TMDs for spin-1 hadrons

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24th International Spin Symposium,

(In-person/Online) Matsue, Japan, October 18-22, 2021,

https://indico2.riken.jp/event/3082/

Recent papers on spin-1:

(1) W. Cosyn, Yu-Bing Dong, SK, M. Sargsian, PRD 95 (2017) 074036.

(2) SK and Qin-Tao Song, PRD 94 (2016) 054022; 101 (2020) 054011 & 094013;
PRD 103 (2021) 014025 (this talk);
JHEP 09 (2021) 141 (Song's talk on Oct. 19).

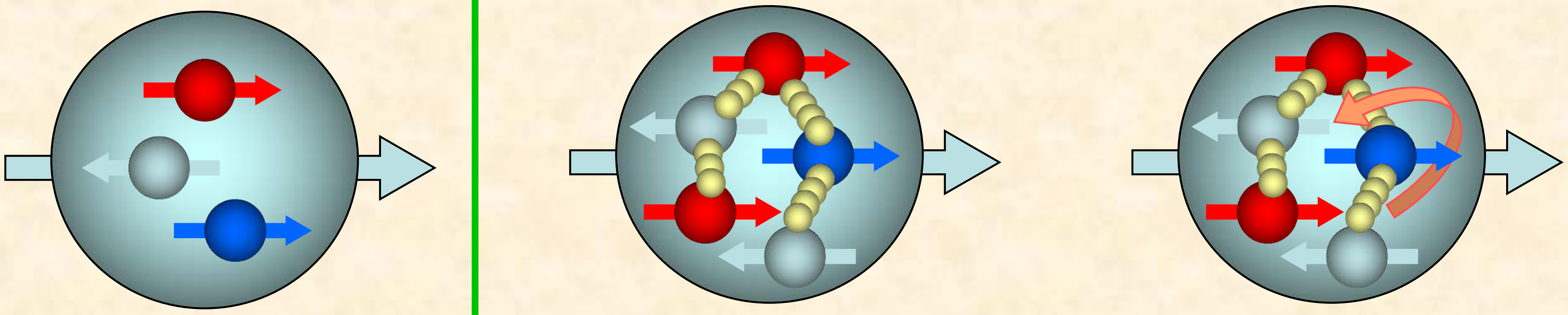
(3) (NICA) A. Arbuzov *et al*., Prog. Nucl. Part. Phys. 119 (2021) 103858.

October 20, 2021

Contents

Nucleon spin

Almost none of nucleon spin is carried by quarks! → **Nucleon spin puzzle!?**

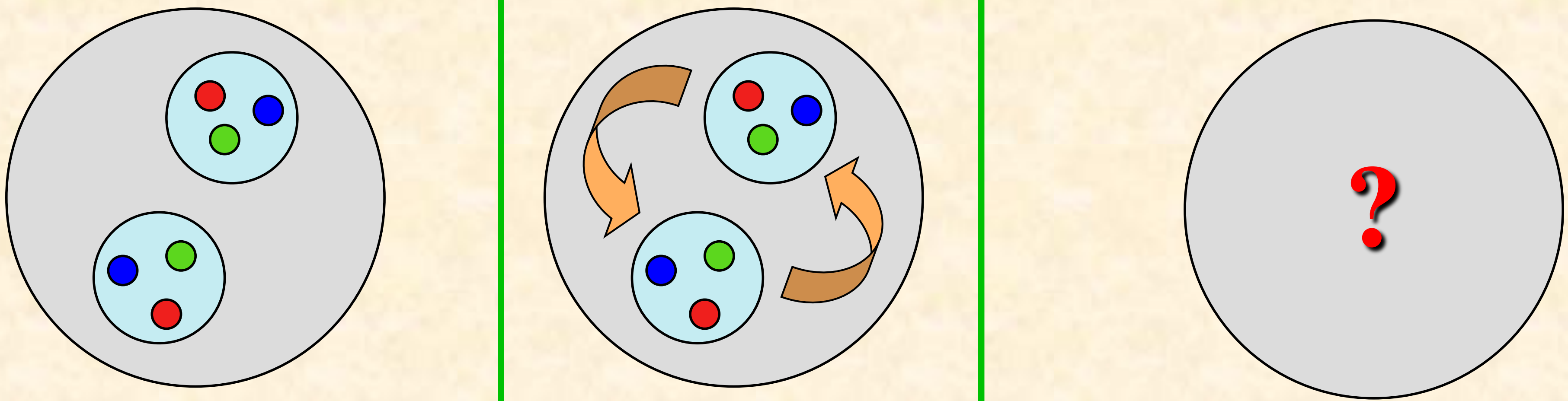


Naïve Quark Model | Sea-quarks and gluons? | Orbital angular momenta ?

"old" standard model

Tensor structure b_1 (*e.g.* deuteron)

Tensor-structure puzzle!?



only S wave $b_1 = 0$

S + D waves

standard model $b_1 \neq 0$

$b_1^{\text{experiment}} \neq b_1^{\text{“standard model”}}$

Experimental projects and personal works on spin-1 physics

Year	Experimental projects	Personal studies of SK with collaborators
	1988: EMC spin puzzle on proton	
1990	1989: Hoodbhoy-Jaffe-Manohar on b_{1-4} [1983: Frankufurt-Strikman]	1990: Close and SK on b_1 sum rule
2000	1998: Courant's BNL report on polarized-deuteron acceleration at RHIC	1999: Hino and SK on formalism of p-d Drell-Yan
	2005: HERMES measurement on b_1	2008: SK on projection operators for b_{1-4}
2010	2011: JLab proposal on b_1	2010: SK on determination of tensor-polarized PDFs
	2016: JLab LoI on gluon transversity	2016: SK and Song on tensor-polarized PDFs in p-d Drell-Yan
		2017: Cosyn, Dong, SK, Sargsian, on convolution estimate on b_1
2020	2021: NICA paper on deuteron	2020: SK and Song on gluon transversity in p-d Drell-Yan,
	2022: Fermilab proposal on deuteron	2021: on TMDs and PDFs up to twist 4, twist-2 relation and sum rule for PDFs
	2025 ~ : JLab, Fermilab, NICA, LHCspin, ···	
2030	2030's : EIC/EicC, ···	

Deuteron spin-1 physics will be developed significantly in 2020's and 2030's.

