The $g_T(x)$ contribution to single spin asymmetry in SIDIS

Sanjin Benić (University of Zagreb)

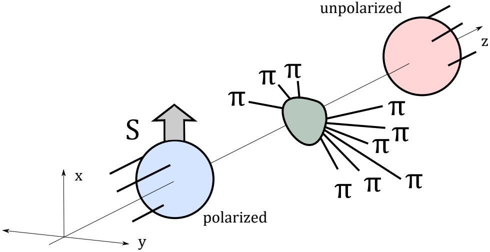
SB, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027
SB, Hatta, Kaushik, Li 2109.05440

SPIN 2021, Matsue, Japan, 18-22 October 2021

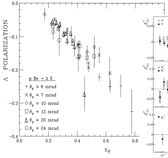
Croatian Science Foundation

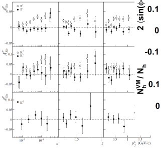
Transverse single spin asymmetry

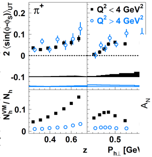
- hadron production is left-right asymmetric

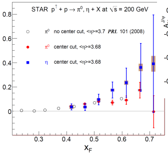


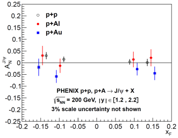
- observed in *ep*, *pp* etc..







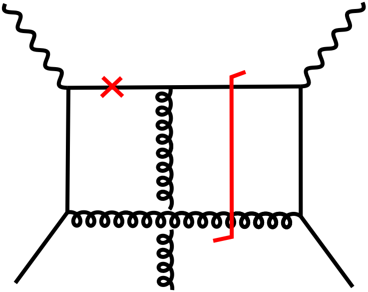




Origin of (T)SSA?

- amplitude is complex - SSA sensitive to the phase

  $\rightarrow$ interference contributions



- we need twist-3 quantities such as quark-gluon-quark correlations

Origin of SSA?

- some known sources in collinear framework

ETQS functions: soft gluonic pole, soft fermionic pole, hard pole...

(TMD: closely related to the Sivers function)
Efremov, Teryaev, Sov. J. Nucl. Phys. **36**, 140 (1982)
Qiu, Sterman, Phys. Rev. D **59**, 014004 (1999)

twist-3 fragmentation functions

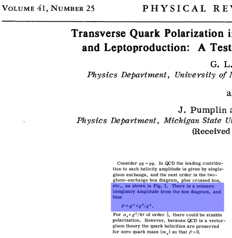
(TMD: closely related to the Collins function)
Yuan, Zhou Phys. Rev. Lett. **103** (2009) 052001
Kang, Yuan, Zhou Phys. Lett. B **691** (2010) 243
Kanazawa, Koike Phys. Rev. D **88** (2013) 074022

...

$\rightarrow$ require new PDFs and/or FFs..

Revisiting a 40 year old estimate

- the first pQCD estimate of SSA: Kane, Pumplin, Repko (1978) consider the two-loop box diagram



VOLUME 41, NUMBER 25 PHYSICAL REVIEW LETTERS 18 DECEMBER 1978

Transverse Quark Polarization in Large-p_T Reactions, e^+e^- Jets, and Leptoproduction: A Test of Quantum Chromodynamics

G. L. Kane
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and

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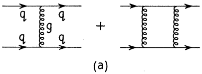
(Received 5 July 1978)

Consider $qq \to qq$. In QCD the leading contribution to each helicity amplitude is given by single-gluon exchange, and the next order is the two-gluon–exchange box diagram, plus crossed box, etc., as shown in Fig. 1. There is a nonzero imaginary amplitude from the box diagram, and thus

$$P \propto g^4 \times g^2/g^4.$$

For $\alpha_s = g^2/4\pi$ of order $\frac{1}{3}$, there could be sizable polarization. However, because QCD is a vector-gluon theory the quark helicities are preserved for zero quark mass (m_q) so that $P = 0$.



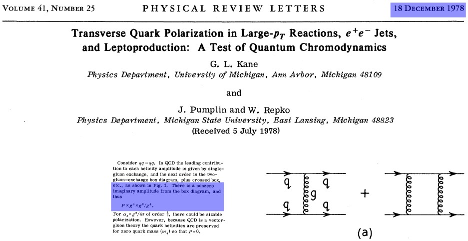
$$A_N \sim \frac{\alpha_s m_q}{P_{hT}}$$

$\rightarrow$ believed to be negligible because $m_q \rightarrow 0$

$\rightarrow$ for $>$ 40 years there has been no attempt to go beyond this simple parametric estimate!

