

Factorized Approach to QED Radiation in Semi-inclusive Deep Inelastic Scattering



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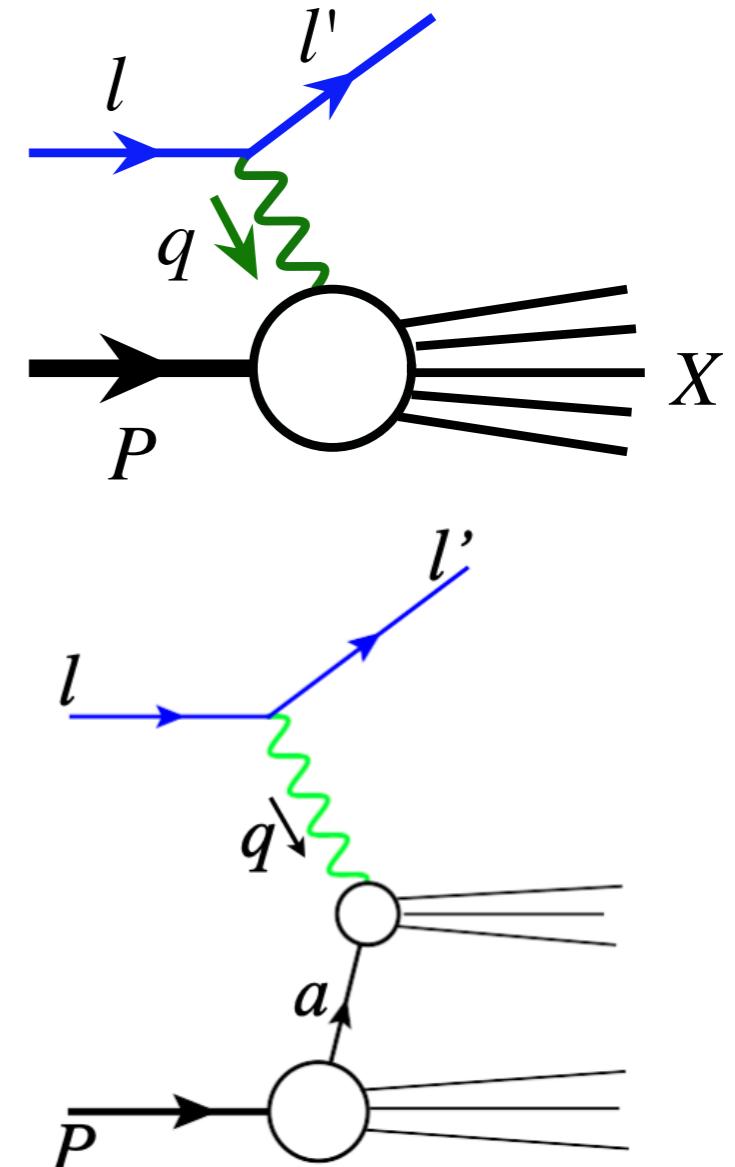
Lepton-Hadron Deep Inelastic Scattering

Inclusive DIS at a large momentum transfer $Q \gg \Lambda_{\text{QCD}}$

- dominated by the scattering of the lepton off an active quark/parton
- not sensitive to the dynamics at a hadronic scale $\sim 1/\text{fm}$
- collinear factorization: $\sigma \propto H(Q) \otimes \phi_{a/P}(x, \mu^2)$
- overall corrections suppressed by $1/Q^n$

QCD factorization

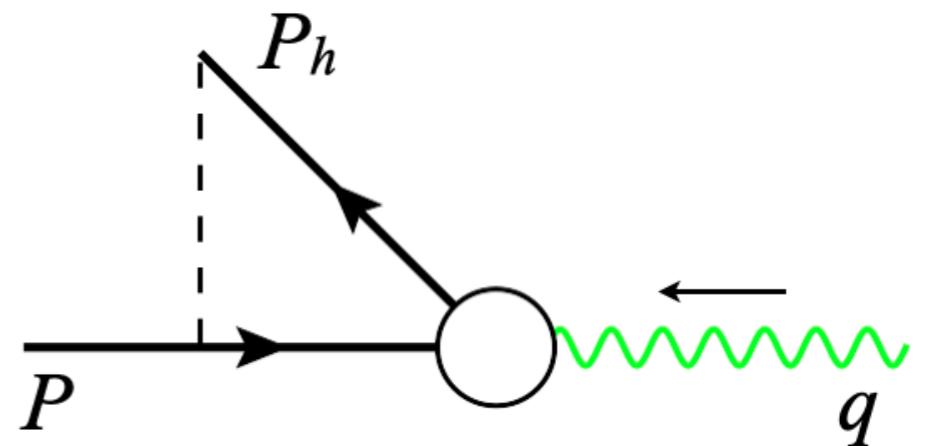
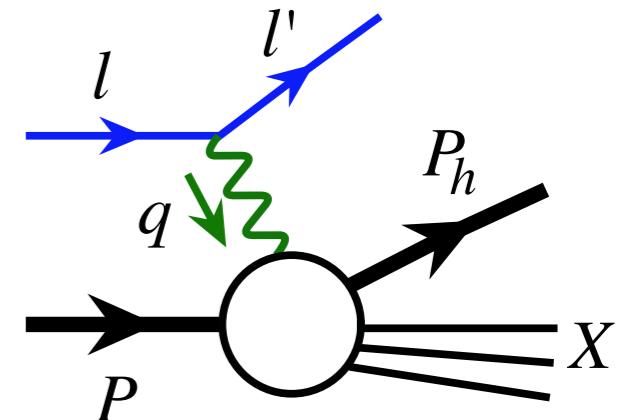
- provides the probe to “see” quarks, gluons and their dynamics indirectly
- predictive power relies on
 - precision of the probe
 - universality of $\phi_{a/P}(x, \mu^2)$



Semi-inclusive Deep Inelastic Scattering

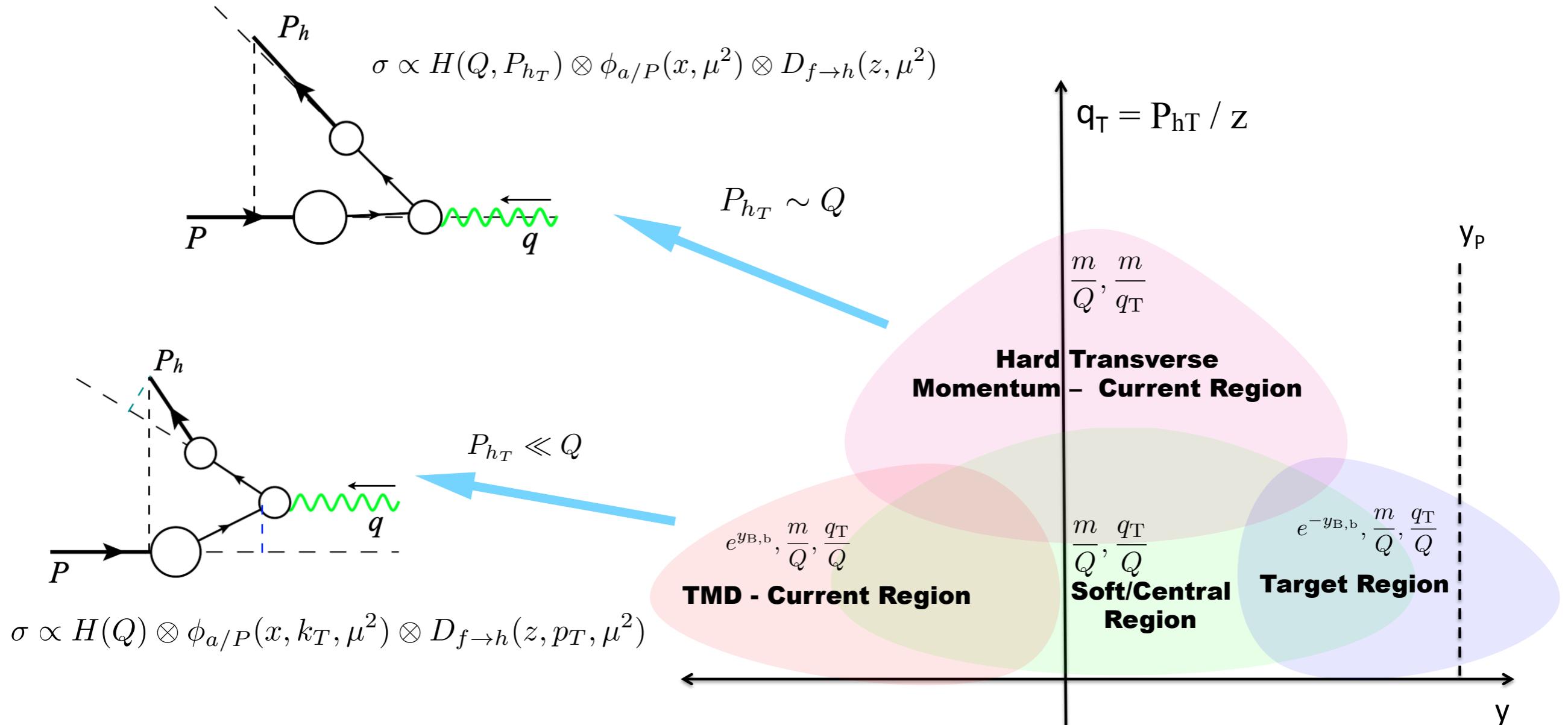
Semi-inclusive DIS: a final state hadron (P_h) is identified

- enable us to explore the emergence of color neutral hadrons from colored quarks/gluons
- flavor dependence by selecting different types of observed hadrons: pions, kaons, ...
- a large momentum transfer Q provides a short-distance probe
- an additional and adjustable momentum scale P_{h_T}



SIDIS Kinematic Regions

Sketch of kinematic regions of the produced hadron



P_{hT} is defined in the photon-hadron frame

[Figure from JHEP10(2019)122]