Standard model prediction for b_1 of deuteron

$$b_1(x) = \int \frac{dy}{y} \delta_T f(y) F_1^N(x/y, Q^2), \qquad y = \frac{M p\cdot q}{M_N P\cdot q} \simeq \frac{2p^-}{P^-}$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y) + f^-(y)}{2}$$

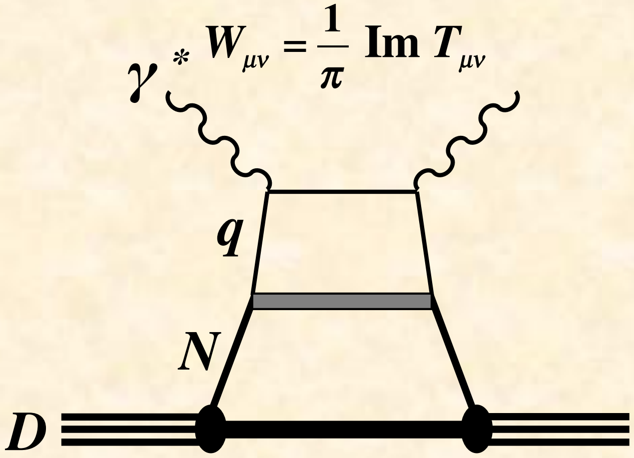
$$= \int d^3p \; y \left[-\frac{3}{4\sqrt{2}\pi} \phi_0(p)\phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] (3\cos^2\theta - 1)\, \delta\left(y - \frac{p\cdot q}{M_N \nu} \right)$$

S-D term D-D term

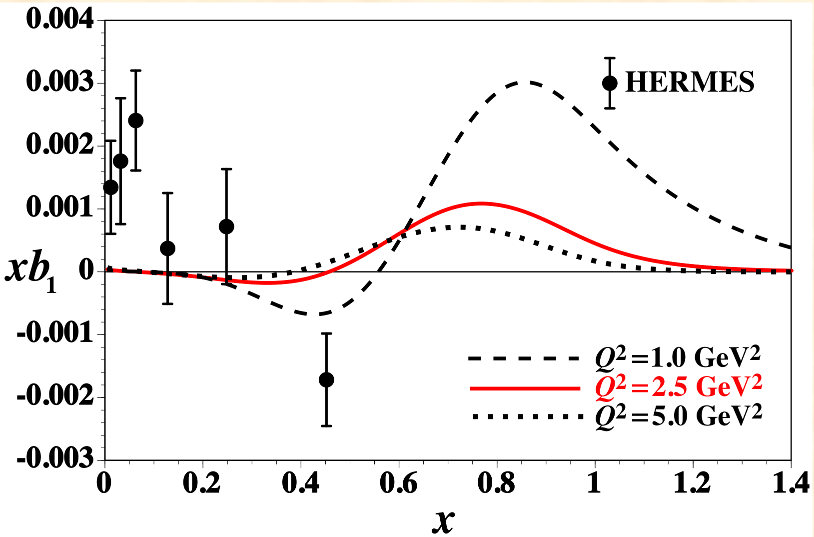
Nucleon momentum distribution:

$$f^H(y) \equiv f^H_\uparrow(y) + f^H_\downarrow(y) = \int d^3p \; y \, |\phi^H(\vec{p})|^2 \delta\left(y - \frac{E - p_z}{M_N} \right)$$

D-state admixture: $\phi^H(\vec{p}) = \phi^H_{\ell=0}(\vec{p}) + \phi^H_{\ell=2}(\vec{p})$

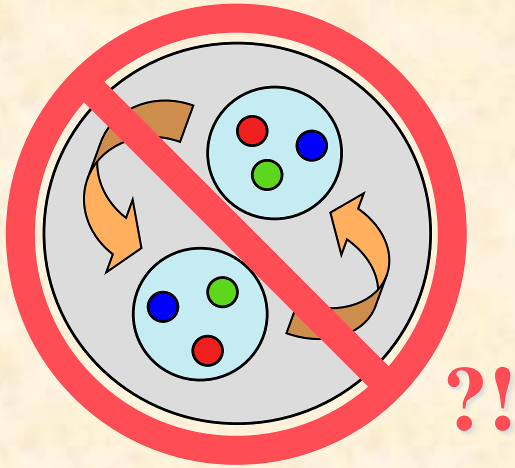


Standard model of the deuteron



W. Cosyn, Yu-Bing Dong, SK, M. Sargsian, Phys. Rev. D 95 (2017) 074036.

$|b_1(\text{theory})| \ll |b_1(\text{HERMES})|$ at $x < 0.5$



S + D waves

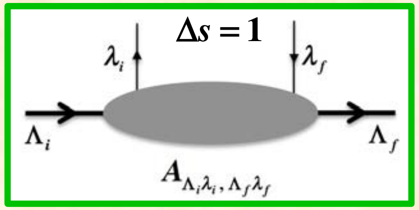
Standard convolution model does not work for the deuteron tensor structure!?

Gluon transversity $\Delta_T g$

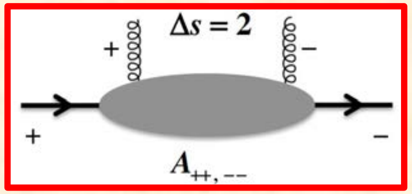
Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, conservation $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ +\frac{1}{2}+\frac{1}{2}\right) - A\left(+\frac{1}{2}-\frac{1}{2},\ +\frac{1}{2}-\frac{1}{2}\right)$

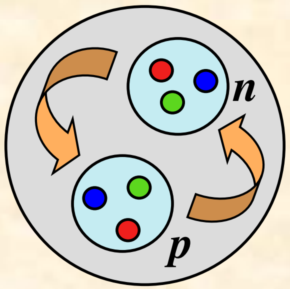
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ -\frac{1}{2}-\frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \to \lambda_f = -\frac{1}{2}$ quark spin flip ($\Delta s = 1$)



Gluon transversity in deuteron: $\Delta_T g(x) \sim A(+1+1,\ -1-1)$, ~~$A\left(+\frac{1}{2}+1,\ -\frac{1}{2}-1\right)$~~ not possible for nucleon



Note: Gluon transversity does not exist for spin-1/2 nucleons.



