Twist-3 gluon fragmentation contribution to hyperon polarization and its frame independence

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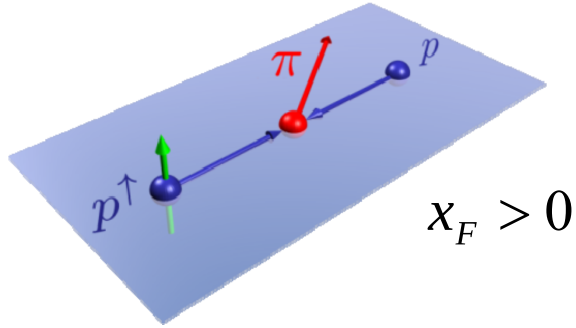
Cf. Phys. Rev. D **104**, no.5, 054023 (2021) [arXiv:2107.03113 [hep-ph]].

Phys. Rev. D **101**, no.5, 054017 (2020) [arXiv:1912.11199 [hep-ph]].

⋆ Transverse Single Spin Asymmetry (SSA)

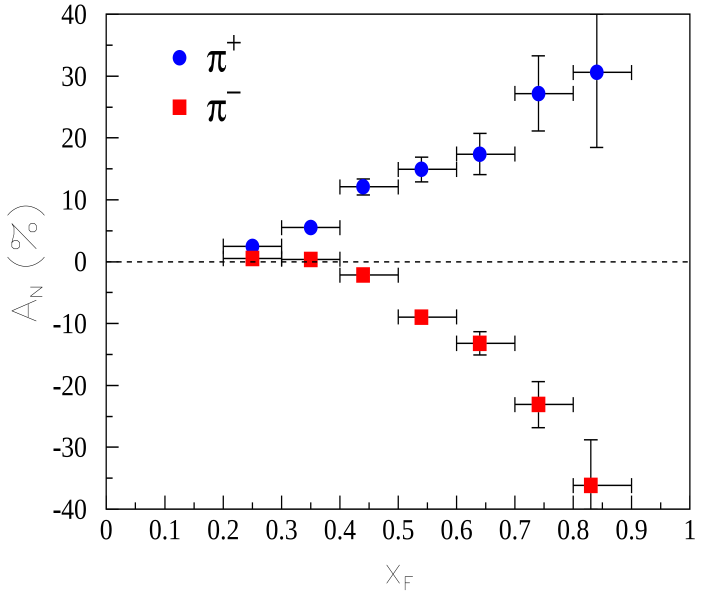
- $p^{\uparrow}p \to \pi X \qquad A_N = \dfrac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$



- FNAL-E704('91)($\sqrt{s} = 20$ GeV), RHIC($\sqrt{s} = 200$, 62 GeV):
  $A_N \sim 0.3$ at large $x_F = 2p_{\|}/\sqrt{s}$.

