SPIN2021

Matsue, Japan

24th International Spin Symposium

October 18 -22, 2021

GENERAL HELICITY FORMALISM FOR TWO-HADRON PRODUCTION IN e^+e^- COLLISIONS AND THE Λ POLARIZING FRAGMENTATION FUNCTION

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in collaboration with
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PRD **102**, 054001 (2020)

JHEP **10**, 078 (2021)

OUTLINE

- ❑ $e^+e^- \rightarrow h_1\ h_2 + X$: theory
  - ✓ TMD helicity formalism & FFs for Spin-½ hadrons
  - ✓ Transverse Λ polarization
- ❑ Phenomenology
  - ✓ Fit of Belle data and extraction of the polarizing FF
  - ✓ Predictions for SIDIS@EIC
- ❑ Concluding remarks

$$e^+e^- \rightarrow \; h_1(P_1)\, h_2(P_2) + X$$

- Complete results for the azimuthal and polarization observables
  Boer, Jakob, Mulders 1997 - Pitonyak, Schlegel, Metz 2014
- TMD factorization for small relative transverse momenta w.r.t. Q^2
  Collins 2011 - Echevarria, Idilbi, Scimemi 2012
- Formulation in the helicity formalism: partonic interpretation
  UD, Murgia, Zaccheddu 2021

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- Formulation in the helicity formalism: partonic interpretation
  UD, Murgia, Zaccheddu 2021
- $e^+e^- \rightarrow h_1(P_1)$ jet X

  ✓ TMD factorization, role of soft factors, universality *Kang, Shao, Zhao 2020 – Boglione,Simonelli 2021 - Gamberg, Kang, Shao, Terry, Zhao 2021*

  ✓ access to the intrinsic transverse momentum within a simplified TMD model (see also *Anselmino, Kishore, Mukherjee 2019*)

➢ Improved TMD analysis *UD, Gamberg, Murgia, Zaccheddu 2021*

[talk by M. Zaccheddu]

$$e^+e^- \to h_1(P_1)\, h_2(P_2) + X$$

Master formula [TMD factorization]

Helicity density matrix

$$\rho^{h_1,S_1}_{\lambda_{h_1},\lambda'_{h_1}} \rho^{h_2,S_2}_{\lambda_{h_2},\lambda'_{h_2}} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta\, dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$

$$= \sum_{q_c,\bar{q}_d} \sum_{\{\lambda\}} \frac{1}{32\pi s} \frac{1}{4} \hat{M}_{\lambda_c\lambda_d,\lambda_a\lambda_b} \hat{M}^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b} \hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp 1}) \hat{D}^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp 2})$$

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Helicity density matrix

Unpol x-sec

$$\rho^{h_1,S_1}_{\lambda_{h_1},\lambda'_{h_1}} \rho^{h_2,S_2}_{\lambda_{h_2},\lambda'_{h_2}} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta\, dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$

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Helicity scatt.
Amplitude
(hard)

$$e^+e^- \rightarrow h_1(P_1)\, h_2(P_2) + X$$

Master formula [TMD factorization]

Helicity density matrix

Unpol x-sec

Fragm. Funct. (soft)

$$\rho^{h_1,S_1}_{\lambda_{h_1},\lambda'_{h_1}}\rho^{h_2,S_2}_{\lambda_{h_2},\lambda'_{h_2}}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta\, dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$

$$= \sum_{q_c,\bar{q}_d}\sum_{\{\lambda\}}\frac{1}{32\pi s}\frac{1}{4}\hat{M}_{\lambda_c\lambda_d,\lambda_a\lambda_b}\hat{M}^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b}\hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp 1})\hat{D}^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp 2})$$

Helicity scatt. Amplitude (hard)

$e^+e^- \to \; h_1(P_1)\, h_2(P_2) + X$

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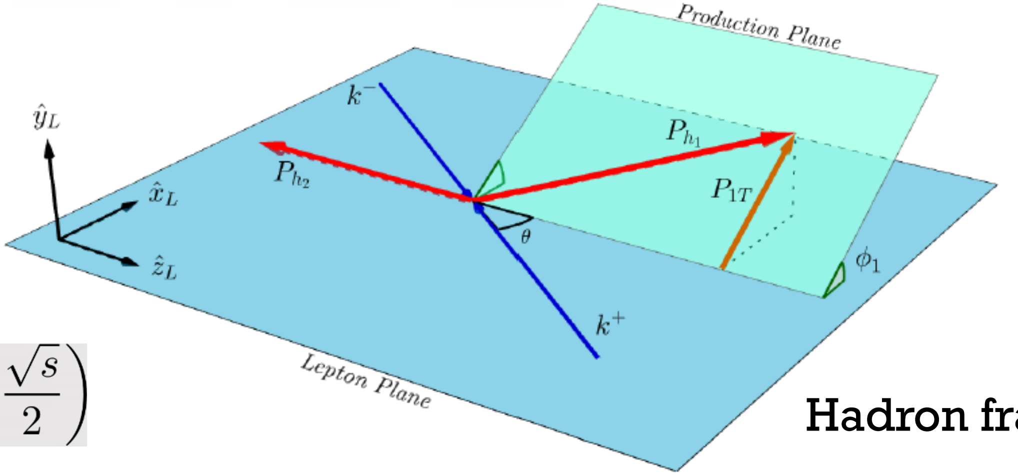
Scaling variables

$z_p = 2|\mathbf{P}_h|/\sqrt{s}$ $\quad z_h = 2E_h/\sqrt{s}$

quark/antiquark momenta

$$q_2 \simeq \left(\frac{\sqrt{s}}{2}, -\frac{p_{\perp 2}}{z_{p_2}}\cos\varphi_2, -\frac{p_{\perp 2}}{z_{p_2}}\sin\varphi_2, -\frac{\sqrt{s}}{2}\right)$$

$\mathbf{q}_1 = -\mathbf{q}_2$



Hadron frame

$e^+e^- \to h_1(P_1)\, h_2(P_2) + X$

$$\rho^{h_1,S_1}_{\lambda_{h_1},\lambda'_{h_1}}\,\rho^{h_2,S_2}_{\lambda_{h_2},\lambda'_{h_2}}\,\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta\, dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$

$$= \sum_{q_c,\bar q_d}\sum_{\{\lambda\}}\frac{1}{32\pi s}\frac{1}{4}\hat M_{\lambda_c\lambda_d,\lambda_a\lambda_b}\hat M^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b}\hat D^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp 1})\hat D^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp 2})$$

