Extracting the Λ polarizing Fragmentation Function from Belle e^+e^- data within the TMD formalism

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Istituto Nazionale di Fisica Nucleare

Motivations and Contents

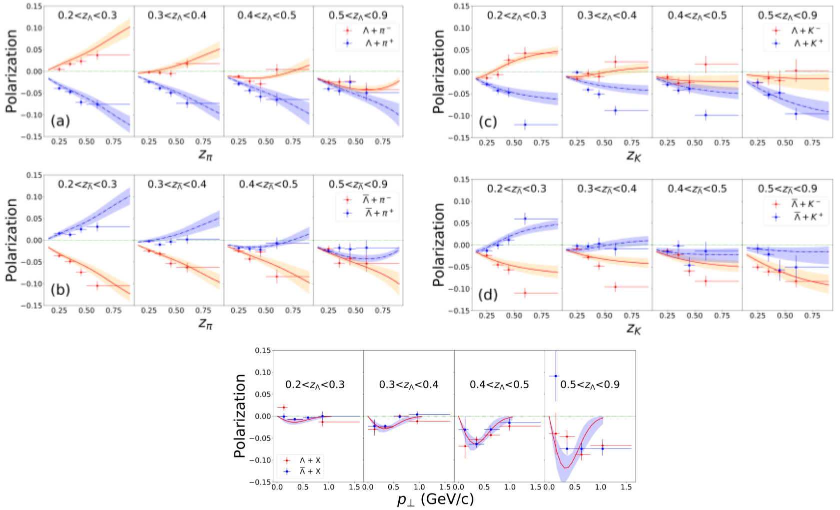
- $e^+e^- \to h^{\uparrow}{}_1 h_2 X$
- $e^+e^- \to \mathrm{h}_1^{\uparrow}(\mathrm{jet})X$

Processes used to study and analyze the Belle data in order to extract the Λ polarizing FF in a simplified TMD approach.

See U. D'Alesio's talk

Belle Data:
[Y. Guan et al., Phys. Rev. Lett. 122. 042001 (2019)]

Our previous analysis:
[D'Alesio, Murgia, Zaccheddu, Phys. Rev. D 102, 054001 (2020)]



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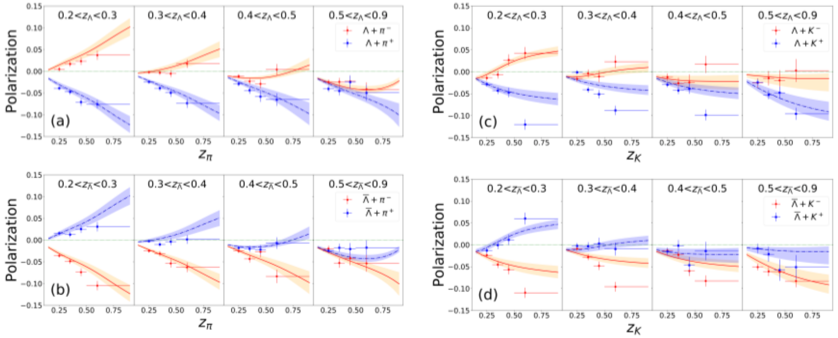

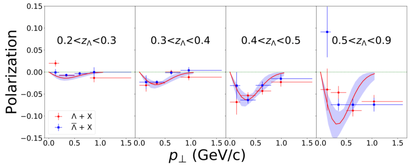

Reanalyze the Belle data within a proper TMD factorization scheme using the evolution equations for TMD FFs

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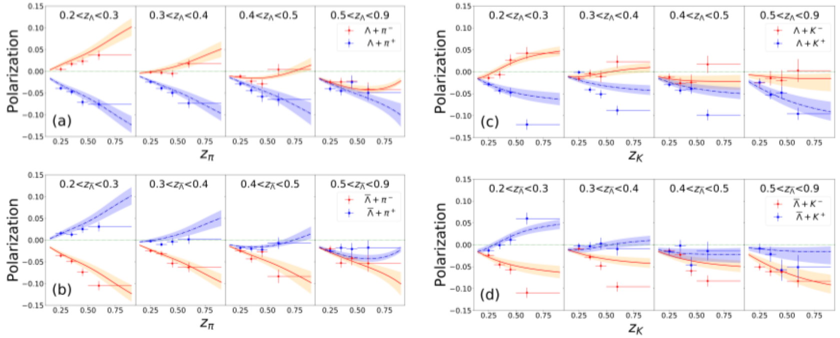

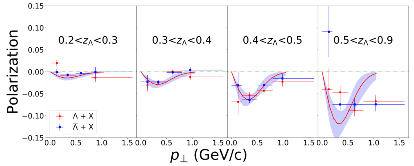

Reanalyze the Belle data within a proper TMD factorization scheme using the evolution equations for TMD FFs

Contents:

- Polarization: $e^+e^- \to h^{\uparrow}{}_1 h_2 X$
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- Convolutions, TMDs Evolution Equations and Models
- Preliminary Results
- Conclusions

