

SPIN2021  
Matsue, October 19th, 2021

# Towards leading-twist T-odd TMD gluon distributions

Francesco Giovanni Celiberto

ECT\*/FBK Trento & INFN-TIFPA

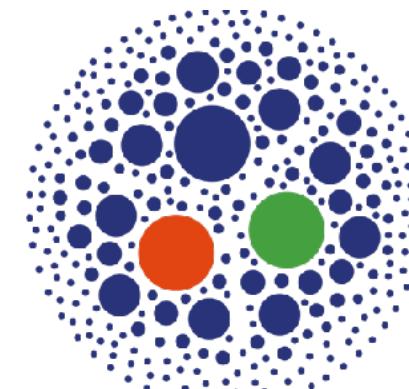
In collaboration with A. Bacchetta and M. Radici



EUROPEAN CENTRE FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS



Trento Institute for  
Fundamental Physics  
and Applications



HAS QCD  
HADRONIC STRUCTURE AND  
QUANTUM CHROMODYNAMICS

# Gluon TMDs: gauge links and modified universality

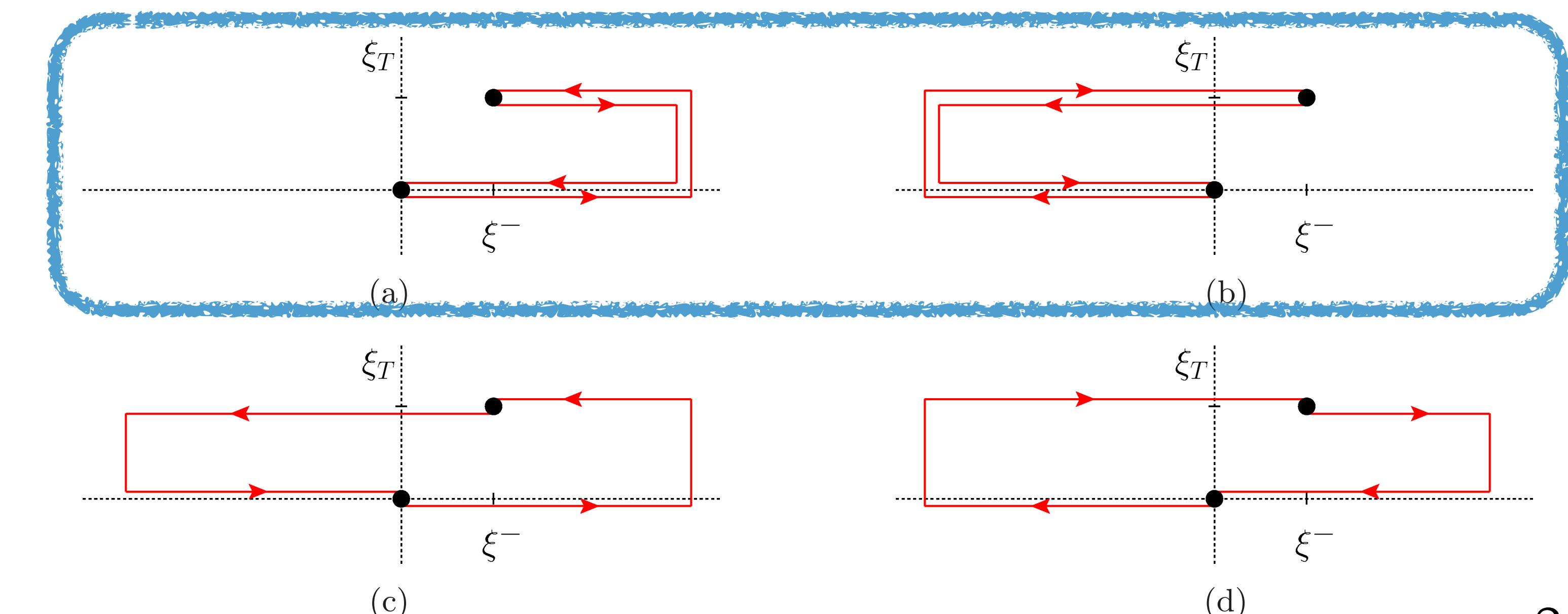
- \* **Single-spin asymmetries** → process dependence of TMDs via **gauge links**
- \* **Color flow** → integration paths of gauge links calculable
- \* Gluon TMDs → more complicated structure with respect to quark **staple links**
- \* **Factorization-preserving** processes → two main kinds of **modified universality**
- \* Different classes of processes → distinct gluon TMDs, **not related** to each other

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## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]

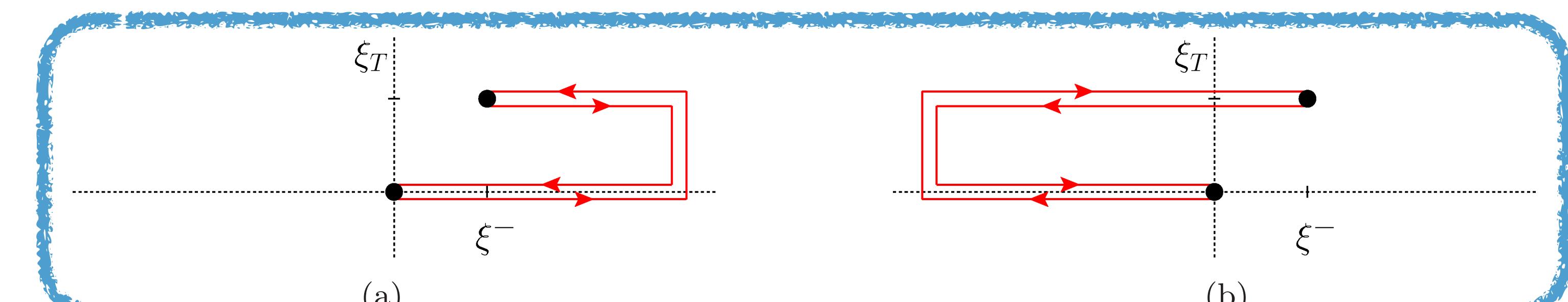


# Gluon TMDs: gauge links and modified universality

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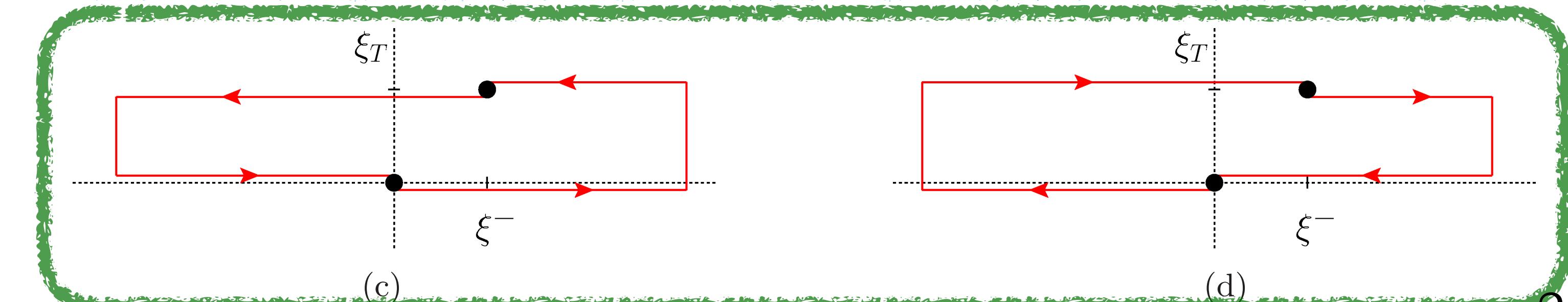
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## Dipole (DP)

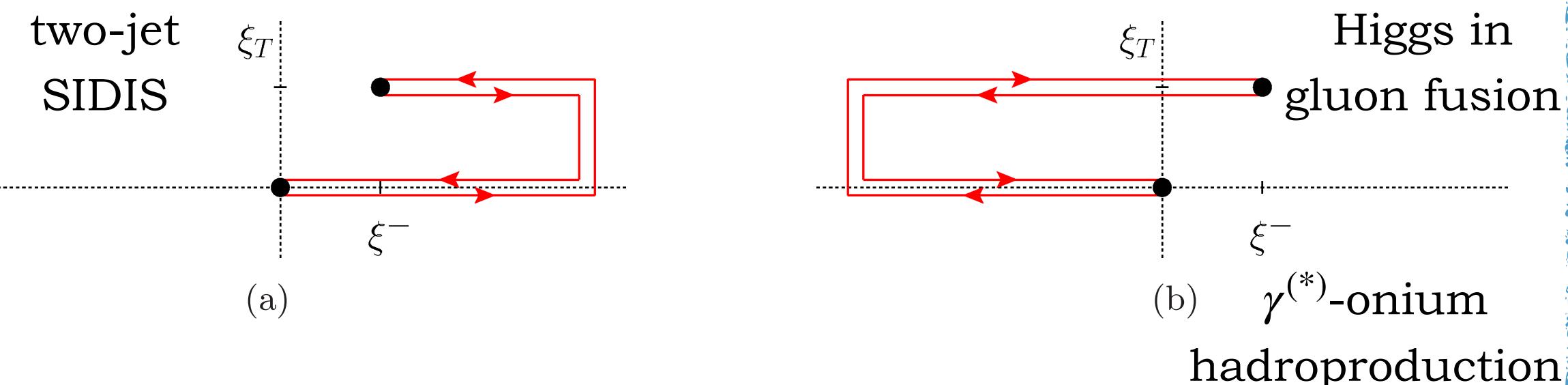
(c) [ + , - ] or (d) [ - , + ]



# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]

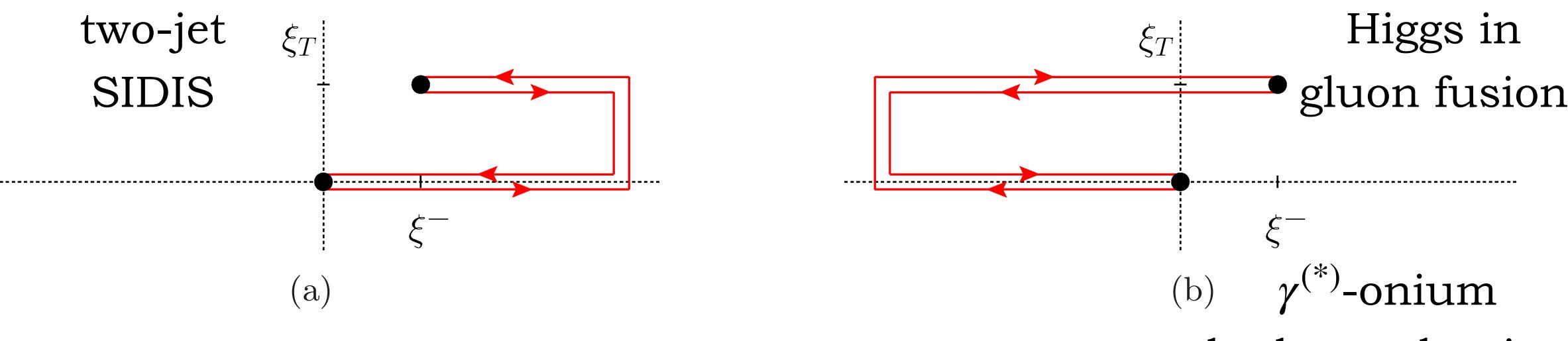


- \* Color flow annihilated within final/initial state
- \*  $f$ -type gluon TMDs  $\rightarrow f^{abc}$  color structure
- \* Modified universality:
$$f_1^{[+,+]} = f_1^{[-,-]},$$
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$
- \* Phenomenology: Higgs, quarkonia or  $\gamma\gamma$  in  $pp$ , two-jet SIDIS, heavy-quark pair SIDIS

# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

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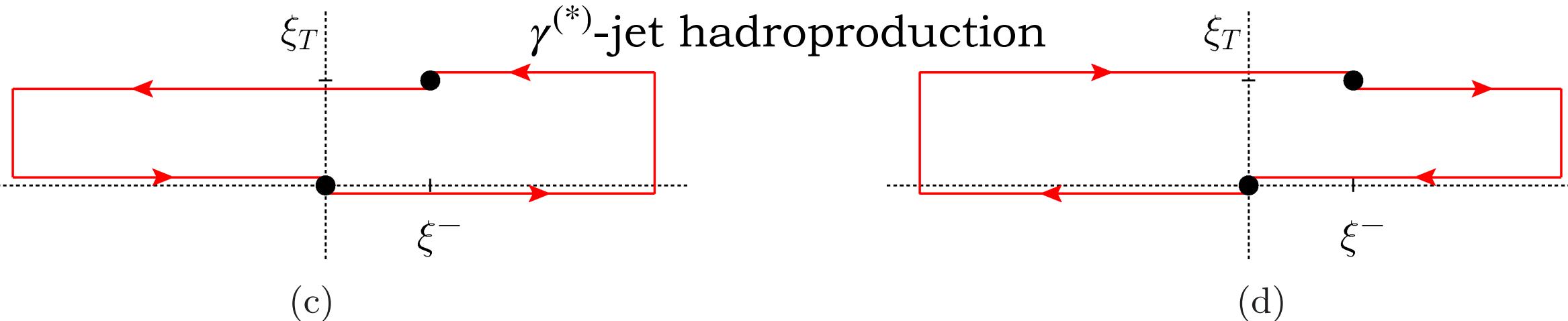


- \* Color flow annihilated within final/initial state
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- \* Phenomenology: Higgs, quarkonia or  $\gamma\gamma$  in  $pp$ , two-jet SIDIS, heavy-quark pair SIDIS

## Dipole (DP)

(c) [ + , - ] or (d) [ - , + ]



- \* Color flow involving both initial and final states
- \*  $d$ -type gluon TMDs  $\rightarrow d^{abc}$  color structure
- \* Modified universality:
$$f_1^{[+,-]} = f_1^{[-,+]},$$

$$f_{1T}^{\perp[+,-]} = -f_{1T}^{\perp[-,+]}$$
- \* Phenomenology: single hadron or  $\gamma^{(*)}\text{-jet}$  hadroproduction, SIDIS or Drell-Yan (subleading)

Gauge link  $\rightarrow$  two main independent sets of TMDs, **not related** to each other

# **T-even and T-odd gluon TMD PDFs at leading-twist**

gluon pol.

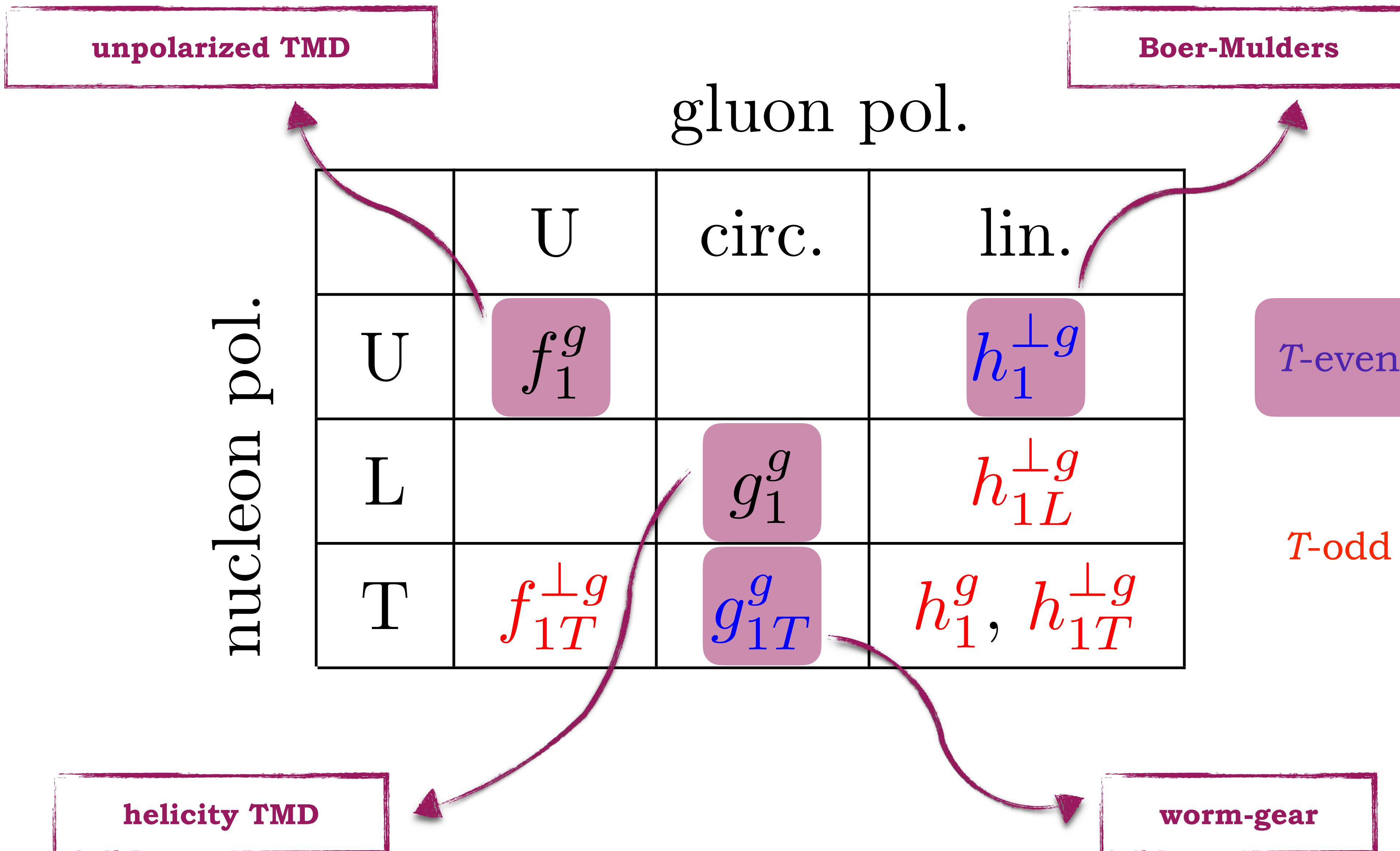
	U	circ.	lin.
U	$f_1^g$		$h_1^{\perp g}$
L		$g_1^g$	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g, h_{1T}^{\perp g}$

*T-even*

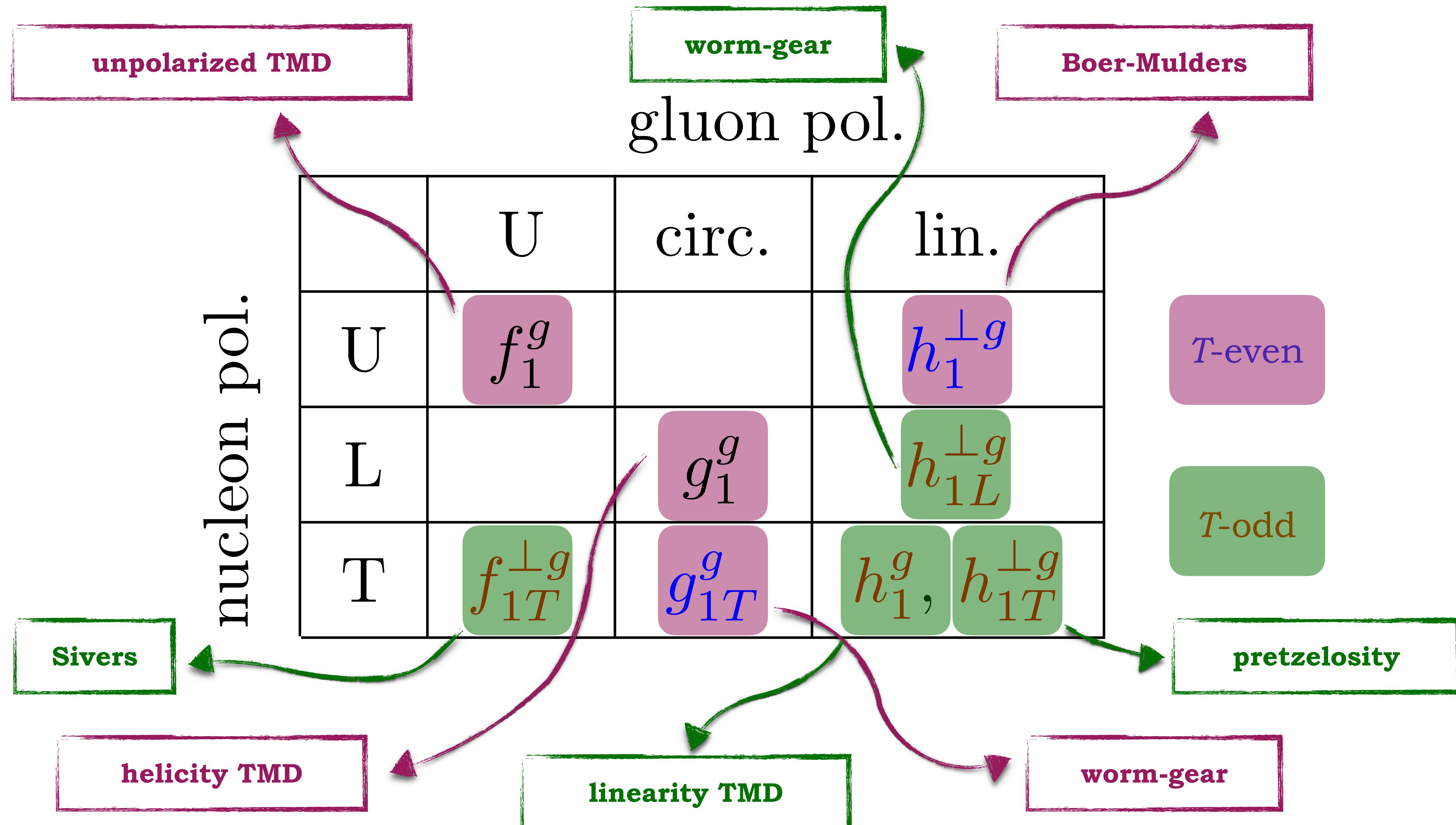
*T-odd*

nucleon pol.

# T-even and T-odd gluon TMD PDFs at leading-twist

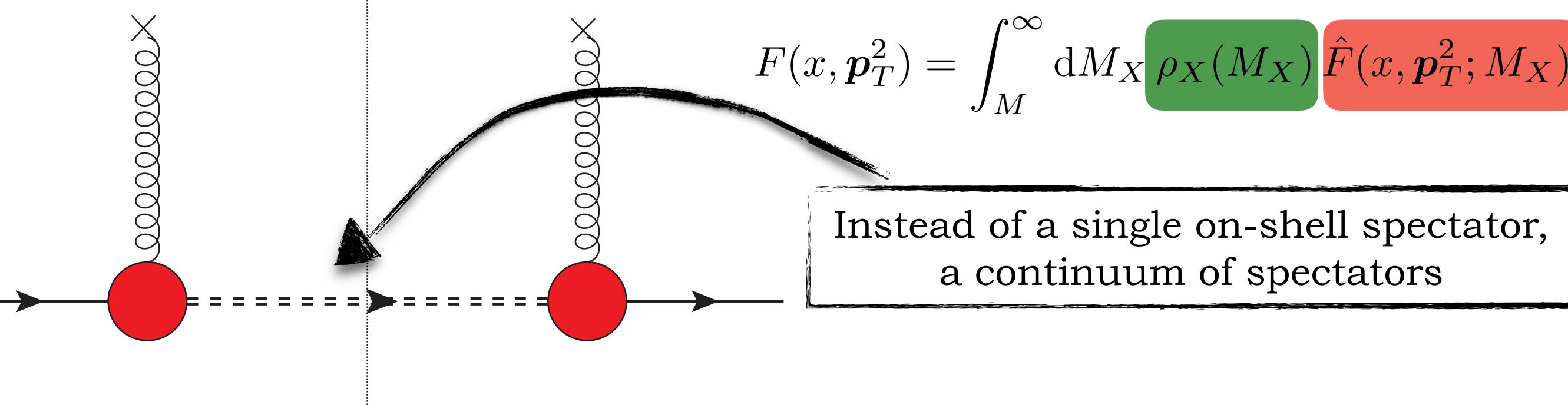


# T-even and T-odd gluon TMD PDFs at leading-twist



# Our model at a glance

## Spectator-system spectral-mass function



Spectral function **learns** small- and moderate- $x$  info  
encoded in **NNPDF** collinear parametrizations

(NNPDF3.1sx + NNPDFpol1.1)

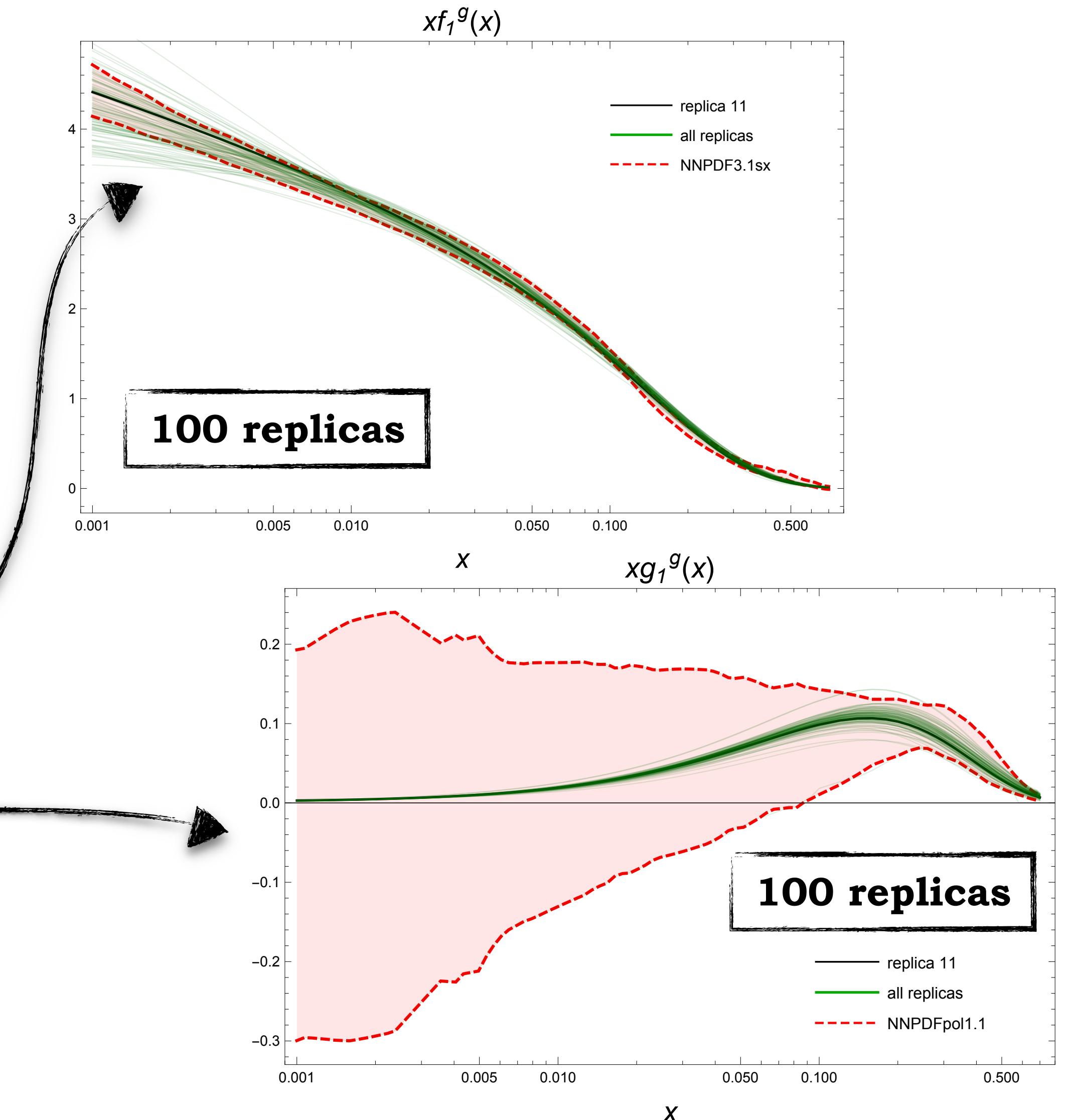
**Simultaneous fit** of  $f_1$  and  $g_1$  PDFs

Inclusion of small- $x$  resummation effects (**BFKL**)

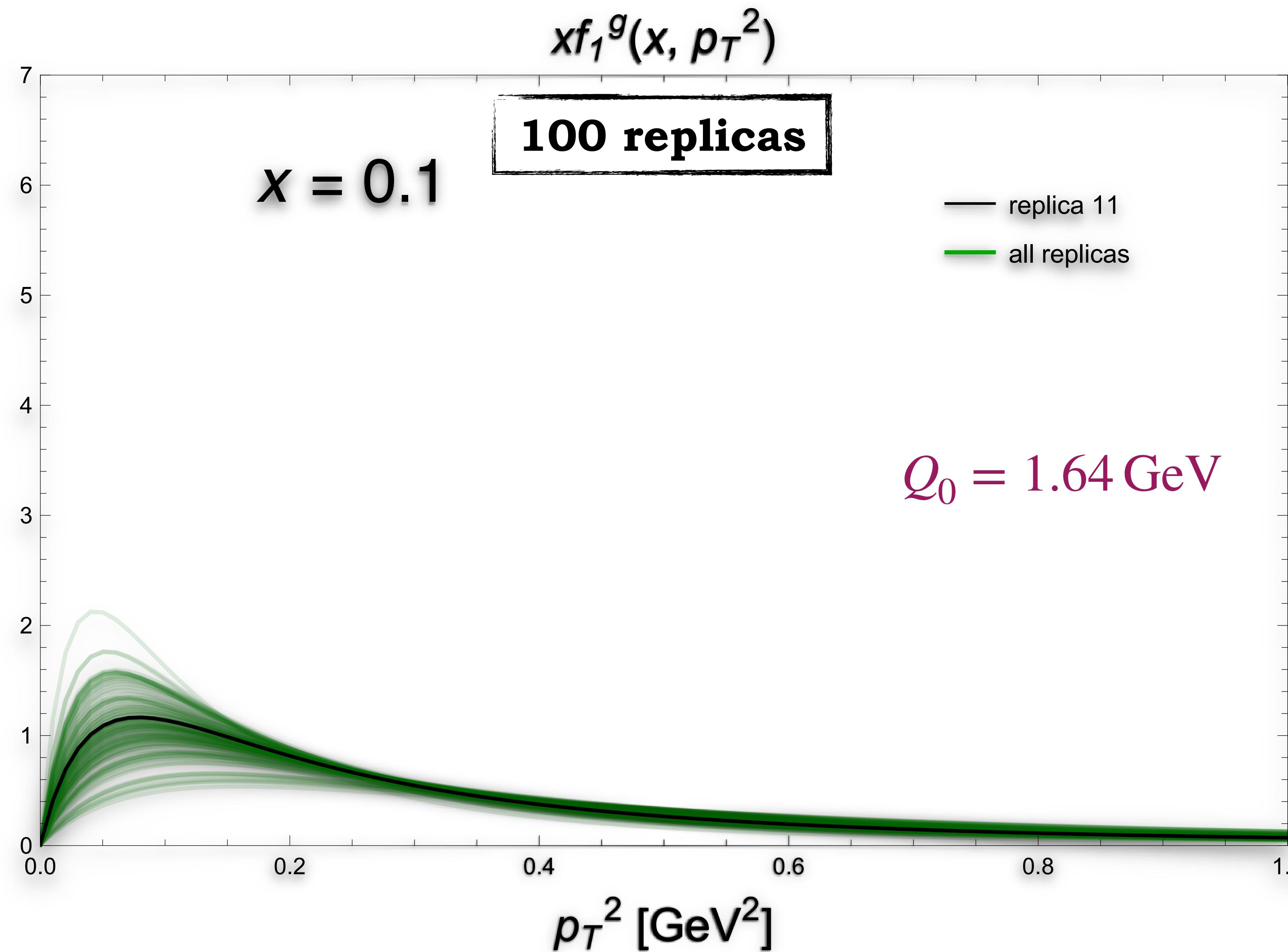
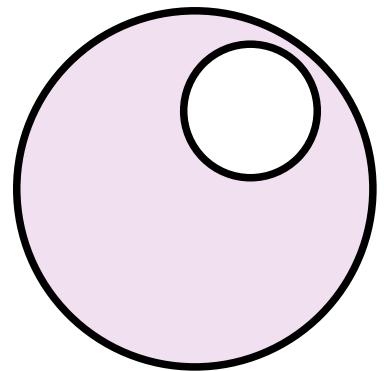
Calculation of all leading-twist  $T$ -even gluon TMDs