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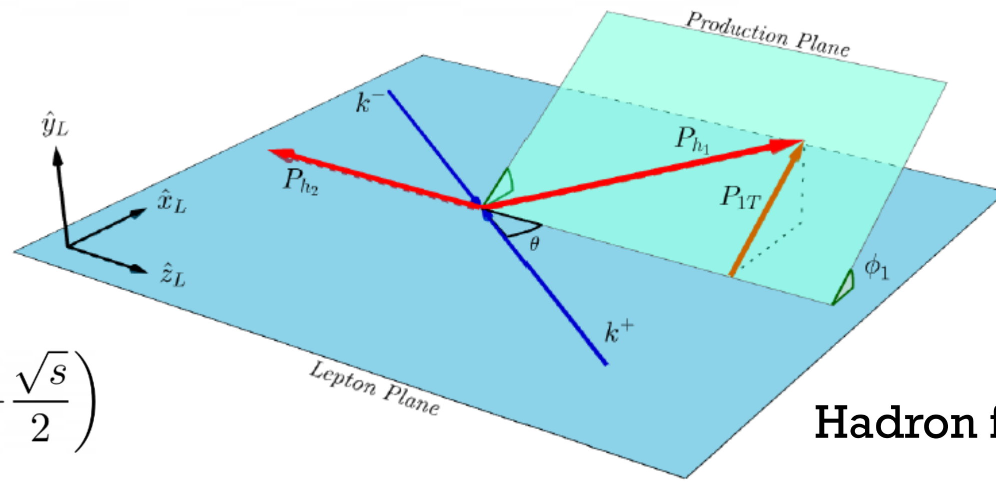

Hadron frame

$$\frac{d\sigma^{e^+e^-\to h_1(S_1)h_2(S_2)\,X}}{d\cos\theta\, dz_1 dz_2 d^2\mathbf{P}_{1T}} = \int d^2\mathbf{p}_{\perp 2}\, d^2\mathbf{p}_{\perp 1}\,\delta^2(\mathbf{p}_{\perp 1} - \mathbf{P}_1 + \mathbf{p}_{\perp 2}\, z_{p_1}/z_{p_2})\,\frac{d\sigma^{e^+e^-\to h_1(S_1)h_2(S_2)\,X}}{d\cos\theta\, dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$

QUARK TMD-FFS FOR SPIN-1/2 HADRONS

Hadron helicity density matrix

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h}\hat{D}_{h/q,s_q}(z,\mathbf{p}_\perp) = \sum_{\lambda_q,\lambda'_q}\rho^{q}_{\lambda_q,\lambda'_q}\hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{p}_\perp)$$

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h} = \frac{1}{2}\begin{pmatrix} 1+P^h_Z & P^h_X - iP^h_Y \\ P^h_X + iP^h_Y & 1-P^h_Z \end{pmatrix}$$

QUARK TMD-FFS FOR SPIN-1/2 HADRONS

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$$\mathbf{P}^h = (P^h_T\cos\phi_h, P^h_T\sin\phi_h, P^h_Z)$$

polarization components
in the hadron helicity frame

QUARK TMD-FFS FOR SPIN-1/2 HADRONS

Hadron helicity density matrix

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h}\hat{D}_{h/q,s_q}(z,\mathbf{p}_\perp) = \sum_{\lambda_q,\lambda'_q} \rho^{q}_{\lambda_q,\lambda'_q}\hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{p}_\perp)$$

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$$\begin{aligned}
(P^h_J\,\hat{D}_{h/q,s_T}) &= \Delta\hat{D}^{h/q}_{S_J/s_T} = \hat{D}^{h/q}_{S_J/s_T} - \hat{D}^{h/q}_{-S_J/s_T} \equiv \Delta\hat{D}^{h/q}_{S_J/s_T}(z,\mathbf{p}_\perp) \\
(P^h_J\,\hat{D}_{h/q,s_z}) &= \Delta\hat{D}^{h/q}_{S_J/s_z} = \hat{D}^{h/q}_{S_J/+} - \hat{D}^{h/q}_{-S_J/+} \equiv \Delta\hat{D}^{h/q}_{S_J/+}(z,\mathbf{p}_\perp) \\
\hat{D}_{h/q,s_T} &= \hat{D}_{h/q}(z,p_\perp) + \frac{1}{2}\Delta\hat{D}_{h/s_T}(z,\mathbf{p}_\perp)
\end{aligned}$$

8 TMD-FFs

QUARK TMD-FFS FOR SPIN-1/2 HADRONS

quark helicity density matrix

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h}\,\hat{D}_{h/q,s_q}(z,\mathbf{p}_\perp) = \sum_{\lambda_q,\lambda'_q} \rho^q_{\lambda_q,\lambda'_q}\,\hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{p}_\perp)$$

$$\rho^q_{\lambda_q,\lambda'_q} = \frac{1}{2}\begin{pmatrix} 1+P^q_L & P^q_T e^{-i\phi_{s_q}} \\ P^q_T e^{i\phi_{s_q}} & 1-P^q_L \end{pmatrix}$$

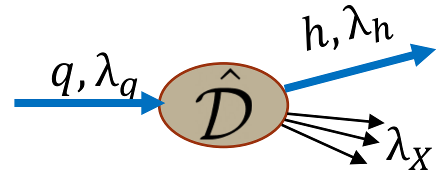
QUARK TMD-FFS FOR SPIN-1/2 HADRONS

quark helicity density matrix

product of fragm. amplitudes

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h}\,\hat{D}_{h/q,s_q}(z,\mathbf{p}_\perp) = \sum_{\lambda_q,\lambda'_q} \rho^{q}_{\lambda_q,\lambda'_q}\,\hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{p}_\perp)$$

$$\sum\!\!\!\!\!\!\int_{X,\lambda_X} \hat{D}_{\lambda_h,\lambda_X;\lambda_q}(z,\mathbf{p}_\perp)\,\hat{D}^{*}_{\lambda'_h,\lambda_X;\lambda'_q}(z,\mathbf{p}_\perp)$$



QUARK TMD-FFS FOR SPIN-1/2 HADRONS

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h}\,\hat{D}_{h/q,s_q}(z,\mathbf{p}_\perp) = \sum_{\lambda_q,\lambda'_q} \rho^q_{\lambda_q,\lambda'_q}\,\hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{p}_\perp)$$

D^{++}_{++}, D^{++}_{--}, D^{++}_{+-}, D^{+-}_{++}, D^{+-}_{+-}, D^{+-}_{-+}

real complex real (q) / imm (g)