Revisiting a 40 year old estimate

- the first pQCD estimate of SSA: Kane, Pumplin, Repko (1978) consider the two-loop box diagram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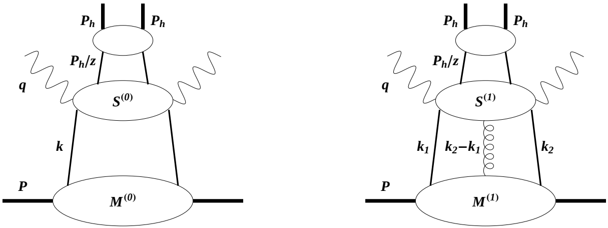
→ for $>$ 40 years there has been no attempt to go beyond this simple parametric estimate!

- **this work**: explicit computation

SB, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027
SB, Hatta, Kaushik, Li 2109.05440

Hadronic tensor



- hadronic tensor $W_{\mu\nu} = \int_z \frac{D(z)}{z^2} w_{\mu\nu}$

$$w_{\mu\nu} = \int_k M^{(0)}(k) S^{(0)}_{\mu\nu}(k) + \int_{k_1,k_2} M^{(1)}_\sigma(k_1,k_2) S^{(1)\sigma}_{\mu\nu}(k_1,k_2)$$

- two parton: $M^{(0)} \sim \langle P, S|\bar{\psi}\psi|P, S\rangle$
- three parton: $M^{(1)}_\sigma \sim \langle P, S|\bar{\psi}A_\sigma\psi|P, S\rangle$

Hadronic tensor

- hard part - collinear expansion

$$k^\mu = xP^\mu + k_T^\mu \qquad S_{\mu\nu}^{(0)}(k) = S_{\mu\nu}^{(0)}(xP) + \frac{dS_{\mu\nu}^{(0)}}{dk_T^\alpha}(xP)k_T^\alpha + \dots$$

- soft part

$$\begin{aligned} M^{(0)}(x) \sim\ & \not{P} f(x) + M_N e(x) + M_N (S\cdot n)\not{P}\gamma_5 \Delta f(x) \\ +\ & M_N^2 (S\cdot n)[\not{P}, \not{n}]\gamma_5 h_L(x) + [\not{P}, \not{S}_T]\gamma_5 h_1(x) + \\ +\ & M_N \not{S}_T \gamma_5 g_T(x) \end{aligned}$$

- no contribution from $g_T(x)$ at Born level

substitution $M^{(0)}(x) \to \gamma_5 \not{S}_\perp g_T(x)/2$ does not contribute to the cross section: the spinor trace $\mathrm{Tr}[\cdots]$ involving γ_5 produces the factor i to the first term in (26) and the result is contracted with the real tensor $L_{\mu\nu}$ of (20) to derive the contribution to (19); but this yields a real quantity for the cross section only when another factor i is provided by the hard part $S^{(0)}(xp, q, P_h/z)$, which is impossible for the Born subprocess. Similarly, it is straightforward to see that the second term

Eguchi, Koike, Tanaka Nucl. Phys. B **763**, 198, (2007)

- higher-orders $\longrightarrow$ non-zero result

SB, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027

Hadronic tensor

- all-order gauge invariant result

$$w_{\mu\nu} = \frac{M_N}{2}\int_x g_T(x)\mathrm{Tr}\left[\gamma_5 \not{S}_T S^{(0)}_{\mu\nu}(x)\right]$$
$$- \frac{M_N}{4}\int_x \tilde{g}(x)\mathrm{Tr}\left[\gamma_5 \not{P} S_T^\alpha \left.\frac{\partial S^{(0)}_{\mu\nu}(k)}{\partial k_T^\alpha}\right|_{k=xP}\right]$$
$$+ \frac{iM_N}{4}\int_{x_1,x_2}\mathrm{Tr}\left[\left(\not{P}\epsilon^{\alpha P n S_T}\frac{G_F(x_1,x_2)}{x_1-x_2} + i\gamma_5 \not{P} S_T^\alpha \frac{\tilde{G}_F(x_1,x_2)}{x_1-x_2}\right)S^{(1)}_{\mu\nu\alpha}(x_1,x_2)\right]$$

Ratcliffe Nucl. Phys. B **264**, 493 (1986)
SB, Hatta, Li, Yang Phys. Rev. D **100** (2019) 9, 094027

- intrinsic $g_T \sim \langle\bar{\psi}\psi\rangle$
- kinematical $\tilde{g} \sim \langle\bar{\psi}\partial^\mu\psi\rangle$
- dynamical $G_F \sim \langle\bar{\psi}F^{\mu\nu}\psi\rangle$

Wandzura-Wilczek approximation

- g_T and $\tilde{g}$ have a twist-two piece

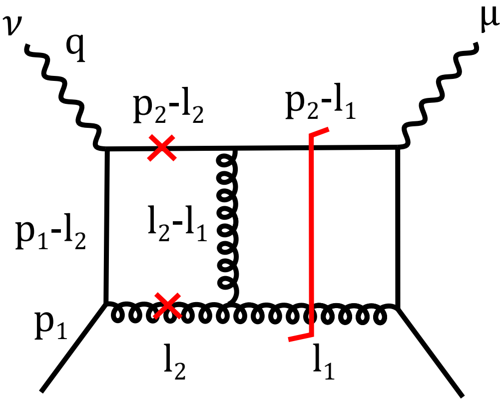
$$g_T(x) = \int_x^1 \frac{dx_1}{x_1} \Delta q(x_1) + \dots \quad \tilde{g}(x) = -2x \int_x^1 \frac{dx_1}{x_1} \Delta q(x_1) + \dots .$$

