Lattice results on twist-3 parton distributions

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In collaboration with

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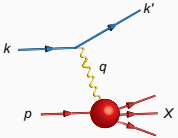
Motivation

- Hadron structure described by a tower of partonic distributions

$$f = \underset{\text{twist-2}}{f^{(0)}} + \frac{1}{Q}\underset{\text{twist-3}}{f^{(1)}} + \frac{1}{Q^2}f^{(2)} + \dots$$



- f: classified according to their twist - order in $1/Q$ at which $f^{(i)}$ appear in factorization theorem of structure functions

$$\sigma_{DIS}(x, Q^2) = \sum_i \left[H^i_{DIS} \otimes f_i\right](x, Q^2)$$

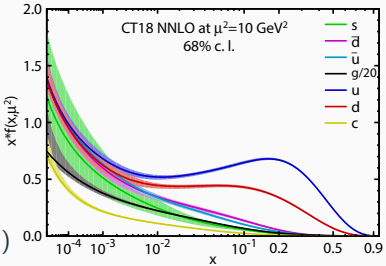
Twist-2 case:

- probabilistic interpretation
- some twist-2 PDFs are very well known

Twist-3 case:

- information about qgq correlations
- kinematically suppressed
- challenging to probe experimentally (JLAB 12 GeV upgrade, EIC)

Unpolarized twist-2 PDFs



[Kovarik et al., 2019, Rev.Mod.Phys. 92 (2020) 4, 045003]

Our lattice work on twist-3 distributions

- Isovector proton twist-3 PDFs: $\boxed{g_T^{u-d}(x), h_L^{u-d}(x)}$

Polarization	PDF	Operator
U	$e(x)$	1
L	$h_L(x)$	σ^{ij}
T	$g_T(x)$	$\gamma_5\gamma_i$

* $g_T^{u-d}(x)$: $\perp$ force acting on the active quark in DIS off a transversely polarized nucleon after it has absorbed the virtual photon [M.Burkardt, PRD 88 (2013) 114502]
* $h_L^{u-d}(x)$: chiral odd, accessible in e.g. Drell-Yan process [R. Jaffe, PRL 67 (1991) 552-555], proton-proton collisions [Y. Koike et al., PLB 759 (2016) 75,],. . .

Interesting aspects to address:

1. Are twist-3 effects sizeable?
2. Wandzura-Wilczek approximation

[S. Wandzura and F. Wilczek, Phys. Lett.72B, 195(1977)][R.L. Jaffe and X. Ji, PRL 67 (1991) 552-555]

E.g.: $g_T(x) \stackrel{?}{=} \int_x^1 dy\, \frac{g_1(y)}{y}$ $\quad g_1(y)$: helicity twist-2 PDF

- Twist-3 GPD $\boxed{\tilde{G}_2(x,\xi,t)}$

$$[\tilde{H} + \tilde{G}_2](x, \xi = 0, Q^2 = 0) = g_T(x)$$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105]

Access twist-3 PDFs through the quasi-distribution approach [X.Ji, Phys. Rev. Lett.110(2013) 262002]

- PDFs computed through purely space-like correlation functions

$$\tilde{\Phi}^{\Gamma}(x,\mu,P_3) = \int \frac{dz}{4\pi} e^{-ixP_3 z} \underbrace{\langle N(P_3)|\bar{\psi}(0)\Gamma W(0,z)\psi(z)|N(P_3)\rangle}_{\text{matrix elements of fast moving nucleons}}{}_{\mu}$$

* μ: renormalization scale
* $z = (0,0,0,z)$
* $\vec{P} = (0,0,P_3)$: proton boost

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- For sufficiently large P_3 ($P_3 >> M$), quasi-PDFs matched to light-cone PDFs

  - twist-2 case: factorization to all orders
    many works between 2013-2020
  - twist-3 case: proved factorization to 1-loop order
    [S.Bhattacharya, K.Cichy, M.Constantinou, A.Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 3, 034005, Phys.Rev.D 102 (2020) 114025]
    [V.M. Braun et al., 2021, JHEP 05 (2021) 086]