## Link with collinear factorization

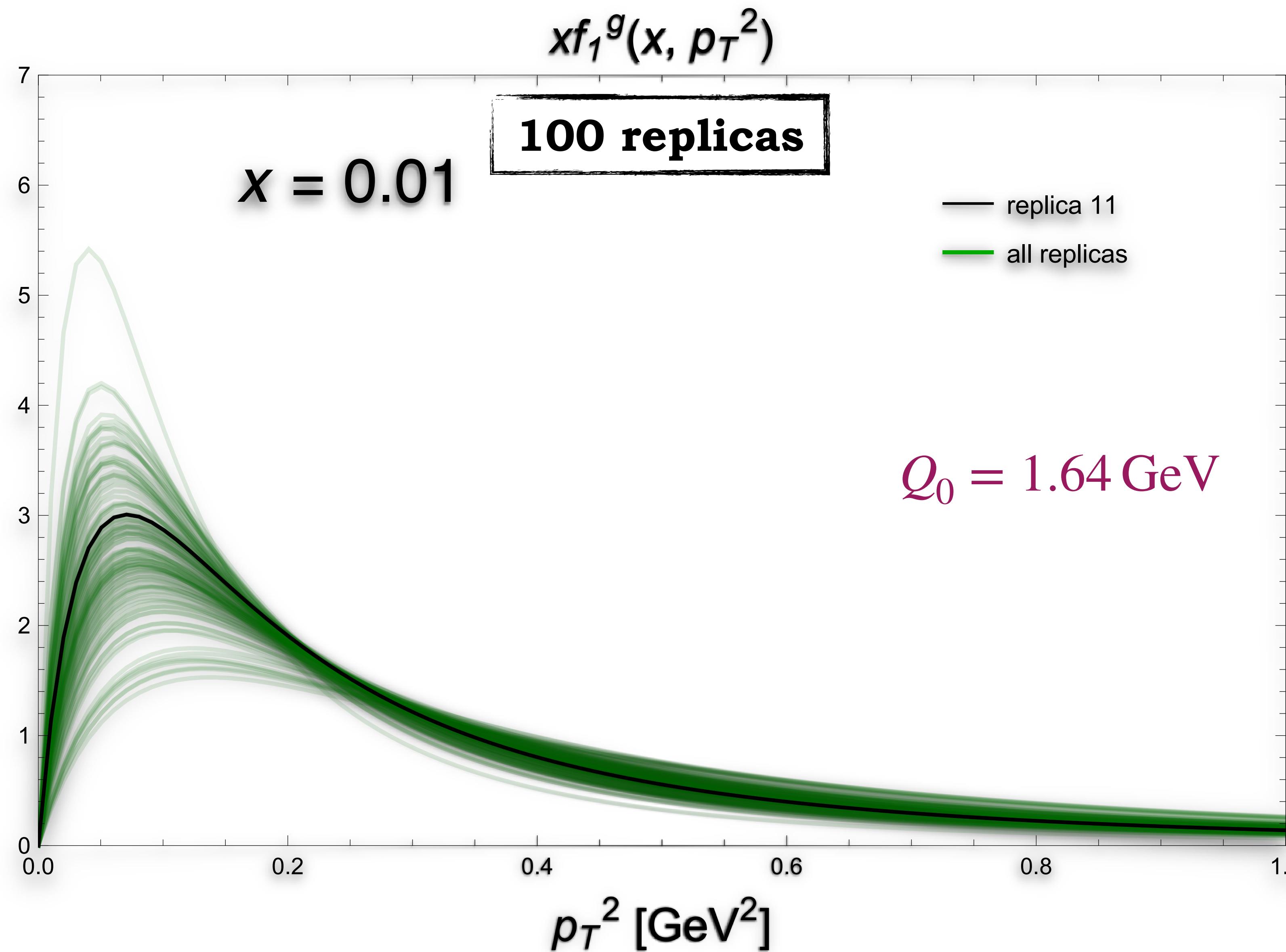
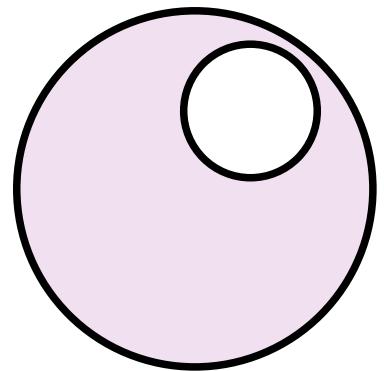
$p_T$ -integrated TMDs **have to** reproduce PDFs  
at the lowest scale ( $Q_0$ ) *before* evolution



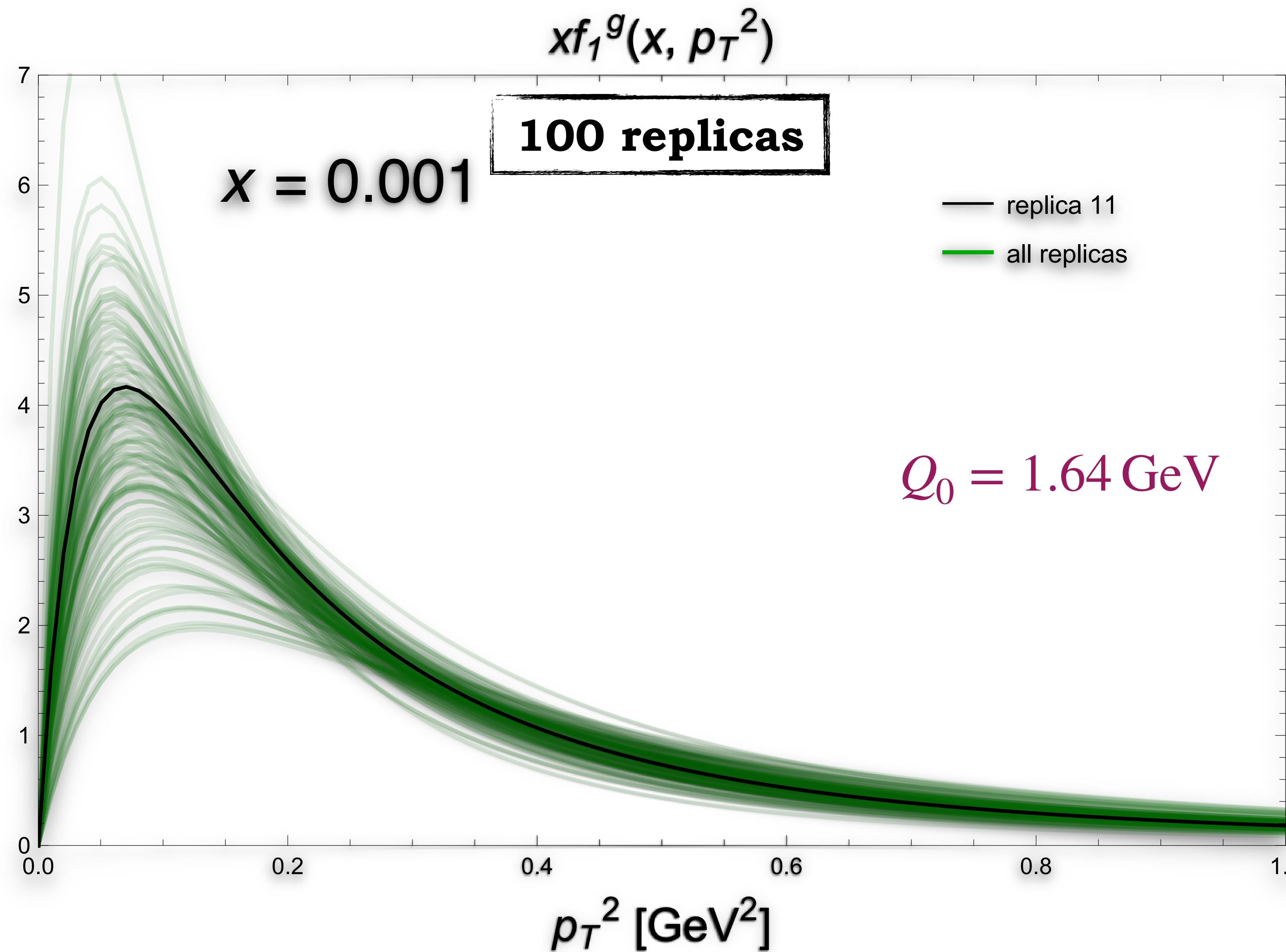
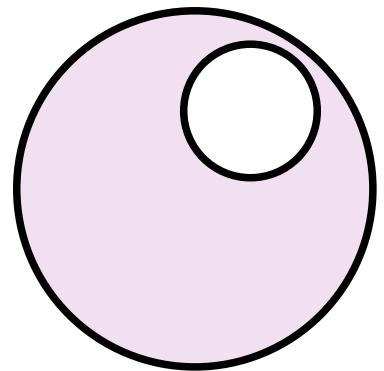
# Unpolarized gluon TMD



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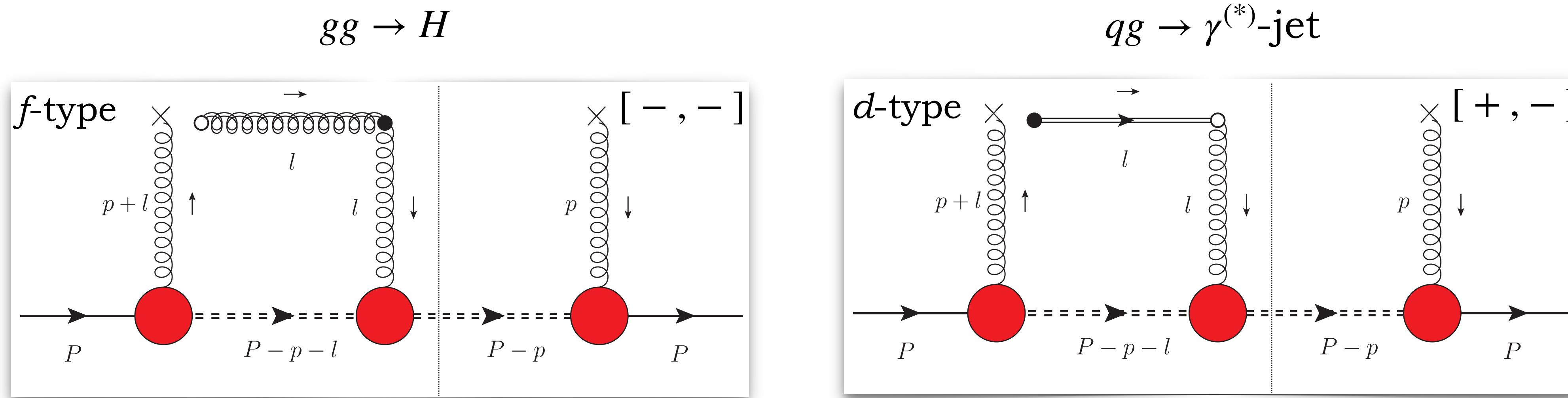
**...towards twist-2  
T-odd gluon TMDs**

# **T-odd gluon TMDs in a spectator model**

- No residual gluon-spectator interaction at tree level
- *Interference with one-gluon exchange (eikonal)*

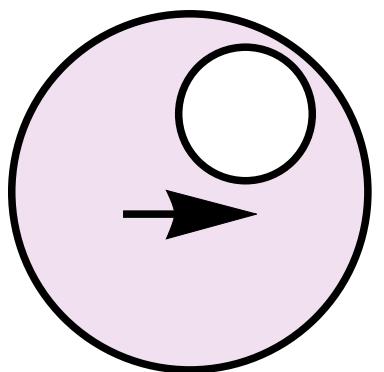
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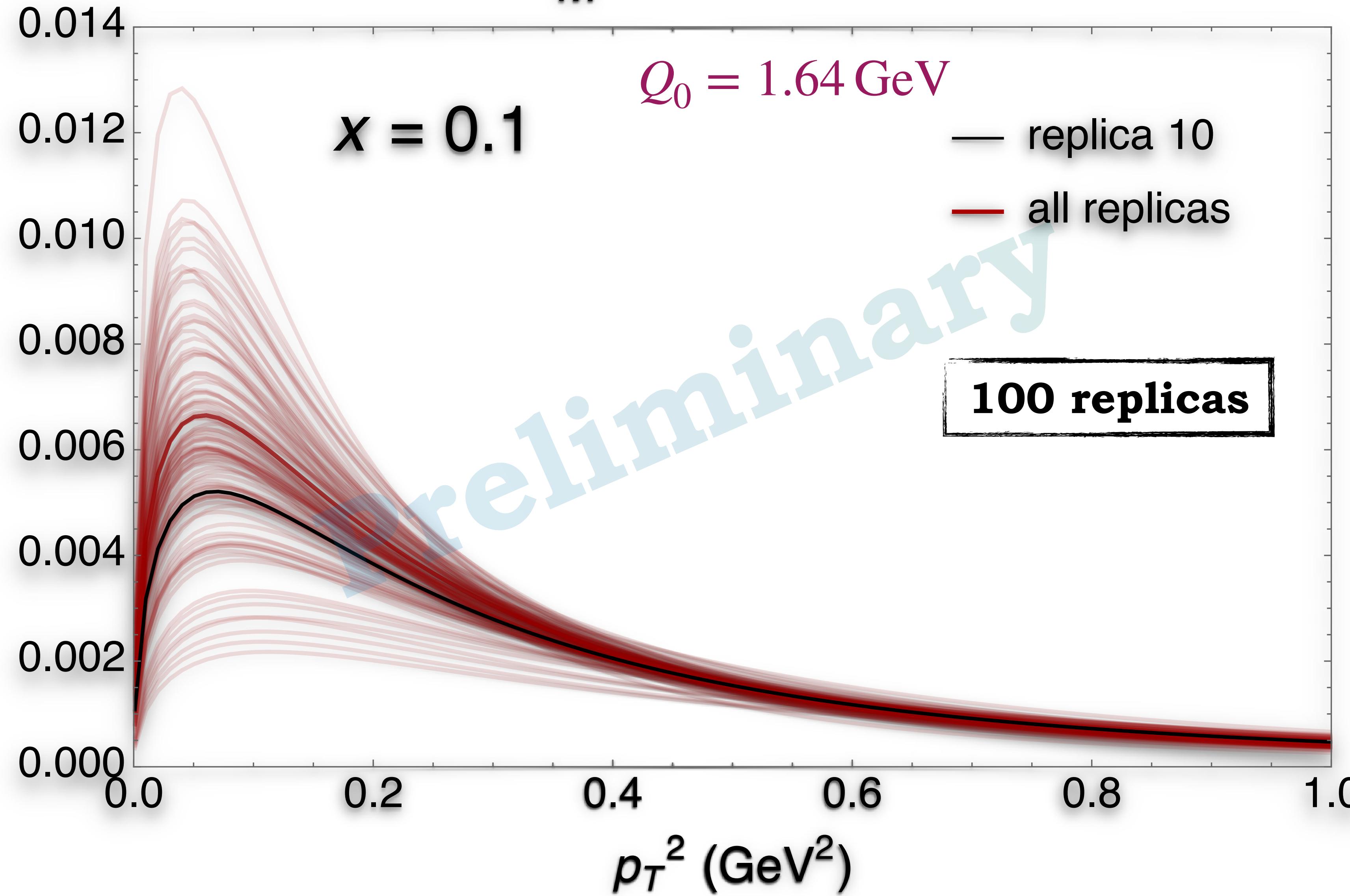


- Leading-twist one-gluon-exchange of the gauge-link operator
- Sensitivity to WW/DP structures
- Calculation of **Sivers** function *underway!*

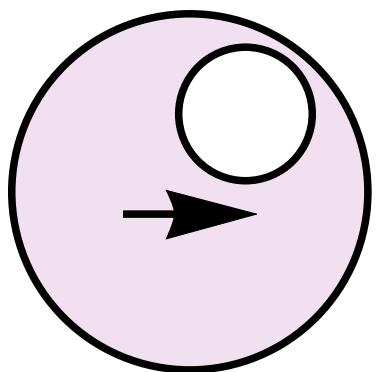
# *f*-type Sivers gluon TMD



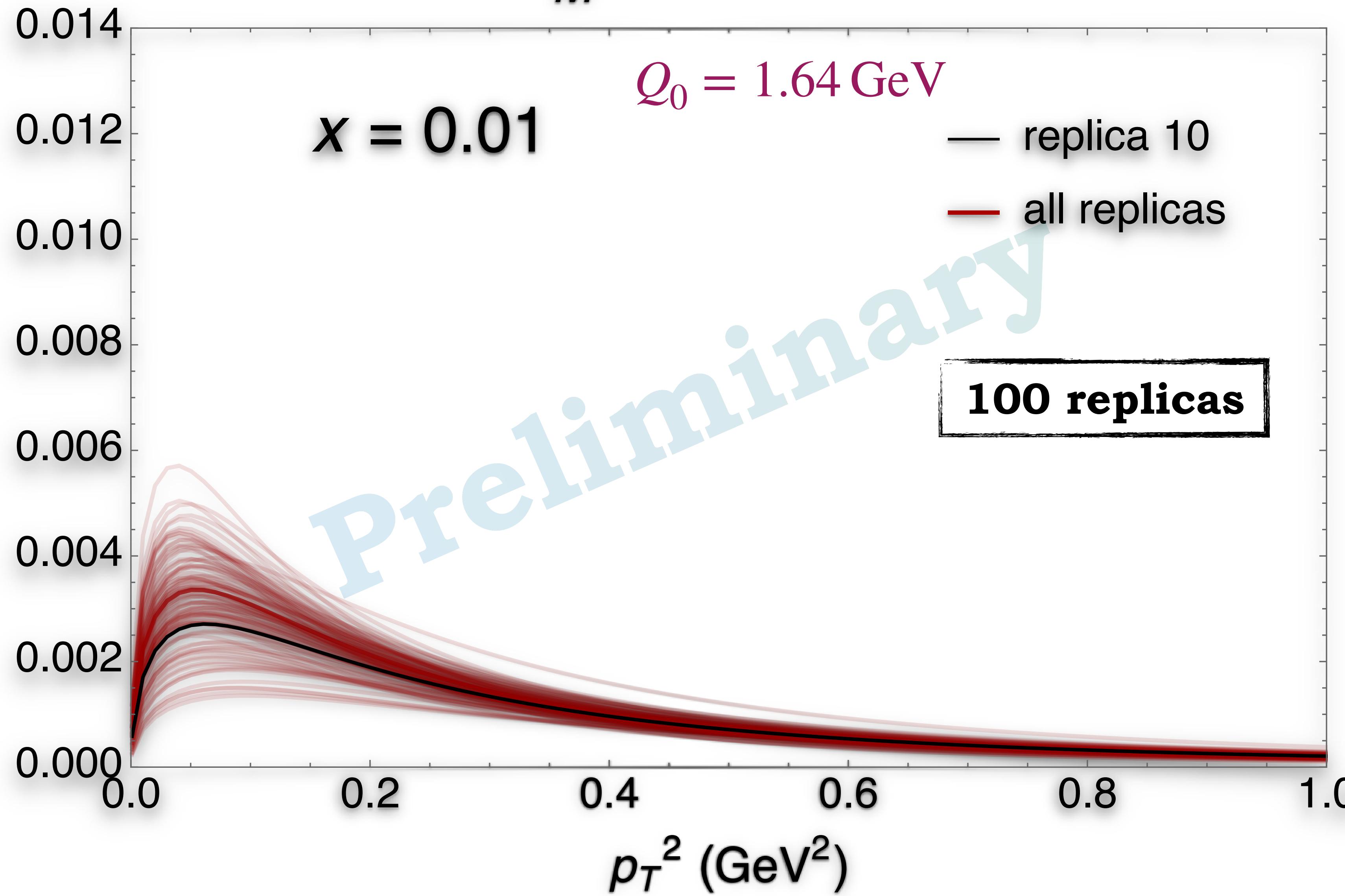
$$x \frac{p_T}{M} f_{1\tau}^{\perp[+,+]}(x, p_T^2)$$



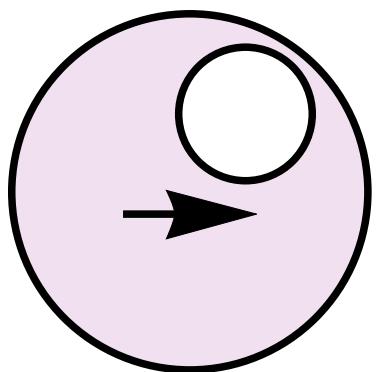
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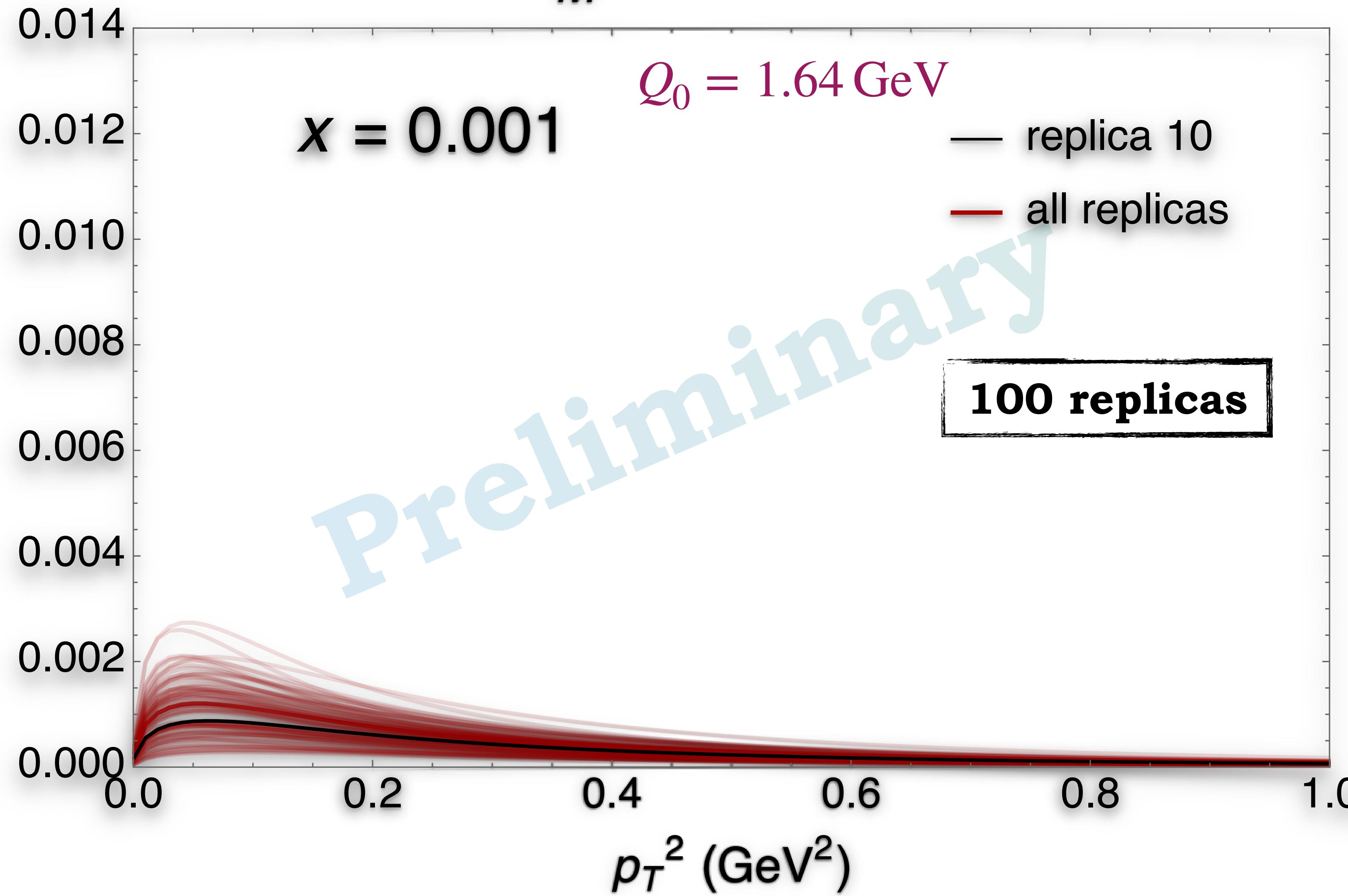
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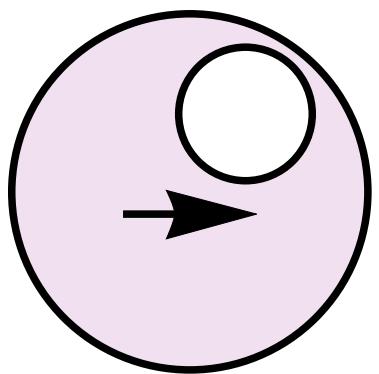
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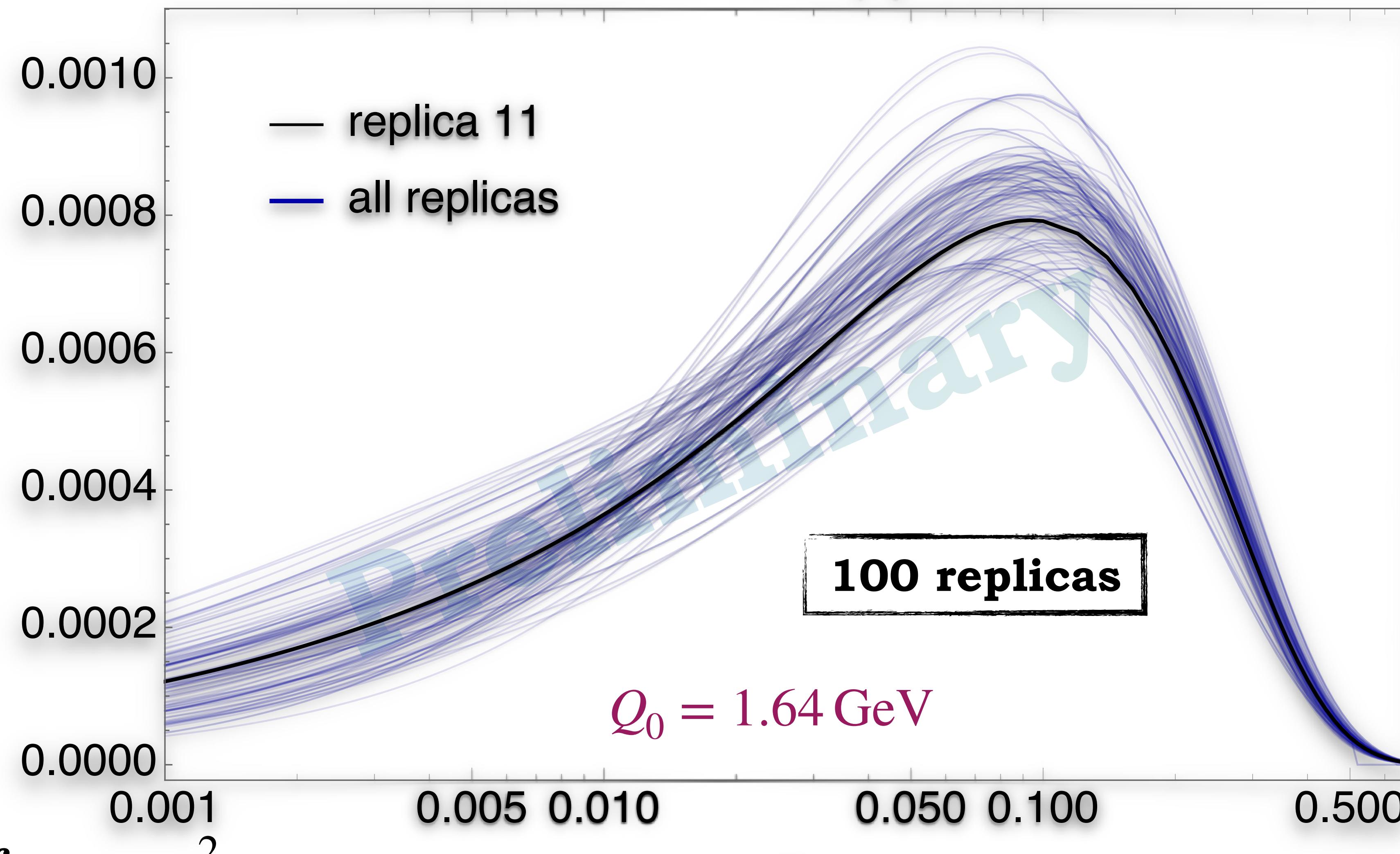
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# *f*-type Qiu-Sterman twist-3 gluon PDF

