

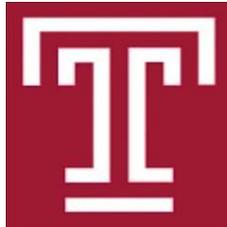
# First global extraction of the worm-gear TMD $g_{1T}$



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BNL & Temple U.

22 October 2021



In Collaboration with:

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**Gregory Penn** (Yale U./ Temple U.)

**Daniel Pitonyak** (Lebanon Valley College)



SPIN2021



*Matsue, Japan*



# Outline



- **Introduction**
- **Theoretical predictions**
- **Overview of fitting procedure: Monte-Carlo technique**
- **Final results and comparison with theoretical predictions**
- **Summary/ Outlook**



# Introduction

Quark polarization  $\longrightarrow$

Nucleon  
polarization



	U	L	T
U	$f_1$		$h_1^\perp$
L		$g_{1L}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	$h_1, h_{1T}^\perp$



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Nucleon polarization



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U	$f_1$		$h_1^\perp$
L		$g_{1L}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	$h_1, h_{1T}^\perp$

“Worm-gear TMD”: One of the least known TMDs





# Extraction of $g_{1T}$ TMD from HERMES, COMPASS & JLab data

SB, Kang, Metz, Penn, Pitonyak, arXiv: 2110.10253 (2021)



## Theoretical predictions

1.

2.



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## Theoretical predictions

1. Large  $N_c$  analysis: (Pobylitsa, hep-ph/ 0301236)

$$g_{1T}^u(x, \vec{k}_\perp^2) = - g_{1T}^d(x, \vec{k}_\perp^2) + 1/N_c\text{-suppressed}$$

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$$\overset{\text{Large-}N_c}{\text{approx.}} g_{1T}^u(x, \vec{k}_\perp^2) \approx \ominus g_{1T}^d(x, \vec{k}_\perp^2) + 1/N_c\text{-suppressed}$$

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### 2. Wandzura-Wilczek-type (WW-type) relation: (Avakian et. al., 0709.3253, Kanazawa et. al., 1512.07233, ... )

$$g_{1T}^{(1)q}(x) \equiv \int d^2 \vec{k}_\perp \left( \frac{k_\perp^2}{2M^2} \right) g_{1T}^q(x, \vec{k}_\perp^2) \stackrel{\text{EOM}}{=} x \int_x^1 \frac{dy}{y} g_1^q(y) + x \tilde{g}_T^q(x)$$



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**Semi-inclusive Deep Inelastic Scattering:**  $\ell(l) + N(P, S) \rightarrow \ell'(l') + h(P_h, S_h) + X$

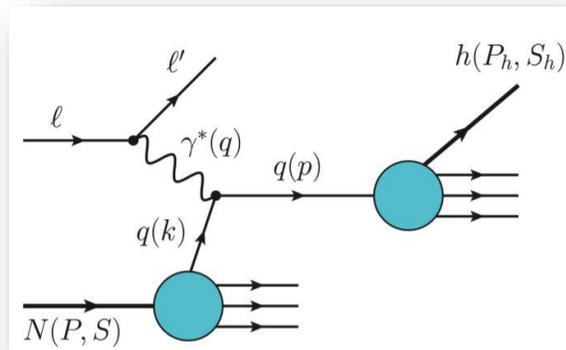


Fig. courtesy:  
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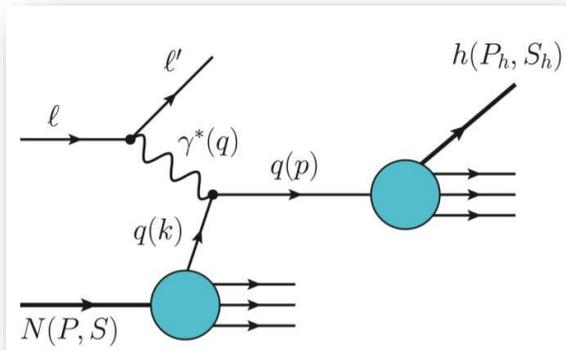


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**Model-independent decomposition of cross-section: (Bacchetta et. al. 2007, ...)**

$$\frac{d\sigma}{dx dy d\phi_S dz_h d\phi_h dP_{hT}^2} = \frac{\alpha_{\text{em}}^2}{x y Q^2} \left\{ \left(1 - y + \frac{1}{2}y^2\right) F_{UU} + \lambda_l |\vec{S}_\perp| y \left(1 - \frac{1}{2}y\right) \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \dots \right\}$$

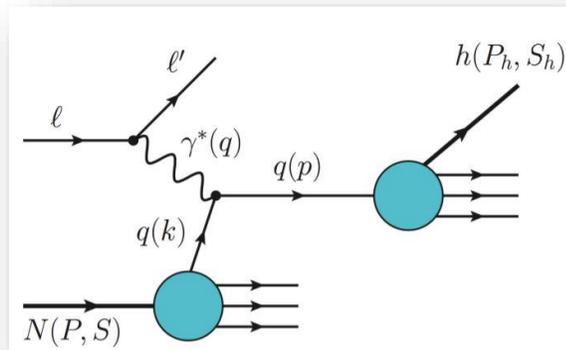


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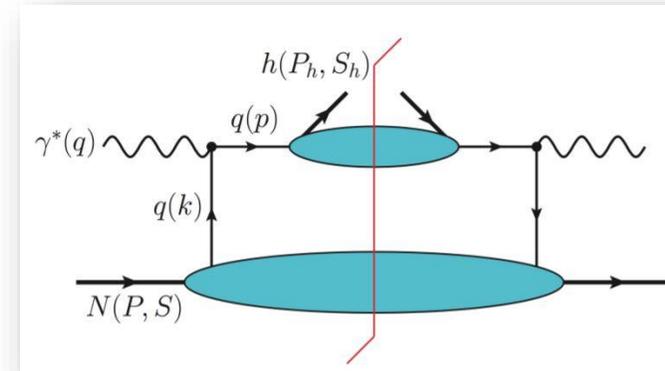


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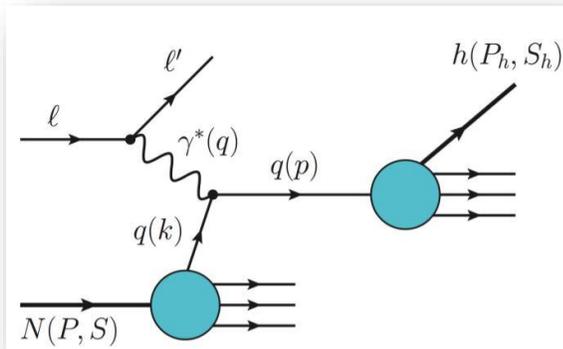


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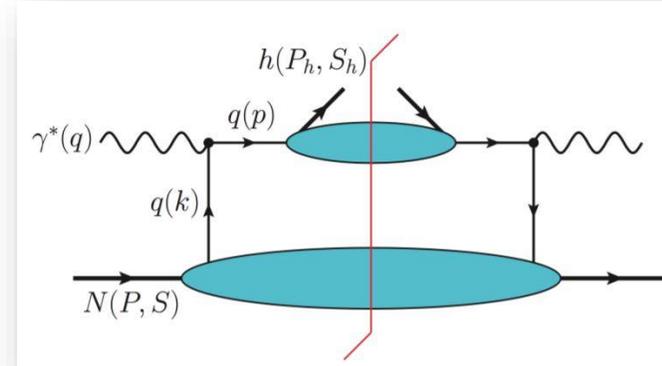


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**Connection between structure functions and TMDs: (Bacchetta et. al. 2007, ...)**

$$F_{UU} = C \left[ f_1(x, \vec{k}_\perp^2) D_1(z, \vec{P}_\perp^2) \right] \quad F_{LT}^{\cos(\phi_h - \phi_S)} = C \left[ \frac{\vec{P}_{hT} \cdot \vec{k}_\perp}{|\vec{P}_{hT}| M} g_{1T}(x, \vec{k}_\perp^2) D_1(z, \vec{P}_\perp^2) \right]$$

$$C[w f D] = x \sum_q e_q^2 \int d^2 \vec{k}_\perp \int d^2 \vec{P}_\perp \delta^{(2)}(z \vec{k}_\perp + \vec{P}_\perp - \vec{P}_{hT}) w(\vec{k}_\perp, \vec{P}_\perp) f^q(x, \vec{k}_\perp^2) D^q(z, \vec{P}_\perp^2)$$



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## Parameterization of $g_{1T}$



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**Gaussian ansatz:**

$$g_{1T}^q(x, \vec{k}_\perp^2, Q^2) = g_{1T}^{(1)q}(x, Q^2) \frac{2M_N^2 e^{-\frac{\vec{k}_\perp^2}{\pi \langle k_\perp^2 \rangle}}}{\pi (\langle k_\perp^2 \rangle)^2}$$

$$q = (u, d)$$



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**where,** 
$$g_{1T}^{(1)}(x, Q^2) = \frac{n}{\int_0^1 dy y^{\alpha+1} (1-y)^\beta f_1(y, Q_0^2)} x^\alpha (1-x)^\beta f_1(x, Q^2)$$



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- **Fix TMD width:**  $\frac{\langle k_{\perp}^2 \rangle|_{g_1}}{\langle k_{\perp}^2 \rangle|_{f_1}} \approx 0.76$  **Lattice QCD Hagler et. al., hep-lat/ 0908.1283 (See also Bastami et. al., 1807.10606 that uses this idea to get  $\langle k_{\perp}^2 \rangle|_{g_{1T}}$ )**



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$$\approx \frac{\langle k_{\perp}^2 \rangle|_{g_{1T}}}{\langle k_{\perp}^2 \rangle|_{f_1}} \quad \xrightarrow{\langle k_{\perp}^2 \rangle|_{f_1} = 0.53}$$

Cammarota et. al., arXiv 2002.08384



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$$\therefore \langle k_{\perp}^2 \rangle|_{g_{1T}} \approx 0.40$$



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Helicity & unpolarized PDFs have similar large-x behavior

$$g_1^q(x)|_{x \rightarrow 1} \propto f_1^q(x)|_{x \rightarrow 1}$$

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• Fix TMD width:

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• Set alphas equal:

$$\alpha^d = \alpha^u$$

• Fix beta from WW approximation:

$$g_{1T}^{(1)q}(x) \stackrel{x \rightarrow 1}{\approx} (1-x) f_1^q(x) \longrightarrow \therefore \beta^u = \beta^d = 1$$



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$$g_{1T}^q(x, \vec{k}_{\perp}^2, Q^2)$$

**3 free parameters:**

$$q = (u, d)$$

$$n^u, n^d, \alpha$$

where,  $g_{1T}^{(1)q}(x, Q^2)$

$$(1-x)^{\beta} f_1(x, Q^2)$$

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$$\langle k_{\perp}^2 \rangle \Big|_{g_{1T}} \approx 0.40$$

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## Experimental data

Dataset	Target	Identified hadron	No. of points
<b>HERMES</b>	$p$	$\pi^+$	26
Airapetian et. al., arXiv: 2007.07755		$\pi^-$	26
		$\pi^0$	8
<b>COMPASS</b>	$p$	$h^+ \approx (\pi^+, K^+)$	33
Parsamyan, PoS: QCDEV2017		$h^- \approx (\pi^-, K^-)$	31
<b>JLab</b>	$n$	$\pi^+$	2
Huang, arXiv: 1108.0489		$\pi^-$	2

**Cut:**

$$\frac{q_T}{Q} < 0.50$$

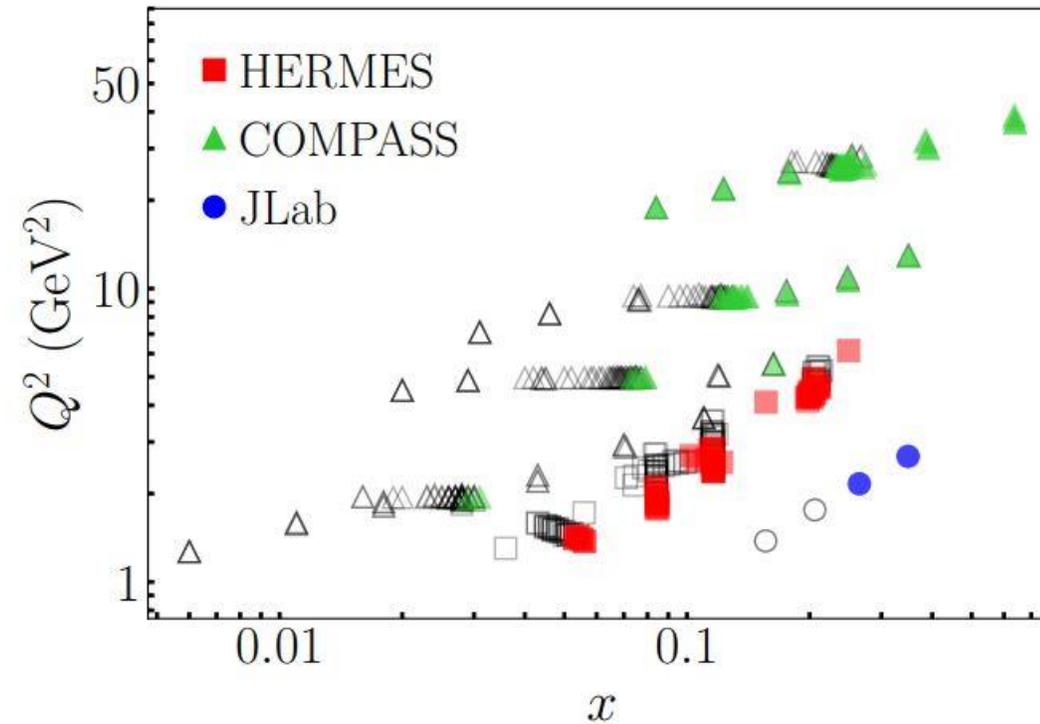


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## Experimental data

Dataset
<b>HERMES</b> Airapetian et. al., arXiv: 2007.07755
<b>COMPASS</b> Parsamyan, PoS: QCDEV2017
<b>JLab</b> Huang, arXiv: 1108.0489



Cut:

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**Fitting procedure: Monte-Carlo technique**



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**Theory**



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## **Fitting procedure: Monte-Carlo technique**

**Theory**

**Fit to exp. data**





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## Fitting procedure: Monte-Carlo technique

Theory

Fit to exp. data

Minimize:

$$\chi^2 = \sum_{H+C+J} \frac{(\text{exp. data} - \text{theory})^2}{(\text{exp. error})^2}$$



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## Fitting procedure: Monte-Carlo technique

Theory

Fit to exp. data

Minimize **weighted chi-squared**:

$$\chi^2 = \sum_{\text{H+C}} \frac{(\text{exp. data} - \text{theory})^2}{(\text{exp. error})^2} + w \sum_{\text{J}} \frac{(\text{exp. data} - \text{theory})^2}{(\text{exp. error})^2}$$