LaMET

$$\tilde{\Phi}^{\Gamma}(x,\mu,P_3) = \int_{-1}^{+1} \frac{dy}{|y|} C\left(\frac{x}{y},\mu\right) \Phi(y,\mu) + \mathcal{O}\left(\frac{1}{P_3^2}\right)$$

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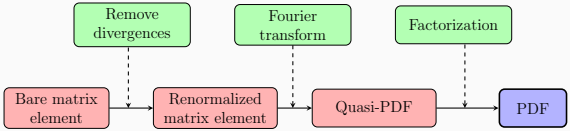
- Road map



Lattice setup

- Configurations of $N_f = 2 + 1 + 1$ flavors & clover term [ETM collaboration]

N_f	$L^3 \times T$	lattice spacing a	m_π	$m_\pi L$
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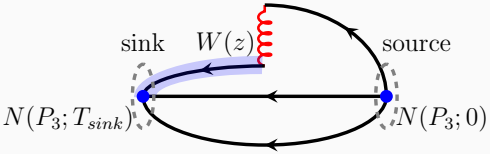
* $\Gamma = \sigma^{12}$ for $h_L(x)$



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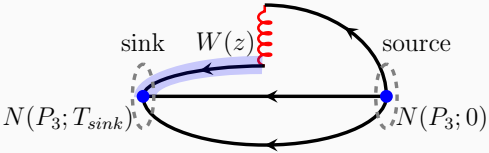

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Lattice techniques:

* sequential inversions for all-to-all propagator (here $T_{sink} = 12a \simeq 1.12$ fm)
* Momentum smearing to improve overlap with proton boosted state [G. Bali et al., Phys.Rev.D 93 (2016) 9, 094515]

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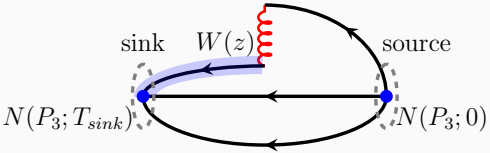

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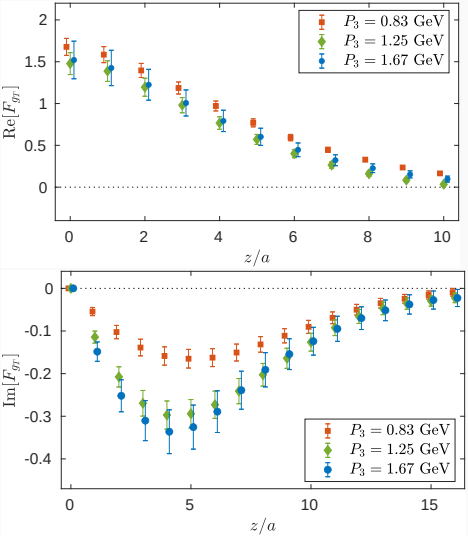
- Momentum dependence of the lattice PDFs studied at three proton boosts