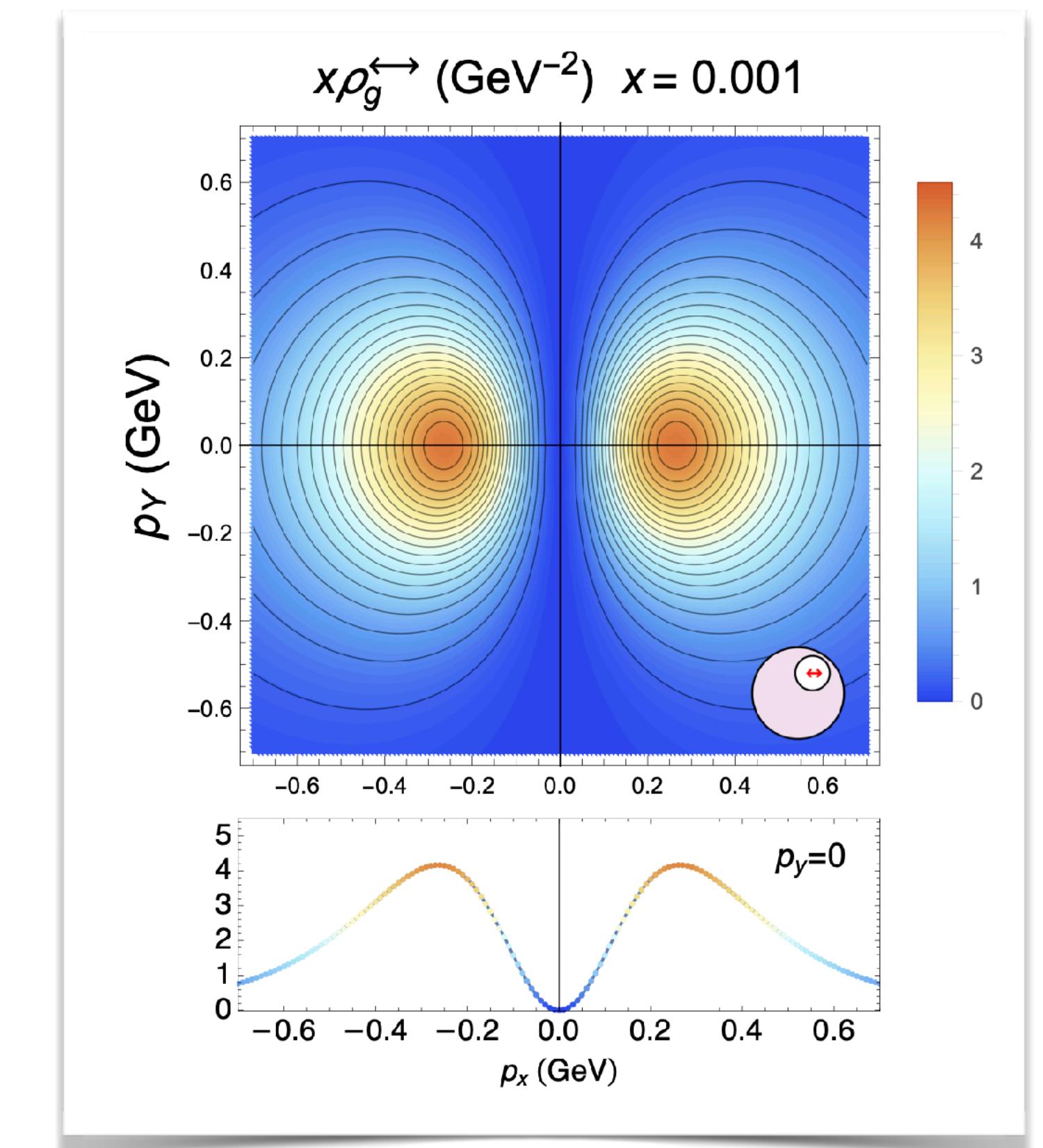
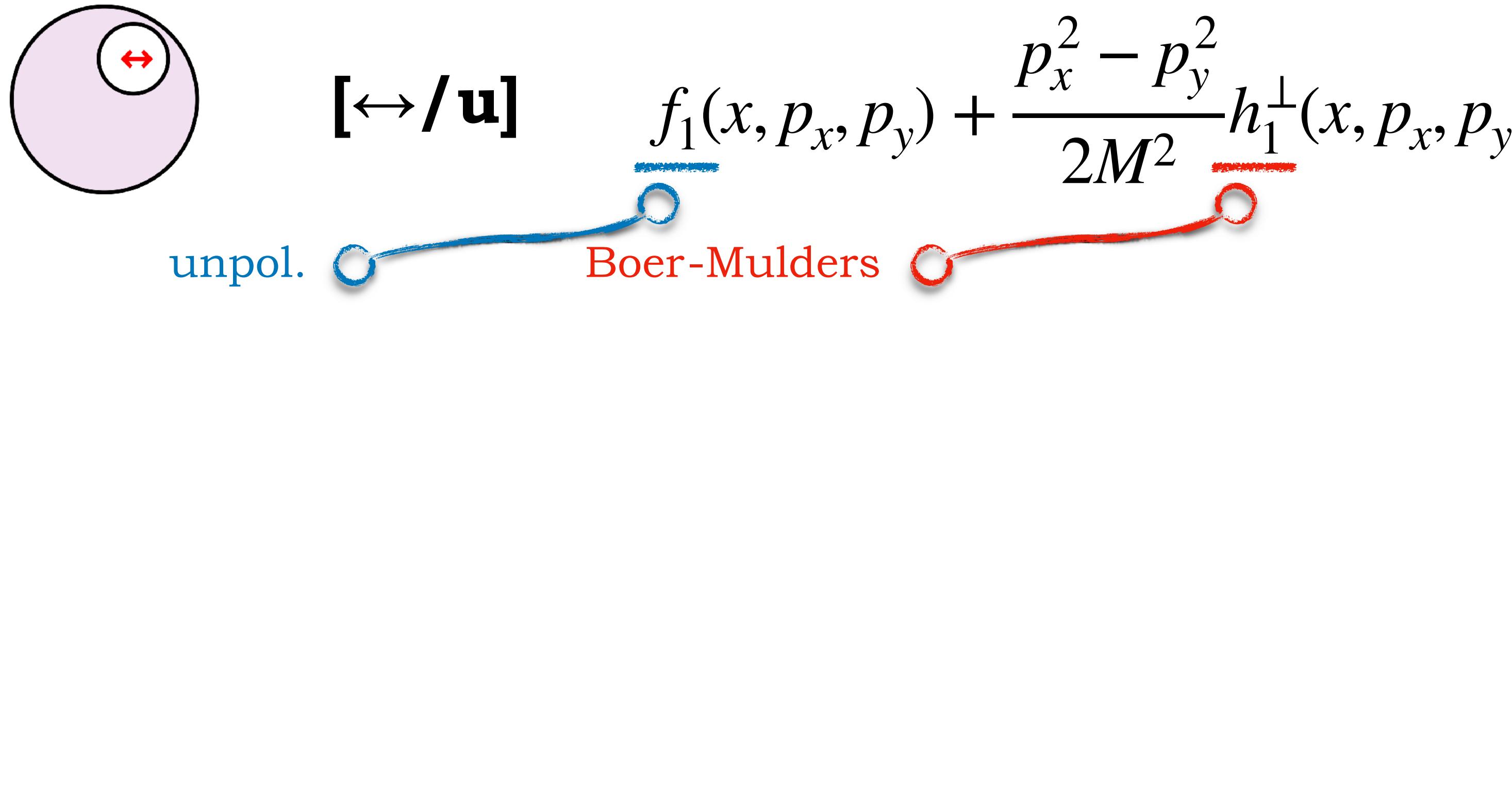


$xf_{1T}^{\perp(f)}(x)$

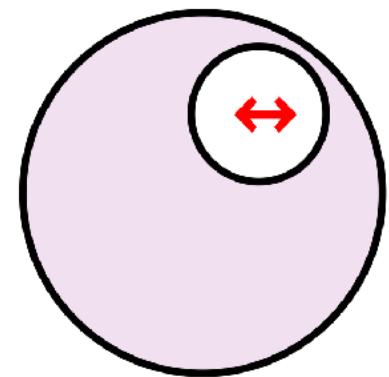


**...gluon TMDs  
in unpolarized  
*pp* collisions**

# Boer-Mulders effect in unpolarized $pp$ collisions



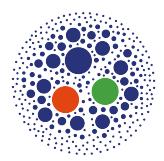
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$[\leftrightarrow / \mathbf{u}]$

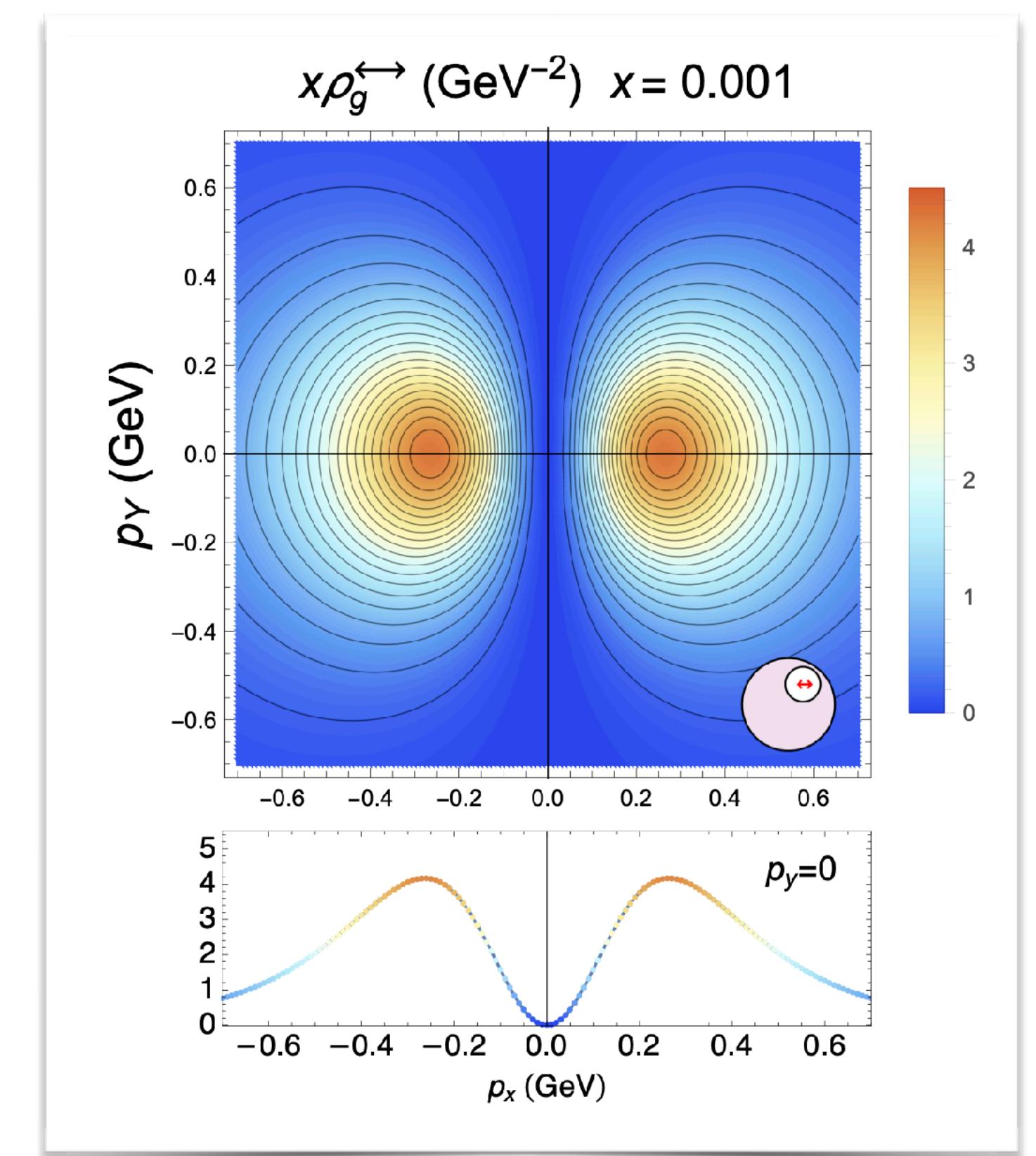
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.      Boer-Mulders



(Pseudo)scalar Higgs  $p_T$ -distribution

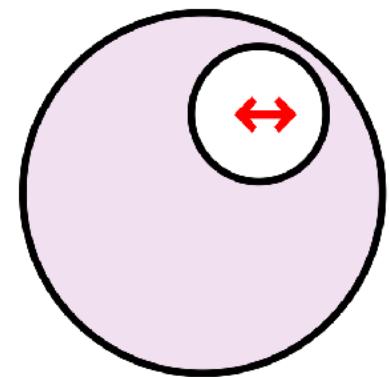
$$\begin{aligned} \frac{E d\sigma^{H(A)}}{d^3\vec{q}} \Big|_{q_T \ll m_H} &= \frac{\pi\sqrt{2}G_F}{128m_H^2 S} \left(\frac{\alpha_s}{4\pi}\right)^2 |\mathcal{A}_{H(A)}(\tau)|^2 \\ &\times \left( \mathcal{C}[f_1^g f_1^g] \pm \mathcal{C}[w_H h_1^{\perp g} h_1^{\perp g}] \right) + \mathcal{O}\left(\frac{q_T}{m_H}\right) \end{aligned}$$



🔗 [D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]

(Higgs+jet angular distributions) 🔗 [D. Boer, C. Pisano (2015)]

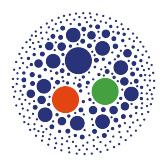
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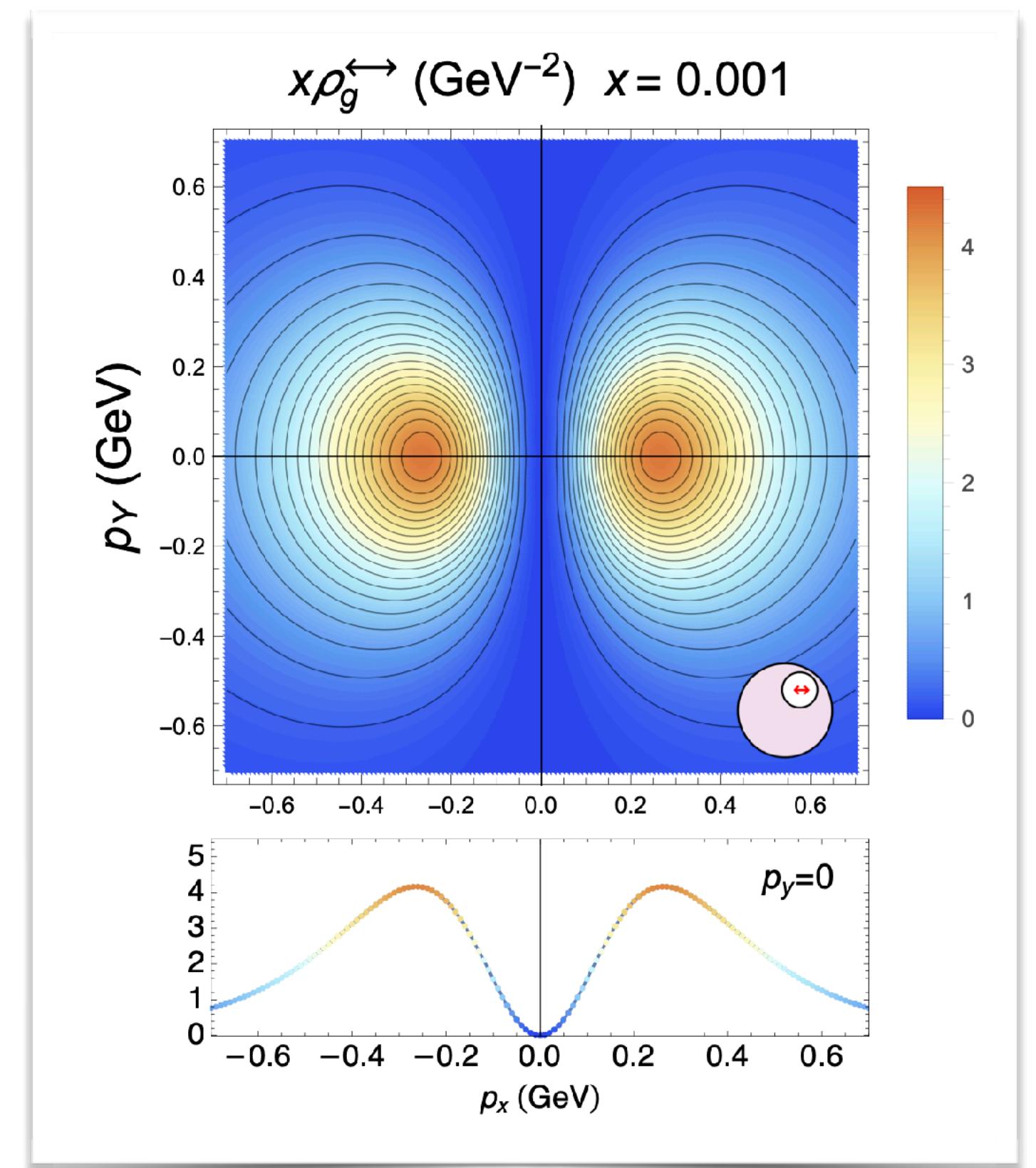
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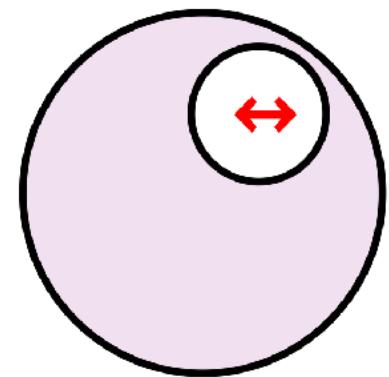
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## Model prediction at low- $x$

$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$

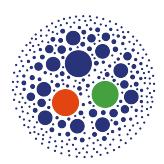
# Boer-Mulders effect in unpolarized $pp$ collisions



$\text{[}\leftrightarrow/\mathbf{u}\text{]}$

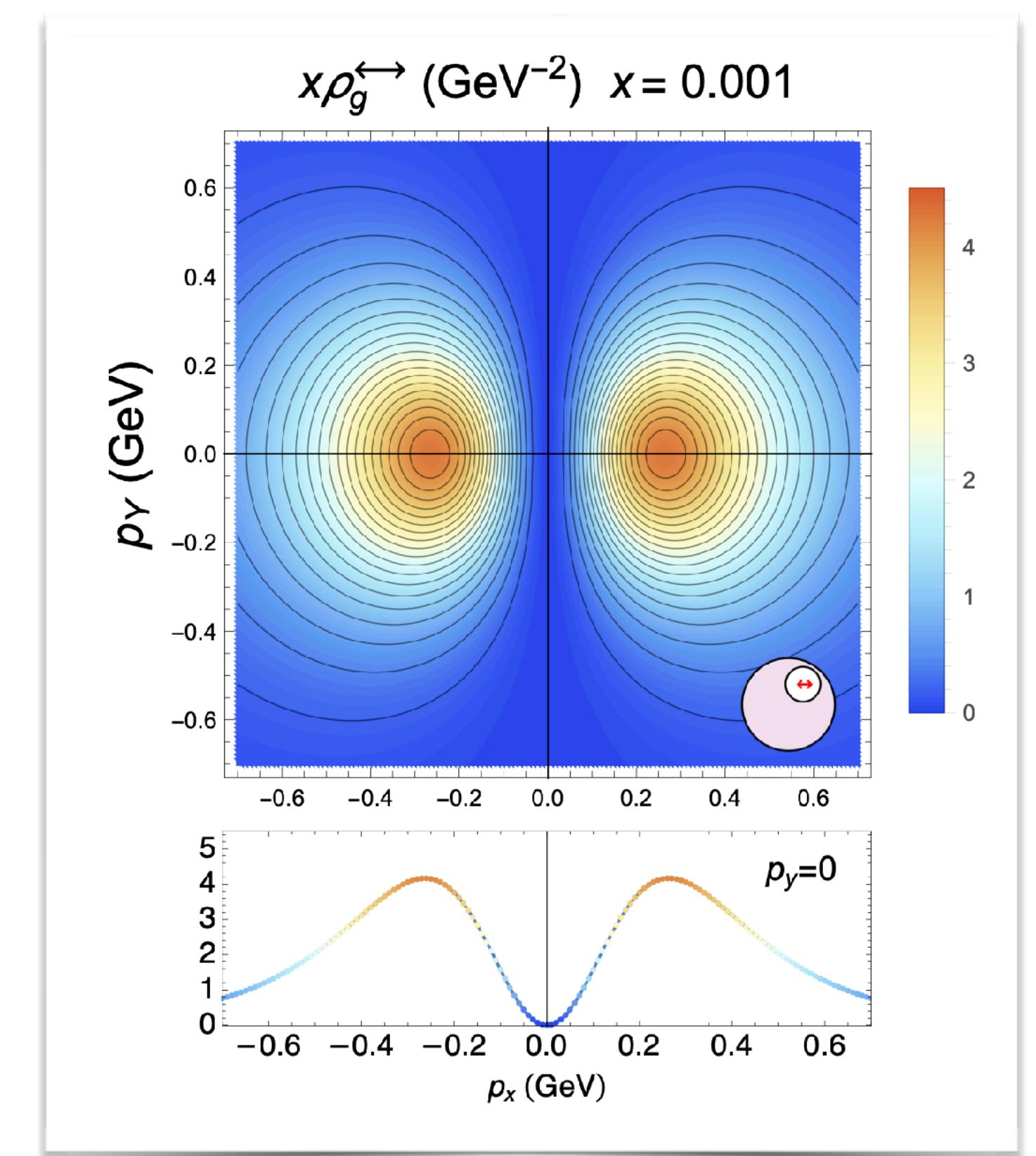
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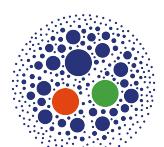


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HEF regime (linear low- $x$  evolution)

$$f_1^g(x, p_T^2) = h_1^{\perp g}(x, p_T^2) + \text{higher twist}$$

# $\eta_{b,c}$ production in unpolarized $pp$ collisions

# TMD phenomenology: from JLab to the LHC

Andrea Signori

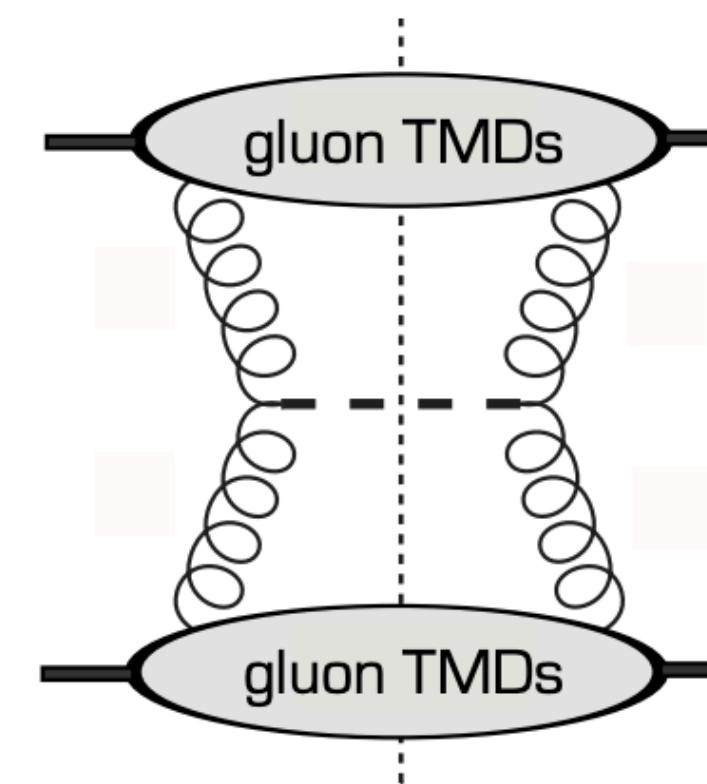
# Spatial and momentum tomography of hadrons and nuclei

INT 17-3  
Sent: 25/2017

# **NRQCD**

$$\frac{\text{CS}}{\text{CO}} \sim \frac{1}{\nu^4}$$

# gluon TMD PDFs



### pseudoscalar quarkonium production:

$p\ p \rightarrow \eta_b\ X$  M = 9.39 GeV

$p\ p \rightarrow \eta_c\ X$  M = 2.98 GeV

[see also talk by C. Pisano week 4]

unpolarized cross section  
at low transverse momentum  
for (pseudo)scalar state

$$\frac{d\sigma}{dg_T} \sim \Phi_A^U \Phi_B^U |\mathcal{M}|^2$$

$$\sim \mathcal{C}[ f_1^{g/A} f_1^{g/B} \dots ]$$

unpolarized gluons

$$\pm c [ h_1^{\perp g/A} h_1^{\perp g/B} ]$$

lin. polarized gluons

# Checkpoints and further steps

- Systematic calculation of all **initial-scale** twist-2  $T$ -even gluon TMDs
- Spectral mass to catch small- and large- $x$  effects
- Simultaneous fit** of  $f_1$  and  $g_1$  PDFs via **replica method**

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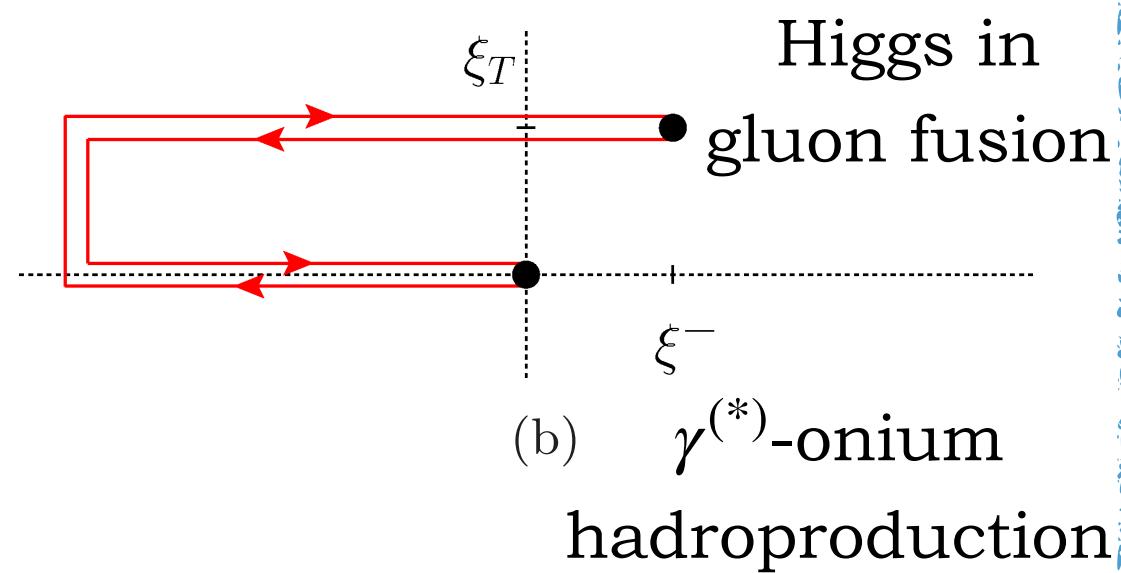
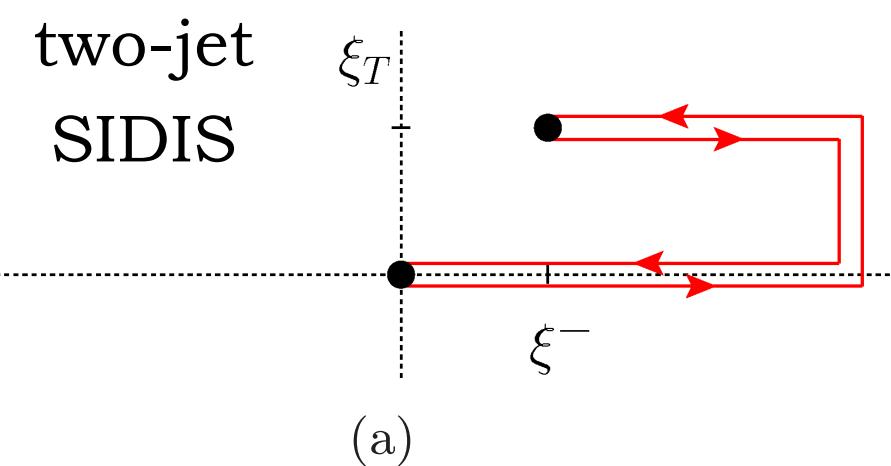
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- Twist-2  $T$ -odd gluon TMDs (**Sivers**, etc.) in progress!
- Inclusion of standard CSS evolution almost done!
- Pheno: **spin asymmetries**, **pseudodata** and **impact studies**
- Evolution: extension to quark TMDs in the same framework
- Explorative studies on gauge-link sensitivity and factorization