(Echevarria, Kang, Terry, arXiv: 2009.10710)

Give JLab data weight similar to  
HERMES & COMPASS data



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## **Fitting procedure: Monte-Carlo technique**

**Theory**

**Fit to exp. data**

**Generate pseudo-data**

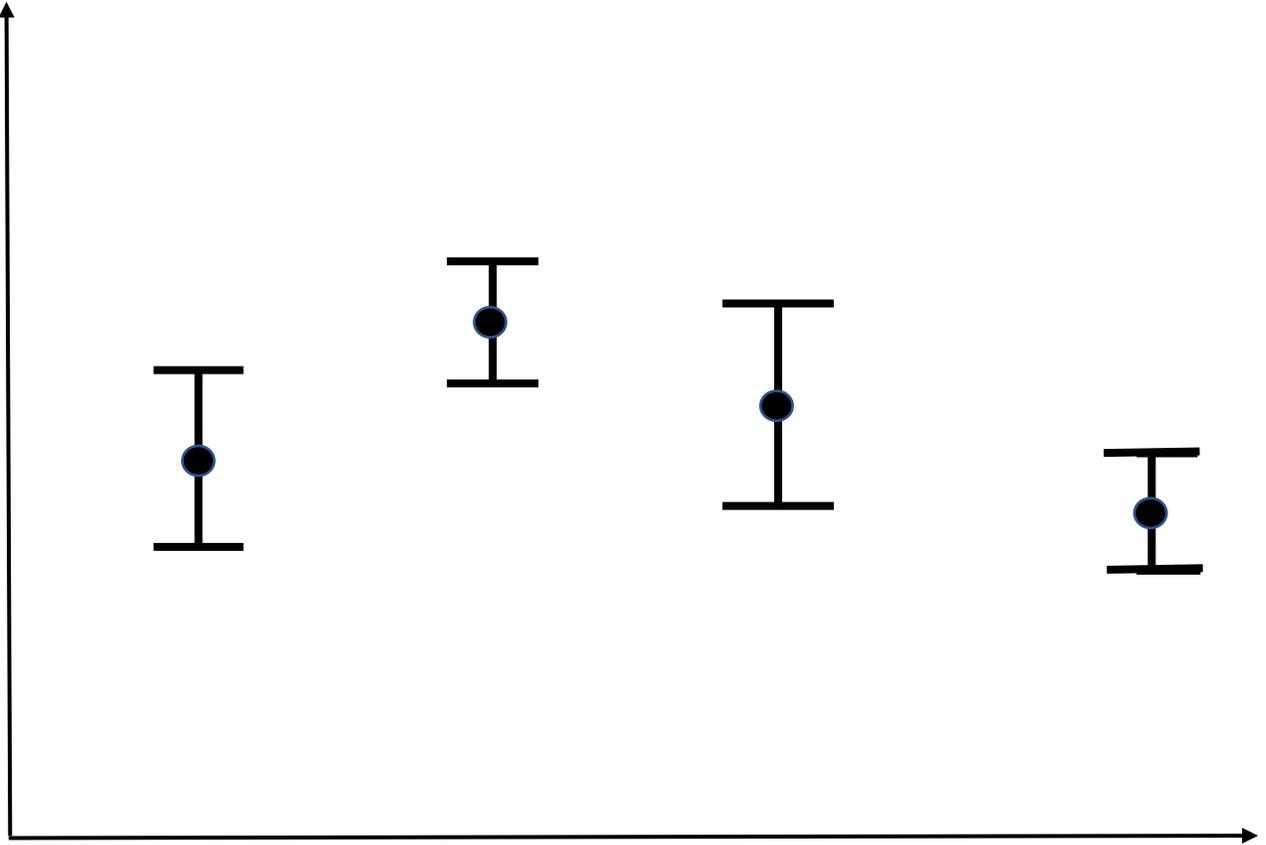
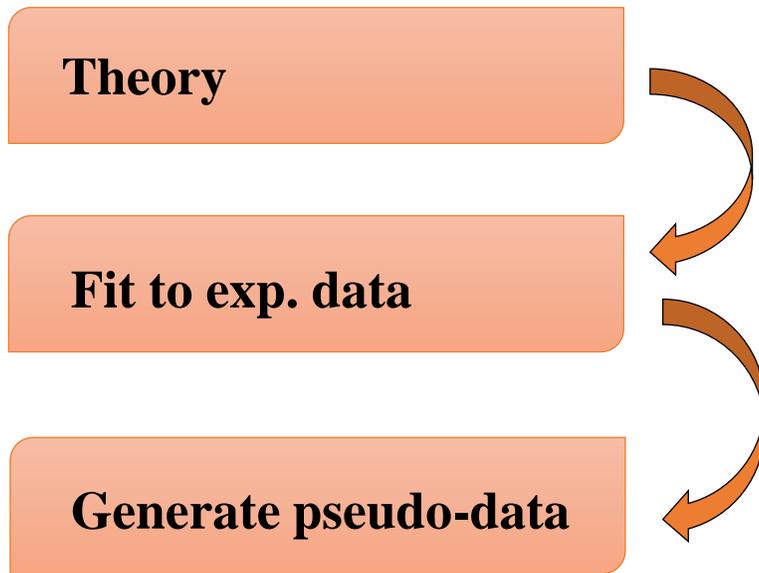




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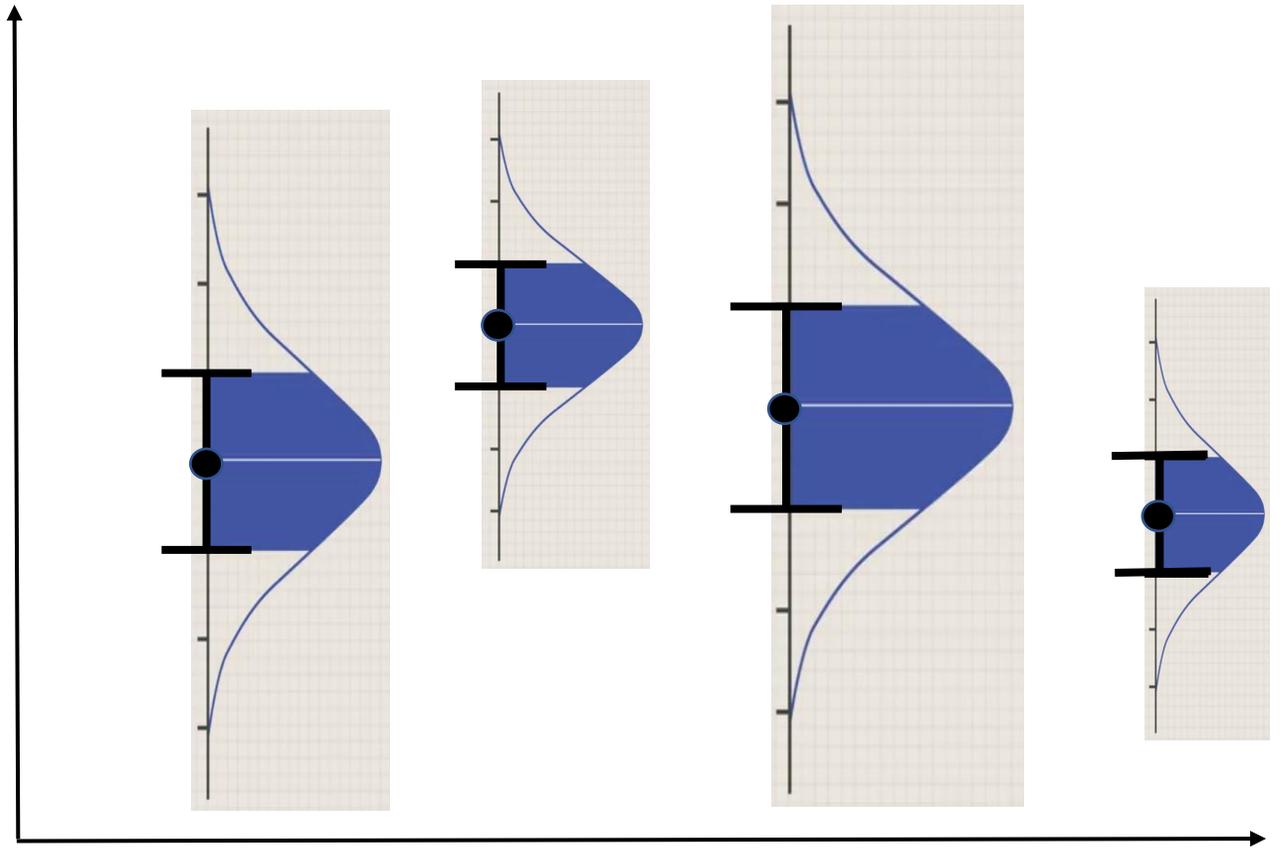
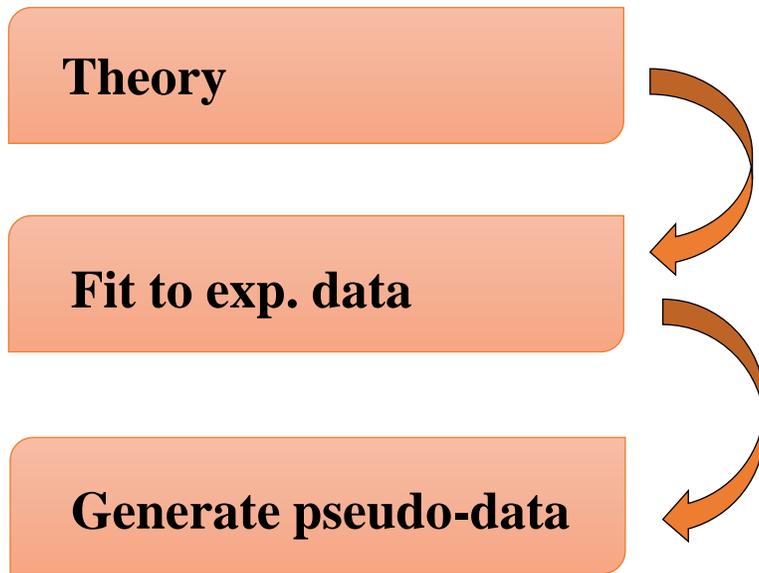


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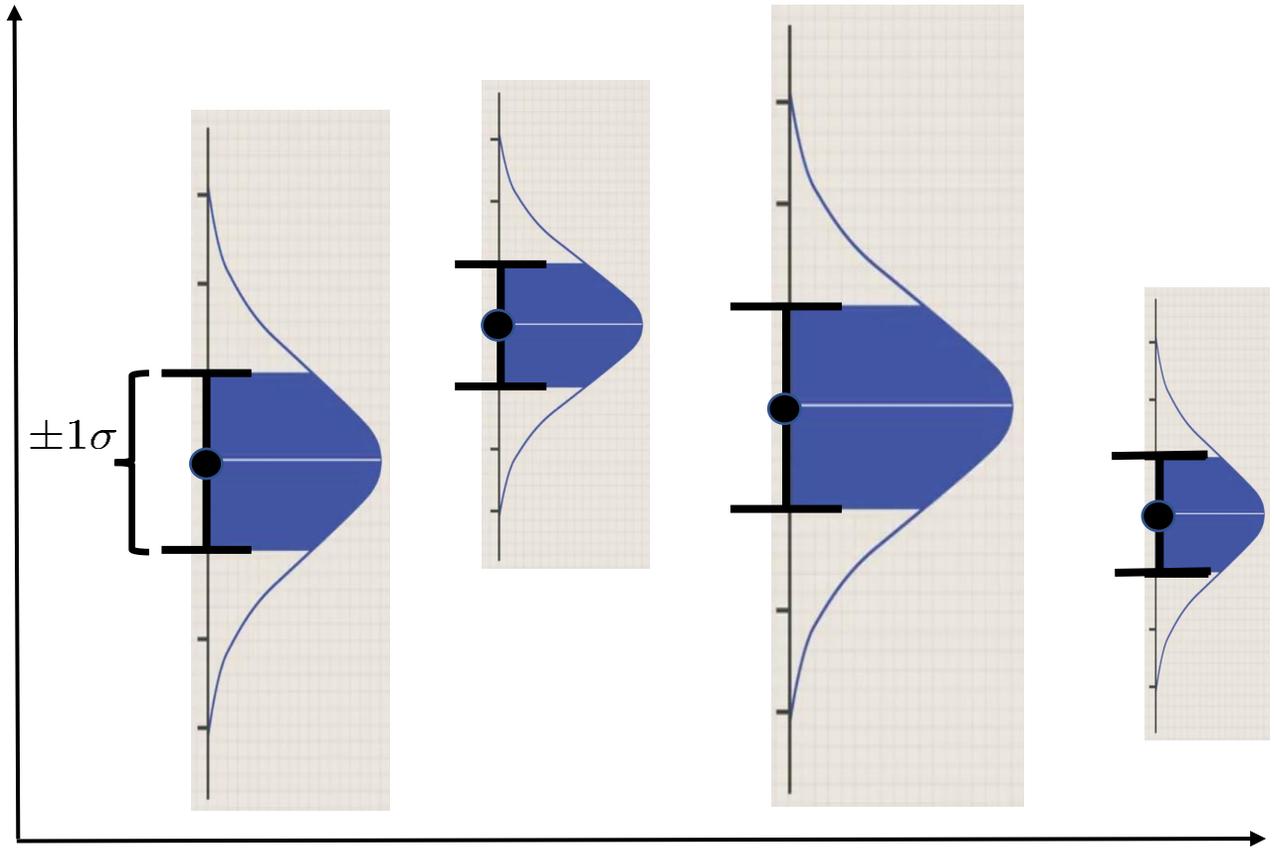
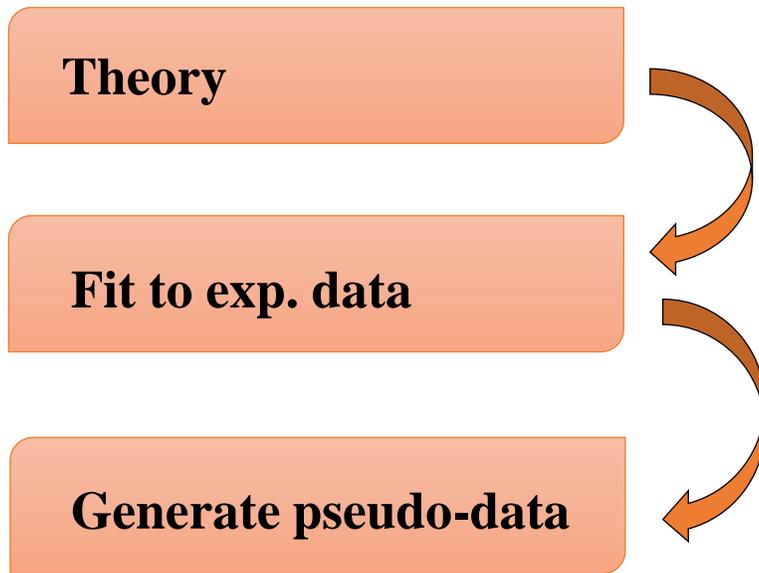