P_3[GeV]	N_{conf}	N_{meas}
0.83	194	1552
1.25	731	11696
1.67	1644	105216

Results: $g_T(x)$

From lattice matrix elements to $g_T^{u-d}(x)$

Bare matrix elements



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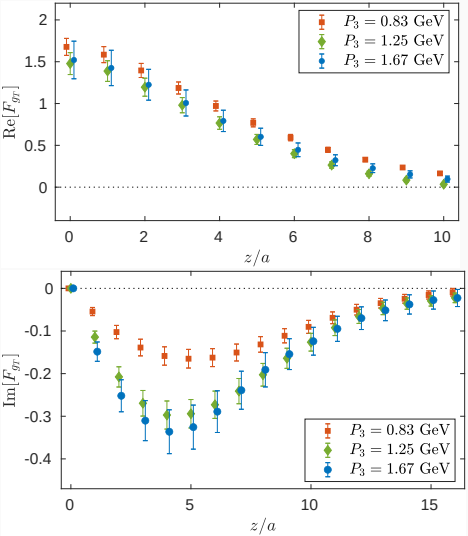
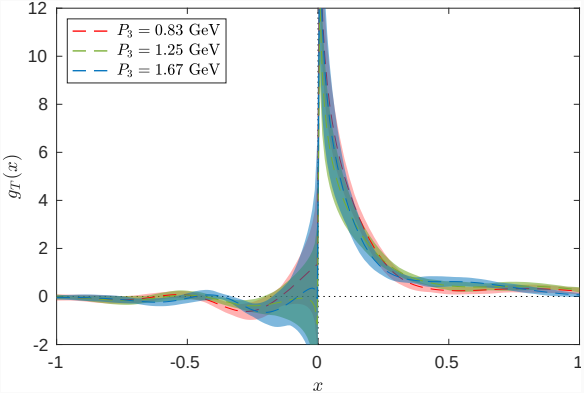


Reconstructed distributions



Antiquarks **Quarks**

(Upon renormalization, F.T and matching)

[S.Bhattacharya, K.Cichy, M. Constantinou,A. Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 11]

Comparison between $g_1^{u-d}(x)$ and $g_T^{u-d}(x)$

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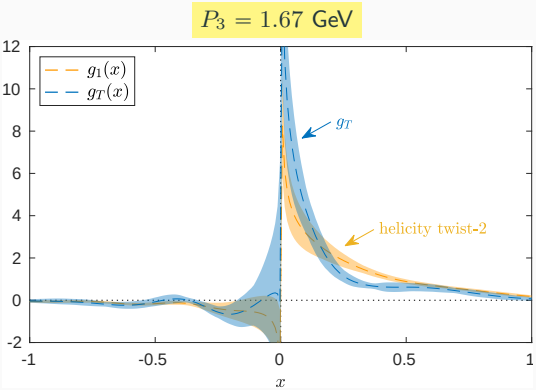

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- $g_1(x)$ extracted on the same gauge ensemble
- g_1 and g_T compatible for antiquarks, $\bar{u} - \bar{d}$
- g_T dominant at small positive x ($x \lesssim 0.2$)
  $\rightarrow$ twist-3 contributions may be sizable

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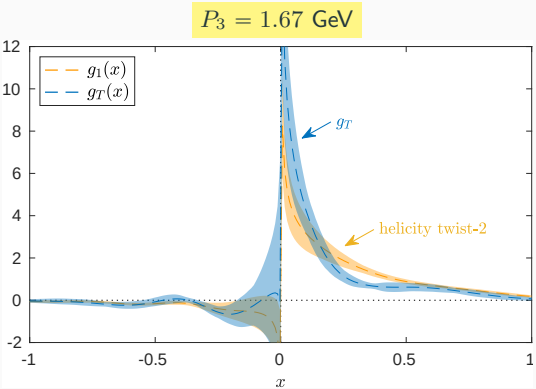

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Burkhardt-Cottingham sum rule:

[H. Burkhardt and W. N. Cottingham, Annals Phys.56,453 (1970)]

$$\int_{-1}^{1} dx\, g_1(x) = \int_{-1}^{1} dx\, g_T(x)$$

(lattice) $\int_{-1}^{1} dx\, g_1(x) - \int_{-1}^{1} dx\, g_T(x) = 0.01(20)$

Wandzura-Wilczek approximation for g_T^{u-d}

- Wandzura-Wilczek relation:

$$g_T(x) = \int_x^1 dy \, \frac{g_1(y)}{y} + g_T^{twist-3}(x) \quad \text{(on the light cone)}$$

