The gluon Sivers function in heavy quark pair and heavy flavor dijet production at the EIC

DingYu Shao
Fudan University

SPIN2021 online Oct 20, 2021

Gluon Sivers function (GSF)

- **Gauge link dependent gluon TMDs**

$$\Gamma^{[U,U']\mu\nu}(x, p_T; n) = \int \frac{d\xi \cdot P d^2\xi_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P, S | F^{n\mu}(0)\, U_{[0,\xi]} F^{n\nu}(\xi) U'_{[\xi,0]} | P, S \rangle \Big|_{\mathrm{LF}}$$

- **GSF: T-odd object; two gauge links; process dependence more involved**
- **For any process GSF can be expressed in terms of two functions:**

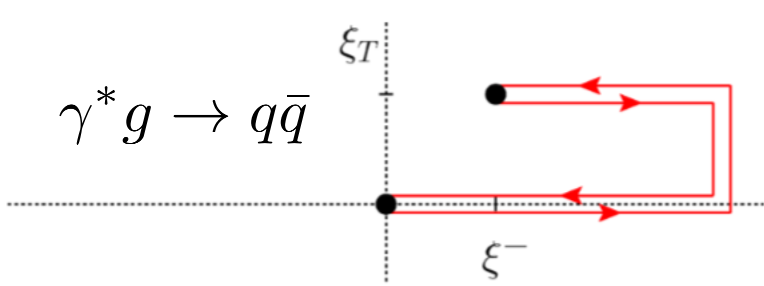
(Buffing, Mukherjee, Mulders'13)

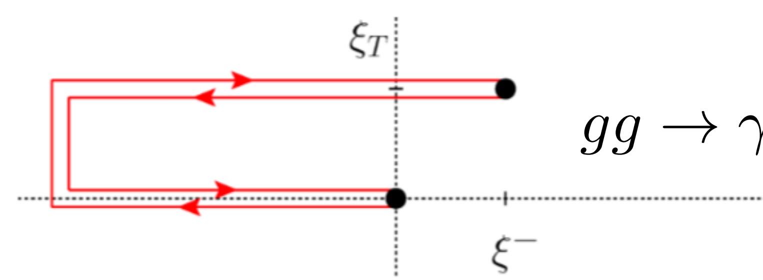
$$f_{1T}^{\perp g[U]}\left(x, \mathbf{k}_\perp^2\right) = \sum_{c=1}^{2} C_{G,c}^{[U]} f_{1T}^{\perp g(Ac)}\left(x, \mathbf{k}_\perp^2\right)$$

calculable for each channel

- $f_{1T}^{\perp g(f)}$ **f-type, C-even**
- $f_{1T}^{\perp g(d)}$ **d-type, C-odd**

$\gamma^* g \to q\bar{q}$





$gg \to \gamma\gamma$

$$f_{1T}^{\perp g[e\,p^\uparrow \to e'\,Q\bar{Q}\,X]}\left(x, p_T^2\right) = -f_{1T}^{\perp g[p^\uparrow\,p \to \gamma\,\gamma\,X]}\left(x, p_T^2\right)$$

Gluon Sivers function (GSF)

- **Theory constrain from Burkardt's sum rule: sum of the transverse momenta of quarks and gluons in a transversely polarized nucleon is zero**
- **Various pp scattering processes suggested to probe GSF**

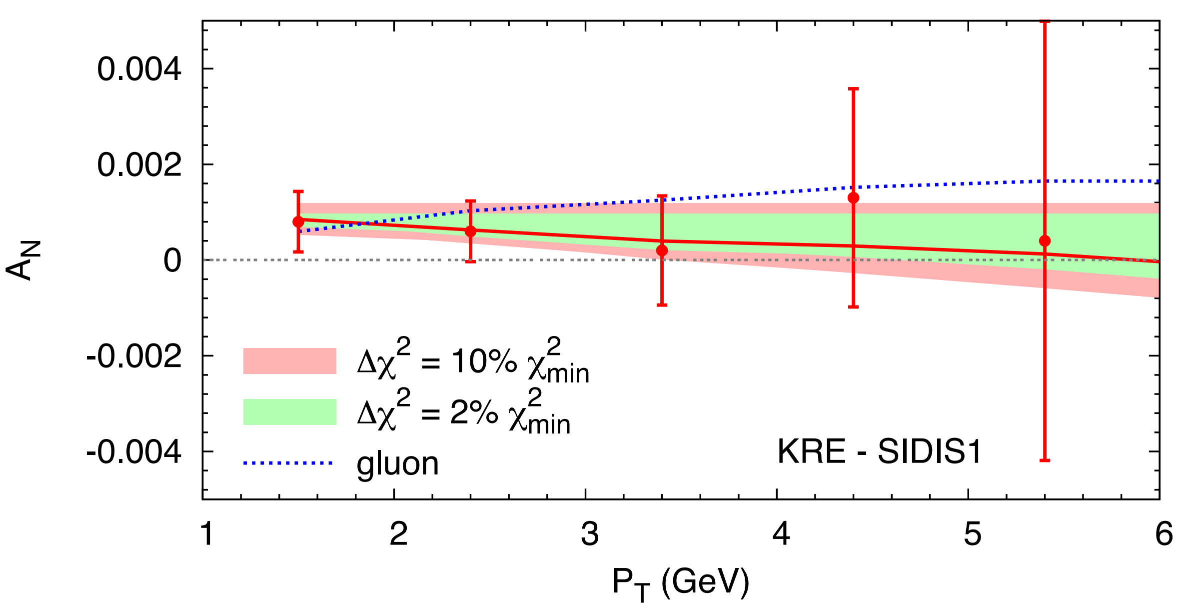
see a review (Boer, Lorce, Pisano, Zhou '15)

$$p^\uparrow p \to \text{jet jet}X \qquad p^\uparrow p \to DX \qquad p^\uparrow p \to \gamma X \qquad p^\uparrow p \to \gamma \text{jet}X \qquad p^\uparrow p \to \text{jet}X \qquad p^\uparrow p \to \pi \text{jet}X$$