FNAL-E704

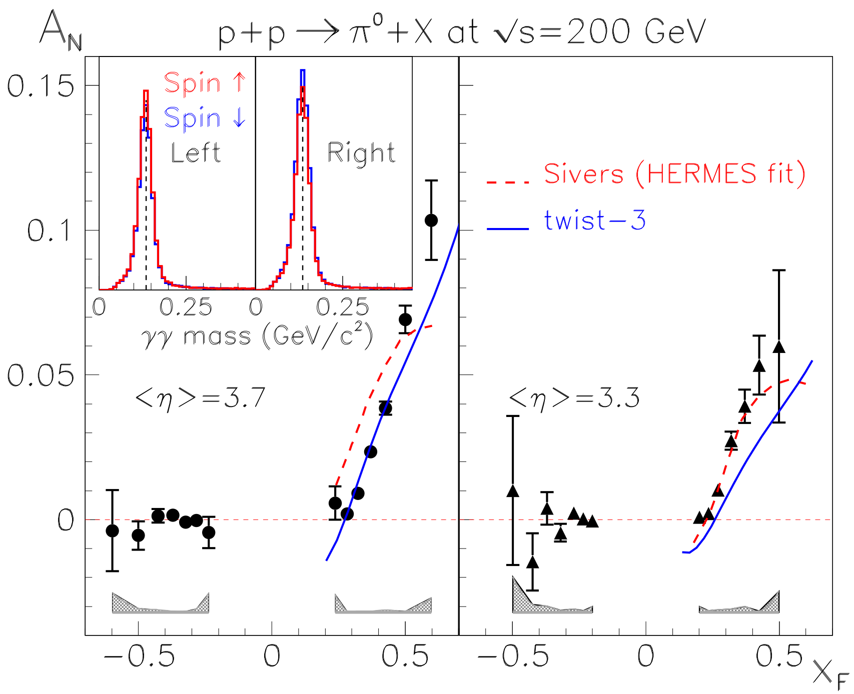
$\sqrt{s} = 20$ GeV



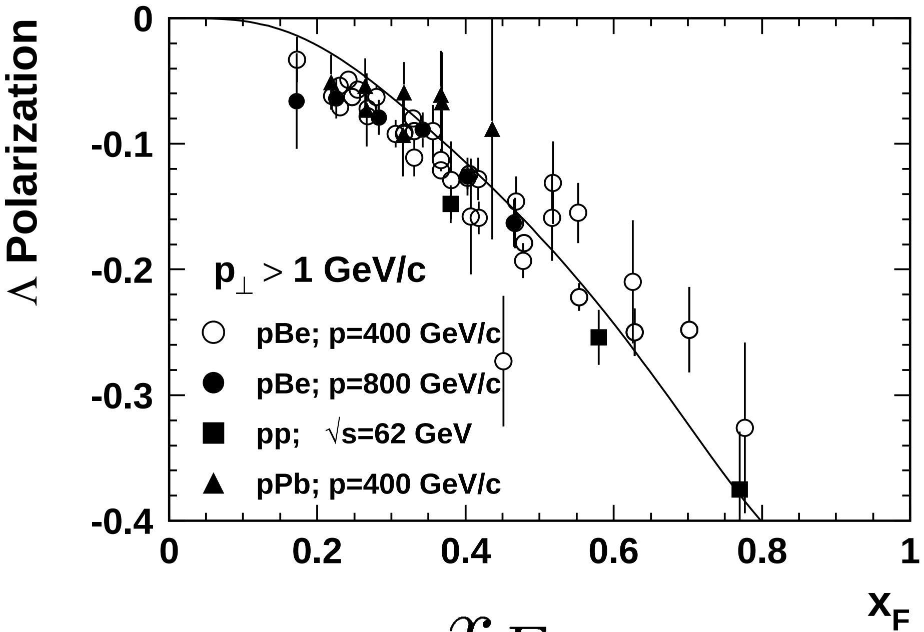
P.L. B264 ('91) 462
P.L. B261 ('91) 201

RHIC-STAR, PRL92('04)
hep-ex/0801.2990, PRL 101(2008)

$\sqrt{s} = 200$ GeV



- $pp \to \Lambda^{\uparrow} X$ (in 80's and 90's)



x_F

Also for other hyperons.

But the collinear twist-2 pQCD gives:

$$\text{SSAs} \sim \frac{\alpha_s m_q}{P_T}: \text{ small!} \quad \text{(Kane } et\ al. \text{ ('78))}$$

⋆ SSAs in the collinear factorization

(Efremov-Teryaev, Qiu-Sterman, Eguchi-YK-Tanaka,..)

· Describes SSA as a twist-3 observable in the large-P_T region ($P_T \sim Q \gg \Lambda_{\rm QCD}$).

$$\sigma = \underbrace{F_2 \otimes \hat{\sigma}_2}_{\text{twist-2}} + \underbrace{\left(\frac{M_N}{Q}\right) F_3 \otimes \hat{\sigma}_3}_{\text{twist-3}} + \underbrace{\left(\frac{M_N}{Q}\right)^2 F_4 \otimes \hat{\sigma}_4}_{\text{twist-4}} + \cdots$$

· Twist-3 DF and FF ($\sim$ multi-parton correlation functions) in the nucleon or in the fragmenting processes cause SSAs.

· **Twist-3 DF and FF are process independent.** ($\leftrightarrow$ **TMD functions**)

· **Applicable to $p^\uparrow p \to \pi X$, $pp \to \Lambda^\uparrow X$.** ($\leftrightarrow$ **TMD factorization not valid.**)

$\star$ LO cross section in the collinear twist-3 framework

- $p^{\uparrow}p \to \pi X$: Completed and used to analyze RHIC data.

· Twist-3 distribution in $p^{\uparrow}$ (Sivers type)

Qiu, Sterman(1998) Kouvaris, Qiu, Vogelsang, Yuan(2006) Koike, Tomita(2009)

Beppu, Kanazawa, Koike, Yoshida(2014)

· Twist-3 distribution in p (Boer-Mulders type)

Kanazawa, Koike(2000)

· Twist-3 fragmentation for π (chiral-odd, Collins type)

Metz, Pitonyak(2013) → **Major source of A_N^{π} of RHIC data.**

Kanazawa, YK, Metz, Pitonyak (2014)

- $pp \to \Lambda^{\uparrow} X$

  · Twist-3 distribution in p (Boer-Mulders type)

  Kanazawa, Koike(2001)   Zhou, Yuan, Liang(2008)   Koike, Yabe, Yoshida(2015)

  · Twist-3 fragmentation for $\Lambda^{\uparrow}$ (chiral-even)

  Quark FF: Koike, Metz, Pitonyak, Yabe, Yoshida(2017)

  Gluon FF: Koike, Yabe, Yoshida (2021)   → **This talk.**

  Requires a new formalism.

- **Phenomenological relevance**

  · **SSAs for $p^{\uparrow}p \to \pi X$ and $pp \to \Lambda^{\uparrow} X$ show similar behavior.**
  i.e. increase at large x_F.

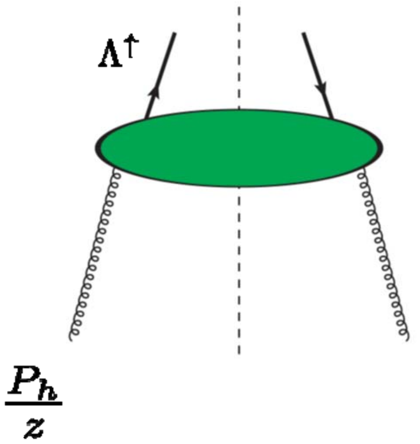
  → Twist-3 FF may be also important in $pp \to \Lambda^{\uparrow} X$.

  · **Twist-3 gluon FF could be important.**
  -Gluons are ample in the collision environment.
  -Twist-3 quark and gluon FFs for $\Lambda^{\uparrow}$ mix under renormalization.