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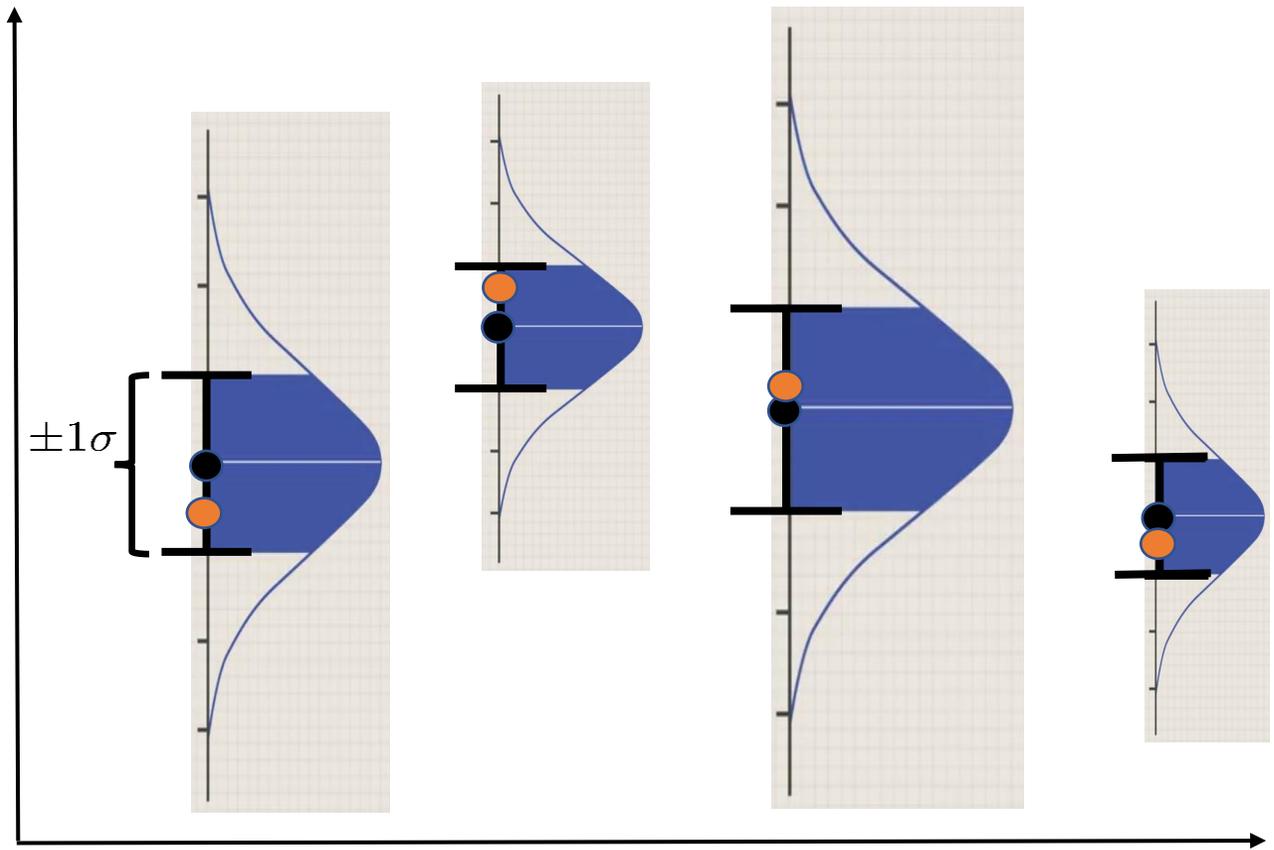
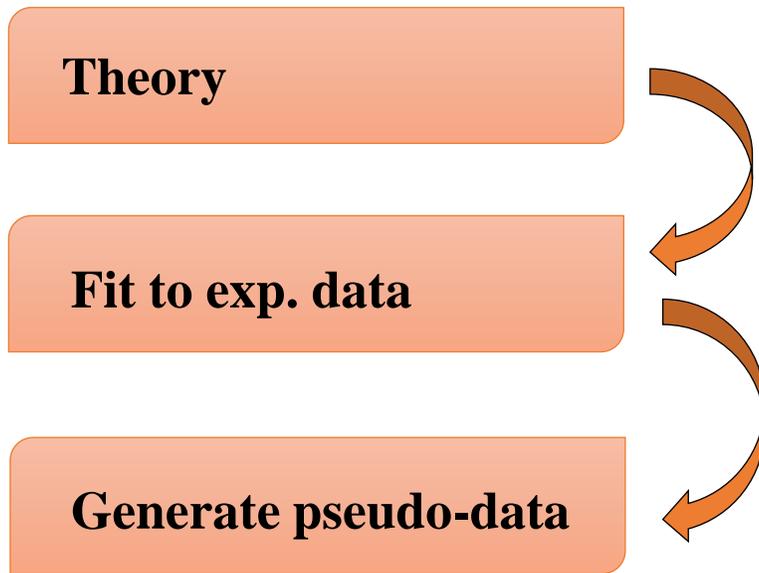


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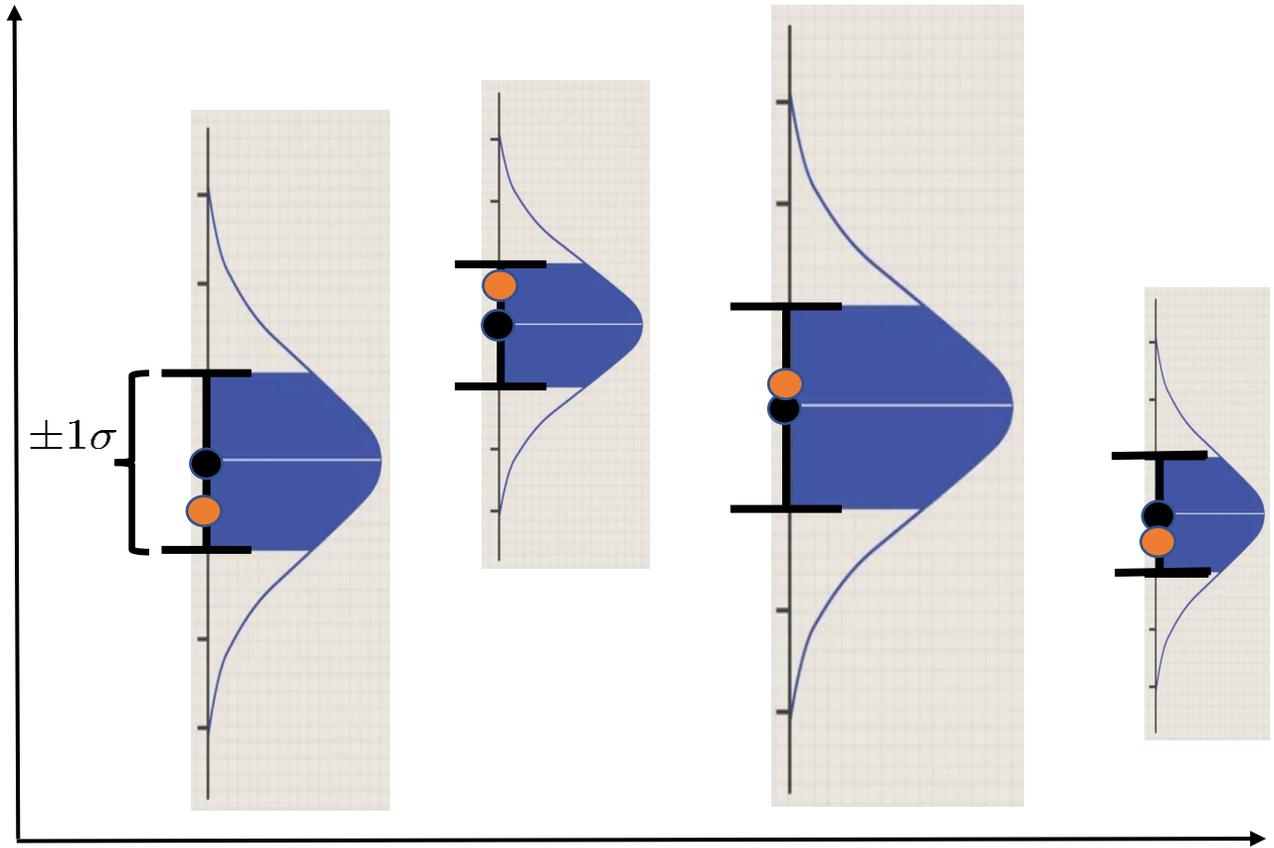
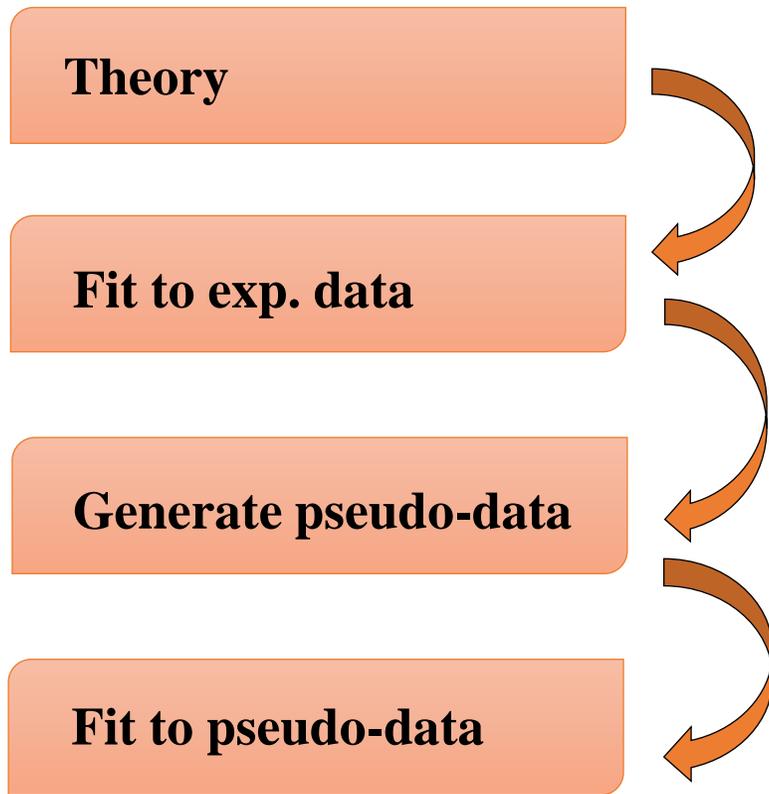


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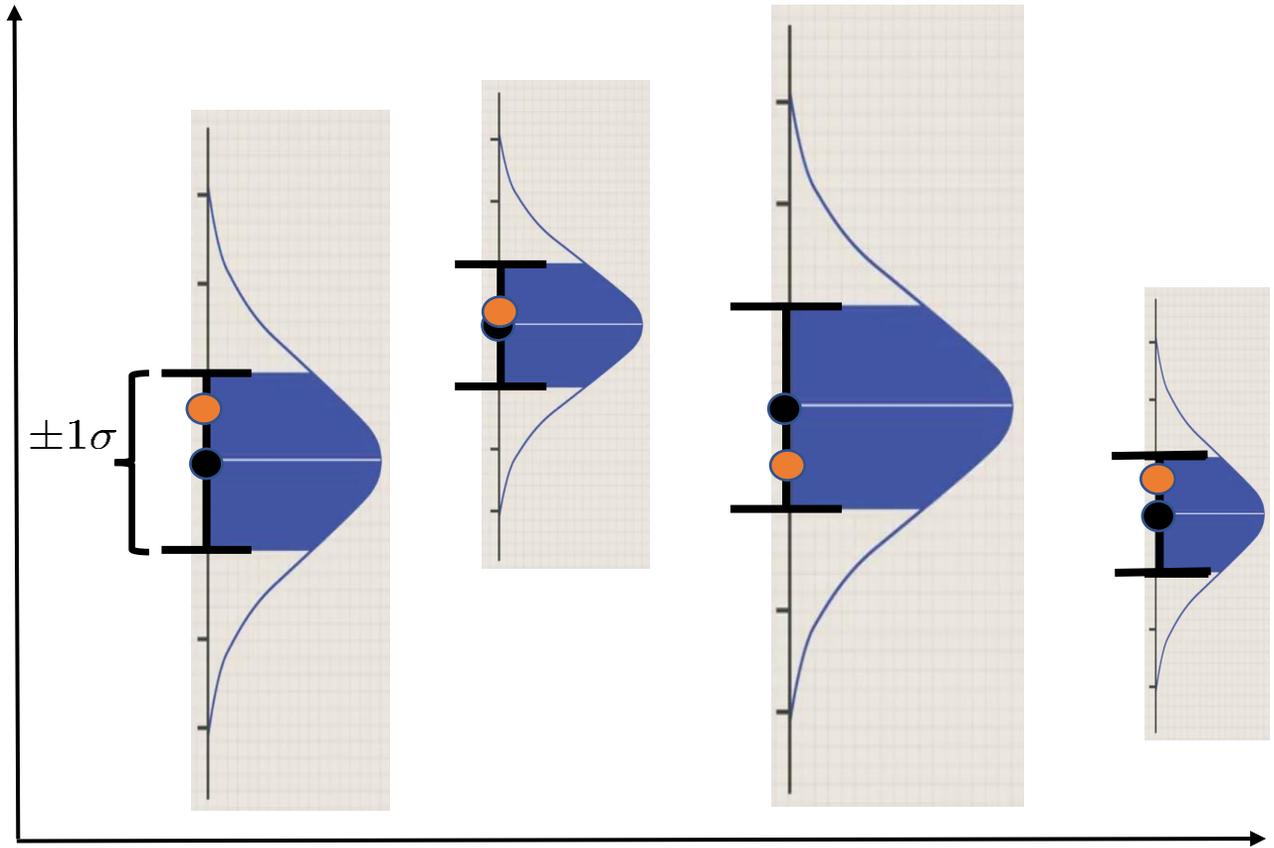
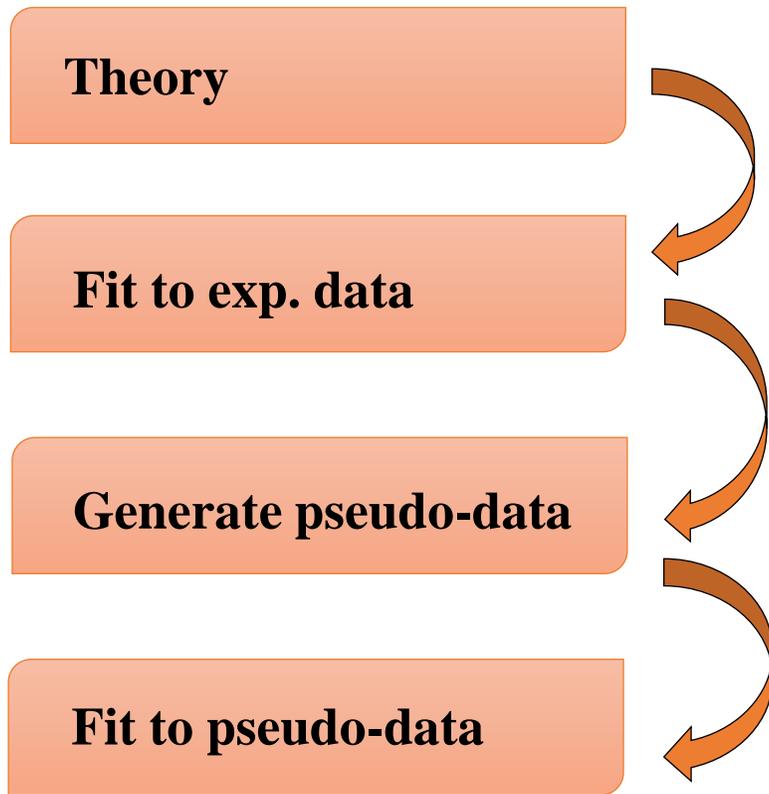


