarity with model by Matevosyan, Kotzinian and Thomas (2017)]

Iteration of $^3P_0$

Case with only *pseudoscalar* mesons (π, K, η^0)



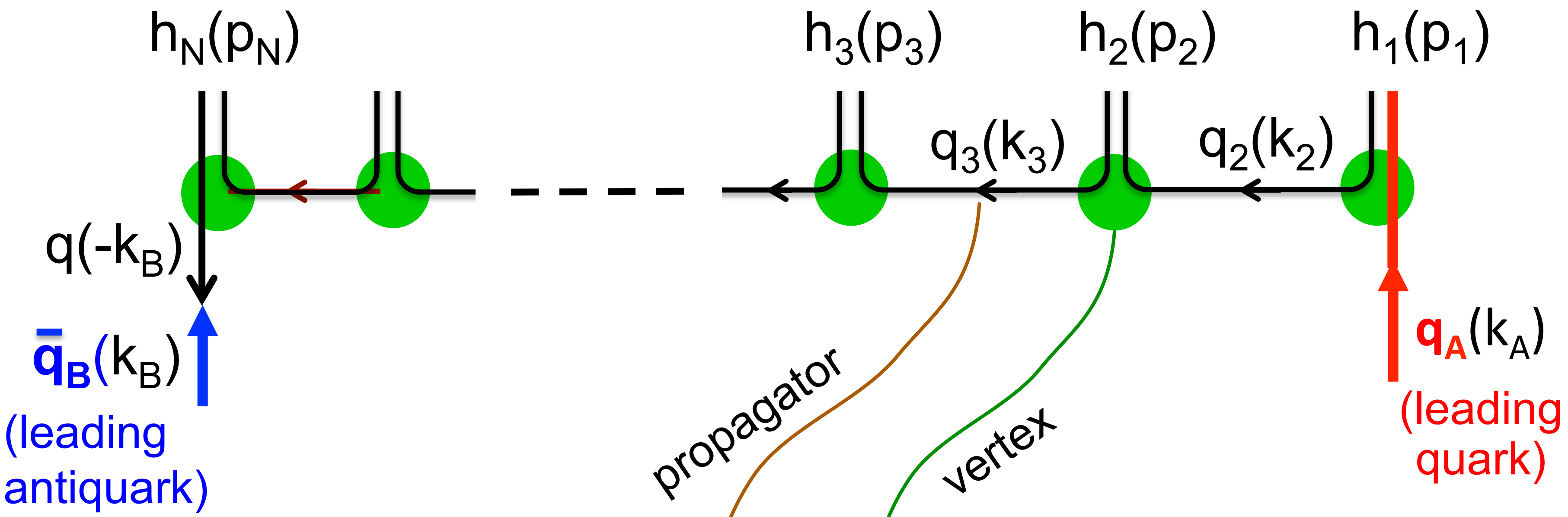
- quark spins are correlated two-by-two

  → propagates the spin information along the quark chain (with some loss)
- Collins effects are alternate in rank
- large positive Collins effect for the *unfavored* meson
- large *dihadron asymmetry* or *Relative Collins Effect*

$\left.\begin{array}{l} \\ \\ \end{array}\right\}$ OK with experiment

> *This description is classical ! Let us look for a quantum one*

Quantum approach : the multiperipheral model



Multiperipheral amplitude :

