Lattice results on twist-3 parton distributions

Aurora Scapellato

TEMPLE UNIVERSITY

In collaboration with

Shohini Bhattacharya[1], Krzysztof Cichy[2], Martha Constantinou[3]
Andreas Metz[3], Fernanda Steffens[4]

BNL[1], Adam Mickiewicz University[2], Temple University[3], University of Bonn[4]

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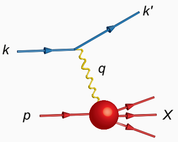
- Hadron structure described by a tower of partonic distributions



$$f \quad f^{(0)} \quad \frac{1}{Q} f^{(1)} \quad \frac{1}{Q^2} f^{(2)} \quad \dots$$

$f^{(0)}$ ↓ twist-2 $\quad$ $f^{(1)}$ ↓ twist-3

- f: classified according to their twist - order in $1/Q$ at which $f^{(i)}$ appear in factorization theorem of structure functions

$$\sigma_{DIS}(x, Q^2) = \sum_i H^i_{DIS} \otimes f_i(x, Q^2)$$

Twist-2 case:

probabilistic interpretation

some twist-2 PDFs are very well known

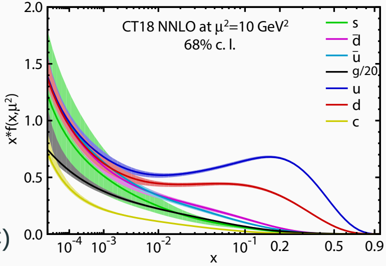
Twist-3 case:

information about *qgq* correlations

kinematically suppressed

challenging to probe experimentally (JLAB 12 GeV upgrade, EIC)

Unpolarized twist-2 PDFs



[Kovarik et al., 2019, Rev.Mod.Phys. 92 (2020) 4, 045003]

Our lattice work on twist-3 distributions

- Isovector proton twist-3 PDFs: $g_T^{u-d}(x), h_L^{u-d}(x)$

Polarization	PDF	Operator
U	$e(x)$	1
L	$h_L(x)$	σ^{ij}
T	$g_T(x)$	$\gamma_5\gamma_i$

$g_T^{u-d}(x)$: ⊥ force acting on the active quark in DIS off a transversely polarized nucleon after it has absorbed the virtual photon [M.Burkardt, PRD 88 (2013) 114502]

$h_L^{u-d}(x)$: chiral odd, accessible in e.g. Drell-Yan process [R. Jaffe, PRL 67 (1991) 552-555], proton-proton collisions [Y. Koike et al., PLB 759 (2016) 75,],. . .

Interesting aspects to address:

1. Are twist-3 effects sizeable?
2. Wandzura-Wilczek approximation

[S. Wandzura and F. Wilczek, Phys. Lett.72B, 195(1977)][R.L. Jaffe and X. Ji, PRL 67 (1991) 552-555]

E.g.: $g_T(x) \approx \int_x^1 dy\, \frac{g_1(y)}{y}$ $g_1(y)$: helicity twist-2 PDF

- Twist-3 GPD $\tilde{G}_2(x, \xi, t)$

$$[\tilde H + \tilde G_2](x, \xi \to 0, Q^2 \to 0) = g_T(x)$$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105]

- PDFs computed through purely space-like correlation functions

$$\tilde{\Phi}^{\Gamma}(x,\mu,P_3) \quad \int \frac{dz}{4\pi} e^{\; ixP_3 z} \underbrace{\langle N(P_3)|\bar{\psi}(0)\Gamma W(0,z)\psi(z)|N(P_3)\rangle}_{\text{matrix elements of fast moving nucleons}}{}_{\mu}$$

μ: renormalization scale $z \quad (0,0,0,z)$ $P \quad (0,0,P_3)$: proton boost

Access twist-3 PDFs through the quasi-distribution approach [X.Ji, Phys. Rev. Lett.110(2013) 262002]

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- For sufficiently large P_3 $(P_3 \gg M)$, quasi-PDFs matched to light-cone PDFs