- SSA from twist-two PDFs and twist-two FFs

$$\Delta\sigma \sim \Delta q \otimes H \otimes D_1$$

- Sivers and Collins asymmetry NOT from Sivers and Collins functions
- Δq - twist-two helicity PDF
- D_1 - twist-two unpolarized FF

$S^{(0)}_{\mu\nu}$ at two loops - return of the box



$\rightarrow$ can get a phase

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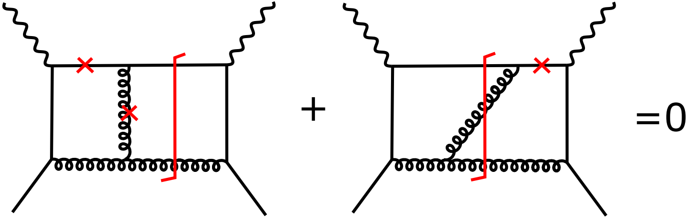
$S_{\mu\nu}^{(0)}$ at two loops - return of the box



$\rightarrow$ real-virtual cancellation

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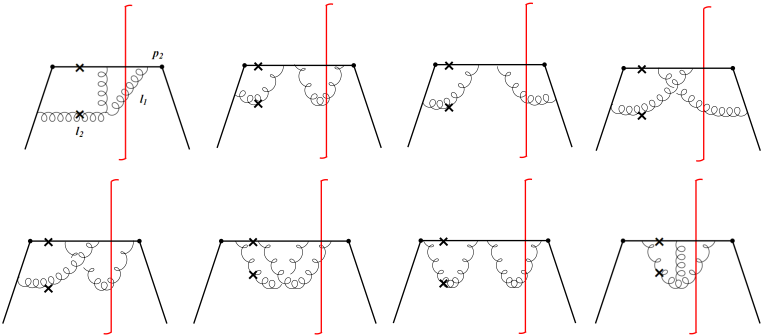
$S^{(0)}_{\mu\nu}$ at two loops

- check all possible cuts in all possible diagrams
  $\rightarrow$ total of 12 diagrams



(8 above + box + mirror of 1st, 2nd and 5th)

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SSA from TMDs - beyond Sivers and Collins

- up to two loops

Worm-gear
collinear limit: $g_{1T}^{(1)}(x) \sim \tilde{g}(x)$

Sivers

$$
\begin{aligned}
d\sigma \; = \; & f_{1T}^{\perp} \otimes H^{(0)}_{\gamma^-,\gamma^+} \otimes D_1 + f_{1T}^{\perp} \otimes H^{(1)}_{\gamma^-,\gamma^x} \otimes D^{\perp} + f_{1T}^{\perp} \otimes H^{(2)}_{\gamma^-,\gamma_5\gamma^x} \otimes G^{\perp} \\
& + g_{1T} \otimes H^{(2)}_{\gamma_5\gamma^-,\gamma^+} \otimes D_1 + g_{1T} \otimes H^{(1)}_{\gamma_5\gamma^-,\gamma_5\gamma^y} \otimes G^{\perp} + g_{1T} \otimes H^{(2)}_{\gamma_5\gamma^-,\gamma^y} \otimes D^{\perp} \\
& + h_1 \otimes H^{(0)}_{\gamma_5\sigma^{y-},\gamma_5\sigma^{y+}} \otimes H_1^{\perp} + h_1 \otimes H^{(1)}_{\gamma_5\sigma^{y-},\gamma_5\sigma^{yx}} \otimes H^{*} + h_1 \otimes H^{(2)}_{\gamma_5\sigma^{y-},I} \otimes E^{*} \\
& + e_T \otimes H^{(1)}_{\gamma_5,\gamma_5\sigma^{y+}} \otimes H_1^{\perp} + e_T^{\perp} \otimes H^{(2)}_{I,\gamma_5\sigma^{y+}} \otimes H_1^{\perp} \\
& + f_T \otimes H^{(1)}_{\gamma^y,\gamma^+} \otimes D_1 + g_T \otimes H^{(2)}_{\gamma_5\gamma^y,\gamma^+} \otimes D_1 \\
& + h_T^{\perp} \otimes H^{(1)}_{\gamma_5\sigma^{yx},\gamma_5\sigma^{y+}} \otimes H_1^{\perp} + h_T \otimes H^{(1)}_{\gamma_5\sigma^{-+},\gamma_5\sigma^{y+}} \otimes H_1^{\perp},
\end{aligned}
$$