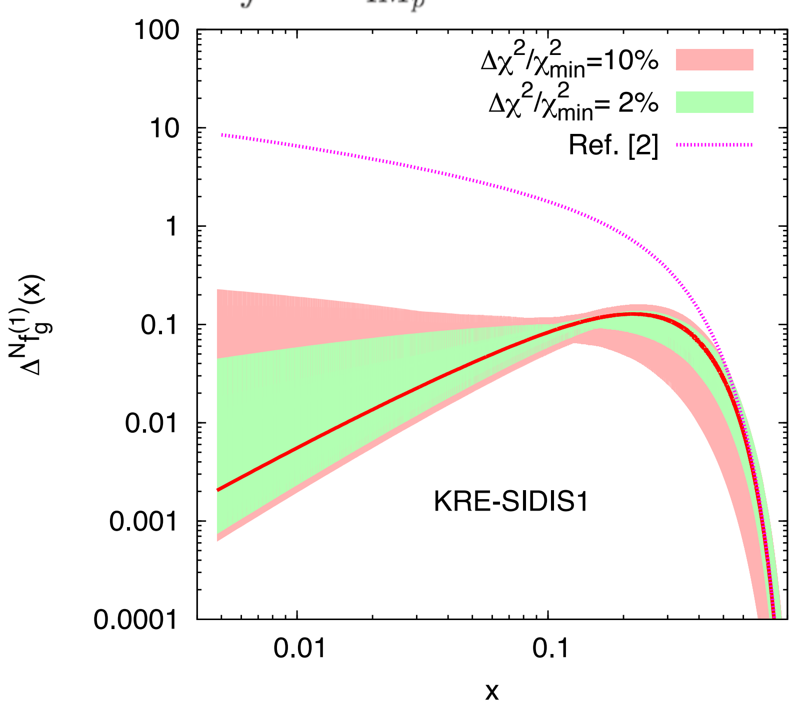
$$p^\uparrow p \to \eta_{c/b}X \qquad p^\uparrow p \to Q\overline{Q}X \qquad p^\uparrow p \to D^0\overline{D}^0X \qquad p^\uparrow p \to J/\psi\gamma X \qquad p^\uparrow p \to J/\psi J/\psi X$$

- **Within generalized parton model, first estimate of the GSF** D'Alesio, Murgia, Pisano '15
  - **twist-3 collinear factorization, indirect constrain on GSF**

$A_N(p^\uparrow p \to \pi X)$



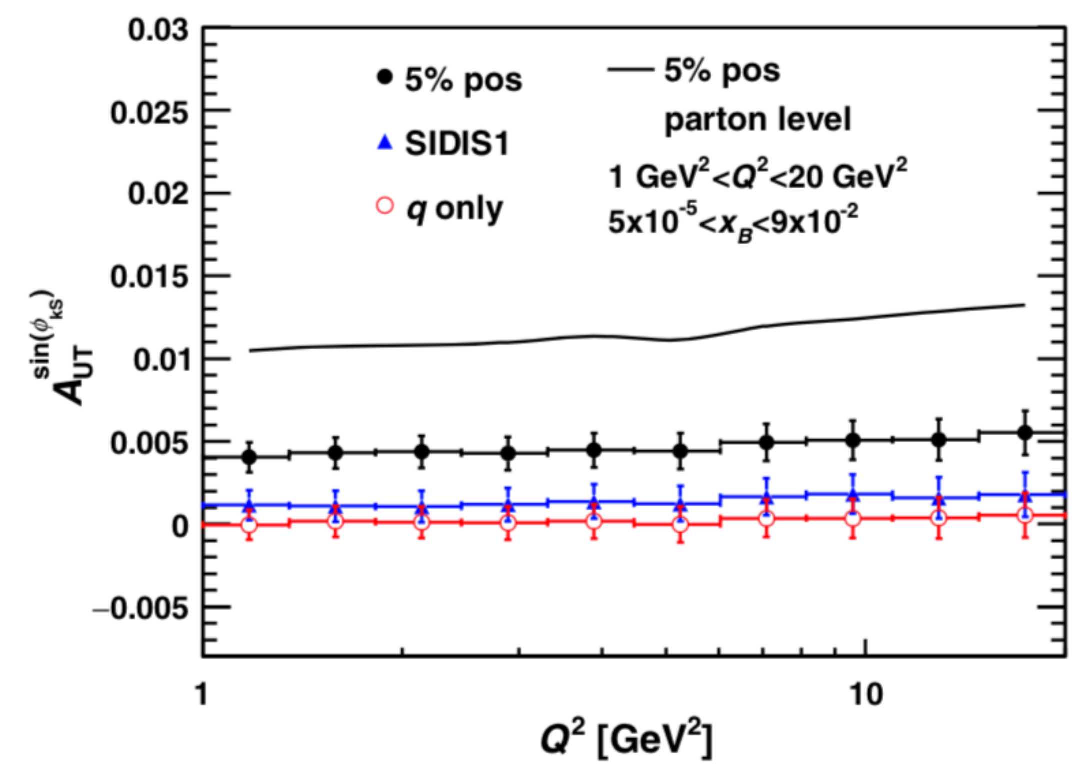
$$\Delta^N f^{(1)}_{g/p^\uparrow}(x) \equiv \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{4M_p} \Delta^N f_{g/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)g}(x)$$



GSF and spin asymmetry at the EIC

- At the EIC the spin asymmetry of the heavy quark pair and dijet production in parton model has been studied in Boer, Mulders, Pisano and Zhou '16
- Accessing of GSF via high-p_T dihadron, open charm and dijet has been investigated using PYTHIA and reweighing methods in Zheng, Aschenauer, Lee, Xiao, Yin '18
  - They find that the dijet process is the most promising channel
- Spin asymmetry from GSF in parton model

$$A_N^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\frac{A_0^T}{A_0^U} = \frac{|\boldsymbol{q}_T|}{M_p}\frac{f_{1T}^{\perp g}\left(x, \boldsymbol{q}_T^2\right)}{f_1^g\left(x, \boldsymbol{q}_T^2\right)}$$