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- Approximation: $\boxed{g_T(x) \stackrel{?}{=} \int_x^1 dy\, \frac{g_1(y)}{y} = g_T^{WW}(x)}$

$$d_2 = \int dx\, 3x^2 [g_T(x) - g_T^{WW}(x)] \sim \mathcal{O}(10^{-3})$$

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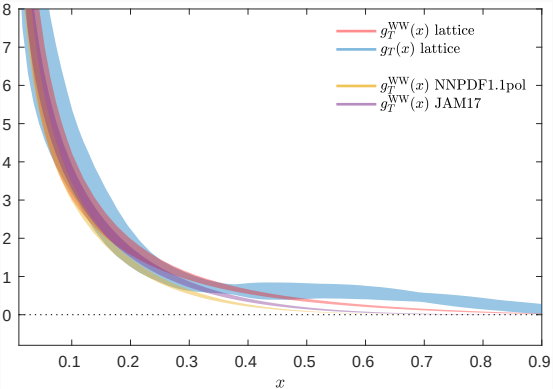
- agreement between g_T and g_T^{WW} for $x \lesssim 0.5$ within uncertainties
- still violations are possible up to $30 - 40\%$
- similar violations are observed in experimental analysis at JLab [A. Accardi et al., JHEP11 (2009) 093]

$P_3 = 1.67$ GeV

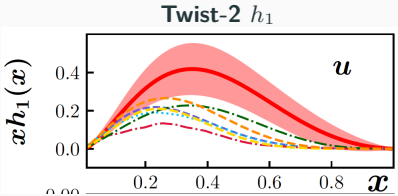


[S.Bhattacharya, K.Cichy, M. Constantinou,A. Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 11]

Beyond $g_T(x)$... The $h_L(x)$ case

Extraction of $h_L(x)$ PDF

- $h_L(x)$ contains the leading twist-2 h_1 transversity PDF
- No experimental data on $h_L(x)$



[JAM, Phys.Rev.D 102 (2020) 5, 054002]

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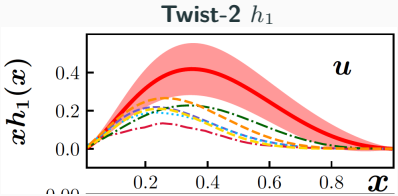
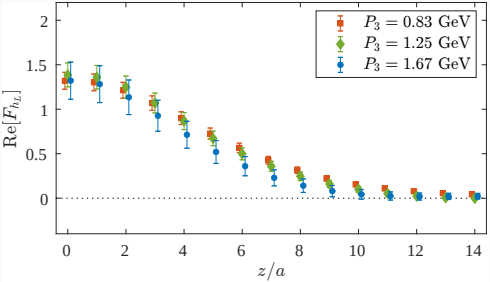
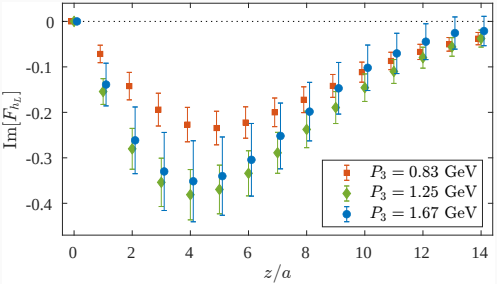


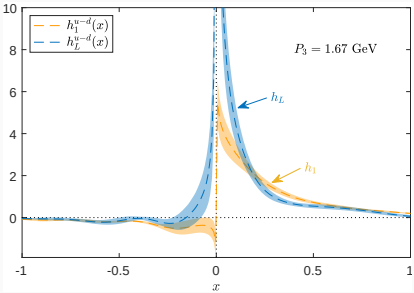
[JAM, Phys.Rev.D 102 (2020) 5, 054002]

h_L from quasi-PDFs $\longrightarrow$ $F_{h_L}(P_3, z) = \langle N(P_3)|\bar{\psi}(0)\sigma^{12}\tau_3 W(0,z)\psi(z)|N(P_3)\rangle$





Results for $h_L(x)$ and twist-2 $h_1(x)$



[S.Bhattacharya, K.Cichy, M. Constantinou,A. Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 11]

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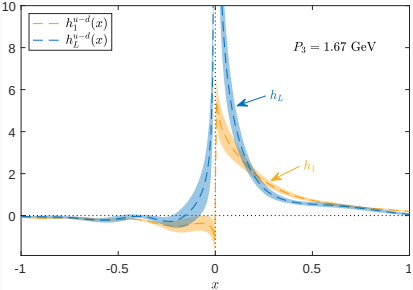