Polarization 2-h: double hadron production

$$P_T^h(z_1, z_2) = \frac{\int d^2\mathbf{q}_T\, F_{TU}^{\sin(\phi_1-\phi_S)}}{\int d^2\mathbf{q}_T\, F_{UU}} = \frac{M_{h_1} \int d^2\mathbf{q}_T\, \mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\, \widetilde{\bar{D}}\right]}{\int d^2\mathbf{q}_T\, \mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right]}$$

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$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q) \sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(q_T\, b_T) \widetilde{D}_{1T}^{\perp(1)}(z_1, b_*; \zeta_1, \mu_b) \widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$
$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right) \tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right) \gamma_K(g(\mu')) \right] \right\}$$
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Different $g_K(b_T)$ functions

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Polarization 2-h: double hadron production

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\,\widetilde{\widetilde{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(q_T\, b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\widetilde{D}}(z_2, b_*; \zeta_2, \mu_b)$$

$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$

$$\times M_{D_1}^{\perp}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\widetilde{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\widetilde{D}}(z_2, b_*; \zeta_2, \mu_b)$$

$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$

$$\times M_{D_1}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

Polarization 2-h: double hadron production

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\widetilde{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(q_T\, b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\widetilde{D}}(z_2, b_*; \zeta_2, \mu_b)$$

$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$

$$\times M_{D_1}^{\perp}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\widetilde{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\widetilde{D}}(z_2, b_*; \zeta_2, \mu_b)$$

$$\times \exp\left\{ \ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$

$$\times M_{D_1}(b_T)\, M_{D_2}(b_T)\exp\left\{ - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$J_{0,1}(b_T q_T)$ Bessel Function

$H_{0,1}(b_T q_T)$ Struve Function

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_1(b_T\, q_T) = \frac{\pi q_{T_{max}}}{2b_T}\{J_1(b_T\, q_{T_{max}})H_0(b_T\, q_{T_{max}}) - J_0(b_T\, q_{T_{max}})H_1(b_T\, q_{T_{max}})\}$$

Polarization 2-h: double hadron production

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(q_T\, b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1, b_*;\zeta_1,\mu_b)\widetilde{\bar{D}}(z_2, b_*;\zeta_2,\mu_b)$$
$$\times \exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*,\mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1,g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}^{\perp}(b_T)\, M_{D_2}(b_T)\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}(z_1, b_*;\zeta_1,\mu_b)\widetilde{\bar{D}}(z_2, b_*;\zeta_2,\mu_b)$$
$$\times \exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*,\mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1,g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
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$J_{0,1}(b_T q_T)$ Bessel Function

$H_{0,1}(b_T q_T)$ Struve Function

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_1(b_T\, q_T) = \frac{\pi q_{T_{max}}}{2b_T}\{J_1(b_T\, q_{T_{max}})H_0(b_T\, q_{T_{max}}) - J_0(b_T\, q_{T_{max}})H_1(b_T\, q_{T_{max}})\}$$

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_0(b_T\, q_T) = \frac{q_{T_{max}}}{b_T} J_1(b_T\, q_{T_{max}})$$

Polarization 2-h: double hadron production

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(q_T\, b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$
$$\times \exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
$$\times M_{D_1}^{\perp}(b_T)\, M_{D_2}(b_T)\exp\left\{-g_K(b_T)\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)\right\}$$

$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right] = \mathcal{H}^{(e^+e^-)}(Q)\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}(z_1, b_*; \zeta_1, \mu_b)\widetilde{\bar{D}}(z_2, b_*; \zeta_2, \mu_b)$$
$$\times \exp\left\{\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(1, g(\mu')) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(g(\mu'))\right]\right\}$$
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$$\int_0^{q_{T_{max}}} dq_T\, q_T J_1(b_T\, q_T) = \frac{\pi q_{T_{max}}}{2b_T}\left\{J_1(b_T\, q_{T_{max}})H_0(b_T\, q_{T_{max}}) - J_0(b_T\, q_{T_{max}})H_1(b_T\, q_{T_{max}})\right\}$$

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_0(b_T\, q_T) = \frac{q_{T_{max}}}{b_T}J_1(b_T\, q_{T_{max}})$$

$$q_{Tmax} = Q * \eta$$
η = [0,2 – 0,3]

Polarization: single hadron with thrust

$$\mathcal{P}(z_1, p_\perp) = \frac{d\Delta\sigma/dz_1 d^2 p_\perp}{d\sigma/dz_1 d^2 p_\perp}$$

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$$\frac{d\Delta\sigma}{dz_1 d^2 p_\perp} = \sigma_0 \sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(b_T\, q_T) \widetilde{D}_{1T}^{\perp(1)}(z_1, \mu_b)\, U_{NG}(\mu_{b_*}, Q)$$
$$\times \exp\left\{ \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D(1, g(\mu')) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K(g(\mu')) \right] \right\}$$
$$\times M_{D_1}^{\perp}(b_T)\ \exp\left\{ - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}} \right) \right\}$$

$$\frac{d\sigma}{dz_1 d^2 p_\perp} = \sigma_0 \sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T) \widetilde{D}(z_1, \mu_b)\, U_{NG}(\mu_{b_*}, Q)$$
$$\times \exp\left\{ \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D(1, g(\mu')) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K(g(\mu')) \right] \right\}$$
$$\times M_{D_1}(b_T)\ \exp\left\{ - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}} \right) \right\}$$