Collins

collinear limit: $g_T(x)$

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SSA at two loops - gluonic channel

- a gluonic analog of $g_T(x)$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | F^{n\alpha}(0)[0,\lambda n] F^{n\beta}(\lambda n) | PS \rangle = -\frac{1}{2} x G(x) g_T^{\alpha\beta}$$
$$+ \frac{i}{2} x \Delta G(x) \epsilon^{nP\alpha\beta}(n \cdot S) + i M_N x \mathcal{G}_{3T}(x) \epsilon^{n\alpha\beta S_T} + \dots$$

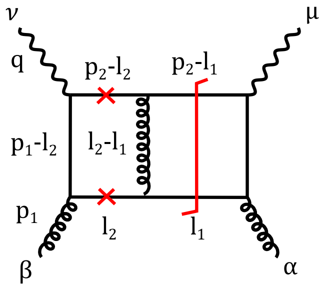
Ji Phys. Lett. B **289** (1992) 137-142
Hatta, Tanaka, Yoshida JHEP **02** (2013) 003

- by a completely analogous computation

$$w_{\mu\nu} = i M_N \int \frac{dx}{x} \mathcal{G}_{3T}(x) \epsilon^{n\alpha\beta S_\perp} S^{(0)\alpha'\beta'}_{\mu\nu}(p_1) \omega_{\alpha'\alpha} \omega_{\beta'\beta}$$
$$- i M_N \int \frac{dx}{x^2} \tilde{g}(x) \left(g_\perp^{\beta\lambda} \epsilon^{\alpha P n S_\perp} - g_\perp^{\alpha\lambda} \epsilon^{\beta P n S_\perp} \right) \left(\frac{\partial S^{(0)}_{\mu\nu\alpha\beta}(k)}{\partial k^\lambda} \right)_{k=p_1} + \dots$$

Hatta, Kanazawa, Yoshida Phys. Rev. D **88** (2013) 1 014037
SB, Hatta, Kaushik, Li 2109.05440

SSA at two loops - gluonic channel



- WW approx

$$\mathcal{G}_{3T}(x) = \frac{1}{2}\int_x^1 \frac{dx_1}{x_1}\Delta G(x_1) + \ldots$$

$$\tilde{g}(x) = \frac{x^2}{2}\int_x^1 \frac{dx_1}{x_1}\Delta G(x_1) + \ldots$$

- SSA from twist-two PDFs and FFs

$$\Delta\sigma = \Delta G \otimes H^{(2)} \otimes D_1$$

- ΔG - twist-two gluon helicity

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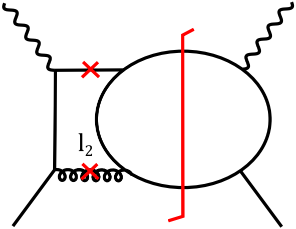
Hard coefficients



- l_2 gluon can be collinear to the proton
  $\rightarrow$ divergence is cancelled between $S^{(0)}_{\mu\nu}$ and $dS^{(0)}_{\mu\nu}/dk^{\alpha}_T$

$$\frac{d^6\Delta\sigma}{dx_B dQ^2 dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2 \alpha_s^2 M_N}{16\pi^2 x_B^2 S_{ep}^2 Q^2} \sum_k \mathcal{A}_k \mathcal{S}_k \int_{x_{min}}^1 \frac{dx}{x} \int_{z_{min}}^1 \frac{dz}{z}$$
$$\times \delta\left(\frac{q_T^2}{Q^2} - \left(1 - \frac{1}{\hat{x}}\right)\left(1 - \frac{1}{\hat{z}}\right)\right)$$
$$\times \sum_f e_f^2 \left[D_f(z) x^2 g'_{Tf}(x) \Delta\hat{\sigma}^{qq}_{Dk} + D_f(z) x g_{Tf}(x) \Delta\hat{\sigma}^{qq}_k + (qg\,\text{channel}) + (gq\,\text{channel})\right]$$

SB, Hatta, Kaushik, Li 2109.05440

Hard coefficients

- all hard coefficients $\Delta\hat{\sigma}$ extracted in completely closed form expressions

$$\Delta\hat\sigma^{qq}_{D8}=\frac{(N_c^2-1)\,\hat x\hat z}{2N_c^2Q(1-\hat z)^2}\left[(1-\hat z)\left(1-\hat x+\hat z-3\hat x\hat z+N_c^2(1-\hat x-\hat z+3\hat x\hat z)\right)+2(1-2\hat x)\,\hat z\log(\hat z)\right],$$

$$\Delta\hat\sigma^{qq}_{D9}=\frac{(N_c^2-1)(1-\hat x)\,\hat x\hat z}{2N_c^2q_T(1-\hat z)^2}\left[(1-\hat z)\left(3N_c^2(1-\hat z)+3\hat z-1\right)-2(1-2\hat z)\log(\hat z)\right],$$