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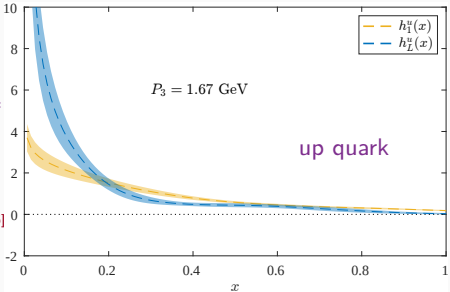
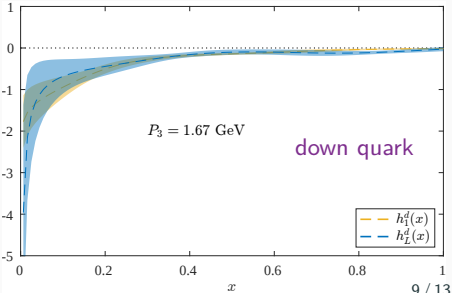
- h_1^{u-d} dominant only for $0.3 \lesssim x \lesssim 0.5$
- up-quark especially dominant for $x < 0.5$
- differences between h_L and h_1 mostly come from the up quark

What about for the single quark flavors?*

*quark loops here neglected - found very small for $h_1(x)$

[C.Alexandrou at al.,arXiv:2106.16065]





Wandzura-Wilczek approximation for $h_L(x)$

$$h_L(x) = 2x \int_x^1 \frac{h_1(y)}{y^2} dy + h_L^{twist-3}(x)$$

[R.L. Jaffe and Xiangdong Ji, Nucl. Phys. B 375 (1992) 527—560]

Test: $\boxed{\mathsf{h}_L(x) \stackrel{?}{=} 2x \int_x^1 dy \, \frac{h_1(y)}{y^2} \equiv h_L^{WW}(x)}$

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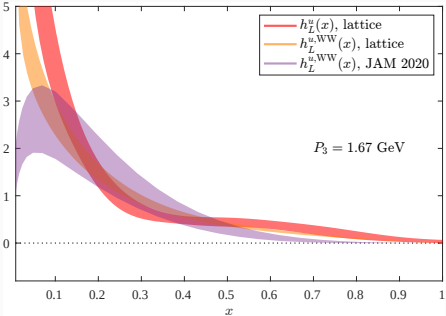
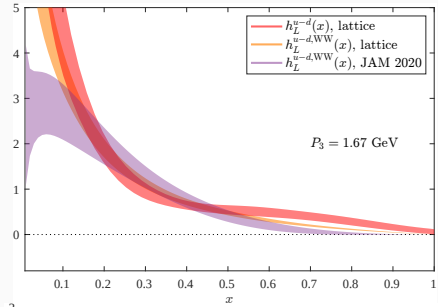
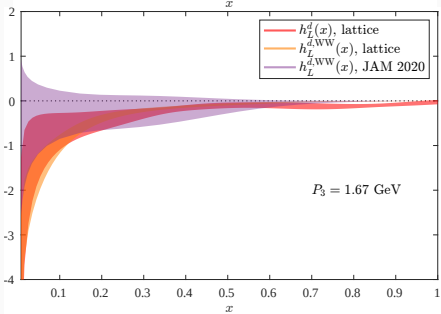
- h_L seems to be largely determined by its twist-2 counterpart within current uncertainties







From twist-3 PDFs...to twist-3 GPDs

Twist-3 GPDs (axial-vector case)