Polarization: single hadron with thrust

$$\mathcal{P}(z_1, p_\perp) = \frac{d\Delta\sigma/dz_1 d^2p_\perp}{d\sigma/dz_1 d^2p_\perp}$$

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$$\times M_{D_1}^{\perp}(b_T)\, \exp\left\{ -g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}} \right) \right\}$$

$$\frac{d\sigma}{dz_1 d^2p_\perp} = \sigma_0 \sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\, \widetilde{D}(z_1, \mu_b)\, U_{NG}(\mu_{b_*}, Q)$$
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Polarization: single hadron with thrust

$$\mathcal{P}(z_1, p_\perp) = \frac{d\Delta\sigma/dz_1 d^2 p_\perp}{d\sigma/dz_1 d^2 p_\perp}$$

Z.-B. Kang, D.Y. Shao, F. Zhao, J. High Energy Phys. 12 (2020) 127

L. Gamberg et al., Phys.Lett.B 818 (2021) 136371

$$\frac{d\Delta\sigma}{dz_1 d^2 p_\perp} = \sigma_0 \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T^2 J_1(b_T\, q_T) \widetilde{D}_{1T}^{\perp(1)}(z_1, \mu_b) U_{NG}(\mu_{b_*}, Q)$$
$$\times \exp\left\{ \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D(1, g(\mu')) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K(g(\mu')) \right] \right\}$$
$$\times M_{D_1}^{\perp}(b_T) \exp\left\{ -g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}}\right) \right\}$$

$$U_{NG}(\mu_{b_*}, Q) = \exp\left[-C_A C_F \frac{\pi^2}{3} u^2 \frac{1+(au)^2}{1+(bu^c)} \right]$$

M. Dasgupta, G.P. Salam, Phys. Lett. B 512 (2001) 323

$$\frac{d\sigma}{dz_1 d^2 p_\perp} = \sigma_0 \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T J_0(b_T\, q_T) \widetilde{D}(z_1, \mu_b) U_{NG}(\mu_{b_*}, Q)$$
$$\times \exp\left\{ \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*, \mu_{b_*}) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D(1, g(\mu')) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K(g(\mu')) \right] \right\}$$
$$\times M_{D_1}(b_T) \exp\left\{ -g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1}}{\sqrt{\zeta_{1,0}}}\right) \right\}$$

$$u = \frac{1}{\beta_0} \ln\left[\frac{\alpha_s(\mu_b)}{\alpha_s(Q)}\right]$$

$g_K(b_T)$: 2 hadrons

$$g_2 \ln\left(\frac{b_T}{b_*}\right) \ln\left(\frac{Q}{Q_0}\right)$$

$g_2 = 0{,}84$
$Q_0^2 = 2{,}4\ GeV^2$
[1]

$$\frac{\alpha_s(C_1/b_*)C_F}{\pi} \ln(1 + b_T^2/b_{max}^2) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$$

[2]

$$g_0(b_{max})\left(1 - \exp\left[-\frac{C_F \alpha_s(\mu_{b_*}) b_T^2}{\pi g_0(b_{max}) b_{max}^2}\right]\right) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$$

$g_0 = 0{,}55$
$b_{max} = 0{,}8$
[2]