$$\Delta\hat\sigma^{qq}_{1}=\frac{N_c^2-1}{2N_c^2q_T(1-\hat z)\,\hat z}\Big[(1-\hat z)(\hat z\hat x(\hat x(3+10\hat z)-3(1+\hat z))-1)$$
$$+N_c^2\hat z\left(\hat x^2(3+2\hat z(5\hat z-6))-1-3\hat x(1-\hat z)^2\right)+6\hat x(2\hat x-1)\,\hat z^2\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{qq}_{2}=\frac{(N_c^2-1)\,\hat x}{N_c{}^2q_T(1-\hat z)}\left[(1-\hat z)\left((1+N_c^2)(-1+\hat x)+(N_c^2-1)(1-3\hat x)\,\hat z\right)+2(2\hat x-1)\,\hat z\log(\hat z)\right],$$

$$\Delta\hat\sigma^{qq}_{3}=\frac{(N_c^2-1)\,\hat x}{4N_c^2Q(1-\hat x)(1-\hat z)^2}\Big[(1-\hat z)\Big\{(1-\hat x)\left(5\hat x+N_c^2(2-11\hat x)\right)\hat z-(1+N_c^2)(1-\hat x)^2$$
$$-(N_c^2-1)(1+\hat x(14\hat x-13))\,\hat z^2\Big\}-2\hat z\left(1-\hat z-\hat x(5-4\hat x-8(1-\hat x)\,\hat z)\right)\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{qq}_{4}=\frac{(N_c^2-1)\,\hat x}{2N_c^2q_T(1-\hat z)^2}\Big[(1-\hat z)\Big\{3(1-\hat z)^2+\hat x(-3+(5-4\hat z)\,\hat z)$$
$$-N_c^2(1-\hat z)(2-3\hat z+\hat x(-2+4\hat z))\Big\}-2\left(\hat x-(1-\hat z)^2-2\hat x(1-\hat z)\,\hat z\right)\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{qq}_{8}=\frac{(N_c^2-1)\hat x}{4N_c^2Q(1-\hat x)(1-\hat z)^2}\Big[(1+N_c^2)(1-\hat x)^2+(1-\hat x)\left(5-10\hat x+N_c^2(6\hat x-5)\right)\hat z$$
$$-\left(9+\hat x(15\hat x-26)-N_c^2(7+9(\hat x-2)\,\hat x)\right)\hat z^2$$
$$-(N_c^2-1)(3+\hat x(4\hat x-9))\,\hat z^3-2\hat z\left(\hat x(7-4\hat x-2\hat z)+\hat z-3\right)\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{qq}_{9}=\frac{(N_c^2-1)\,\hat x}{2N_c^2q_T(1-\hat z)^2}\Big[(1-\hat z)\left(2N_c^2(1-\hat x(1-\hat z)-2\hat z)(1-\hat z)-3+\hat z(9-4\hat z)+\hat x(3-(7-2\hat z)\,\hat z)\right)$$
$$-2\left(1+(\hat z-3)\,\hat z+\hat x(2\hat z-1)\right)\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{qg}_{D8}=\frac{(N_c^2-1)\,\hat x}{2N_c^2Q\hat z}\left[\hat z\left(-2+\hat z-N_c^2\hat z+\hat x\left(4-3\hat z+N_c^2(-2+3\hat z)\right)\right)-2(1-2\hat x)(1-\hat z)\log(1-\hat z)\right],$$

$$\Delta\hat\sigma^{qg}_{D9}=\frac{(N_c^2-1)(1-\hat x)\,\hat x(1-\hat z)}{2N_c{}^2q_T\hat z^2}\left[\hat z\left(2+3(N_c^2-1)\,\hat z\right)+(2-4\hat z)\log(1-\hat z)\right],$$

$$\Delta\hat\sigma^{qg}_{1}=\frac{N_c^2-1}{2N_c^2q_T\hat z^2}\Big[\hat z\left(1+\hat x^2(-13+23\hat z-10\hat z^2)+3\hat x(2-3\hat z+\hat z^2)\right)$$
$$-N_c^2(1-\hat z)\left(-1-3\hat x\hat z^2+\hat x^2(1-8\hat z+10\hat z^2)\right)-6\hat x(-1+2\hat x)(1-\hat z)^2\log(1-\hat z)\Big],$$

$$\Delta\hat\sigma^{qg}_{2}=\frac{(N_c^2-1)\,\hat x(1-\hat z)}{N_c^2q_T\hat z^2}\left[\hat z\left(2-\hat z+N_c^2\hat z-\hat x\left(4-3\hat z+N_c{}^2(-2+3\hat z)\right)\right)+2(1-2\hat x)(1-\hat z)\log(1-\hat z)\right],$$