S + D waves

$b_1\ (\delta_T q,\ \delta_T g) \neq 0 \ \Leftrightarrow$ still $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?

TMDs and PDFs for spin-1 hadrons

S. Kumano and Qin-Tao Song,
Phys. Rev. D 103 (2021) 014025.

Twsit-2 TMDs for spin-1/2 nucleons and spin-1 hadrons

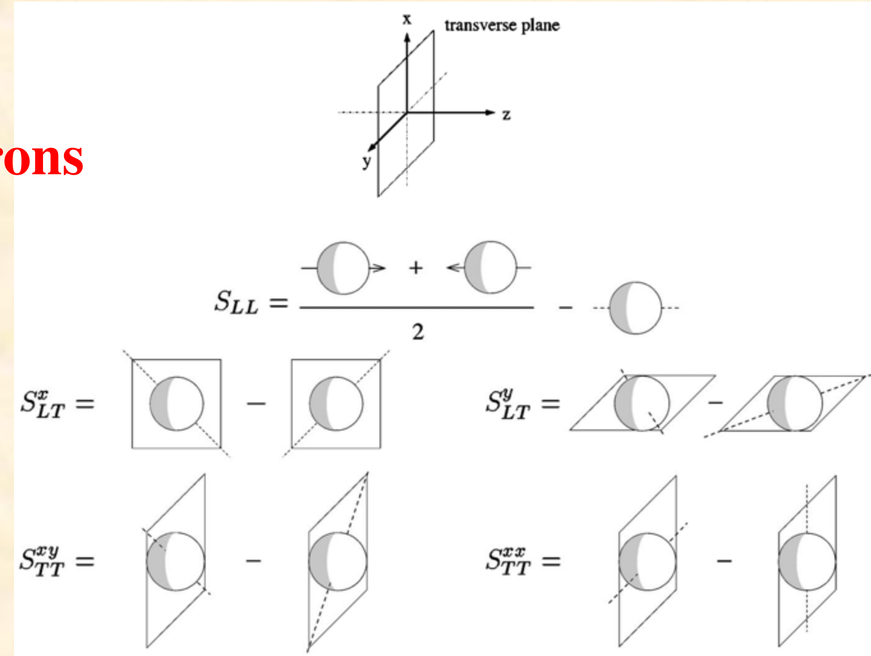
Twist-2 TMDs

Bacchetta-Mulders, PRD 62 (2000) 114004.

Hadron \ Quark	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1]$, $[h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}]$, $[h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}]$, $[h_{1TT}^\perp]$

Spin-1/2 nucleon (rows U, L, T)

Spin-1 hadrons (rows LL, LT, TT)



Twist-2 collinear PDFs

$[\cdots]$= chiral odd

Hadron \ Quark	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					
L			$g_{1L}\,(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}\,(b_1)$					
LT						*1
TT						

*1 Because of the time-reversal invariance, the collinear PDF $h_{1LT}(x)$ vanishes. However, since the time-reversal invariance cannot be imposed in the fragmentation functions, we should note that the corresponding fragmentation function $H_{1LT}(z)$ should exist as a collinear fragmentation function. (see our PRD paper for the details)

TMD correlation functions for spin-1 hadrons

$$\Phi_{ij}(k,P,T) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \left\langle P,T \middle| \bar{\psi}_j(0) W(0,\xi) \psi_i(\xi) \middle| P,T \right\rangle$$

$$W(0,\xi) = P \exp\left[-ig \int_0^\xi d\xi \cdot A(\xi) \right]$$

Spin vector: $S^\mu = S_L \frac{P^+}{M} \bar{n}^\mu - S_L \frac{M}{2P^+} n^\mu + S_T^\mu$

Tensor: $T^{\mu\nu} = \frac{1}{2}\left[\frac{4}{3} S_{LL} \frac{(P^+)^2}{M^2} \bar{n}^\mu \bar{n}^\nu + \frac{P^+}{M} \bar{n}^{\{\mu} S_{LT}^{\nu\}} - \frac{2}{3} S_{LL} \left(\bar{n}^{\{\mu} n^{\nu\}} - g_T^{\mu\nu} \right) + S_{TT}^{\mu\nu} - \frac{M}{2P^+} n^{\{\mu} S_{LT}^{\nu\}} + \frac{1}{3} S_{LL} \frac{M^2}{(P^+)^2} n^\mu n^\nu \right]$

Tensor part (twist-2): Bacchetta, Mulders, PRD 62 (2000) 114004

$$\Phi(k,P,T) = \left(\frac{A_{13}}{M} I + \frac{A_{14}}{M^2} \not{P} + \frac{A_{15}}{M^2} \not{k} + \frac{A_{16}}{M^3} \sigma_{\rho\sigma} P^\rho k^\sigma \right) k_\mu k_\nu T^{\mu\nu} + \left[A_{17}\gamma_\nu + \left(\frac{A_{18}}{M} P^\rho + \frac{A_{19}}{M} k^\rho \right) \sigma_{\nu\rho} + \frac{A_{20}}{M^2} \varepsilon_{\tau\nu\rho\sigma} P^\rho k^\sigma \gamma^\tau \gamma_5 \right] k_\mu T^{\mu\nu}$$

Tensor part (twist-2, 3, 4): n^μ dependent terms are added for up to twist 4.

[For the spin-1/2 nucleon: Goeke, Metzand, Schlegel, PLB 618 (2005) ,90; Metz, Schweitzer, Teckentrup, PLB 680 (2009) 141.]