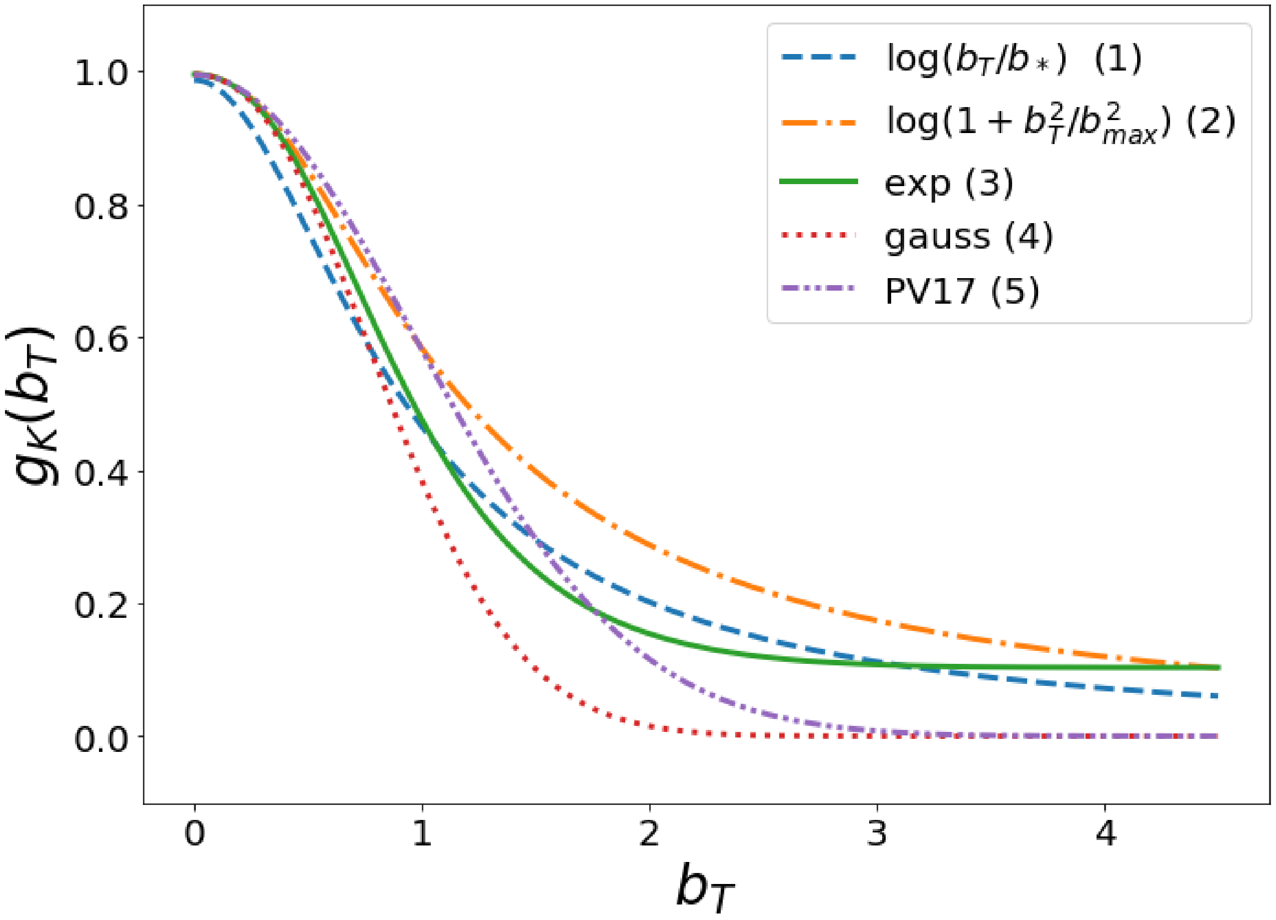
$$\frac{C_F}{\pi} \frac{b_T^2}{b_{max}^2} \alpha_s(\mu_{b_*}) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$$

[2]

$$g_2\, b_T^2/2 \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$$

$g_2 = 0{,}13$
[3]

$$\exp\left\{ - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_1 \zeta_2}}{\sqrt{\zeta_{1,0} \zeta_{2,0}}}\right)\right\}$$



[1] C.A. Aidala, B. Field, L.P. Gamberg, T.C. Rogers, Phys. Rev. D 89 (2014) 094002
P. Sun, J. Isaacson, C.P. Yuan, F. Yuan, Int. J. Mod. Phys. A 33 (2018) 1841006

[2] J. Collins, T. Rogers,Phys.Rev.D 91 (2015) 7, 074020

[3] A. Bacchetta et al, JHEP 06 (2017) 081

Models

PV 17 [3]

$$M_D(b_T) = \frac{g_3\, e^{-b_T^2 \frac{g_3}{4z^2}} + \frac{\lambda_F}{z^2} g_4^2 (1 - g_4 \frac{g_4}{4z^2}) e^{-b_T^2 \frac{g_4}{4z^2}}}{g_3 + \frac{\lambda_F}{z^2} g_4^2}$$

[3] A. Bacchetta et al, JHEP 06 (2017) 081

Gaussian

$$M_D(b_T) = \exp\left(- \frac{\langle p_\perp^2 \rangle b_T^2}{4 z_p^2} \right)$$

Power Law

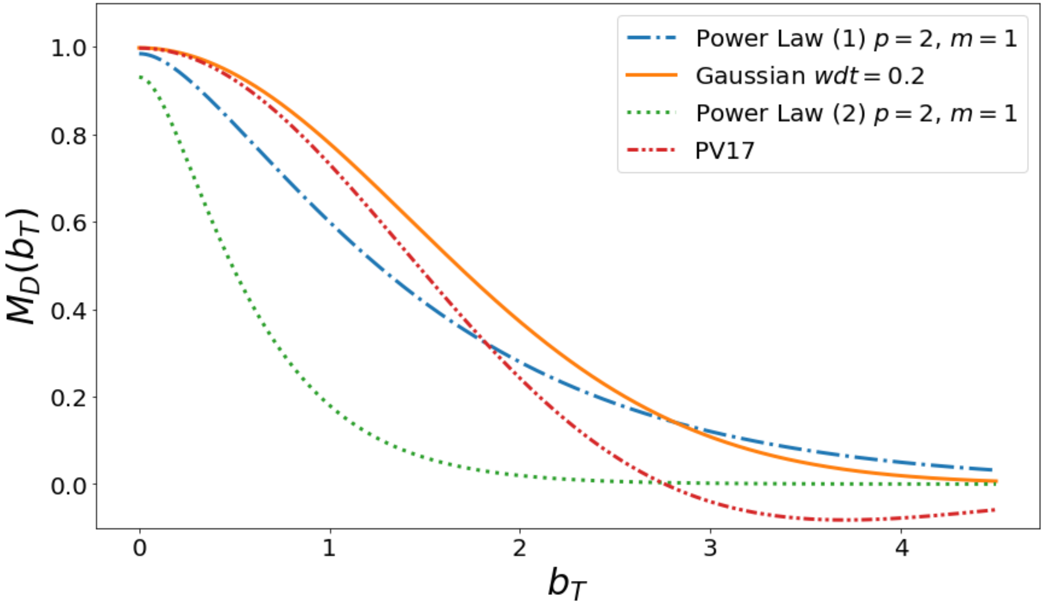
$$M_D^\perp(b_T, p, m) = \frac{2^{2-p}}{\Gamma(p-1)} (b_T m)^{p-1} K_{p-1}(b_T m) \quad (1)$$

