TMD cross-section factorization for dijet production at the EIC

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Outline

Dijet production

- Kinematic region
- Cross-section factorization
- New TMD Soft Function

Heavy meson pair production

- Cross-section factorization

ζ-prescription & evolution

Plots

Based on the work published by

Rafael F. del Castillo, Miguel G. Echevarría, Yiannis Makris & Ignazio Scimemi

https://arxiv.org/abs/2008.07531v4

...and following work soon to be published

Motivation

- Gluon transverse momentum dependent distributions (TMDs) are difficult to access due to the lack of clean processes where the factorization of the cross-section holds and incoming gluons constitute the dominant effect.

- We consider two processes which are presently attracting increasing attention

$$\ell + h \to \ell' + J_1 + J_2 + X$$

Dijet

$$\ell + h \to \ell' + H + \bar{H} + X$$

Heavy-meson

Dominguez, Xiao, Yuan, 2013

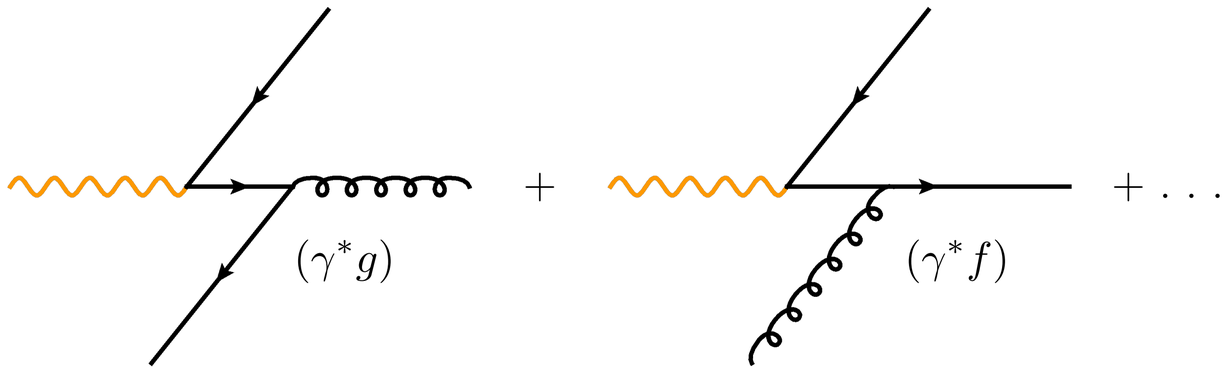
Boer, Brodsky, Mulders, Pisano, 2011

Zhang, 2017

Dijet production

$$\ell + h \to \ell' + J_1 + J_2 + X$$

dijet LO process:



- Sensitive of polarized and unpolarized TMDPDFs
- Experimental observation should be possible in the future EIC Page, Chu, Aschenauer, 2020
- Jets here described have $p_T \in [2, 40]$ GeV and are found in the central rapidity region
- Factorization within SCET