Kumano-Song-2021, for the details see PRD 103 (2021) 014025

$$\Phi(k,P,T \mid n) = \left(\frac{A_{13}}{M} I + \frac{A_{14}}{M^2} \not{P} + \frac{A_{15}}{M^2} \not{k} + \frac{A_{16}}{M^3} \sigma_{\rho\sigma} P^\rho k^\sigma \right) k_\mu k_\nu T^{\mu\nu} + \left[A_{17}\gamma_\nu + \left(\frac{A_{18}}{M} P^\rho + \frac{A_{19}}{M} k^\rho \right) \sigma_{\nu\rho} + \frac{A_{20}}{M^2} \varepsilon_{\tau\nu\rho\sigma} P^\rho k^\sigma \gamma^\tau \gamma_5 \right] k_\mu T^{\mu\nu}$$

Bacchetta -Mulders

$$+ \left(\frac{B_{21} M}{P\cdot n} k_\mu + \frac{B_{22} M^3}{(P\cdot n)^2} n_\mu \right) n_\nu T^{\mu\nu} + i\gamma_5 \varepsilon_{\mu\tau\rho\sigma} P^\rho \left(\frac{B_{23}}{(P\cdot n) M} k^\tau n^\sigma k_\nu + \frac{B_{24} M}{(P\cdot n)^2} k^\tau n^\sigma n_\nu \right) T^{\mu\nu}$$

$$+ \left[\frac{B_{25}}{P\cdot n} \not{n} k_\mu k_\nu + \left(\frac{B_{26} M^2}{(P\cdot n)^2} \not{n} + \frac{B_{28}}{P\cdot n} \not{P} + \frac{B_{30}}{P\cdot n} \not{k} \right) k_\mu n_\nu + \left(\frac{B_{27} M^4}{(P\cdot n)^3} \not{n} + \frac{B_{29} M^2}{(P\cdot n)^2} \not{P} + \frac{B_{31} M^2}{(P\cdot n)^2} \not{k} \right) n_\mu n_\nu + \frac{B_{32} M^2}{P\cdot n} \gamma_\mu n_\nu \right] T^{\mu\nu}$$

$$- \left[\varepsilon_{\mu\tau\rho\sigma} \gamma^\tau P^\rho \left(\frac{B_{34}}{P\cdot n} n^\sigma k_\nu + \frac{B_{33}}{P\cdot n} k^\sigma n_\nu + \frac{B_{35} M^2}{(P\cdot n)^2} n^\sigma n_\nu \right) + \varepsilon_{\lambda\tau\rho\sigma} k^\lambda \gamma^\tau P^\rho n^\sigma \left(\frac{B_{36}}{P\cdot n M^2} k_\mu k_\nu + \frac{B_{37}}{(P\cdot n)^2} k_\mu n_\nu + \frac{B_{38} M^2}{(P\cdot n)^3} n_\mu n_\nu \right) \right] \gamma_5 T^{\mu\nu}$$

$$+ \varepsilon_{\mu\tau\rho\sigma} k^\tau P^\rho n^\sigma \left(\frac{B_{39}}{(P\cdot n)^2} k_\nu + \frac{B_{40} M^2}{(P\cdot n)^3} n_\nu \right) \not{n} \gamma_5 T^{\mu\nu}$$

$$+ \sigma_{\rho\sigma} \left[P^\rho k^\sigma \left(\frac{B_{41}}{(P\cdot n) M} k_\mu n_\nu + \frac{B_{42} M}{(P\cdot n)^2} n_\mu n_\nu \right) + P^\rho n^\sigma \left(\frac{B_{43}}{(P\cdot n) M} k_\mu k_\nu + \frac{B_{44} M}{(P\cdot n)^2} k_\mu n_\nu + \frac{B_{45} M^3}{(P\cdot n)^3} n_\mu n_\nu \right) \right] T^{\mu\nu}$$

$$+ \sigma_{\rho\sigma} \left[k^\rho n^\sigma \left(\frac{B_{46}}{(P\cdot n) M} k_\mu k_\nu + \frac{B_{47} M}{(P\cdot n)^2} k_\mu n_\nu + \frac{B_{48} M^3}{(P\cdot n)^3} n_\mu n_\nu \right) \right] T^{\mu\nu} + \sigma_{\mu\sigma} \left[n^\sigma \left(\frac{B_{49} M}{P\cdot n} k_\nu + \frac{B_{50} M^3}{(P\cdot n)^2} n_\nu \right) + \left(\frac{B_{51} M}{P\cdot n} P^\sigma + \frac{B_{52} M}{P\cdot n} k^\sigma \right) n_\nu \right] T^{\mu\nu}$$

New terms in our paper

From this correlation function, new tensor-polarized TMDs are defined in twist-3 and 4 in addition to twist-2 ones.

Terms associated with $n = \frac{1}{\sqrt{2}}(1,\ 0,\ 0,\ -1)$

Twist-3 TMDs for spin-1 hadrons

$$\Phi^{[\Gamma]}(x, k_T, T) \equiv \frac{1}{2}\mathrm{Tr}\left[\Phi^{[\Gamma]}(x, k_T, T)\,\Gamma\right] = \frac{1}{2}\mathrm{Tr}\left[\int dk^- \Phi(k, P, T\,|\,n)\,\Gamma\right], \quad F(x, k_T^2) \equiv F'(x, k_T^2) - \frac{k_T^2}{2M^2}F^\perp(x, k_T^2)$$

$$\Phi^{[\gamma^i]}(x, k_T, T) = \frac{M}{P^+}\left[f_{LL}^\perp(x, k_T^2)\frac{S_{LL}k_T^i}{M} + f'_{LT}(x, k_T^2)S_{LT}^i - f_{LT}^\perp(x, k_T^2)\frac{k_T^i S_{LT}\cdot k_T}{M^2} - f'_{TT}(x, k_T^2)\frac{S_{TT}^{ij}k_{Tj}}{M} + f_{TT}^\perp(x, k_T^2)\frac{k_T^i k_T\cdot S_{TT}\cdot k_T}{M^3}\right]$$