**Backup  
slides**

# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

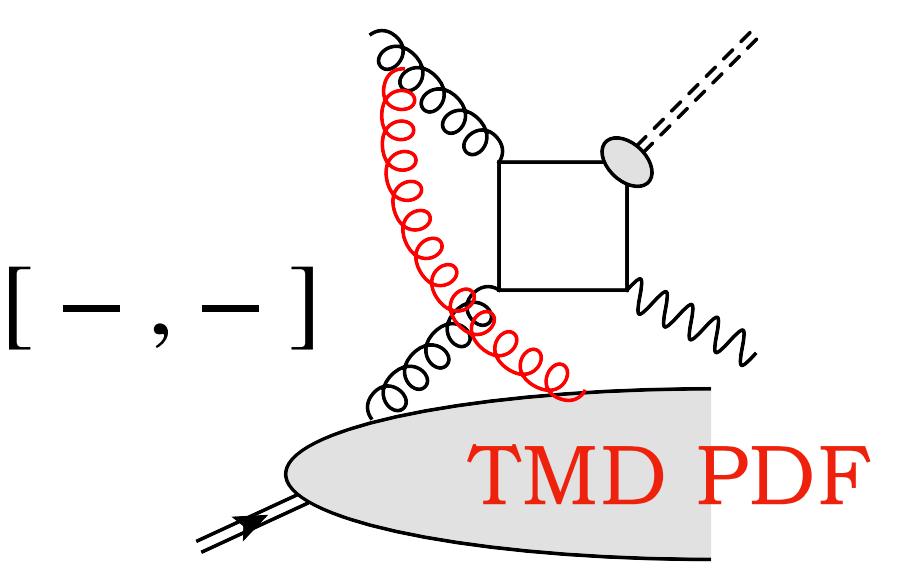
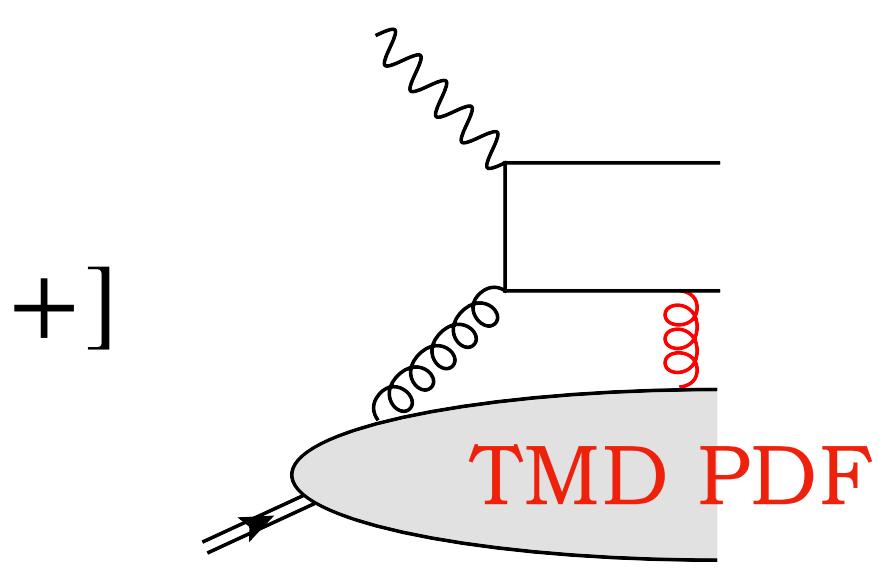
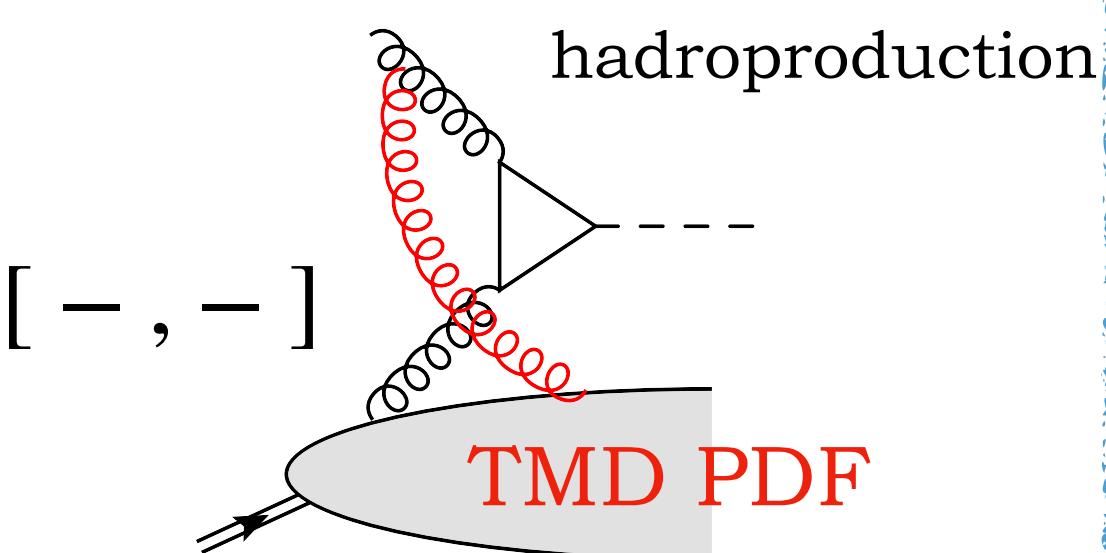
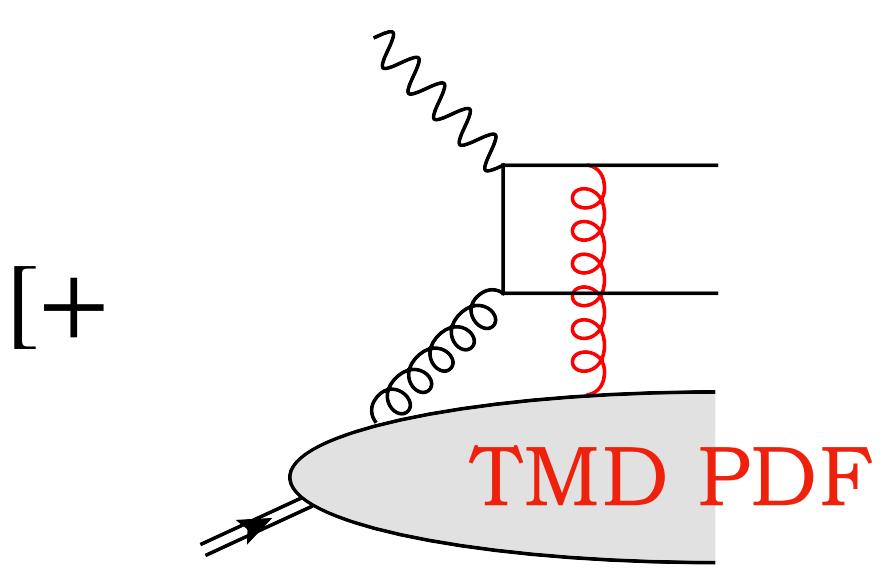
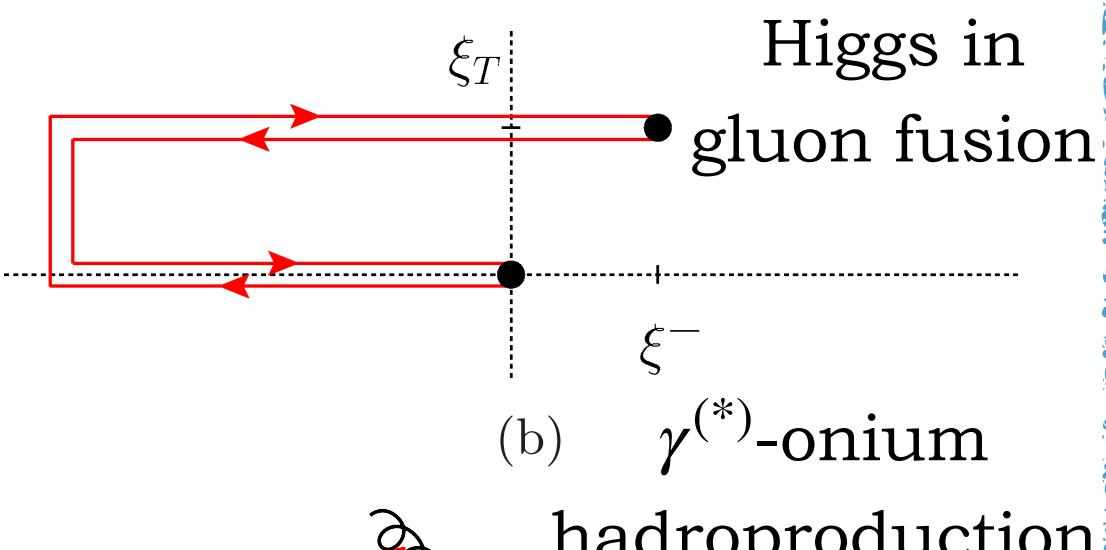
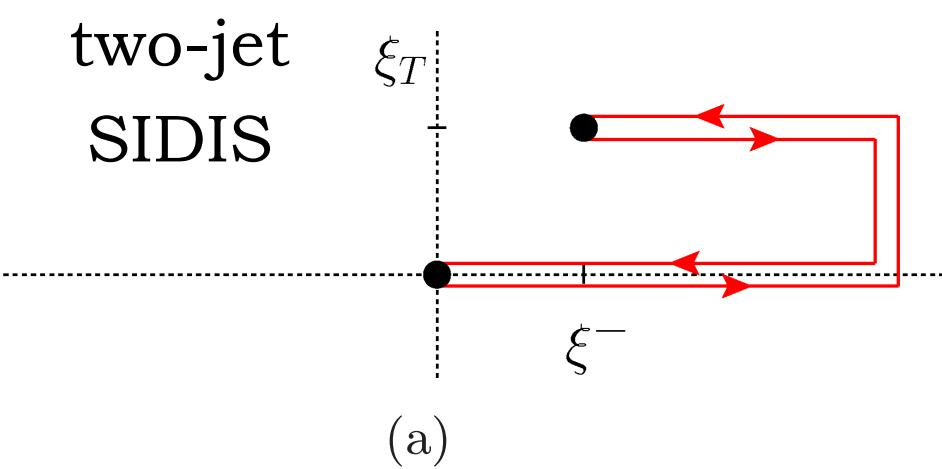
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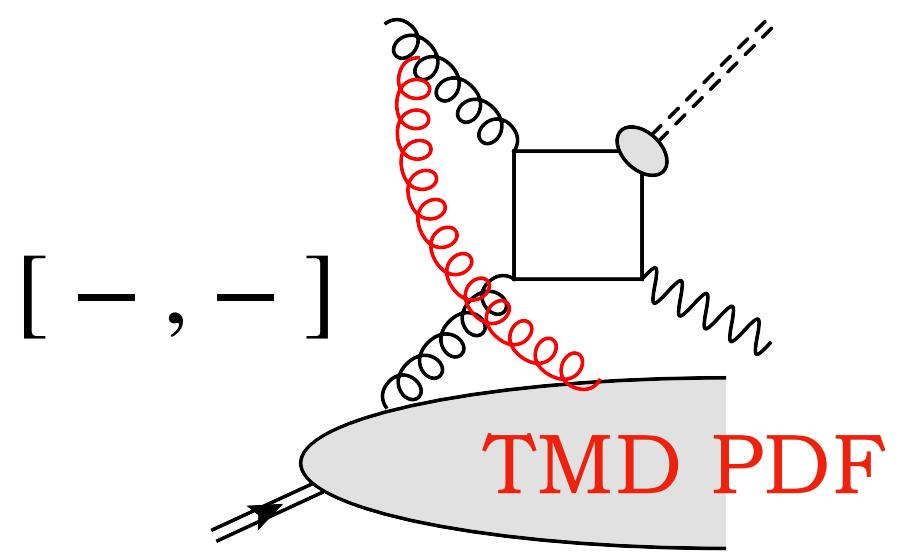
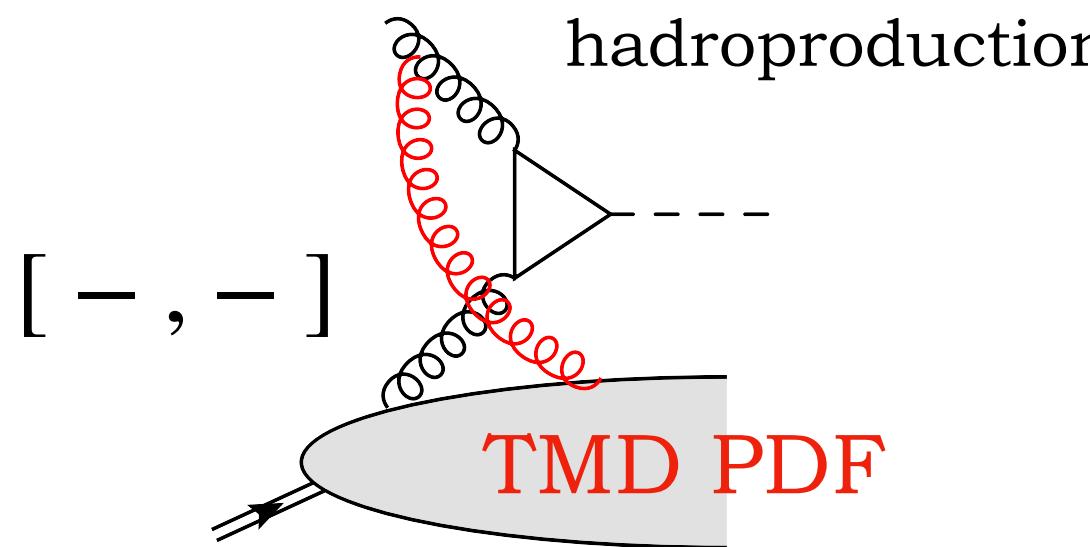
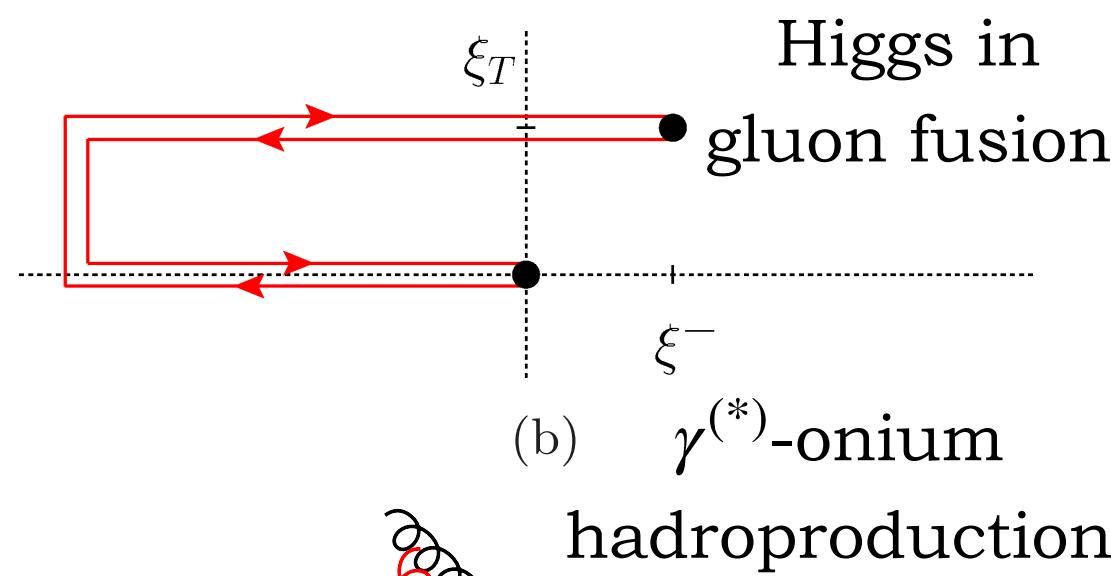
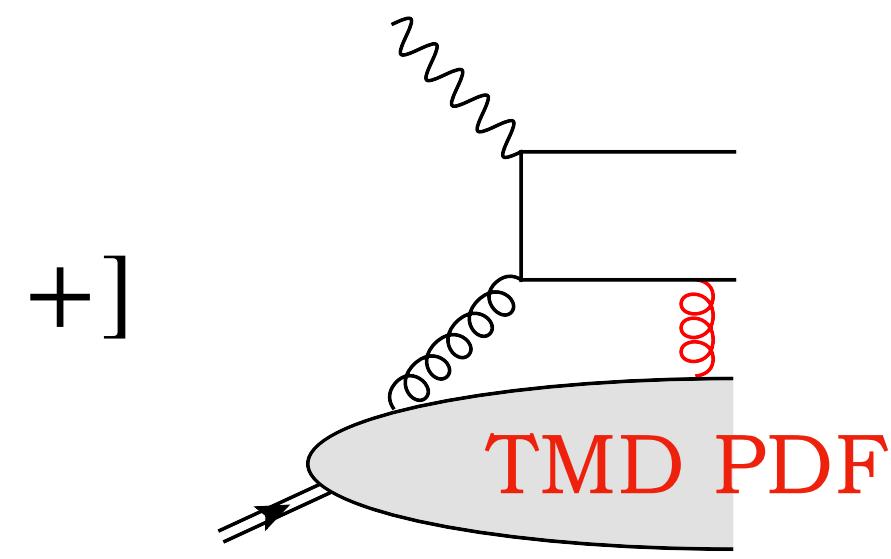
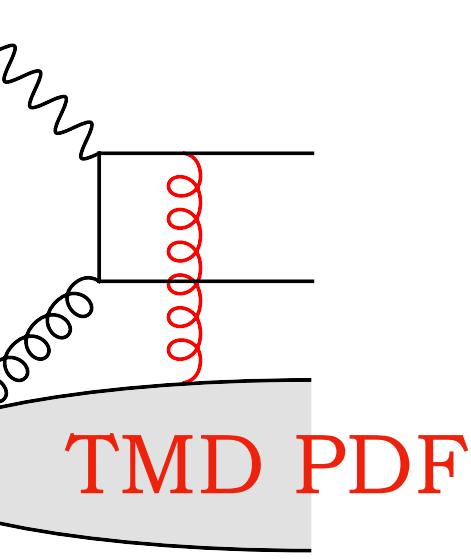
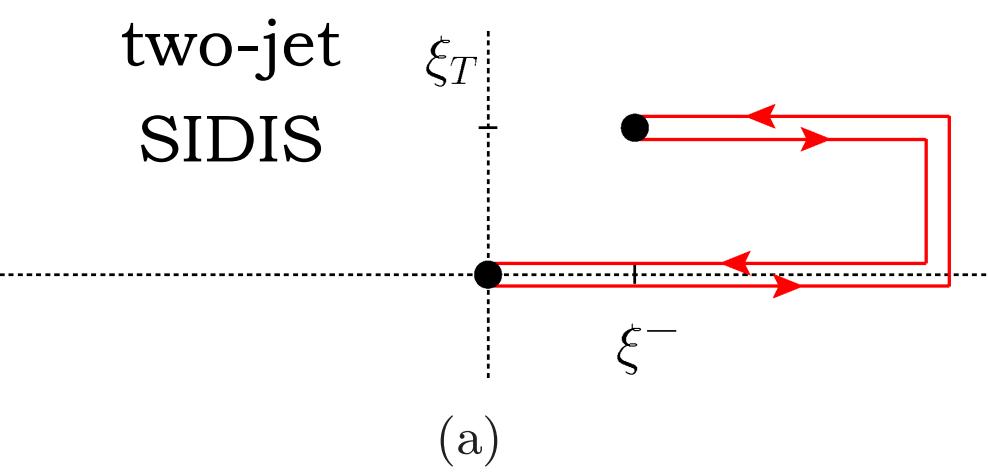
(a) [ + , + ] or (b) [ - , - ]



# Accessing WW and DP gluon TMDs

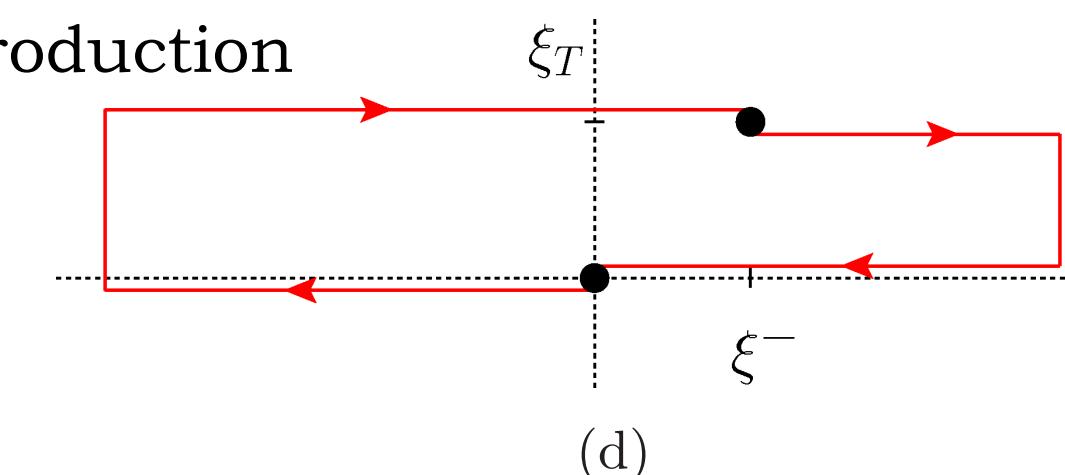
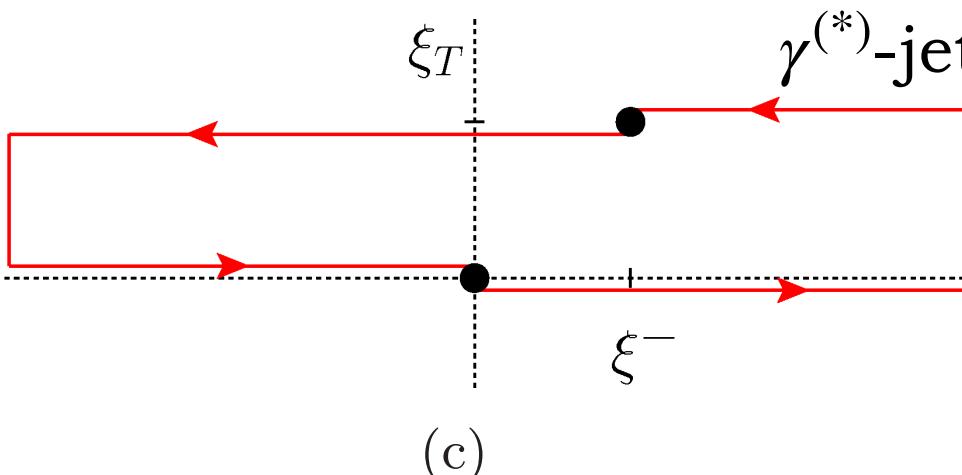
## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]



## Dipole (DP)

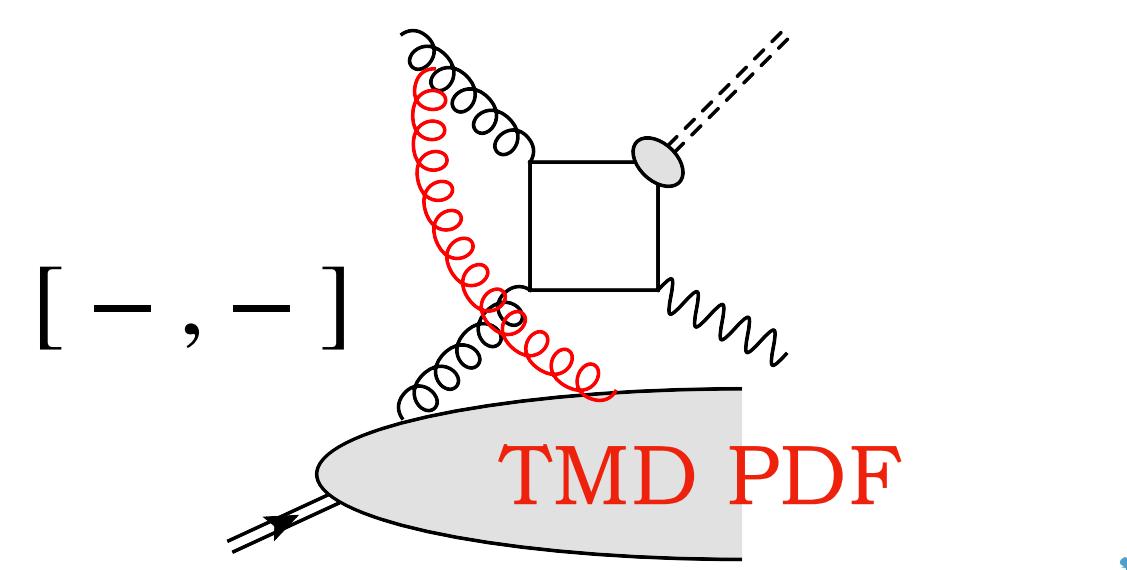
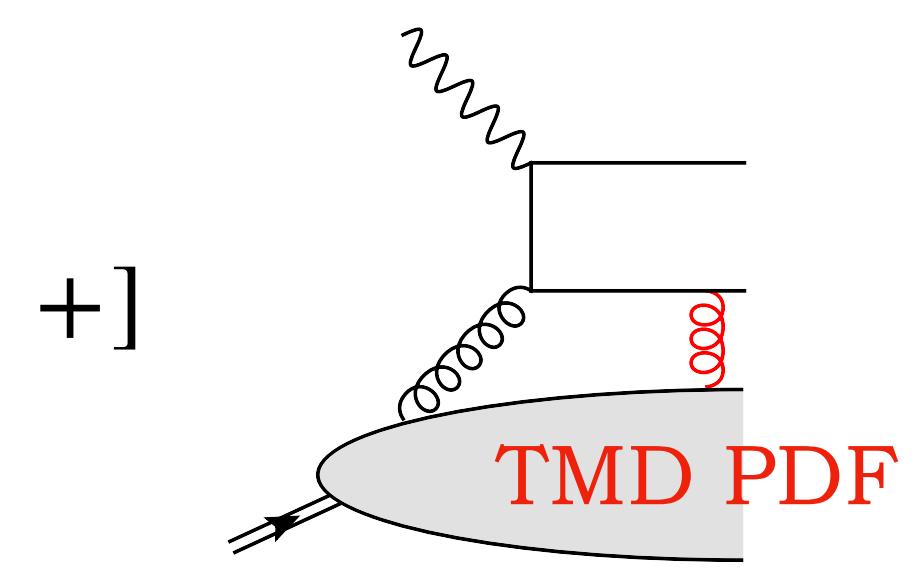
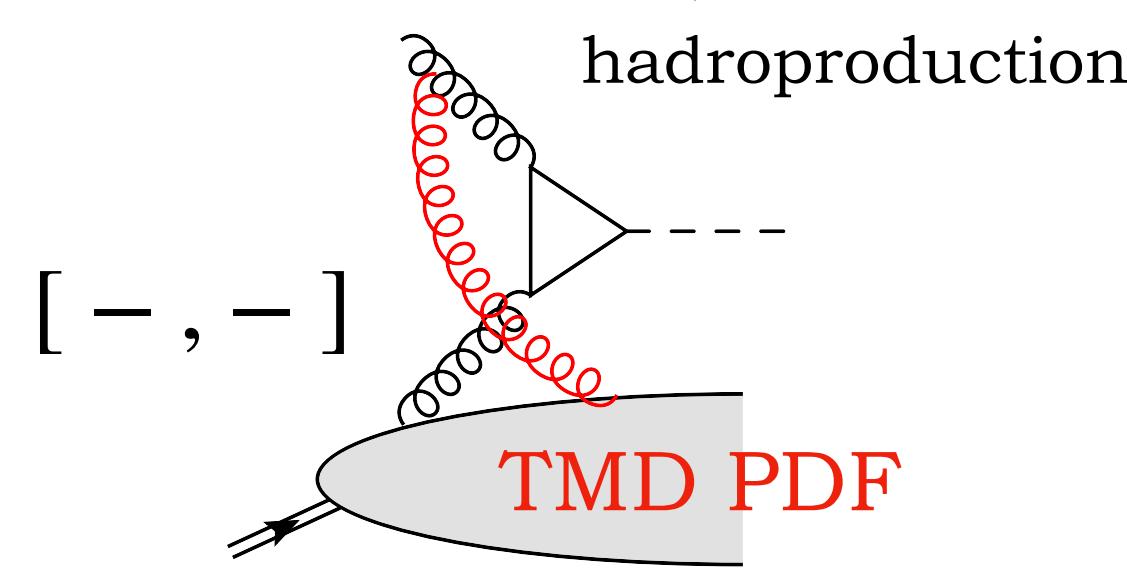
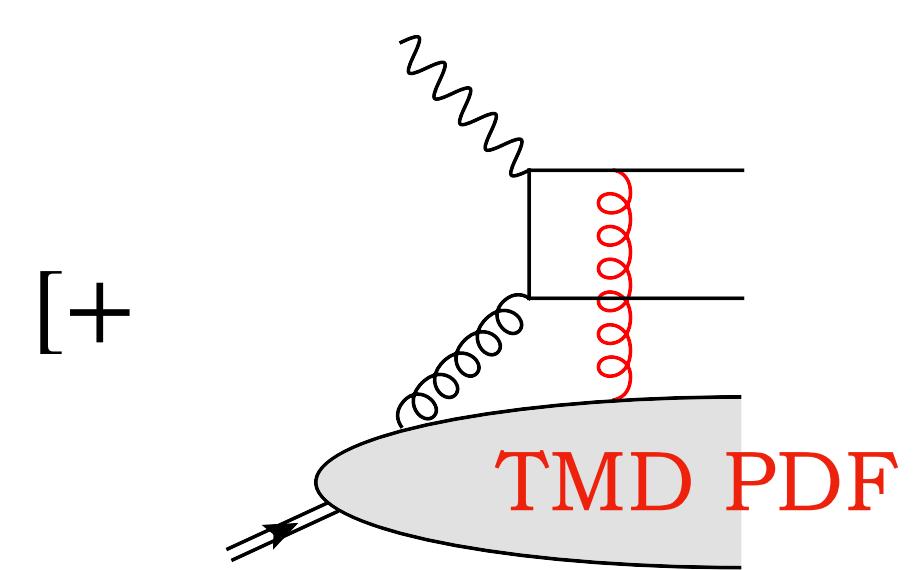
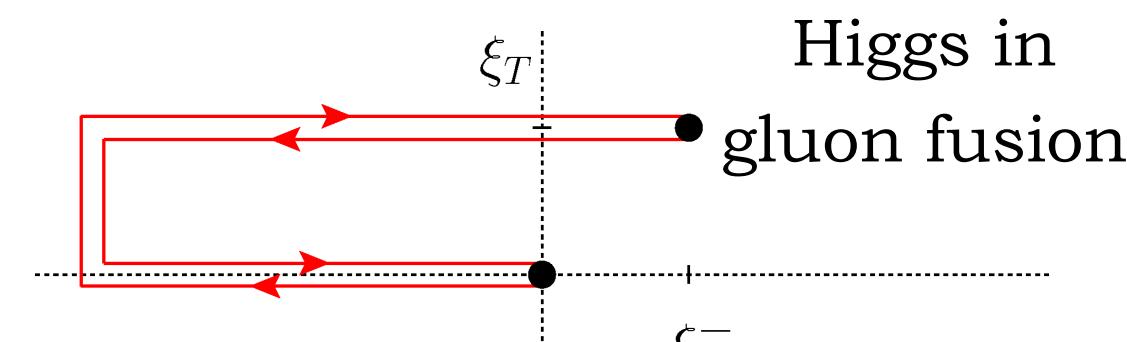
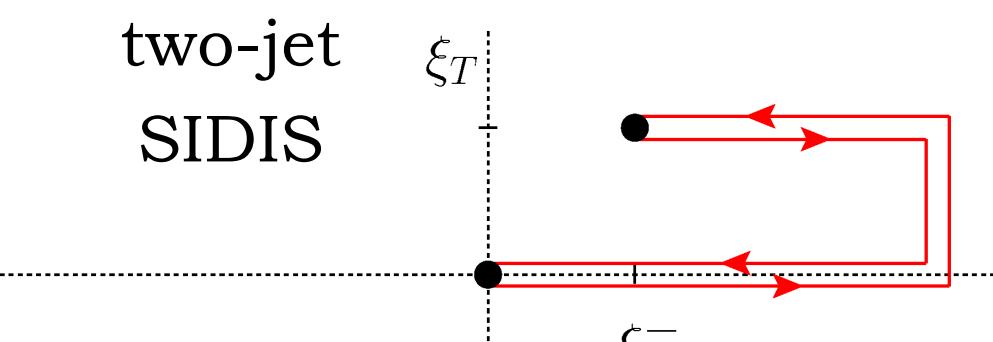
(c) [ + , - ] or (d) [ - , + ]



# Accessing WW and DP gluon TMDs

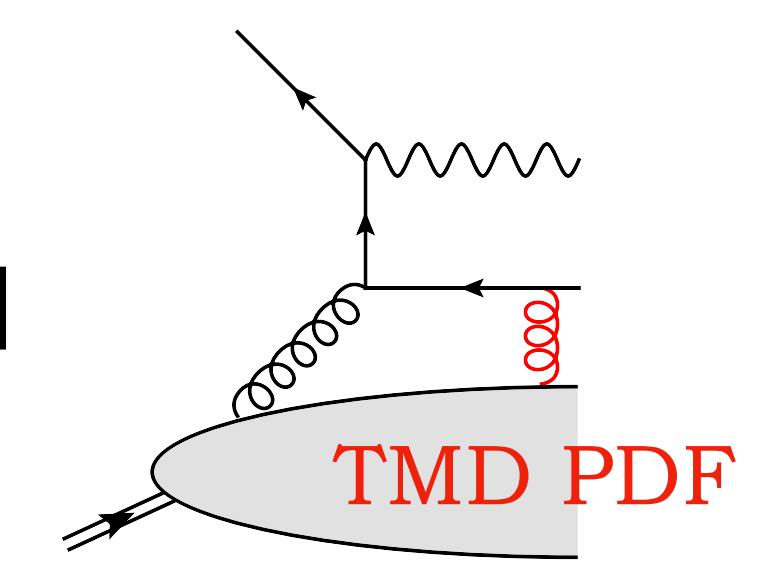
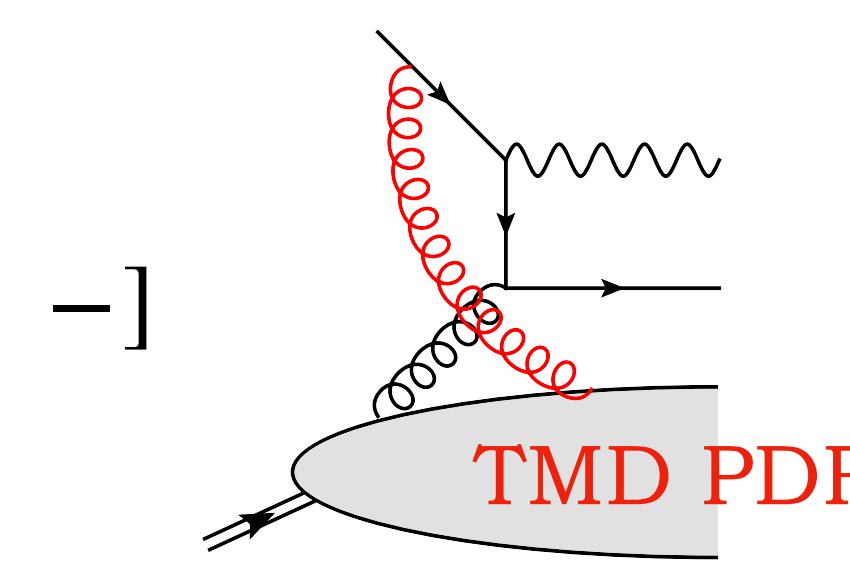
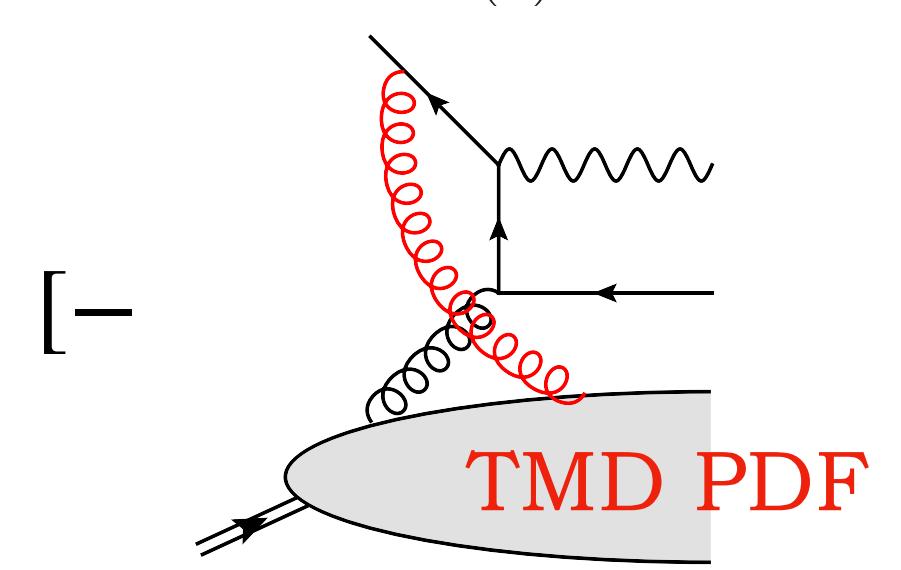
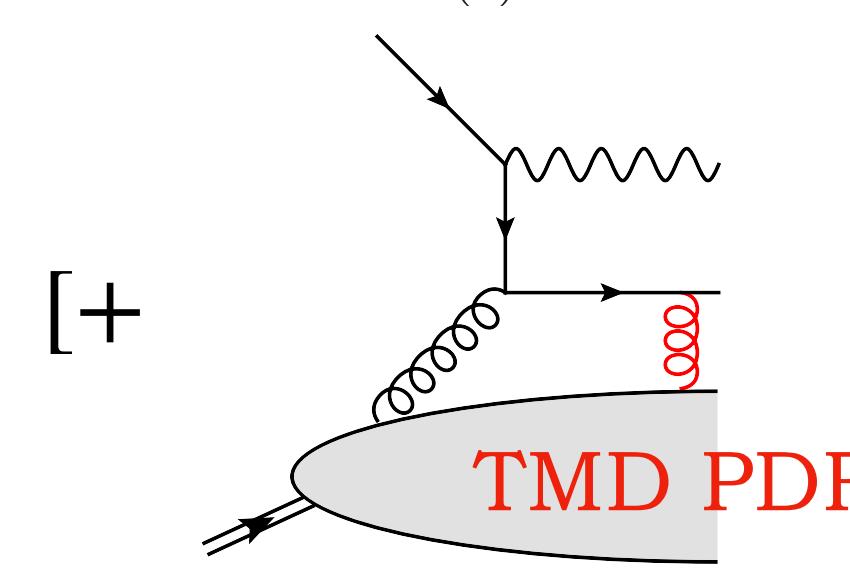
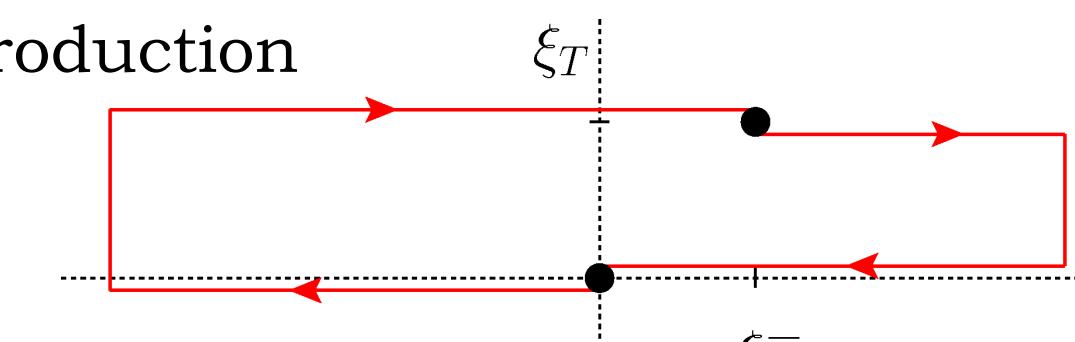
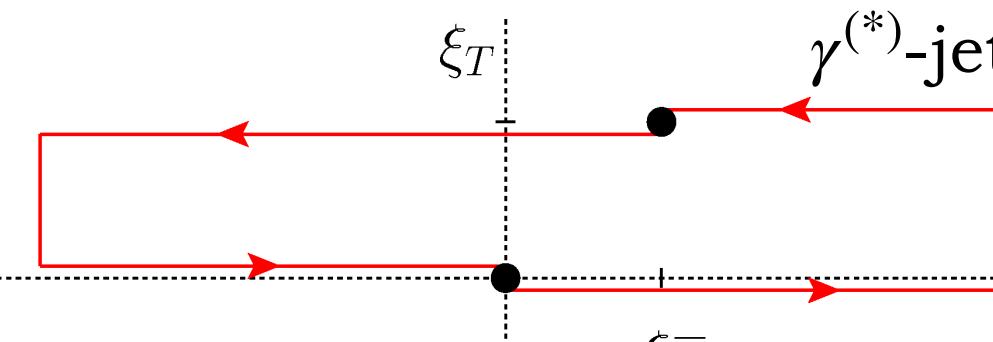
## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]