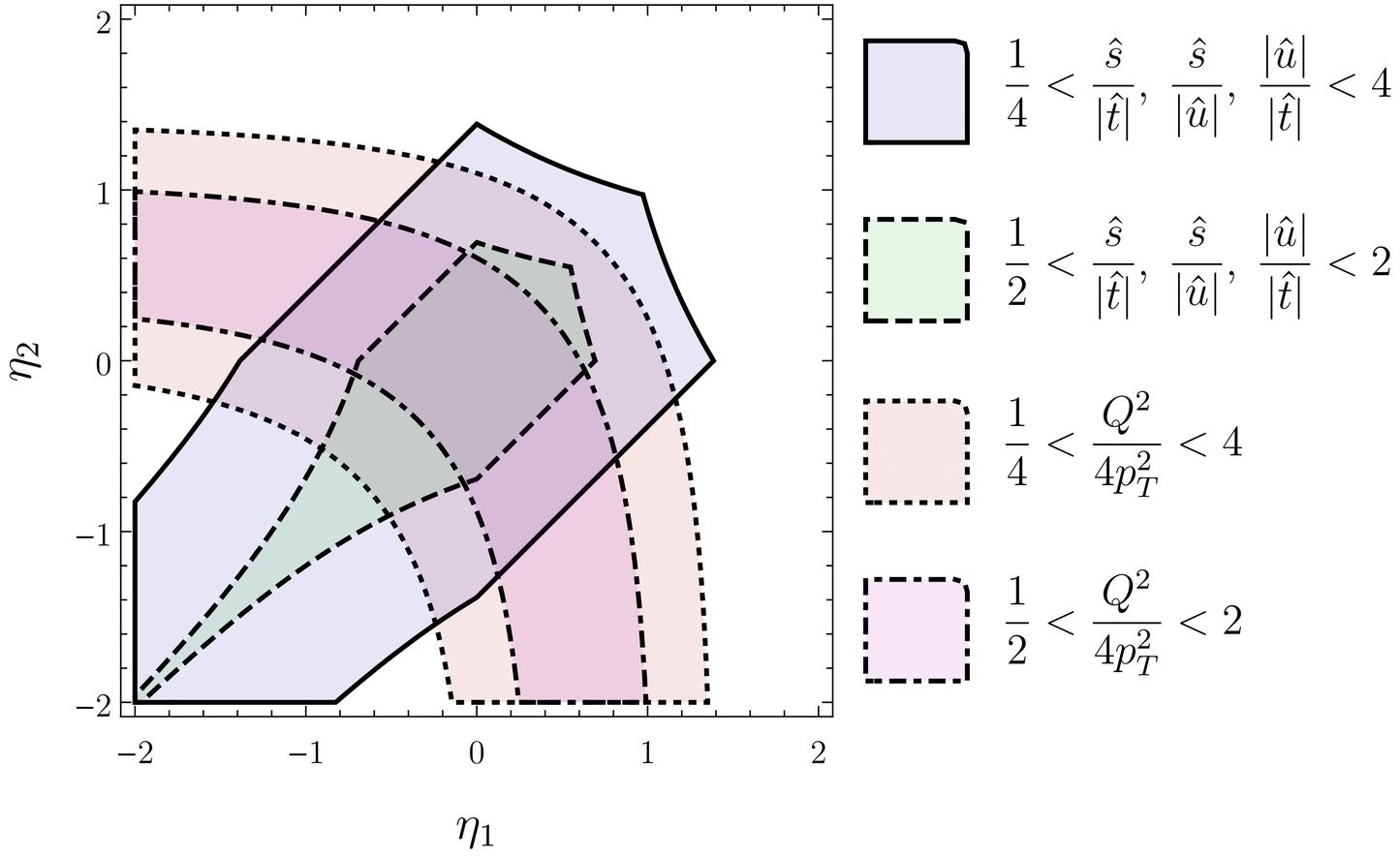
Kinematic region

Dijet production

$$\boldsymbol{r}_T = \boldsymbol{p}_{1T} + \boldsymbol{p}_{2T}$$

$$p_T = \frac{|\boldsymbol{p}_{1T}| + |\boldsymbol{p}_{2T}|}{2}$$

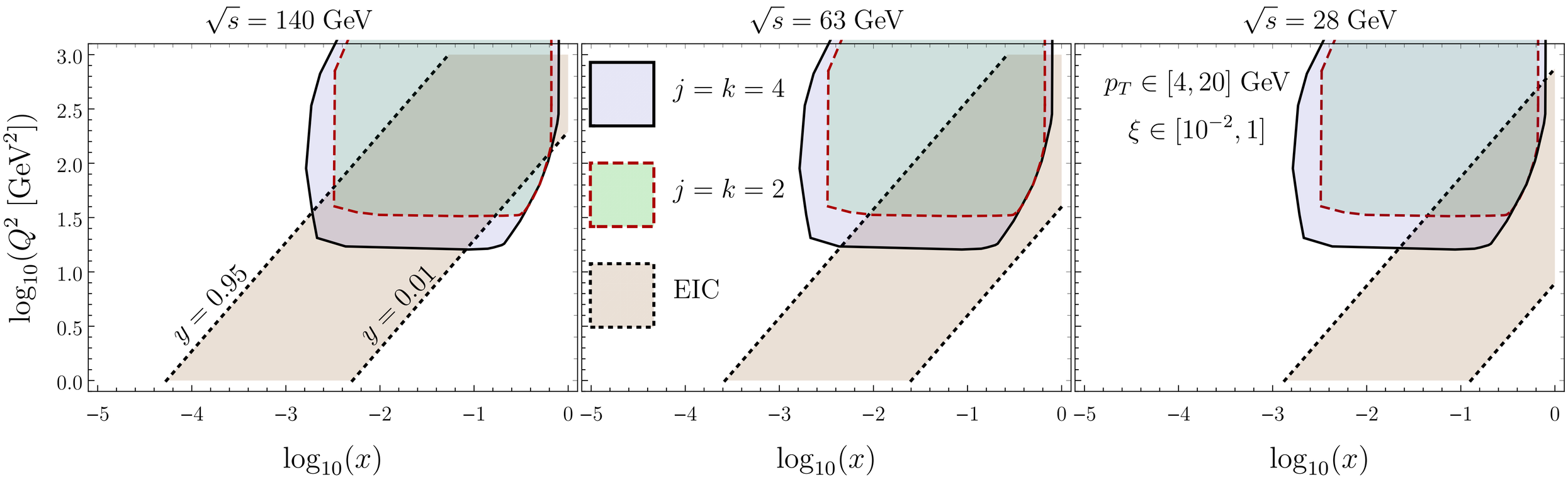
$$|\boldsymbol{r}_T| \ll p_T$$



Factorization holds for $|\boldsymbol{r}_T| \ll p_T$ and for the central rapidity region

Kinematic region vs EIC coverage

Dijet production



$$\frac{1}{j} < \frac{\hat{s}}{|\hat{t}|}, \frac{\hat{s}}{|\hat{u}|}, \frac{|\hat{u}|}{|\hat{t}|} < j \qquad \frac{1}{k} < \frac{Q^2}{4p_T^2} < k$$

Overlapping increases with higher beam energies

Cross-section factorization

Dijet production

$$\frac{d\sigma}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T}$$

We measure over

- x Bjorken variable
- η_i jet pseudorapidity
- p_T transverse momentum
- $\boldsymbol{r}_T$ transverse momentum imbalance

$(\gamma^* g)$

$$\frac{d\sigma(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sum_f H^{\mu\nu}_{\gamma^* g \to f\bar{f}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T) F_{g,\mu\nu}(\xi, \boldsymbol{b}, \mu, \zeta_1)$$
$$\times S_{\gamma g}(\boldsymbol{b}, \eta_1, \eta_2, \mu, \zeta_2) \Big(\mathcal{C}_f(\boldsymbol{b}, R, \mu) J_f(p_T, R, \mu)\Big) \Big(\mathcal{C}_{\bar{f}}(\boldsymbol{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu)\Big)$$

$(\gamma^* f)$

$$\frac{d\sigma^U(\gamma^* f)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sigma_0^{fU} \sum_f H^U_{\gamma^* f \to g\bar{f}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T) F_f(\xi, \boldsymbol{b}, \mu, \zeta_1)$$
$$\times S_{\gamma f}(\boldsymbol{b}, \zeta_2, \mu) \Big(\mathcal{C}_g(\boldsymbol{b}, R, \mu) J_g(p_T, R, \mu)\Big) \Big(\mathcal{C}_{\bar{f}}(\boldsymbol{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu)\Big)$$

Unpolarized & linearly polarized cross-section

Dijet production

$$F_g^{\mu\nu}(\xi, \boldsymbol{b}) = f_1(\xi, \boldsymbol{b}) \frac{g_T^{\mu\nu}}{d-2} + h_1^{\perp}(\xi, \boldsymbol{b}) \left(\frac{g_T^{\mu\nu}}{d-2} + \frac{b^\mu b^\nu}{\boldsymbol{b}^2} \right)$$

$$H^{\mu\nu}_{\gamma^* g \to f\bar{f}} = \sigma_0^{gU} \, H^U_{\gamma^* g \to f\bar{f}} \frac{g_T^{\mu\nu}}{d-2} + \sigma_0^{gL} \, H^L_{\gamma^* g \to f\bar{f}} \left(- \frac{g_T^{\mu\nu}}{d-2} + \frac{v_{1T}^\mu \, v_{2T}^\nu + v_{2T}^\mu \, v_{1T}^\nu}{2 \, v_{1T} \cdot v_{2T}} \right)$$

Unpolarized cross-section

$$\frac{d\sigma^U(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sigma_0^{gU} \sum_f H^U_{\gamma^* g \to f\bar{f}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b} \cdot \boldsymbol{r}_T) \, f_1(\xi, \boldsymbol{b}, \mu, \zeta_1)$$
$$\times \, S_{\gamma g}(\boldsymbol{b}, \zeta_2, \mu) \Big(\mathcal{C}_f(\boldsymbol{b}, R, \mu) J_f(p_T, R, \mu) \Big) \Big(\mathcal{C}_{\bar{f}}(\boldsymbol{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu) \Big)$$

Linearly polarized cross-section

$$\frac{d\sigma^L(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sigma_0^{gL} \sum_f H^L_{\gamma^* g \to f\bar{f}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b} \cdot \boldsymbol{r}_T) \, h_1^{\perp}(\xi, \boldsymbol{b}, \mu, \zeta_1)$$
$$\times \, \frac{s_{\boldsymbol{b}}^2 - c_{\boldsymbol{b}}^2}{2} \, S_{\gamma g}(\boldsymbol{b}, \zeta_2, \mu) \Big(\mathcal{C}_f(\boldsymbol{b}, R, \mu) J_f(p_T, R, \mu) \Big) \Big(\mathcal{C}_{\bar{f}}(\boldsymbol{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu) \Big)$$

New soft function

n - incoming beam direction

v_1 - jet 1 direction

v_2 - jet 2 direction

Soft function

$$\hat{S}_{\gamma g}(\boldsymbol{b}) = \frac{1}{C_F C_A}\langle 0|\mathcal{S}_n^\dagger(\boldsymbol{b},-\infty)_{ca'}\mathrm{Tr}\Big[S_{v_2}(+\infty,\boldsymbol{b})T^{a'}S_{v_1}^\dagger(+\infty,\boldsymbol{b})$$

$$\times\, S_{v_1}(+\infty,0)T^a S_{v_2}^\dagger(+\infty,0)\Big]\mathcal{S}_n(0,-\infty)_{ac}|0\rangle$$

$$\hat{S}_{\gamma f} = \hat{S}_{\gamma g}(n \leftrightarrow v_2)$$

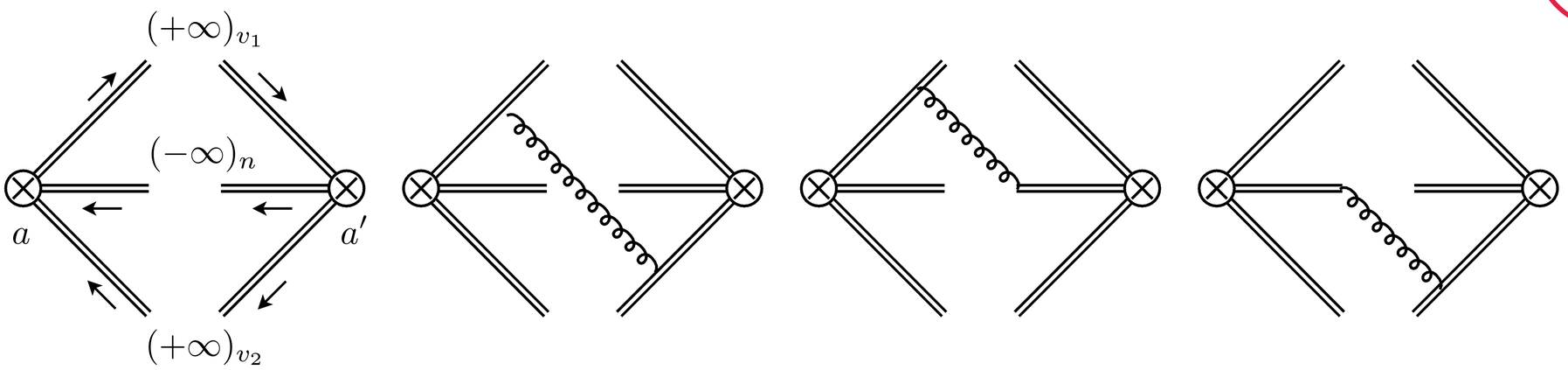
Wilson lines

$$S_v(+\infty,\xi) = P\exp\Big[-ig\int_0^{+\infty} d\lambda\; v\cdot A(\lambda v+\xi)\Big] \qquad S_{\bar{v}}^\dagger(+\infty,\xi) = P\exp\Big[ig\int_0^{+\infty} d\lambda\; \bar{v}\cdot A(\lambda \bar{v}+\xi)\Big]$$

$$S_n(+\infty,\xi) = \lim_{\delta^+\to 0} P\exp\Big[-ig\int_0^{+\infty} d\lambda\; n\cdot A(\lambda n+\xi)e^{-\delta^+\lambda}\Big]$$

δ - regulator !!!