$$\Phi^{[1]}(x, k_T, T) = \frac{M}{P^+}\left[e_{LL}(x, k_T^2)S_{LL} - e_{LT}^\perp(x, k_T^2)\frac{S_{LT}\cdot k_T}{M} + e_{TT}^\perp(x, k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\right]$$

$$\Phi^{[i\gamma_5]}(x, k_T, T) = \frac{M}{P^+}\left[e_{LT}(x, k_T^2)\frac{S_{LT\mu}\varepsilon_T^{\mu\nu}k_{T\nu}}{M} - e_{TT}(x, k_T^2)\frac{S_{TT\mu\rho}k_T^\rho \varepsilon_T^{\mu\nu}k_{T\nu}}{M^2}\right]$$

$$\Phi^{[\gamma^i\gamma_5]}(x, k_T, T) = \frac{M}{P^+}\left[-g_{LL}^\perp(x, k_T^2)\frac{S_{LL}\varepsilon_T^{ij}k_{Tj}}{M} - g'_{LT}(x, k_T^2)\varepsilon_T^{ij}S_{LTj} + g_{LT}^\perp(x, k_T^2)\frac{\varepsilon_T^{ij}k_{Tj}S_{LT}\cdot k_T}{M^2} + g'_{TT}(x, k_T^2)\frac{\varepsilon_T^{ij}S_{TTjl}k_T^l}{M} - g_{TT}^\perp(x, k_T^2)\frac{\varepsilon_T^{ij}k_{Tj}k_T\cdot S_{TT}\cdot k_T}{M^3}\right]$$

$$\Phi^{[\sigma^{-+}]}(x, k_T, T) = \frac{M}{P^+}\left[h_{LL}(x, k_T^2)S_{LL} - h_{LT}(x, k_T^2)\frac{S_{LT}\cdot k_T}{M} + h_{TT}(x, k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\right]$$

$$\Phi^{[\sigma^{ij}]}(x, k_T, T) = \frac{M}{P^+}\left[h_{LT}^\perp(x, k_T^2)\frac{S_{LT}^i k_T^j - S_{LT}^j k_T^i}{M} - h_{TT}^\perp(x, k_T^2)\frac{S_{TT}^{il}k_{Tl}k_T^j - S_{TT}^{jl}k_{Tl}k_T^i}{M^2}\right]$$

***2, *3** Because of the time-reversal invariance, the collinear PDFs $g_{LT}(x)$ and $h_{LL}(x)$ do not exist. However, the corresponding new collinear fragmentation functions $G_{LT}(z)$ and $H_{LL}(z)$ should exist. (see our PRD paper for the details)

Quark \ Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f^\perp$ [e]			$g^\perp$		[h]
L		$f_L^\perp$ [e_L]	$g_L^\perp$		[h_L]	
T		$f_T, f_T^\perp$ [$e_T, e_T^\perp$]	$g_T, g_T^\perp$		[h_T], [$h_T^\perp$]	
LL	$f_{LL}^\perp$ [e_{LL}]			$g_{LL}^\perp$		[h_{LL}]
LT	$f_{LT}, f_{LT}^\perp$ [$e_{LT}, e_{LT}^\perp$]			$g_{LT}, g_{LT}^\perp$		[h_{LT}], [$h_{LT}^\perp$]
TT	$f_{TT}, f_{TT}^\perp$ [$e_{TT}, e_{TT}^\perp$]			$g_{TT}, g_{TT}^\perp$		[h_{TT}], [$h_{TT}^\perp$]

Quark \ Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	[e]					
L					[h_L]	
T			g_T			
LL	[e_{LL}]					*3
LT	f_{LT}			*2		
TT						

New TMDs [· · ·]= chiral odd **New collinear PDFs**

Twist-4 TMDs for spin-1 hadrons

$$\Phi^{[\Gamma]}(x, k_T, T) \equiv \frac{1}{2}\mathrm{Tr}\left[\Phi^{[\Gamma]}(x, k_T, T)\,\Gamma\right] = \frac{1}{2}\mathrm{Tr}\left[\int dk^- \Phi(k, P, T \mid n)\,\Gamma\right], \quad F(x, k_T^2) \equiv F'(x, k_T^2) - \frac{k_T^2}{2M^2}F^\perp(x, k_T^2)$$

$$\Phi^{[\gamma^-]}(x, k_T, T) = \frac{M^2}{P^{+2}}\left[f_{3LL}(x, k_T^2)S_{LL} - f_{3LT}(x, k_T^2)\frac{S_{LT}\cdot k_T}{M} + f_{3TT}(x, k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\right]$$

$$\Phi^{[\gamma^-\gamma_5]}(x, k_T, T) = \frac{M^2}{P^{+2}}\left[g_{3LT}(x, k_T^2)\frac{S_{LT\mu}\varepsilon_T^{\mu\nu}k_{T\nu}}{M} + g_{3TT}(x, k_T^2)\frac{S_{TT\mu\rho}k_T^\rho\varepsilon_T^{\mu\nu}k_{T\nu}}{M^2}\right]$$

$$\Phi^{[\sigma^{i-}]}(x, k_T, T) = \frac{M^2}{P^{+2}}\left[h_{3LL}^\perp(x, k_T^2)\frac{S_{LL}k_T^i}{M} + h'_{3LT}(x, k_T^2)S_{LT}^i - h_{3LT}^\perp(x, k_T^2)\frac{k_T^i S_{LT}\cdot k_T}{M^2} - h'_{3TT}(x, k_T^2)\frac{S_{TT}^{ij}k_{Tj}}{M} + h_{3TT}^\perp(x, k_T^2)\frac{k_T^i k_T\cdot S_{TT}\cdot k_T}{M^3}\right]$$

*4 Because of the time-reversal invariance, $h_{3LT}(x)$ does not exist; however, the corresponding new collinear fragmentation function $H_{3LT}(z)$ should exist because the time-reversal invariance does not have to be imposed.