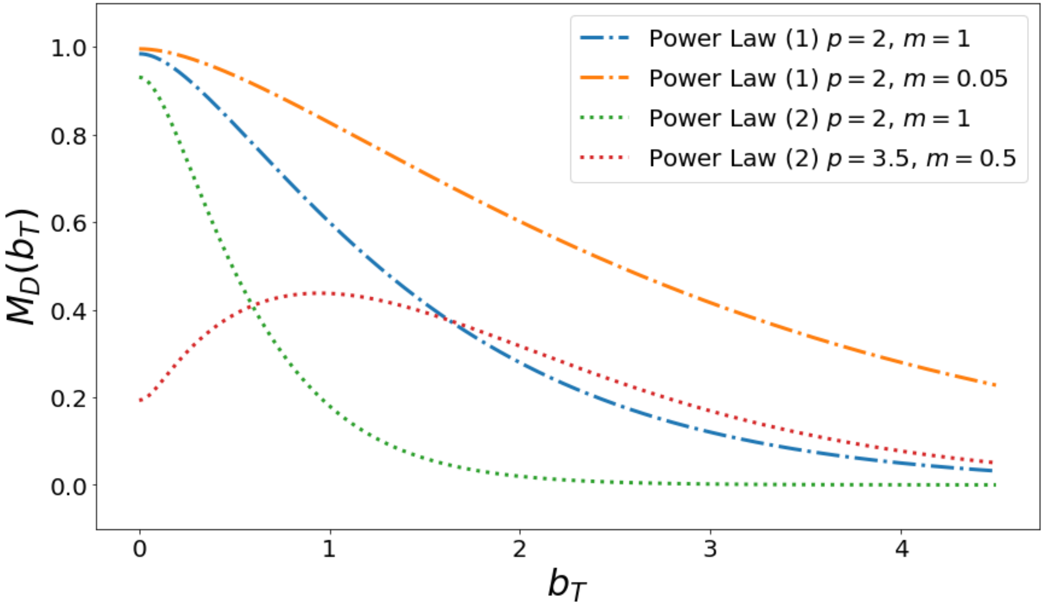
$$M_D^\perp(b_T, p, m) = \frac{2^{2-p}}{\Gamma(p-1)} (b_T m / z_p)^{p-1} K_{p-1}(b_T m / z_p) \quad (2)$$

Unpolarized π/K

Unpolarized Λ

Polarized Λ





Preliminary results: 2-h data Fit

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5\ - 0.9]$ bin excluded → 96 data points

Fitted 8 parameters

$b_{max} = 0{,}8$

$\tilde{\tilde{D}}(z_2; \mu_b)$ Unpolarized FF : Gaussian

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$\tilde{\tilde{D}}(z_2; \mu_b)$ Unpolarized FF : Gaussian

$$\widetilde{D}_{1T}^{\perp(1)}(z_1; \mu_b) : M_D(b_T) = \exp\left(-\frac{\langle p_\perp^2 \rangle b_T^2}{4 z_p^2}\right)$$

Gaussian model		
$g_K(b_T, b_{max}) \ln\left(\frac{\sqrt{\zeta_1 \zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)$	χ^2_{dof}	q_{Tmax}/Q
$g_2 \ln\left(\frac{b_T}{b_*}\right) \ln\left(\frac{Q}{Q_0}\right)$	1,286	0,27
$\frac{\alpha_s(C_1/b_*)C_F}{\pi} \ln(1 + b_T^2/b_{max}^2) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,363	0,27
$g_0(b_{max})\left(1 - \exp\left[-\frac{C_F \alpha_s(\mu_{b_*}) b_T^2}{\pi g_0(b_{max}) b_{max}^2}\right]\right) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,286	0,27
$\frac{C_F}{\pi} \frac{b_T^2}{b_{max}^2} \alpha_s(\mu_{b_*}) \ln\left(\frac{Q^2 z_1 z_2}{M_{h1} M_{h2}}\right)$	1,264	0,27