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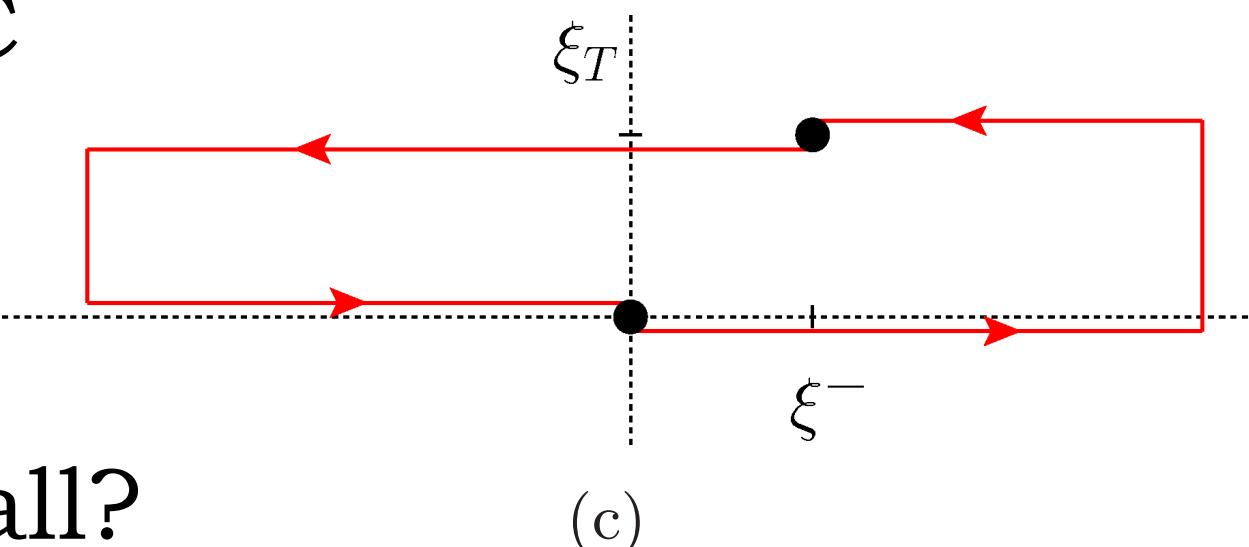
(c) [ + , - ] or (d) [ - , + ]



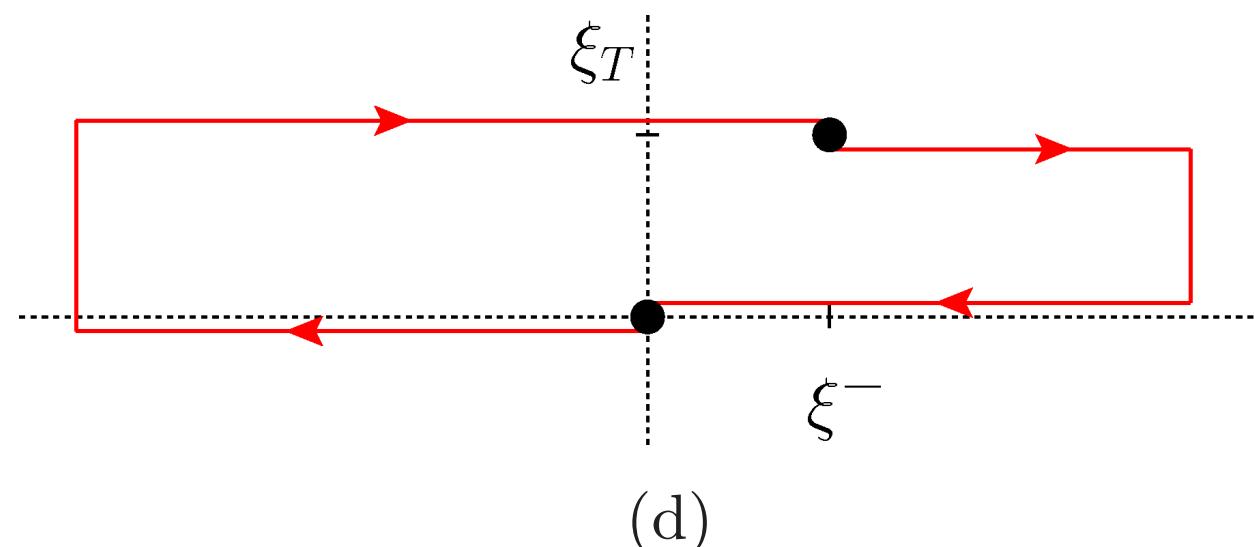
# Dihadron hadroproduction and factorization breaking

- \* Proof of factorization violation  [T. J. Rogers, P. J. Mulders (2010)]

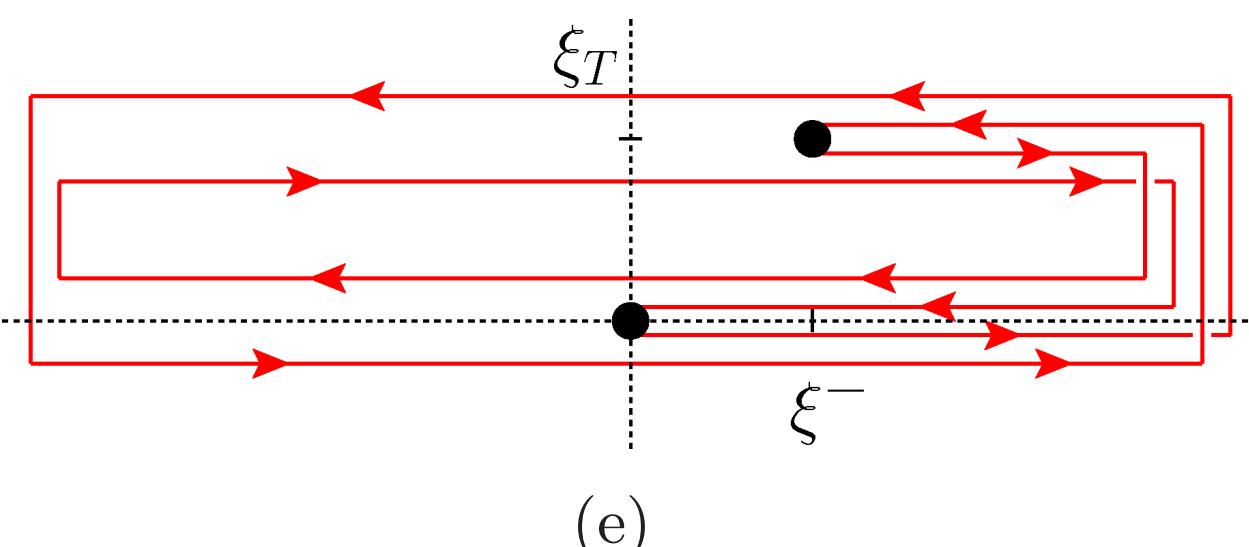
- \* Assumed factorization in SCET and CGC



- \* Significance of low- $x$  studies

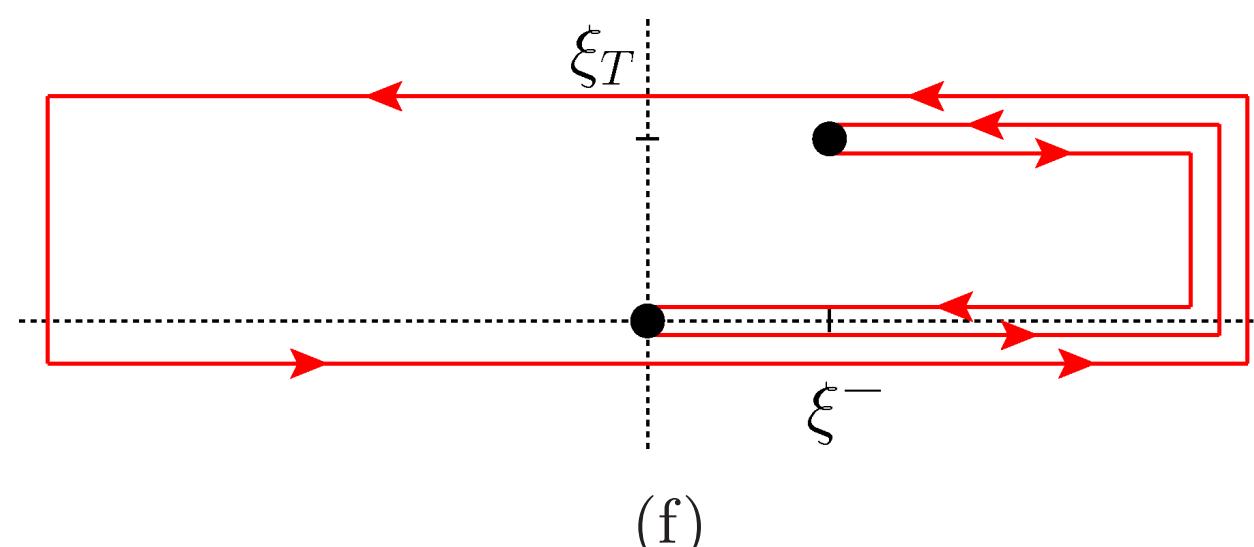


- \* Size of factorization-breaking effects small?



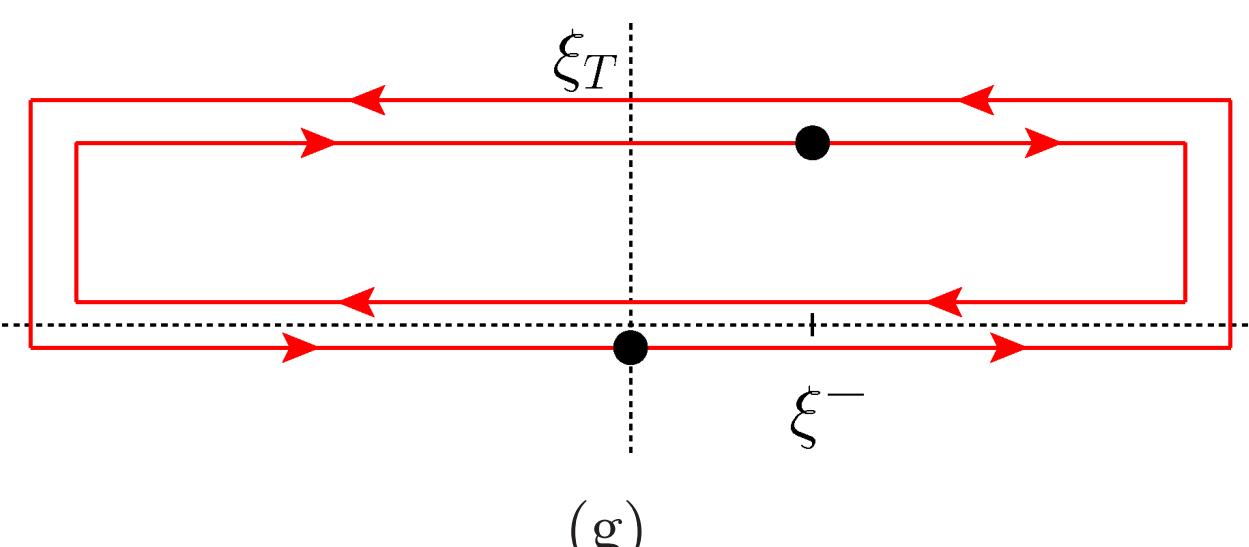
- \* DP TMDs:

(c)  $[+, -]$  and (d)  $[-, +]$

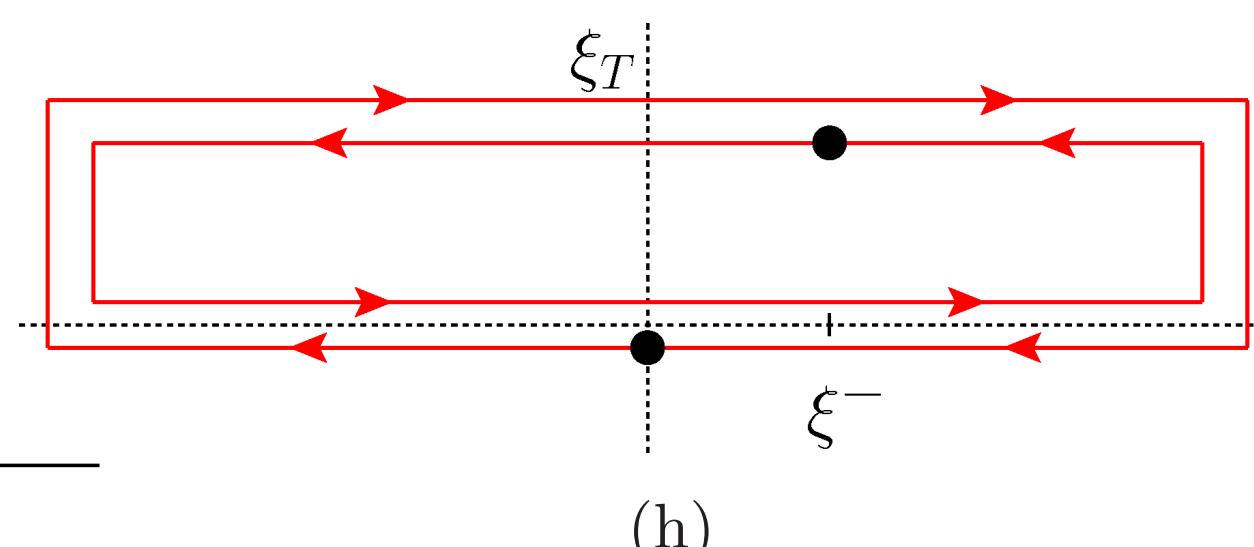
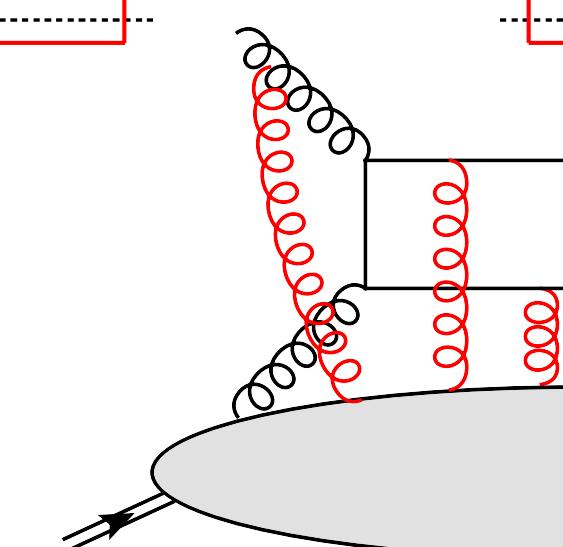


- \* Appearance of new gauge **loop links**:

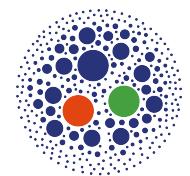
(e)  $[+ \square, + \square]$ , (f)  $[+, + \square]$ ,



(g)  $[\square, \square]$ , and (h)  $[\square, \square]$

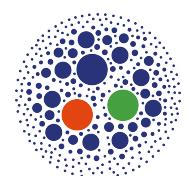
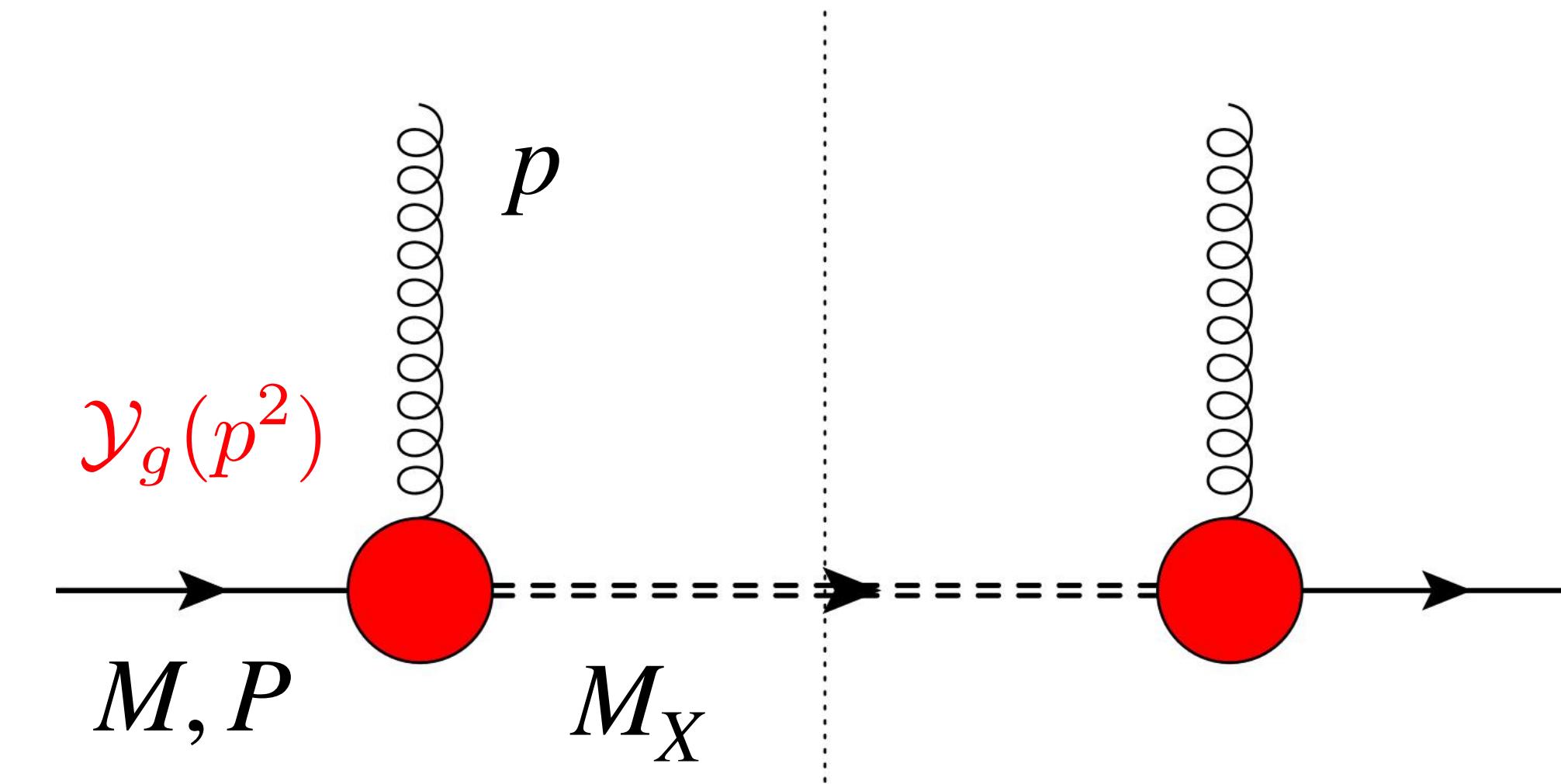


# Assumptions of the model



## Effective vertex

Lowest Fock state:  
**tri-quark spectator**  
on-shell and  
with mass  $M_X$



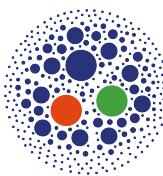
## Spin-1/2 spectator (gluon)

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} Tr \left[ (\not{P} + M) \frac{1 + \gamma^5 \$}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \gamma_g^{\rho*} \gamma_{g\sigma}(\not{P} - \not{p} + M) \right]$$

$$\gamma_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$

mimics proton form factors  
(conserved EM current  
of a free nucleon)

# Assumptions of the model



## Link with collinear factorization

$p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution

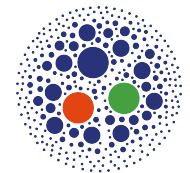


## Dipolar form factor(s)

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

1. Cancels singularity of gluon propagator
2. Suppresses effects of high  $p_T$
3. Compensates log divergences arising from  $p_T$ -integration
4. Adds three more parameters:  $\kappa_{1,2}$  and  $\Lambda_X$

# Assumptions of the model



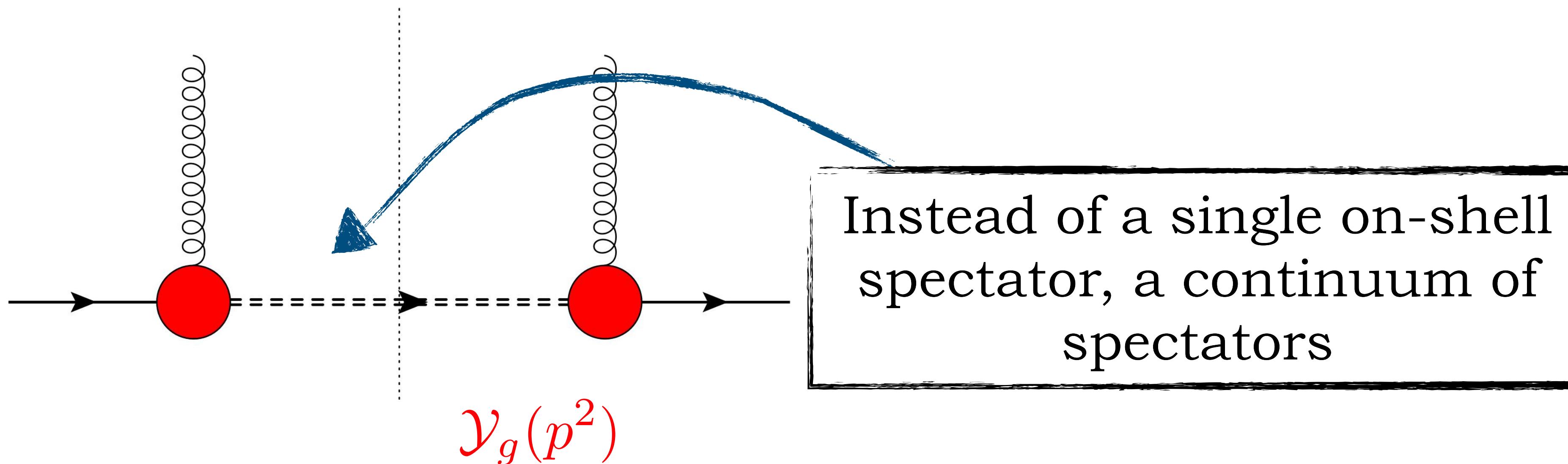
## Spectator-system spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectral-mass function

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



# Assumptions of the model



## Spectator-system spectral-mass function

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spectral-mass function

spectator-model TMD

∅ [Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X(M_X; \{X^{(\text{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\}) = \mu^{2a} \left[ \frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

low- $x$  (high- $\mu^2$ ) tail  $\propto (a - b)$

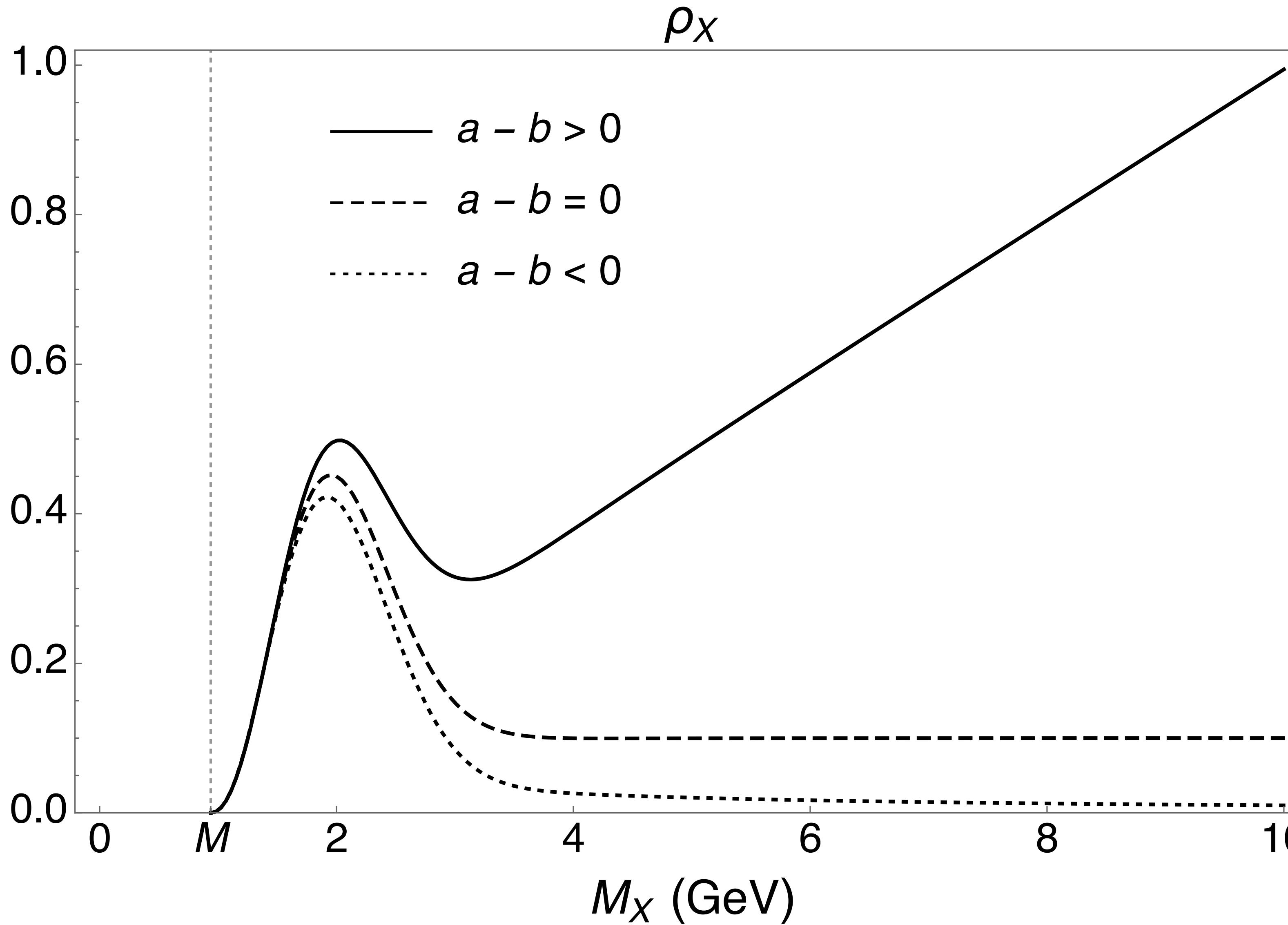
$q\bar{q}$  contributions energetically available at large  $M_X$