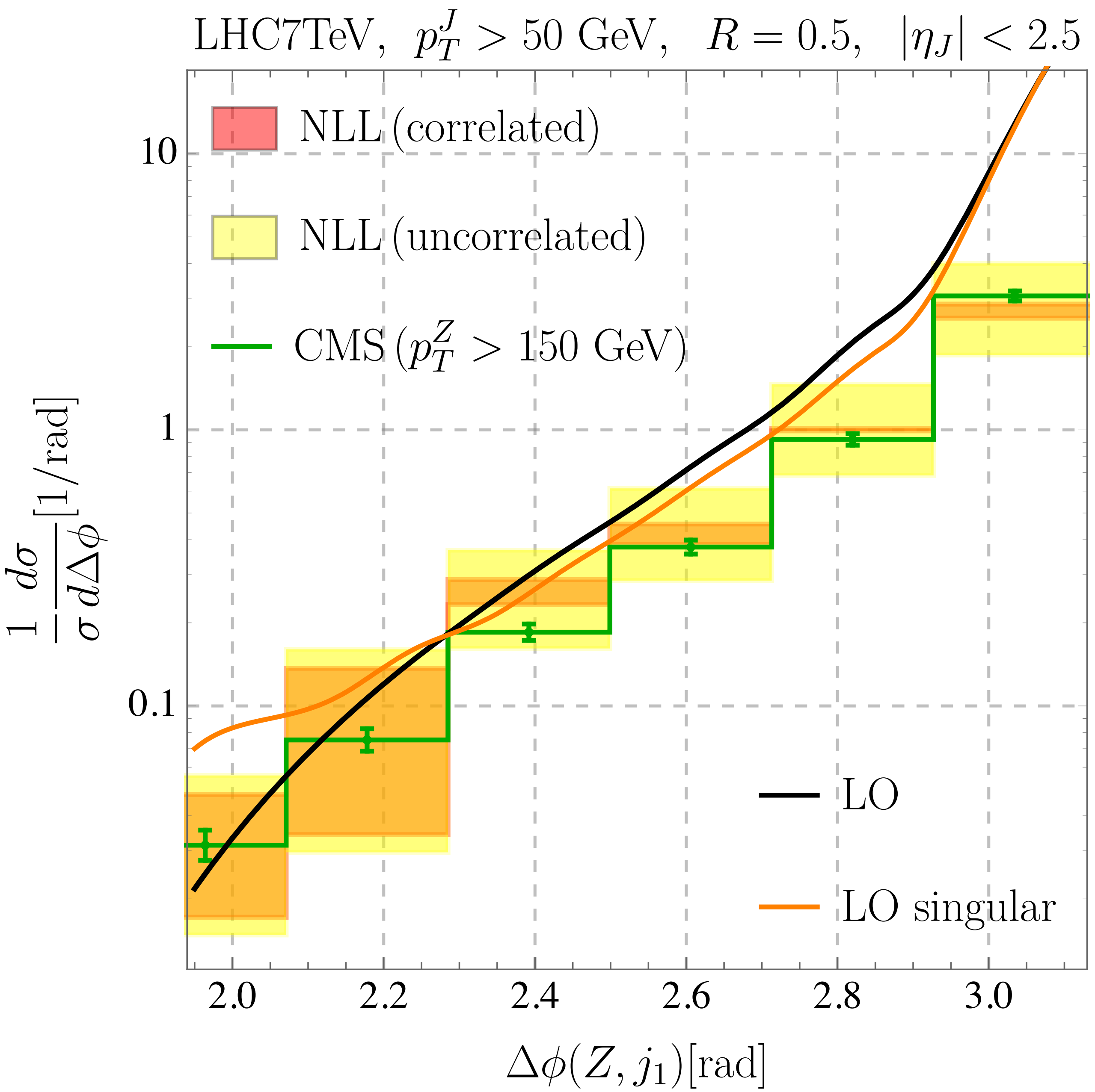
- positivity bounds Mulders, Rodrigues '20

$$\frac{|\boldsymbol{p}_T|}{M_p}\left|f_{1T}^{\perp g}\left(x, \boldsymbol{p}_T^2\right)\right| \leq f_1^g\left(x, \boldsymbol{p}_T^2\right)$$

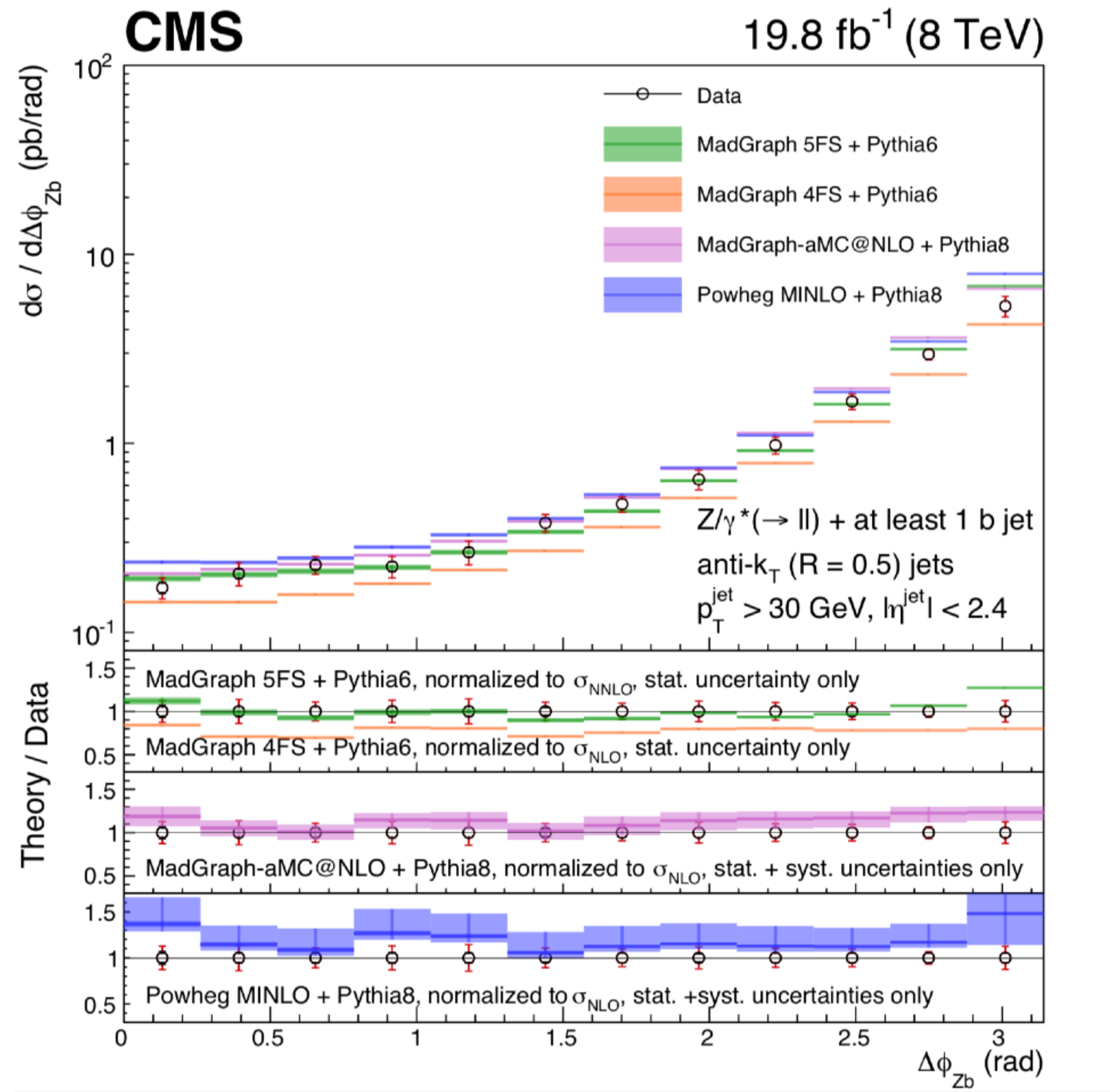
- We recently develop TMD factorization framework for open charm and heavy flavor dijet production, and study spin asymmetry after including QCD evolution. (Kang, Reiten, DYS, Terry '20 JHEP; Kang, Lee, DYS in progress)