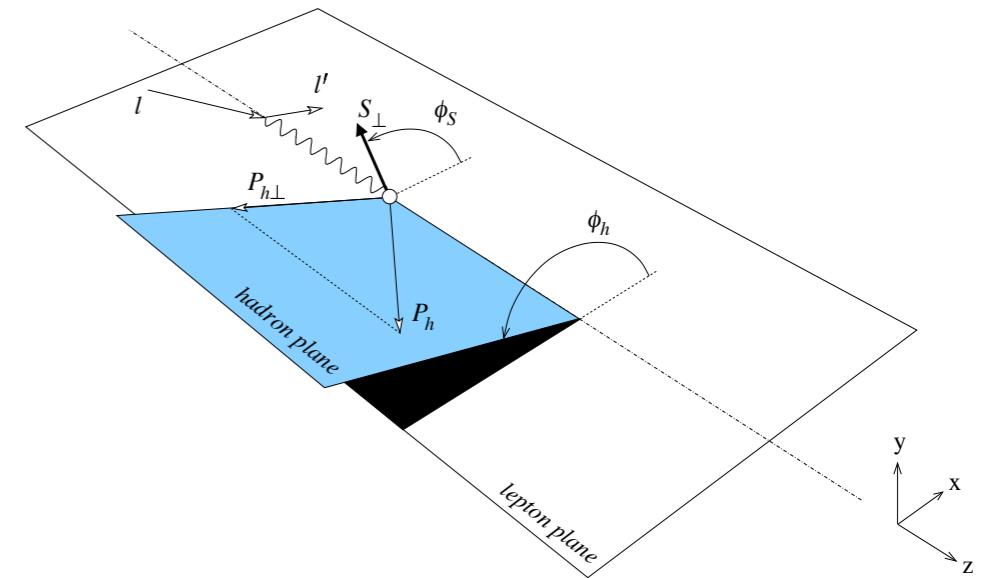
SIDIS in Trento Convention

SIDIS differential cross section

18 structure functions $F(x_B, z, Q^2, P_{hT})$,
(one photon exchange approximation)

$$\frac{d^6\sigma}{dx_B dy dz dP_{hT}^2 d\phi_h d\phi_S}$$

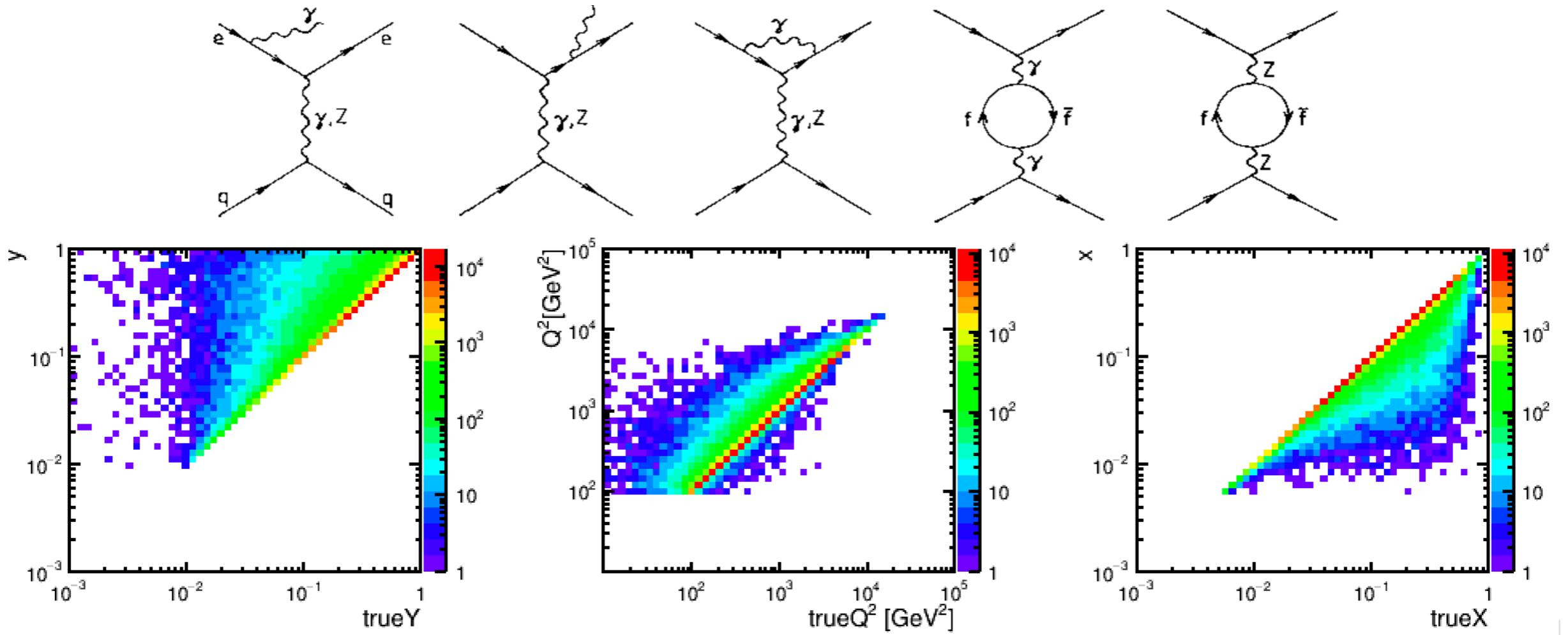
$$\begin{aligned}
 &= \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right) \\
 &\times \left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} F_{UU}^{\cos \phi_h} \cos \phi_h + \epsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h + \lambda_e \sqrt{2\epsilon(1-\epsilon)} F_{LU}^{\sin \phi_h} \sin \phi_h \right. \\
 &+ S_L \left[\sqrt{2\epsilon(1+\epsilon)} F_{UL}^{\sin \phi_h} \sin \phi_h + \epsilon F_{UL}^{\sin 2\phi_h} \sin 2\phi_h \right] + \lambda_e S_L \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} F_{LL}^{\cos \phi_h} \cos \phi_h \right] \\
 &+ S_T \left[\left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \sin(\phi_h - \phi_S) + \epsilon F_{UT}^{\sin(\phi_h + \phi_S)} \sin(\phi_h + \phi_S) \right. \\
 &+ \epsilon F_{UT}^{\sin(3\phi_h - \phi_S)} \sin(3\phi_h - \phi_S) \quad + \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin \phi_S} \sin \phi_S + \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin(2\phi_h - \phi_S)} \sin(2\phi_h - \phi_S) \left. \right] \\
 &+ \lambda_e S_T \left[\sqrt{1-\epsilon^2} F_{LT}^{\cos(\phi_h - \phi_S)} \cos(\phi_h - \phi_S) \right. \\
 &+ \sqrt{2\epsilon(1-\epsilon)} F_{LT}^{\cos \phi_S} \cos \phi_S + \sqrt{2\epsilon(1-\epsilon)} F_{LT}^{\cos(2\phi_h - \phi_S)} \cos(2\phi_h - \phi_S) \left. \right\}
 \end{aligned}$$



[Trento conventions, PRD70,117504 (2004)]

Need to know the photon-hadron frame.

Kinematics with Radiative Effects



[Figures from X. Chu at 2nd EIC YR workshop]

*Kinematic experience
by the parton*



*Kinematic reconstructed
from observed momenta*

QED radiation will have significant impact due to kinematic shift, although α is small.

Traditional Method to Handle QED Radiation

Radiative correction (RC) to Born kinematics:

$$\sigma_{\text{measured}} = \sigma_{\text{No QED radiation}} \otimes \eta_{\text{RC}} \xrightarrow{\text{RC factor}}$$

“In many nuclear physics experiments, radiative corrections quickly become a dominant source of systematics. In fact, the uncertainty on the corrections might be the dominant source for high-statistics experiment”

— EIC Yellow Report

Problems or challenges:

The determination of RC factor relies on Monte Carlo simulation.

Usually depends on the physics we want to extract, hence introducing bias.

Also depends on experimental acceptance.

increasingly difficult for reactions beyond inclusive DIS, e.g. SIDIS ...

Multidimensional kinematic shift, challenge to decouple 18 structure functions.

Almost impossible to determine the virtual photon event by event, and thus the *true photon-hadron frame*.

Problematic to define P_{hT} and azimuthal angles, essential for TMD physics.

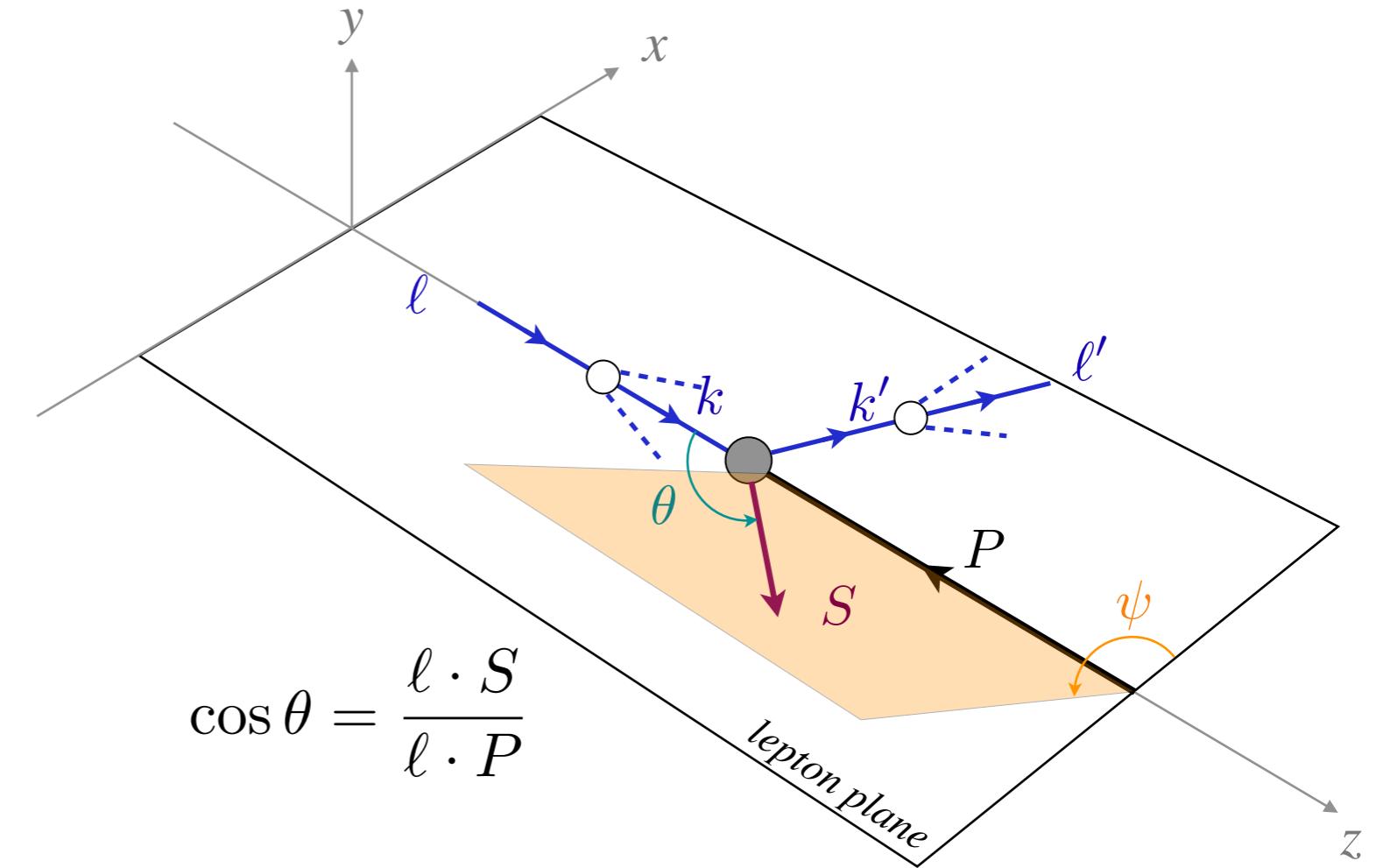
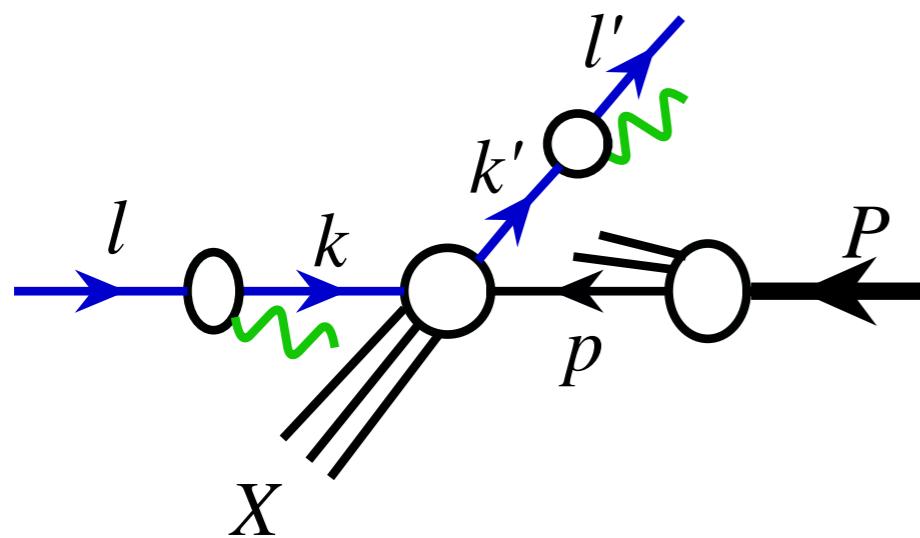
Basic Ideas of Our Approach

- Do not try to invent any scheme to treat QED radiation to match Born kinematics. — No radiative correction!
- Generalize the QCD factorization to include Electroweak theory, resum the logarithmic enhanced QED contributions.
 - QED radiation is part of the production cross sections.
 - treat QED radiation in the same way as QCD radiation is treated.
- Same systematically improvable treatment of QED contributions for both inclusive DIS and SIDIS.