⋆ Complete set of Twist-3 gluon FFs

- **intrinsic twist-3**

YK, K. Yabe, S. Yoshida, Phys. Rev. D101, 054017 (2020)



$$\widehat{\Gamma}^{\alpha\beta}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F_a^{w\beta}(0)|hX\rangle\langle hX|F_a^{w\alpha}(\lambda w)|0\rangle$$

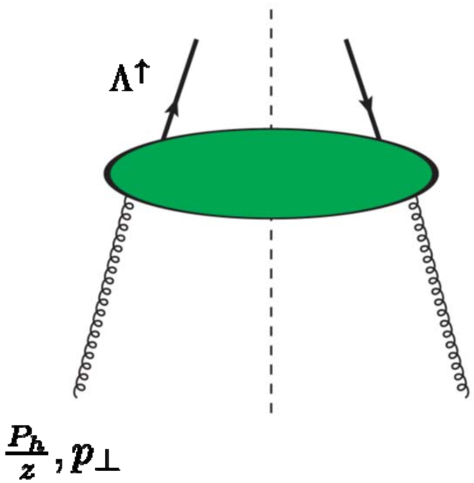
$$= -iM_h \epsilon^{P_h w S_\perp [\alpha} w^{\beta]} \Delta\widehat{G}_{3T}(z) + M_h \epsilon^{P_h w S_\perp \{\alpha} w^{\beta\}} \boxed{\Delta\widehat{G}_{3\bar{T}}(z) +} \cdots$$

contributes to SSAs

P_h, $S_\perp$: momentum and spin of $\Lambda^\uparrow$.

Another lightlike vector w ($P_h \cdot w = 1$) is needed.

- **kinematical twist-3**



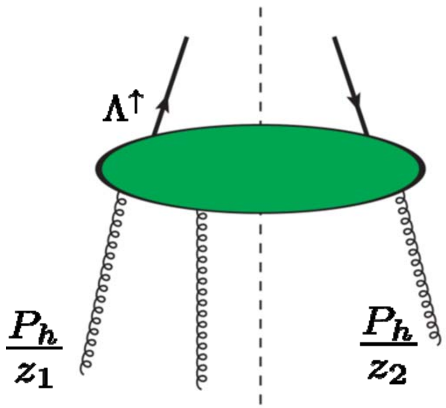
$$\widehat{\Gamma}_\partial^{\alpha\beta\gamma}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F_a^{w\beta}(0)|hX\rangle\langle hX|F_a^{w\alpha}(\lambda w)|0\rangle \overleftarrow{\partial}^\gamma$$

$$= -i\frac{M_h}{2} g_\perp^{\alpha\beta} \epsilon^{P_h w S_\perp \gamma} \boxed{\hat{G}_T^{(1)}(z)} + \frac{M_h}{2} \epsilon^{P_h w \alpha\beta} S_\perp^\gamma \Delta\hat{G}_T^{(1)}(z)$$

$$-i\frac{M_h}{8}\left(\epsilon^{P_h w S_\perp \{\alpha} g_\perp^{\beta\}\gamma} + \epsilon^{P_h w \gamma \{\alpha} S_\perp^{\beta\}}\right) \boxed{\Delta\hat{H}_T^{(1)}(z)} + \cdots$$

$\sim k_\perp^2$-moment of TMD FFs.

- dynamical twist-3



· 3-gluon correlation functions

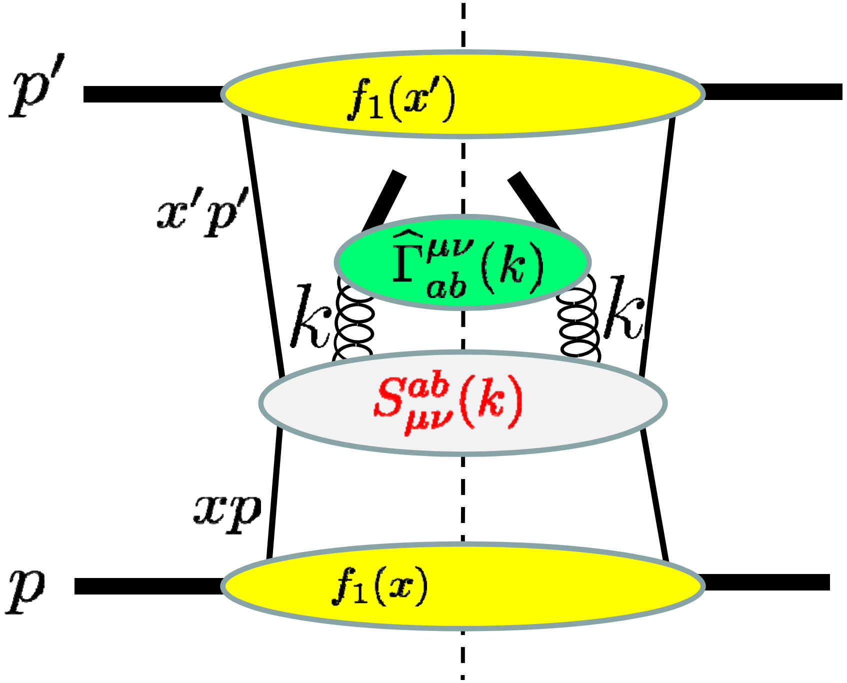
Two different color contractions with the structure constants d^{abc} and if^{abc}.