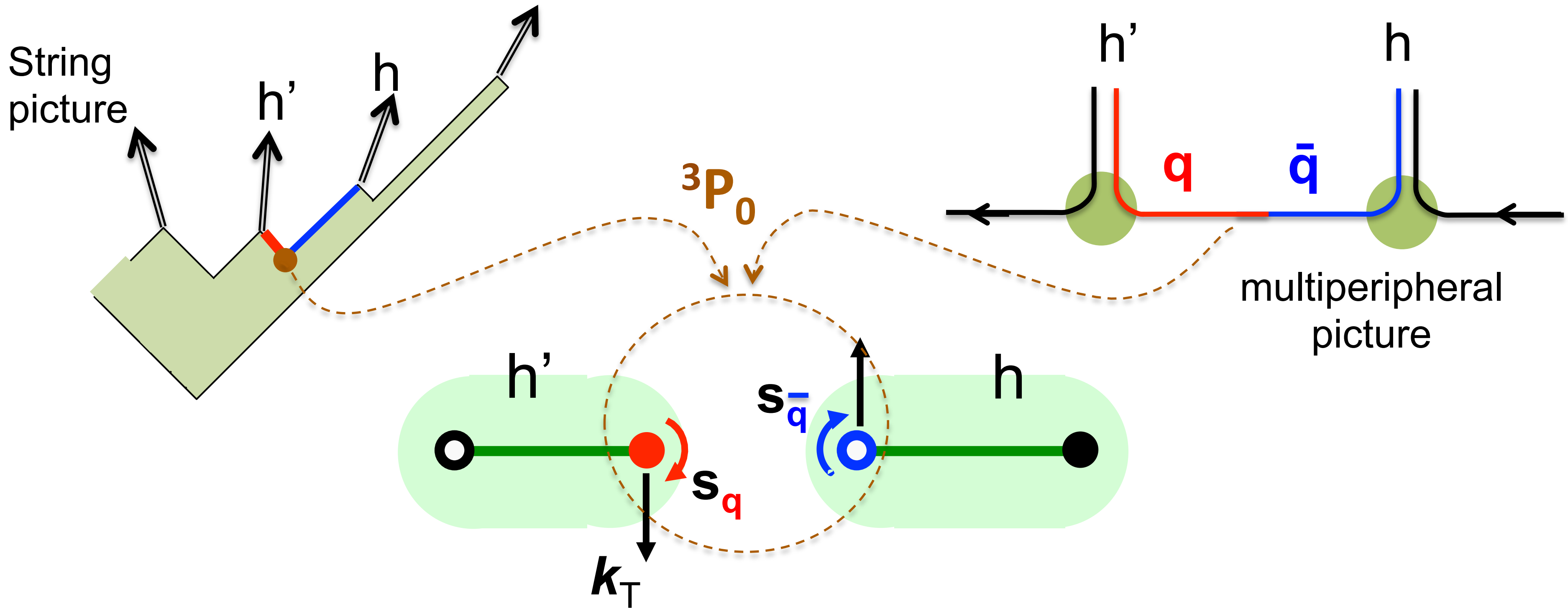
$\Gamma(k_{N+1},p_N,k_N)\ \Delta(k_N)\ \ldots\ldots\ \Delta(k_3)\ \Gamma(k_3,p_2,k_2)\ \Delta(k_2)\ \Gamma(k_2,p_1,k_1)$

sandwiched between **Pauli** spinors $\underline{\chi^\dagger(-\mathbf{S}_B)\,\sigma_z}$ and $\chi(\mathbf{S}_A)$

$\Downarrow$

analogue of the Dirac spinor $v^\dagger(-k,\mathbf{S}')$

Spin part of the propagator. It correlates $\mathbf{s}_{\bar{q}}$ in h and $\mathbf{s}_q$ in h'



$^3P_0$ wave function : $\langle \mathbf{S}_q, \mathbf{S}_{\bar{q}} | ^3P_0 \rangle = \chi^\dagger(-\mathbf{S}_q)\, \sigma_z\, \boldsymbol{\sigma}.\mathbf{k}\; \chi(\mathbf{S}_{\bar{q}})$

During tunneling k_z is not fixed, but imaginary: $k_z \sim \mathbf{i}\,(m_q^2 + \mathbf{k}_T^2)^{1/2}$.

We replace it by a phenomenological *complex mass parameter* μ, with $\mathrm{Im}(\mu) > 0$

$\rightarrow$ The spin propagator is $\sigma_z\, \boldsymbol{\sigma}.\mathbf{k} = \boxed{\mu + \sigma_z\, \boldsymbol{\sigma}.\mathbf{k}_T}$ (analogue of $m+\gamma.p$)