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2008.02895 and 2108.13371

Inclusive DIS with QED



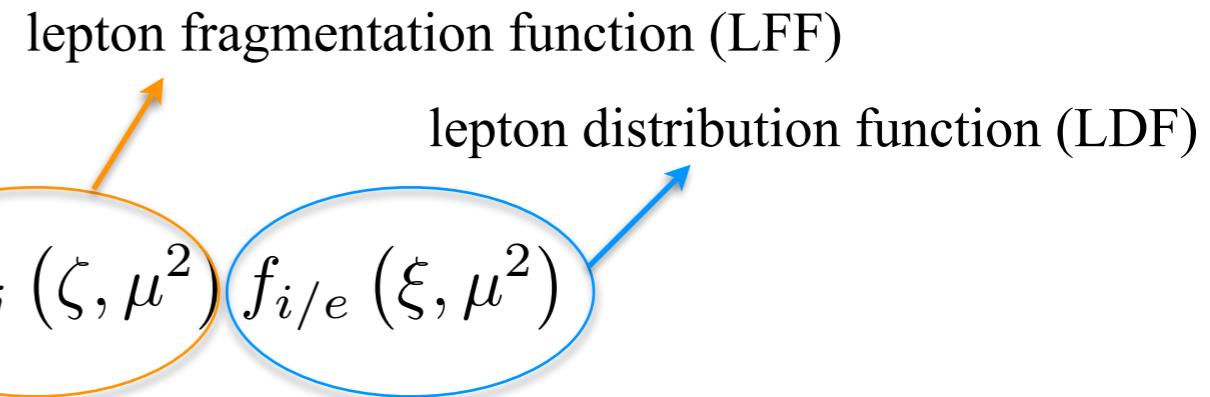
Define inclusive DIS as inclusive lepton scattering with large ℓ'_T

in lepton-hadron frame

Factorized Approach to inclusive DIS

Unpolarized inclusive DIS cross section:

$$E' \frac{d\sigma_{\ell P \rightarrow \ell' X}}{d^3 \ell'} = \frac{1}{2s} \sum_{i,j,a} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^1 \frac{d\xi}{\xi} D_{e/j}(\zeta, \mu^2) f_{i/e}(\xi, \mu^2) \times \int_{x_{\min}}^1 \frac{dx}{x} f_{a/N}(x, \mu^2) \hat{H}_{ia \rightarrow jX}(\xi \ell, xP, \ell'/\zeta, \mu^2) + \dots$$



$$\zeta_{\min} = -\frac{t+u}{s}, \quad \xi_{\min} = -\frac{u}{\zeta s + t}, \quad x_{\min} = -\frac{\xi t}{\zeta \xi s + u}$$

one-photon exchange approximation:

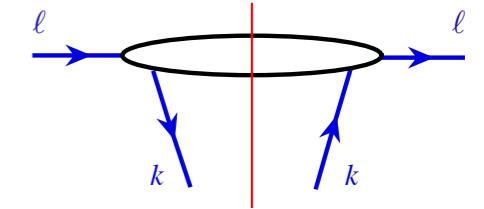
$$\begin{aligned} \frac{d^2\sigma_{\ell P \rightarrow \ell' X}}{dx_B dy} &\approx \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^1 d\xi D_{e/e}(\zeta, \mu^2) f_{e/e}(\xi, \mu^2) \left[\frac{Q^2}{x_B} \frac{\hat{x}_B}{\hat{Q}^2} \right] \\ &\times \frac{4\pi\alpha^2}{\hat{x}_B \hat{y} \hat{Q}^2} \left[\hat{x}_B \hat{y}^2 F_1(\hat{x}_B, \hat{Q}^2) + \left(1 - \hat{y} - \frac{1}{4} \hat{y}^2 \hat{\gamma}^2 \right) F_2(\hat{x}_B, \hat{Q}^2) \right] \end{aligned}$$

$$\hat{Q}^2 = -\hat{q}^2 = \frac{\xi}{\zeta} Q^2, \quad \hat{x}_B = \frac{\hat{Q}^2}{2P \cdot \hat{q}}, \quad \hat{y} = \frac{P \cdot \hat{q}}{P \cdot k}, \quad \hat{\gamma} = \frac{2M \hat{x}_B}{\hat{Q}}$$

LDF and LFF

Lepton distribution function:

$$f_{i/e}(\xi) = \int \frac{dz^-}{4\pi} e^{i\xi \ell^+ z^-} \langle e | \bar{\psi}_i(0) \gamma^+ \Phi_{[0,z^-]} \psi_i(z^-) | e \rangle$$



LO: $f_{i/e}^{(0)}(\xi) = \delta_{ie} \delta(1 - \xi)$

NLO($\overline{\text{MS}}$): $f_{e/e}^{(1)}(\xi, \mu^2) = \frac{\alpha}{2\pi} \left[\frac{1 + \xi^2}{1 - \xi} \ln \frac{\mu^2}{(1 - \xi)^2 m_e^2} \right]_+$

Lepton fragmentation function:

$$D_{e/j}(\zeta) = \frac{\zeta}{2} \sum_X \int \frac{dz^-}{4\pi} e^{i\ell'^+ z^- / \zeta} \text{Tr} [\gamma^+ \langle 0 | \bar{\psi}_j(0) \Phi_{[0,\infty]} | e, X \rangle \langle e, X | \psi_j(z^-) \Phi_{[z^-, \infty]} | 0 \rangle]$$

LO: $D_{e/j}^{(0)}(\zeta) = \delta_{ej} \delta(1 - \zeta)$

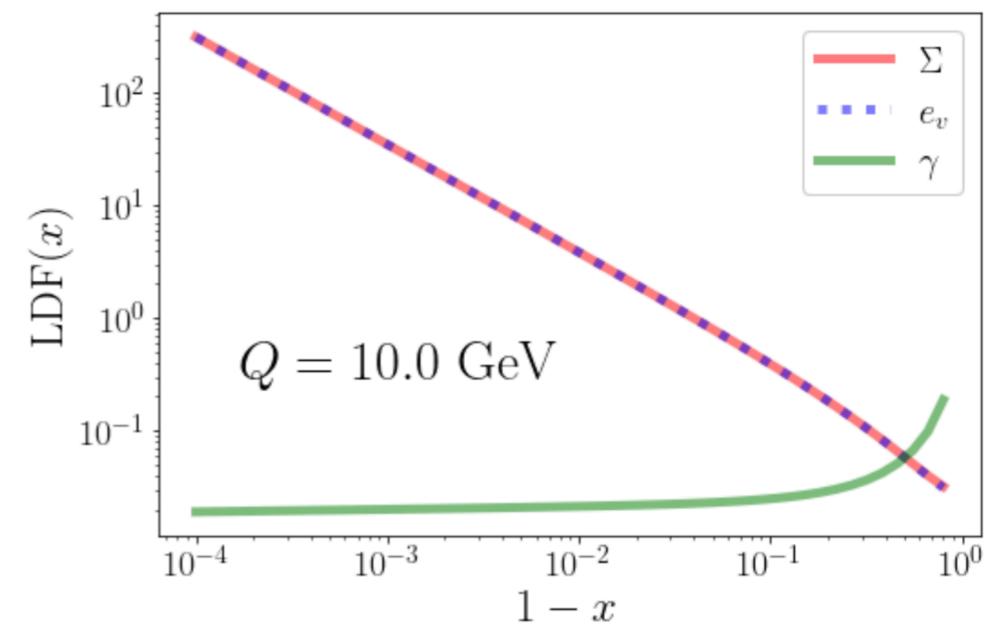
NLO($\overline{\text{MS}}$): $D_{e/e}^{(1)}(\zeta, \mu) = \frac{\alpha}{2\pi} \left[\frac{1 + \zeta^2}{1 - \zeta} \ln \frac{\zeta^2 \mu^2}{(1 - \zeta)^2 m_e^2} \right]_+$

Resum:

$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} f_+ \\ f_\gamma \end{pmatrix} = \begin{pmatrix} P_{ee} & P_{e\gamma} \\ P_{\gamma e} & P_{\gamma\gamma} \end{pmatrix} \otimes \begin{pmatrix} f_+ \\ f_\gamma \end{pmatrix}$$

QED DGLAP evolution

Similar for LFF



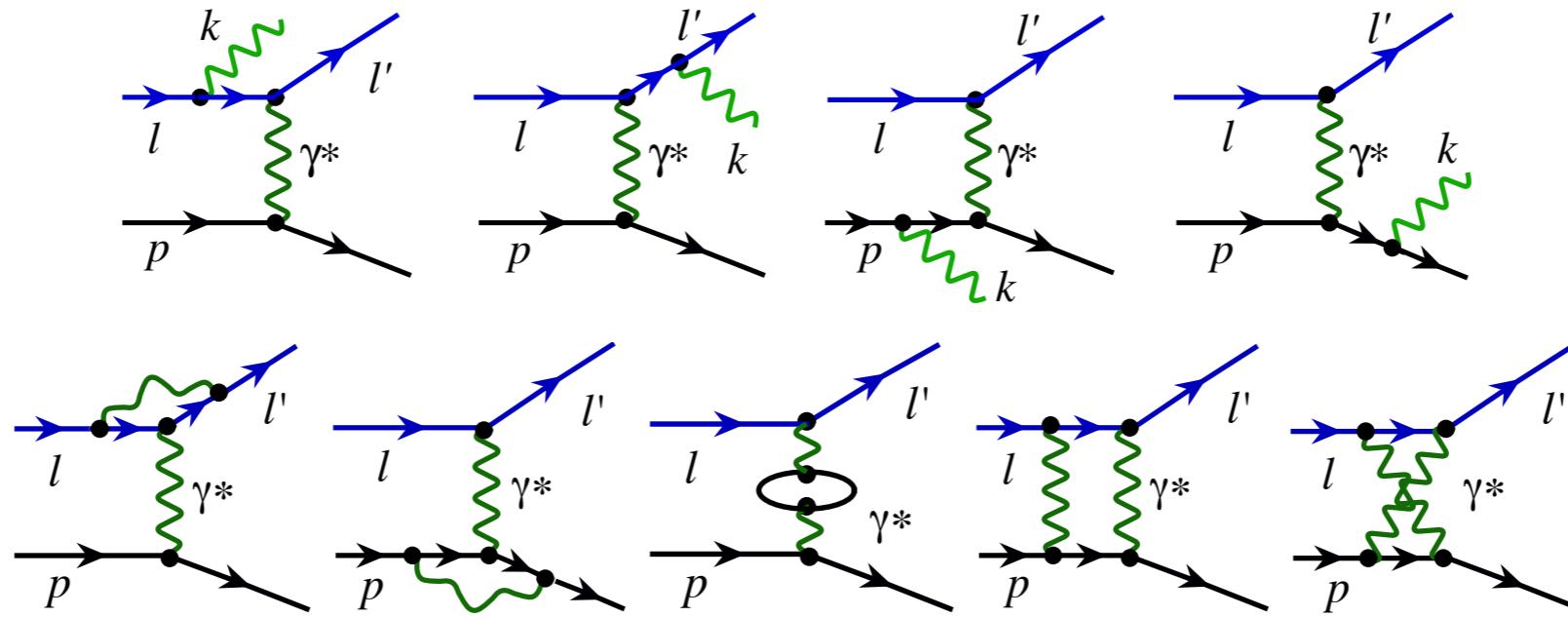
Hard Part of Inclusive DIS

LO:

$$\sigma_{eq}^{(2,0)} = D_{e/e}^{(0)} \otimes f_{e/e}^{(0)} \otimes f_{q/q}^{(0)} \otimes \hat{H}_{eq \rightarrow eX}^{(2,0)} = \hat{H}_{eq \rightarrow eX}^{(2,0)}$$

$$\hat{H}_{eq \rightarrow eX}^{(2,0)} = \frac{4\alpha^2 e_q^2}{\zeta} \left[\frac{(\zeta \xi x s)^2 + (x u)^2}{(\xi t)^2} \right] \delta(\zeta \xi x s + x u + \xi t)$$

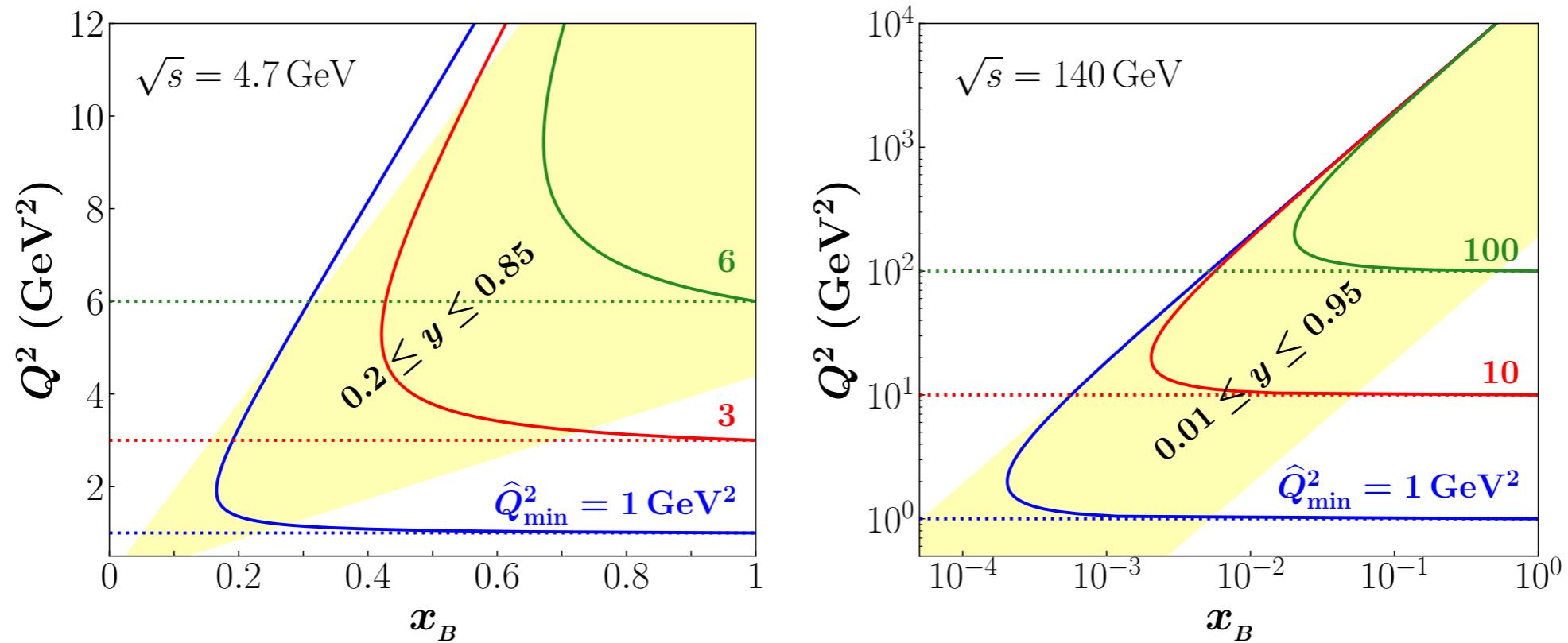
NLO:



$$\hat{H}_{eq \rightarrow eX}^{(3,0)} = \sigma_{eq}^{(3,0)} - D_{e/e}^{(1)} \otimes \hat{H}_{eq \rightarrow eX}^{(2,0)} - f_{e/e}^{(1)} \otimes \hat{H}_{eq \rightarrow eX}^{(2,0)} - f_{q/q}^{(1)} \otimes \hat{H}_{eq \rightarrow eX}^{(2,0)}$$

The Hard Scale

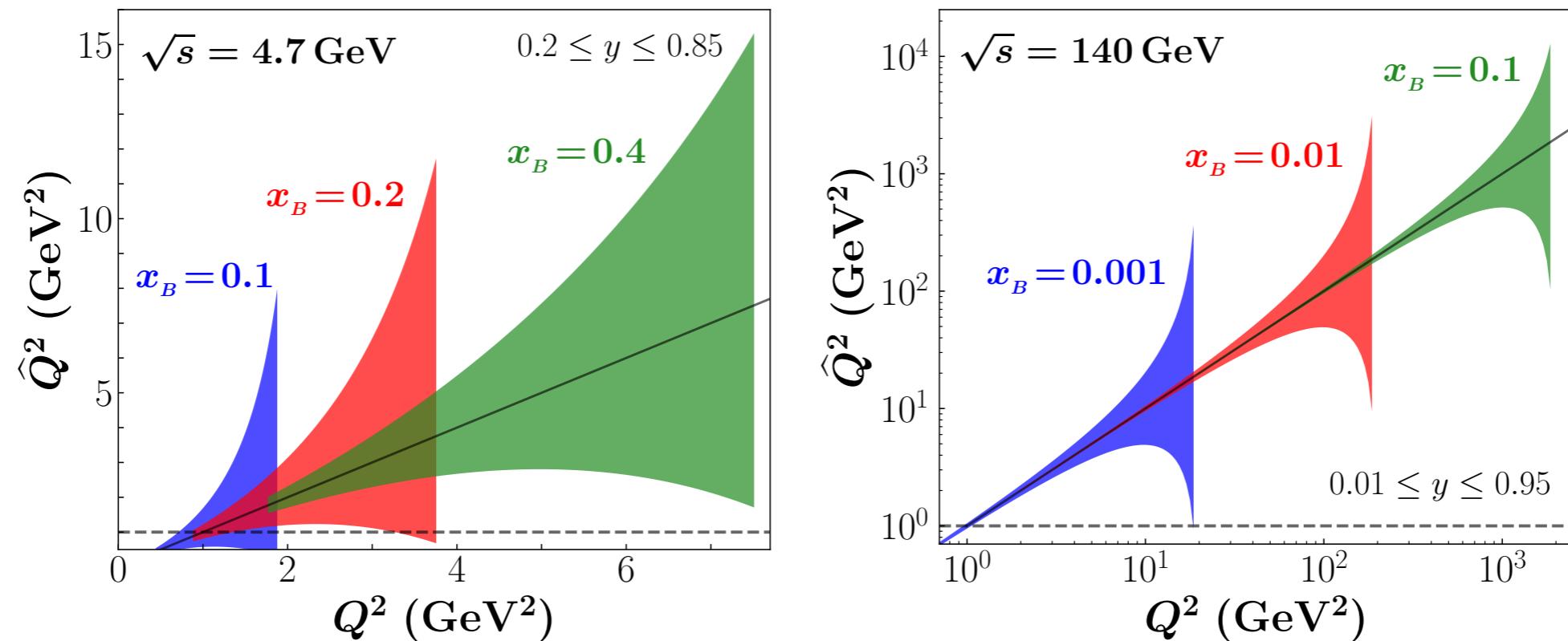
Collision induced QED radiation changes the hard scale from Q^2 to \hat{Q}^2



\hat{Q}^2 has a minimum value $\hat{Q}_{\min}^2 = \frac{1-y}{1-yx_B} Q^2 < Q^2$

The Hard Scale

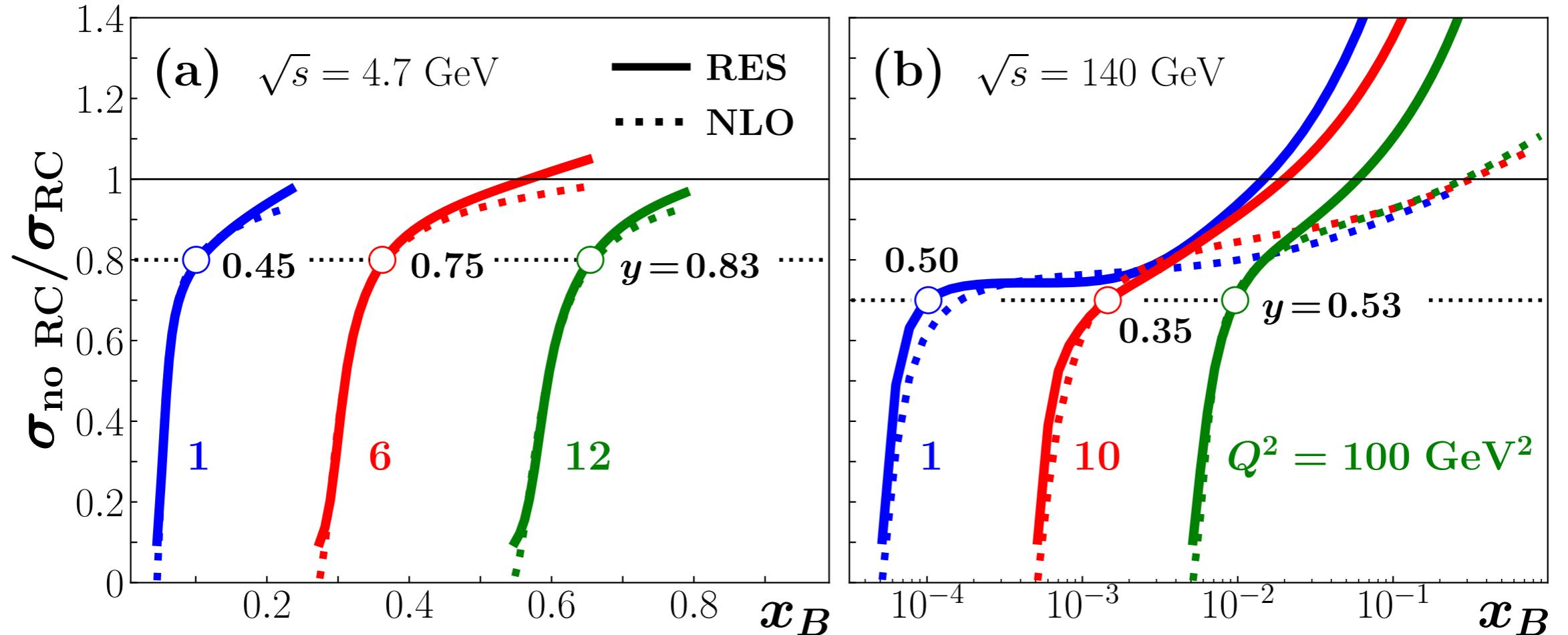
Collision induced QED radiation changes the hard scale from Q^2 to \hat{Q}^2