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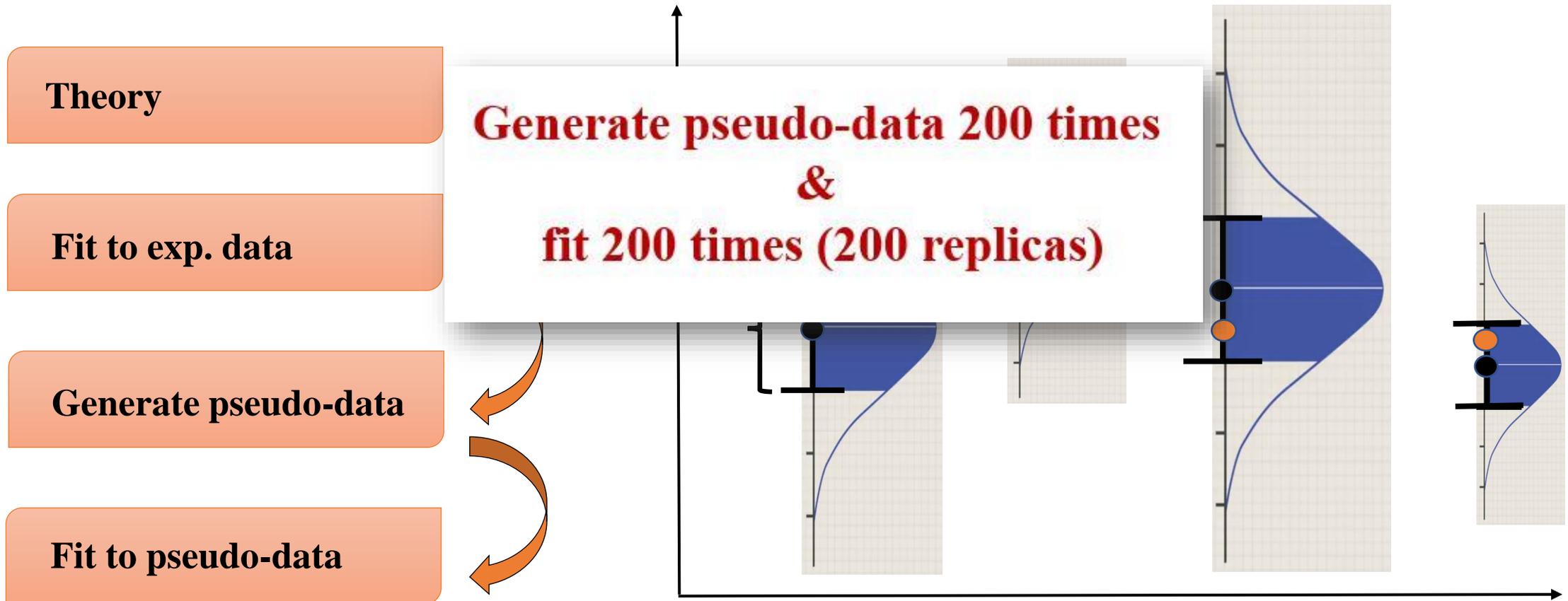




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**Theory versus data**

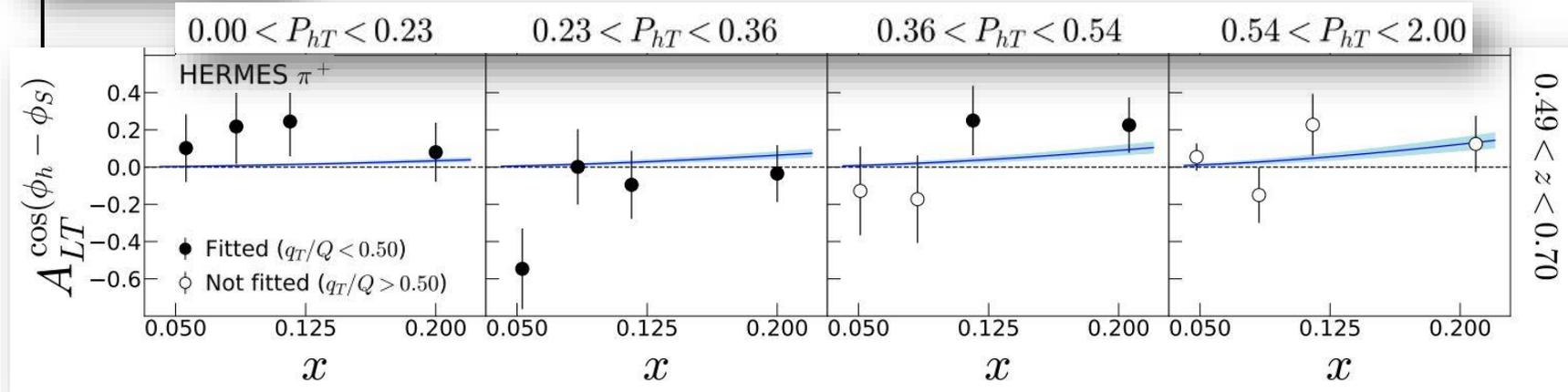


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Sample results

## Theory versus data



Data set	$\chi_w^2/N_{\text{pts.}} _{\text{Main}}$
HERMES $\pi^+$	1.20

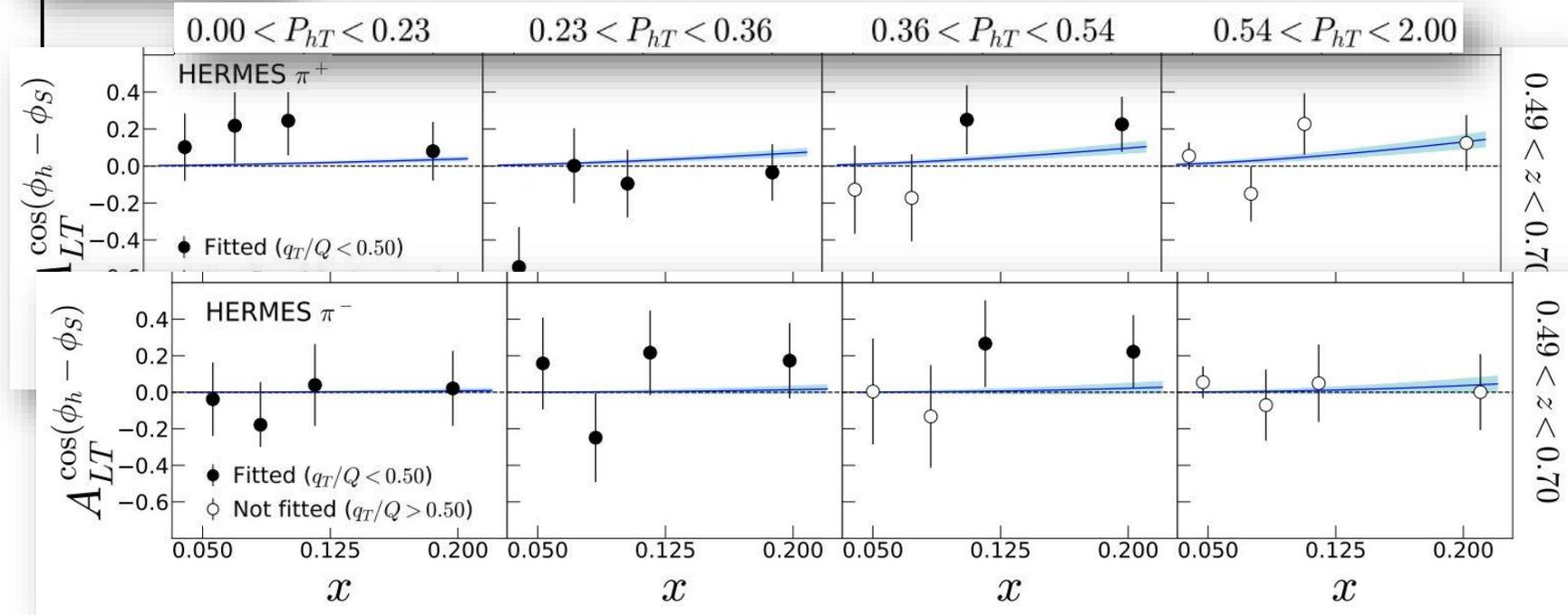


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Sample results

## Theory versus data



Data set	$\chi_w^2/N_{\text{pts.}} _{\text{Main}}$
HERMES $\pi^+$	1.20
HERMES $\pi^-$	0.88

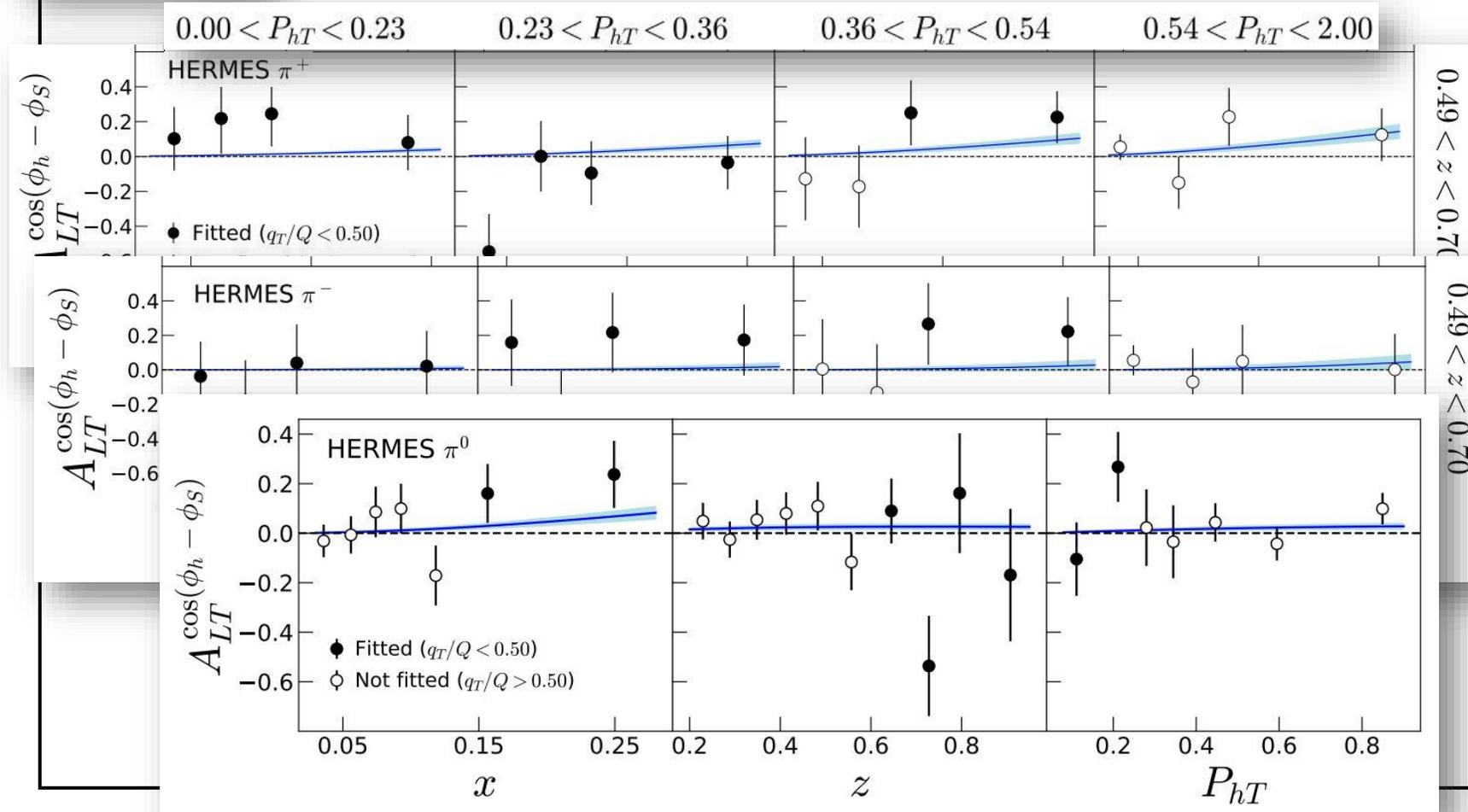


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Sample results

## Theory versus data



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HERMES $\pi^-$	0.88
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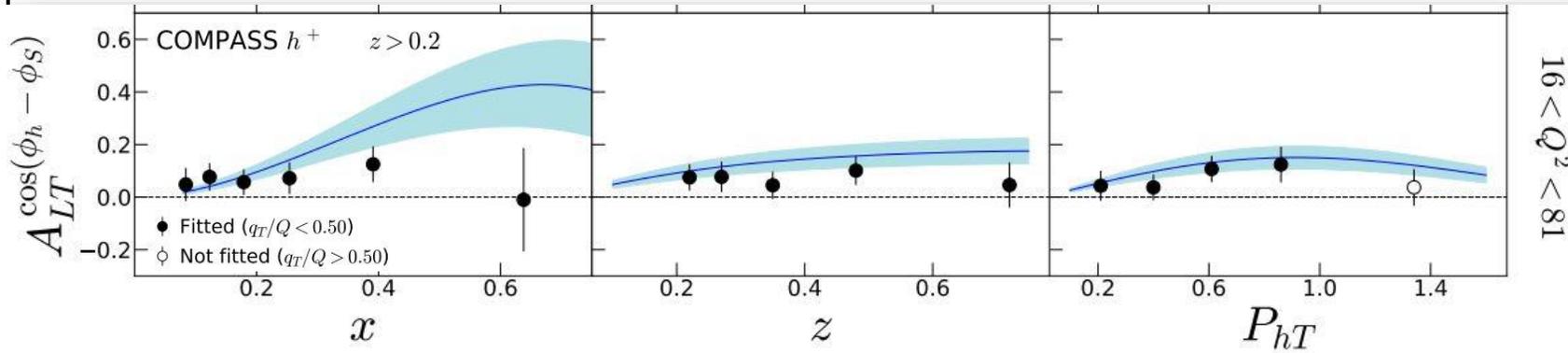