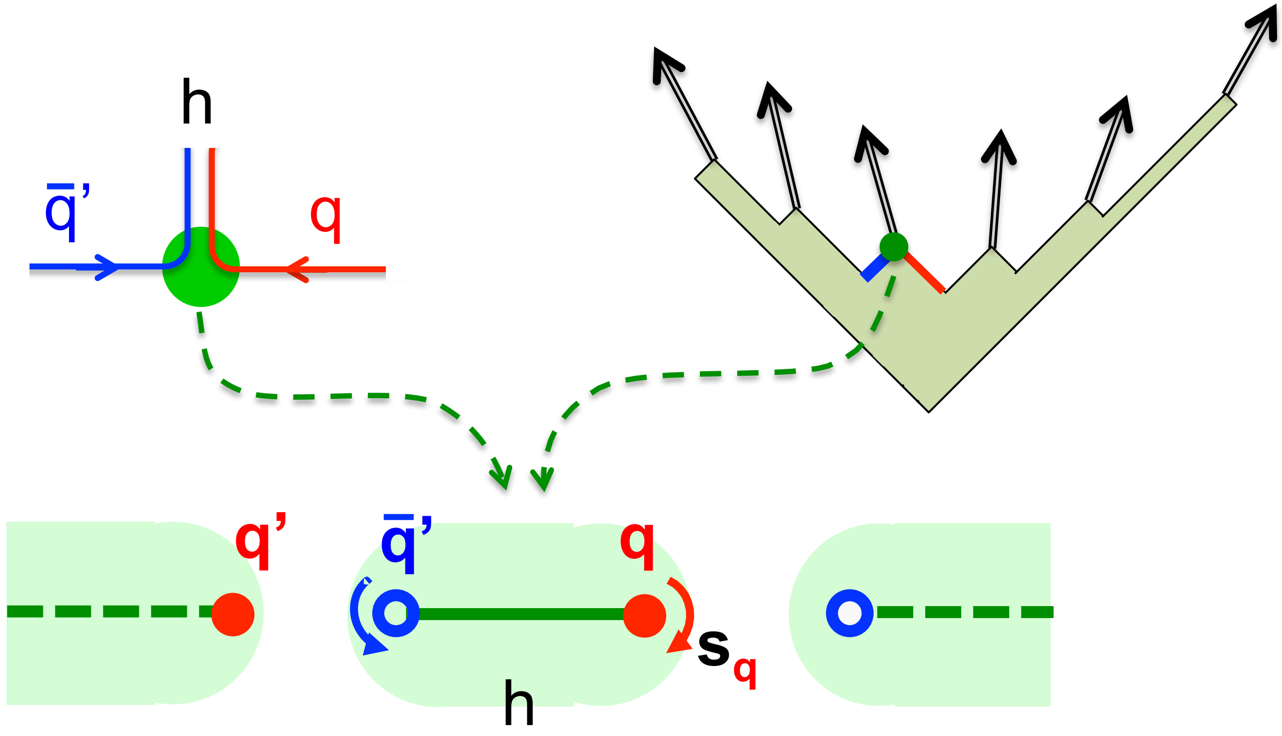
[X.A. Dubna-Spin'09]

The vertex matrix. It correlates $\mathbf{s}_q$ and $\mathbf{s}_{\bar{q}'}$ in the same hadron

1) pseudoscalar meson

↓

spin singlet
$\mathbf{s}_q = -\,\mathbf{s}_{\bar{q}'}$



Correlation implemented by $\Gamma^{(\mathrm{spin})}(PS) = \sigma_z$

analogue of γ_5

The vertex. 2) vector meson (ρ, ω, ϕ)

- it depends on the vector amplitude **V** of the VM.
  The simplest one which satisfy Parity and LR symmetry is

$$\boxed{\Gamma^{(spin)}(VM) = G_L\, V^*_z + G_T\, \mathbf{V}_T^*.\boldsymbol{\sigma}_T\, \sigma_z}$$

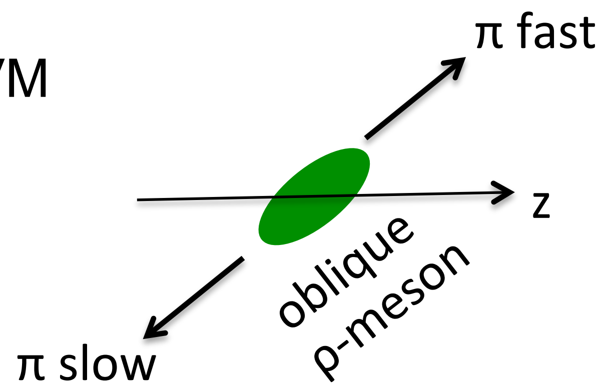
(analogues of $\gamma_\mu V^\mu$ and $\sigma_{\mu\nu} V^\mu p^\nu$ couplings)

G_L and G_T can be complex. New relevant parameters are :

- $|G_L|^2 + 2\,|G_T|^2$ $\rightarrow$ abundance of vector mesons
- $|G_L/G_T|^2$ $\rightarrow$ *alignment*
- $\mathrm{Arg}(G_L/G_T)$ $\rightarrow$ ***oblique*** polarization of the VM