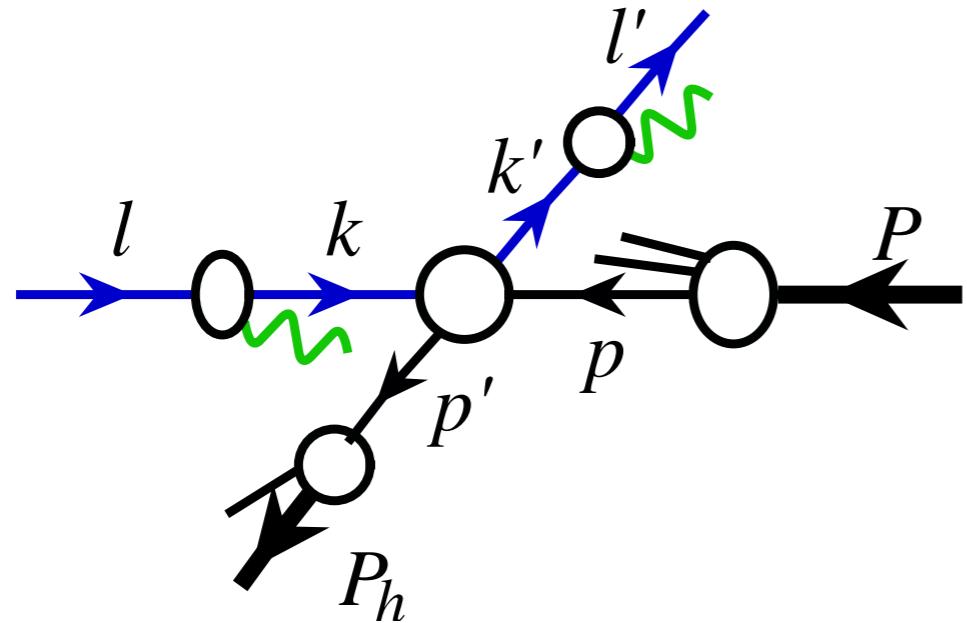
$$\hat{Q}_{\min}^2 = Q^2 \frac{(1 - y)}{(1 - x_B y)}$$

$$\hat{Q}_{\max}^2 = Q^2 \frac{1}{(1 - y + x_B y)}$$

Impact on Inclusive DIS



Semi-inclusive DIS with QED



Define SIDIS as inclusive production of large ℓ'_T lepton plus large P_{hT} hadron.
in lepton-hadron frame

$$\bar{P}_T \equiv |\ell'_T - P_{hT}| / 2 \quad \bar{p}_T \equiv |\ell'_T + P_{hT}|$$

$$\bar{P}_T \gg \bar{p}_T \quad \text{TMD factorization}$$

$$\bar{P}_T \sim \bar{p}_T \quad \text{collinear factorization}$$

SIDIS Cross Section with QED Radiations

Differential cross section

$$d\sigma_{\ell P \rightarrow \ell' P_h X} = \frac{1}{2s} |M_{\ell P \rightarrow \ell' P_h X}|^2 dPS$$

one photon exchange approximation:

$$E_{\ell'} E_{P_h} \frac{d^6 \sigma_{\ell P \rightarrow \ell' P_h X}}{d^3 \ell' d^3 P_h} \approx \frac{\alpha^2}{2s} \int d^4 \hat{q} \left(\frac{1}{\hat{q}^2} \right)^2 \tilde{L}^{\mu\nu}(\ell, \ell', \hat{q}) \tilde{W}_{\mu\nu}(\hat{q}, P, P_h, S) \quad P \rightarrow \text{X}$$

Hadronic tensor: $\tilde{W}_{\mu\nu}(\hat{q}, P, P_h, S) = \sum_{X_h} \int \prod_{i \in X_h} \frac{d^3 p_i}{(2\pi)^3 2E_i} \delta^{(4)} \left(\hat{q} + P - P_h - \sum_{i \in X_h} p_i \right)$

$$\times \langle P, S | J_\mu(0) | P_h X_h \rangle \langle P_h X_h | J_\nu(0) | P, S \rangle$$

Leptonic tensor: $\tilde{L}^{\mu\nu}(\ell, \ell', \hat{q}) \equiv \sum_{X_L} \int \prod_{i \in X_L} \frac{d^3 k_i}{(2\pi)^3 2E_i} \delta^{(4)} \left(\ell - \ell' - \hat{q} - \sum_{i \in X_L} k_i \right)$

$$\times \langle \ell | j^\mu(0) | \ell' X_L \rangle \langle \ell' X_L | j^\nu(0) | \ell \rangle$$

The lowest order recovers no QED radiation expression:

$$\tilde{L}^{\mu\nu(0)}(\ell, \ell', \hat{q}) = 2 (\ell^\mu \ell'^\nu + \ell'^\mu \ell^\nu - \ell \cdot \ell' g^{\mu\nu}) \delta^{(4)}(\ell - \ell' - \hat{q})$$

Lepton SFs in Helicity Basis

Basis vectors and polarization vectors:

$$T^\mu = \frac{\sqrt{\xi_B \zeta_B}}{Q} \tilde{\ell}^\mu + \frac{1}{\sqrt{\xi_B \zeta_B} Q} \tilde{\ell}'^\mu,$$

$$X^\mu = -\frac{\hat{Q} \sqrt{\xi_B \zeta_B}}{Q \sqrt{\hat{\mathbf{q}}_T^2}} \tilde{\ell}^\mu + \frac{\hat{Q}}{Q \sqrt{\xi_B \zeta_B} \sqrt{\hat{\mathbf{q}}_T^2}} \tilde{\ell}'^\mu,$$

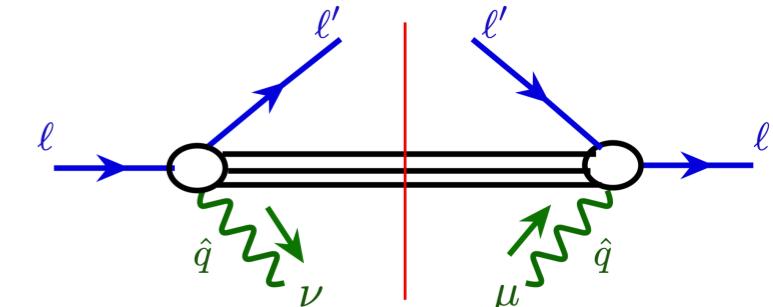
$$Y^\mu = \varepsilon^{\mu\nu\rho\sigma} Z_\nu T_\rho X_\sigma,$$

$$Z^\mu = \frac{\hat{q}^\mu}{Q}$$

$$\epsilon_0^\mu(\hat{q}) = T^\mu,$$

$$\epsilon_+^\mu(\hat{q}) = -\frac{1}{\sqrt{2}} X^\mu - \frac{i}{\sqrt{2}} Y^\mu,$$

$$\epsilon_-^\mu(\hat{q}) = \frac{1}{\sqrt{2}} X^\mu - \frac{i}{\sqrt{2}} Y^\mu,$$



Helicity basis lepton structure functions:

$$\begin{aligned} \tilde{L}^{\mu\nu} &= \epsilon_0^{*\mu} \epsilon_0^\nu L_{00} + (\epsilon_+^{*\mu} \epsilon_+^\nu + \epsilon_-^{*\mu} \epsilon_-^\nu) L_{++} + (\epsilon_+^{*\mu} \epsilon_-^\nu + \epsilon_-^{*\mu} \epsilon_+^\nu) L_{+-} \\ &\quad - \epsilon_0^{*\mu} (\epsilon_+^\nu - \epsilon_-^\nu) L_{0+} - (\epsilon_+^\mu - \epsilon_-^\mu)^* \epsilon_0^\nu L_{+0} \\ &= T^\mu T^\nu L_{00} + (X^\mu X^\nu + Y^\mu Y^\nu) L_{TT} \\ &\quad + (T^\mu X^\nu + T^\nu X^\mu) L_\Delta + (Y^\mu Y^\nu - X^\mu X^\nu) L_{\Delta\Delta}, \end{aligned}$$

Expansion in α :

$$L_{\rho\sigma} = e^2 \sum_{N=0}^{\infty} \left(\frac{\alpha}{\pi} \right)^N L_{\rho\sigma}^{(N)}$$

Leading order: $L_{TT}^{(0)} = 2 \delta(\xi - 1) \delta(\frac{1}{\zeta} - 1) \delta^{(2)}(\hat{\mathbf{q}}_T)$

the other three vanish.

Factorization of Lepton Structure Function

CSS factorization

“W+Y” formalism:

$$L_{TT}(\xi_B, \zeta_B, Q^2, \hat{q}_T^2) = \int \frac{d^2 b}{(2\pi)^2} e^{i \hat{q}_T \cdot b} \widetilde{W}_{TT}(\xi_B, \zeta_B, Q^2, b) + Y_{TT}(\xi_B, \zeta_B, Q^2, \hat{q}_T^2)$$

b-space resummed form:

$$\begin{aligned} \widetilde{W}_{TT}(\xi_B, \zeta_B, Q^2, b) &= 2 \int_{\zeta_B}^1 \frac{d\zeta}{\zeta^2} \int_{\xi_B}^1 \frac{d\xi}{\xi} \left[C_D \left(\frac{\zeta_B}{\zeta}, \alpha \right) D(\zeta, \mu_b^2) \right] \left[C_f \left(\frac{\xi_B}{\xi}, \alpha \right) f(\xi, \mu_b^2) \right] \\ &\times \exp \left\{ - \int_{\mu_b^2}^{\mu_Q^2} \frac{d\mu'^2}{\mu'^2} \left[A(\alpha(\mu')) \ln \frac{\mu_Q^2}{\mu'^2} + B(\alpha(\mu')) \right] \right\} \end{aligned}$$