$$\Delta\hat\sigma^{qg}_{3}=-\frac{(N_c^2-1)\,\hat x}{4N_c{}^2Q(1-\hat x)\,\hat z^2}\Big[\hat z\Big\{(-1+\hat z)\left(2+(N_c^2-1)\,\hat z\right)+\hat x\left(10-24\hat z+13\hat z^2+N_c^2(-4+16\hat z-13\hat z^2)\right)$$
$$+\hat x^2\left(-8+23\hat z-14\hat z^2+N_c^2(4-17\hat z+14\hat z^2)\right)\Big\}$$
$$+2(1-\hat z)\left(-1+\hat x(5-8\hat z)+\hat z+\hat x^2(-4+8\hat z)\right)\log(1-\hat z)\Big],$$

$$\Delta\hat\sigma^{qg}_{4}=-\frac{(N_c^2-1)\,\hat x(1-\hat z)}{2N_c{}^2q_T\hat z^3}\Big[\hat z\left(2+(4N_c^2-3)\,\hat z-3(N_c^2-1)\,\hat z^2+\hat x\left(-2+(3-2N_c{}^2)\,\hat z+4(N_c^2-1)\,\hat z^2\right)\right)$$
$$+2\left((1-\hat z)^2-\hat x(1-2\hat z+2\hat z^2)\right)\log(1-\hat z)\Big],$$

$$\Delta\hat\sigma^{qg}_{8}=\frac{(N_c^2-1)\,\hat x}{4N_c^2Q(1-\hat x)\,\hat z^2}\Big[\hat z\Big\{6-(1+N_c{}^2)\,\hat z+3(N_c^2-1)\,\hat z^2+\hat x\left(-22+8\hat z+9\hat z^2+N_c{}^2(4-9\hat z^2)\right)$$
$$+\hat x^2\left(16-9\hat z-4\hat z^2+N_c^2(-4+3\hat z+4\hat z^2)\right)\Big\}+2(1-\hat z)\left(3+8\hat x^2+\hat z-\hat x(11+2\hat z)\right)\log(1-\hat z)\Big],$$

$$\Delta\hat\sigma^{qg}_{9}=\frac{(N_c^2-1)\,\hat x(1-\hat z)}{2N_c^2q_T\hat z^3}\Big[\hat z\left((4-N_c^2)\,\hat z-4-4(N_c^2-1)\,\hat z^2+\hat x\left(4+3(N_c^2-2)\,\hat z+2(N_c^2-1)\,\hat z^2\right)\right)$$
$$+2\left(-2+2\hat x+3\hat z-4\hat x\hat z+\hat z^2\right)\log(1-\hat z)\Big].$$

$$\Delta\hat\sigma^{gq}_{D8}=\frac{2(1-\hat x)\,\hat x}{N_cQ(1-\hat z)^2\hat z}\left[(1-\hat x)\left(\hat z\log(\hat z)-(1-\hat z)\log(1-\hat z)\right)+\hat x\hat z(1-\hat z)(1-2\hat z)\right],$$

$$\Delta\hat\sigma^{gq}_{D9}=\frac{2(1-\hat x)^2\hat x}{N_cq_T(1-\hat z)^2\hat z^2}\left[(1-\hat z)^2\log(1-\hat z)+\hat z^2\log(\hat z)+\hat z(1-\hat z)(\hat z^2+(1-\hat z)^2)\right],$$

$$\Delta\hat\sigma^{gq}_{1}=\frac{(1-\hat x)}{N_cq_T(1-\hat z)\,\hat z^2}\Big[(1-2\hat z)\left(1+2\hat x^2(1-2\hat z)^2-(1-\hat z)\,\hat z-2\hat x(1-(1-\hat z)\,\hat z)\right)$$
$$+6(1-\hat x)\,\hat x\left((1-\hat z)\log(1-\hat z)-\hat z\log(\hat z)\right)\Big],$$

$$\Delta\hat\sigma^{gq}_{2}=\frac{4(1-\hat x)\,\hat x}{N_cq_T(1-\hat z)\,\hat z^2}\left[(1-\hat z)\left(\hat x(2\hat z-1)\,\hat z+(1-\hat x)\log(1-\hat z)\right)-(1-\hat x)\,\hat z\log(\hat z)\right],$$

$$\Delta\hat\sigma^{gq}_{3}=\frac{\hat x}{N_cQ(1-\hat z)^2\hat z^2}\Big[(1-\hat z)\,\hat z\left(1-2(1-\hat z)\,\hat z+\hat x(-6+(13-12\hat z)\,\hat z)+\hat x^2(5-12(1-\hat z)\,\hat z)\right)$$
$$+(1-\hat x)\left((1-\hat z)(1-\hat z+\hat x(-3+4\hat z))\log(1-\hat z)-\hat z(1-\hat z-\hat x(1-4\hat z))\log(\hat z)\right)\Big],$$