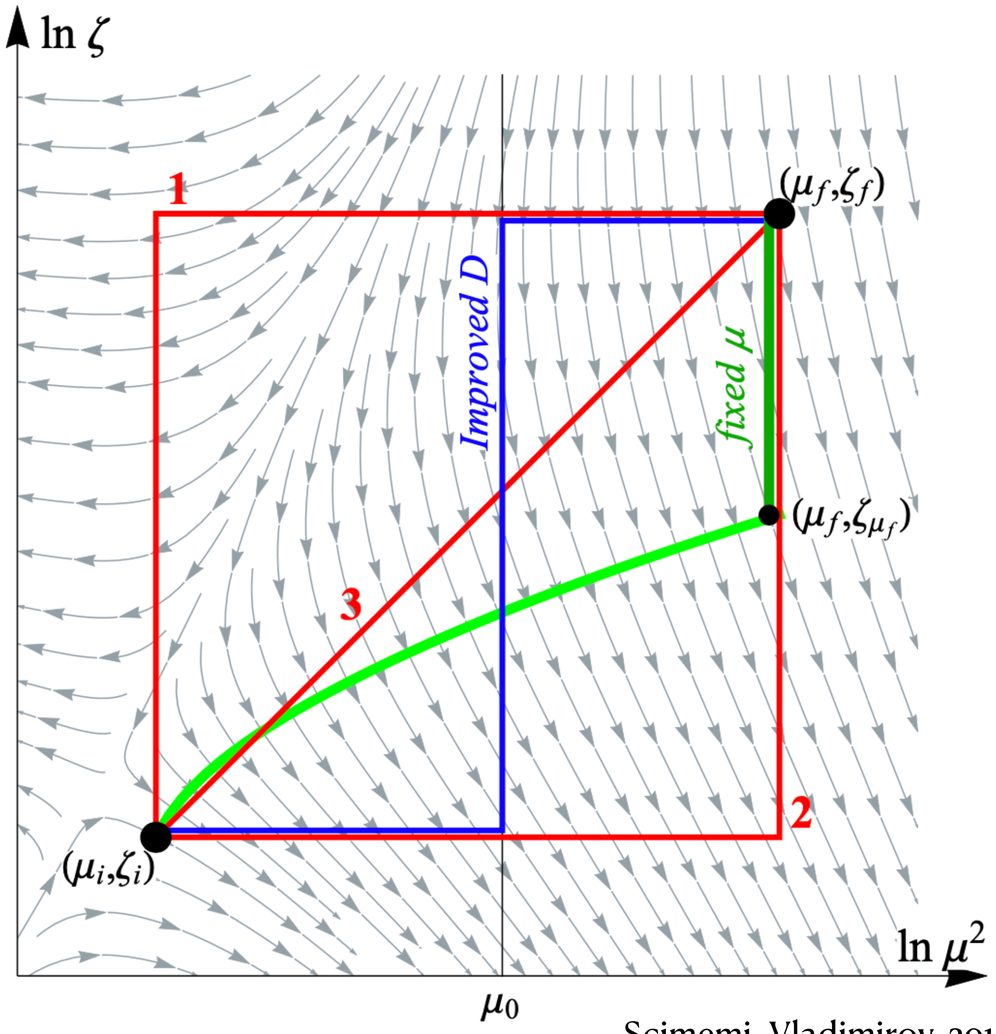
Echevarría, Scimemi, Vladimirov, 2016



+ virtual diagrams at one-loop order...

Evolution, double-scale evolution

fixed μ evolution



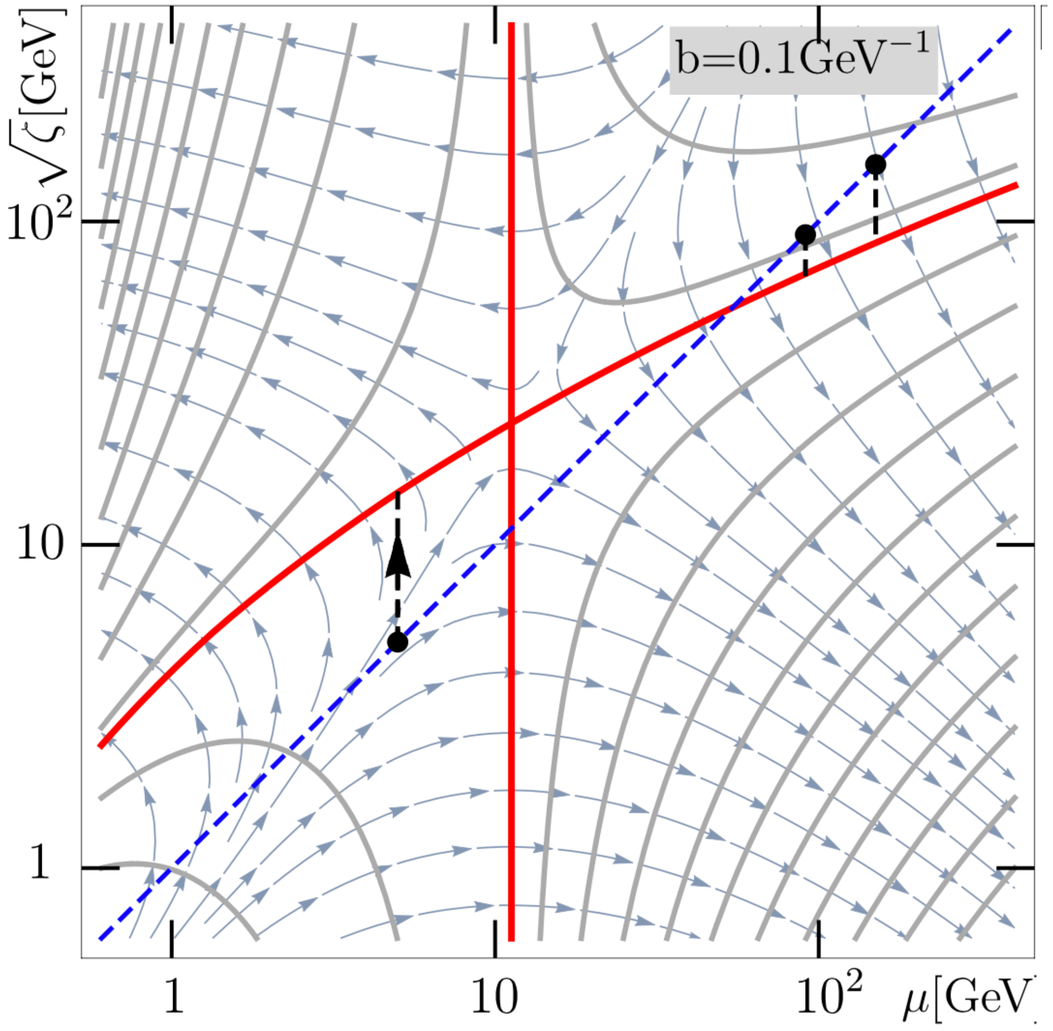
Scimemi, Vladimirov, 2018
Scimemi, Vladimirov, 2020

Describe evolution of functions depending on two scales

TMDPDF | TMDFF | Dijets SF

μ scale	Renormalization scale	ε regulator (DR)
ζ scale	Rapidity scale	δ regulator