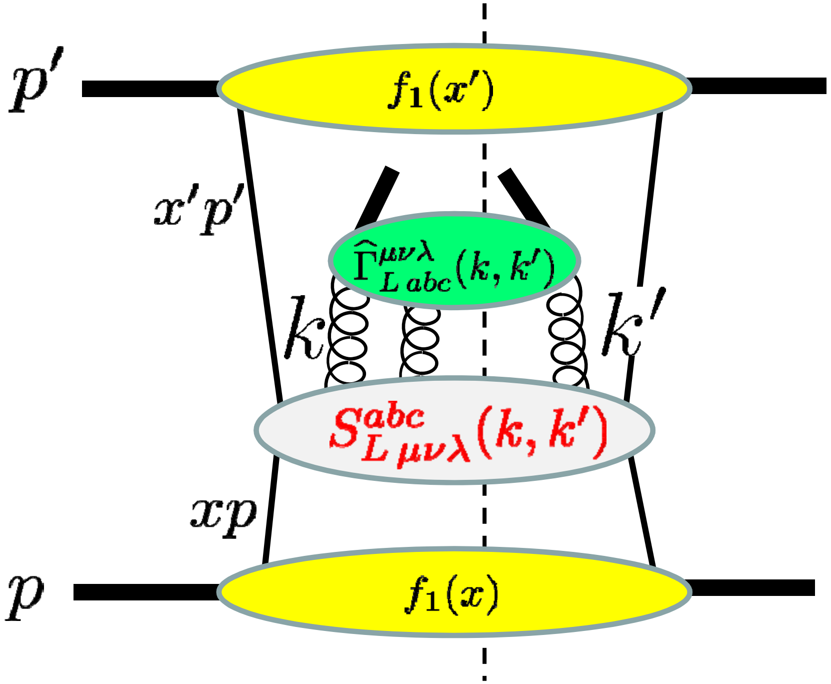
$$\hat{\Gamma}_{FA}^{\alpha\beta\gamma}(\frac{1}{z_1},\frac{1}{z_2})$$
$$=\frac{-if_{abc}}{N^2-1}\sum_X\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{-i\frac{\lambda}{z_1}}e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})}\langle 0|F_b^{w\beta}(0)|hX\rangle\langle hX|F_a^{w\alpha}(\lambda w)gF_c^{w\gamma}(\mu w)|0\rangle$$
$$=-M_h\left(\widehat{N}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\alpha\gamma}\epsilon^{P_h w S_\perp\beta}+\widehat{N}_2\left(\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\beta\gamma}\epsilon^{P_h w S_\perp\alpha}-\widehat{N}_2\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp\gamma}\right)$$

$$\hat{\Gamma}_{FS}^{\alpha\beta\gamma}(\frac{1}{z_1},\frac{1}{z_2})$$
$$=\frac{d_{abc}}{N^2-1}\sum_X\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{-i\frac{\lambda}{z_1}}e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})}\langle 0|F_b^{w\beta}(0)|hX\rangle\langle hX|F_a^{w\alpha}(\lambda w)gF_c^{w\gamma}(\mu w)|0\rangle$$
$$=-M_h\left(\widehat{O}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\alpha\gamma}\epsilon^{P_h w S_\perp\beta}+\widehat{O}_2\left(\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\beta\gamma}\epsilon^{P_h w S_\perp\alpha}+\widehat{O}_2\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp\gamma}\right)$$

-These functions are complex functions. T-odd imaginary parts contribute to SSAs.

⋆ Derivation of the twist-3 gluon FF contribution to the cross section





+ mirror

· Start from correlator of A^μ_a (gauge field).

$\widehat{\Gamma}^{\mu\nu}_{ab}(k) \sim \mathcal{F.T.}\langle 0|A^\nu_b|hX\rangle\langle hX|A^\mu_a|0\rangle$

2-body

$\widehat{\Gamma}^{\mu\nu\lambda}_{abc}(k,k') \sim \mathcal{F.T.}\langle 0|A^\nu_b|hX\rangle\langle hX|A^\mu_a A^\lambda_c|0\rangle$

3-body

$S^{ab}_{\mu\nu}(k)$, $S^{abc}_{\mu\nu\lambda}(k,k')$: Hard parts

- Apply collinear expansion to $S^{ab}_{\mu\nu}(k)$ and $S^{abc}_{L\,\mu\nu\lambda}(k,k')$ with resepect to k and k' around P_h/z and P_h/z'.

- Hard parts satisfy the Ward identities:

$$k^\mu S^{ab}_{\mu\nu}(k) = k^\nu S^{ab}_{\mu\nu}(k) = 0,$$

Gives connection between 3-body and 2-body hard parts.

$$\begin{aligned}
(k'-k)^\lambda S^{abc}_{L\,\mu\nu\lambda}(k,k') &= \frac{-if^{abc}}{N^2-1} S_{\mu\nu}(k') + \boxed{G^{abc}_{\mu\nu}(k,k'),} \\
k^\mu S^{abc}_{L\,\mu\nu\lambda}(k,k') &= \frac{if^{abc}}{N^2-1} S_{\lambda\nu}(k') + \boxed{G^{cba}_{\lambda\nu}(k'-k,k'),} \\
k'^\nu S^{abc}_{L\,\mu\nu\lambda}(k,k') &= 0,
\end{aligned}$$

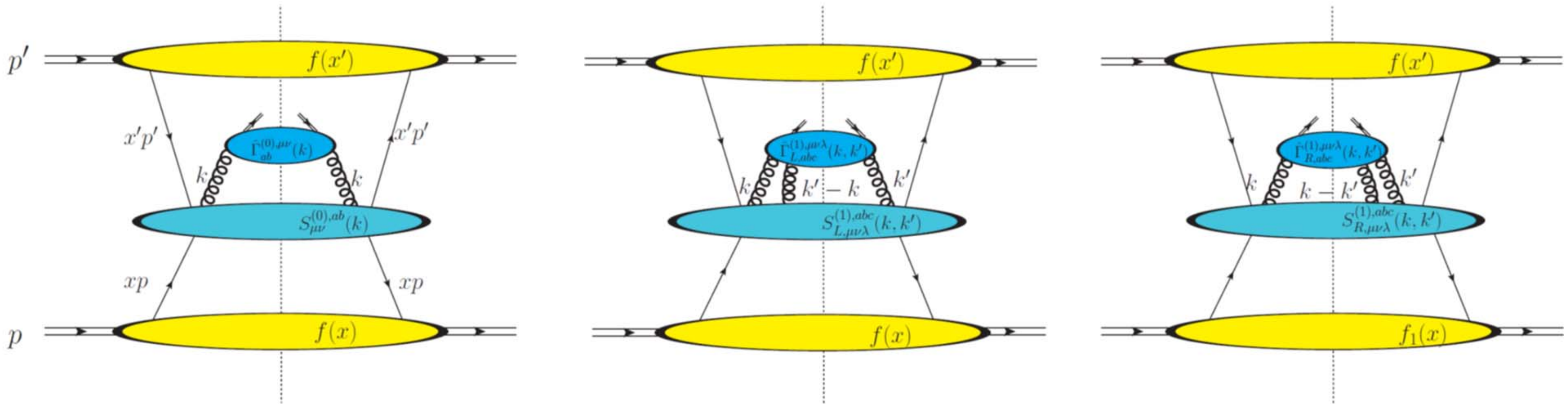
Ghost-like terms do not contribute to the cross section.