Jet TMD studies at the LHC

Chien, DYS & Wu ’19 JHEP



Eur. Phys. J. C 77 (2017) 751



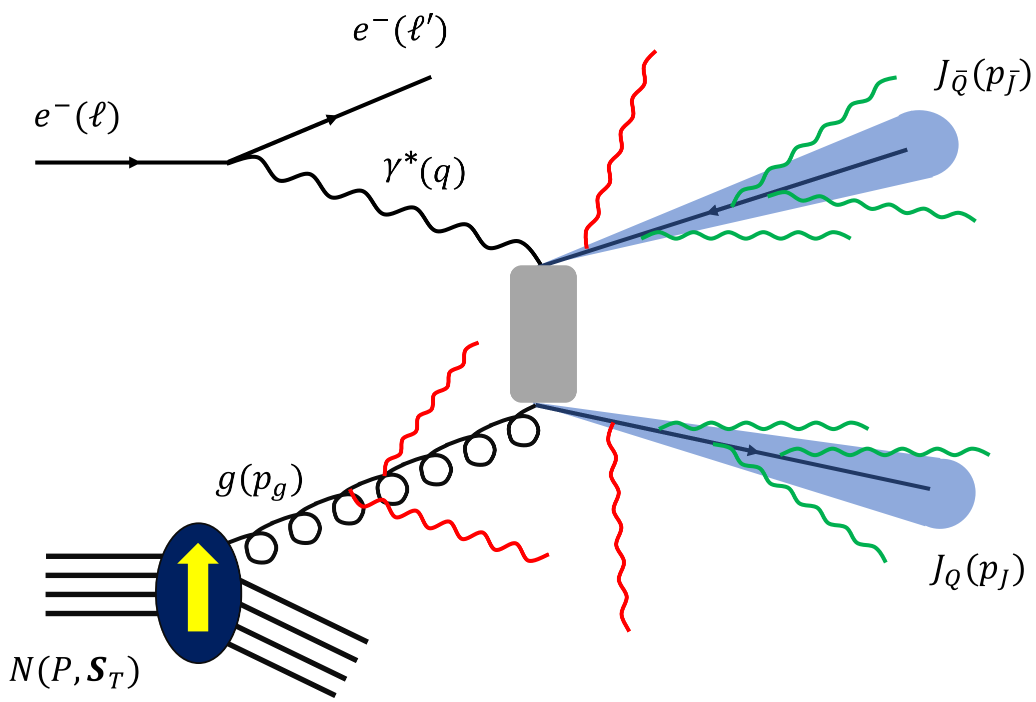
(see also Sun,Yuan,Yuan ’15; Buffing,Kang,Lee,Liu ’18,...)

All-order resummation results are consistent with the LHC data

TMD factorization for heavy-flavor dijet production in DIS

(Kang, Reiten, DYS, Terry '20 JHEP)

$$e(\ell) + N(P, \boldsymbol{S}_T) \to e(\ell') + J_Q(p_J) + J_{\bar{Q}}(p_{\bar{J}}) + X$$



In the Breit frame, the dijet imbalance is defined as $\boldsymbol{q}_T = \boldsymbol{p}_{JT} + \boldsymbol{p}_{\bar{J}T}$

$$q_T R \ll q_T \lesssim m_Q \lesssim p_T R \ll p_T$$

R: Jet radius

M_Q: heavy quark mass

the factorized form of the spin-independent cross section

$$d\sigma^{UU} \sim H(Q, p_T) J_Q(p_T R, m_Q) J_{\bar{Q}}(p_T R, m_Q) S(\boldsymbol{\lambda}_T) f_g(\boldsymbol{k}_T) S^c_Q(\boldsymbol{l}_{QT}) S^c_{\bar{Q}}(\boldsymbol{l}_{\bar{Q}T}) \delta^{(2)}(\boldsymbol{k}_T + \boldsymbol{\lambda}_T + \boldsymbol{l}_{QT} + \boldsymbol{l}_{\bar{Q}T} - \boldsymbol{q}_T)$$

- **Hard, soft and TMD functions are the same as light-jet cases, since p_T>>m_Q**
- **Jet and collinear-soft functions are new, which receive finite quark mass correction**

Heavy quark mass corrections in the evolution equation

Anomalous dimension for the HF quark jet function:

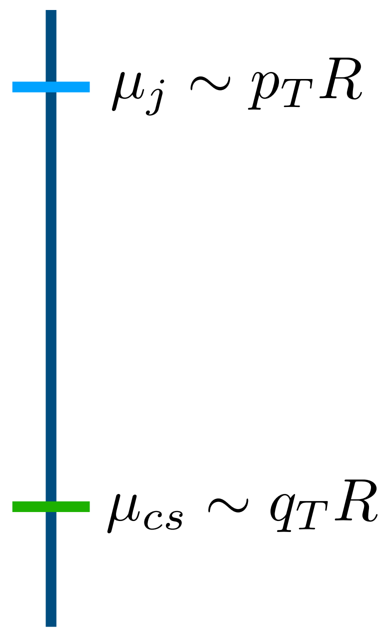
$$\Gamma^{jQ}(\alpha_s) = -C_F\gamma^{\text{cusp}}(\alpha_s)\ln\frac{m_Q^2+p_T^2R^2}{\mu^2} + \gamma^{jQ}(\alpha_s) \qquad \gamma_0^{jQ} = 2C_F\left(3-\frac{2m_Q^2}{m_Q^2+p_T^2R^2}\right)$$