8 real quantities
by parity

QUARK TMD-FFS FOR SPIN-1/2 HADRONS

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h}\,\hat{D}_{h/q,s_q}(z,\mathbf{p}_\perp) = \sum_{\lambda_q,\lambda'_q} \rho^q_{\lambda_q,\lambda'_q}\,\hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{p}_\perp)$$

D^{++}_{++}, D^{++}_{--}, D^{++}_{+-}, D^{+-}_{++}, D^{+-}_{+-}, D^{+-}_{-+}

real | complex | real (q) / imm (g)

8 real quantities
by parity

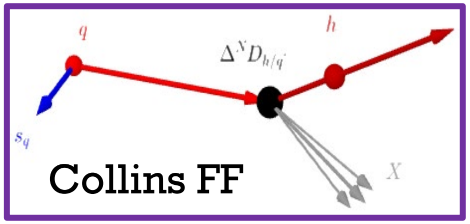
$$\begin{aligned}
\hat{D}_{h/q,s_q} &= (D^{++}_{++} + D^{++}_{--}) + 2P^q_T \mathrm{Im} D^{++}_{+-} \sin(\phi_{s_q} - \phi_h) \\
P^h_Z \hat{D}_{h/q,s_q} &= P^q_L (D^{++}_{++} - D^{++}_{--}) + 2P^q_T \mathrm{Re} D^{++}_{+-} \cos(\phi_{s_q} - \phi_h) \\
P^h_X \hat{D}_{h/q,s_q} &= 2P^q_L \mathrm{Re} D^{+-}_{++} + P^q_T (D^{+-}_{+-} + D^{+-}_{-+}) \cos(\phi_{s_q} - \phi_h) \\
P^h_Y \hat{D}_{h/q,s_q} &= -2\mathrm{Im} D^{+-}_{++} + P^q_T (D^{+-}_{+-} - D^{+-}_{-+}) \sin(\phi_{s_q} - \phi_h)
\end{aligned}$$

8 independent TMD Fragmentation Functions

		Hadron		
	Pol. States	**U**	**L**	**T**
Quark	**U**	$\hat{D}_{h/q}$		$\Delta \hat{D}_{h^\uparrow/q}$
	L		$\Delta \hat{D}^{h/q}_{S_Z/s_L}$	$\Delta \hat{D}^{h/q}_{S_X/s_L}$
	T	$\Delta \hat{D}_{h/q^\uparrow}$	$\Delta \hat{D}^{h/q}_{S_Z/s_T}$	$\Delta \hat{D}^{h/q}_{S_X/s_T}$ / $\Delta^- \hat{D}^{h/q}_{S_Y/s_T}$

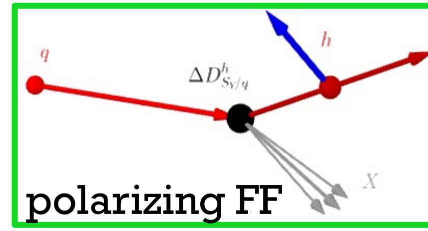
Surviving in the collinear limit

8 independent TMD Fragmentation Functions



T-odd & chiral-odd

UNIVERSAL



T-odd & chiral-even

		Hadron		
	Pol. States	**U**	**L**	**T**
Quark	**U**	$\hat{D}_{h/q}$		$\Delta\hat{D}_{h^\uparrow/q}$
	L		$\Delta\hat{D}^{h/q}_{S_Z/s_L}$	$\Delta\hat{D}^{h/q}_{S_X/s_L}$
	T	$\Delta\hat{D}_{h/q^\uparrow}$	$\Delta\hat{D}^{h/q}_{S_Z/s_T}$	$\Delta\hat{D}^{h/q}_{S_X/s_T}$ / $\Delta^-\hat{D}^{h/q}_{S_Y/s_T}$

OBSERVABLES (EXAMPLES)

Single polariz. cross sect.

$$P_X^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{4s}\sum_q e_q^2\sin^2\theta\Delta D^{h_1/q}_{S_X/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar q^\uparrow}(z_2,p_{\perp 2})\sin(2\varphi_2+\phi_{h_1})$$

$$P_Y^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{2s}\sum_q e_q^2\Big\{(1+\cos^2\theta)\Delta D^{h_1}_{S_Y/q}(z_1,p_{\perp 1})D_{h_2/\bar q}(z_2,p_{\perp 2}) + \frac{1}{2}\sin^2\theta\Delta^- D^{h_1}_{S_Y/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar q^\uparrow}(z_2,p_{\perp 2})\cos(2\varphi_2+\phi_{h_1})\Big\}$$

$$P_Z^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{4s}\sum_q e_q^2\sin^2\theta\Delta D^{h_1/q}_{S_Z/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar q^\uparrow}(z_2,p_{\perp 2})\sin(2\varphi_2+\phi_{h_1})$$

$$\cos\phi_{h_1} \simeq \frac{P_{1T}}{p_{\perp 1}}\cos(\phi_1-\varphi_2) - \frac{z_{p_1}}{z_{p_2}}\frac{p_{\perp 2}}{p_{\perp 1}} \qquad \sin\phi_{h_1} = \frac{P_{1T}}{p_{\perp 1}}\sin(\phi_1-\varphi_2)$$

OBSERVABLES (EXAMPLES)

Single polariz. cross sect.

$$P_X^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{4s}\sum_q e_q^2\sin^2\theta\Delta D_{S_X/s_T}^{h_1/q}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\sin(2\varphi_2+\phi_{h_1})$$

$$P_Y^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{2s}\sum_q e_q^2\Big\{(1+\cos^2\theta)\Delta D_{S_Y/q}^{h_1}(z_1,p_{\perp 1})D_{h_2/\bar{q}}(z_2,p_{\perp 2})$$

$$+\frac{1}{2}\sin^2\theta\Delta^- D_{S_Y/s_T}^{h_1}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\cos(2\varphi_2+\phi_{h_1})\Big\}$$

$$P_Z^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{4s}\sum_q e_q^2\sin^2\theta\Delta D_{S_Z/s_T}^{h_1/q}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\sin(2\varphi_2+\phi_{h_1})$$

convolutions

$$\mathcal{C}[wD\bar{D}] = \sum_q e_q^2\int d^2\mathbf{p}_{\perp 1}d^2\mathbf{p}_{\perp 2}\,\delta^{(2)}\left(\mathbf{p}_{\perp 1}-\mathbf{P}_{1T}+\mathbf{p}_{\perp 2}\,z_{p_1}/z_{p_2}\right)w(\mathbf{p}_{\perp 2},\mathbf{P}_{1T})D(z_1,p_{\perp 1})\bar{D}(z_2,p_{\perp 2})$$