Evolution, ζ-prescription



Scimemi, Vladimirov, 2018
Scimemi, Vladimirov, 2020

fixed μ evolution

Evolution kernel is given by

Saddle point

$$S\left(\boldsymbol{b};\mu_f,\zeta_{2,f}\right)=\exp\left[\int_P\left(\gamma_S\left(\mu,\zeta_2\right)d\ln\mu-\mathcal{D}_S(\mu,b)d\ln\zeta_2\right)\right]S\left(\boldsymbol{b};\overbrace{\mu_0,\zeta_{2,0}}\right)$$

$$\left.\begin{aligned}\frac{d}{d\ln\mu}S(\boldsymbol{b};\mu,\zeta)&=\gamma_S(b;\mu,\zeta)S(\boldsymbol{b};\mu,\zeta)\\ \frac{d}{d\ln\zeta}S(\boldsymbol{b};\mu,\zeta)&=-\mathcal{D}_S(b,\mu)S(\boldsymbol{b};\mu,\zeta)\end{aligned}\right\}\longrightarrow\boxed{\nabla F=\boldsymbol{E}F}$$

$$\boldsymbol{E}=(\gamma_S(b,\mu,\zeta),-\mathcal{D}_S(b,\mu))$$

Equipotential (null-evolution) line is given by $\gamma_S=2\mathcal{D}_S\dfrac{d\ln\zeta_\mu}{d\ln\mu^2}$

gluon channel solution $\zeta_{2,\mu}^{\gamma^*g}(\boldsymbol{b},\mu)=\left(\dfrac{\mu}{\mu_0}\right)^{\frac{2C_F}{C_A}}\zeta_{2,0}e^{v_S(\boldsymbol{b},\mu)}$

perturbative

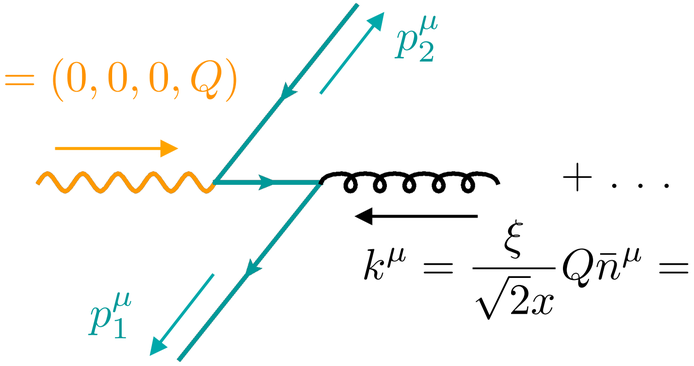
$$\boxed{R_S\big((\mu_0,\zeta_{2,0})\to(\mu_f,\zeta_f)\big)=\left(\frac{\zeta_f}{\zeta_{2,\mu}(\boldsymbol{b},\mu_f)}\right)^{-D_S(\boldsymbol{b},\mu_f)}}$$

Heavy-meson pair production

$$\ell + h \to \ell' + H + \bar{H} + X$$

$$q^\mu = \frac{Q}{\sqrt{2}}(n^\mu - \bar{n}^\mu) = (0, 0, 0, Q)$$

heavy meson pair at LO:



$+ \ldots$

$$k^\mu = \frac{\xi}{\sqrt{2}x} Q\bar{n}^\mu = \frac{\xi}{2x}(Q, 0, 0, -Q)$$

- Experimentally more challenging
- Observation of charmed mesons could be possible

Arratia, Furletova, Hobbs, Olness, Nguyen et al. 2020

Li, Liu, Vitev, 2020

Chudakov, Higinbotham, Hyde, Furletov, Furletova, Nguyen, 2016

Cross-section factorization

Heavy meson pair production

$$\frac{d\sigma(\gamma^* g)}{dx\, d\eta_H\, d\eta_{\bar{H}}\, dp_T\, d\boldsymbol{r}_T}$$

We measure over

- x Bjorken variable
- η_H, $\eta_{\bar{H}}$ heavy meson pseudorapidity
- p_T transverse momentum
- $\boldsymbol{r}_T$ transverse momentum imbalance

$$\frac{d\sigma(\gamma^* g)}{dx\, d\eta_H\, d\eta_{\bar{H}}\, dp_T\, d\boldsymbol{r}_T} = H^{\mu\nu}_{\gamma^* g \to Q\bar{Q}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, F_{g,\mu\nu}(\xi, \boldsymbol{b}, \mu, \zeta_1)$$
$$\times\, S_{\gamma g}(\boldsymbol{b}, \mu, \zeta_2)\, J_{Q\to H}(\boldsymbol{b}, p_T, m_Q, \mu)\, J_{\bar{Q}\to\bar{H}}(\boldsymbol{b}, p_T, m_Q, \mu)$$

Fickinger, Fleming, Kim, Mereghetti, 2016

Region sensitive to TMD	$\lvert\boldsymbol{r}_T\rvert \ll p_T^{H,\bar{H}}$
Factorization for highly boosted heavy mesons	$p_T^H \gg m_H$
We have a new scale	m_Q

Plots for phenomenological analysis

https://teorica.fis.ucm.es/artemide/
https://github.com/vladimirovalexey/artemide-public."

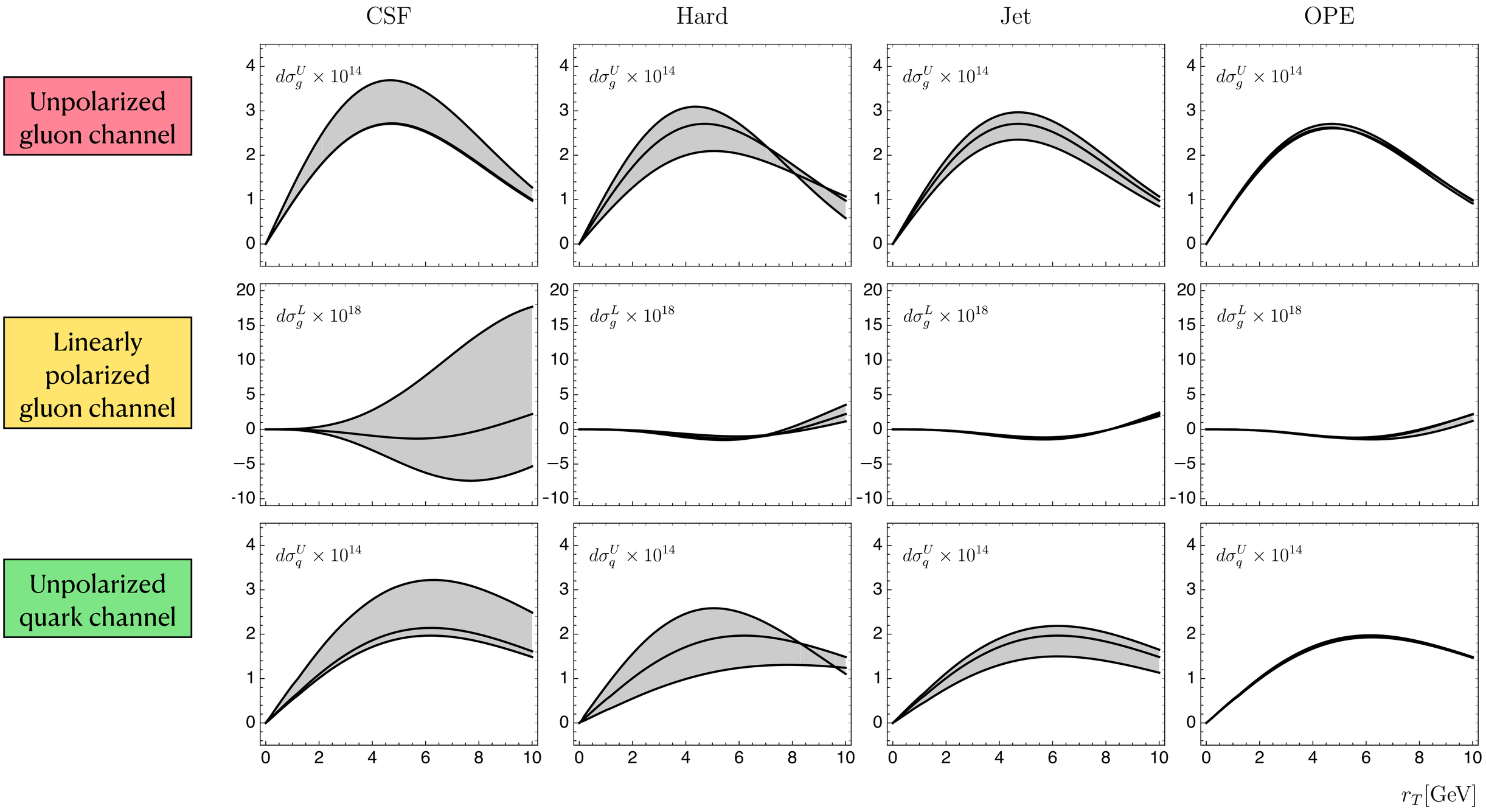
- We use arTeMiDe to obtain the plots
- TMDPDF and TMDFF structure and evolution is included arTeMiDe
- SF double-scale evolution and jet functions included as new modules