LaMET

$$\tilde{\Phi}^{\Gamma}(x,\mu,P_3) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y},\mu\right) \Phi(y,\mu) + O\left(\frac{1}{P_3^2}\right)$$

twist-2 case: factorization to all orders
many works between 2013-2020

twist-3 case: proved factorization to 1-loop order

[S.Bhattacharya, K.Cichy, M.Constantinou, A.Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 3, 034005, Phys.Rev.D 102 (2020) 114025]

[V.M. Braun et al., 2021, JHEP 05 (2021) 086]

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→ matching kernel

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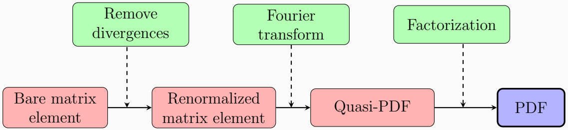
- Road map



Lattice setup

- Configurations of N_f 2 1 1 flavors & clover term [ETM collaboration]

N_f	L^3 T	lattice spacing a	m_π	$m_\pi L$
4	32^3 64	0.093 fm	270 MeV	4

Lattice setup

- Configurations of $N_f = 2+1+1$ flavors & clover term [ETM collaboration]

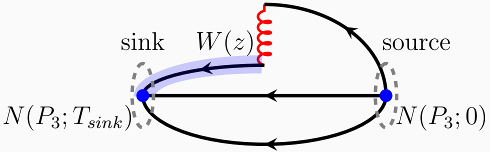
N_f	$L^3 \times T$	lattice spacing a	m_π	$m_\pi L$
4	$32^3 \times 64$	0.093 fm	270 MeV	4

- Matrix elements extracted from:

$$\frac{C^{3pt}(T_{sink};\tau;\Gamma;P_3)}{C^{2pt}(T_{sink};P_3)} \xrightarrow{T_{sink} \gg \tau \gg 0} M(P_3,\Gamma)$$

$\Gamma = \gamma_x\gamma_5, \gamma_y\gamma_5$ for $g_T(x)$

$\Gamma = \sigma^{12}$ for $h_L(x)$



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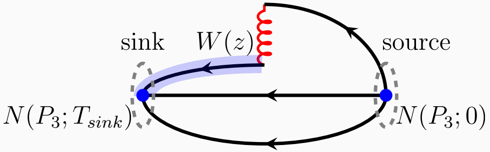

Γ $\gamma_x\gamma_5, \gamma_y\gamma_5$ for $g_T(x)$

Γ σ^{12} for $h_L(x)$

Lattice techniques:

sequential inversions for all-to-all propagator (here T_{sink} $12a$ 1.12 fm)

Momentum smearing to improve overlap with proton boosted state [G. Bali et al., Phys.Rev.D 93 (2016) 9, 094515]

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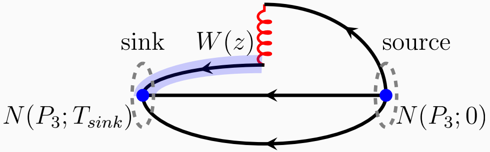

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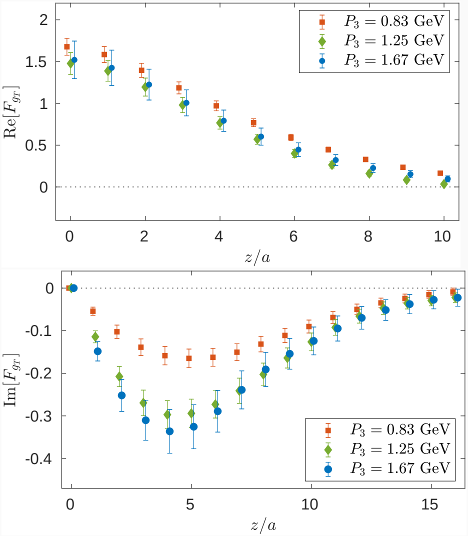
- Momentum dependence of the lattice PDFs studied at three proton boosts