$$\mu^2 = M_X^2 - M^2$$

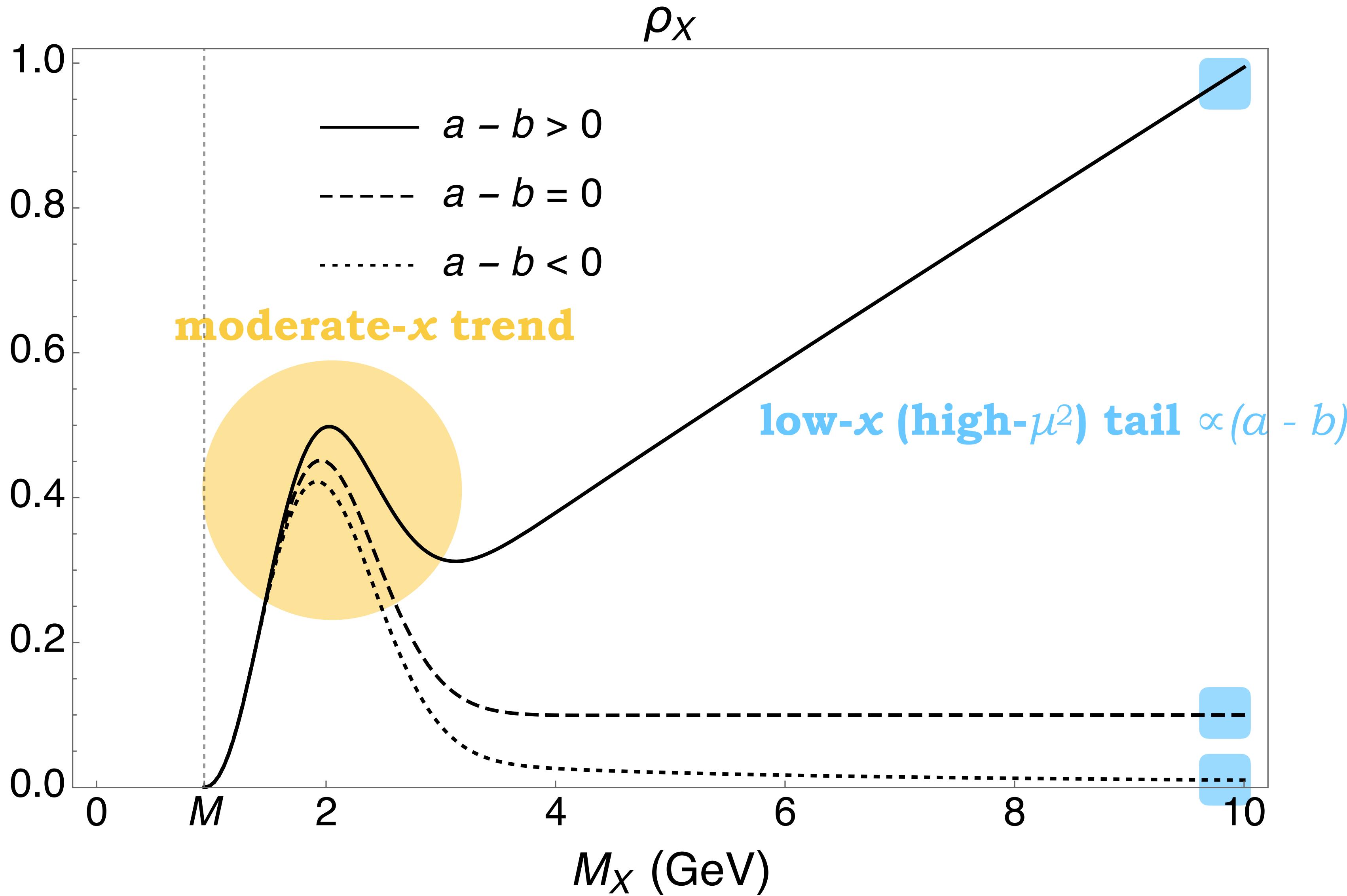
moderate- $x$  trend

pure tri-quark contribution at low  $M_X$

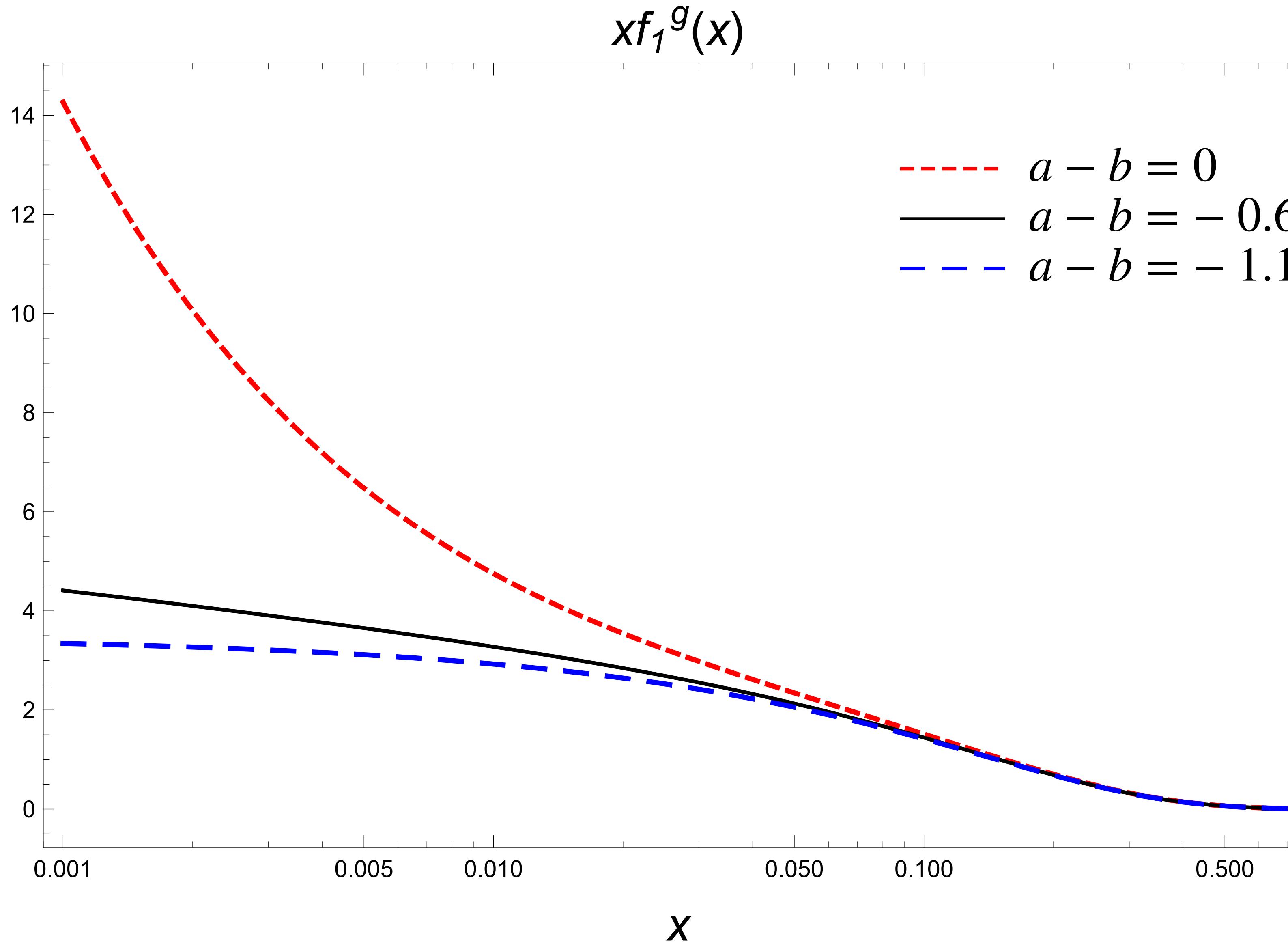
# Spectral function vs $(a - b)$



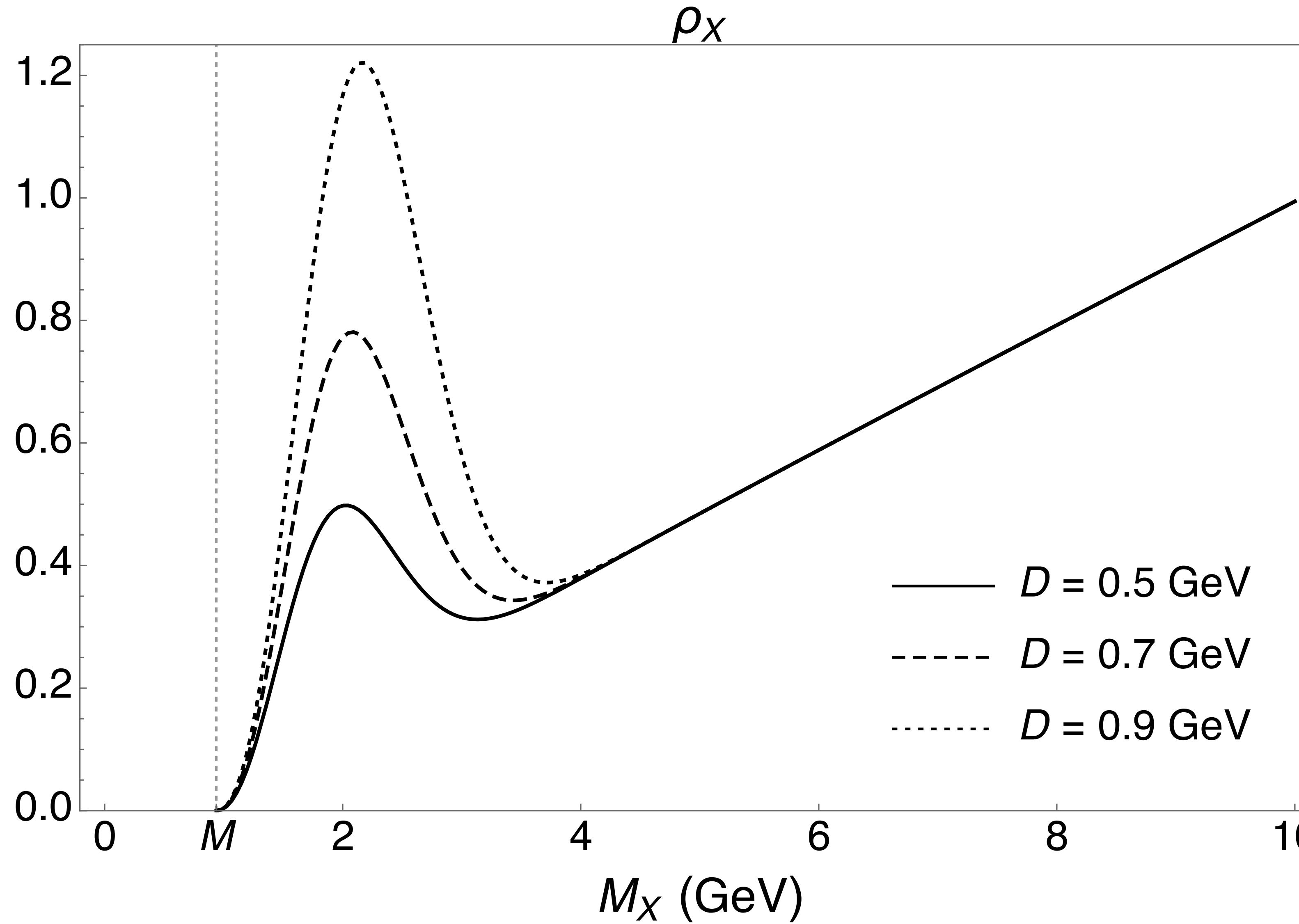
# Spectral function vs $(a - b)$



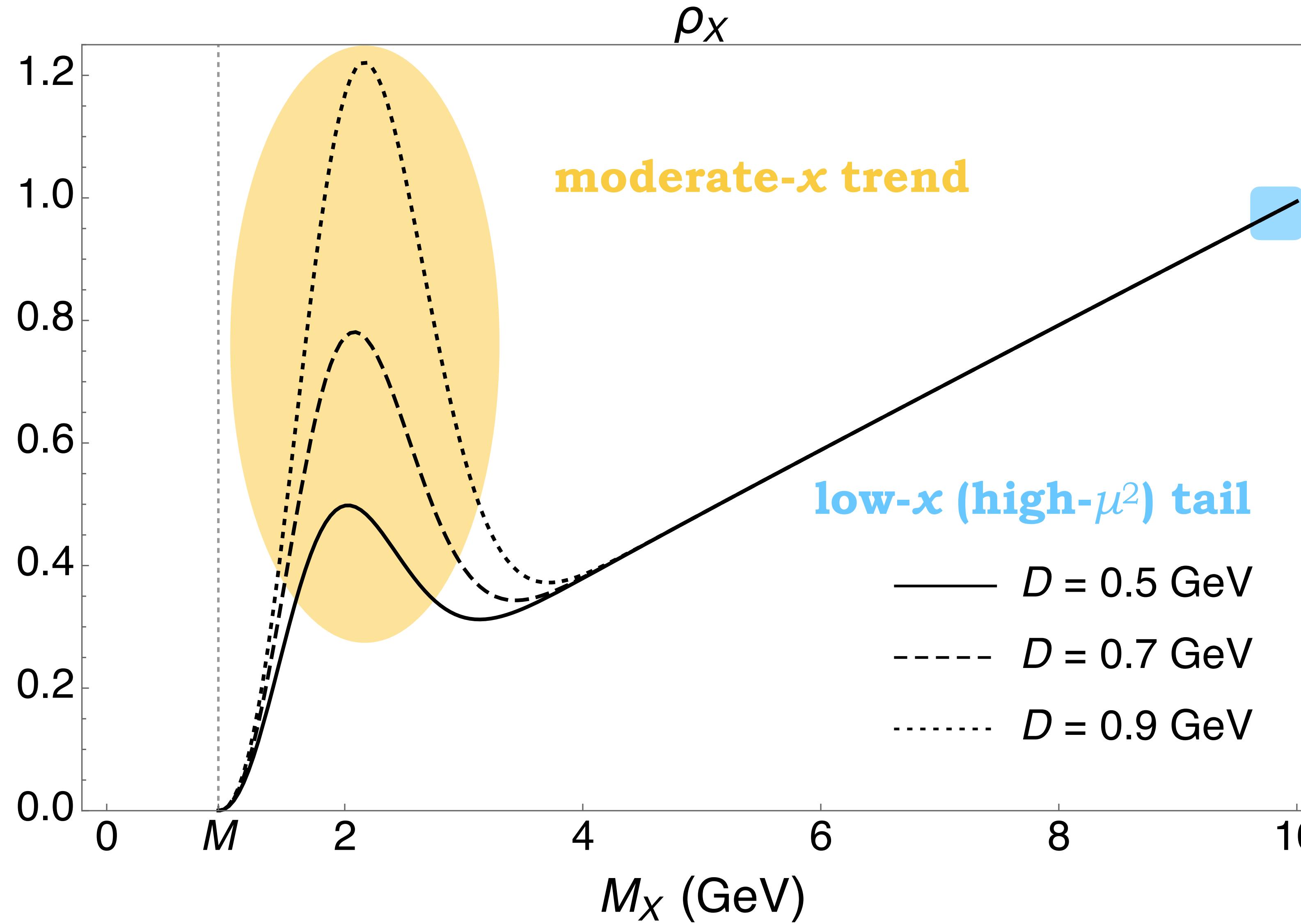
# $xf_1$ collinear PDF vs $(a - b)$



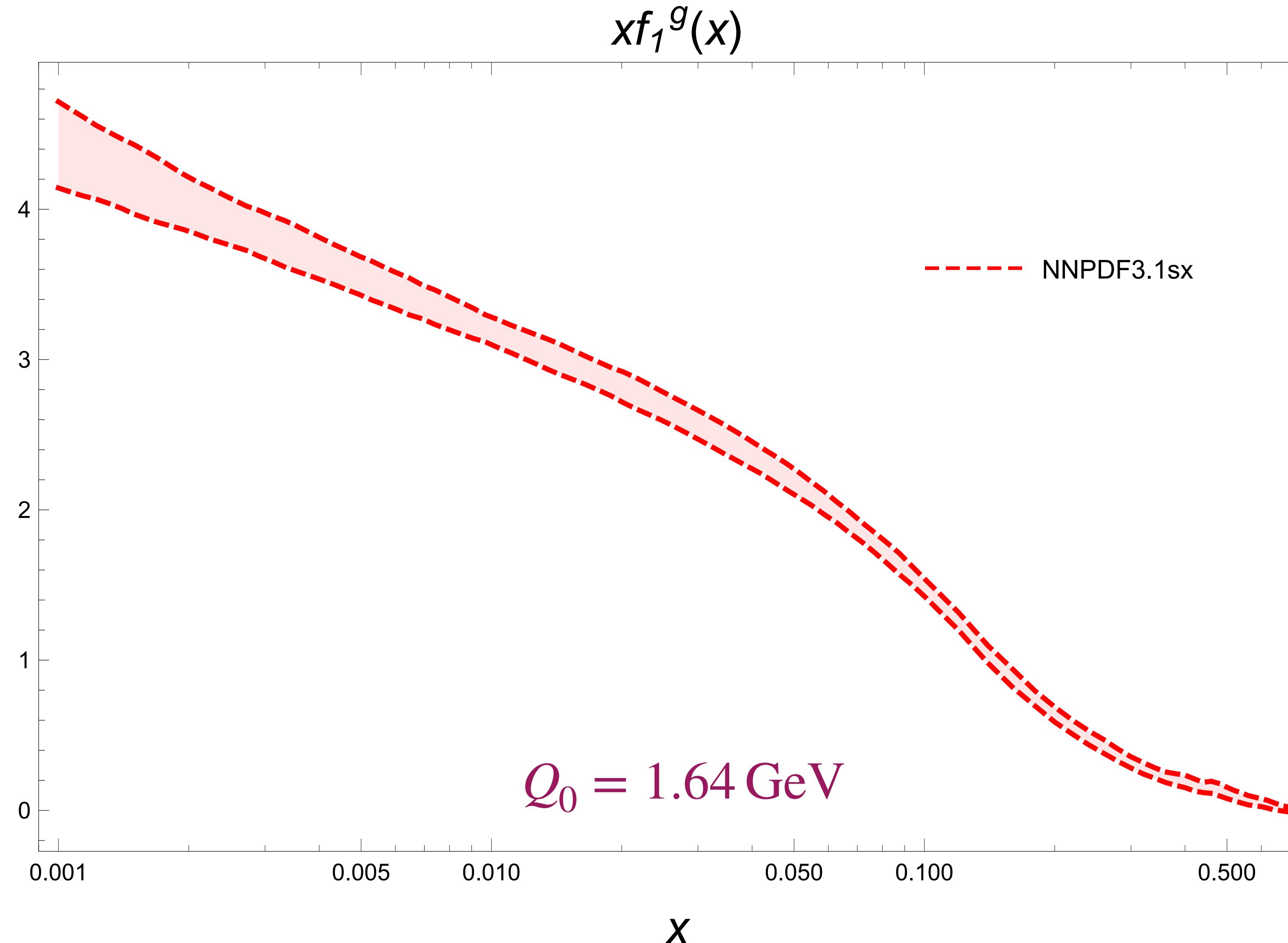
# Spectral function vs $D$



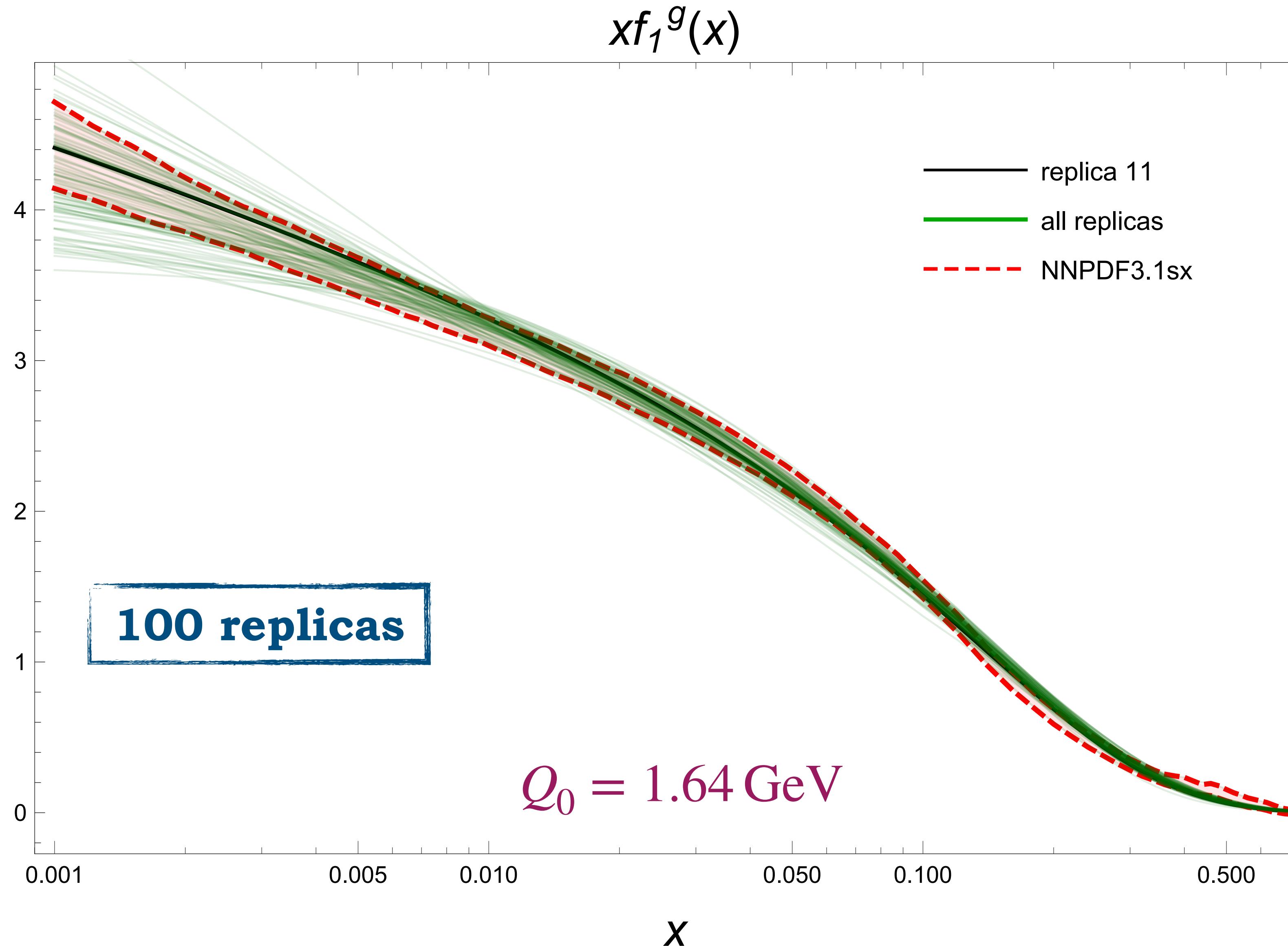
# Spectral function vs $D$



# Unpolarized gluon PDF

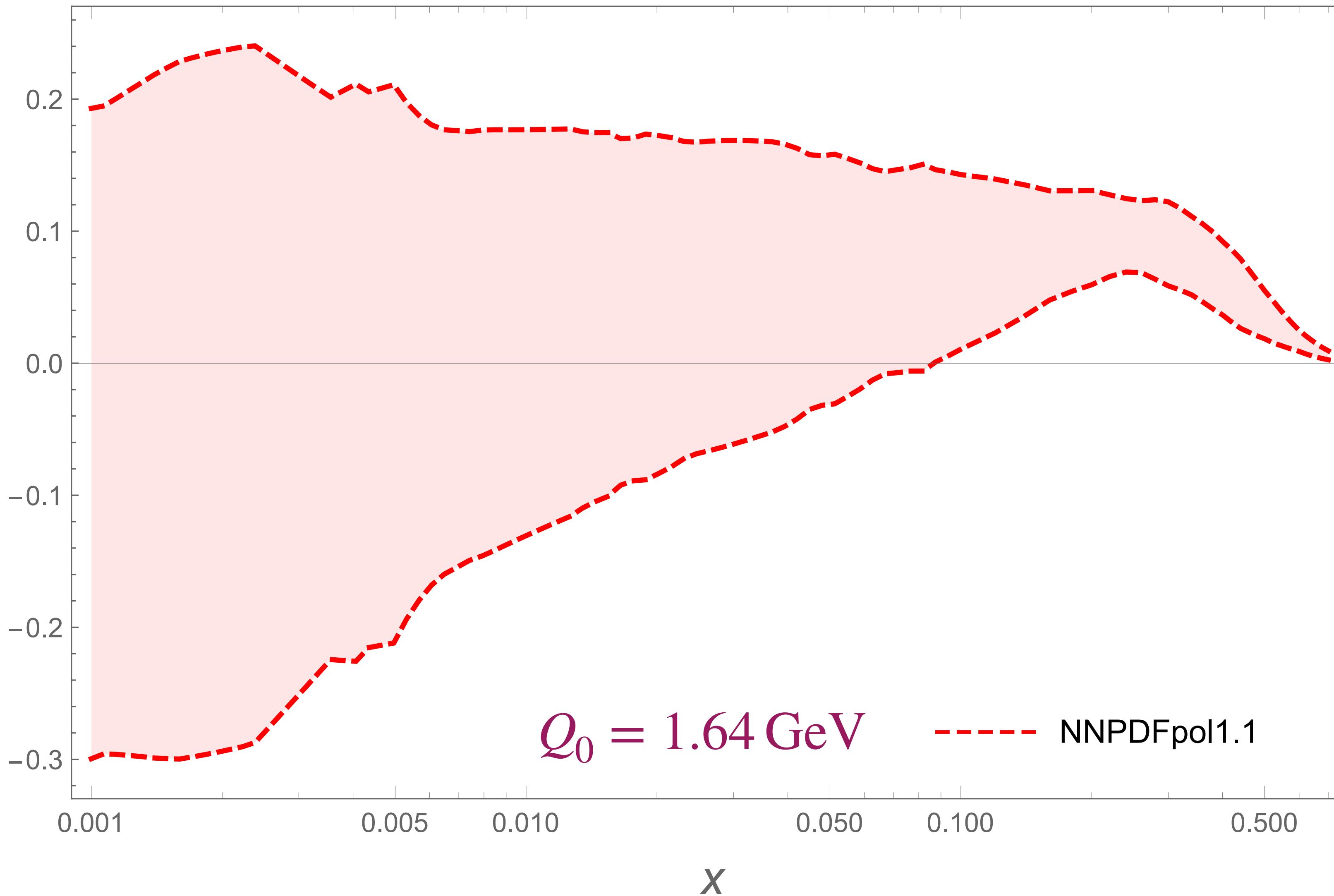


# Unpolarized gluon PDF



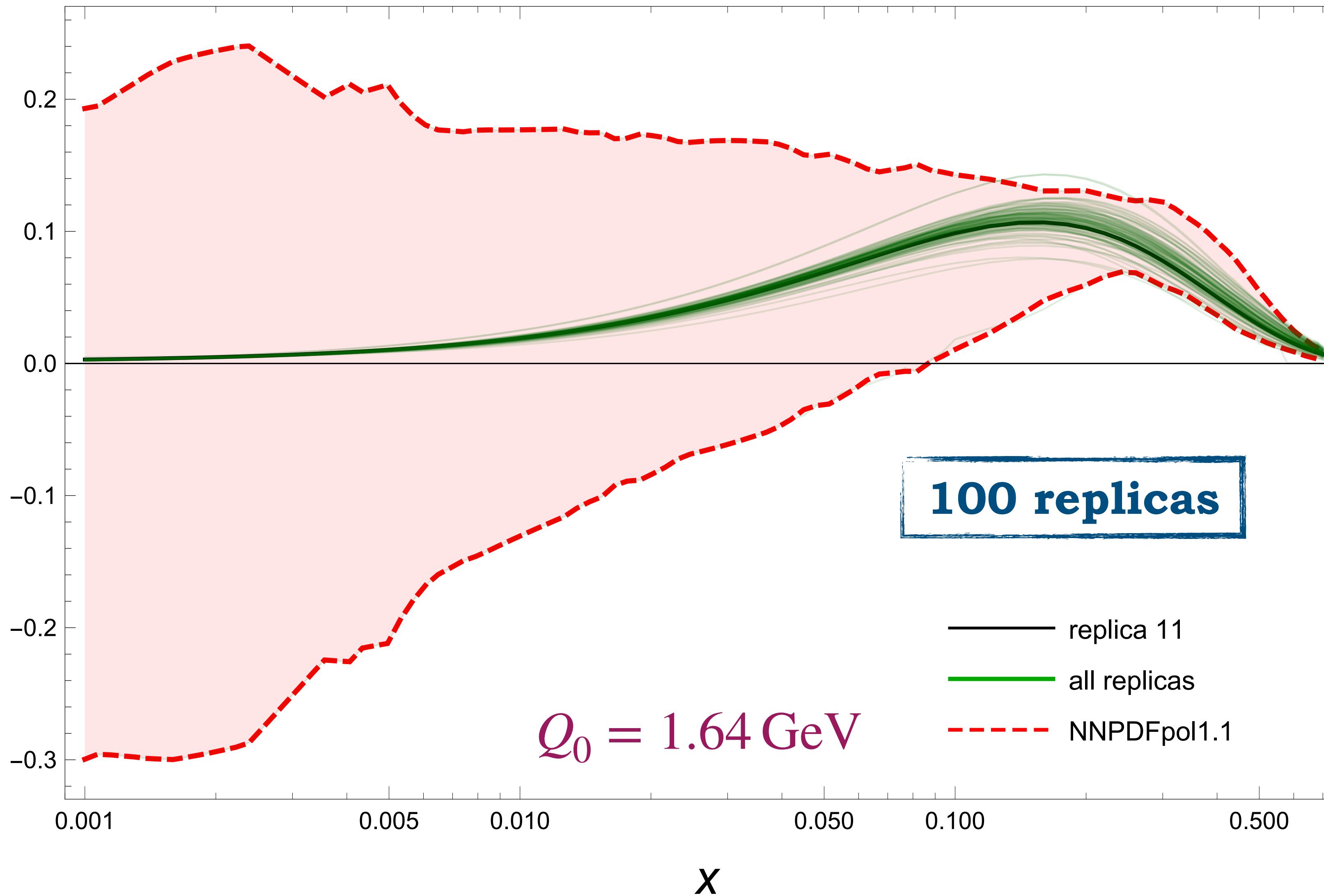
# Helicity gluon PDF

$xg_1^g(x)$



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# Fit specifics

$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for  $g_1$

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$$\langle x \rangle_g = \int_0^1 dx x f_1^g(x, Q_0)$$

$$S_g = \frac{1}{2} \langle 1 \rangle_{\Delta g} = \int_0^1 dx g_1^g(x, Q_0)$$

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Our model @  $Q_0 = 1.64$  GeV

$$\langle x \rangle_g = 0.424(9)$$

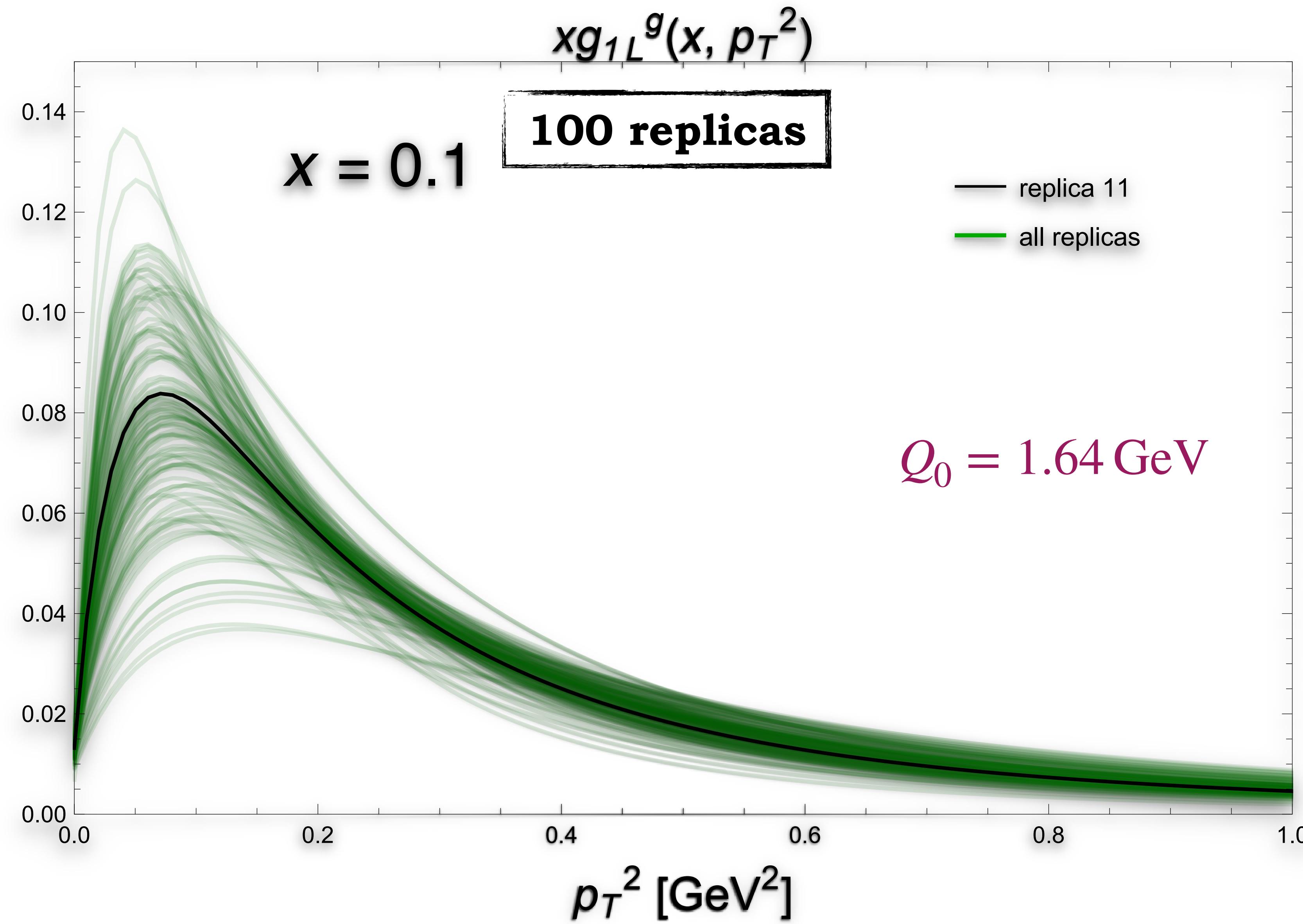
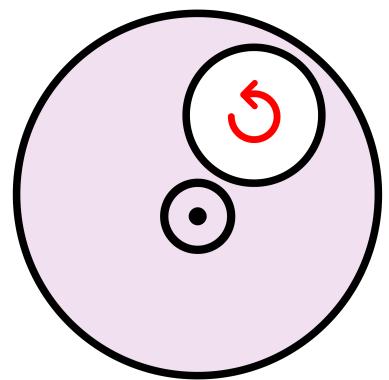
$$\langle S \rangle_g = 0.159(11)$$