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Fitted 8 parameters

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$\tilde{\bar{D}}(z_2;\mu_b)$ Unpolarized FF : Gaussian

$$\tilde{D}_{1T}^{\perp(1)}(z_1;\mu_b) : M_D(b_T) = \exp\left(-\frac{\langle p_\perp^2\rangle b_T^2}{4z_p^2}\right)$$

$$\tilde{D}_{1T}^{\perp(1)}(z_1;\mu_b) : M_D^\perp(b_T,p,m) = \frac{2^{2-p}}{\Gamma(p-1)}(b_T m)^{p-1}K_{p-1}(b_T m)$$

Gaussian model		
$g_K(b_T,b_{max})\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)$	χ^2_{dof}	q_{Tmax}/Q
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$g_0(b_{max})\left(1-\exp\left[-\frac{C_F\alpha_s(\mu_{b_*})b_T^2}{\pi g_0(b_{max})b_{max}^2}\right]\right)\ln\left(\frac{Q^2z_1z_2}{M_{h1}M_{h2}}\right)$	1,286	0,27
$\frac{C_F}{\pi}\frac{b_T^2}{b_{max}^2}\alpha_s(\mu_{b_*})\ln\left(\frac{Q^2z_1z_2}{M_{h1}M_{h2}}\right)$	1,264	0,27

Power Law model		
$g_K(b_T,b_{max})\ln\left(\frac{\sqrt{\zeta_1\zeta_2}}{\sqrt{\zeta_{1,0}\zeta_{2,0}}}\right)$	χ^2_{dof}	q_{Tmax}/Q
$g_2\ln\left(\frac{b_T}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right)$	1,293	0,27
$\frac{\alpha_s(C_1/b_*)C_F}{\pi}\ln(1+b_T^2/b_{max}^2)\ln\left(\frac{Q^2z_1z_2}{M_{h1}M_{h2}}\right)$	1,368	0,27
$g_0(b_{max})\left(1-\exp\left[-\frac{C_F\alpha_s(\mu_{b_*})b_T^2}{\pi g_0(b_{max})b_{max}^2}\right]\right)\ln\left(\frac{Q^2z_1z_2}{M_{h1}M_{h2}}\right)$	1,323	0,27
$\frac{C_F}{\pi}\frac{b_T^2}{b_{max}^2}\alpha_s(\mu_{b_*})\ln\left(\frac{Q^2z_1z_2}{M_{h1}M_{h2}}\right)$	1,283	0,27

Preliminary results: 2-h data Fit

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Fitted 8-9 parameters

Polarized	Unpolarized	g_K	q_{Tmax}/Q	χ^2_{dof}
Power Law (1)	PV17	PV17	0.2	1.32
Power Law (1)	PV17	PV17	0.25	1.29
Power Law (1)	PV17	PV17	0.27	1.29
Power Law (1)	PV17	PV17	0.3	1.32

- All the models give compatible results with almost the same χ^2_{dof}
- Main contribution given by $\widetilde{D}_{1T}^{\perp(1)}(z_1;\mu_b)$ parametrization

Preliminary results: 2-h data Fit

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \ - 0.9]$ bin excluded → 96 data points

Fitted 8-9 parameters

Polarized	Unpolarized	g_K	q_{Tmax}/Q	χ^2_{dof}
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Power Law (1)	PV17	PV17	0.27	1.29
Power Law (1)	PV17	PV17	0.3	1.32

Power Law (1)	Power Law (1)	$\log(1 + b_T^2/b_{max}^2)$	0.27	1.319	$m = 1$
Power Law (1)	Power Law (1)	$\log(1 + b_T^2/b_{max}^2)$	0.27	1.335	m fitted
Power Law (2)	Power Law (1)	$\log(1 + b_T^2/b_{max}^2)$	0.3	1.275	m fitted
Power Law (2)	Power Law (2)	$\log(1 + b_T^2/b_{max}^2)$	0.25	1.203	m fitted
Power Law (2)	Gauss	$\log(1 + b_T^2/b_{max}^2)$	0.25	1.211	m fitted

- All the models give compatible results with almost the same χ^2_{dof}
- Main contribution given by $\widetilde{D}_{1T}^{\perp(1)}(z_1;\mu_b)$ parametrization

Preliminary results: 2-h data Fit

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \;- 0.9]$ bin excluded → 96 data points