OBSERVABLES (EXAMPLES)

Single polariz. cross sect.

$$P_X^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{4s}\sum_q e_q^2\sin^2\theta\Delta D^{h_1/q}_{S_X/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar q^\uparrow}(z_2,p_{\perp 2})\sin(2\varphi_2+\phi_{h_1})$$

$$P_Y^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{2s}\sum_q e_q^2\Big\{(1+\cos^2\theta)\Delta D^{h_1}_{S_Y/q}(z_1,p_{\perp 1})D_{h_2/\bar q}(z_2,p_{\perp 2})$$

$$+\ \frac{1}{2}\sin^2\theta\Delta^- D^{h_1}_{S_Y/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar q^\uparrow}(z_2,p_{\perp 2})\cos(2\varphi_2+\phi_{h_1})\Big\}$$

$$P_Z^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{4s}\sum_q e_q^2\sin^2\theta\Delta D^{h_1/q}_{S_Z/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar q^\uparrow}(z_2,p_{\perp 2})\sin(2\varphi_2+\phi_{h_1})$$

TENSORIAL ANALYSIS

Tensors

$$T^i = \frac{1}{P_{1T}}\int d^2\mathbf{p}_{\perp 2}\, p^i_{\perp 2}\,\Delta D^{h_1}(z_1,p_{\perp 1})\,\Delta D^{h_2}(z_2,p_{\perp 2})$$

$$T^{ij} = \frac{1}{P_{1T}^2}\int d^2\mathbf{p}_{\perp 2}\, p^i_{\perp 2}p^j_{\perp 2}\,\Delta D^{h_1}(z_1,p_{\perp 1})\,\Delta D^{h_2}(z_2,p_{\perp 2})$$

ϕ_1 dependence – S_i scalar functions

$$T^i = \frac{P^i_{1T}}{P_{1T}}S_1(P_{1T})$$

$$T^{ij} = \frac{P^i_{1T}P^j_{1T}}{P^2_{1T}}S_2(P_{1T}) + \delta^{ij}\,S_3(P_{1T})$$

STRUCTURE FUNCTIONS
(EXAMPLES)

Unpol. cross section

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 dz_2 d^2\mathbf{P}_{1T}} = \frac{3\pi\alpha^2}{2s}\left\{(1+\cos^2\theta)F_{UU} + \sin^2\theta\cos(2\phi_1)\boxed{F_{UU}^{\cos(2\phi_1)}}\right\}$$

STRUCTURE FUNCTIONS
(EXAMPLES)

Unpol. cross section

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 dz_2 d^2\mathbf{P}_{1T}} = \frac{3\pi\alpha^2}{2s}\left\{(1+\cos^2\theta)F_{UU} + \sin^2\theta\cos(2\phi_1)F_{UU}^{\cos(2\phi_1)}\right\}$$

Tensorial analysis

$$\begin{aligned}
&\cos(2\phi_1)\,F_{UU}^{\cos(2\phi_1)} \\
&= \sum_q e_q^2 \int d^2\boldsymbol{p}_{\perp 2}\frac{1}{4}\left[\frac{P_{1T}}{p_{\perp 1}}\cos(\phi_1+\varphi_2) - \frac{z_{p_1}}{z_{p_2}}\frac{p_{\perp 2}}{p_{\perp 1}}\cos(2\varphi_2)\right]\Delta^N D_{h_1/q^\uparrow}\Delta^N D_{h_2/\bar{q}^\uparrow} \\
&= \cos(2\phi_1)\,\mathcal{C}\left[\frac{1}{4}\left\{\frac{P_{1T}}{p_{\perp 1}}\hat{\boldsymbol{p}}_{\perp 2}\cdot\hat{\boldsymbol{P}}_{1T} - \frac{z_{p_1}}{z_{p_2}}\frac{p_{\perp 2}}{p_{\perp 1}}\left[2\left(\hat{\boldsymbol{p}}_{\perp 2}\cdot\hat{\boldsymbol{P}}_{1T}\right)^2 - 1\right]\right\}\Delta^N D_{h_1/q^\uparrow}\Delta^N D_{h_2/\bar{q}^\uparrow}\right] \\
&= \cos(2\phi_1)\,\mathcal{C}\left[\left\{\frac{\boldsymbol{p}_{\perp 2}\cdot\boldsymbol{P}_{1T}}{z_1 z_2} - \left[2\left(\frac{\boldsymbol{p}_{\perp 2}\cdot\hat{\boldsymbol{P}}_{1T}}{z_2}\right)^2 - \frac{p_{\perp 2}^2}{z_2^2}\right]\right\}\frac{H_1^\perp \bar{H}_1^\perp}{M_{h_1}M_{h_2}}\right],
\end{aligned}$$

Access to the Collins function

in agreement with *Boer, Jakob, Mulders 1997 - Pitonyak, Schlegel, Metz 2014*

TRANSVERSE POLARIZATION

$$\mathbf{P}_T^{h_1} = P_X^{h_1}\,\hat{\mathbf{X}}_{h_1} + P_Y^{h_1}\,\hat{\mathbf{Y}}_{h_1} = P_{x_L}^{h_1}\,\hat{\mathbf{x}}_L + P_{y_L}^{h_1}\,\hat{\mathbf{y}}_L = P_T^{h_1}(\cos\phi_{S_1}^L\,\hat{\mathbf{x}}_L + \sin\phi_{S_1}^L\,\hat{\mathbf{y}}_L)$$

From the helicity to the lab frame

$$P_T^{h_1}\,\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 dz_2 d^2\mathbf{P}_{1T}} = \frac{3\pi\alpha^2}{4s}\Big\{\Big(1+\cos^2\theta\Big)\sin(\phi_1-\phi_{S_1}^L)F_{TU}^{\sin(\phi_1-\phi_{S_1}^L)} + \sin^2\theta\Big(\sin(\phi_1+\phi_{S_1}^L)F_{TU}^{\sin(\phi_1+\phi_{S_1}^L)} + \sin(3\phi_1-\phi_{S_1}^L)F_{TU}^{\sin(3\phi_1-\phi_{S_1}^L)}\Big)\Big\}$$