P_3[GeV]	N_{conf}	N_{meas}
0.83	194	1552
1.25	731	11696
1.67	1644	105216

Results: $g_T(x)$

From lattice matrix elements to $g_T^{u-d}(x)$

Bare matrix elements



From lattice matrix elements to $g_T^{u\ d}$pxq

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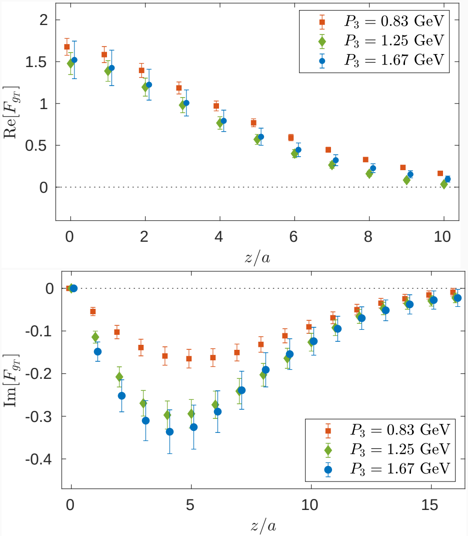
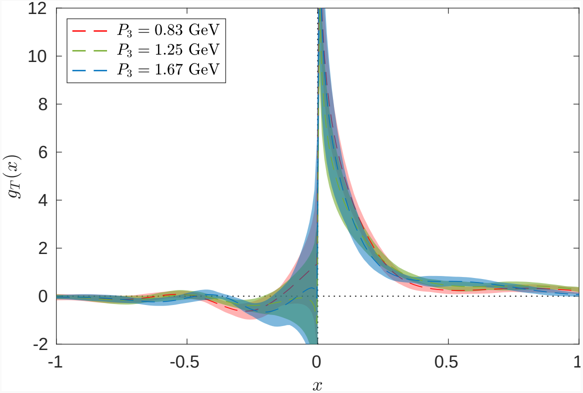


Reconstructed distributions



Antiquarks **Quarks**

(Upon renormalization, F.T and matching)

[S.Bhattacharya, K.Cichy, M. Constantinou,A. Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 11]

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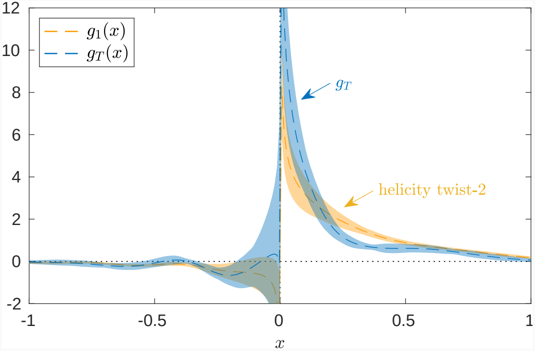
$P_3 = 1.67$ GeV



Antiquarks Quarks

- $g_1(x)$ extracted on the same gauge ensemble
- g_1 and g_T compatible for antiquarks, $\bar{u}-\bar{d}$
- g_T dominant at small positive x ($x \lesssim 0.2$)
  $\Rightarrow$ twist-3 contributions may be sizable

Comparison between g_1^{u} ^{d}pxq and g_T^{u} ^{d}pxq

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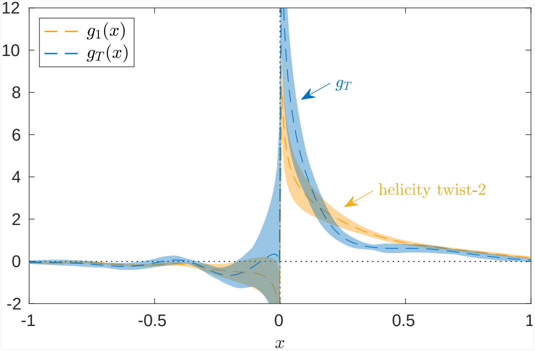

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Burkhardt-Cottingham sum rule:

[H. Burkhardt and W. N. Cottingham, Annals Phys.56,453 (1970)]

$$» _{1}^{1} \quad dx\, g_1 \text{pxq} \qquad » _{1}^{1} \quad dx\, g_T \text{pxq}$$