$p_T = 20\,\mathrm{GeV} \quad (p_T \sim Q)$

$\sqrt{s} = 140\,\mathrm{GeV}$

Integrated over x

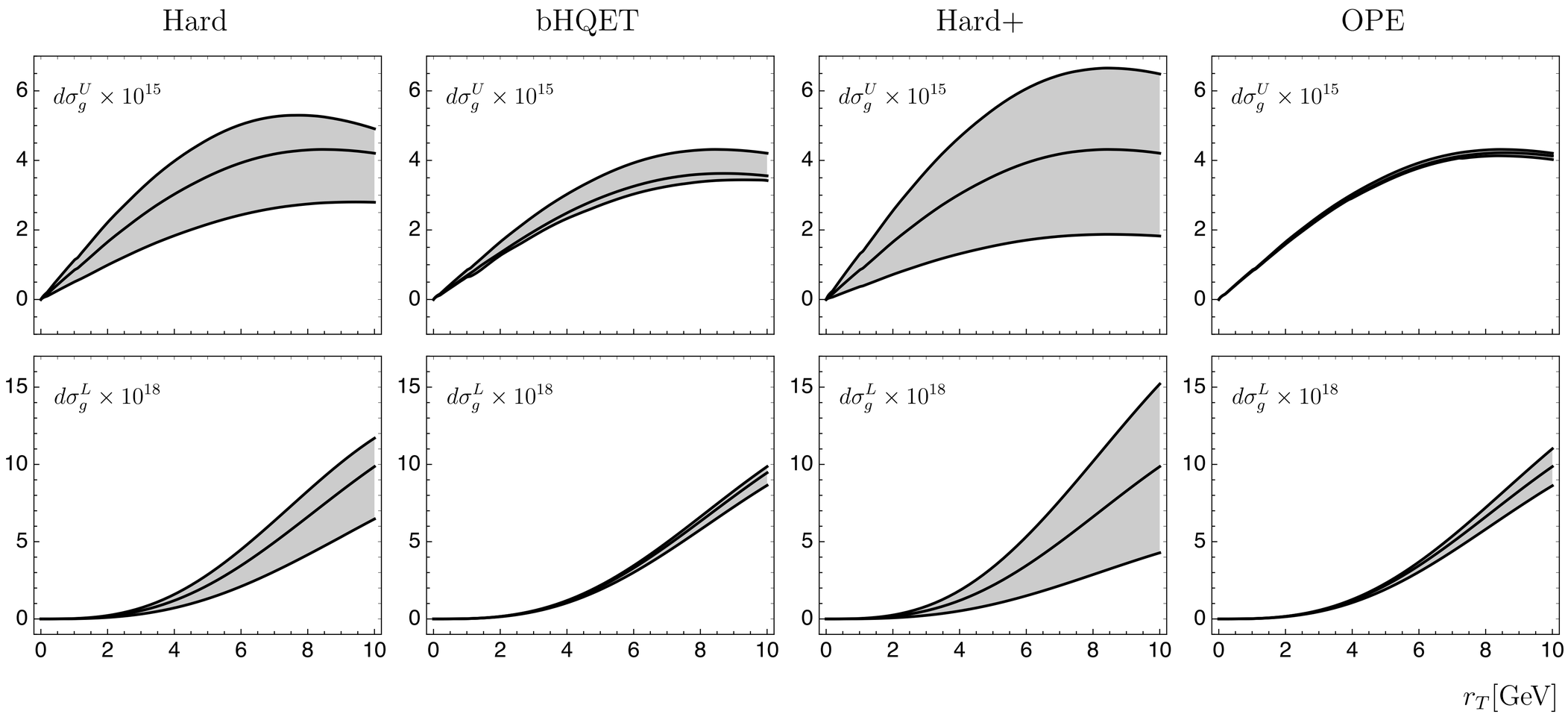
Central rapidity region


CSF
Hard
Jet
OPE
Unpolarized gluon channel
Linearly polarized gluon channel
Unpolarized quark channel
$d\sigma_g^U \times 10^{14}$
$d\sigma_g^L \times 10^{18}$
$d\sigma_q^U \times 10^{14}$
r_T[GeV]

Heavy Meson Pair Production

Unpolarized gluon channel

Linearly polarized gluon channel



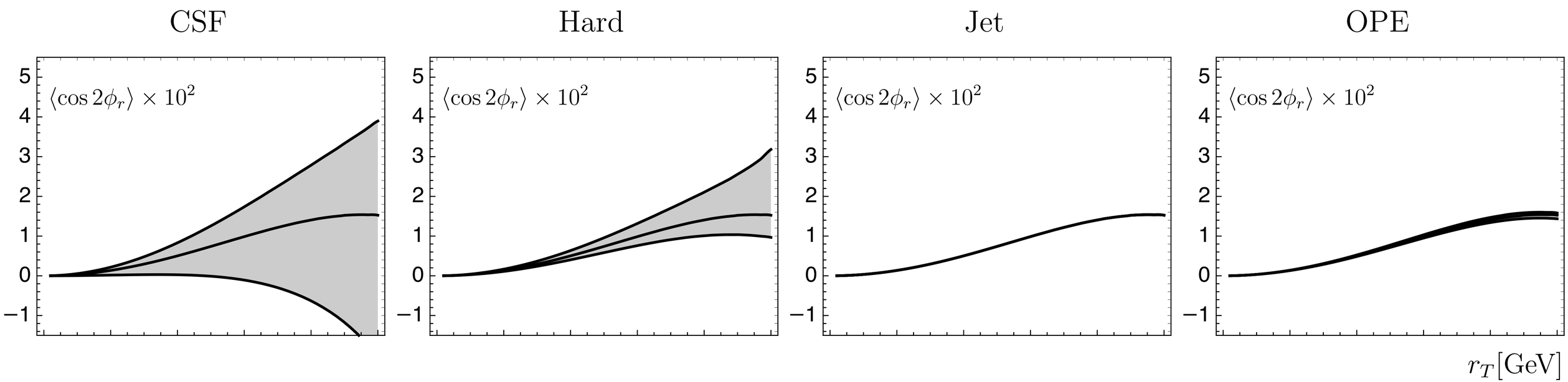
Conclusion

- We have established factorization for dijet and heavy meson pair production
- Can be potentially observed in the future EIC
- We have been able to compute the new TMD Soft Function up to NLO and its anomalous dimension up to three-loops
- Rapidity structure of this new SF allows us to use the ζ-prescription
- The presence of the new SF makes the gluon TMDPDF extraction non-trivial
- Analysis of the numerical result for the cross-section shows the effect of linearly polarized gluon TMDs can be neglected compared to unpolarized gluon TMDs

Thank you for listening!

Backup

$\cos(2\phi_r)$ angular modulation for dijet production



Kinematic definitions

We define three light-like vectors for the incoming beam and the outgoing jets

$$n^\mu = \frac{1}{\sqrt{2}}(1,0,0,1), \qquad n^2 = \bar{n}^2 = 0, \qquad \bar{n}\cdot n = 1$$

$$v_J^2 = \bar{v}_J^2 = 0, \qquad v_J \cdot \bar{v}_J = 1, \quad \text{with } J = 1,2$$

The standard Lorentz invariants

$$Q^2 = -q^2, \qquad x = \frac{Q^2}{2P\cdot q}, \qquad \xi = \frac{k^+}{P^+}$$

The transverse momentum imbalance

$$\boldsymbol{r}_T = \boldsymbol{p}_{1T} + \boldsymbol{p}_{2T}, \qquad\qquad p_T = \frac{|\boldsymbol{p}_{1T}| + |\boldsymbol{p}_{2T}|}{2}$$