TRANSVERSE POLARIZATION

$$\mathbf{P}_T^{h_1} = P_X^{h_1}\,\hat{\mathbf{X}}_{h_1} + P_Y^{h_1}\,\hat{\mathbf{Y}}_{h_1} = P_{x_L}^{h_1}\,\hat{\mathbf{x}}_L + P_{y_L}^{h_1}\,\hat{\mathbf{y}}_L = P_T^{h_1}(\cos\phi_{S_1}^L\,\hat{\mathbf{x}}_L + \sin\phi_{S_1}^L\,\hat{\mathbf{y}}_L)$$

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$F_{TU}^{\sin(\phi_1-\phi_{S_1}^L)}$ → $\dfrac{D_{1T}^{\perp}\bar{D}_1}{M_{h_1}}$ Polarizing FF

$F_{TU}^{\sin(\phi_1+\phi_{S_1}^L)}$ → $\dfrac{H_1\bar{H}_1^{\perp}}{M_{h_2}}$

$F_{TU}^{\sin(3\phi_1-\phi_{S_1}^L)}$ → $\dfrac{H_{1T}^{\perp}\bar{H}_1^{\perp}}{2M_{h_1}^2 M_{h_2}}$

TRANSVERSE POLARIZATION

Projection along the normal to the hadron plane [lab frame] (phenom.)

$$\hat{\boldsymbol{n}} \equiv (\cos\phi_n, \sin\phi_n, 0) = \frac{-\boldsymbol{P}_2 \times \boldsymbol{P}_1}{|\boldsymbol{P}_2 \times \boldsymbol{P}_1|} = -\sin\phi_1 \hat{\boldsymbol{x}}_L + \cos\phi_1 \hat{\boldsymbol{y}}_L \,,$$

$$P_n^{h_1} \equiv \mathbf{P}^{h_1} \cdot \hat{\mathbf{n}} = \mathbf{P}_T^{h_1} \cdot \hat{\mathbf{n}}$$

$$\phi_n = \phi_1 + \frac{\pi}{2}$$

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$$P_n^{h_1} \equiv \mathbf{P}^{h_1} \cdot \hat{\mathbf{n}} = \mathbf{P}_T^{h_1} \cdot \hat{\mathbf{n}}$$

$$\phi_n = \phi_1 + \frac{\pi}{2} \quad \Rightarrow \quad \phi_{S_1}^L = \phi_n$$

$$P_n^{h_1} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{d\cos\theta dz_1 dz_2 d^2\mathbf{P}_{1T}} = \frac{3\pi\alpha^2}{4s} \Big\{ \Big(1 + \cos^2\theta\Big) \Big[- F_{TU}^{\sin(\phi_1 - \phi_{S_1}^L)} \Big] + \sin^2\theta \Big(F_{TU}^{\sin(\phi_1 + \phi_{S_1}^L)} - F_{TU}^{\sin(3\phi_1 - \phi_{S_1}^L)} \Big) \cos(2\phi_1) \Big\}$$

TRANSVERSE POLARIZATION

Projection along the normal to the hadron plane [lab frame] (phenom.)

$$P_n^{h_1}\,\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1dz_2d^2\mathbf{P}_{1T}} = \frac{3\pi\alpha^2}{4s}\Big\{\Big(1+\cos^2\theta\Big)\Big[-F_{TU}^{\sin(\phi_1-\phi^L_{S_1})}\Big] + \sin^2\theta\Big(F_{TU}^{\sin(\phi_1+\phi^L_{S_1})} - F_{TU}^{\sin(3\phi_1-\phi^L_{S_1})}\Big)\cos(2\phi_1)\Big\}$$

By integrating over P_{1T}, the $\cos(2\phi_1)$ term is washed out, leaving

TRANSVERSE POLARIZATION

Projection along the normal to the hadron plane [lab frame] (phenom.)

$$P_n^{h_1}\,\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta\, dz_1 dz_2} = \frac{3\pi\alpha^2}{4s}\Big\{\Big(1+\cos^2\theta\Big)\Big[-F_{TU}^{\sin(\phi_1-\phi_{S_1}^L)}\Big]$$

$$P_n^{h_1}(z_1,z_2) = -\frac{F_{TU}^{\sin(\phi_1-\phi_{S_1}^L)}}{F_{UU}}\,.$$

Direct access to the polarizing FF

PHENOMENOLOGY

- First attempt to describe $P_T(\Lambda)$ data in unpolarized hadron-hadron collisions within a phenomenological TMD model *Anselmino, Boer, UD, Murgia 2001*
- Twist-three approach: *Kanazawa, Koike 2001*

2019

- Belle data for $P_T(\Lambda)$ in $e^+e^- \to \Lambda^\uparrow h + X$ and $e^+e^- \to \Lambda^\uparrow(\text{jet}) + X$

  Guan et al. (Belle Coll.) 2019

GAUSSIAN MODEL

$$D_{h/q}(z, p_\perp) \quad = \quad D_{h/q}(z) \, \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

$$\langle p_\perp^2 \rangle_{\rm pol} = \frac{M_{\rm pol}^2}{M_{\rm pol}^2 + \langle p_\perp^2 \rangle} \langle p_\perp^2 \rangle$$

$$\Delta D_{h^\uparrow/q}(z, p_\perp) \quad = \quad \Delta D_{h^\uparrow/q}(z) \frac{\sqrt{2e}\, p_\perp}{M_{\rm pol}} \, \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle_{\rm pol}}}{\pi \langle p_\perp^2 \rangle}$$

$$\Delta D_{\Lambda^\uparrow/q}(z) = N_q z^{a_q} (1-z)^{b_q} \, \frac{(a_q + b_q)^{(a_q+b_q)}}{a_q^{a_q} b_q^{b_q}} D_{\Lambda/q}(z)$$

FIT OF BELLE DATA

No isospin symmetry constraint

N_u	$= 0.67^{+0.33}_{-0.32}$	N_d	$= -0.65^{+0.32}_{-0.35}$
N_s	$= -1.00^{+0.02}_{-0.00}$	$N_{\rm sea}$	$= -0.40^{+0.18}_{-0.28}$
a_s	$= 2.17^{+0.76}_{-0.71}$		
b_u	$= 3.36^{+3.15}_{-2.31}$	$b_{\rm sea}$	$= 2.14^{+3.06}_{-1.92}$
$\langle p_\perp^2 \rangle_{\rm pol}$	$= 0.056^{+0.026}_{-0.019}$ GeV^2		