Expansion in α :

$$A = \sum_{N=1}^{\infty} \left(\frac{\alpha}{\pi} \right)^N A^{(N)}$$

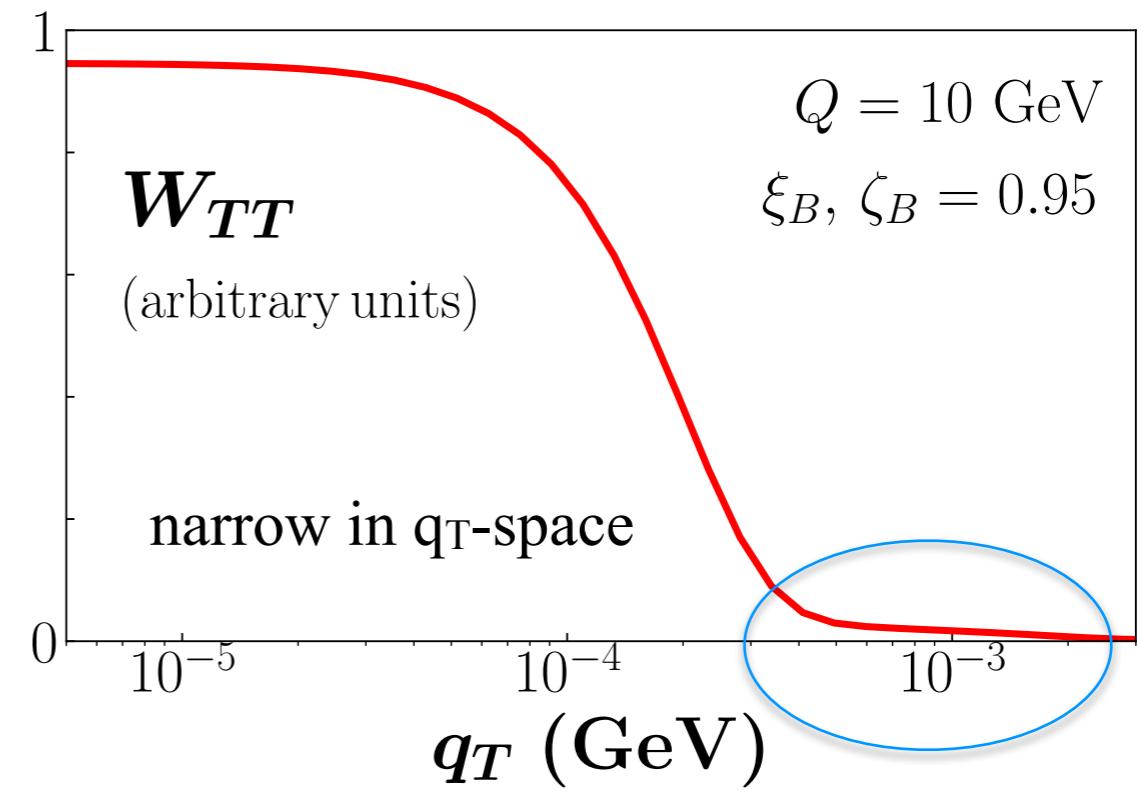
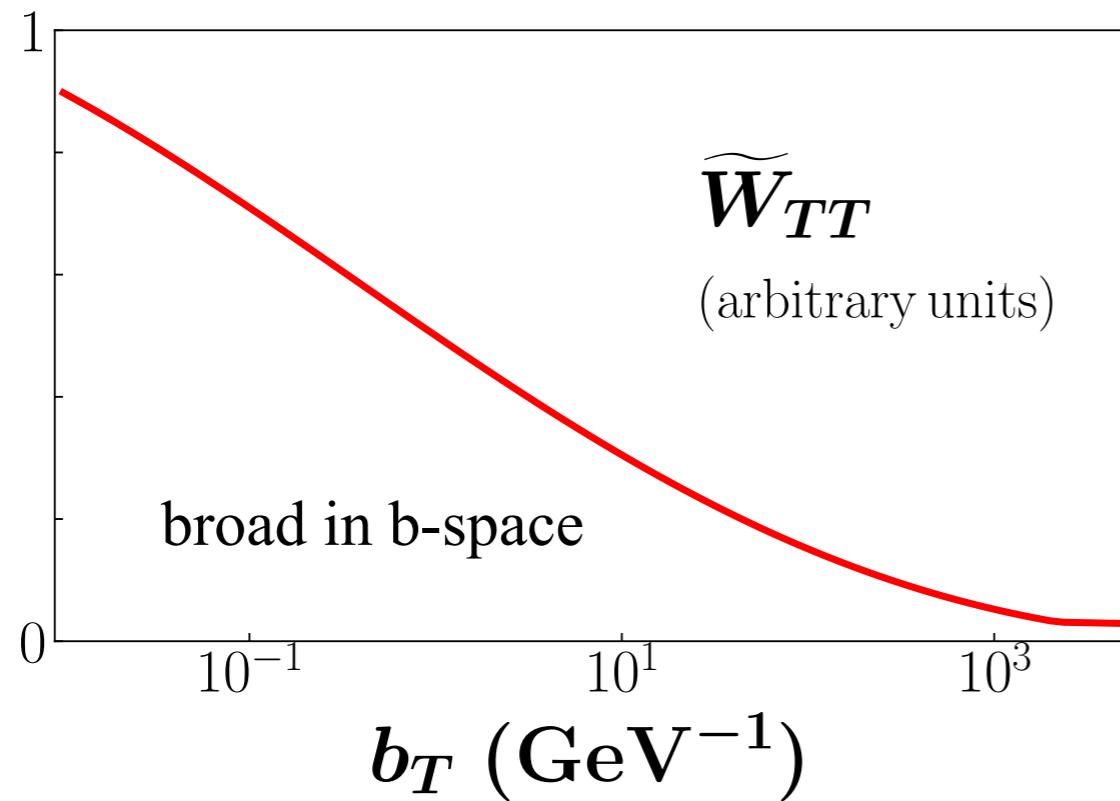
$$B = \sum_{N=1}^{\infty} \left(\frac{\alpha}{\pi} \right)^N B^{(N)}$$

$$C_{f,D} = \sum_{N=0}^{\infty} \left(\frac{\alpha}{\pi} \right)^N C_{f,D}^{(N)}$$

$$\begin{aligned} A^{(1)} &= 1, & C_f^{(0)}(\lambda) &= \delta(1-\lambda) \\ B^{(1)} &= -\frac{3}{2} & C_D^{(0)}(\eta) &= \delta(1-\eta) \\ C_f^{(1)}(\lambda) &= \frac{1}{2}(1-\lambda) - \left(\frac{1+\lambda^2}{1-\lambda} \right)_+ \ln \frac{\mu_{\overline{\text{MS}}}}{\mu_b} - 2\delta(1-\lambda), \\ C_D^{(1)}(\eta) &= \frac{1}{2\eta}(1-\eta) - \frac{1}{\eta} \left(\frac{1+\eta^2}{1-\eta} \right)_+ \ln \frac{\mu_{\overline{\text{MS}}}}{\mu_b} - 2\delta(1-\eta) \end{aligned}$$

Lepton TMD

QED shower generates very small transverse momentum

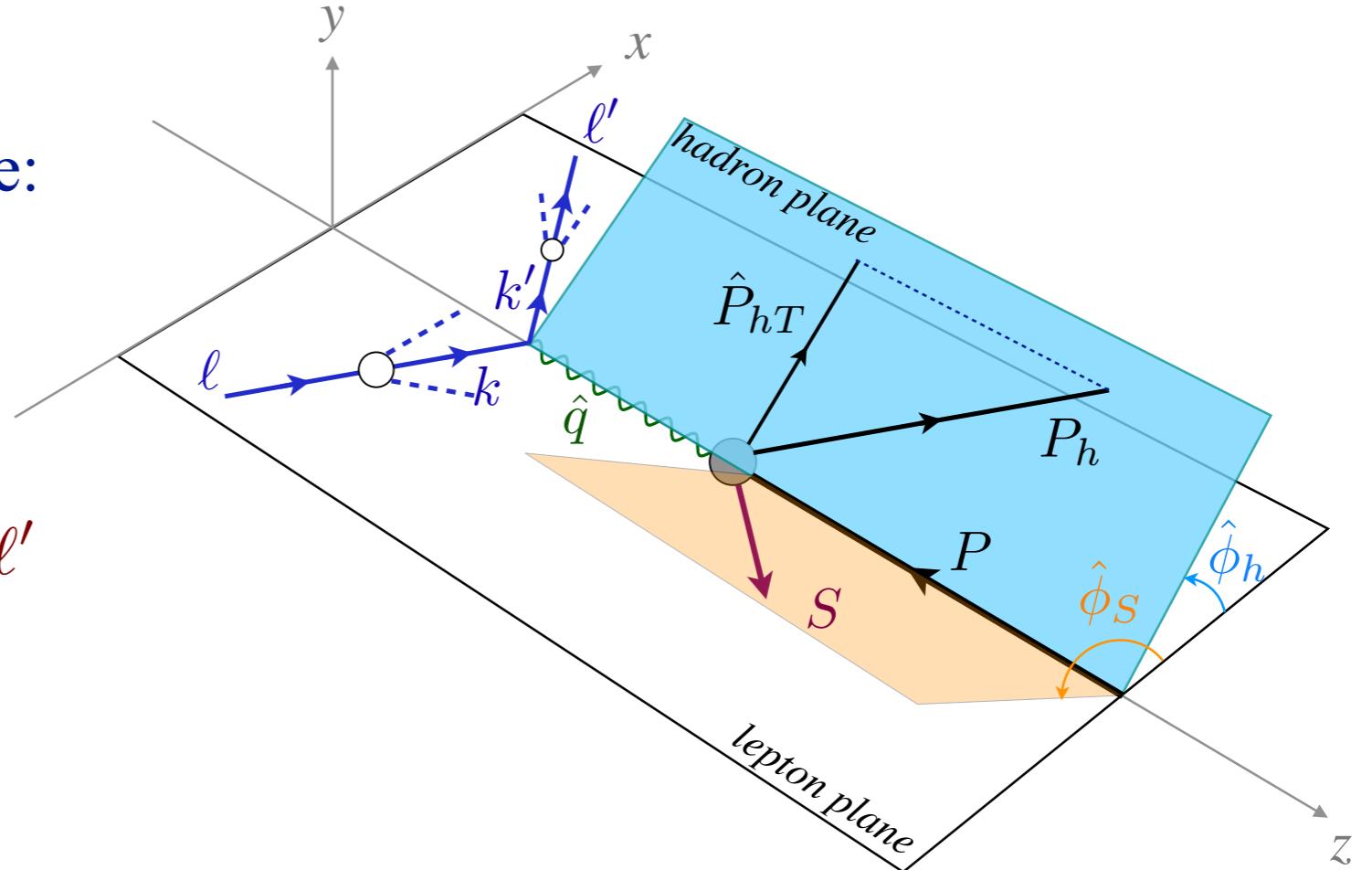


Collinear LDF and LFF are good approximation of lepton TMDs.

Impact on hadron P_{hT} in “photon-hadron frame” is mainly caused by logarithmic enhanced collinear radiation.