$$S_{\mu\nu}(k) \equiv \delta^{ab} S^{ab}_{\mu\nu}(k)$$

⋆ Cross section from twist-3 gluon FFs

YK, K. Yabe and S. Yoshida, Phys. Rev. D **104**, no.5, 054023 (2021)

$$p(p) + p(p') \to \Lambda(P_h) + X$$



intrinsic

$$E_h \frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h} = \frac{1}{16\pi^2 S_E}\int_0^1 \frac{dx}{x} f_1(x)\int_0^1 \frac{dx'}{x'} f_1(x')\Big[\Omega^\mu{}_\alpha \Omega^\nu{}_\beta \int_0^1 dz\, \mathrm{Tr}\left[\hat{\Gamma}^{\alpha\beta}(z) S_{\mu\nu}(P_h/z)\right]$$

$$-i\,\Omega^\mu{}_\alpha \Omega^\nu{}_\beta \Omega^\lambda{}_\gamma \int_0^1 dz\, \mathrm{Tr}\left[\hat{\Gamma}_\partial^{\alpha\beta\gamma}(z) \frac{\partial S_{\mu\nu}(k)}{\partial k^\lambda}\bigg|_{c.l.}\right] + \Re\Big\{ i\,\Omega^\mu{}_\alpha \Omega^\nu{}_\beta \Omega^\lambda{}_\gamma \int_0^1 \frac{dz}{z}\int_z^\infty \frac{dz'}{z'}\left(\frac{1}{1/z - 1/z'}\right)$$

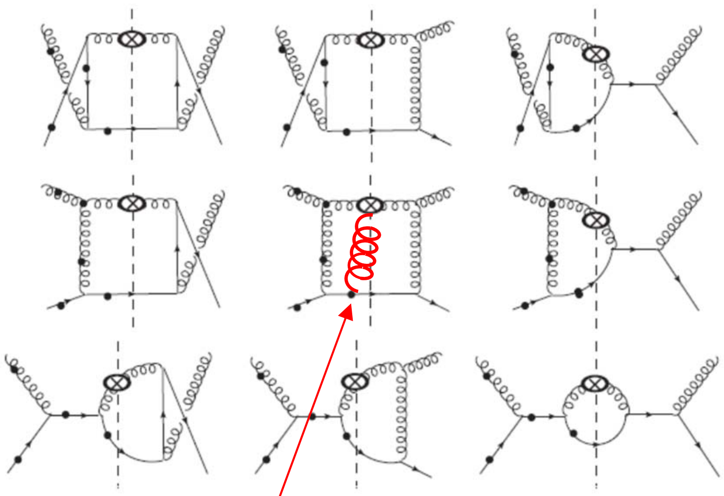
kinematical

$$\times \mathrm{Tr}\left[\left(-\frac{i f^{abc}}{N}\hat{\Gamma}_{FA}^{\alpha\beta\gamma}\left(\frac{1}{z'},\frac{1}{z}\right) + d^{abc}\frac{N}{N^2-4}\hat{\Gamma}_{FS}^{\alpha\beta\gamma}\left(\frac{1}{z'},\frac{1}{z}\right)\right) S^L_{\mu\nu\lambda,abc}(z',z)\right]\Big\}\Big]$$

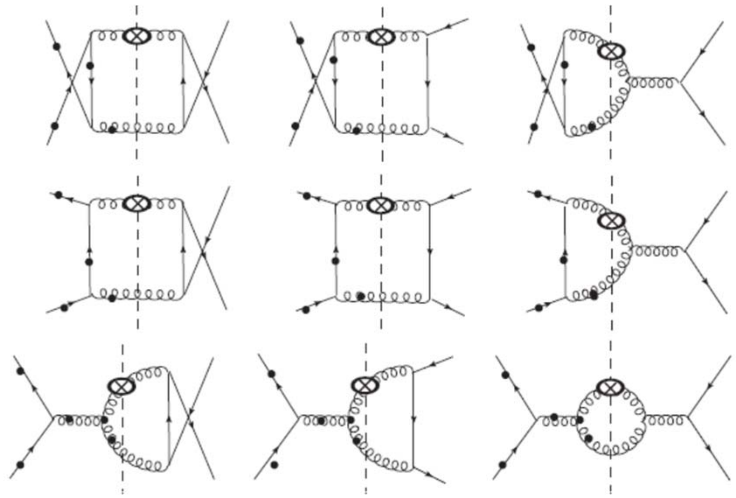
dynamical

-3 channels, $gq \to gq$, $q\bar{q} \to gg$, $gg \to gg$ for pp collision.