$\Lambda + \pi/K$ data fit : $\chi^2_{\rm dof} = 1.26$

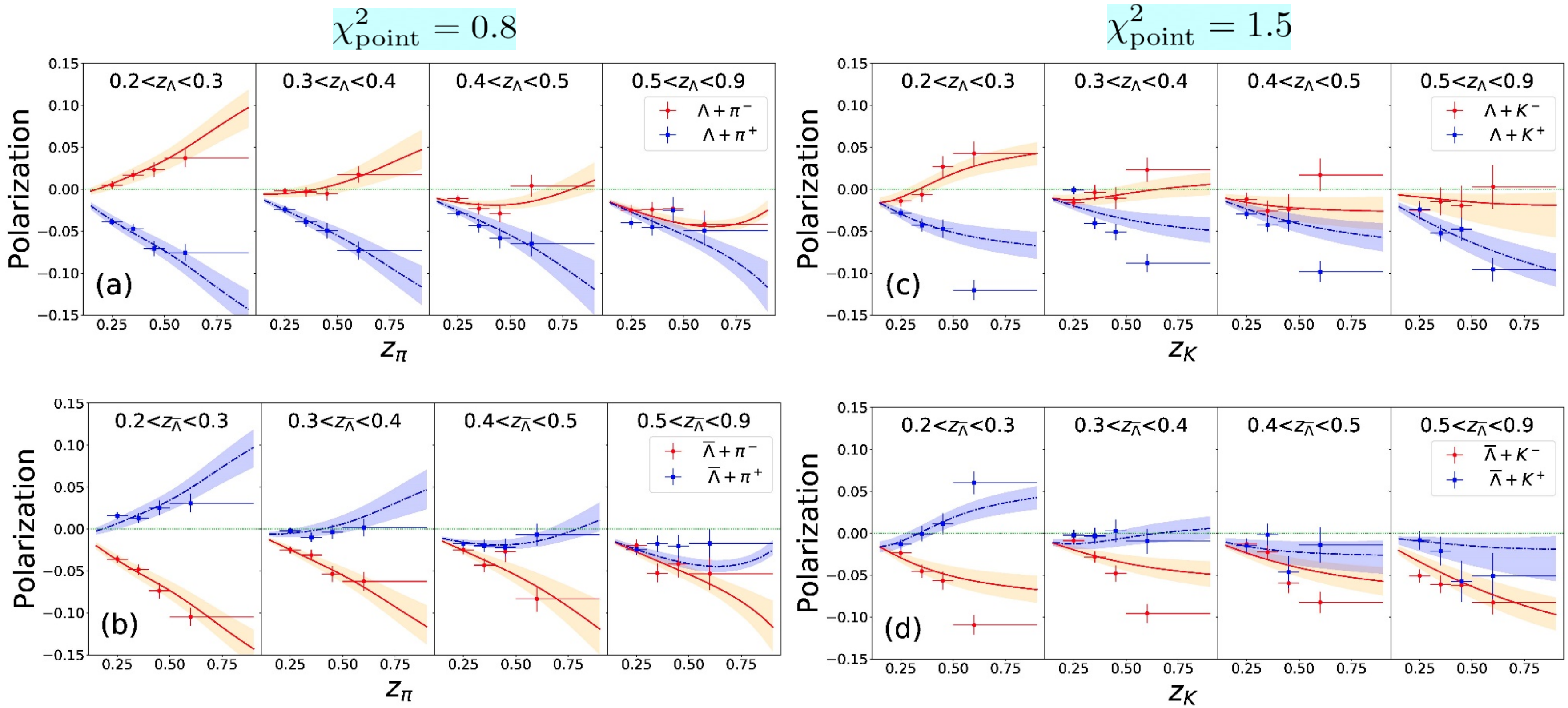
- ✓ 8 parameter fit
- ✓ No TMD evolution (fixed-scale analysis)