Theoretical framework

(DVCS at twist-3 level)

$$\tilde{F}^{\mu}_{q}(x,\xi,\Delta) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle p'|\bar{q}(\frac{\lambda}{2}n)\gamma^{\mu}\gamma_5 W(\frac{\lambda}{2}n,-\frac{\lambda}{2}n)q(-\frac{\lambda}{2}n)|p\rangle$$

$$\tilde{F}^{\mu} = P^{\mu}\frac{\tilde{h}^{+}}{P^{+}}\tilde{H} + P^{\mu}\frac{\tilde{e}^{+}}{P^{+}}\tilde{E} + \Delta^{\mu}_{\perp}\frac{\tilde{b}}{2m}(\tilde{E} + \tilde{G}_1) + \tilde{h}^{\mu}_{\perp}(\tilde{H} + \tilde{G}_2) + \Delta^{\mu}_{\perp}\frac{\tilde{h}^{+}}{P^{+}}\tilde{G}_3 + \tilde{\Delta}^{\mu}_{\perp}\frac{h^{+}}{P^{+}}\tilde{G}_4$$

$$\Delta = p' - p\,,\, P = \frac{p'+p}{2}$$

$$\xi = -\frac{\Delta^{+}}{2P^{+}}\,,\, P^{+} = \frac{P^{0}+P^{3}}{2}$$

4 twist-3 GPDs to disentangle

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

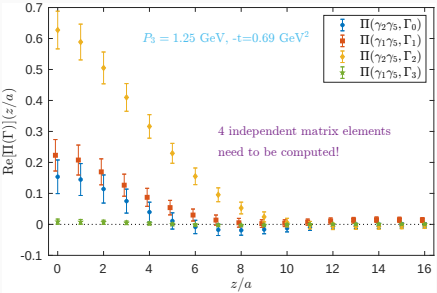
Lattice setup

- Breit frame
- Same ensemble as twist-3 PDFs

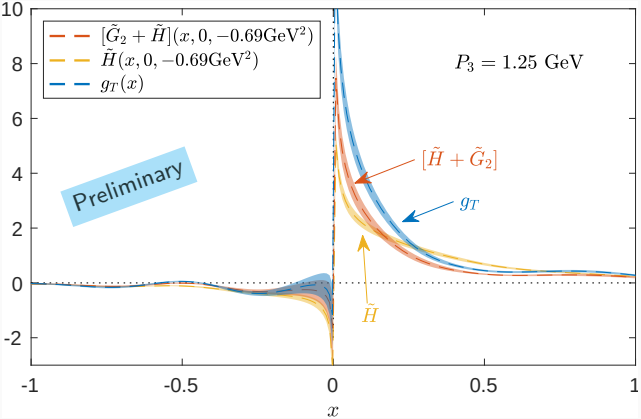
Higher computational cost than PDFs!

P_3[GeV]	$\vec{Q} \times \frac{L}{2\pi}$	$-t$[GeV^2]	ξ	N_{meas}
1.25	$(2,0,0)$	0.69	0	4288
1.25	$(2,2,0)$	1.08	0	8704

many more Q-values will be added [in progress]



x-dependence of twist-3 GPDs ($\xi = 0$)



[S. Bhattacharya, K. Cichy, M. Constantinou, J. Dodson, A. Metz, AS, F. Steffens, (DIS2021) arXiv:2107.12818]

- g_T dominant as expected ($[\tilde{H} + \tilde{G}_2](x, 0, 0) = g_T(x)$)
- $[\tilde{H} + \tilde{G}_2]$ enhanced for small-x with respect to $\tilde{H}$
  (similar behavior as for g_T vs. g_1)

Summary & Outlook

Study of twist-3 PDFs & GPDs

- Lattice investigations can be pursued using quasi-distribution formalism (and also other approaches)
- For the first time, a qualitative comparison between twist-2 and twist-3 PDFs can be made

  → test of the Wandzura-Wilczek approximation for g_T and h_L for $x > 0$

- Several challenges in front of us:
  - can we disentangle the twist-2 and twist-3 parts of g_T and h_L?

    → calculation of quark-gluon-quark matrix elements
    → to what extent numerical results are affected?

    [V.M. Braun, Y. Ji , A. Vladimirov, JHEP 05 (2021) 086]
  - Systematics on the lattice: finite lattice spacing and volume effects, pion mass dependence, ...
- Lots of work, but of great impact in phenomenology in the next few years

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Thank you!