Lattice @  $Q_0 = 2$  GeV

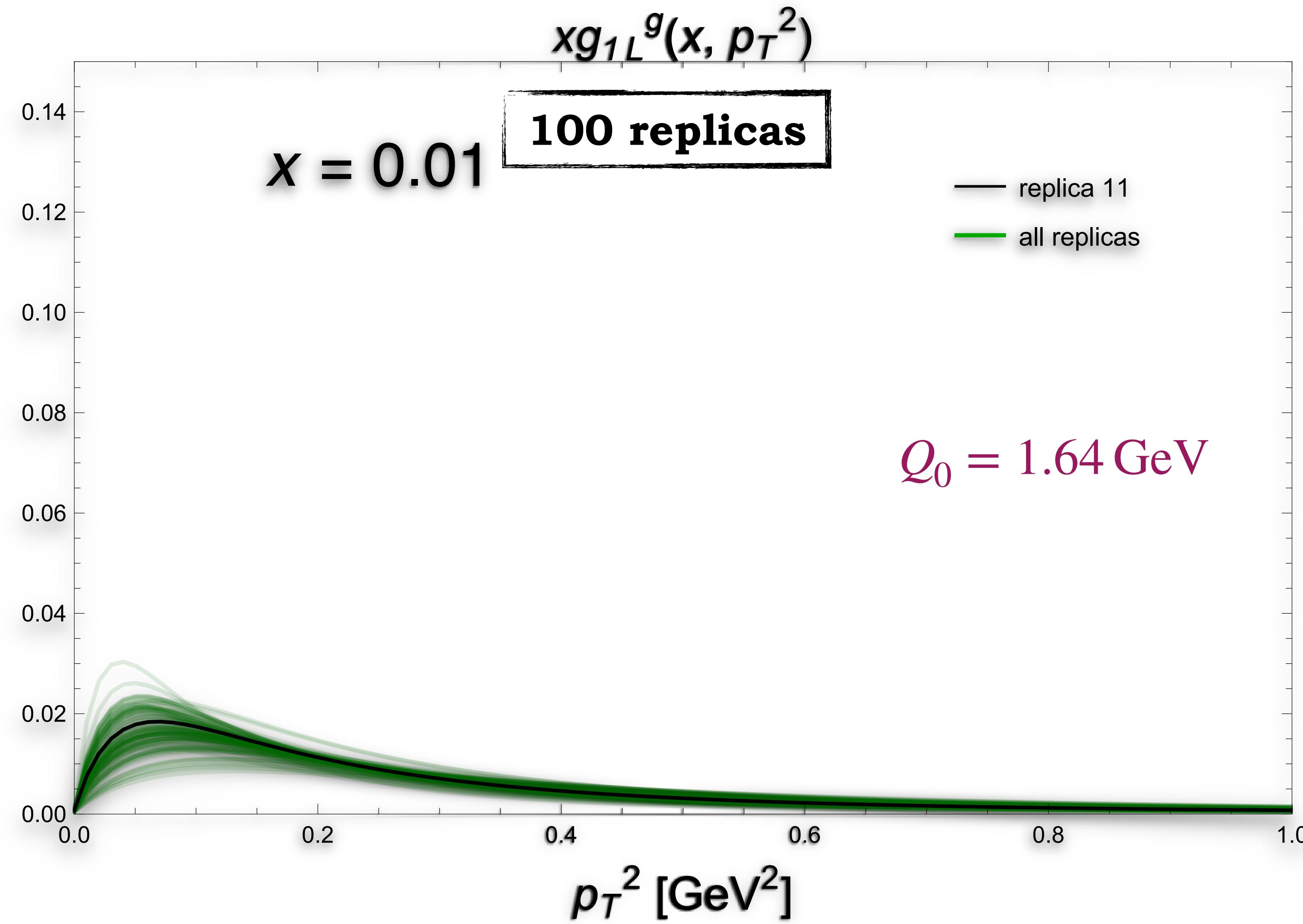
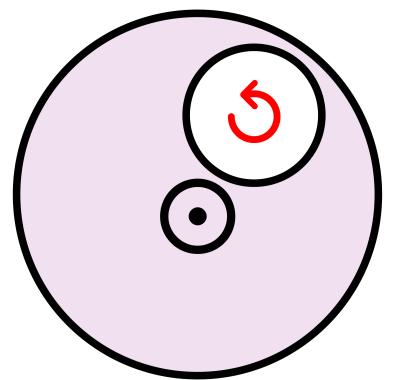
$$\langle x \rangle_g = 0.427(92)$$

$$\langle J \rangle_g = 0.187(46)$$

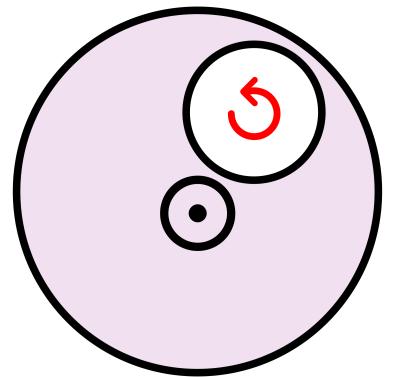
# Helicity gluon TMD



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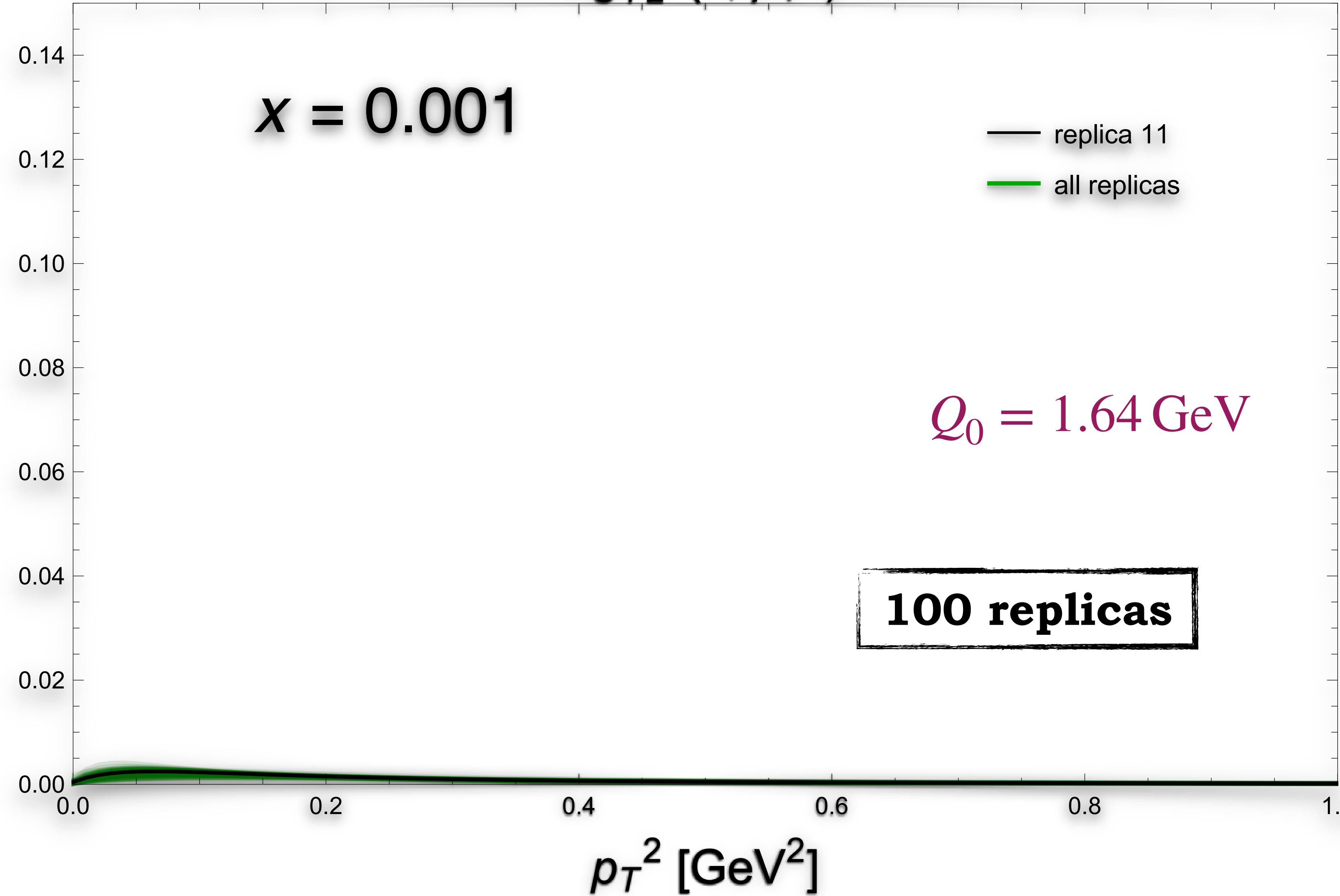
$x g_{1L}^g(x, p_T^2)$

$x = 0.001$

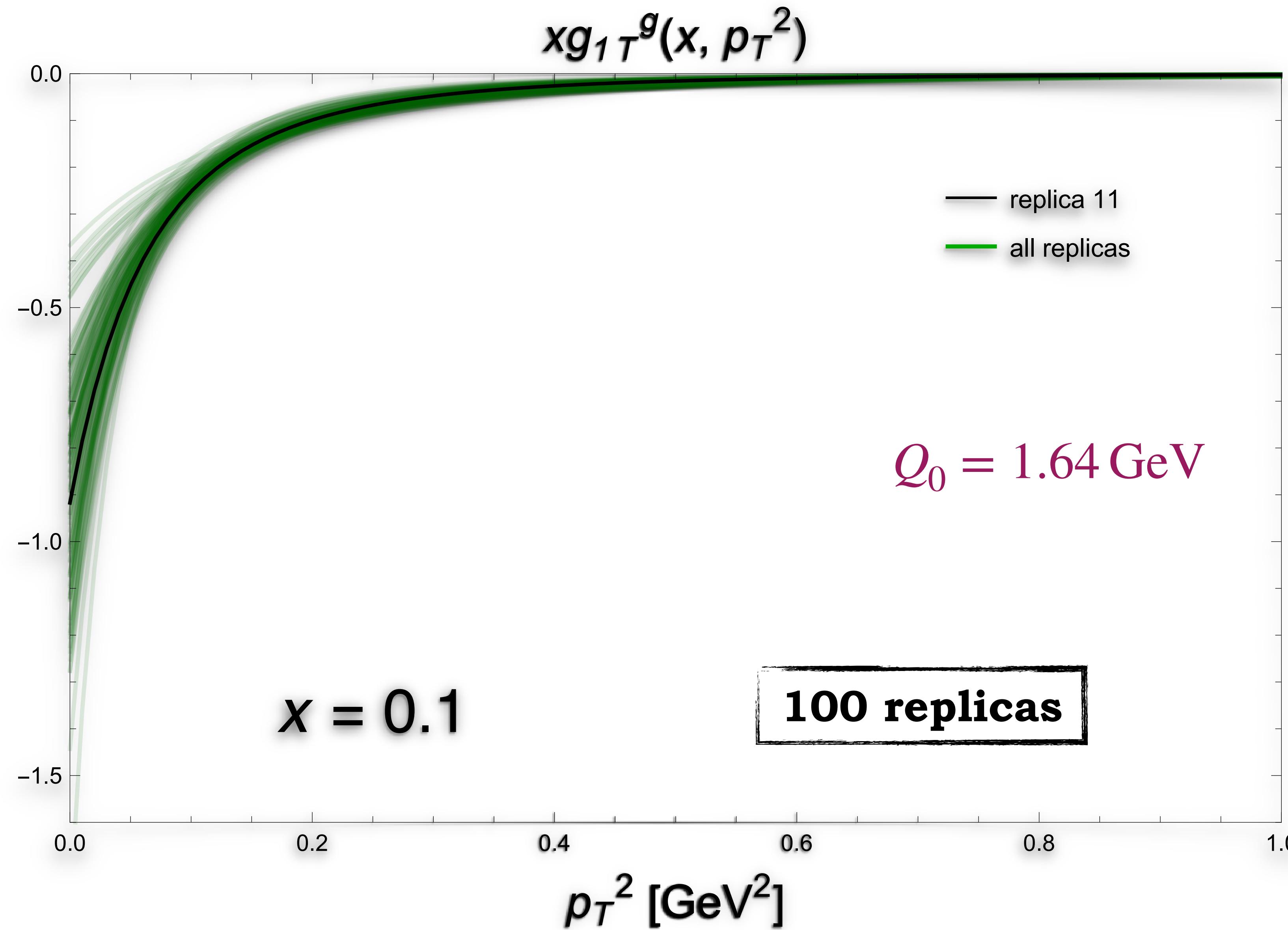
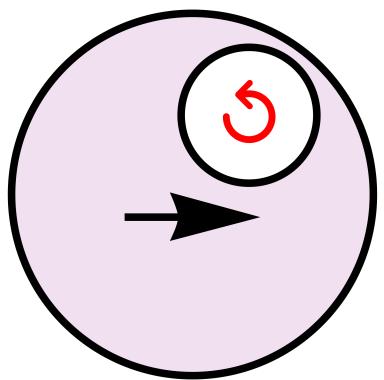
— replica 11  
— all replicas

$Q_0 = 1.64 \text{ GeV}$

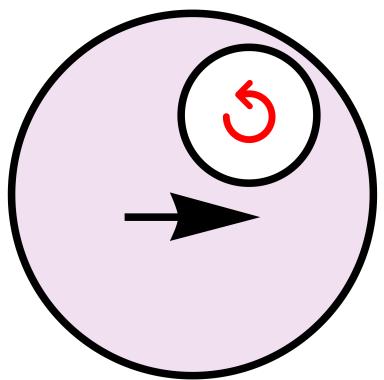
100 replicas



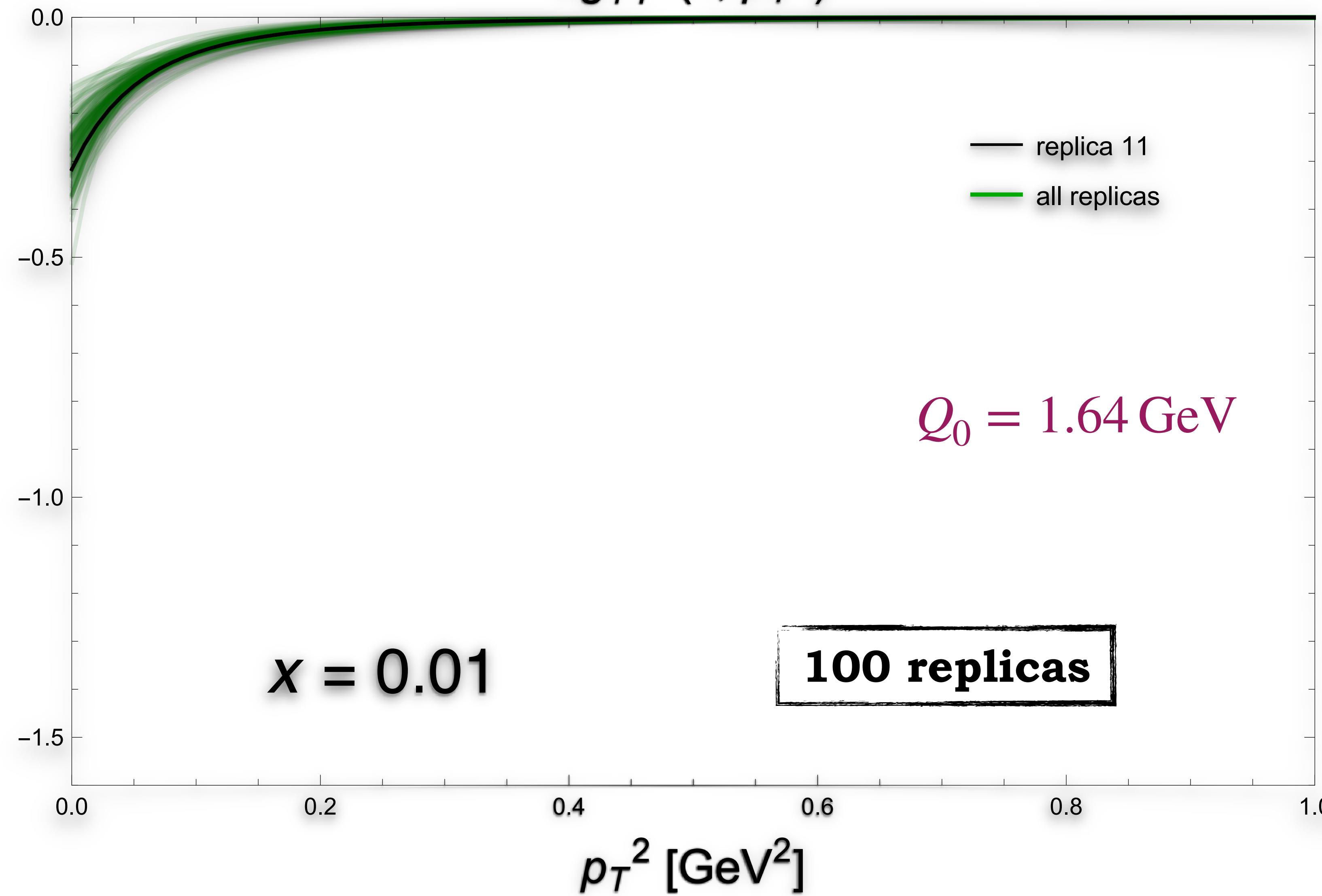
# Worm-gear gluon TMD



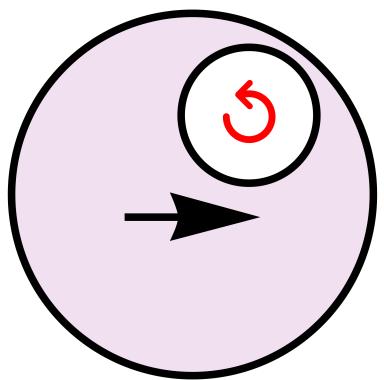
# Worm-gear gluon TMD



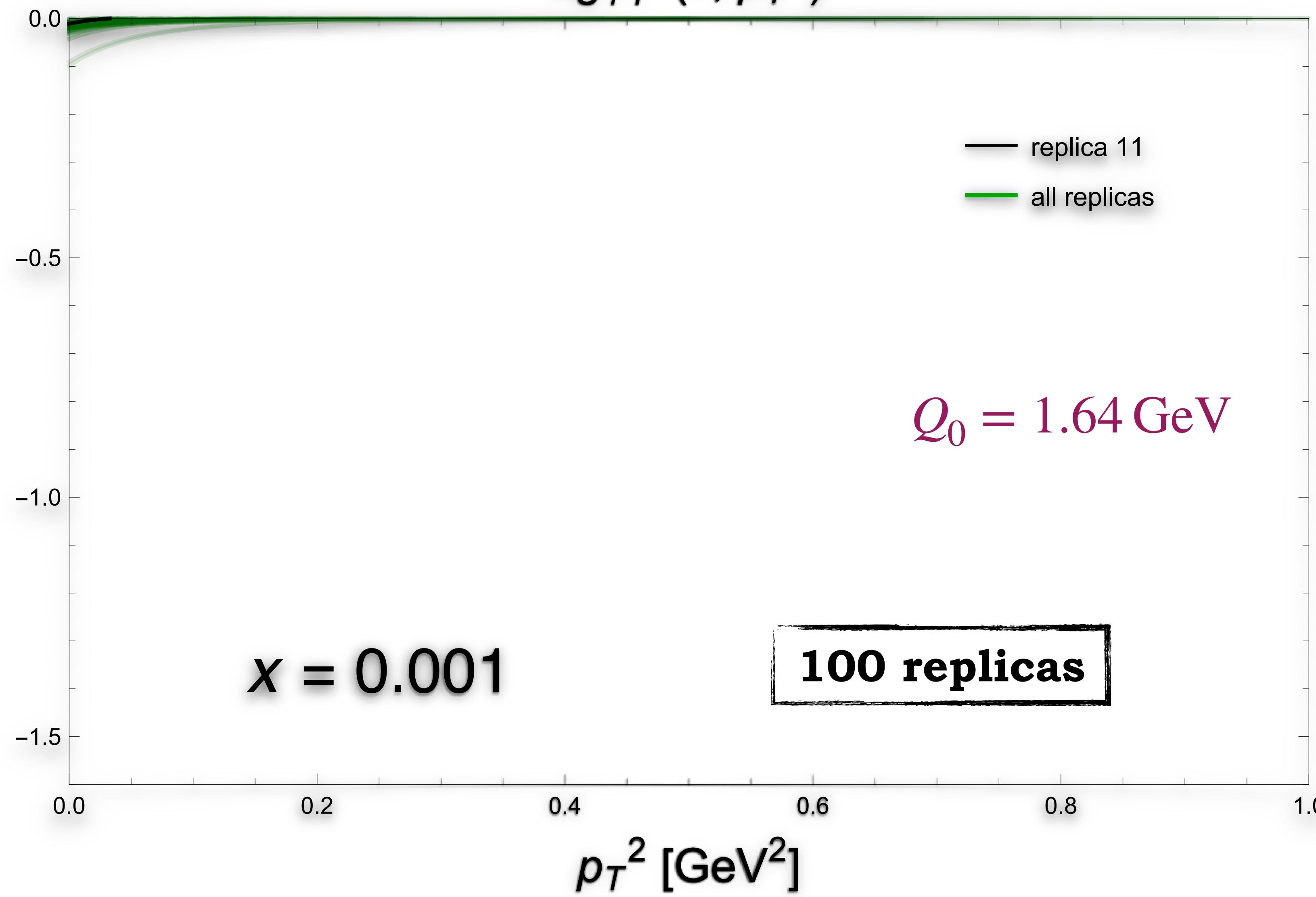
$$xg_1 \tau^g(x, p_T^2)$$



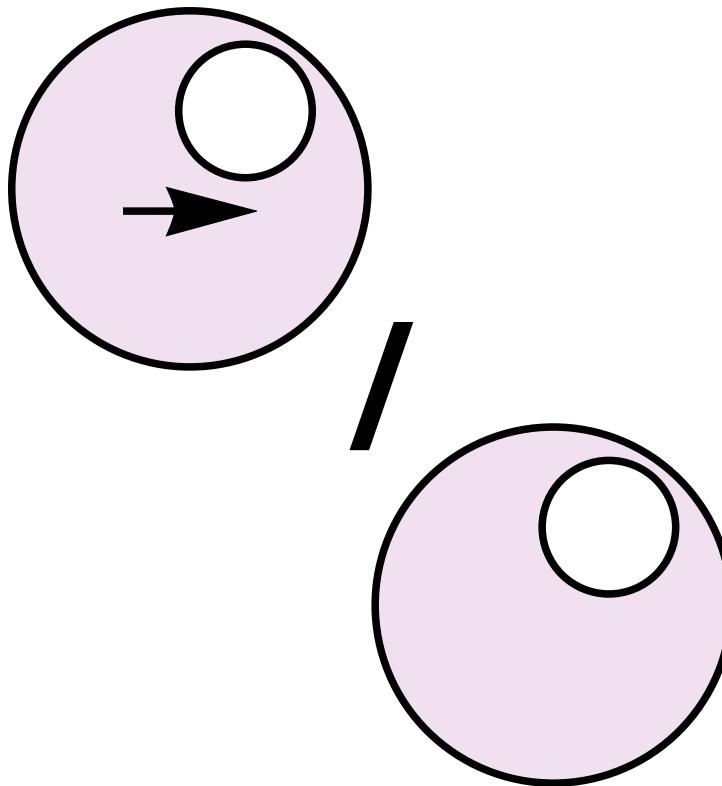
# Worm-gear gluon TMD



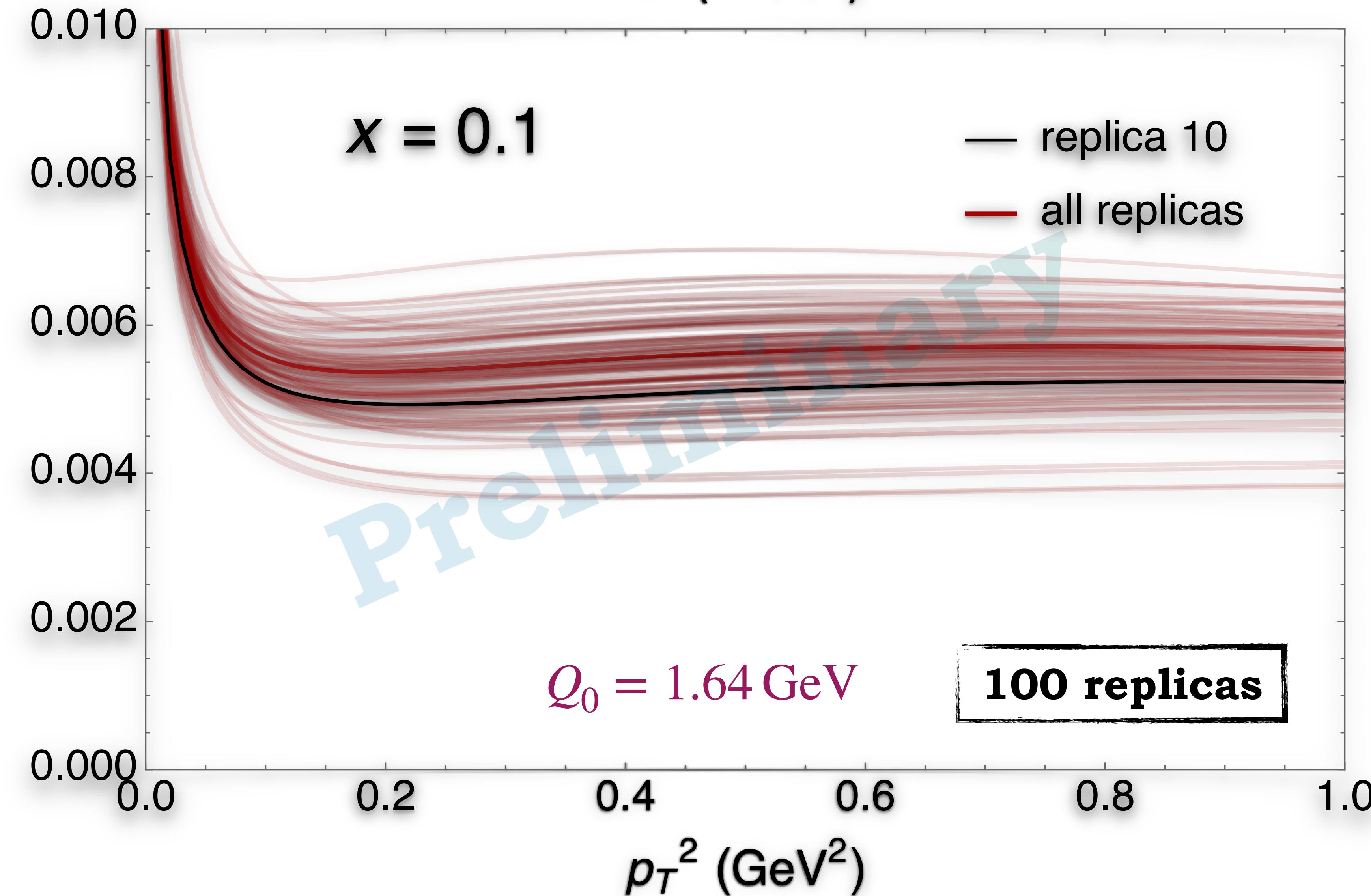
$x g_1 \tau^g(x, p_T^2)$



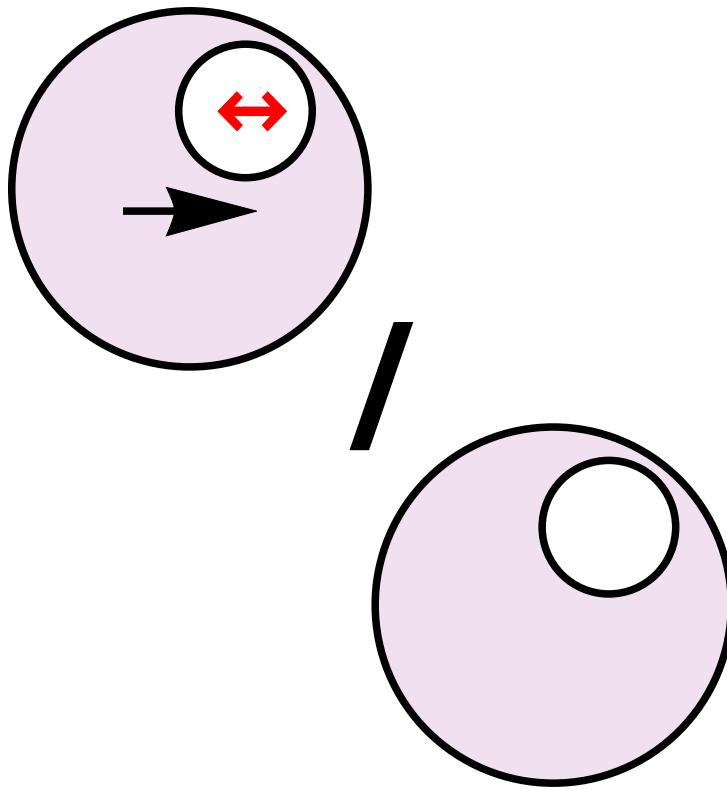
# *f*-type Sivers / unpol.



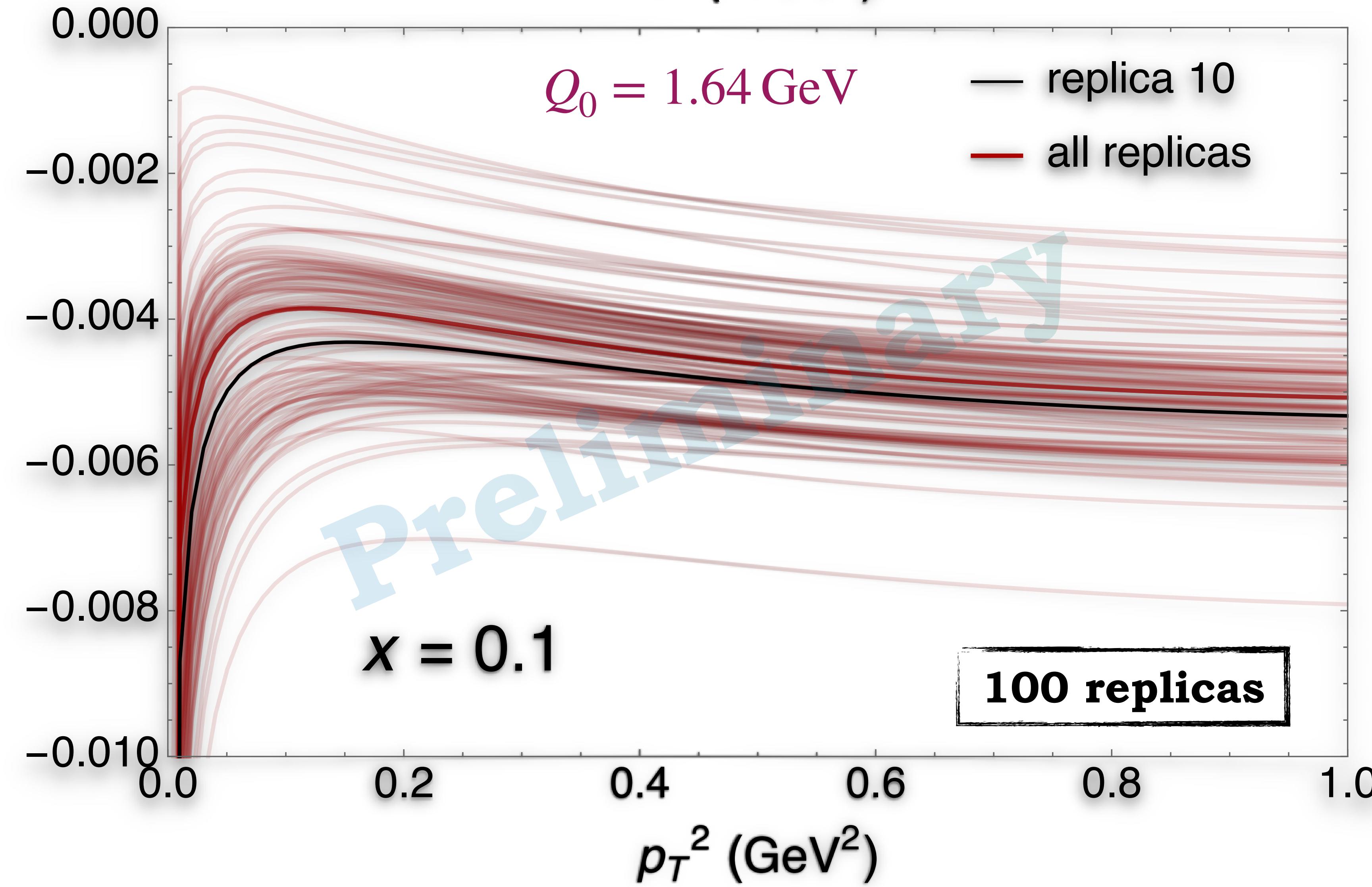
$$\frac{\frac{p_T}{M} f_1 \tau^{\perp[+,+]}(x, p_T^2)}{f_1^g(x, p_T^2)}$$



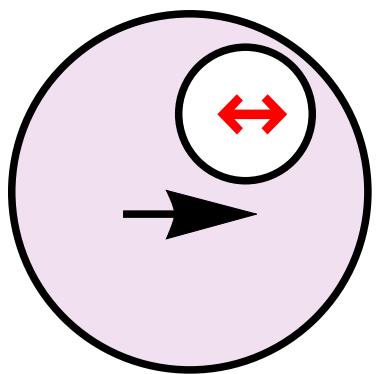
# *f*-type linearity / unpol.