Similar analysis
Callos, Kang, Terry 2020

isospin symmetry constraint

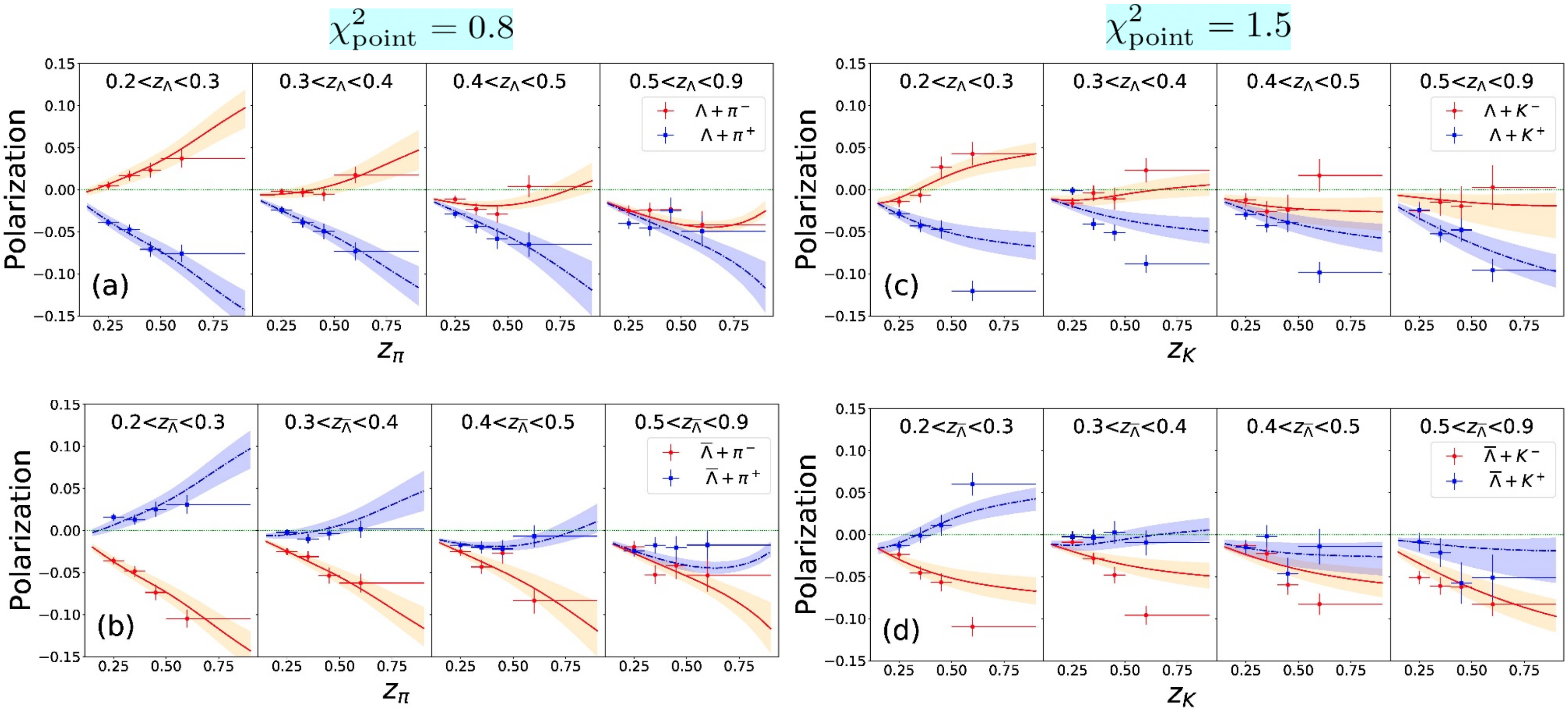
Chen, Liang, Pan, Song, Wei 2021

Associated production



- simpler fits with only two pFFs (*u*=*d*, *s* and/or no sea) ➡ $\chi^2 \sim 2$
- kaon, some tension (unpol FFs?, parametriz. of polFF?)

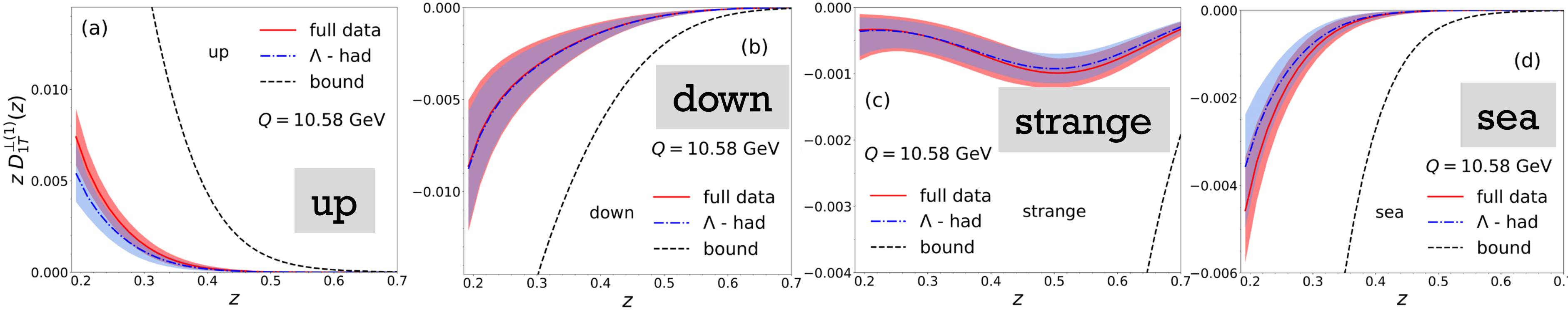
Associated production



- simpler fits with only two pFFs (u=d, s and/or no sea) ➡ $\chi^2 \sim 2$
- kaon, some tension (unpol FFs?, parametriz. of polFF?)
- u, d, s AND isospin symmetry $\rightarrow \chi^2 \sim 2$
- u, d, s plus charm w/wo isospin symm. $\rightarrow \chi^2 \sim 1.27$ [& better χ^2 (K)]

First moments of the polarizing FF

$$D_{1T}^{\perp(1)}(z) = \int d^2\mathbf{p}_\perp \frac{p_\perp}{2zm_h} \Delta D_{h^\uparrow/q}(z, p_\perp)$$



Full-data (including Λ (jet)) and *Λ-had* extractions
Consistency: central lines and (overlapping) uncertainty bands!!!

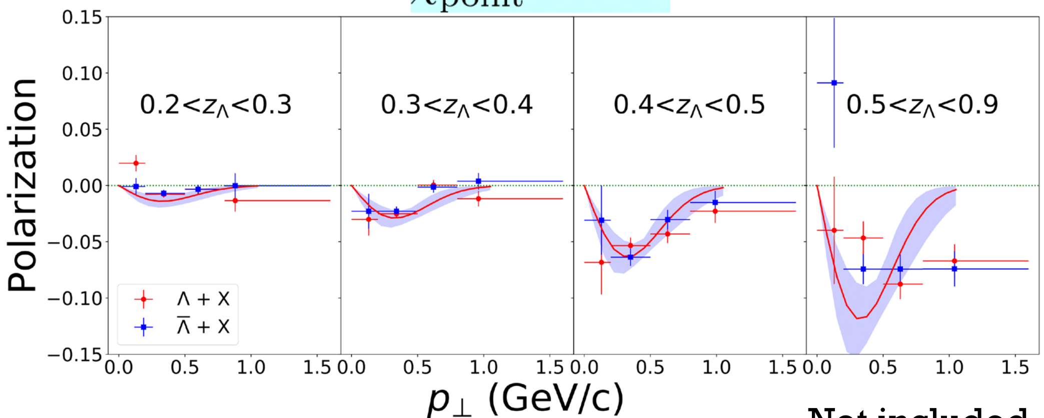
- ❑ $e^+e^- \rightarrow h_1(P_1)$ jet X within a simplified TMD scheme

$$\mathcal{P}_T(z_1, p_{\perp 1}) = \frac{\sum_q e_q^2 \, \Delta D_{h_1^\uparrow/q}(z_1, p_{\perp 1})}{\sum_q e_q^2 \, D_{h_1/q}(z_1, p_{\perp 1})}$$

orthogonal to the thrust plane

Inclusive Λ production [full-data analysis]

$\chi^2_{\rm point} = 2.75$



Not included

Full-data fit leads to $\chi^2_{\rm dof} = 1.94$

- Less good description…but not so bad!
- Theory
  - ✓ Polarization = 0 at $p_\perp = 0$
  - ✓ $P(\Lambda) = P(\bar{\Lambda})$
- not *clearly* visible in the data
- Reanalysis within TMD factorization [next talk by M. Zaccheddu]