$$\Delta\hat\sigma^{gq}_{4}=\frac{2(1-\hat x)\,\hat x}{N_cq_T(1-\hat z)^2\hat z^3}\Big[(1-\hat z)\Big\{\hat z\left(\hat x-2\hat x\hat z(2+\hat z(-3+2\hat z))-(1-\hat z)(1+\hat z(-3+4\hat z))\right)$$
$$-(1-\hat x(1-\hat z))(1-\hat z)\log(1-\hat z)\Big\}-\hat x\hat z^3\log(\hat z)\Big],$$

$$\Delta\hat\sigma^{gq}_{8}=\frac{\hat x}{N_cQ(1-\hat z)^2\hat z^2}\Big[(-1+\hat z)\,\hat z\left(-3+10\hat x-7\hat x^2+(-2+\hat x)\,\hat z+2(1+2(-1+\hat x)\,\hat x)\,\hat z^2\right)$$
$$+(1-\hat x)\left((1-\hat z)(3-3\hat z+\hat x(-5+2\hat z))\log(1-\hat z)-\hat z(1-3\hat z+\hat x(-1+2\hat z))\log(\hat z)\right)\Big],$$

$$\Delta\hat\sigma^{gq}_{9}=\frac{-2(1-\hat x)\,\hat x}{N_cq_T(1-\hat z)^2\hat z^3}\Big[(2+\hat x(-2+\hat z)-2\hat z)(1-\hat z)^2\log(1-\hat z)+\hat z^2\left(1-\hat x-(2-\hat x)\,\hat z\right)\log(\hat z)$$
$$-\hat z(1-\hat z)\left(-2+3\hat z+(-1+\hat z)(-2\hat x+\hat x\hat z+2(-1+\hat x)\,\hat z^2)\right)\Big].$$

- checked QCD and QED gauge invariance

SB, Hatta, Kaushik, Li 2109.05440

Numerical setup

$$A_{UT}^{\sin(\alpha\phi_h+\beta\phi_S)} = \frac{2\int_0^{2\pi} d\phi_h d\phi_S \sin(\alpha\phi_h+\beta\phi_S)[d\sigma(\phi_h,\phi_S) - d\sigma(\phi_h,\phi_S+\pi)]}{\int_0^{2\pi} d\phi_h d\phi_S [d\sigma(\phi_h,\phi_S) + d\sigma(\phi_h,\phi_S+\pi)]}$$

- numerator: $O(\alpha_s^2)$ g_T contribution
- denominator: $O(\alpha_s)$ unpolarized cross section
  Meng, Olness, Soper Nucl. Phys. **B371**, 79, (1992)

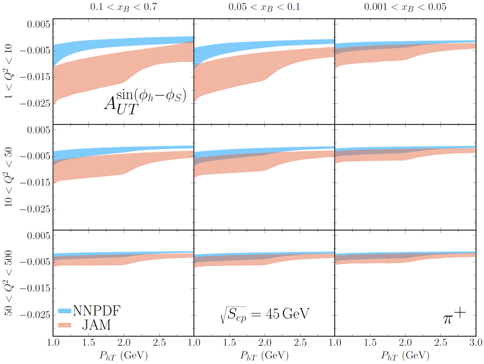
Sivers $A_{UT}^{\sin(\phi_h-\phi_S)}$, Collins $A_{UT}^{\sin(\phi_h+\phi_S)}$, etc...

- $g_T(x)$ and $\mathcal{G}_{3T}(x)$ computed from the WW relation to helicity PDFs

$$g_T(x) = \int_x^1 \frac{dx_1}{x_1}\Delta q(x_1)\,, \qquad \mathcal{G}_{3T}(x) = \frac{1}{2}\int_x^1 \frac{dx_1}{x_1}\Delta G(x_1)$$

- we use most recent global fits for helicity PDFs: NNPDF and JAM

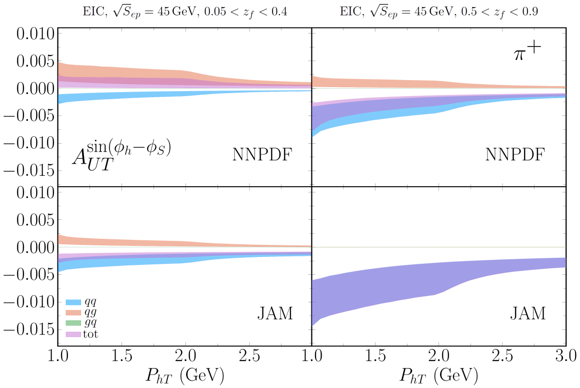
Sivers - EIC predictions



- Sivers up to 2% (JAM) at large x_B and small Q^2
- drop for small x_B:

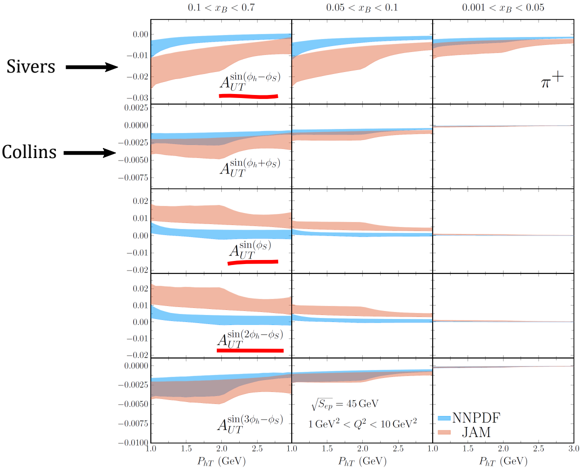
$$A_{UT} \sim \frac{\Delta\sigma}{\sigma} \sim \frac{\alpha_s^2 x g_T(x)}{\alpha_s f(x)} \sim \alpha_s \frac{x \Delta f}{f}$$

Channel breakdown



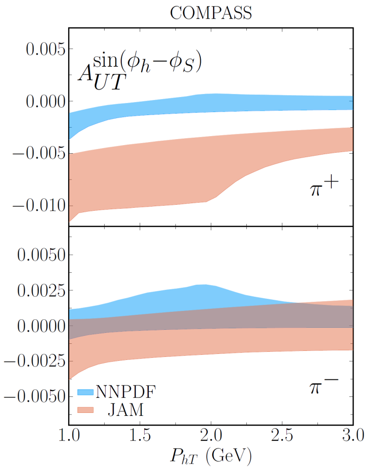
- cancellation between qq and qg channels
- Sivers flips sign going from small to large z_f
- tiny contribution from the gluon initiated (gq) channel

All A_{UT}s @ EIC



- three A_{UT}s at a few percent level:
  $\sin(\phi_h - \phi_S)$ (Sivers), $\sin(\phi_S)$ and $\sin(2\phi_h - \phi_S)$

Sivers and Collins at COMPASS



- Sivers up to 1% (JAM) in magnitude, Collins small
- Sivers asymmetry at COMPASS positive, but for $P_{hT} < 1$ GeV

Conclusions

- new contribution to SSA from $g_T(x) \sim \int_x^1 dx_1 \Delta f(x_1)/x_1$ in SIDIS at two loops

$$\text{KPR}: \; A_N \sim \frac{\alpha_s m_q}{P_{hT}}$$

- involves only (known) twist-2 PDFs and FFs (after WW approx)
- numerical computation in SIDIS reveals up to 2% for the Sivers moment $A_{UT}^{\sin(\phi_h - \phi_S)}$ at the EIC with $P_{hT} > 1$ GeV
- asymmetries drop for small x_B: $\Delta\sigma \sim x g_T(x)$
- expect similar contributions from $g_T(x)$ in pp, DY, open charm etc..
- at $P_{hT} < 1$ GeV we need a TMD computation $\rightarrow$ more sources for SSA possible

Conclusions

- new contribution to SSA from $g_T(x) \sim \int_x^1 dx_1 \Delta f(x_1)/x_1$ in SIDIS at two loops

$$A_N \sim \frac{\alpha_s M_N}{P_{hT}} \frac{x\Delta f(x)}{f(x)}$$

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- numerical computation in SIDIS reveals up to 2% for the Sivers moment $A_{UT}^{\sin(\phi_h-\phi_S)}$ at the EIC with $P_{hT} > 1$ GeV
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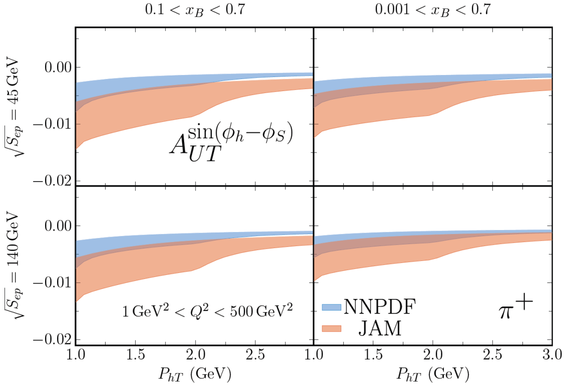
Energy dependence



- a percent level asymmetry persits up to top EIC energies

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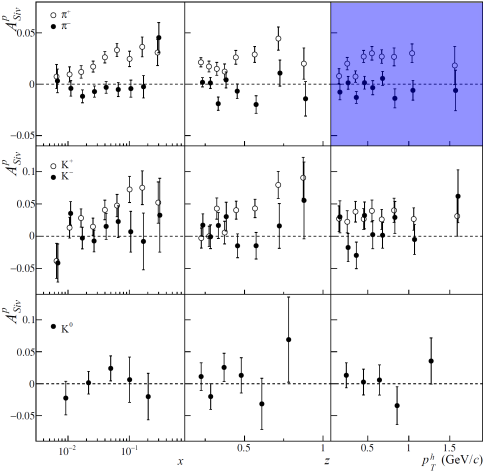
Sivers and Collins at COMPASS



- Sivers asymmetry at COMPASS positive, but for $P_{hT} < 1$ GeV