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Sample results

Theory versus data



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HERMES $\pi^0$	1.94
COMPASS $h^+$	0.97

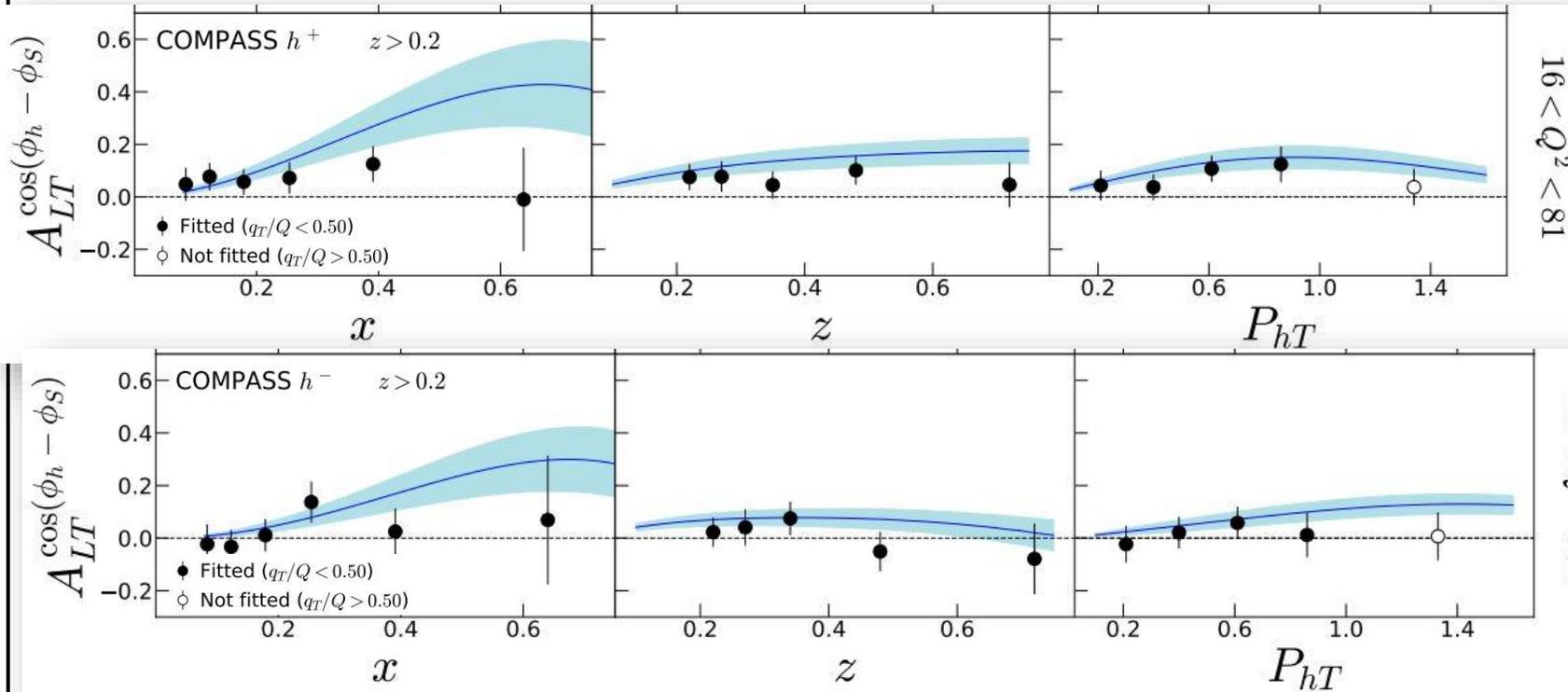


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Sample results

Theory versus data



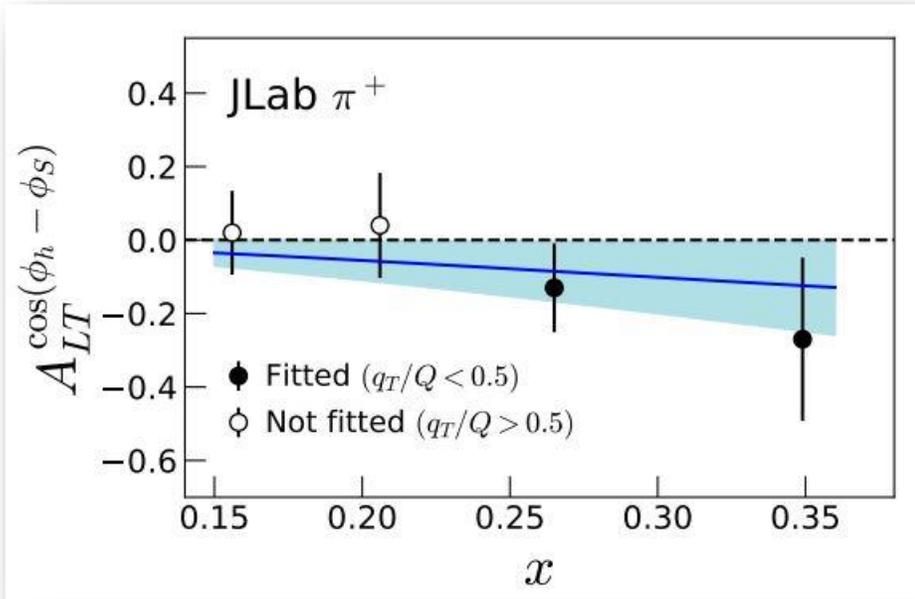
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## Theory versus data



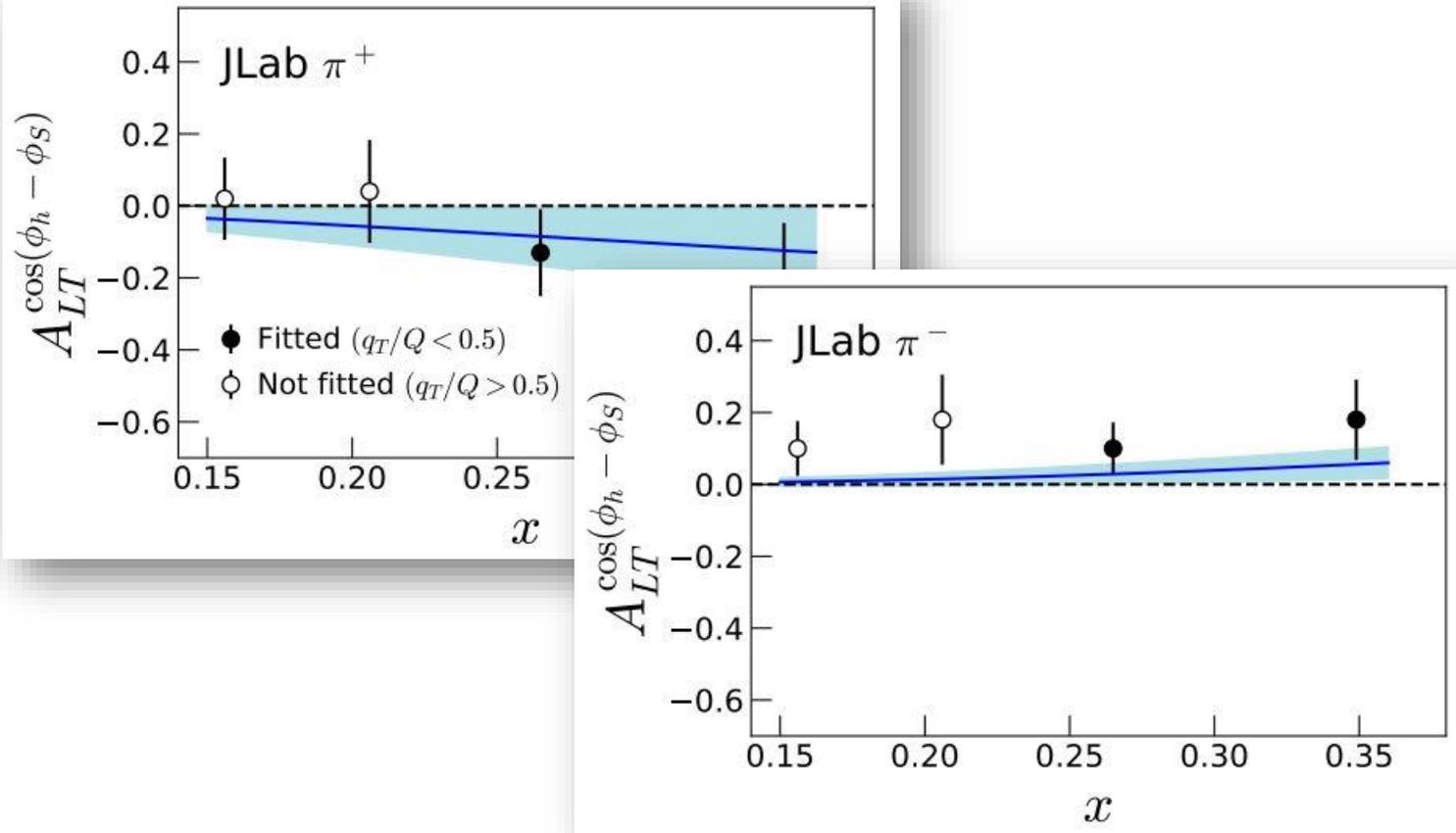
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## Theory versus data



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## Theory versus data

**Strong compatibility between our theory and data**

Data set	$\chi_w^2/N_{\text{pts.}} _{\text{Main}}$
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HERMES $\pi^0$	1.94
COMPASS $h^+$	0.97
COMPASS $h^-$	0.71
JLab $\pi^+$	0.31
JLab $\pi^-$	1.13
<b>Global</b>	<b>0.86</b>



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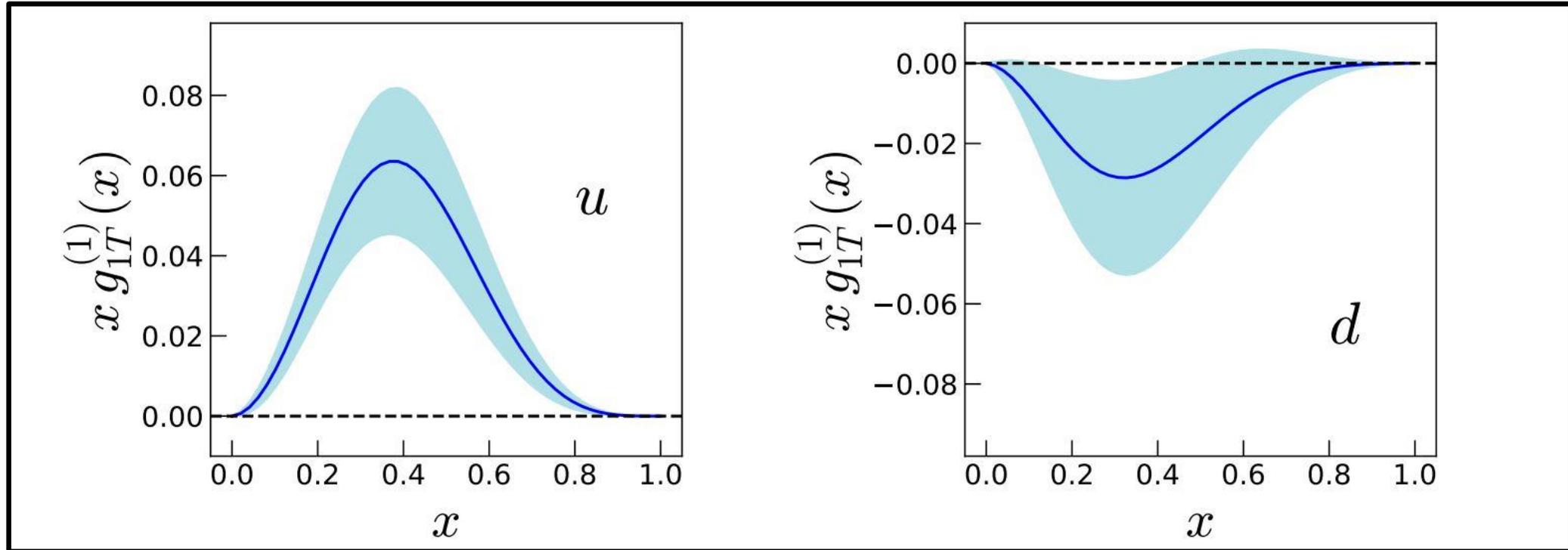
## Results for the x-dependence



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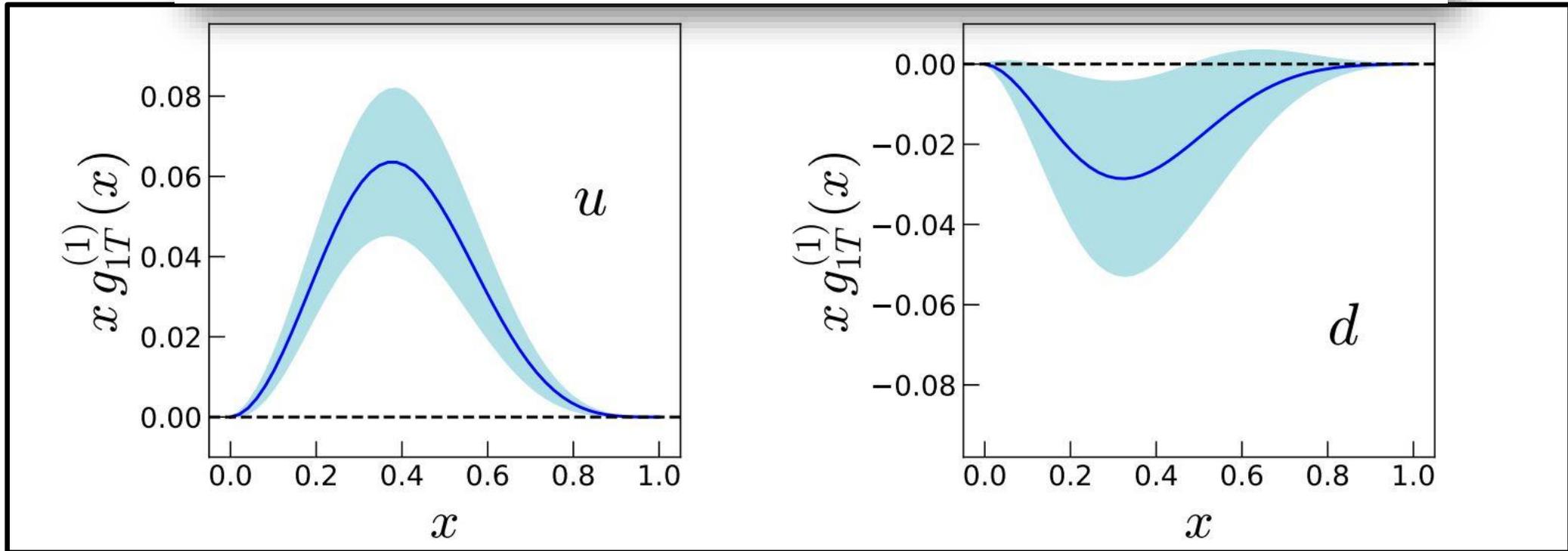
- Up quark distribution is positive
- Down quark distribution is negative