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}], [h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}], [h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}], [h_{3TT}^\perp]$

New TMDs

$[\cdots]$= chiral odd

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					
L			g_{3L}			
T					$[h_{3T}]$	
LL	f_{3LL}					
LT						*4
TT						

New collinear PDFs

TMDs and their sum rules for spin-1 hadrons

Twist-2 TMDs

Hadron \ Quark	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

Time-reversal invariance in colliear corrlation functions (PDFs)

$$\int d^2k_T \Phi_{\text{T-odd}}(x, k_T^2) = 0$$

Sum rules for the TMDs of spin-1 hadrons

$$\int d^2k_T \, h_{1LT}(x, k_T^2) = 0, \qquad \int d^2k_T \, g_{LT}(x, k_T^2) = 0,$$

$$\int d^2k_T \, h_{LL}(x, k_T^2) = 0, \qquad \int d^2k_T \, h_{3LT}(x, k_T^2) = 0$$

For example, in the twist-4

$$\int d^2k_T \, h_{3LT}(x, k_T^2) \equiv \int d^2k_T \left[h'_{3LT}(x, k_T^2) - \frac{k_T^2}{2M^2} h_{3LT}(x, k_T^2) \right] = 0$$

$$\Phi^{[\sigma^{i-}]} = \frac{M^2}{P^{+2}} \Big[h_{3LL}^\perp(x, k_T^2) S_{LL} \frac{k_T^i}{M} + h'_{3LT}(x, k_T^2) S_{LT}^i - h_{3LT}^\perp(x, k_T^2) \frac{k_T^i S_{LT} \cdot k_T}{M^2}$$
$$- h'_{3TT}(x, k_T^2) \frac{S_{TT}^{ij} k_{Tj}}{M} + h_{3TT}^\perp(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \frac{k_T^i}{M} \Big]$$

Twist-3 TMDs

Hadron \ Quark	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f^\perp$ $[e]$			$g^\perp$		$[h]$
L		$f_L^\perp$ $[e_L]$	$g_L^\perp$		$[h_L]$	
T		$f_T, f_T^\perp$ $[e_T, e_T^\perp]$	$g_T, g_T^\perp$		$[h_T], [h_T^\perp]$	
LL	$f_{LL}^\perp$ $[e_{LL}]$			$g_{LL}^\perp$		$[h_{LL}]$
LT	$f_{LT}, f_{LT}^\perp$ $[e_{LT}, e_{LT}^\perp]$			$g_{LT}, g_{LT}^\perp$		$[h_{LT}], [h_{LT}^\perp]$
TT	$f_{TT}, f_{TT}^\perp$ $[e_{TT}, e_{TT}^\perp]$			$g_{TT}, g_{TT}^\perp$		$[h_{TT}], [h_{TT}^\perp]$

Twist-4 TMDs

Hadron \ Quark	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}], [h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}], [h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}], [h_{3TT}^\perp]$

New fragmentations for spin-1 hadrons

Corresponding fragmentation functions exist for the spin-1 haddrons
simply by changing function names and kinematical variables.

TMD distribution functions: $f, \; g, \; h, \; e \; ; \quad x, \; k_T, \; S, \; T, \; M, \; n, \; \gamma^+, \; \sigma^{i+}$

$$\Downarrow$$

TMD fragmentation functions: $D, \; G, \; H, \; E \; ; \; z, \; k_T, \; S_h, \; T_h, \; M_h, \; \bar{n}, \; \gamma^-, \; \sigma^{i-}$

Collinear fragmentation functions:
X. Ji, Phys. Rev. D 49, 114 (1994).

PDFs for spin-1 hadrons

Twist-2 PDFs

Quark / Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					
L			$g_{1L}\,(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}\,(b_1)$					
LT						*1
TT						

*1: $h_{1LT}(x)$, *2: $g_{LT}(x)$, *3: $h_{LL}(x)$, *4: $h_{3LT}(x)$

Because of the time-reversal invariance, the collinear PDF vanishes. However, since the time-reversal invariance cannot be imposed in the fragmentation functions, we should note that the corresponding fragmentation function should exist as a collinear fragmentation function.

We derived analogous relations to Wandzura-Wilczek relation and Burkhardt-Cottingham sum rule for f_{LT} and f_{1LL}. (→Song's talk)

SK and Qin-Tao Song, JHEP 09 (2021) 141

Twist-3 PDFs

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					*3
LT	f_{LT}			*2		
TT						

Twist-4 PDFs

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					
L			g_{3L}			
T					$[h_{3T}]$	
LL	f_{3LL}					
LT						*4
TT						

Twist-2 relation and sum rule

Skip this page:
Song's talk on Oct.19,
SK and Qin-Tao Song,
JHEP 09 (2021) 141.

- **Twist-3 matrix element in terms of tensor-polarized PDFs**

$$\left\langle P,T \left| \bar\psi(0)(\partial^\mu\gamma^\alpha - \partial^\alpha\gamma^\mu)\psi(\xi) \right| P,T \right\rangle = 2MS^\alpha_{LT}\int_-^1 dx\, e^{-ixP^+\xi^-}\left[-\frac{3}{2}f_{1LL}(x) + f_{LT}(x) - \frac{d}{dx}\{xf_{LT}(x)\}\right]$$

- **Twist-3 operator in terms of gluon field tensor**

$$\xi_\mu\left[\bar\psi(0)(\gamma^\alpha\partial^\mu - \gamma^\mu\partial^\alpha)\psi(\xi)\right] = g\int_0^1 dt\,\bar\psi(0)\left\{i\left(t-\frac{1}{2}\right)G^{\alpha\mu}(t\xi) - \frac{1}{2}\gamma_5\tilde G^{\alpha\mu}(t\xi)\right\}\xi_\mu\not\xi\psi(\xi)$$