SIDIS with Collinear Factorized QED

True photon-hadron frame:



$$\hat{q} = \xi \ell - \frac{1}{\zeta} \ell' \neq \ell - \ell'$$

$$E_{\ell'} E_{P_h} \frac{d^6 \sigma_{\ell(\lambda_\ell) P(S) \rightarrow \ell' P_h X}}{d^3 \ell' d^3 P_h} \approx \sum_{i,j,\lambda_k} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} D_{e/j}(\zeta) \int_{\xi_{\min}}^1 d\xi f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)$$

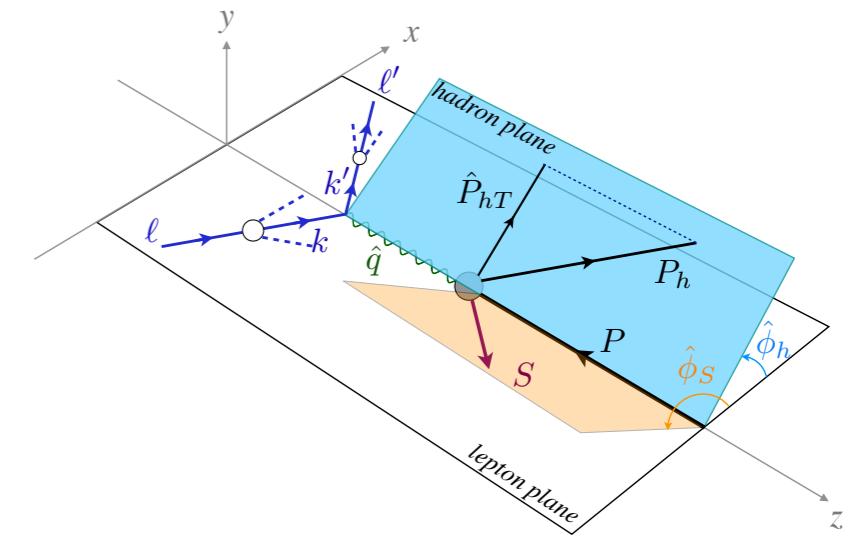
$$\times \left[E_{k'} E_{P_h} \frac{d^6 \hat{\sigma}_{k(\lambda_k) P(S) \rightarrow k' P_h X}}{d^3 k' d^3 P_h} \right]_{k=\xi \ell, k'=\ell'/\zeta}$$

$$E_{k'} E_{P_h} \frac{d^6 \hat{\sigma}_{k(\lambda_k) P(S) \rightarrow k' P_h X}}{d^3 k' d^3 P_h} = \left(\frac{4 \hat{x}_B}{\hat{Q}^2} \sqrt{\hat{z}^2 - \left(\hat{\gamma} \hat{P}_{hT} / \hat{Q} \right)^2} \right) \frac{d^6 \hat{\sigma}_{k(\lambda_k) P(S) \rightarrow k' P_h X}}{d \hat{x}_B d \hat{y} d \hat{\phi}_S d \hat{z} d \hat{\phi}_h d \hat{P}_{hT}^2}$$

SIDIS with Collinear QED Factorization

$$\frac{d^6\sigma}{dx_B dy dz dP_{hT}^2 d\phi_h d\phi_S}$$

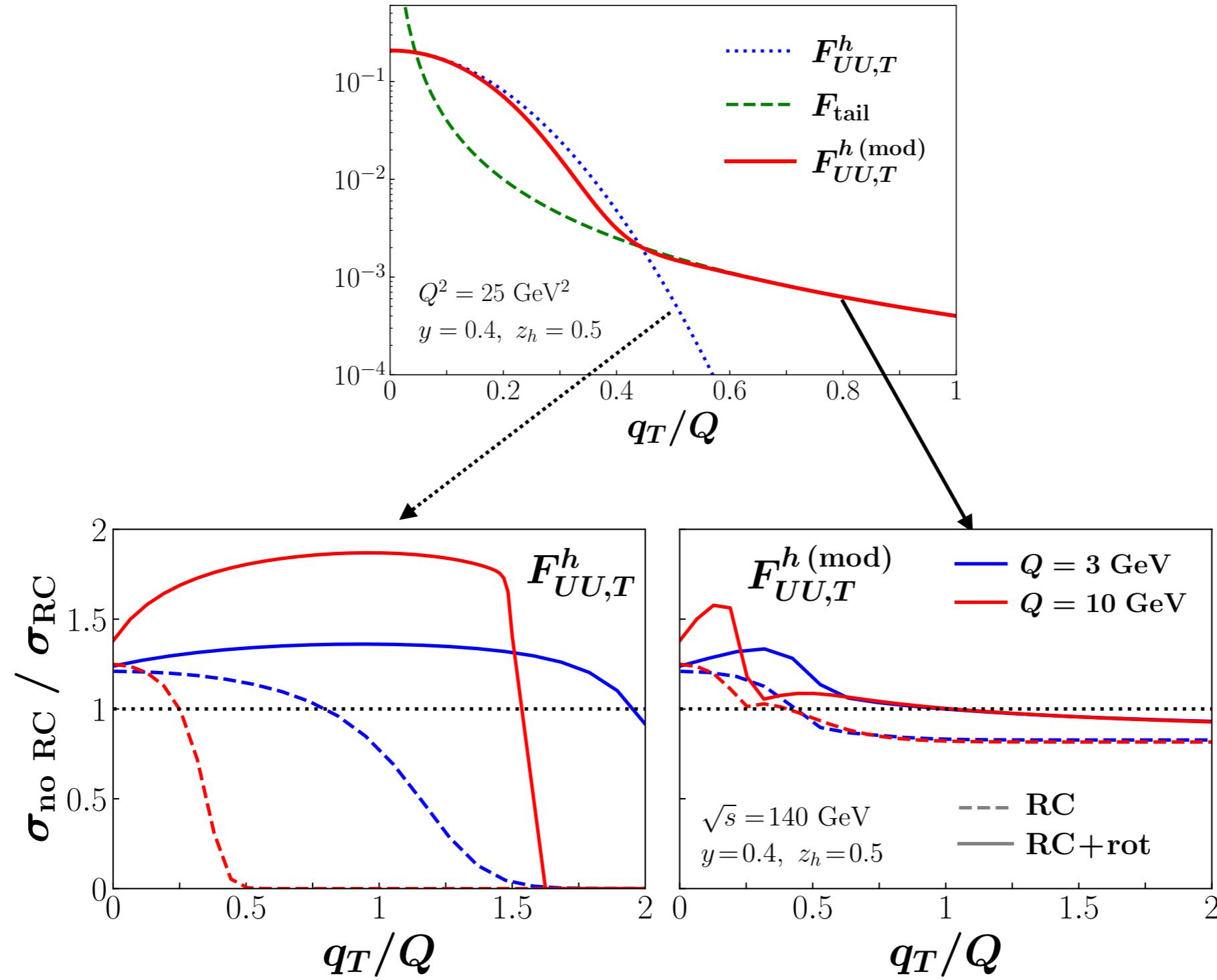
$$\begin{aligned}
 &= \left[\frac{\alpha^2}{x_B y Q^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x_B} \right) \right. \\
 &\quad \times \{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} F_{UU}^{\cos \phi_h} \cos \phi_h + \epsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h + \lambda_e \sqrt{2\epsilon(1-\epsilon)} F_{LU}^{\sin \phi_h} \sin \phi_h \right. \\
 &\quad + S_L \left[\sqrt{2\epsilon(1+\epsilon)} F_{UL}^{\sin \phi_h} \sin \phi_h + \epsilon F_{UL}^{\sin 2\phi_h} \sin 2\phi_h \right] + \lambda_e S_L \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} F_{LL}^{\cos \phi_h} \cos \phi_h \right] \\
 &\quad + S_T \left[\left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \sin(\phi_h - \phi_S) + \epsilon F_{UT}^{\sin(\phi_h + \phi_S)} \sin(\phi_h + \phi_S) \right. \\
 &\quad \left. + \epsilon F_{UT}^{\sin(3\phi_h - \phi_S)} \sin(3\phi_h - \phi_S) + \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin \phi_S} \sin \phi_S + \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin(2\phi_h - \phi_S)} \sin(2\phi_h - \phi_S) \right] \\
 &\quad + \lambda_e S_T \left[\sqrt{1-\epsilon^2} F_{LT}^{\cos(\phi_h - \phi_S)} \cos(\phi_h - \phi_S) \right. \\
 &\quad \left. \left. + \sqrt{2\epsilon(1-\epsilon)} F_{LT}^{\cos \phi_S} \cos \phi_S + \sqrt{2\epsilon(1-\epsilon)} F_{LT}^{\cos(2\phi_h - \phi_S)} \cos(2\phi_h - \phi_S) \right] \right\} \\
 &\quad \{Q^2, x_B, y, \gamma, \epsilon, z, P_{hT}, \phi_h, \phi_S, S_T, S_L\} \rightarrow \{\hat{Q}^2, \hat{x}_B, \hat{y}, \hat{\gamma}, \hat{\epsilon}, \hat{z}, \hat{P}_{hT}, \hat{\phi}_h, \hat{\phi}_S, \hat{S}_T, \hat{S}_L\}
 \end{aligned}$$



$$\left. \otimes f_{e/e}(\xi) \otimes D_{e/e}(\zeta) \left(\frac{\hat{x}_B}{x_B \xi \zeta} \right) \rightarrow \text{Jacobian between the two frames}$$

$\hat{Q}^2, \hat{x}_B, \hat{z}, \hat{P}_{hT}, \hat{\phi}_h, \hat{\phi}_S, \hat{S}_T, \hat{S}_L$ are functions of $\xi, \zeta, Q^2, x_B, z, P_{hT}, \phi_h, \phi_S, S_T, S_L$

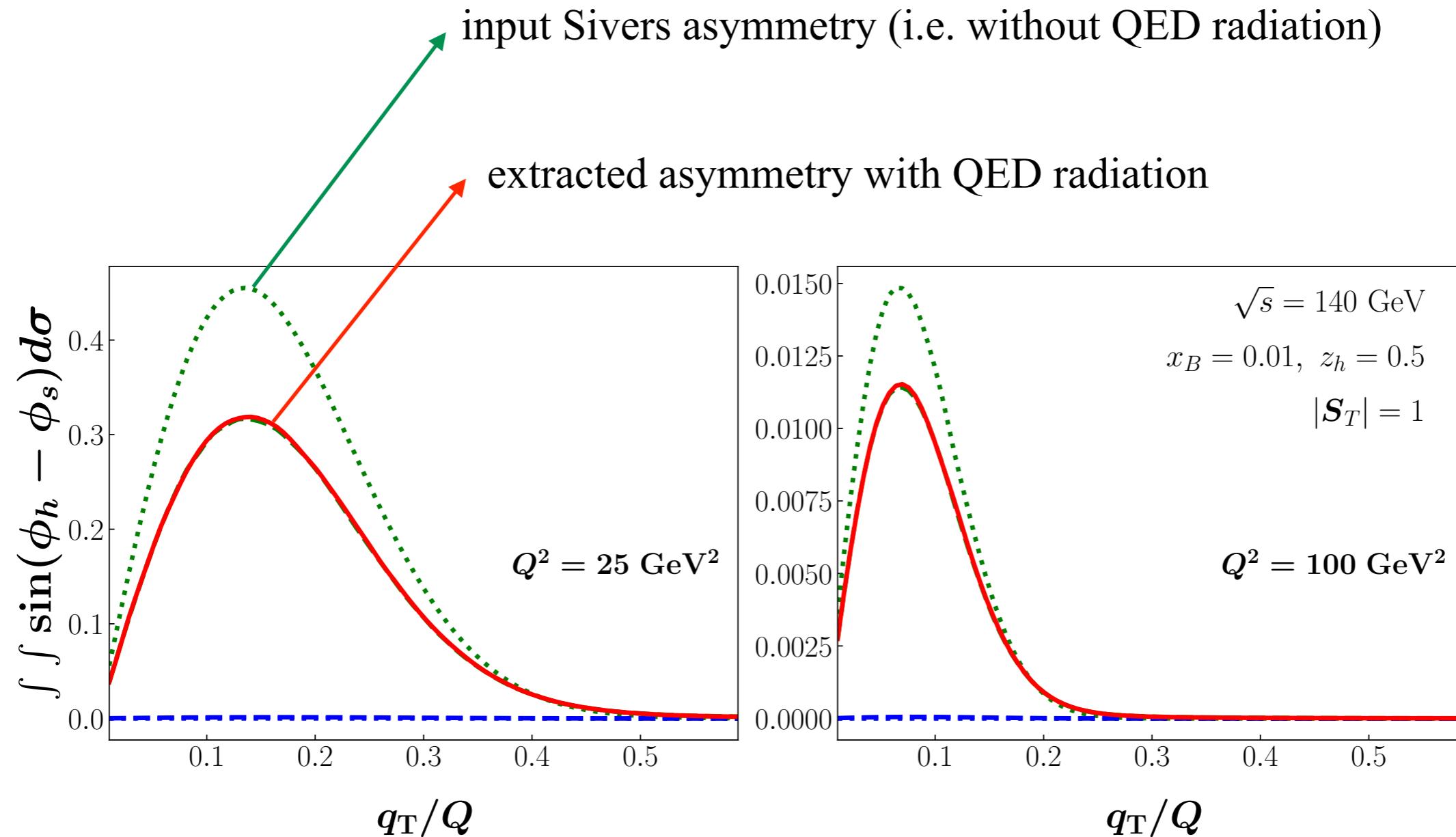
Impact of QED Effects: P_{hT} Distribution



Radiative correction factor depends on the hadronic physics we want to extract.