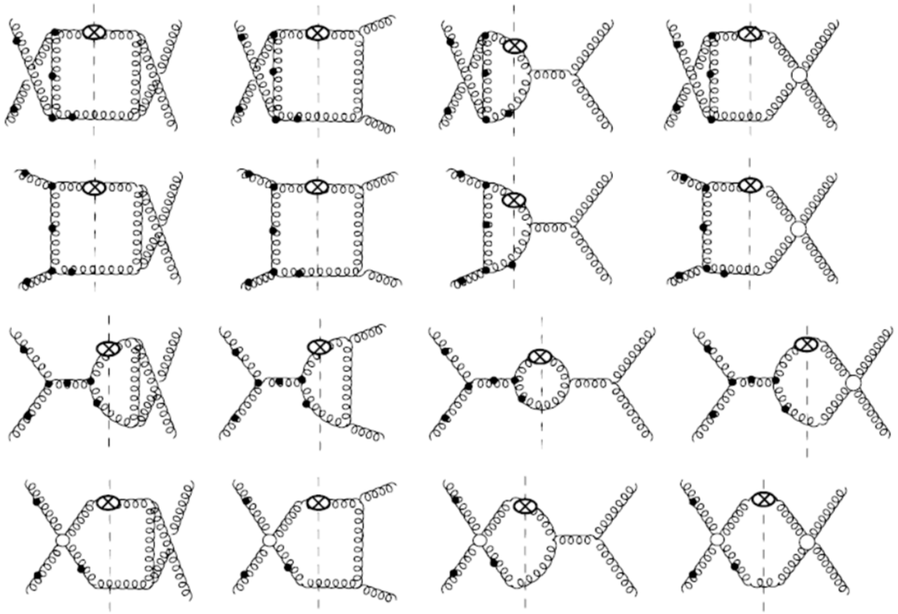
- $qg \to gq$ channel



- $q\bar{q} \to gg$ channel



- $gg \to gg$ channel

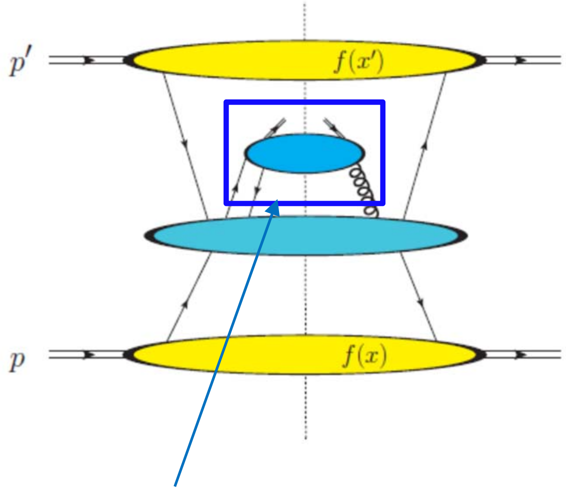


· For 3-body diagrams, an extra gluon line needs to be supplied.

-Each partonic hard cross section is still not gauge- and frame- independent.

Consider the $q\bar{q}g$ contribution together.

(Reason will become clear later.)



$$\tilde{\Delta}^{\alpha}_{ij}\left(\frac{1}{z_1},\frac{1}{z_2}\right) = \frac{1}{N}\sum_X\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{-i\frac{\lambda}{z_1}}e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})}\langle 0|F_a^{w\alpha}(\mu w)|hX\rangle\langle hX|\bar{\psi}_j(\lambda w)t^a\psi_i(0)|0\rangle$$

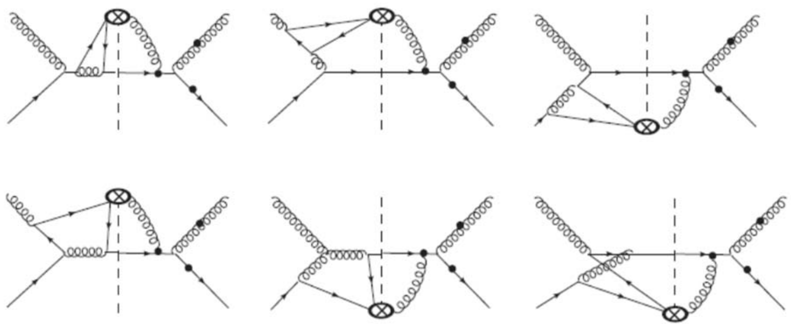
$$= M_h\left[e^{\alpha P_h w S_\perp}(\not{P}_h)_{ij}\tilde{D}_{FT}\left(\frac{1}{z_1},\frac{1}{z_2}\right) + iS_\perp^\alpha(\gamma_5\not{P}_h)_{ij}\tilde{G}_{FT}\left(\frac{1}{z_1},\frac{1}{z_2}\right)\right]$$

These functions are also complex dynamical FFs.

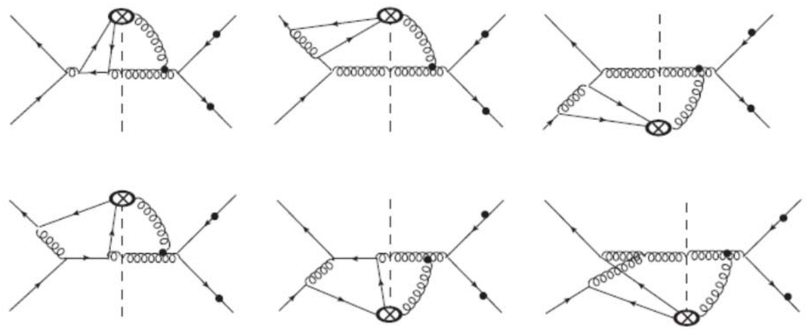
$\rightarrow$ imaginary part gives hyperon polarization.