$\downarrow$

source of *z-ordered*
di-hadron S_T asymmetry



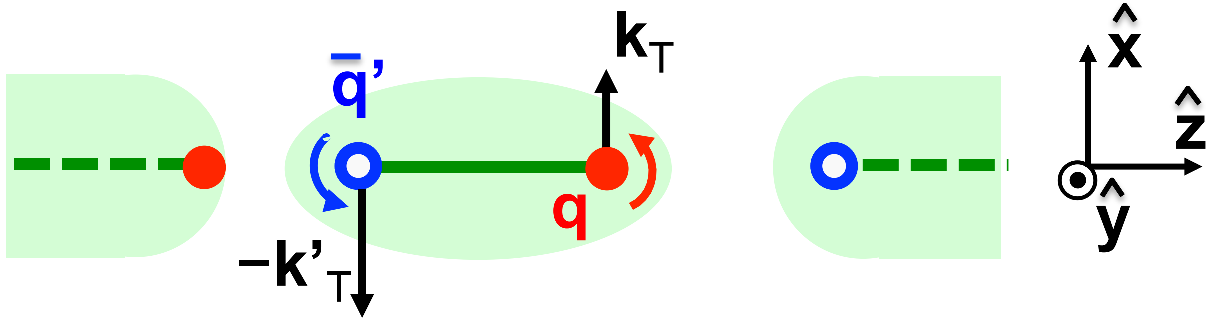
Effects linked to **V**

real **V** (linear polarization)
and $\mathbf{S_q} = \hat{\mathbf{y}}$ (spining anticlockwise)

a) $\mathbf{V} = \hat{\mathbf{x}}$ or $\hat{\mathbf{z}}$

→ $s_y(\text{VM}) = \pm 1$

$s_y(\bar{\mathbf{q}}') = s_y(\mathbf{q}) = \pm \frac{1}{2}$



VM of rank 1: Collins effect **opposite** to that of pseudoscalar

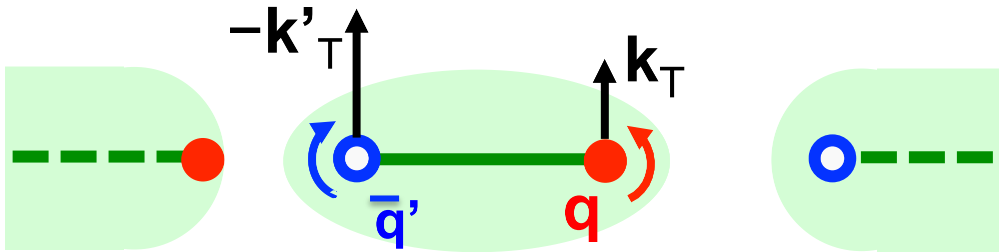
[Jan Czyzewski '96]

VM of rank > 1: **smaller** $\langle \mathbf{p}_T^2 \rangle$ than for pseudoscalars.

b) $\mathbf{V} = \hat{\mathbf{y}}$

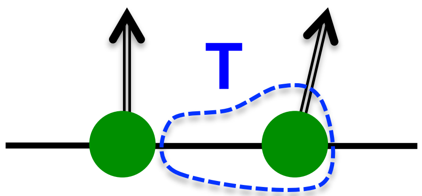
→ $S_y(\text{VM}) = 0$

$s_y(\bar{\mathbf{q}}') = - s_y(\mathbf{q})$



→ Same $\langle \mathbf{p}_T^2 \rangle$ and Collins asymmetry as pseudoscalars (any rank)

The full splitting amplitude



$$\mathbf{T}(p,k) = \boldsymbol{\Delta}(k') \,\boldsymbol{\Gamma}(k',p,k)$$

$$= (F_{\mathbf{Lund}})^{1/2} \times (\mu + \sigma_z \,\boldsymbol{\sigma}.\mathbf{k'}_T) \times \begin{cases} \sigma_z & \text{(PS meson)} \\ (G_L \, V_z^* + G_T \, \mathbf{V}_T^*.\boldsymbol{\sigma}_T \, \sigma_z) & \text{(V-meson)} \end{cases}$$

spinless Lund fragmentation function :

$$F_{\mathbf{Lund}}(\underset{\downarrow \; p^+/k^+}{Z}, \mathbf{p}_T) = N \; \underbrace{(1-Z)^a}_{\text{quark action}} \; \underbrace{\exp\{-b_L(m^2+\mathbf{p}_T^2)/Z\}}_{\text{string area factor}} \; \underbrace{\exp\{-b_T \, \mathbf{k'}_T^2\}}_{\text{tunneling factor}}$$