Kinematic definitions

We can rewrite them in terms of the Born level kinematics

$$Q = 2p_T \cosh(\eta_-) \exp(\eta_+), \qquad \xi = 2x \cosh(\eta_+) \exp(-\eta_+)$$

$$\eta_\pm = \frac{\eta_1 \pm \eta_2}{2}$$

The parsonic Mandelstam variables are given by

$$\hat{s} = (q+k)^2 = +4p_T^2 \cosh^2(\eta_-)$$

$$\hat{t} = (q-p_2)^2 = -4p_T^2 \cosh(\eta_-) \cosh(\eta_+) \exp(\eta_1)$$

$$\hat{u} = (q-p_1)^2 = -4p_T^2 \cosh(\eta_-) \cosh(\eta_+) \exp(\eta_2)$$

$$\hat{s} + \hat{t} + \hat{u} = -Q^2$$

Zero-bin subtraction

- We need to subtract the zero-bin from the TMD beam function
- The zero-bin corresponds to the two-direction back-to-back soft function (the one used in Drell-Yan or SIDIS). Here, we use the subtraction as done in Echevarría, Idilbi, Scimemi, 2013.
- We can reorganize the zero-bin to obtain rapidity divergence-free function as expressed in the cross-section factorization
- This leads to the universal TMDPDF and a rapidity divergence-free new TMD soft function and the introduction of the scale ζ

$$\boxed{\begin{aligned} B_i^{\text{un.}}(\xi, \boldsymbol{b}, \mu, k^-/\delta^-) \quad &\longrightarrow \quad F_i(\xi, \boldsymbol{b}, \mu, \zeta_1) \\ \hat{S}_{\gamma i}(\boldsymbol{b}, \mu, \sqrt{A_n}\,\delta^+) \quad &\longrightarrow \quad S_{\gamma i}(\boldsymbol{b}, \mu, \zeta_2) \end{aligned}}$$

Zero-bin subtraction

Due to the rapidity divergencies structure of the two-direction soft function (zero-bin) can be split as

$$S(\boldsymbol{b},\mu,\sqrt{\delta^+\delta^-}) = S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^+\nu)S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^-/\nu)$$

ν arbitrary positive number

$$\left(S_i^{\rm bare}(\boldsymbol{b},\delta)\right)^{\frac{1}{2}} = 1 + a_s C_i\Big\{-\frac{2}{\epsilon^2} + \frac{4}{\epsilon}\ln\left(\frac{\sqrt{2}\,\delta}{\mu}\right) + \ln\left(B\mu^2 e^{2\gamma_E}\right)\left[4\ln\left(\frac{\sqrt{2}\,\delta}{\mu}\right) + \ln\left(B\mu^2 e^{2\gamma_E}\right)\right] + \frac{\pi^2}{6}\Big\}$$

In this way we defined the rapidity divergence-free objects

Universal TMDPDF

$$F_i(\xi,\boldsymbol{b},\mu,\zeta_1) = \frac{B_i^{\rm un.}(\xi,\boldsymbol{b},\mu,k^-/\delta^-)}{S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^-/\nu)}\Bigg|_{\sqrt{2}\,k^-/\nu\to\sqrt{\zeta_1}}$$

Rapidity divergence-free new soft function

$$S_{\gamma i}(\boldsymbol{b},\mu,\zeta_2) = \frac{\hat{S}_{\gamma i}(\boldsymbol{b},\mu,\sqrt{A_n}\,\delta^+)}{S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^+\nu)}\Bigg|_{\nu/\sqrt{2A_n}\to\sqrt{\zeta_2}}$$

ζ scale associated with the δ-regulator and zero-bin split

Zero-bin subtraction

ζ scale

The scale can be removed from the final result by introducing the constrain

$$\zeta_1\,\zeta_2 = \frac{(k^-)^2}{A_n} = \frac{\hat{u}\,\hat{t}}{\hat{s}}$$

In the Breit-frame this leads to

$$\zeta_1\,\zeta_2 = p_T^2$$

Notice that ζ_1 has square mass dimension, ζ_2 is dimensionless $\longrightarrow$ $\zeta_1 = p_T^2$, $\zeta_2 = 1$ natural way of choosing the scale

Procedure totally analogous to the one used in Drell-Yan or SIDIS

This allows to use ζ-prescription for TMDPDF and SF evolution

Scimemi, Vladimirov, 2020

Zero-bin subtraction

Subtracted soft function, finite result

$(\gamma^* g)$ - channel

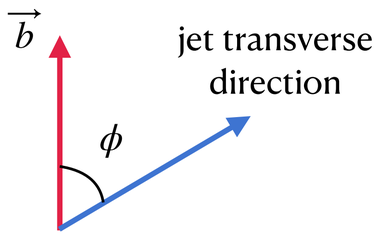
$$S_{\gamma g}(\boldsymbol{b}, \mu, \zeta_2) = 1 + a_s \Big\{ C_F \Big[\frac{\pi^2}{3} + 2\ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 4\mathrm{Li}_2(1 + A_{\boldsymbol{b}}) \Big]$$

$$+ C_A \Big[-2\ln\big(B\mu^2 e^{2\gamma_E}\big) \boxed{\ln\zeta_2} - \ln^2(-A_{\boldsymbol{b}}) - \frac{\pi^2}{3} - 2\mathrm{Li}_2(1 + A_{\boldsymbol{b}}) \Big] \Big\} + \mathcal{O}(a_s^2)$$

with... $$A_{\boldsymbol{b}} = \frac{(v_1 \cdot v_2)}{2\,(v_1 \cdot \hat{b})\,(v_2 \cdot \hat{b})} = -\frac{\hat{s}}{4\,p_T^2\,c_b^2}$$

Consistency check

Dijet-production



$$\frac{d}{d\ln\mu}G(\mu) = \gamma_G(\mu)\,G(\mu)$$

$(\gamma^* g)$-channel $\quad \gamma_{H_{\gamma g}} + \gamma_{S_{\gamma g}} + \gamma_{F_g} + 2\gamma_{J_f} + \gamma_{\mathcal{C}_1} + \gamma_{\mathcal{C}_2} + \gamma_\alpha = 0$

$(\gamma^* f)$-channel $\quad \gamma_{H_{\gamma f}} + \gamma_{S_{\gamma f}} + \gamma_{F_f} + \gamma_{J_f} + \gamma_{J_g} + \gamma_{\mathcal{C}_f} + \gamma_{\mathcal{C}_g} + \gamma_\alpha = 0$

The sum of all anomalous dimensions should cancel for each channel

ζ-logs

ϕ-logs

$$\gamma^{[1]}_{S_{\gamma g}} = 4\Big\{ -C_A \ln\zeta_2 + 2C_F\Big[\ln\big(B\mu^2\, e^{2\gamma_E}\big) - \ln\hat{s} + \ln p_T^2 + \ln\big(4c_{\boldsymbol{b}}^2\big)\Big]\Big\}\,,$$

$$\gamma^{[1]}_{S_{\gamma f}} = 4\Big\{(C_F + C_A)\Big[\ln\big(B\mu^2 e^{2\gamma_E}\big) - \ln\hat{s} + \ln p_T^2 + \ln\big(4c_{\boldsymbol{b}}^2\big)\Big] + (C_F - C_A)\Big[\ln\Big(\frac{\hat{t}}{\hat{u}}\Big) - \kappa(v_f)\Big] - C_F \ln\zeta_2\Big\}$$

$$\gamma^{[1]}_{F_i} = 4C_i\Big[-\ln\Big(\frac{\zeta_1}{\mu^2}\Big) + \gamma_i\Big]\,,$$

$$\gamma^{[1]}_{\mathcal{C}_g} = 4C_A\Big[-\ln\Big(B\mu^2\, e^{2\gamma_E}\Big) + \ln R^2 - \ln\big(4c_{\boldsymbol{b}}^2\big) + \kappa(v_g)\Big]$$

$$\gamma^{[1]}_{\mathcal{C}_i} = 4C_F\Big[-\ln\Big(B\mu^2\, e^{2\gamma_E}\Big) + \ln R^2 - \ln\big(4c_{\boldsymbol{b}}^2\big) + \kappa(v_i)\Big]$$

$$\kappa(v_f) = -\kappa(v_{\bar{f}}) = -\kappa(v_g) = i\pi\,\mathrm{sign}(c_{\boldsymbol{b}})$$

They cancel !!!