⋆ Diagrams for $q\bar{q}g$-correlations

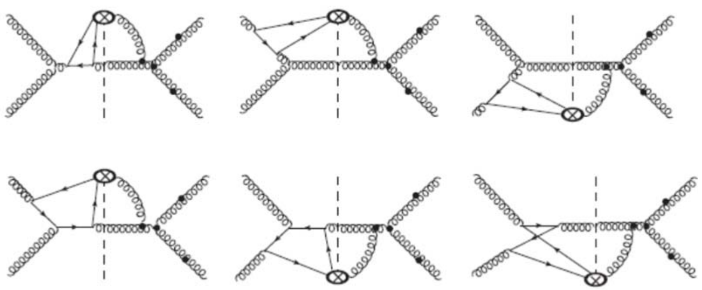
- $qg \to gq$ channel



- $q\bar{q} \to gg$ channel



- $gg \to gg$ channel



⋆ QCD EOM relation

YK, K. Yabe, S. Yoshida, Phys. Rev. D101, 054017 (2020)

-Intrinsic, kinematical, and dynamical twist-3 FFs are not independent.

QCD equation of motion

$$D_\alpha F_a^{\alpha w}(\lambda w) = -g\bar{\psi}(\lambda w)\not{w}t^a\psi(\lambda w)$$

QCD EOM relation

dynamical FFs

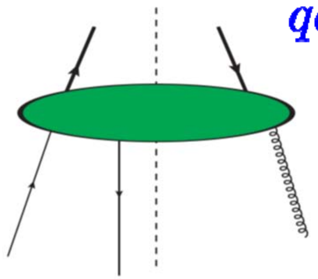
$$\frac{1}{z}\Delta\widehat{G}_{3\bar{T}}(z) = \int_0^{1/z} d\left(\frac{1}{z'}\right)\left(\frac{1}{\frac{1}{z}-\frac{1}{z'}}\right)\Im\left\{2\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)+\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)-\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\}$$

intrinsic FF

$$+\frac{1}{2}\left(\widehat{G}_T^{(1)}(z)+\Delta\widehat{H}_T^{(1)}(z)\right)-\frac{2}{C_F}\Im\int_0^{1/z} d\left(\frac{1}{z_1}\right)\widetilde{D}_{FT}\left(\frac{1}{z_1},\frac{1}{z_1}-\frac{1}{z}\right)$$

kinematical FFs

$q\bar{q}g$ FF



-This relation shows one needs to consider the $q\bar{q}g$-contribution together.

→ Gauge-invarinace of the cross section is guaranteed.

⋆ Lorentz invariance relation (+EOM relation)

-Consequence of the Lorentz invariance property of the correlation functions for FFs. (Nonlocal version of OPE.)

-Derivatives of the kinematical FFs can be represented in terms of the kinematical FFs themselves and the dynamical FFs.

$$\begin{aligned}\frac{1}{z}\frac{\partial}{\partial(1/z)}[\widehat{G}_T^{(1)}(z)] &= 2\widehat{G}_T^{(1)}(z)-\frac{4}{C_F}\int_0^{1/z} d\left(\frac{1}{z'}\right)\Im\widetilde{D}_{FT}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\\ &+ 4\int_0^{1/z} d\left(\frac{1}{z'}\right)\frac{1}{\frac{1}{z}-\frac{1}{z'}}\Im\left\{\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)-\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\}\\ &+ \frac{2}{z}\int_0^{1/z} d\left(\frac{1}{z'}\right)\frac{1}{\left(\frac{1}{z}-\frac{1}{z'}\right)^2}\Im\left\{\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)+\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)-2\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\}\end{aligned}$$

$$\begin{aligned}\frac{1}{z}\frac{\partial}{\partial(1/z)}[\Delta\widehat{H}_T^{(1)}(z)] &= 4\Delta\widehat{H}_T^{(1)}(z)-\frac{8}{C_F}\int_0^{1/z} d\left(\frac{1}{z'}\right)\Im\widetilde{D}_{FT}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\\ &+ 8\int_0^{1/z} d\left(\frac{1}{z'}\right)\frac{1}{\frac{1}{z}-\frac{1}{z'}}\Im\left\{\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)+\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\right\}\\ &+ \frac{4}{z}\int_0^{1/z} d\left(\frac{1}{z'}\right)\frac{1}{\left(\frac{1}{z}-\frac{1}{z'}\right)^2}\Im\left\{\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)+\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\right\}\end{aligned}$$

- Partonic hard cross sections depend on w, which brings frame dependence of the twist-3 cross section.

- By eliminating the derivatives of the kinematical FFs by LIRs, we found that all the partonic hard cross sections for each twist-3 FF have the common dependence on w which is actually w-independent:

i.e., the cross section takes the form of

$$P_h^0 \frac{d\sigma(S_\perp)}{d^3P_h} = \frac{2\alpha_s^2 M_\Lambda}{s^2} \left(p\cdot P_h \epsilon^{wP_hp'S_\perp} - p'\cdot P_h \epsilon^{wP_hpS_\perp} \right) \int \frac{dx}{x} f_1(x) \int \frac{dx'}{x'} f_1(x') \int \frac{dz}{z^2} \delta(\hat{s}+\hat{t}+\hat{u}) \left[\frac{\Delta\hat{G}_{3\bar{T}}(\frac{1}{z})}{z} \hat{\sigma} + \cdots \right]$$

$$\left(p\cdot P_h \epsilon^{wP_hp'S_\perp} - p'\cdot P_h \epsilon^{wP_hpS_\perp} \right) = -\epsilon^{P_hpp'S_\perp}.$$ independent of w

Frame independence of the cross section is achieved!

⋆ Summary

- Twist-3 gluon FF contribution to $pp \to \Lambda^{\uparrow} X$ was calculated.
  $\to$ LO cross section completed.

- Thanks to EOM relations and LIRs, the twist-3 cross section satisfies the color gauge invariance and the frame independence.

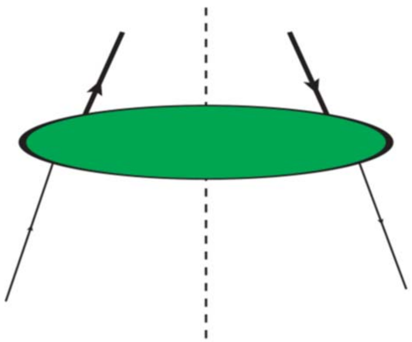
- LO formalism for all types of twist-3 quark/gluon distribution/fragmentation functions has been well established, together with the complete sets of EOM relations and LIRs.