[A. Kerbizi, X.A., A. Martin, arXiv:2109.06124]

Splitting function and polarization of q'

- Splitting function for the process $q \to h + q'$:

$$F(Z, \mathbf{p}_T) = \mathrm{trace}\ \{\mathbf{T}\ \rho(q)\ \mathbf{T}^\dagger\}$$

- For a *vector meson* $\mathbf{T} = \mathbf{T}_\alpha V_\alpha$ (α = x, y, or z)

  If the VM polarization is not analysed : $F(Z, \mathbf{p}_T) = \Sigma_\alpha\ \mathrm{trace}\ \{\mathbf{T}_\alpha\ \rho(q)\ \mathbf{T}_\alpha^\dagger\}$

- The spin density matrix of q' is

$$\rho(q') = \begin{cases} \mathbf{T}\ \rho(q)\ \mathbf{T}^\dagger / \mathrm{trace}\ \{\mathbf{T}\ \rho(q)\ \mathbf{T}^\dagger\} & \text{if } h = \text{pseudo-scalar meson} \\ \mathbf{T}_\alpha\ \rho(q)\ \mathbf{T}_\alpha^\dagger / \mathrm{trace}\ \{\mathbf{T}_\beta\ \rho(q)\ \mathbf{T}_\beta^\dagger\} & \text{if } h = \textit{not analysed}\ \text{VM} \end{cases}$$

  q' is polarized both by *spin transfer* from q and by $\mathbf{k}'_T$ - $\mathbf{s}_{q'}$ correlations.

- If the VM is *analysed* we need to calculate $\rho(q')$ in an other way.
  This is the case when the momenta of the decay products are known
  (second slide below)

Polarization and decay of the vector meson

- The VM is polarized with the density matrix $\rho_{\alpha\beta}(h) =$ trace{ $\mathbf{T}_\alpha\, \rho(q)\, \mathbf{T}^\dagger_\beta$ }
  (recall : $\mathbf{T} = \mathbf{T}_\alpha\, V_\alpha$)

$\downarrow$

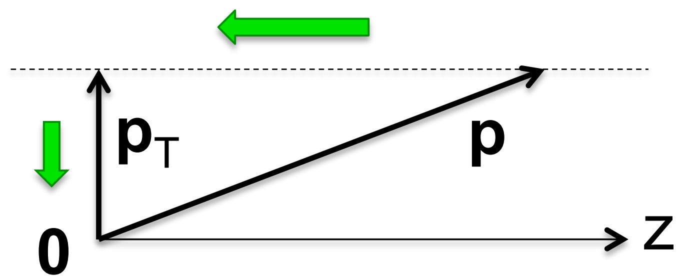
(summation over the spin of q')

→ Its decay is **anisotropic**. Let $\mathbf{r}$ be the relative momentum of the decay products and $\langle \mathbf{r}|M|\alpha\rangle$ the decay amplitude. The angular distribution of $\mathbf{r}$ is

$$\langle \mathbf{r}|M|\alpha\rangle \;\; \rho_{\alpha\beta}(h) \;\; \langle \beta|M^\dagger|\mathbf{r}\rangle$$

- **Choice of frame :** $\mathbf{r}$ is measured in the VM rest frame. But the VM can be put at rest by different Lorentz transformations, hence different "frames" (*helicity frame*, *Jackson frame*, etc), which differ by Wigner rotations.

LR symmetry imposes the composition of two boosts :



Polarization of the left-over quark **q'** following a VM decay

- the spins of the VM and of q' are *entangled*.

$\longrightarrow$ The polarization of q' depends on the directions of the decay products

$\longrightarrow$ In the Monte Carlo code, we **cannot** treat ***separately*** the decay of the VM and the fragmentation of q'.