(lattice) $» _{1}^{1} \quad dx\, g_1 \text{pxq} \qquad » _{1}^{1} \quad dx\, g_T \text{pxq} \qquad$ 0.01p20q

Wandzura-Wilczek approximation for g_T^{u-d}

- Wandzura-Wilczek relation:

$$g_T(x) = \int_x^1 dy\, \frac{g_1(y)}{y} + g_T^{twist-3}(x) \quad \text{(on the light cone)}$$

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- Approximation: $$\boxed{g_T(x) \overset{?}{\approx} \int_x^1 dy \frac{g_1(y)}{y} \equiv g_T^{WW}(x)}$$

$$d_2 = \int_0^1 dx\, 3x^2 [g_T(x) - g_T^{WW}(x)] = O(10^{-3})$$

[D. Flay et al., PRD 94, 5 (2016) 052003]

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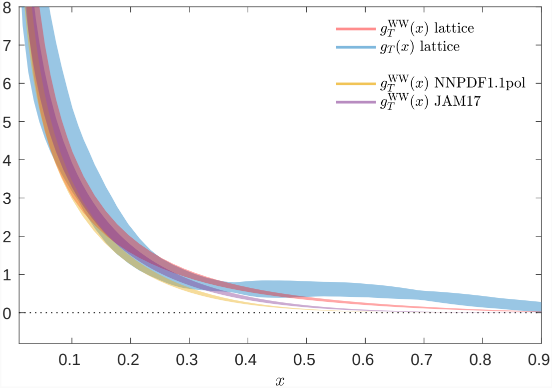
- agreement between g_T and g_T^{WW} for $x \lesssim 0.5$ within uncertainties
- still violations are possible up to 30 − 40%
- similar violations are observed in experimental analysis at JLab [A. Accardi et al., JHEP11 (2009) 093]

$P_3 = 1.67$ GeV



[S.Bhattacharya, K.Cichy, M. Constantinou, A. Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 11]

Beyond $g_T(x)$... The $h_L(x)$ case

Extraction of $h_L(x)$ PDF

Twist-2 h_1

- $h_L(x)$ contains the leading twist-2 h_1 transversity PDF
- No experimental data on $h_L(x)$

[JAM, Phys.Rev.D 102 (2020) 5, 054002]

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h_L from quasi-PDFs $\longrightarrow$ $F_{h_L}(P_3; z) = \langle N(P_3)| \bar{\psi}(0) \gamma^{12}\gamma_3 W(0; z) \psi(z) |N(P_3)\rangle$

[S.Bhattacharya, K.Cichy, M. Constantinou,A. Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 11]

- h_1^{u-d} dominant only for $0.3 \lesssim x \lesssim 0.5$

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- h_1^{u-d} dominant only for $0.3 \lesssim x \lesssim 0.5$
- up-quark especially dominant for $x \gtrsim 0.5$
- differences between h_L and h_1 mostly come from the up quark

What about for the single quark flavors?

quark loops here neglected - found very small for $h_1(x)$

[C.Alexandrou at al., arXiv:2106.16065]

up quark

down quark

$$h_L(x) = 2x \int_x^1 \frac{h_1(y)}{y^2} dy + h_L^{twist-3}(x)$$

[R.L. Jaffe and Xiangdong Ji, Nucl. Phys. B 375 (1992) 527—560]

Test: $$h_L(x) \stackrel{?}{=} 2x \int_x^1 dy \, \frac{h_1(y)}{y^2} \equiv h_L^{WW}(x)$$

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Test: $$h_L(x) \stackrel{?}{\approx} 2x \int_x^1 dy \, \frac{h_1(y)}{y^2} \equiv h_L^{WW}(x)$$

- h_L seems to be largely determined by its twist-2 counterpart within current uncertainties

From twist-3 PDFs...to twist-3 GPDs

Twist-3 GPDs (axial-vector case)