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**For a better flavor separation, we need more precise neutron data**



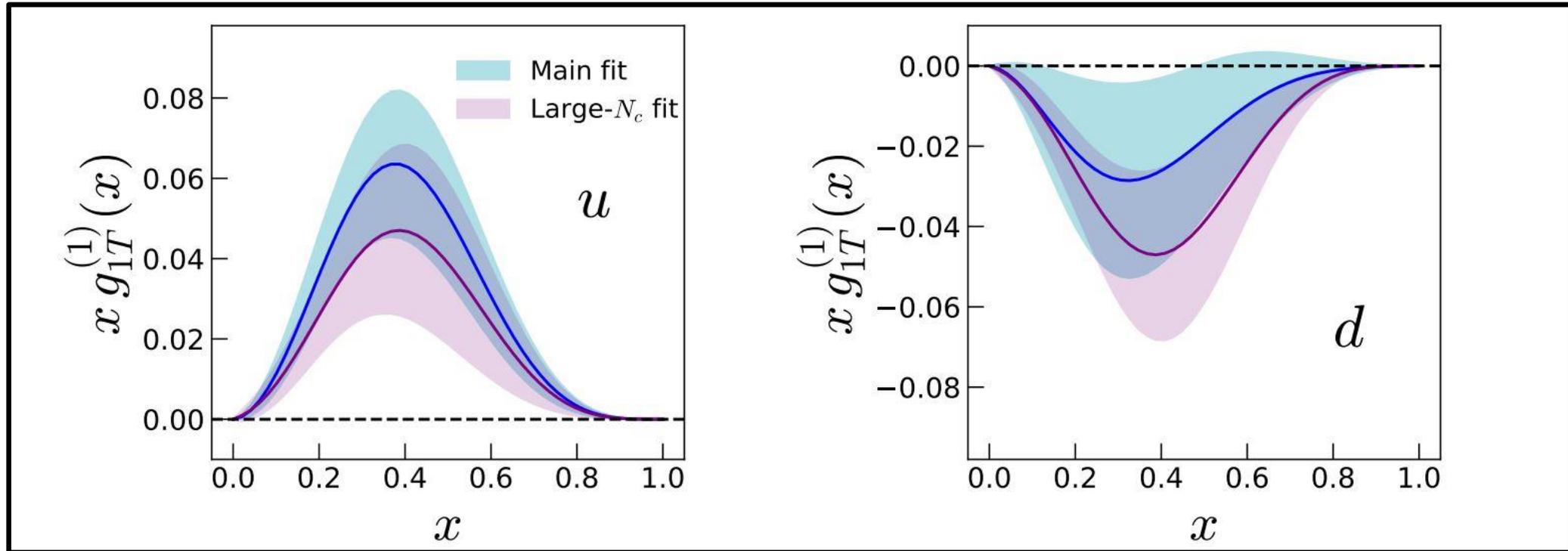
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## Test of theoretical predictions



- Qualitative agreement with large- $N_c$  fit

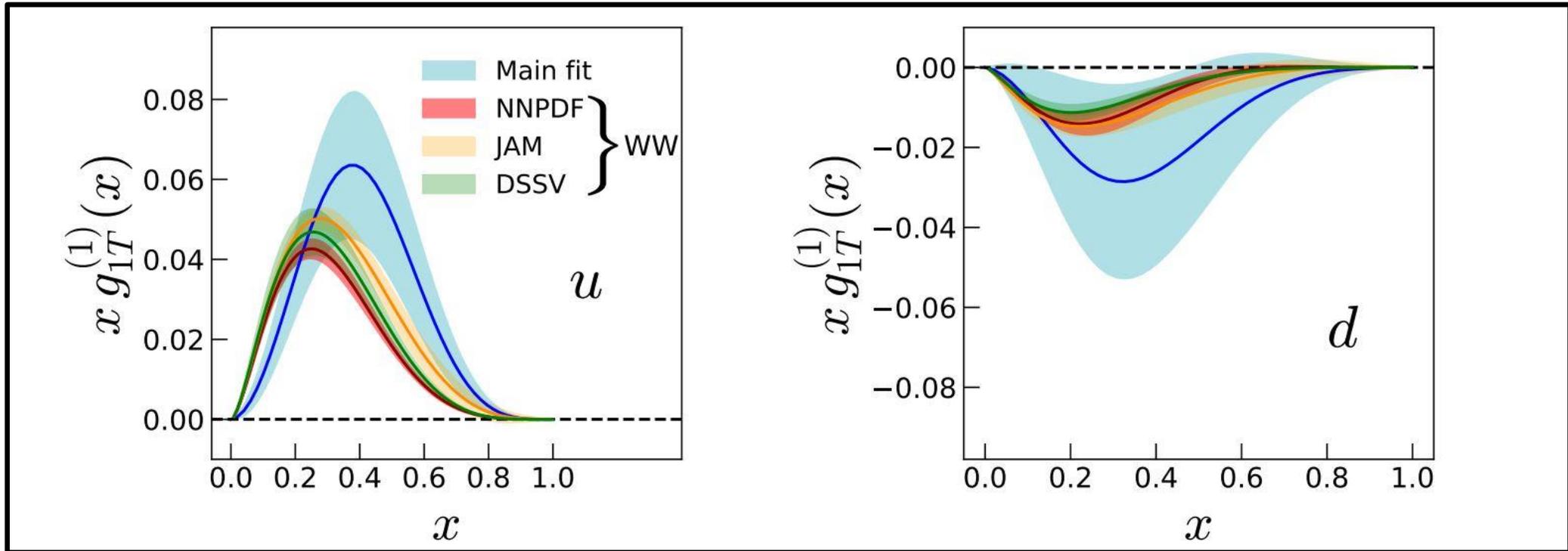
- Slight preference to violate large- $N_c$  approx.



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## Test of theoretical predictions



- Qualitative agreement with WW-type approx.
- Hints of slight violation of WW-type approx.



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**Violation of existing theoretical predictions?**



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## Violation of existing theoretical predictions?

Summary of $\chi_w^2/N_{\text{pts.}}$					
Data set	$\chi_w^2/N_{\text{pts.}} _{\text{Main}}$	$\chi_w^2/N_{\text{pts.}} _{\text{Large-}N_c}$	$\chi_w^2/N_{\text{pts.}} _{\text{NNPDF}}$	$\chi_w^2/N_{\text{pts.}} _{\text{JAM}}$	$\chi_w^2/N_{\text{pts.}} _{\text{DSSV}}$



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HERMES $\pi^+$	1.20	1.23			
HERMES $\pi^-$	0.88	0.88			
HERMES $\pi^0$	1.94	2.01			
COMPASS $h^+$	0.97	0.51			
COMPASS $h^-$	0.71	0.53			
JLab $\pi^+$	0.31	0.06			
JLab $\pi^-$	1.13	2.23			



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Similar or better chi-squared for some data sets cal predictions?

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HERMES $\pi^+$	1.20		1.19	1.19	1.19
HERMES $\pi^-$	0.88		0.85	0.85	0.85
HERMES $\pi^0$	1.94		1.98	1.95	1.96
COMPASS $h^+$	0.97		0.71	1.02	0.89
COMPASS $h^-$	0.71		0.71	0.81	0.80
JLab $\pi^+$	0.31		0.81	0.78	0.96
JLab $\pi^-$	1.13		1.15	0.93	0.93



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JLab $\pi^-$	1.13	2.23	1.15	0.93	0.93
<b>Global</b>	<b>0.86</b>	<b>0.99</b>	<b>0.95</b>	<b>0.94</b>	<b>0.97</b>

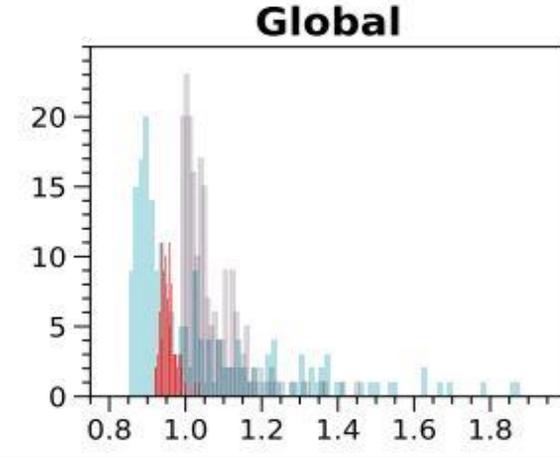
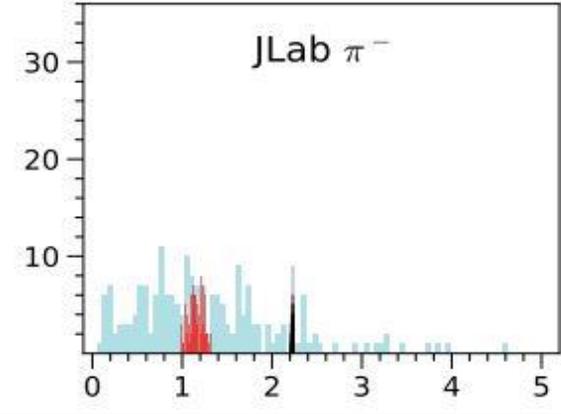
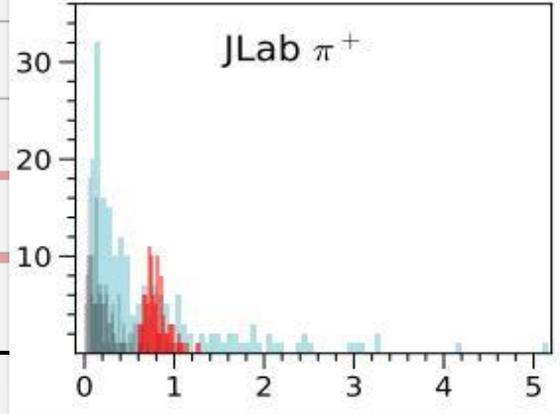
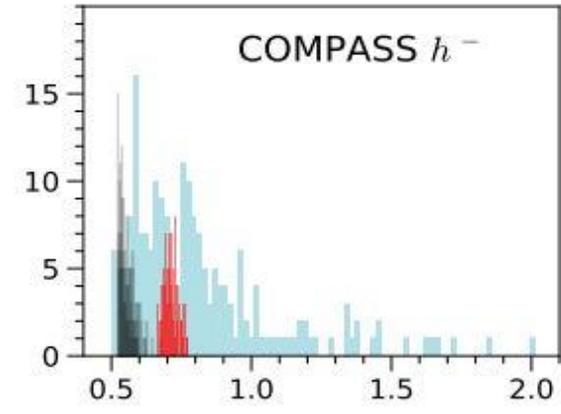
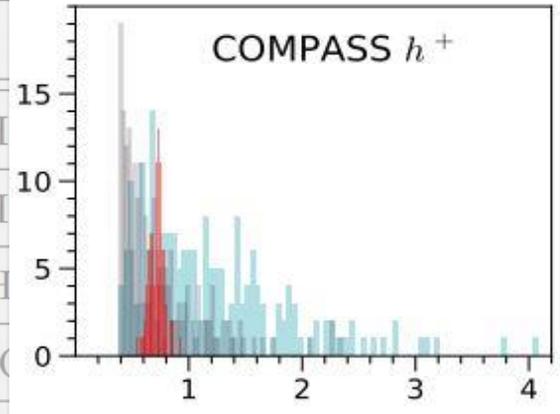
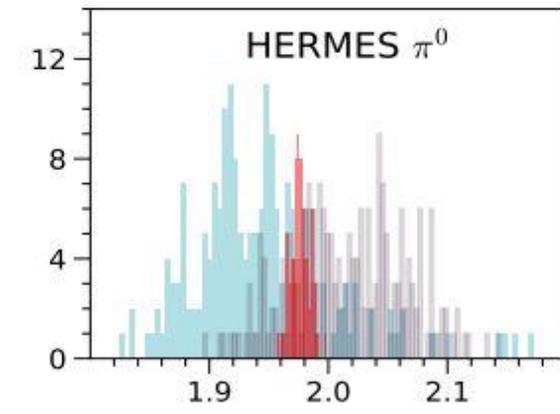
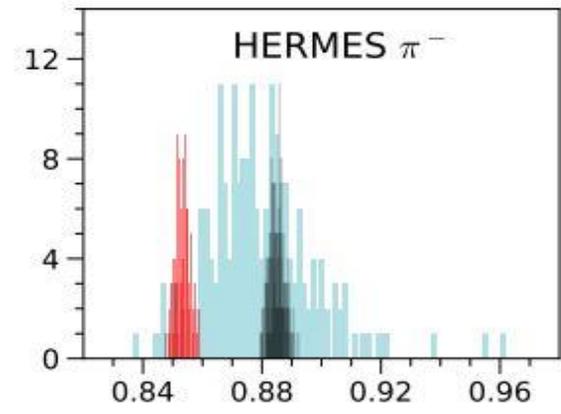
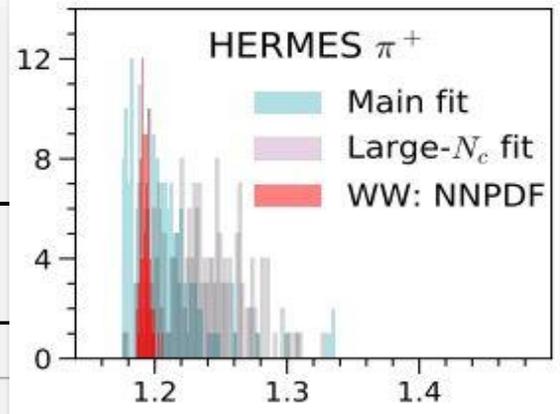
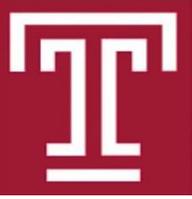