- **Matrix element of field tensor in terms of twist-3 multiparton distribution functions**

$$\int\frac{d(P\cdot\xi)}{2\pi}e^{ix_1P\cdot\xi}\langle P,T|g\int_0^1 dt\,\bar\psi(0)\left\{i\left(t-\frac{1}{2}\right)G^{\mu\nu}(t\xi) - \frac{1}{2}\gamma_5\tilde G^{\mu\nu}(t\xi)\right\}\xi_\mu\not\xi\psi(\xi)|P,T\rangle_{\xi^+=\vec\xi_T=0}$$

$$= -2MS^\nu_{LT}\mathcal{P}\int_0^1 dx_2\frac{1}{x_1-x_2}\left[\frac{\partial}{\partial x_1}\{F_{G,LT}(x_1,x_2)+G_{G,LT}(x_1,x_2)\} + \frac{\partial}{\partial x_2}\{F_{G,LT}(x_2,x_1)+G_{G,LT}(x_2,x_1)\}\right]$$

$$x\frac{df_{LT}(x)}{dx} = -\frac{3}{2}f_{1LL}(x) - f_{LT}^{(HT)}(x),$$ **Higher-twist:** $$f_{LT}^{(HT)}(x) = -\mathcal{P}\int_0^1 dy\frac{1}{x-y}\left[\frac{\partial}{\partial x}\{F_{G,LT}(x,y)+G_{G,LT}(x,y)\} + \frac{\partial}{\partial y}\{F_{G,LT}(y,x)+G_{G,LT}(y,x)\}\right]$$

$$\to f_{LT}(x) = \frac{3}{2}\int_x^{\varepsilon(x)}\frac{dy}{y}f_{1LL}(y) + \int_x^{\varepsilon(x)}\frac{dy}{y}f_{LT}^{(HT)}(y), \quad \varepsilon(x) = \frac{i}{\pi}P\int_{-\infty}^{\infty}dy\frac{1}{y}e^{-ixy} = \begin{cases} +1 & x>0 \\ -1 & x<0 \end{cases}$$

Define $f^+(x) = f(x) + \bar f(x) = f(x) - f(-x), \quad f = f_{1LL},\ f_{LT},\ f_{LT}^{(HT)}, \quad x>0$

$$\to f^+_{LT}(x) = \frac{3}{2}\int_x^1\frac{dy}{y}f^+_{1LL}(y) + \int_x^1\frac{dy}{y}f_{LT}^{(HT)+}(y)$$ $\to$ Twist-2 relation: $$f^+_{LT}(x) = \frac{3}{2}\int_x^1\frac{dy}{y}f^+_{1LL}(y)$$

If we define $f_{2LT}(x) = \frac{2}{3}f_{LT}(x) - f_{1LL}(x)$,

$$f^+_{2LT}(x) = -f^+_{1LL}(x) + \int_x^1\frac{dy}{y}f^+_{1LL}(y) + \frac{2}{3}\int_x^1\frac{dy}{y}f_{LT}^{(HT)+}(y)$$ $\to$ Twist-2 relation: $f^+_{2LT}(x) = -f^+_{1LL}(x) + \int_x^1\frac{dy}{y}f^+_{1LL}(y)$, Wandzura-Wilczek like

$\to$ Sum rule: $\int_0^1 dx\, f^+_{2LT}(x) = 0$, Burkhardt-Cottingham like

If the parton-model sum rule without the tensor-polarized antiquark distributions $\int_0^1 dx\, f^+_{1LL}(x) = \frac{2}{3}\int_0^1 dx\, b_1^+(x) = 0$ is valid, $\to$ Sum rule: $\int_0^1 dx\, f^+_{LT}(x) = 0$

$$\int dx\, b_1^D(x) = \lim_{t\to 0} -\frac{5}{12}\frac{t}{M^2}F_Q(t) + \sum_i e_i^2\int dx\,\delta_T\bar q_i(x)$$
$= 0$?

F. E. Close and SK, PRD 42 (1990) 2377.

Summary on the twist-2 relation and sum rule

$g_2(x) = -g_1(x) + \int_x^1\frac{dy}{y}g_1(y)$ (Wandzura-Wilczek relation), $\int_0^1 dx\, g_2(x) = 0$ (Burkhardt-Cottingham sum rule)

For tensor-polarized spin-1 hadrons, we obtained

$f^+_{2LT}(x) = -f^+_{1LL}(x) + \int_x^1\frac{dy}{y}f^+_{1LL}(y)$, $\int_0^1 dx\, f^+_{2LT}(x) = 0$, $f_{2LT}(x) \equiv \frac{2}{3}f_{LT}(x) - f_{1LL}(x)$

$\int_0^1 dx\, f^+_{LT}(x) = 0$ if $\int_0^1 dx\, f^+_{1LL}(x) = \frac{2}{3}\int_0^1 dx\, b_1^+(x) = 0$

Existence of multiparton distribution functions: $F_{G,LT}(x_1,x_2)$, $G_{G,LT}(x_1,x_2)$, $H^\perp_{G,LL}(x_1,x_2)$, $H_{G,TT}(x_1,x_2)$

Future prospects and summary

Spin-1 deuteron experiments from the middle of 2020’s

JLab

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

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J. Pierce

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Proposal (approved),
Experiment: middle of 2020’s

Fermilab

The Transverse Structure of the Deuteron with Drell-Yan

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Proposal,
Fermilab-PAC: January, 2022
Experiment: 2020’s

NICA

Progress in Particle and Nuclear Physics 119 (2021) 103858

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

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journal homepage: www.elsevier.com/locate/ppnp

Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

A. Arbuzov, A. Bacchetta, M. Butenschoen, F.G. Celiberto, U. D’Alesio, M. Deka, I. Denisenko, M.G. Echevarria, A. Efremov, N.Ya. Ivanov, A. Guskov, A. Karpishkov, Ya. Klopot, B.A. Kniehl, A. Kotzinian, S. Kumano, J.P. Lansberg, Keh-Fei Liu, F. Murgia, M. Nefedov, B. Parsamyan, C. Pisano, M. Radici, A. Rymbekova, V. Saleev, A. Shipilova, Qin-Tao Song, O. Teryaev