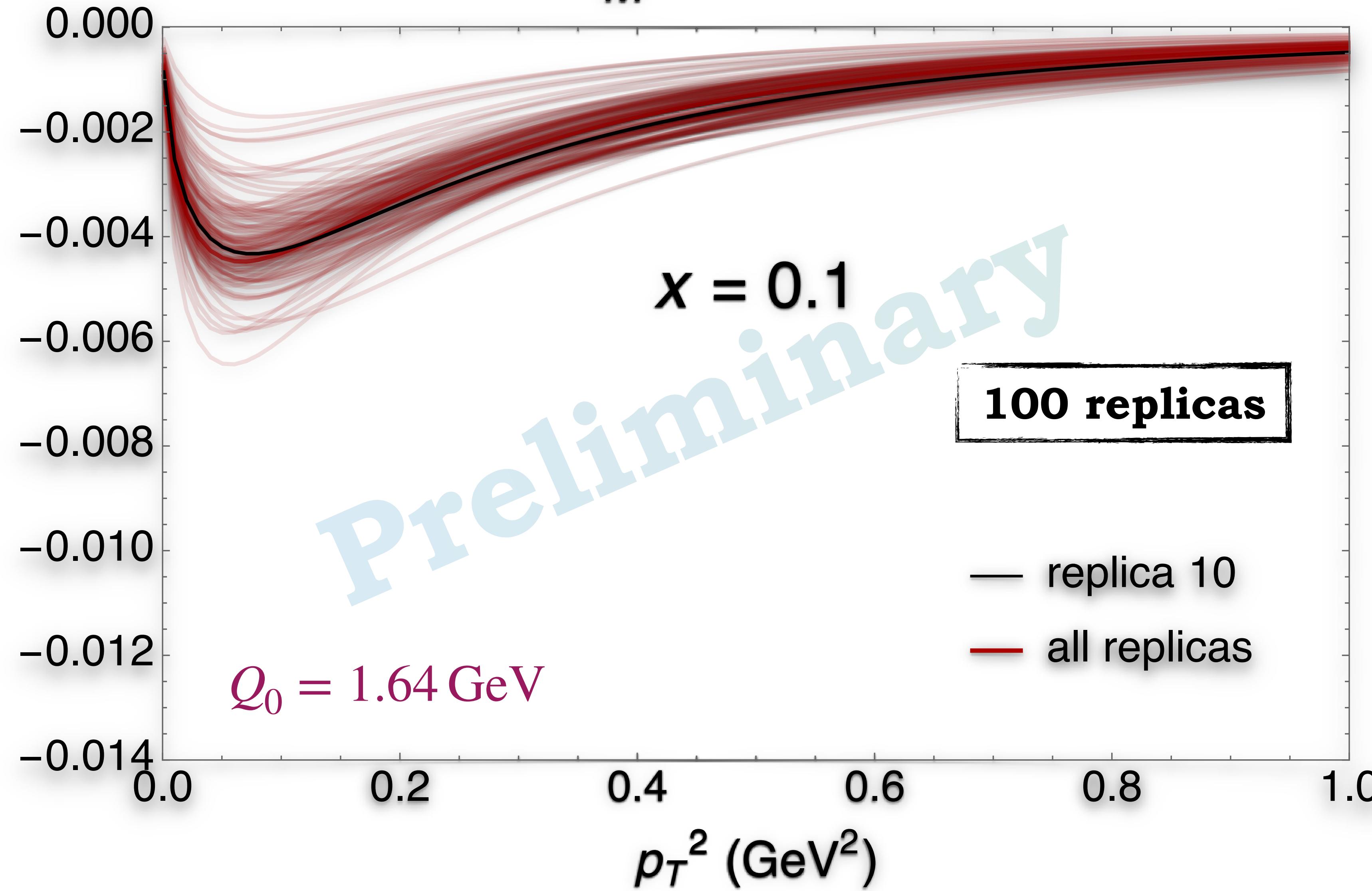
$$\frac{\frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)}{f_1^g(x, p_T^2)}$$



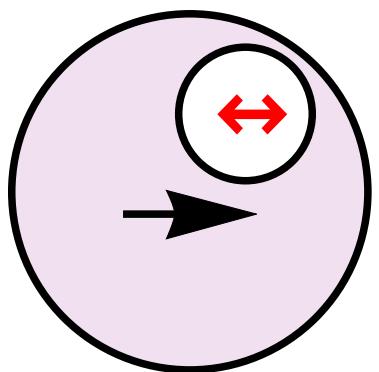
# *f*-type Linearity gluon TMD



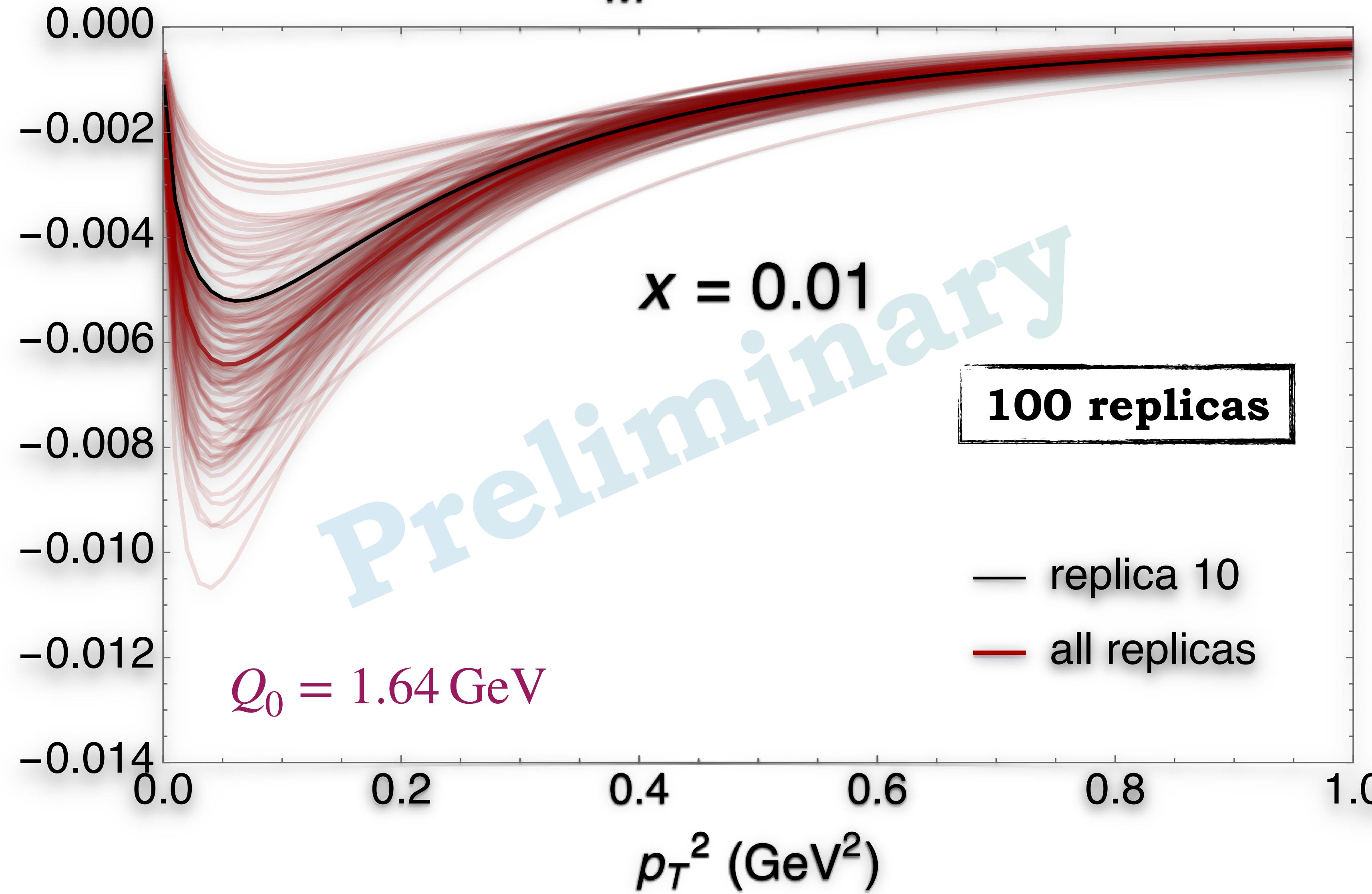
$$x \frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)$$



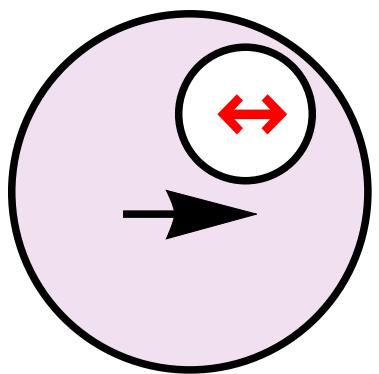
# *f*-type Linearity gluon TMD



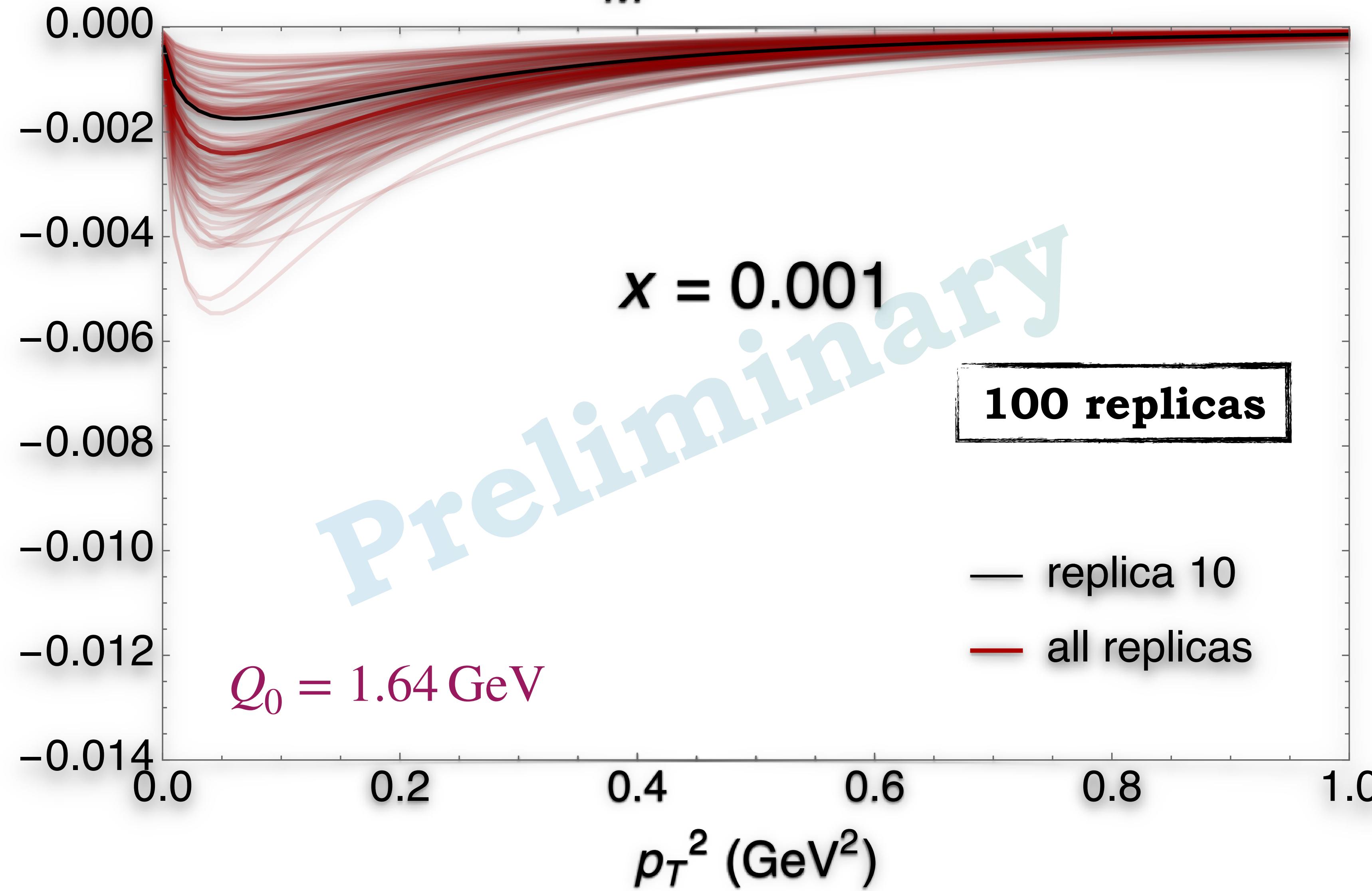
$$x \frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)$$



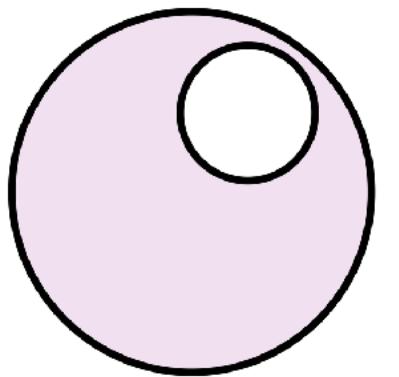
# *f*-type Linearity gluon TMD



$$x \frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)$$



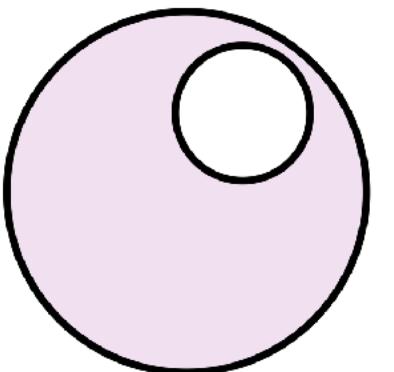
# $\rho$ -densities



**Unpolarized [u/u]**

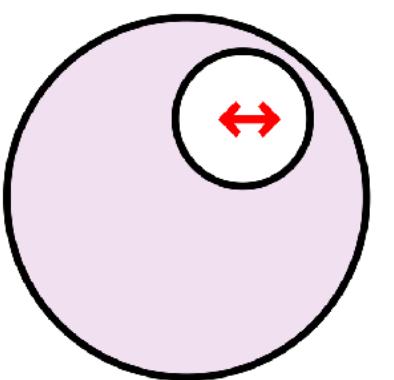
$$f_1(x, p_x, p_y)$$

# $\rho$ -densities



**Unpolarized [u/u]**

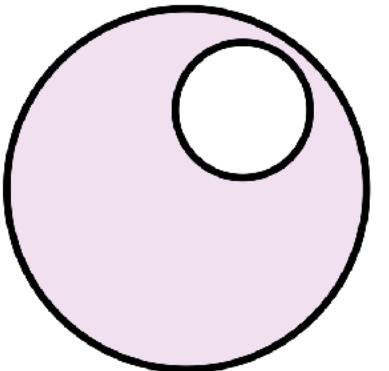
$$f_1(x, p_x, p_y)$$



**Boer-Mulders [ $\leftrightarrow/u$ ]**

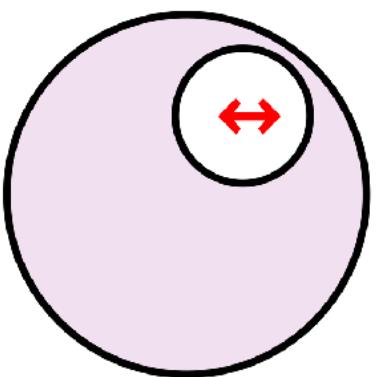
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

# $\rho$ -densities



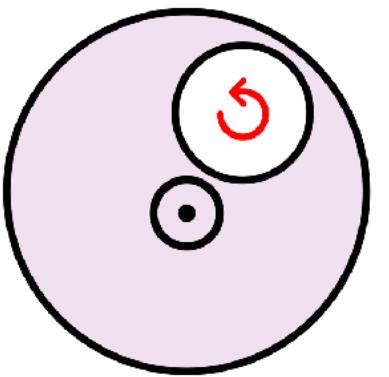
**Unpolarized** [u/u]

$$f_1(x, p_x, p_y)$$



**Boer-Mulders** [ $\leftrightarrow/u$ ]

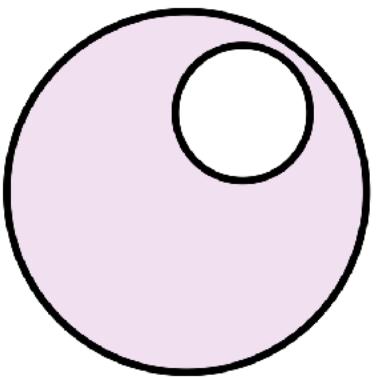
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



**Helicity** [ $\cup/+$ ]

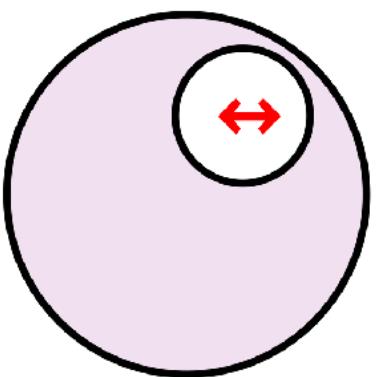
$$\frac{1}{2} \left[ f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$

# $\rho$ -densities



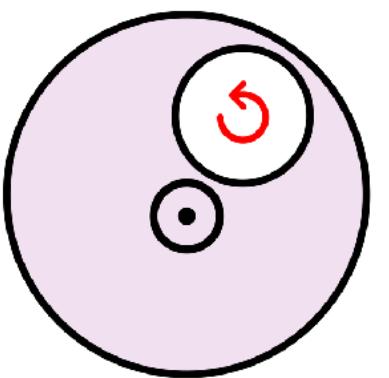
**Unpolarized** [u/u]

$$f_1(x, p_x, p_y)$$



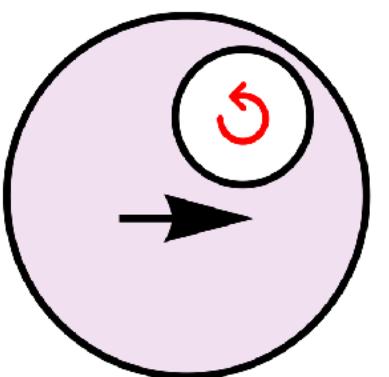
**Boer-Mulders** [ $\leftrightarrow/u$ ]

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



**Helicity** [ $\cup/+$ ]

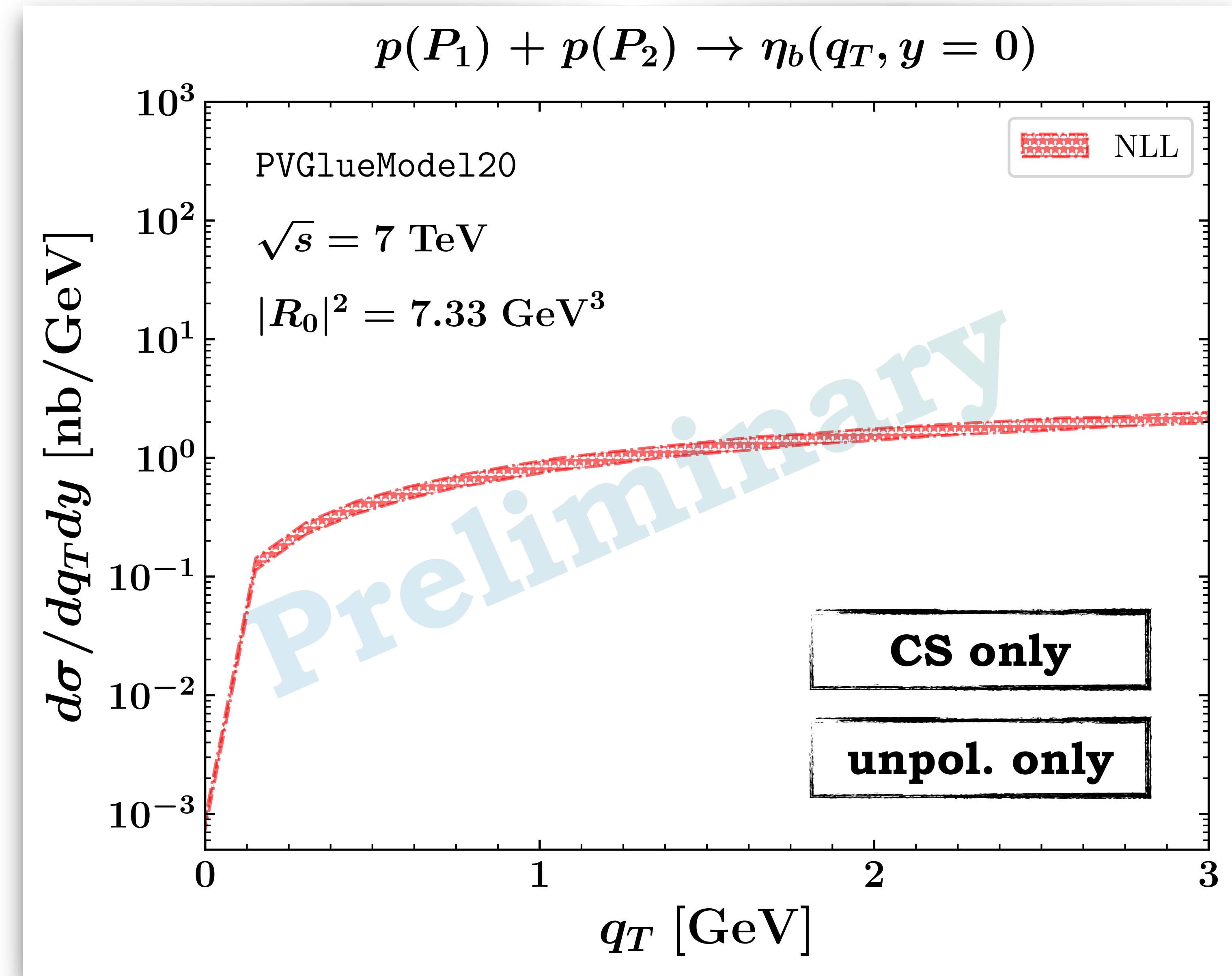
$$\frac{1}{2} \left[ f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$



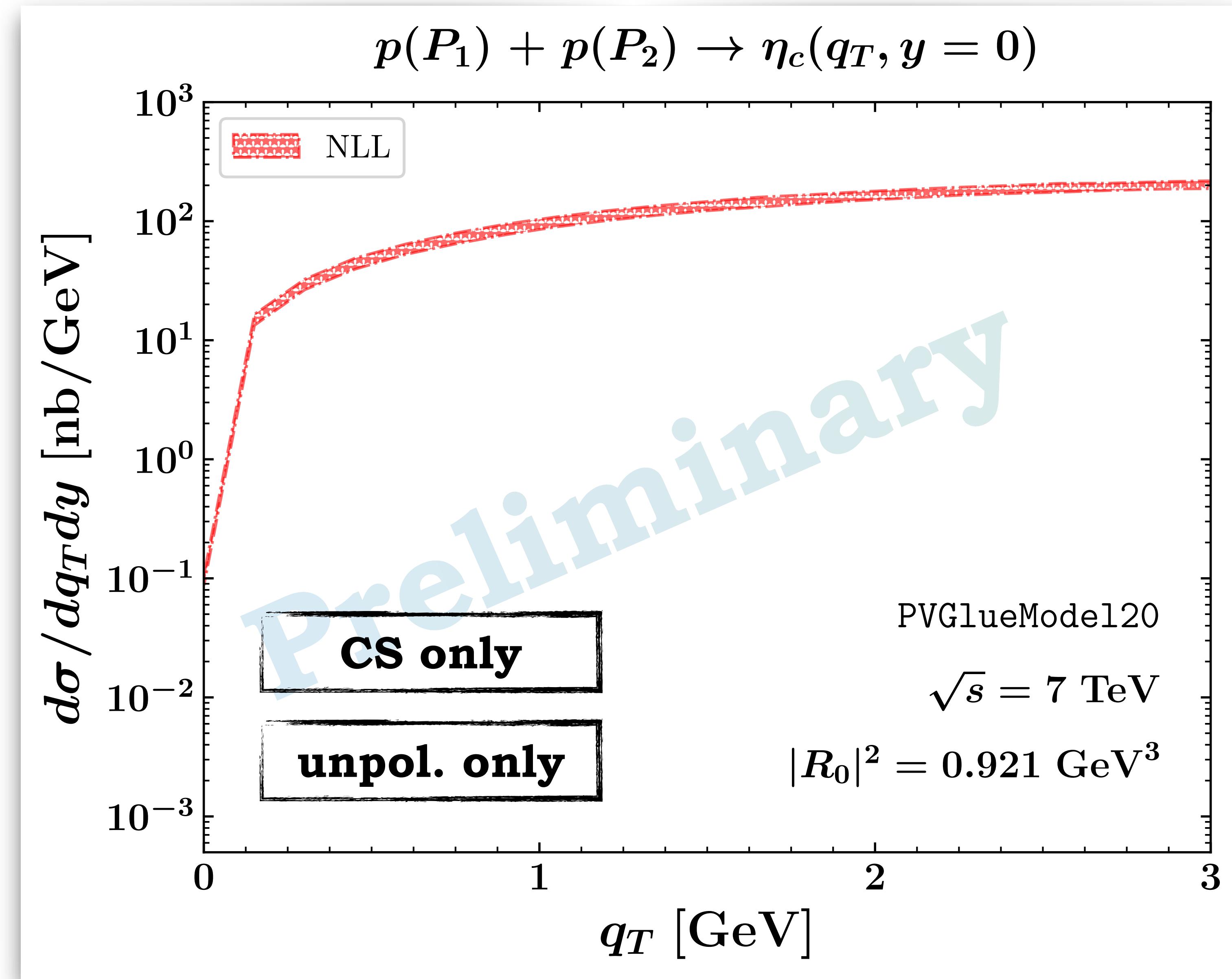
**Worm-gear** [ $\cup/\rightarrow$ ]

$$f_1(x, p_x, p_y) - \frac{p_x}{M} g_{1T}(x, p_x, p_y)$$

# $\eta_b$ production @ 7TeV LHC

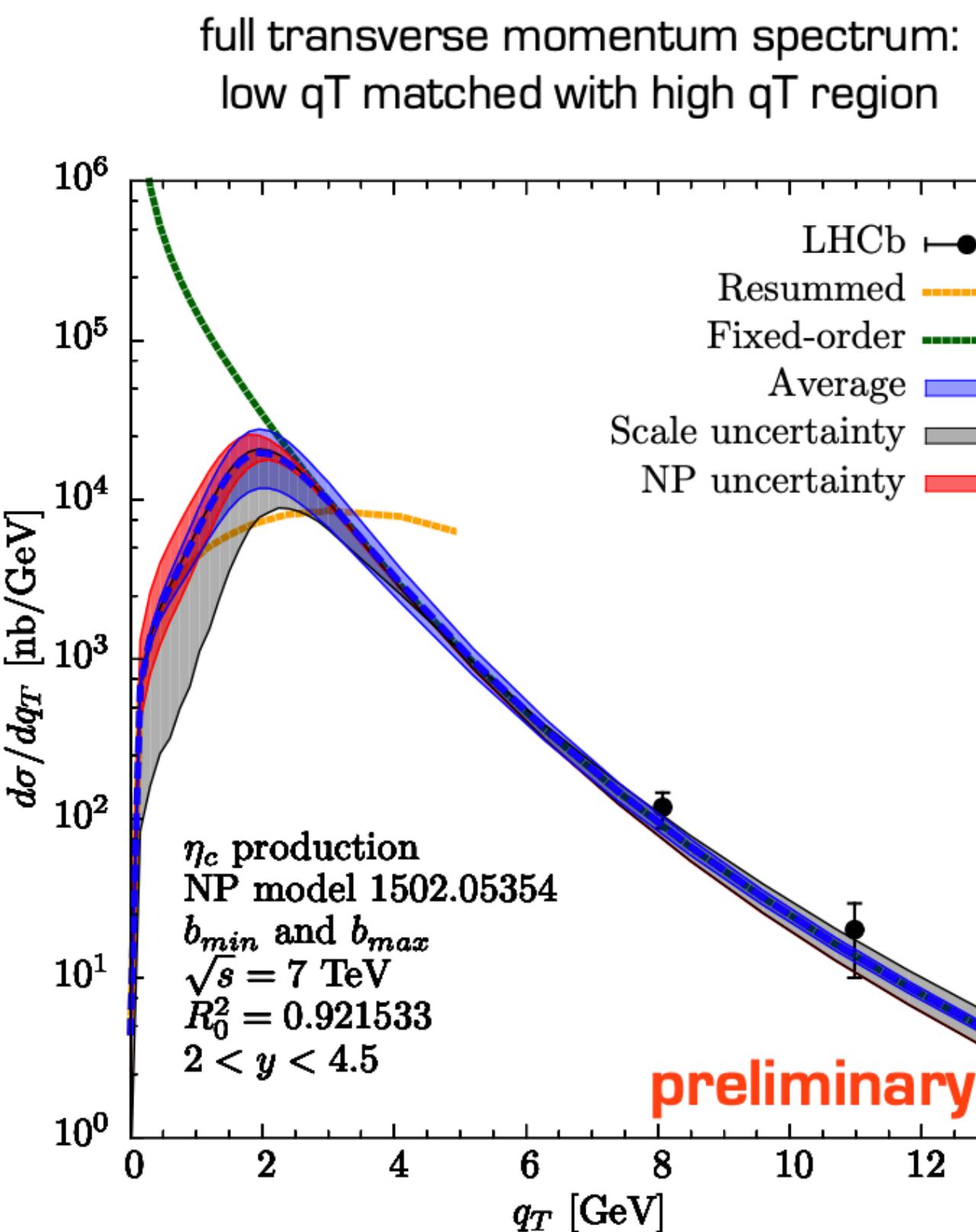


# $\eta_c$ production @ 7TeV LHC



# $\eta_c$ production @ 7TeV LHC

## $\eta_c$ production at LHC



blue band: uncertainty from matching

grey band: scale uncertainty

red band: nonpert. uncertainty

$$S_{NP}(\bar{b}_T) = - \left[ \frac{a_1}{2} + \frac{a_2}{2} \ln Q^2 \right] \bar{b}_T^2$$

$a_i = 0.5$  GeV $^2$ , var. 50%, envelope

both for unpolarized and  
linearly polarized distributions

the formalism is in good shape!

we need the data at low qT