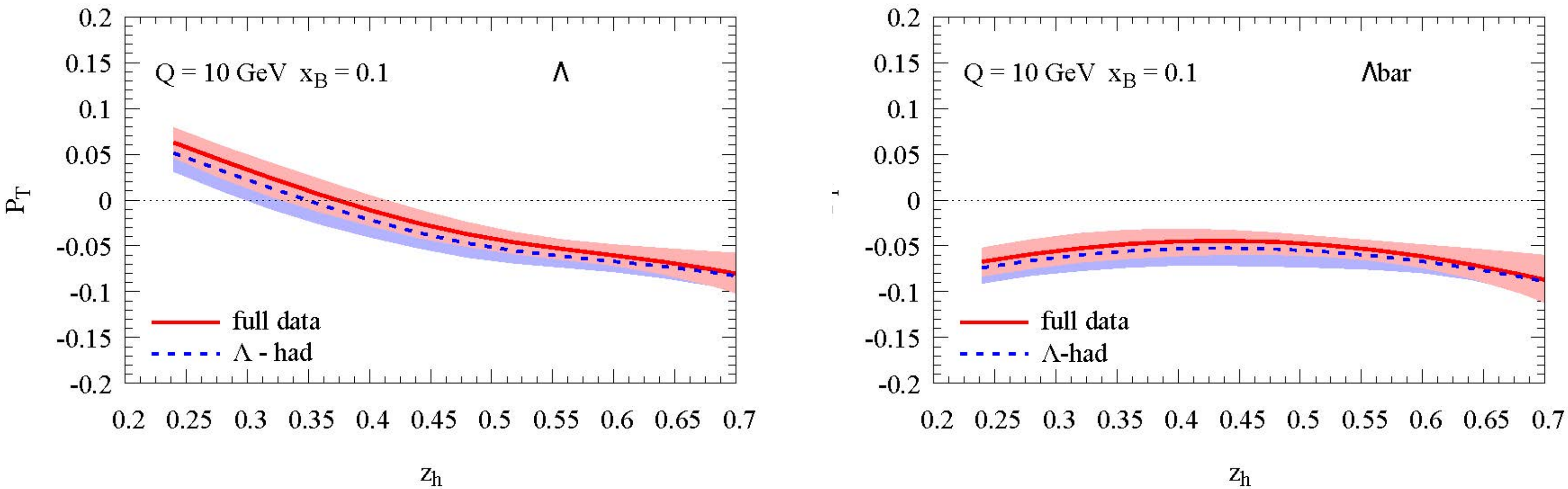
TRANSVERSE Λ POLARIZATION IN SIDIS

$$P_T(x_B, z_h) = \frac{\sqrt{2e\pi}}{2M_p}\frac{\langle p_\perp^2\rangle_{pol}^2}{\langle p_\perp^2\rangle}\frac{1}{\sqrt{\langle p_\perp^2\rangle_{pol} + \xi_p^2\langle k_\perp^2\rangle}} \times \frac{\sum_q e_q^2 f_{q/p}(x_B)\Delta D_{\Lambda^\uparrow/q}(z_h)}{\sum_q e_q^2 f_{q/p}(x_B) D_{\Lambda/q}(z_h)}$$

Transverse w.r.t. the target-Λ plane

$$\xi_p = z_h\left(1 - \frac{m_h^2}{z_h^2 Q^2}\frac{x_B}{1-x_B}\right)$$

EIC kinematics



CONCLUDING REMARKS

- General helicity formalism for $e^+e^- \rightarrow h_1\ h_2 + X$ AND LT quark (and gluon) TMD-FFs for spin-1/2 hadrons
- Fit of Belle e^+e^- data on transverse Λ polarization and extraction of the polarizing FF
- Polarizing FF
  - ✓ three different valence polFFs (up, down and strange) + sea
  - ✓ relative sign between up and down polFFs
- SIDIS measurements @EIC important to complement these findings and to check the universality of the polFF

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NEXT

- TMD analysis of inclusive production and role of charm and isospin symmetry (ongoing)

THANKS for the ATTENTION

BACK-UP SLIDES

FIT OF BELLE DATA

Data selection:

- $\Lambda + \pi/K$: $z_{\pi,K} = [0.5 - 0.9]$ excluded → 96/128 data points
- Λ(jet) : $z_\Lambda = [0.5 - 0.9]$ excluded → 24/32 data points

- **Unpol. FFs**: DSS07 for π/K , AKK08 for Λ

$$\langle p_\perp^2 \rangle = 0.2 \ \text{GeV}^2$$

$$D_{\Lambda/\bar{q}}(z_p) = (1 - z_p) D_{\Lambda/q}(z_p)$$

flavour separation

- **Polarizing FF**

$$\Delta D_{\Lambda^\uparrow/q}(z) = N_q z^{a_q} (1 - z)^{b_q} \frac{(a_q + b_q)^{(a_q + b_q)}}{a_q^{a_q} b_q^{b_q}} D_{\Lambda/q}(z)$$

$|N_q| \leq 1$ Positivity bound

✓ No TMD evolution (fixed-scale analysis)

CORRESPONDENCE WITH AMSTERDAM NOTATION

Projection of the correlator $\Delta(z, \mathbf{k}_T)$

$(-z\,\mathbf{k}_T) \;=\; \mathbf{p}_\perp$

$$D_1(z,p_\perp) = D^{++}_{++} + D^{++}_{--} = D_{h/q}$$

$$\frac{p_\perp}{zM_h} D^\perp_{1T}(z,p_\perp) = -2\,\mathrm{Im}D^{+-}_{++} = \Delta D^h_{S_Y/q} = \Delta D_{h^\uparrow/q}$$

Polarizing FF

$$G_{1L}(z,p_\perp) = D^{++}_{++} - D^{++}_{--} = \Delta D^{h/q}_{S_Z/s_L}$$

$$\frac{p_\perp}{zM_h} G_{1T}(z,p_\perp) = 2\,\mathrm{Re}D^{++}_{+-} = \Delta D^{h/q}_{S_X/s_L}$$

$$\frac{p_\perp}{zM_h} H^\perp_{1L}(z,p_\perp) = 2\,\mathrm{Re}D^{+-}_{++} = \Delta D^{h/q}_{S_Z/s_T}$$

$$\frac{2p_\perp}{zM_h} H^\perp_1(z,p_\perp) = 4\,\mathrm{Im}D^{++}_{+-} = \Delta D_{h/q^\uparrow}$$

Collins FF

$$H_1(z,p_\perp) = D^{+-}_{+-}$$

$$\frac{p^2_\perp}{z^2M^2_h} H^\perp_{1T}(z,p_\perp) = 2\,D^{+-}_{-+}$$

Extension of the Trento Convention results

Bacchetta, UD, Diehl, Miller 2004