Prog. Nucl. Part. Phys.
119 (2021) 103858,
Experiment: middle of 2020’s

LHCspin

LHC spin

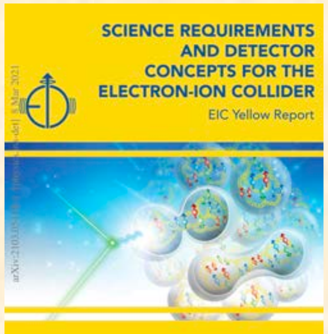
CERN-ESPP-Note-2018-111

The LHCSpin Project

C. A. Aidala[1], A. Bacchetta[2,3], M. Boglione[4,5], G. Bozzi[2,3], V. Carassiti[6,7], M. Chiosso[4,5], R. Cimino[8], G. Ciullo[6,7], M. Contalbrigo[6,7], U. D’Alesio[9,10], P. Di Nezza[8], R. Engels[11], K. Grigoryev[11], D. Keller[12], P. Lenisa[6,7], S. Liuti[12], A. Metz[13], P.J. Mulders[14,15], F. Murgia[10], A. Nass[11], D. Panzieri[5,16], L. L. Pappalardo[6,7], B. Pasquini[2,3], C. Pisano[9,10], M. Radici[3], F. Rathmann[11], D. Reggiani[17], M. Schlegel[18], S. Scopetta[19,20], E. Steffens[21], A. Vasilyev[22]

arXiv:1901.08002,
Experiment: ~2028

2030’s EIC/EicC



R. Abdul Khalek *et al*.
arXiv:2103.05419.

D. P. Anderle *et al*.,
Front. Phys. 16 (2021) 64701.

Frontiers of Physics

Front. Phys. 16(6), 64701 (2021)

https://doi.org/10.1007/s11467-021-1062-0

REVIEW ARTICLE

Electron-ion collider in China

Daniele P. Anderle, Valerio Bertone, Xu Cao, Lei Chang, Ningbo Chang, Gu Chen, Xurong Chen, Zhuojun Chen, Zhufang Cui, Lingyun Dai, Weitian Deng, Minghui Ding, Xu Feng, Chang Gong, Longcheng Gui, Feng-Kun Guo, Chengdong Han, Jun He, Tie-Jiun Hou, Hongxia Huang, Yin Huang, Krešimir Kumerički, L. P. Kaptari, Demin Li, Hengne Li, Minxiang Li, Xueqian Li, Yutie Liang, Zuotang Liang, Chen Liu, Chuan Liu, Guoming Liu, Jie Liu, Liuming Liu, Xiang Liu, Tianbo Liu, Xiaofeng Luo, Zhun Lyu, Boqiang Ma, Fu Ma, Jianping Ma, Yugang Ma, Lijun Mao, Cédric Mezrag, Hervé Moutarde, Jialun Ping, Sixue Qin, Hang Ren, Craig D. Roberts, Juan Rojo, Guodong Shen, Chao Shi, Qintao Song, Hao Sun, Paweł Sznajder, Enke Wang, Fan Wang, Qian Wang, Rong Wang, Ruiru Wang, Taofeng Wang, Wei Wang, Xiaoyu Wang, Xiaoyun Wang, Jiajun Wu, Xinggang Wu, Lei Xia, Bowen Xiao, Guoqing Xiao, Ju-Jun Xie, Yaping Xie, Hongxi Xing, Hushan Xu, Nu Xu, Shusheng Xu, Mengshi Yan, Wenbiao Yan, Wencheng Yan, Xinhu Yan, Jiancheng Yang, Yi-Bo Yang, Zhi Yang, Deliang Yao, Zhihong Ye, Peilin Yin, C.-P. Yuan, Wenlong Zhan, Jianhui Zhang, Jinlong Zhang, Pengming Zhang, Yifei Zhang, Chao-Hsi Chang, Zhenyu Zhang, Hongwei Zhao, Kuang-Ta Chao, Qiang Zhao, Yuxiang Zhao, Zhengguo Zhao, Liang Zheng, Jian Zhou, Xiang Zhou, Xiaorong Zhou, Bingsong Zou, Liping Zou

Summary: our spin-1 TMD and PDF studies

TMDs of spin-1 hadrons

- TMDs: interdisciplinary field of physics
- We proposed new 30 TMDs and 3 PDFs in twist 3 and 4.
- New sum rules for TMDs.
- New TMD fragmentation functions.

Twist-3 TMD: $f_{LL}^{\perp}, e_{LL}, f_{LT}, f_{LT}^{\perp}, e_{1T}, e_{1T}^{\perp}, f_{TT}, f_{TT}^{\perp}, e_{TT}, e_{TT}^{\perp},$
$g_{LL}^{\perp}, g_{LT}, g_{LT}^{\perp}, g_{TT}, g_{TT}^{\perp}, h_{1L}, h_{LT}, h_{LT}^{\perp}, h_{TT}, h_{TT}^{\perp}$

Twist-4 TMD: $f_{3LL}, f_{3LT}, f_{3TT}, g_{3LT}, f_{3TT}, h_{3LL}^{\perp}, h_{3LT}, h_{3LT}^{\perp}, h_{3TT}, h_{3TT}^{\perp}$

Twist-3 PDF: e_{LL}, f_{LT}

Twist-4 PDF: f_{3LL}

Sum rules: $\int d^2k_T\, g_{LT}(x, k_T^2) = \int d^2k_T\, h_{LL}(x, k_T^2) = \int d^2k_T\, h_{3LL}(x, k_T^2) = 0$

TMD distribution functions: $f,\ g,\ h,\ e\ ;\quad x,\ k_T,\ S,\ T,\ M,\ n,\ \gamma^+,\ \sigma^{i+}$

$\Downarrow$

TMD fragmentation functions: $D,\ G,\ H,\ E\ ;\quad z,\ k_T,\ S_h,\ T_h,\ M_h,\ \bar{n},\ \gamma^-,\ \sigma^{i-}$

The End

The End