Fitted 8-9 parameters

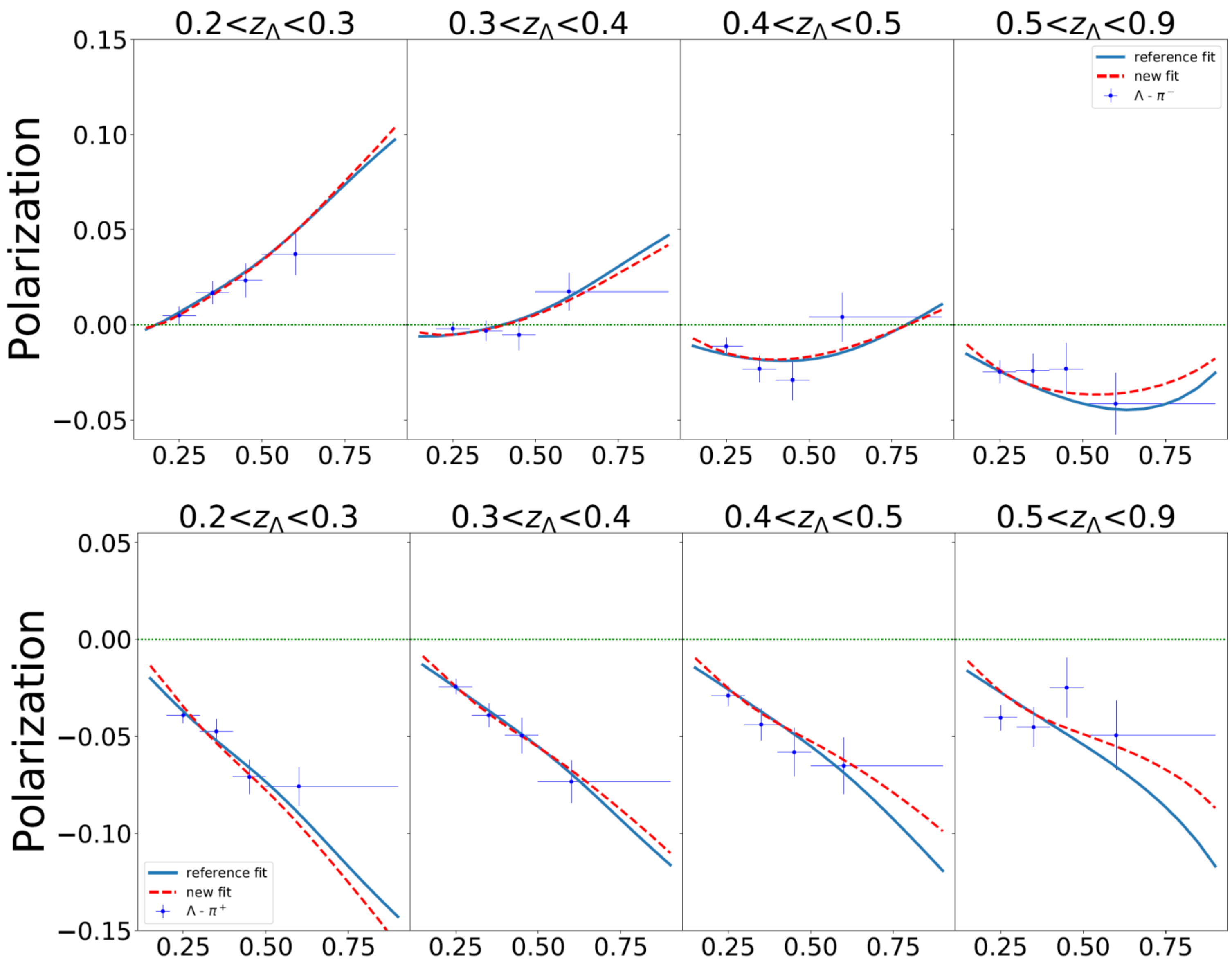
Polarized	Unpolarized	g_K	q_{Tmax}/Q	χ^2_{dof}
Power Law (1)	PV17	PV17	0.2	1.32
Power Law (1)	PV17	PV17	0.25	1.29
Power Law (1)	PV17	PV17	0.27	1.29
Power Law (1)	PV17	PV17	0.3	1.32

Power Law (1)	Power Law (1)	$\log(1 + b_T^2/b_{max}^2)$	0.27	1.319	$m = 1$
Power Law (1)	Power Law (1)	$\log(1 + b_T^2/b_{max}^2)$	0.27	1.335	m fitted
Power Law (2)	Power Law (1)	$\log(1 + b_T^2/b_{max}^2)$	0.3	1.275	m fitted
Power Law (2)	Power Law (2)	$\log(1 + b_T^2/b_{max}^2)$	0.25	1.203	m fitted
Power Law (2)	Gauss	$\log(1 + b_T^2/b_{max}^2)$	0.25	1.211	m fitted

Best χ^2_{dof}

- All the models give compatible results with almost the same χ^2_{dof}
- Main contribution given by $\widetilde{D}_{1T}^{\perp(1)}(z_1;\mu_b)$ parametrization