Refactorization of heavy-quark fragmentation

Heavy meson pair production

We use heavy-quark jet function to describe the fragmentation of heavy mesons from heavy quarks. In the limit $r_T \ll p_T$ there are two scales that need to be resummed

$$\mu_+ = m_Q, \qquad \text{and} \qquad \mu_{\mathcal{J}} = m_Q \frac{r_T}{p_T}$$

To do this we use **bHQET (boosted heavy quark effective theory)** to factorize the jet function into a hard matching coefficient and a TMD matrix element

$$J_{Q\to H}(\boldsymbol{b}, p_T, m_Q, \mu) = H_+(m_Q, \mu)\mathcal{J}_{Q\to H}\left(\boldsymbol{b}, \frac{m_Q}{p_T}, \mu\right)$$

Appears for the first time

Known up to two-loops

Refactorization of heavy-quark fragmentation

Heavy meson pair production

Up to one loop order we find

$$H_+(m_Q,\mu) = 1 + \frac{\alpha_s}{4\pi} C_F \left\{ \ln\left(\frac{\mu^2}{m_Q^2}\right) + \ln^2\left(\frac{\mu^2}{m_Q^2}\right) + 8 + \frac{\pi^2}{6} \right\}$$

$$\gamma_+ = \frac{\alpha_s C_F}{\pi} \left\{ \frac{1}{2} - \ln\left(\frac{m_Q^2}{\mu^2}\right) \right\}$$

$$\mathcal{J}^{\text{bare}}_{Q\to Q}\left(\boldsymbol{b}, \frac{m_Q}{p_T}\right) = 1 + \frac{\alpha_s C_F}{\pi} \left\{ -\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\left[1 - 2\ln\mathcal{R}\right] + \ln\mathcal{R} - \ln^2\mathcal{R} - \frac{5\pi^2}{24} \right\}$$

$$\gamma_{\mathcal{J}} = \frac{\alpha_s C_F}{\pi} \left\{ 1 - 2\ln\mathcal{R} \right\}$$

with... $\mathcal{R} = -\frac{i\, p_T \mu\, e^{\gamma_E} (\boldsymbol{v}\cdot\boldsymbol{b})}{m_Q |\boldsymbol{v}|}$

We have separated scales and can now be resummed

Anomalous dimension are consistent $\gamma_{\mathcal{J}} + \gamma_+ = \gamma_J + \gamma_{\mathcal{C}_f}$ $\longrightarrow$ Consistent !!!

Connection to the fragmentation shape function

Heavy meson pair production

We can see that the shape function is related to the heavy meson jet function

$$\mathcal{J}_{Q\to H}(\boldsymbol{b}) = \frac{m_H}{\sqrt{2}\,p_H^-}\tilde{S}_{Q\to H}\Big(\tau \to \frac{\boldsymbol{v}\cdot\boldsymbol{b}}{\sqrt{2}}\Big)$$

- We can check our NLO calculation (finite terms)
- We can get the jet function AD up to two loops

Fickinger, Fleming, Kim, Mereghetti, 2016

This sum is known up to three-loops...

$$\gamma_{S_{\gamma g}} = -\left(\gamma_{H_{\gamma g}} + \gamma_{F_g} + \gamma_\alpha + \gamma_{\mathcal{J}}(\boldsymbol{v}_1) + \gamma_{\mathcal{J}}(\boldsymbol{v}_2) + 2\gamma_+\right)$$

Soft function

anomalous dimension

$$\gamma_{S_{\gamma g}} = \gamma_{\text{cusp}}\Big[2C_F \ln\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) - C_A \ln\zeta_2\Big] + \delta\gamma_{S_{\gamma g}}$$

$$\delta\gamma_{S_{\gamma g}}^{[1]} = 0$$

$$\delta\gamma_{S_{\gamma g}}^{[2]} = C_F\Big[C_A\Big(\frac{1616}{27} - \frac{22}{9}\pi^2 - 56\,\zeta_3\Big) + n_f T_F\Big(-\frac{448}{27} + \frac{8}{9}\pi^2\Big)\Big]$$

$$\delta\gamma_{S_{\gamma g}}^{[3]} = \ldots$$

Connection to the fragmentation shape function

Heavy meson pair production

Shape function is defined as Fickinger, Fleming, Kim, Mereghetti, 2016

$$S_{Q\to H}(\omega) = \frac{1}{2N_c}\sum_X \langle 0|\delta(\omega - i\sqrt{2}\,\bar{v}\cdot\partial)W_v^\dagger h_{v\beta_+}|H_\beta X\rangle\langle H_\beta X|\bar{h}_{v,\beta_+}W_v\frac{\not{\bar{v}}}{\sqrt{2}}|0\rangle$$

and its Fourier transformation...

$$\tilde{S}_{Q\to H}(\tau) = \int d\omega \exp(i\omega\tau)S_{Q\to H}(\omega)$$

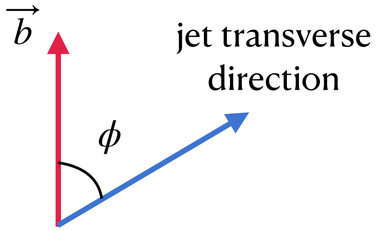
$$= \frac{1}{2\,m_H N_C}\sum_X \langle 0|\exp\Big(-\sqrt{2}\,\tau\,\bar{v}\cdot\partial\Big)\,W_v^\dagger h_{v\beta_+}|XH\rangle\langle XH|\bar{h}_{Qv}W_v\frac{\not{\bar{v}}}{\sqrt{2}}|0\rangle$$

We can see that it is related to the bHQET jet function

$$\mathcal{J}_{Q\to H}(\boldsymbol{b}) = \frac{m_H}{\sqrt{2}\,p_H^-}\tilde{S}_{Q\to H}\Big(\tau\to\frac{\boldsymbol{v}\cdot\boldsymbol{b}}{\sqrt{2}}\Big)$$

- We can check our NLO calculation (finite terms)
- We can get the jet function AD up to two loops

ϕ-dependent part



SF (and CSF) depends on the angle between the transverse plane and the jet direction ϕ

Our approach is to separate the ϕ-dependent part of the evolution and integrate it

$$\gamma_S(b,\mu,\zeta,\phi) = \bar{\gamma}_S(b,\mu,\zeta) + \gamma_S^{\phi}(\phi)$$

$$S\left(\boldsymbol{b};\mu_f,\zeta_{2,f}\right) = \exp\left[\int_{\mu_0}^{\mu_f}\left(\gamma_S^{\phi}(\phi)d\ln\mu\right)\right]\exp\left[\int_P\left(\bar{\gamma}_S(\mu,\zeta_2)d\ln\mu - \mathcal{D}_S(\mu,b)d\ln\zeta_2\right)\right]S\left(\boldsymbol{b};\mu_0,\zeta_{2,0}\right)$$