Anomalous dimension for the HF collinear-soft function

$$\Gamma^{cs_Q}(\alpha_s) = C_F\gamma^{\text{cusp}}(\alpha_s)\ln\frac{R^2\mu_b^2}{\mu^2} + \gamma^{cs_Q}(\alpha_s) \qquad \gamma_0^{cs_Q} = -4C_F\left[2\ln\left[-2i\cos(\phi_b-\phi_J)\right] - \frac{m_Q^2}{m_Q^2+p_T^2R^2} - \ln\frac{m_Q^2+p_T^2R^2}{p_T^2R^2}\right]$$

Heavy-quark mass dependence cancels out in

$$\Gamma^{j_Q} + \Gamma^{cs_Q} = \Gamma^{j_q} + \Gamma^{cs_q}$$



Heavy quark mass will contribute the RG evolution between jet and collinear-sot function

RG evolution and resummation

- **Resummation formula:**

$$\frac{d\sigma^{UU}}{dQ^2 dy d^2\boldsymbol{q}_T dy_J d^2\boldsymbol{p}_T} = H(Q, p_T, y_J, \mu_h) \int_0^\infty \frac{bdb}{2\pi} J_0(b\, q_T) f_{g/N}(x_g, \mu_{b*})$$
$$\times \exp\left[-\int_{\mu_{b*}}^{\mu_h} \frac{d\mu}{\mu} \Gamma^h(\alpha_s) - 2\int_{\mu_{b*}}^{\mu_j} \frac{d\mu}{\mu} \Gamma^{j_Q}(\alpha_s) - 2\int_{\mu_{b_*}}^{\mu_{cs}} \frac{d\mu}{\mu} \Gamma^{cs_Q}(\alpha_s) \right]$$
$$\times \exp\left[-S_{\rm NP}(b, Q_0, n\cdot p_g) \right]$$

- **b*-prescription to avoid Landau pole** $b_* = b/\sqrt{1+b^2/b_{\rm max}^2}$ $\quad \mu_{b_*} = 2e^{-\gamma_E}/b_*$

- **Non-perturbative model:** $S_{\rm NP}(b, Q_0, n\cdot p_g) = g_1^f b^2 + \frac{g_2}{2}\frac{C_A}{C_F} \ln\frac{n\cdot p_g}{Q_0} \ln\frac{b}{b_*}$

Sun, Isaacson,Yuan,Yuan '14

- **Typical scales:** $\mu_h \sim p_T, \quad \mu_j \sim R p_T, \quad \mu_{cs} \sim R\mu_{b*}$

Spin dependent cross section

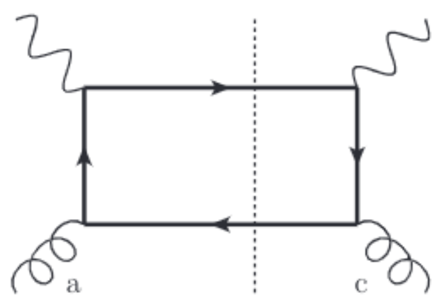
- **Resummation formula:**

$$\frac{d\sigma^{UT}(\boldsymbol{S}_T)}{dQ^2dyd^2\boldsymbol{q}_Tdy_Jd^2\boldsymbol{p}_T}=\sin(\phi_q-\phi_s)\,H(Q,p_T,y_J,\mu_h)\int_0^\infty\frac{b^2db}{4\pi}J_1(b\,q_T)f_{1T,g/p}^{\perp}(x_g,\mu_{b*})$$
$$\times\exp\left[-\int_{\mu_{b*}}^{\mu_h}\frac{d\mu}{\mu}\Gamma^h(\alpha_s)-2\int_{\mu_{b*}}^{\mu_j}\frac{d\mu}{\mu}\Gamma^j(\alpha_s)-2\int_{\mu_{b_*}}^{\mu_{cs}}\frac{d\mu}{\mu}\Gamma^{cs}(\alpha_s)\right]$$
$$\times\exp\left[-S_{\rm NP}^{\perp}(b,Q_0,n\cdot p_g)\right]$$

- **Polarized hard function: For the polarized process, we must consider the attachment of an additional gluon from gauge link in GSF**

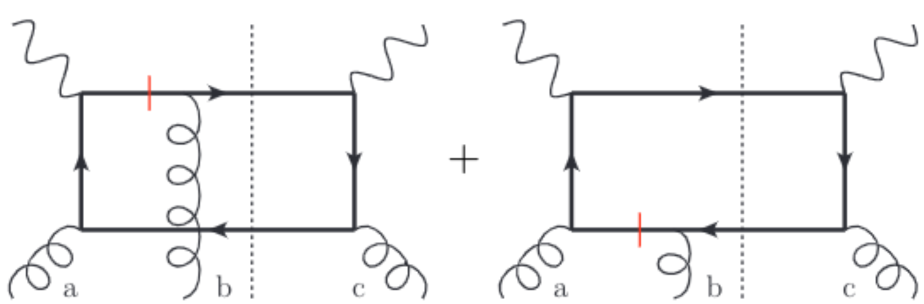
Qiu, Vogelsange, Yuan '07;
Kang, Lee, DYS, Terry, '20 JHEP …

unpolarized:



$$=C_u\,\delta^{ac}\,h^{(0)}(Q,p_T,y_J,\mu)$$

polarized:



$$=(C_1+C_2)\left(-if^{abc}\right)h^{(0)}(Q,p_T,y_J,\mu)$$

polarized and unpolarized hard functions are the same $C_1+C_2=C_u$

f-type gluon Sivers function

Numerical results

Anti-k$_T$, R=0.6

c-jets: $5\,\mathrm{GeV} < p_T < 10\,\mathrm{GeV}, \quad |\eta_J| < 4.5,$

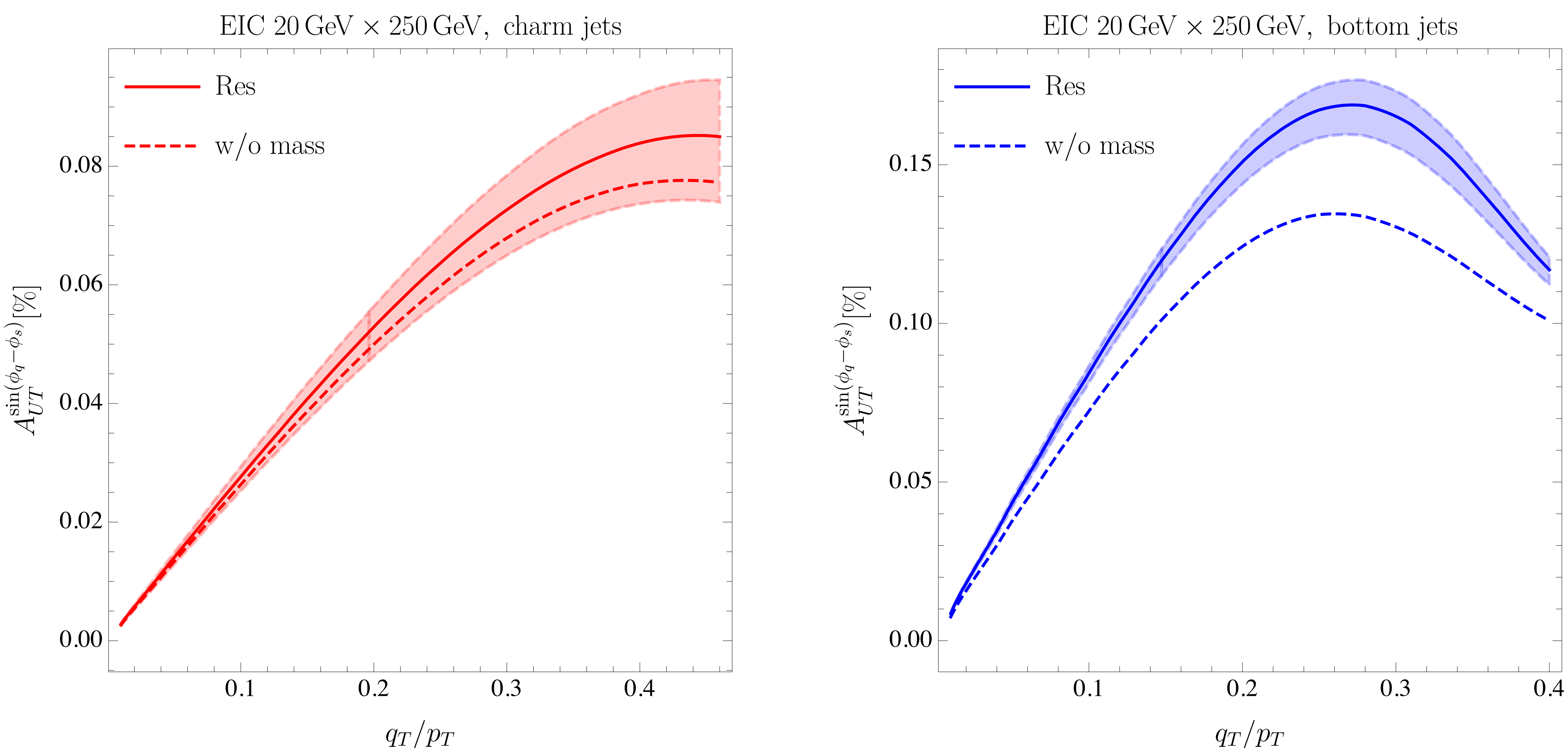
b-jets: $10\,\mathrm{GeV} < p_T < 15\,\mathrm{GeV}, \quad |\eta_J| < 4.5,$

$$d\sigma(\boldsymbol{S}_T) = d\sigma^{UU} + \sin(\phi_q - \phi_s)d\sigma^{UT}$$

$$A_{UT}^{\sin(\phi_q-\phi_s)} = \frac{d\sigma^{UT}}{d\sigma^{UU}}$$

GSF: SIDIS1 set

D'Alesio, Murgia, Pisano '15



Heavy quark mass can give sizable corrections to the predicted asymmetry

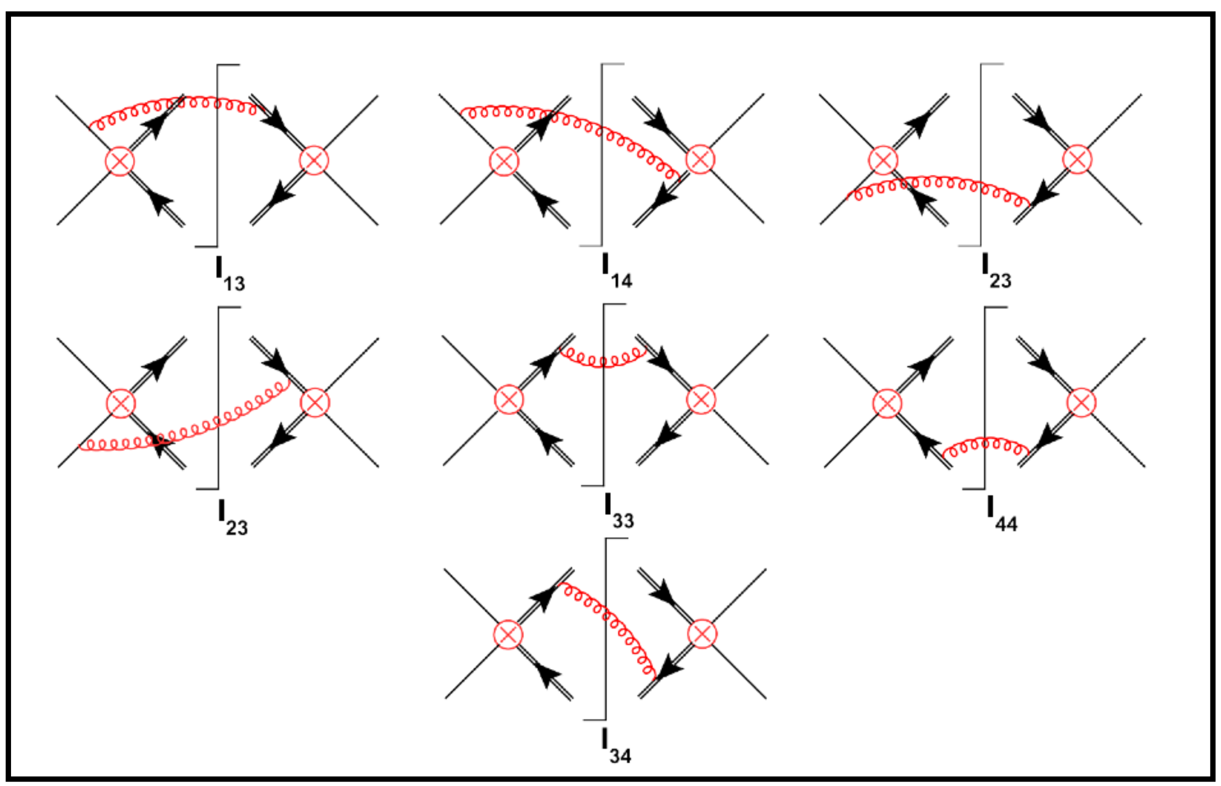
TMD resummation for heavy quark pair at the LHC