# Extraction of $g_{1T}$ TMD from HERMES, COMPASS & JLab data

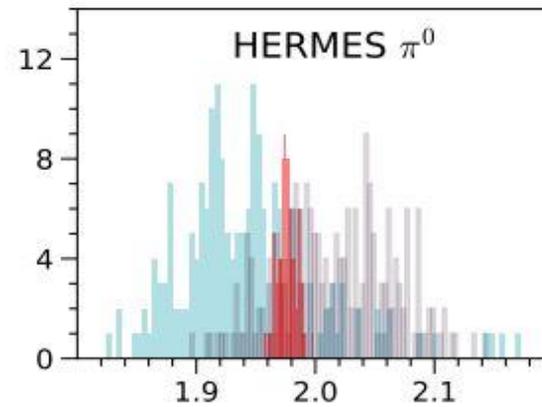
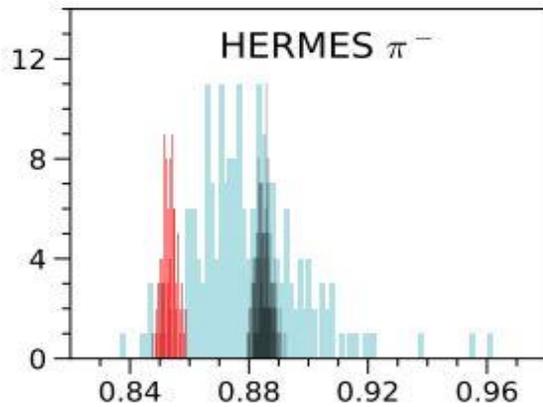
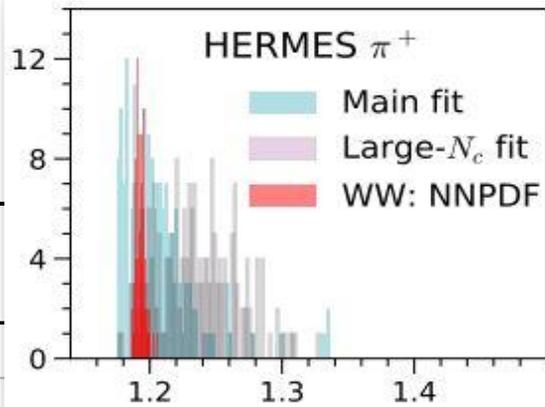
SB, Kang, Metz, Penn, Pitonyak, arXiv: 2110.10253 (2021)

## Violation of existing theoretical predictions?

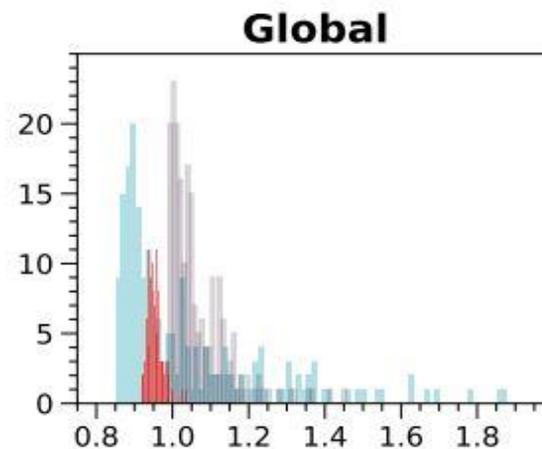
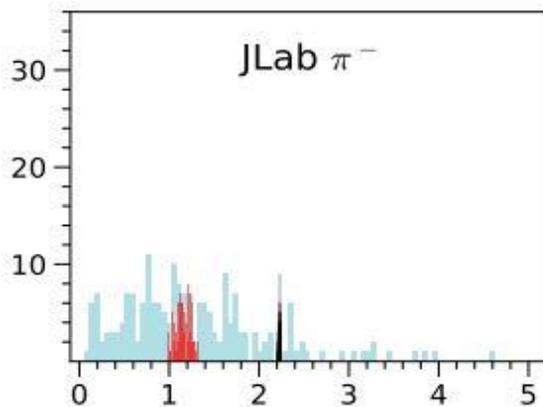
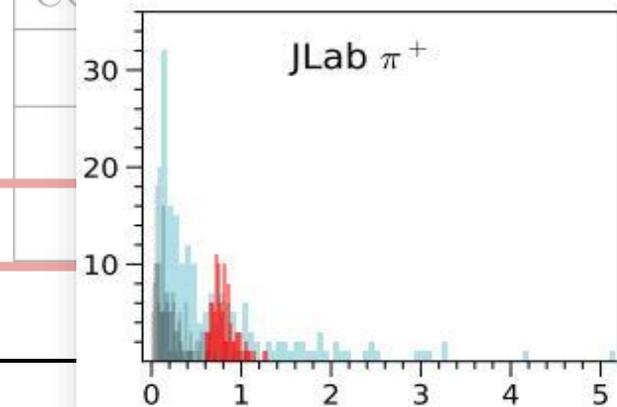
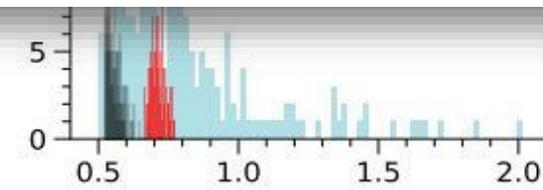
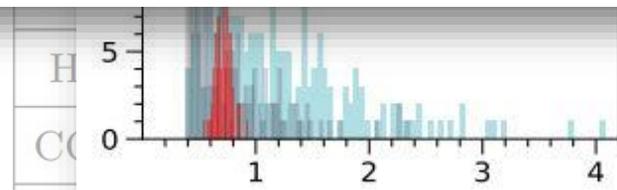
Summary of $\chi_w^2/N_{\text{pts.}}$					
Data set	$\chi_w^2/N_{\text{pts.}} _{\text{Main}}$	$\chi_w^2/N_{\text{pts.}} _{\text{Large-}N_c}$	$\chi_w^2/N_{\text{pts.}} _{\text{NNPDF}}$	$\chi_w^2/N_{\text{pts.}} _{\text{JAM}}$	$\chi_w^2/N_{\text{pts.}} _{\text{DSSV}}$
HERMES $\pi^+$	1.20	1.23	1.19	1.19	1.19
HERMES $\pi^-$	0.88	0.88	0.85	0.85	0.85
HERMES $\pi^0$	1.94	2.01	1.98	1.95	1.96
COMPASS $h^+$	0.97	0.51	0.71	1.02	0.89
COMPASS $h^-$	<b>Our global chi-squared is consistently better</b>				0.80
JLab $\pi^+$	0.31	0.06	0.81	0.78	0.96
JLab $\pi^-$	1.13	2.23	1.15	0.93	0.93
<b>Global</b>	<b>0.86</b>	<b>0.99</b>	<b>0.95</b>	<b>0.94</b>	<b>0.97</b>

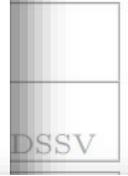
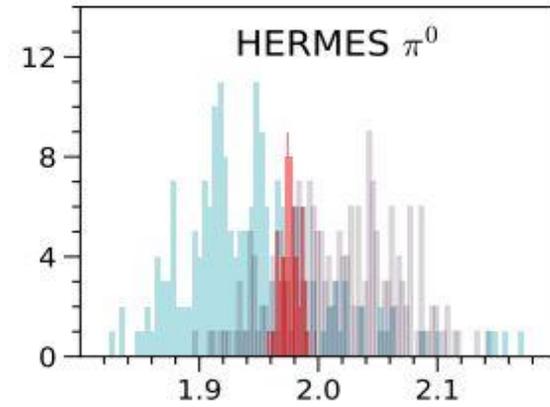
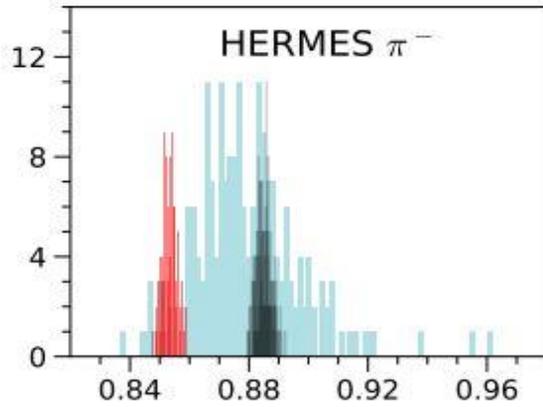
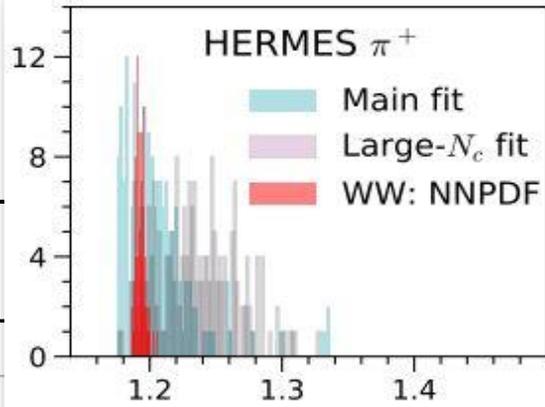


DSSV



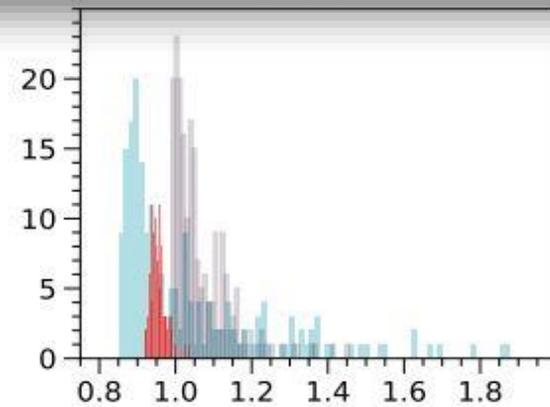
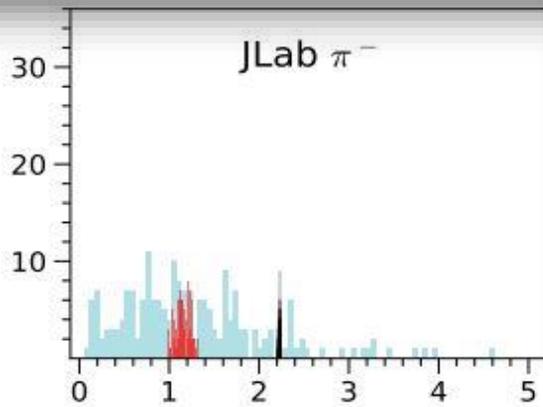
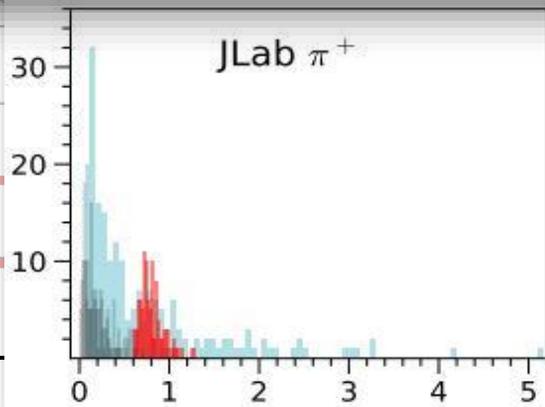
**Statistically no significant differences between all the scenarios**





**Statistically no significant differences between all the scenarios**

**Hence, at present data is compatible with large- $N_c$  & WW-type approx.**





# Extraction of $g_{1T}$ TMD from HERMES, COMPASS & JLab data

SB, Kang, Metz, Penn, Pitonyak, arXiv: 2110.10253 (2021)



## Comparison with lattice QCD results



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## Comparison with lattice QCD results

Calculation of worm-gear shift:

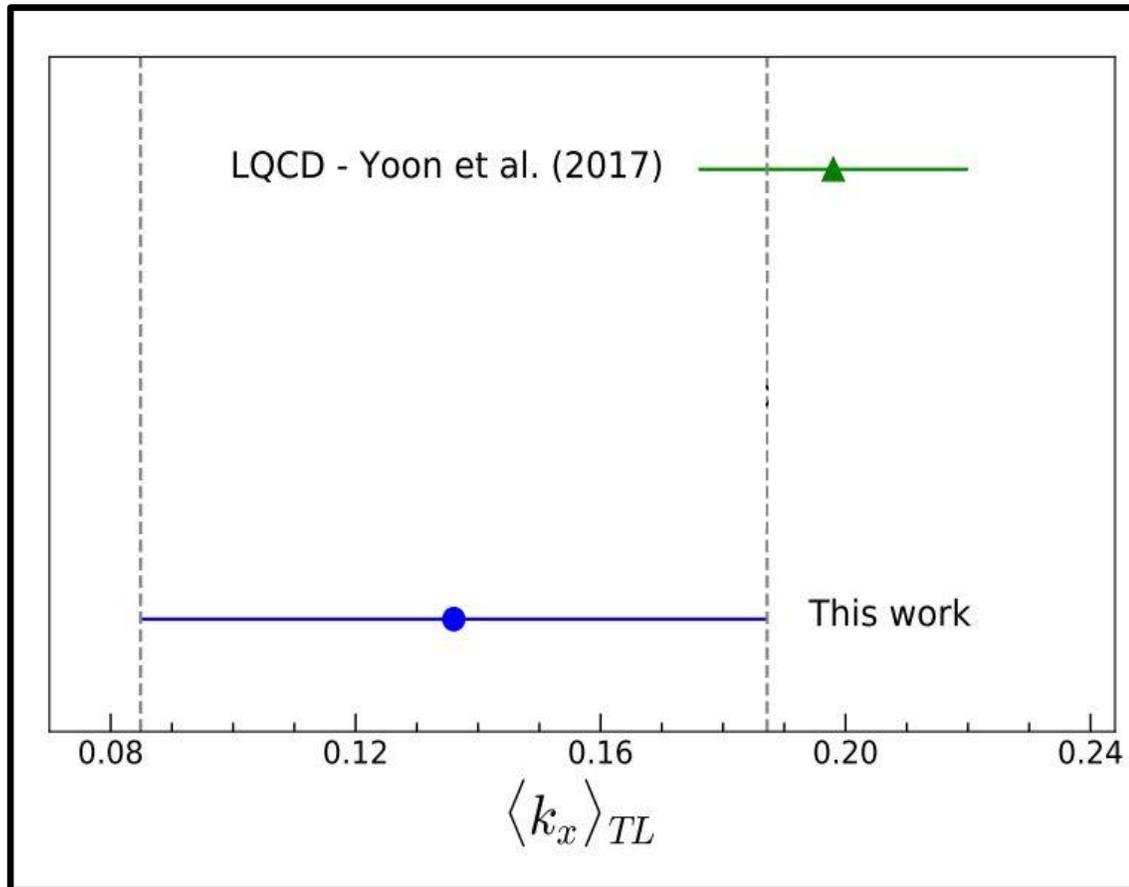
$$[\langle k_x \rangle_{TL}](Q^2) \equiv M \frac{\int_0^1 dx \left[ g_{1T}^{(1)u}(x, Q^2) - g_{1T}^{(1)d}(x, Q^2) \right]}{\int_0^1 dx \left[ f_1^u(x, Q^2) - f_1^d(x, Q^2) \right]}$$