Theoretical framework

(DVCS at twist-3 level)

$$\tilde{F}_q^{\mu}(x,\xi,\Delta) \quad \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi} e^{i\lambda x}\langle p'|\bar{q}(\tfrac{\lambda}{2}n)\gamma^{\mu}\gamma_5 W(\tfrac{\lambda}{2}n, \tfrac{\lambda}{2}n)q(\tfrac{\lambda}{2}n)|p\rangle$$

$$\tilde{F}^{\mu} \quad P^{\mu}\frac{\tilde{h}}{P}\tilde{H} \quad P^{\mu}\frac{\tilde{e}}{P}\tilde{E} \quad \Delta_{\perp}^{\mu}\frac{\tilde{b}}{2m}\tilde{E} \quad \tilde{G}_1 \quad \tilde{h}_{\perp}^{\mu}\tilde{H} \quad \tilde{G}_2 \quad \Delta_{\perp}^{\mu}\frac{\tilde{h}}{P}\tilde{G}_3 \quad \tilde{\Delta}_{\perp}^{\mu}\frac{h}{P}\tilde{G}_4$$

$$\Delta \quad p' \quad p, \; P \quad \frac{p' \quad p}{2}$$
$$\xi \quad \frac{\Delta}{2P}, \; P \quad \frac{P^0 \quad P^3}{2}$$

4 twist-3 GPDs to disentangle

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105]
[F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

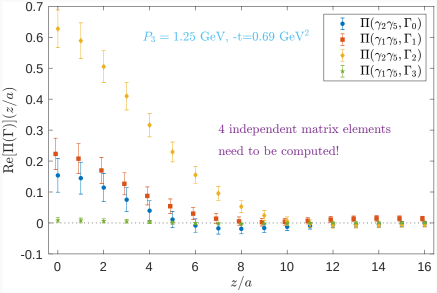
Lattice setup

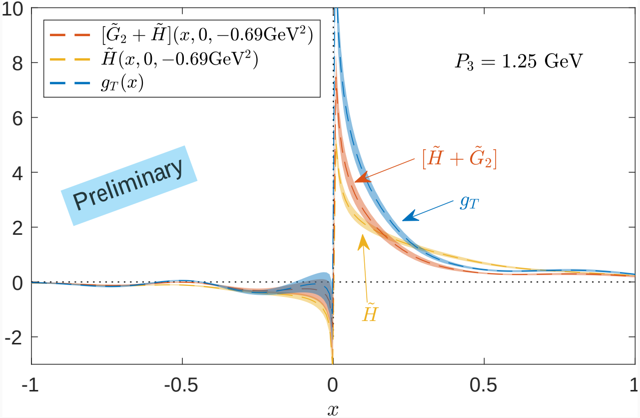
Higher computational cost than PDFs!

- Breit frame
- Same ensemble as twist-3 PDFs

P_3[GeV]	$Q \quad \frac{L}{2\pi}$	t[GeV^2]	ξ	N_{meas}
1.25	(2, 0, 0)	0.69	0	4288
1.25	(2, 2, 0)	1.08	0	8704

many more Q-values will be added [in progress]





[S. Bhattacharya, K. Cichy, M. Constantinou, J. Dodson, A. Metz, AS, F. Steffens, (DIS2021) arXiv:2107.12818]

- g_T dominant as expected ($[\tilde{H} + \tilde{G}_2](x, 0, 0) = g_T(x)$)
- $[\tilde{H} + \tilde{G}_2]$ enhanced for small-x with respect to $\tilde{H}$ (similar behavior as for g_T vs. g_1)

Summary & Outlook

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Study of twist-3 PDFs & GPDs

- Lattice investigations can be pursued using quasi-distribution formalism (and also other approaches)
- For the first time, a qualitative comparison between twist-2 and twist-3 PDFs can be made

  Ñ test of the Wandzura-Wilczek approximation for g_T and h_L for x ¡ 0

- Several challenges in front of us:

  can we disentangle the twist-2 and twist-3 parts of g_T and h_L?

  Ñ calculation of quark-gluon-quark matrix elements

  Ñ to what extent numerical results are affected?

  [V.M. Braun, Y. Ji , A. Vladimirov, JHEP 05 (2021) 086]

  Systematics on the lattice: finite lattice spacing and volume effects, pion mass dependence, ...

- Lots of work, but of great impact in phenomenology in the next few years

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