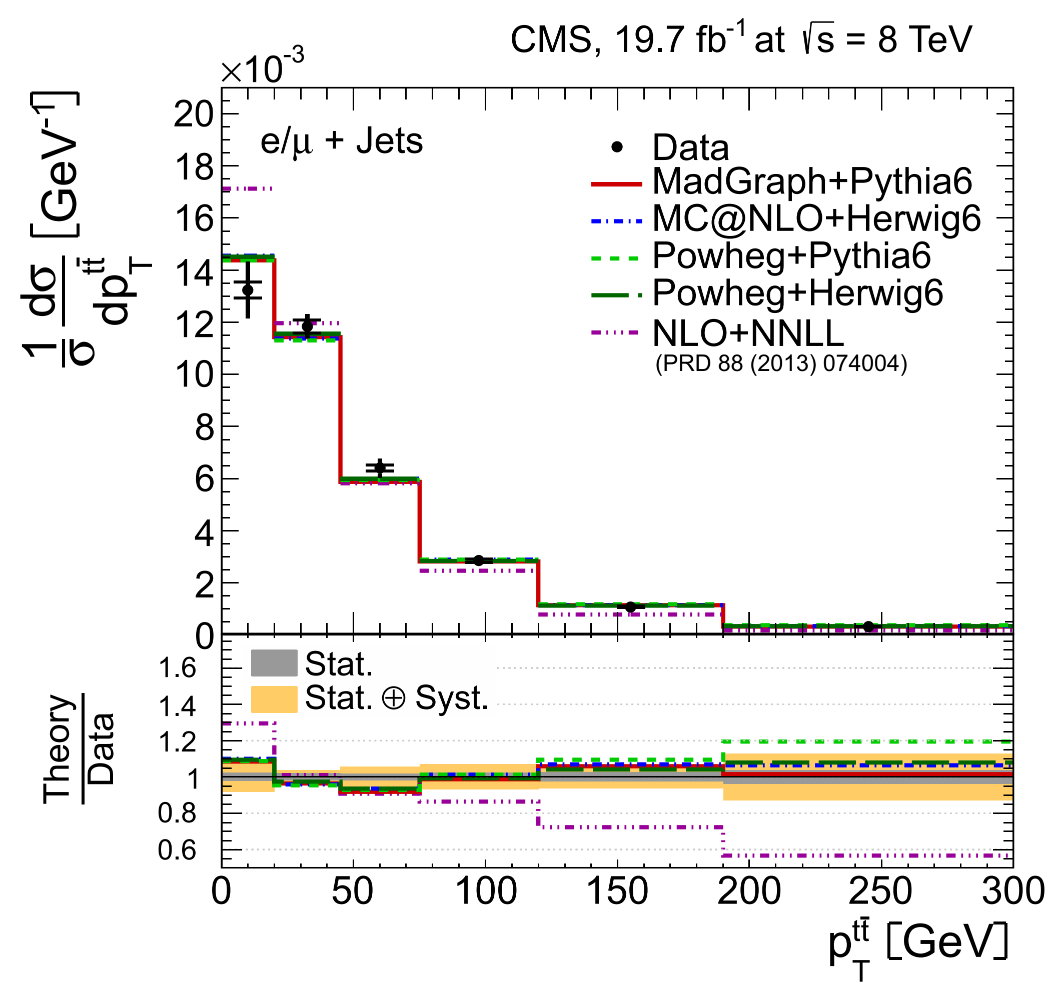
(Li, Li, DYS, Yang, Zhu ’12 PRL ’13 PRD & Catani, Grazinni & Sargsyan ’18)

NNLL predictions for top quark pair production in the small transverse momentum region

$$\frac{d^4\sigma}{dq_T^2\,dy\,dM\,d\cos\theta} = \sum_{i=q,\bar{q},g} \frac{8\pi\beta_t}{3sM}\,\frac{1}{2}\int x_T dx_T J_0(x_T q_T)\left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{i\bar{i}}(x_T^2,\mu)} B_{i/N_1}(\xi_1, x_T^2, \mu) B_{\bar{i}/N_2}(\xi_2, x_T^2, \mu)$$
$$\times\, \mathrm{Tr}[\boldsymbol{H}_{i\bar{i}}(M, m_t, \cos\theta, \mu)\underline{\boldsymbol{S}_{i\bar{i}}(L_\perp, M, m_t, \cos\theta, \mu)}].$$

soft function matrix account for the soft gluon emissions from the final massive states





TMD resummation for heavy quark pair at the EIC

Kang, Lee & Shao in progress

We consider the heavy quark pair production at the EIC in the back-to-back limit

$$e^-(l) + P(P) \to e^-(l') + Q(p_Q) + \bar{Q}(p_{\bar{Q}}) + X$$

Factorization formula $q_T \ll p_T \sim m_Q$

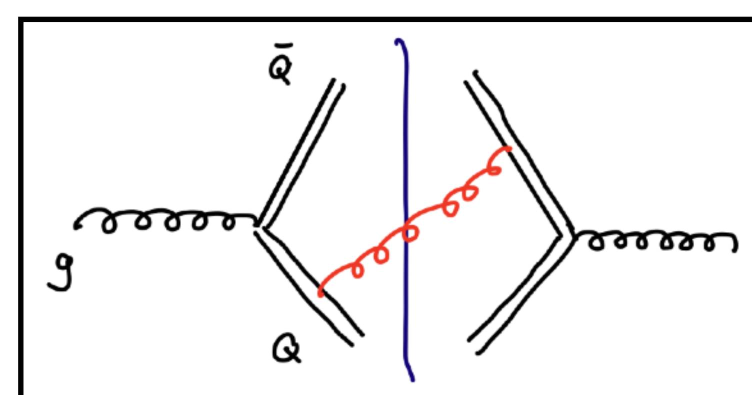
$$\frac{d\sigma_A}{d^2\boldsymbol{q}_T} \propto \int d^2\boldsymbol{k}_T d^2\boldsymbol{\lambda}_T \delta^{(2)}(\boldsymbol{k}_T + \boldsymbol{\lambda}_T - \boldsymbol{q}_T) x\, \Phi_g^{\mu\nu}(x, \boldsymbol{k}_T, \mu, \nu) H_{\mu\nu}(\mu)\, S^{\text{global}}(\boldsymbol{\lambda}_T, \phi_Q, \mu, \nu)$$