Impact of QED Effects: Azimuthal Asymmetries

Sivers asymmetry:



Asymmetry amplitude is affected by QED radiation.

Impact of QED Effects: Azimuthal Asymmetries

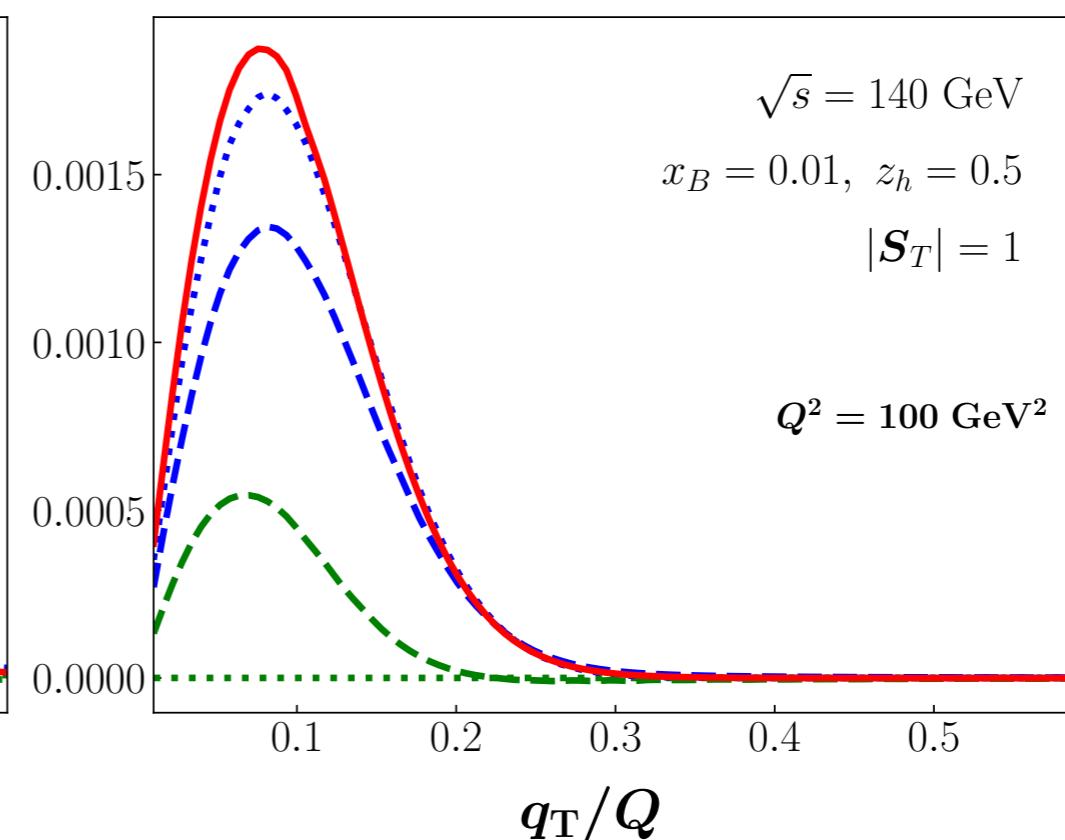
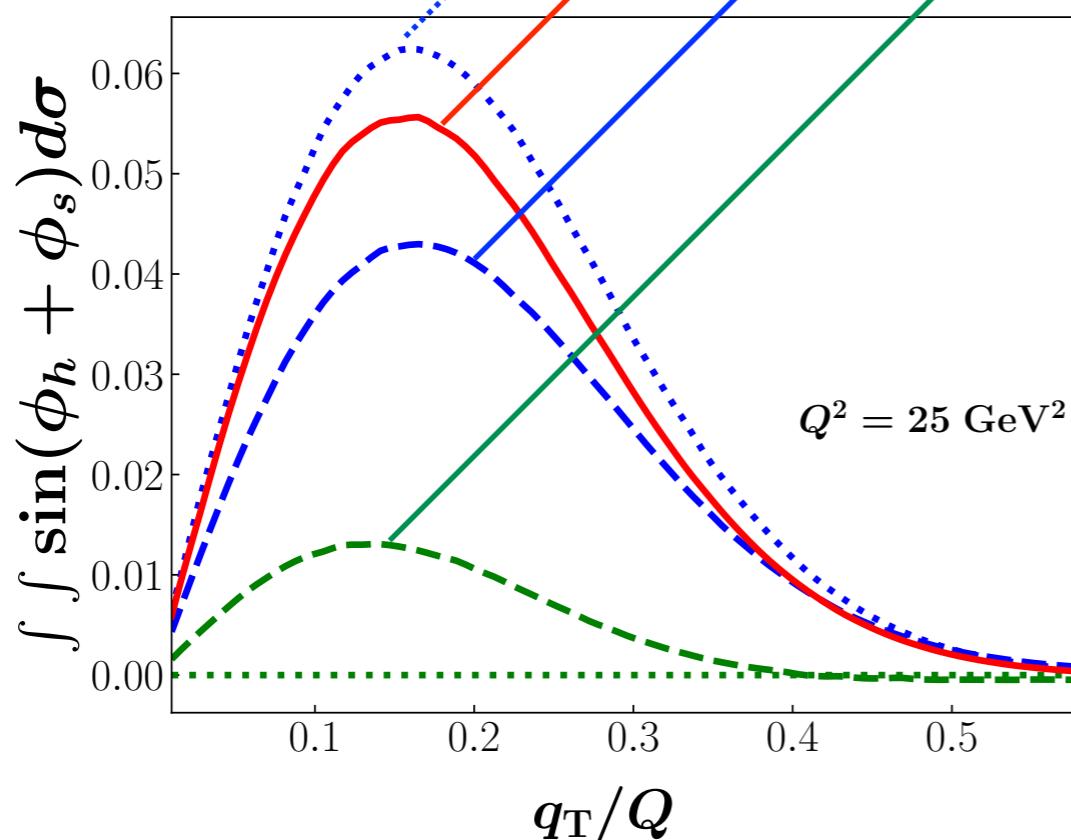
Collins asymmetry:

input Collins asymmetry (i.e. without QED radiation)

extracted Collins asymmetry

Collins asymmetry with QED radiation

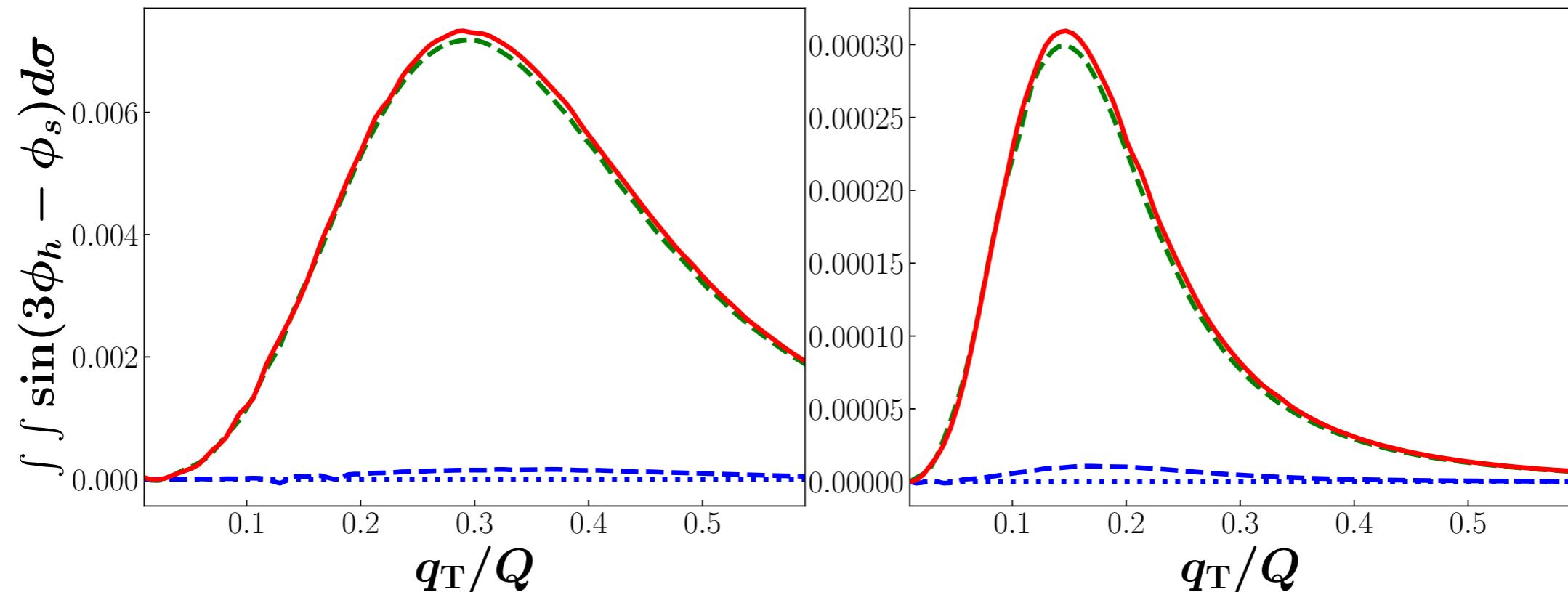
leakage from Sivers asymmetry



Azimuthal modulations mix with each other.

Impact of QED Effects: Azimuthal Asymmetries

Pretzelosity asymmetry:



Input pretzelosity is zero. Extracted pretzelosity asymmetry is all from the leakage of other asymmetries (Sivers, Collins).

Summary

- QED radiation effects are important in SIDIS, and hence precise extractions of TMDs.
 - Experimental “photon-hadron frame” does not coincide with the *true photon hadron frame*, where the factorization works.
 - Almost impossible to determine/reconstruct the *true photon hadron frame* event by event.
 - Challenge to match to Born kinematics without introducing model/theory bias.
- We propose a factorized approach to treat QED radiations.
 - Treat QED radiation as a part of the production cross section.
 - Generalize QCD factorization to include QED. All perturbatively calculable hard parts are IR safe.
 - Transverse momentum generated by QED shower is small, and one can apply collinear factorization for the leptonic tensor.
 - Huge and nontrivial effects on P_{hT} dependence and azimuthal modulations.

Thank you!