R_S^{ϕ} → Integrate over ϕ

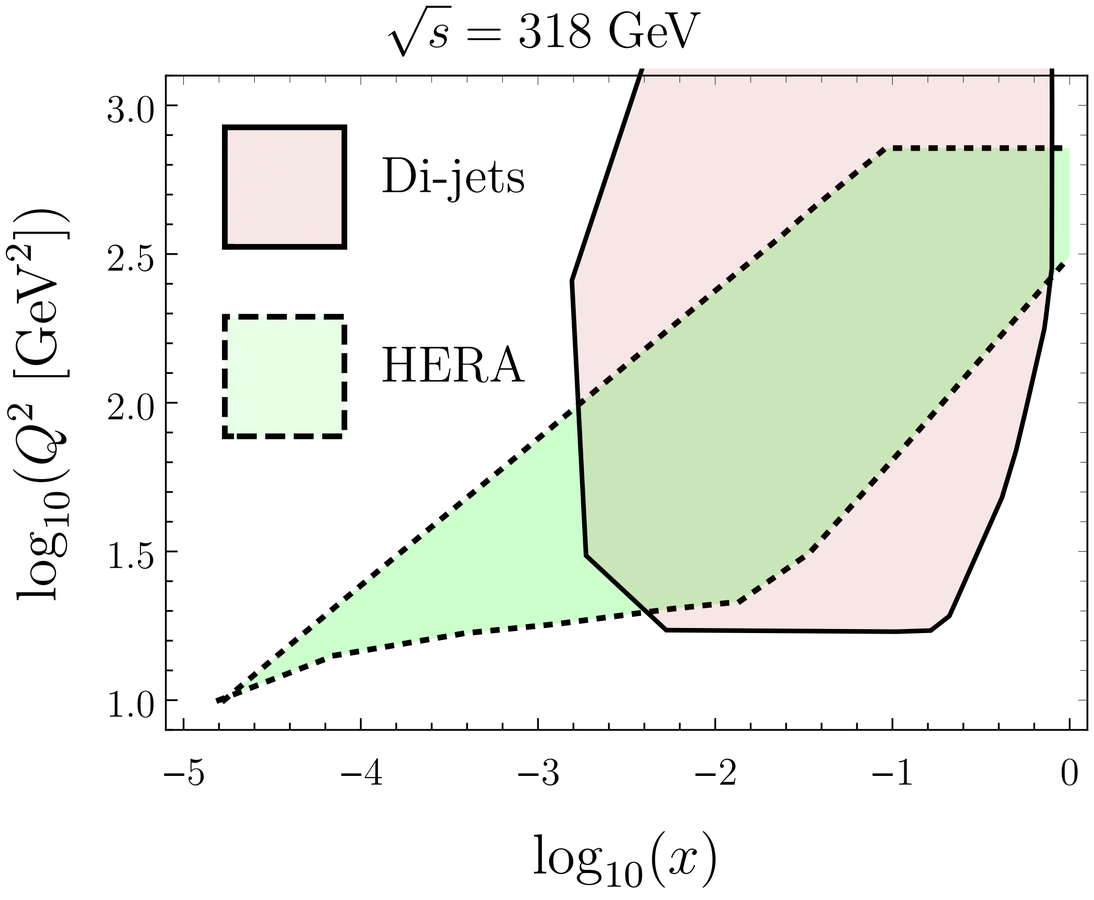
R_S → ζ-prescription

...and we do the same for CSF

This ϕ-dependent factors should be integrated along with the ϕ-dependent terms of the perturbative result

Kinematic region vs HERA coverage

Dijet production



New soft function

$$A_{\boldsymbol{b}} = \frac{(v_1 \cdot v_2)}{2\,(v_1 \cdot \hat{b})\,(v_2 \cdot \hat{b})} = -\frac{\hat{s}}{4\,p_T^2\,c_b^2}$$

$$A_n = \frac{(v_1 \cdot v_2)}{2\,(v_1 \cdot n)(v_2 \cdot n)}$$

Bare soft function

$$\hat{S}^{\text{bare}}_{\gamma g}(\boldsymbol{b}) = \hat{S}^{\text{finite}}_{\gamma g}(\boldsymbol{b}) + a_s\Big\{C_A\Big[-\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\Big(2\ln\Big(\frac{\sqrt{2}\,\delta^+}{\mu}\Big) + \ln(2A_n)\Big)\Big] + 2C_F\Big[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\Big(\frac{B\,\mu^2\,e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big)\Big]\Big\}$$

$$\hat{S}^{\text{bare}}_{\gamma f}(\boldsymbol{b}) = \hat{S}^{\text{finite}}_{\gamma f}(\boldsymbol{b}) + a_s\Big\{C_A\Big[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\Big(\ln\frac{(n\cdot v_1)(\boldsymbol{v}_2\cdot\boldsymbol{b})}{(n\cdot v_2)(\boldsymbol{v}_1\cdot\boldsymbol{b})} + \ln\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big)\Big)\Big] + \frac{4}{\epsilon}C_F\ln\Big(-\frac{i\,\boldsymbol{v_1}\cdot\boldsymbol{b}\,\delta^+ e^{\gamma_E}}{n\cdot v_1}\Big)\Big\}$$

Finite soft function

$$\hat{S}^{\text{finite}}_{\gamma g}(\boldsymbol{b}) = 1 + a_s\Big\{C_A\Big[\ln\big(B\,\mu^2 e^{2\gamma_E}\big)\Big(\ln\big(B\mu^2 e^{2\gamma_E}\big) + 4\ln\Big(\frac{\sqrt{2}\,\delta^+}{\mu}\Big) + 2\ln(2A_n)\Big) - \ln^2(-A_{\boldsymbol{b}}) - \frac{\pi^2}{6} - 2\text{Li}_2(1+A_{\boldsymbol{b}})\Big] + C_F\Big[\frac{\pi^2}{3} + 2\ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 4\text{Li}_2(1+A_{\boldsymbol{b}})\Big]\Big\},$$

$$\hat{S}^{\text{finite}}_{\gamma f}(\boldsymbol{b}) = 1 + a_s\Big\{C_A\Big[\frac{\pi^2}{6} + \ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 2\text{Li}_2(1+A_{\boldsymbol{b}}) + 2\ln\big(B\mu^2 e^{2\gamma_E}\big)\ln\frac{(n\cdot v_1)(\boldsymbol{v}_2\cdot\boldsymbol{b})}{(n\cdot v_2)(\boldsymbol{v}_1\cdot\boldsymbol{b})}\Big] + 4C_F\ln\big(B\mu^2 e^{2\gamma_E}\big)\ln\Big(-\frac{i\,\boldsymbol{v_1}\cdot\boldsymbol{b}\,\delta^+ e^{\gamma_E}}{n\cdot v_1}\Big)\Big\}$$

$$A_{\boldsymbol{b}} = \frac{(v_1 \cdot v_2)}{2\,(v_1 \cdot \hat{b})\,(v_2 \cdot \hat{b})} = -\frac{\hat{s}}{4\,p_T^2\,c_b^2}$$

$$A_n = \frac{(v_1 \cdot v_2)}{2\,(v_1 \cdot n)(v_2 \cdot n)}$$

Zero-bin subtraction

Subtracted soft function

Finite soft function

$$S_{\gamma g}(\boldsymbol{b}, \mu, \zeta_2) = 1 + a_s \Big\{ C_F \Big[\frac{\pi^2}{3} + 2\ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 4\mathrm{Li}_2(1 + A_{\boldsymbol{b}}) \Big]$$
$$+ C_A \Big[-2\ln\big(B\mu^2 e^{2\gamma_E}\big)\ln\zeta_2 - \ln^2(-A_{\boldsymbol{b}}) - \frac{\pi^2}{3} - 2\mathrm{Li}_2(1 + A_{\boldsymbol{b}}) \Big] \Big\} + \mathcal{O}(a_s^2)$$

$$S_{\gamma f}(\boldsymbol{b}, \mu, \zeta_2) = 1 + a_s \Big\{ C_A \Big[\frac{\pi^2}{6} + \ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 2\mathrm{Li}_2(1 + A_{\boldsymbol{b}}) + 2\ln\big(B\mu^2 e^{2\gamma_E}\big)\ln\frac{(n \cdot v_1)(\boldsymbol{v}_2 \cdot \boldsymbol{b})}{(n \cdot v_2)(\boldsymbol{v}_1 \cdot \boldsymbol{b})} \Big]$$
$$+ C_F \ln\big(B\mu^2 e^{2\gamma_E}\big) \Big[\ln\big(B\mu^2 e^{2\gamma_E}\big) - 2\ln\zeta_2 + 2\ln\Big(\frac{2(n \cdot v_2)}{(v_1 \cdot v_2)(n \cdot v_2)}\Big) - \frac{\pi^2}{6} + 4\ln(-i\,\boldsymbol{v}_1 \cdot \hat{\boldsymbol{b}}) \Big] \Big\} + \mathcal{O}(a_s^2),$$

Renormalization function

$$Z^S_{\gamma g}(\boldsymbol{b}, \mu, \zeta_2) = 1 + a_s \Big\{ C_F \Big[\frac{4}{\epsilon^2} + \frac{4}{\epsilon}\ln\Big(\frac{B\mu^2\, e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) \Big] - C_A \frac{2}{\epsilon}\ln\zeta_2 \Big\} + \mathcal{O}(a_s^2)$$

$$Z^S_{\gamma f}(\boldsymbol{b}, \mu, \zeta_2) = 1 + a_s \Big\{ C_A \Big[\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\Big(\ln\frac{(n \cdot v_1)(\boldsymbol{v}_2 \cdot \boldsymbol{b})}{(n \cdot v_2)(\boldsymbol{v}_1 \cdot \boldsymbol{b})} \Big) + \ln\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) \Big]$$
$$+ \frac{2}{\epsilon} C_F \Big[\ln\big(B\mu^2 e^{2\gamma_E}\big) - \ln\zeta_2 + \ln\Big(\frac{2(n \cdot v_2)}{(v_1 \cdot v_2)(n \cdot v_2)}\Big) + 2\ln(-i\,\boldsymbol{v}_1 \cdot \hat{\boldsymbol{b}}) \Big] \Big\} + \mathcal{O}(a_s^2)$$