gluon TMDs | Hard | soft

Heavy quark mass corrections are included in both hard and soft functions

E.g. soft radiation between two heavy quarks

$$\omega_{QQ} = \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{\mu_{\boldsymbol{b}}^2}\right)^{\epsilon}\left(-\frac{2}{\epsilon} + f_{QQ}\right)$$



$$f_{Q\bar{Q}} = -\frac{1+\beta_Q^2}{2\beta_Q}\,\text{sign}\,(c_{bQ})\left[L_\zeta\left[\zeta\left(c_{bQ}, \alpha_Q\right), \alpha_Q\right] - L_\zeta\left[\zeta\left(-c_{bQ}, \alpha_Q\right), \alpha_Q\right]\right]$$

$$\beta_Q = \sqrt{1 - \frac{4m_Q^2}{\hat{s}}}, \qquad \alpha_Q = \cos^2(\phi_b + \phi_Q)\sin^2\theta_Q, \quad c_{bQ} = \frac{\beta_Q}{\sqrt{1-\beta_Q^2}}\cos(\phi_b + \phi_Q)\sin\theta_Q,$$

Numerical results

Spin-dependent cross section

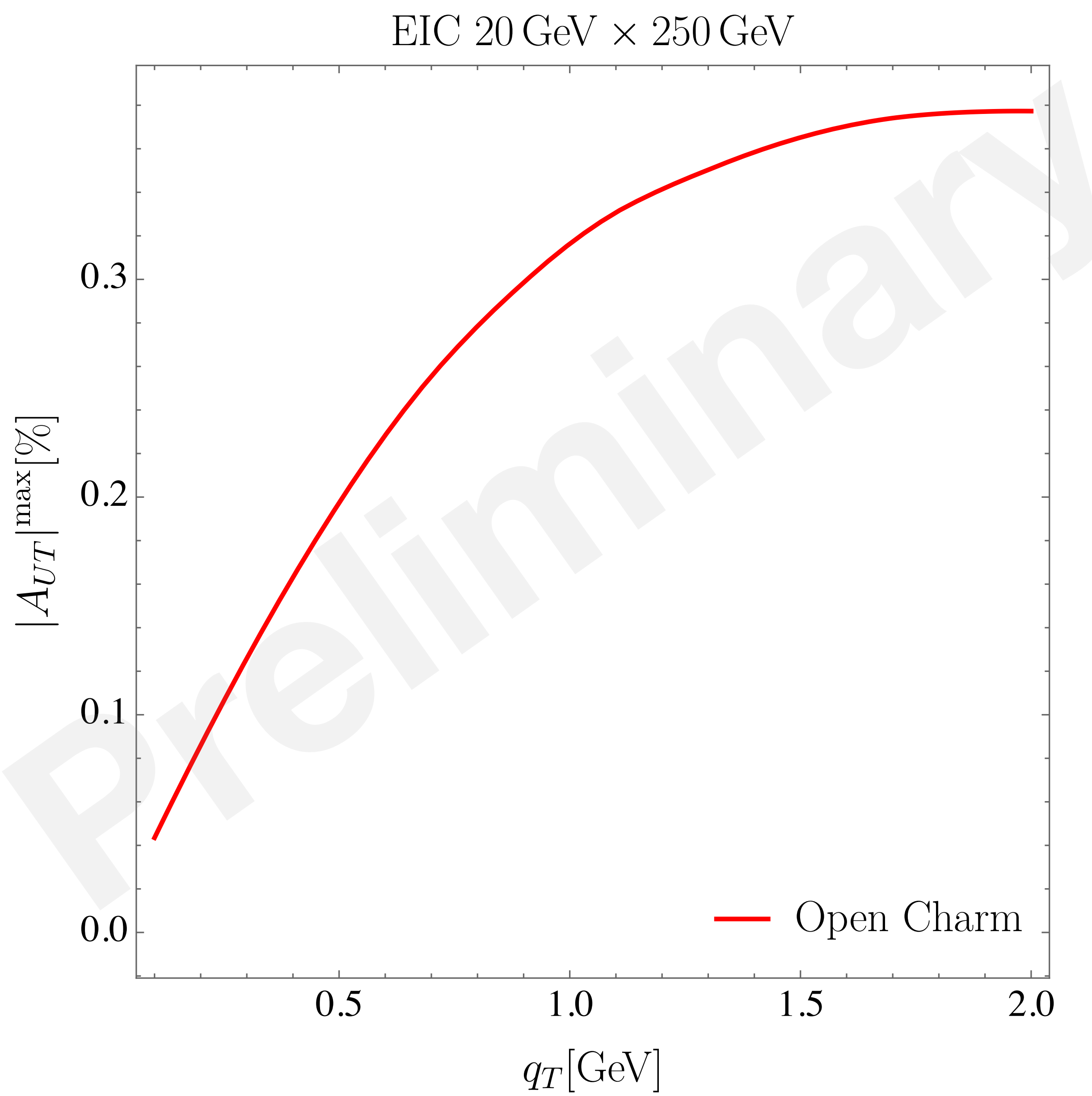
$$d\sigma(\boldsymbol{S}_T) = d\sigma^{UU} + \sin(\phi_q - \phi_s) d\sigma^{UT}$$

$$A_{UT}^{\sin(\phi_q-\phi_s)} = \frac{d\sigma^{UT}}{d\sigma^{UU}}$$

Positivity bounds Mulders, Rodrigues '20

$$\frac{|\boldsymbol{p}_T|}{M_p}\left|f_{1T}^{\perp g}\left(x, \boldsymbol{p}_T^2\right)\right| \leq f_1^g\left(x, \boldsymbol{p}_T^2\right)$$

Sivers asymmetry is significantly reduced after including the QCD evolution



Conclusion

- **Heavy quark pair and dijet production can be used to study gluon TMDs at the EIC**
- **We develop the TMD factorization formalism for heavy flavor dijet and heavy quark pair production in electron polarized proton collisions**
- **In both cases we consider heavy quark mass correction in the factorization formula, as well as the associated evolution equations**
- **We generate predictions for the Sivers asymmetry in these two processes**
- **In heavy flavor dijet process, we find the mass correction could be important for future global analyses for bottom dijet production**
- **In open charm process, we find the QCD evolution reduces the size of the asymmetry**

Thank you