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- Consistency between lattice results & our main fit result



## Caveats in LQCD calculations:

- **No definite scale:**  $Q \approx \frac{1}{a}$
- **Limits  $\hat{\zeta} \rightarrow \infty$  &  $b_T \rightarrow 0$  cannot be taken**

## MES, COMPASS & JLab data

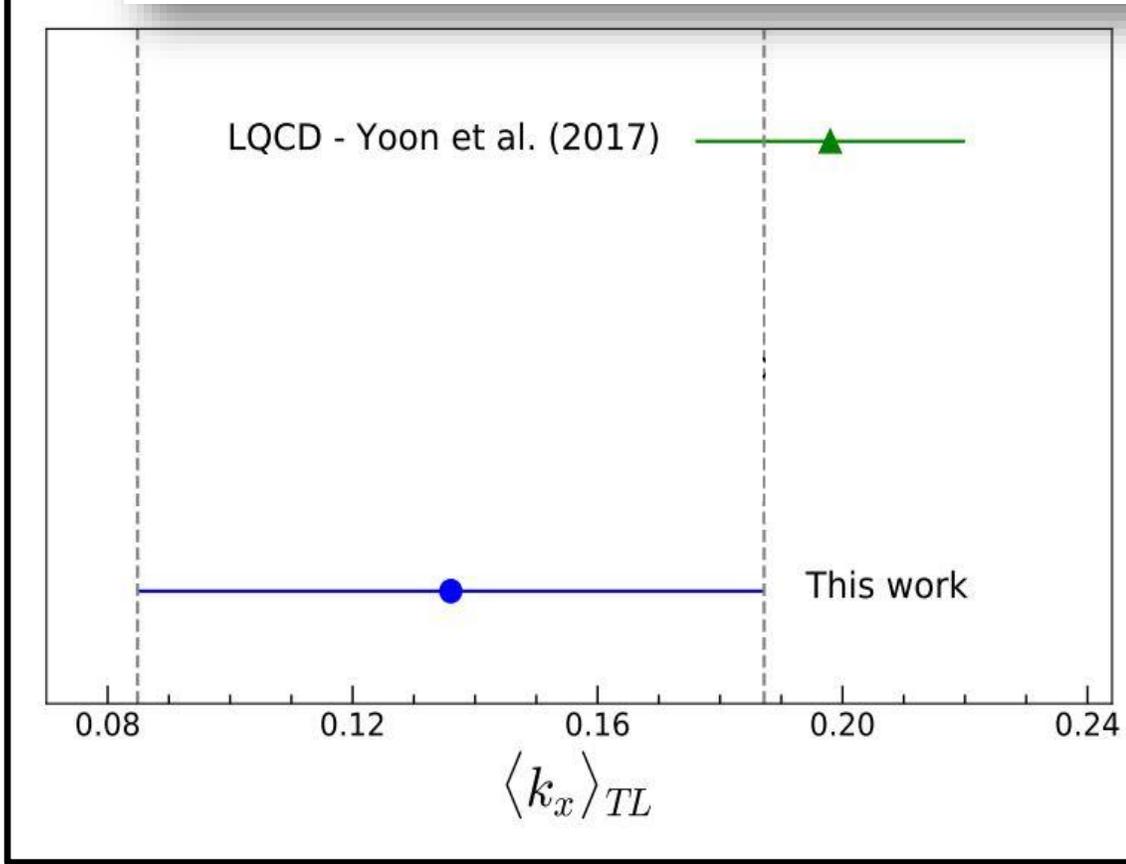
ak, arXiv: 2110.10253 (2021)

### QCD results

Calculation of worm-gear shift:

$$[\langle k_x \rangle_{TL}](Q^2) \equiv M \frac{\int_0^1 dx [g_{1T}^{(1)u}(x, Q^2) - g_{1T}^{(1)d}(x, Q^2)]}{\int_0^1 dx [f_1^u(x, Q^2) - f_1^d(x, Q^2)]}$$

- **Consistency between lattice results & our main fit result**
- **It is encouraging that lattice QCD & exp. data are in reasonable agreement**



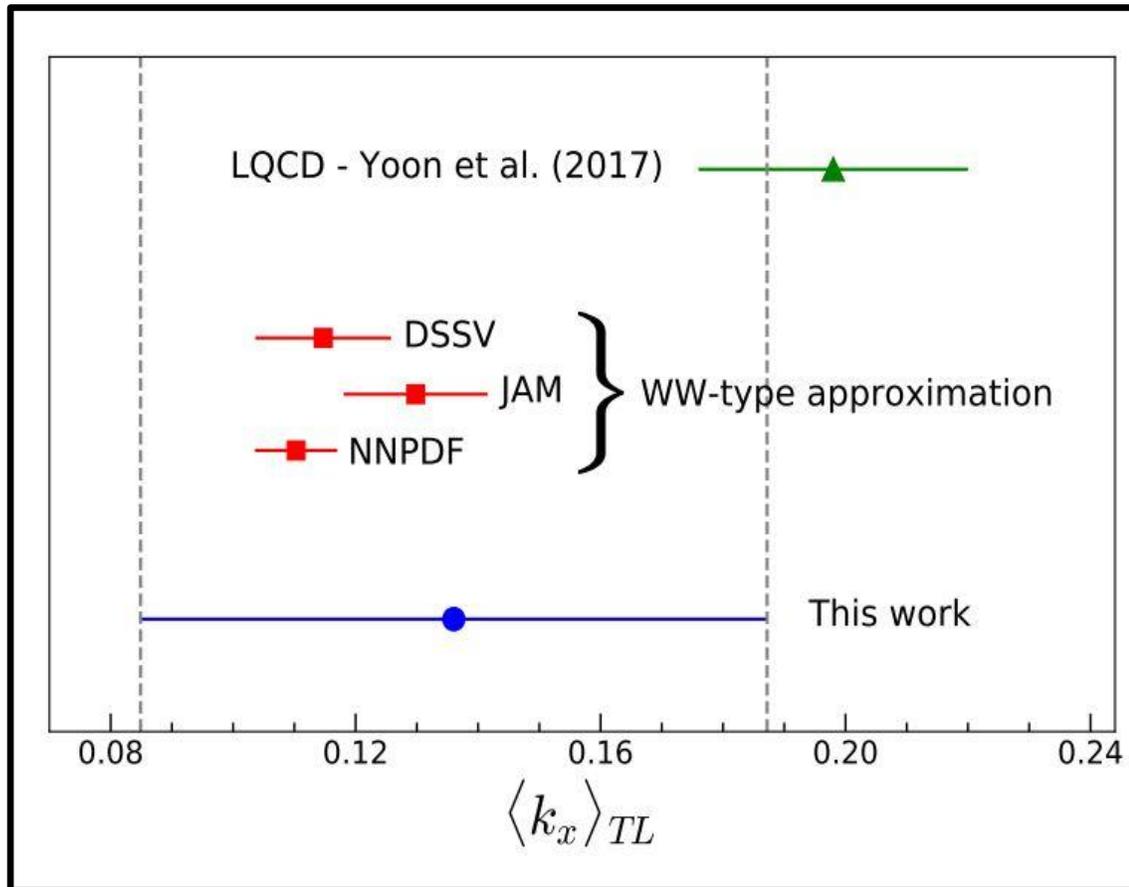


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## Comparison with lattice QCD results



## Calculation of worm-gear shift:

$$[\langle k_x \rangle_{TL}](Q^2) \equiv M \frac{\int_0^1 dx \left[ g_{1T}^{(1)u}(x, Q^2) - g_{1T}^{(1)d}(x, Q^2) \right]}{\int_0^1 dx \left[ f_1^u(x, Q^2) - f_1^d(x, Q^2) \right]}$$

- **Consistency between lattice results & our main fit result**
- **It is encouraging that lattice QCD & exp. data are in reasonable agreement**
- **Consistency between results from WW-type approx. & our main fit result**



# Summary/ Outlook

## Summary

- We have shown our final results for  $g_{1T}$ , obtained from a simultaneous fitting to HERMES, COMPASS & JLab data on SIDIS
- Qualitative agreements with large- $N_c$  & WW-type approximation
- Although there is an indication of a slight violation of both large- $N_c$  & WW-type approximation, the data is not precise enough to affirm the degree of violation
- Encouraging agreement in the worm-gear shift with lattice QCD results

## Outlook

- Extend analysis to extract  $h_{1L}^\perp, \dots$

# Backup slides

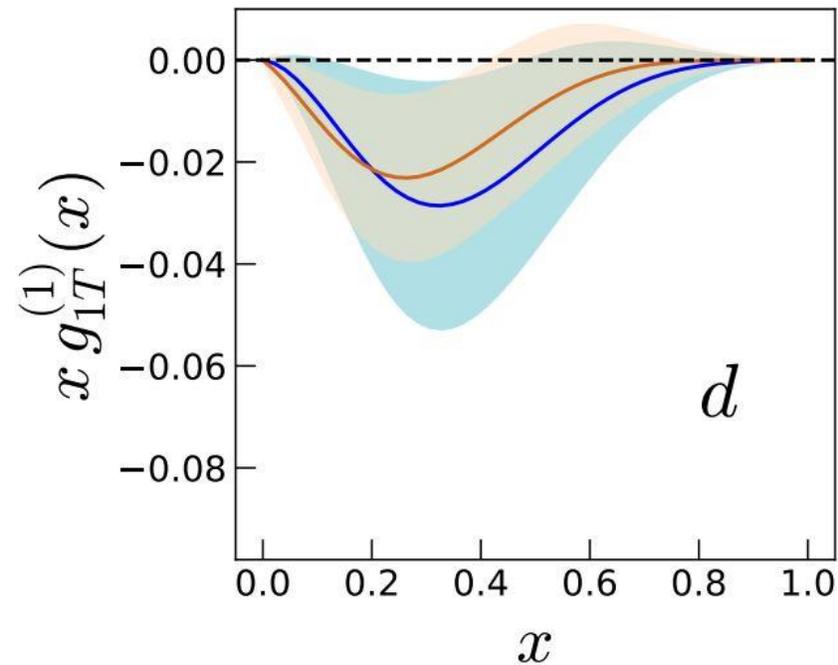
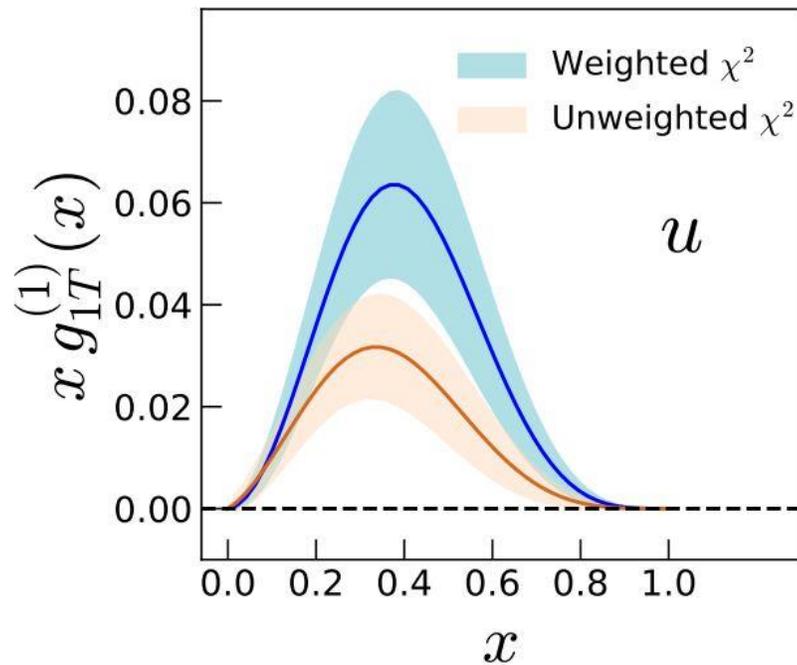


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## Weighted versus unweighted methods



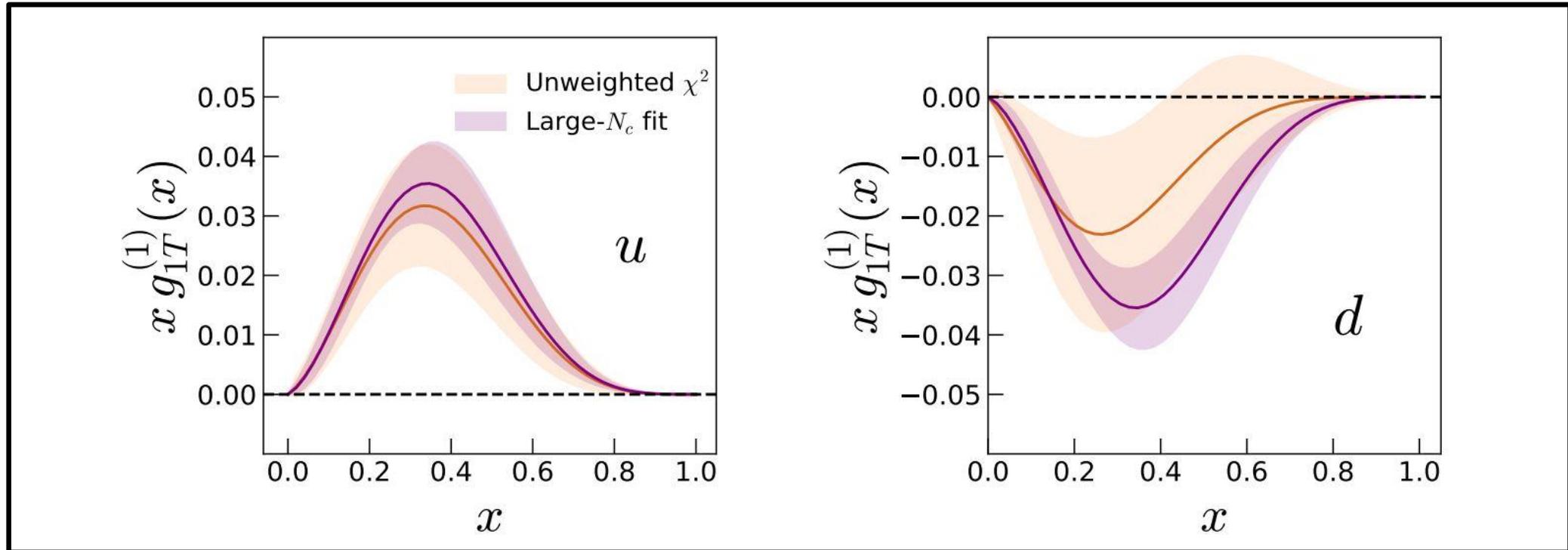


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## Additional plots from the unweighted method





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## Additional plots from the unweighted method