- Future studies
  -Numerical estimate of three types of cross sections for $pp \to \Lambda^{\uparrow} X$.
  -Phenomenological analysis of experimental data.
  Hopefully RHIC can provide data!

Backups

⋆ Twist-3 FFs for π and $\Lambda^\uparrow$

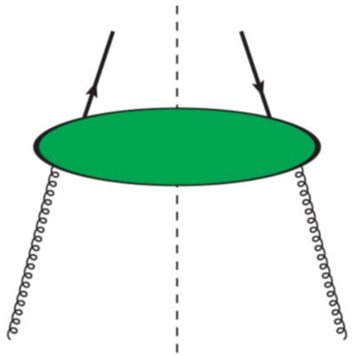
quark fragmentation



$$\mathcal{F}.\mathcal{T}.\langle 0|\psi_i|hX\rangle\langle hX|\bar{\psi}_j|0\rangle$$

exists in both processes

gluon fragmentation



$$\mathcal{F}.\mathcal{T}.\langle 0|F^{\alpha w}|hX\rangle\langle hX|F^{\beta w}|0\rangle$$

exists only in $\Lambda^\uparrow$ production

$$-M_\Lambda(w^\alpha\epsilon^{\beta P_h w \boxed{S_\perp}}+w^\beta\epsilon^{\alpha P_h w S_\perp})\Delta\hat{G}_{3\bar{T}}(z)$$

Spin vector is needed

π: **chiral-odd**

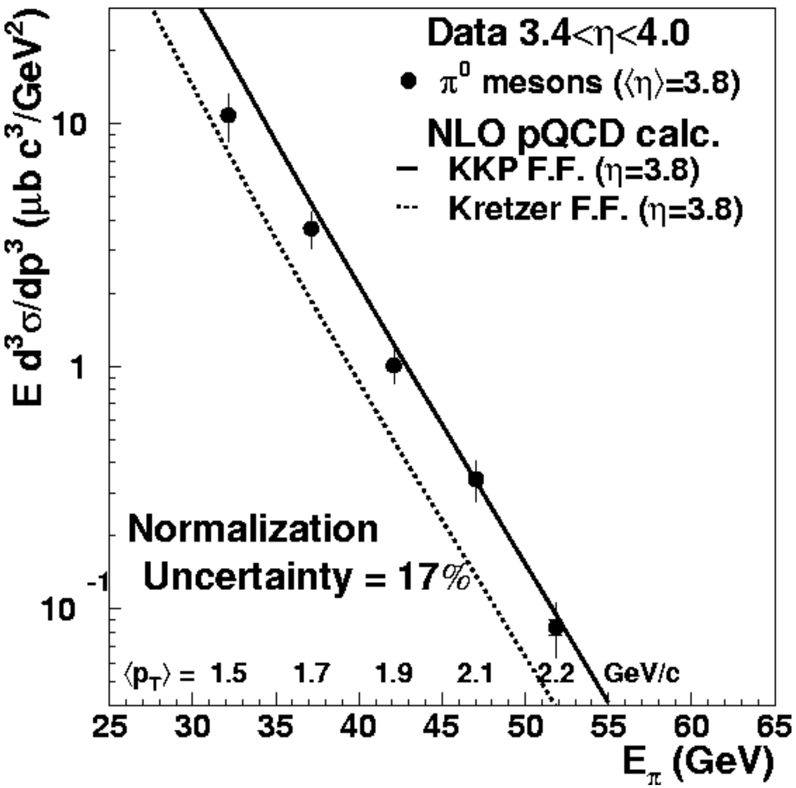
Metz, Pitonyak(2013)

$\Lambda^\uparrow$: **chiral-even**

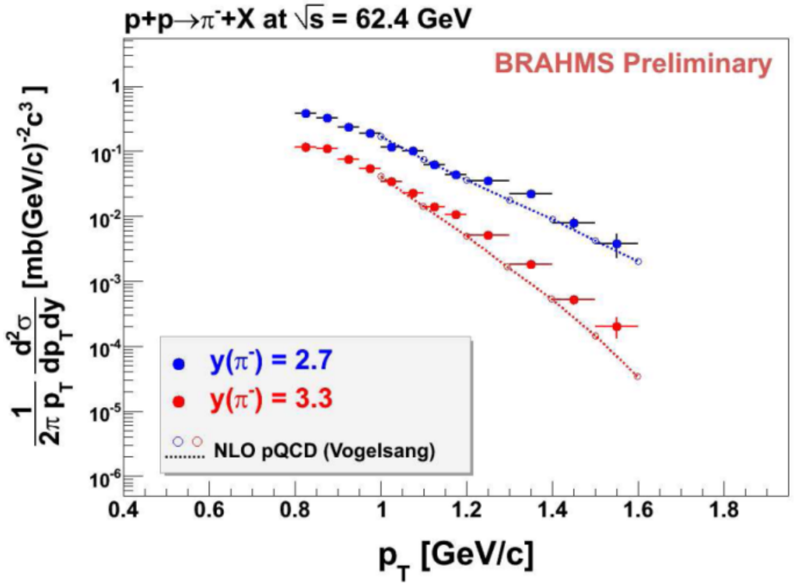
Koike, Metz, Pitonyak, Yabe, Yoshida(2017)

• Analysis of unpolarized cross section in terms of the collinear factorization (NLO).

★ RHIC-STAR ('04)



★ RHIC-BRAHMS ('07)



NLO works perfectly well at RHIC energy!