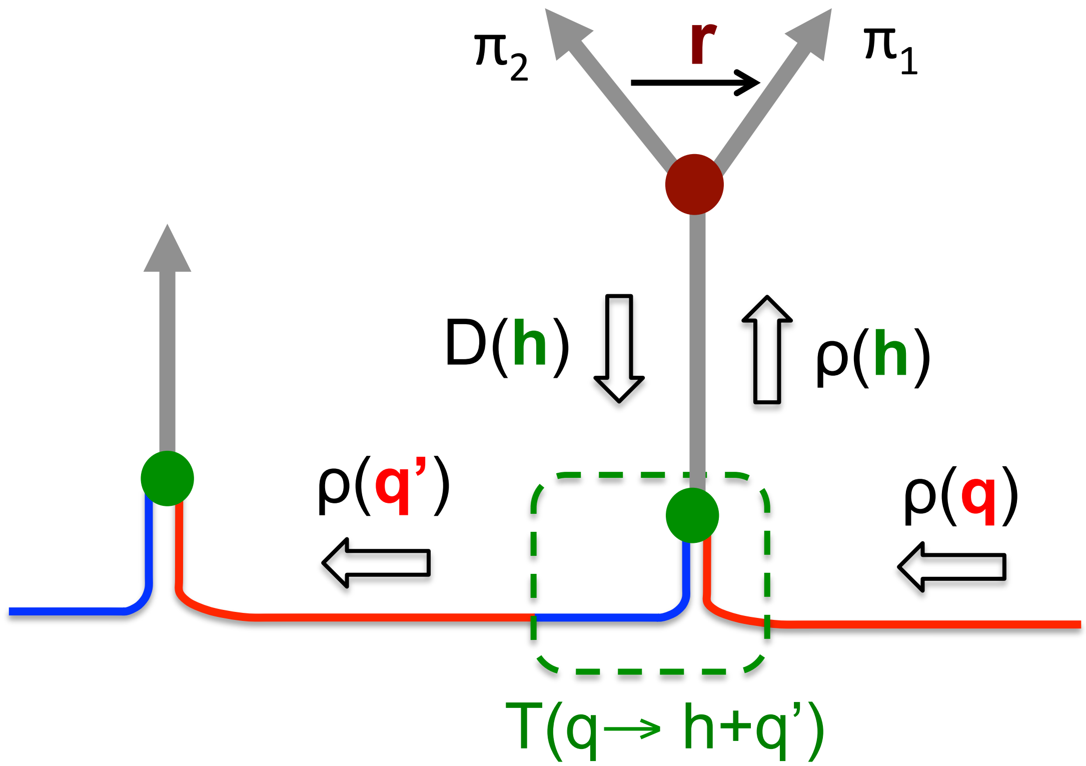
We must use the *algorithm of Collins and Knowles* (1988) :

1) calculate $\rho_{\alpha\beta}(h) = \mathrm{trace}\{ \mathbf{T}_\alpha\, \rho(q)\, \mathbf{T}^\dagger_\beta \}$
2) generate the relative momentum $\mathbf{r}$ the decay products with the angular distribution $\langle \mathbf{r}|M|\alpha\rangle\, \rho_{\alpha\beta}(h)\, \langle\beta|M^\dagger|\mathbf{r}\rangle$

(treated last slide)

3) calculate the *decay matrix* $D_{\beta\alpha}(h) = \langle\beta|M^\dagger|\mathbf{r}\rangle\, \langle \mathbf{r}|M|\alpha\rangle$
4) deduce $\rho(q') = D_{\beta\alpha}(h)\, \mathbf{T}_\alpha\, \rho(q)\, \mathbf{T}^\dagger_\beta$

Flow of the spin information during and after VM emission and decay



- $D_{\beta\alpha}$ carries the information about the direction of **r** "**backward in time**", from the **decay** event to the **splitting** event (in violation of classical causality)

Main conclusions

- We extended the Lund model of string fragmentation by including the spin degree of freedom, respecting the rules of quantum mechanics.
- The $^3P_0$ mechanism, in agreement with experimental Collins asymmetry, is implemented by the factor $\mu + \sigma_z\, \boldsymbol{\sigma}.\mathbf{k}_T$ of the quark propagator.
- Vector mesons have been included. They have opposite Collins effect w.r.t. pions, in accordance with Czyzewski's prediction, and smaller $\langle \mathbf{p}_T^2 \rangle$ (this is a ''hidden spin'' effect).
- A complex value of the parameter G_L/G_T leads to an *oblique* polarization of the vector mesons, resulting in a dihadron spin asymmetry.
- In a Monte Carlo code, the anisotropic decay of a polarized VM must be generated **before** continuing the recursive quark fragmentation (Collins and Knowles algorithm). Due to a quantum entanglement, the polarization of the leftover quark depends on the directions of the decay products.
- The translation of this model in a Monte Carlo code and the comparison with experimental data is presented by Albi Kerbizi in the next talk.

Thank you for your patience !!!