Lambda – pion: fit comparison



Bin excluded
$z_\pi = [0.5 - 0.9]$

Power Law Model

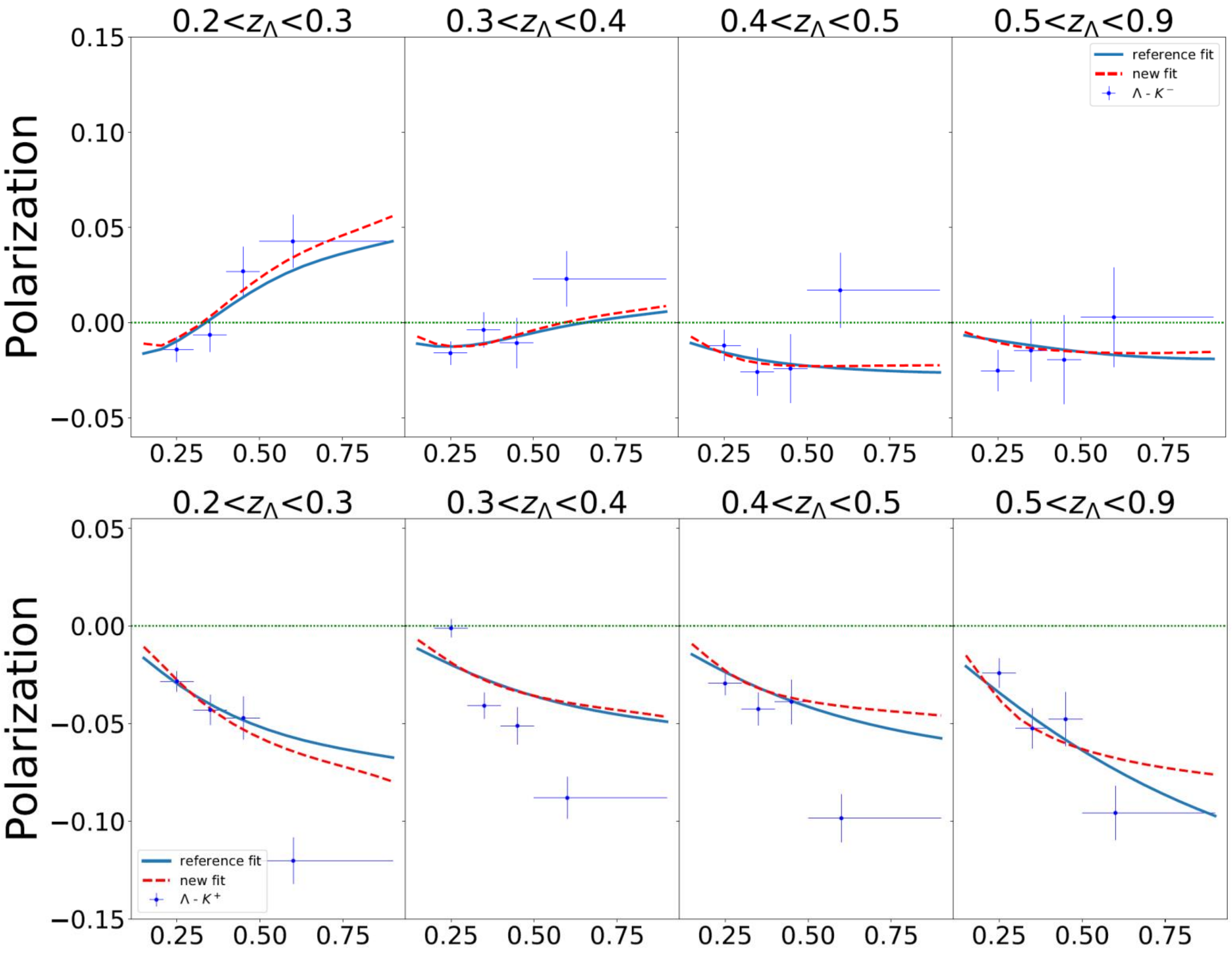
$\chi^2_{dof} = 1.203$

- New fit
- Reference Fit [4]

[4] D'Alesio, Murgia, Zaccheddu, Phys. Rev. D 102, 054001 (2020)

Preliminary Results

Lambda – kaon: fit comparison



Bin excluded
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Power Law Model

$\chi^2_{dof} = 1.203$

- New fit
- Reference Fit [4]

[4] D'Alesio, Murgia, Zaccheddu, Phys. Rev. D 102, 054001 (2020)

Preliminary Results

Preliminary results: both data set Fit

Data selection:
- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \ - 0.9]$ bin excluded → 96 data points
- $\Lambda(jet)$: $z_{\Lambda} = [0.2 \ - 0.3]$ bin excluded → 23 data points

Preliminary results: both data set Fit

Data selection:
- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \ - 0.9]$ bin excluded → 96 data points
- $\Lambda(jet)$: $z_{\Lambda} = [0.2 \ - 0.3]$ bin excluded → 23 data points

$\widetilde{D}_{1T}^{\perp(1)}(z_1;\mu_b)$ Power Law Model

$$M_D^{\perp}(b_T, p, m) = \frac{2^{2-p}}{\Gamma(p-1)} (b_T m)^{p-1} K_{p-1}(b_T m)$$

m = 1 (fixed)
p to be fitted

Preliminary results: both data set Fit

Data selection:

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m = 1 (fixed)

p to be fitted

$\widetilde{\widetilde{D}}(z_2;\mu_b)$: Gaussian for all unpolarized hadrons

$$M_D(b_T) = \exp\left(-\frac{\langle p_\perp^2\rangle b_T^2}{4z_p^2}\right)$$

width = 0.2

$$g_2 \ln\left(\frac{b_T}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right)$$

$g_2 = 0{,}84$

$Q_0^2 = 2{,}4\ GeV^2$

Q=10,58 GeV

Fitted 8 parameters

Preliminary results: both data set Fit

Data selection:
- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \; - 0.9]$ bin excluded → 96 data points
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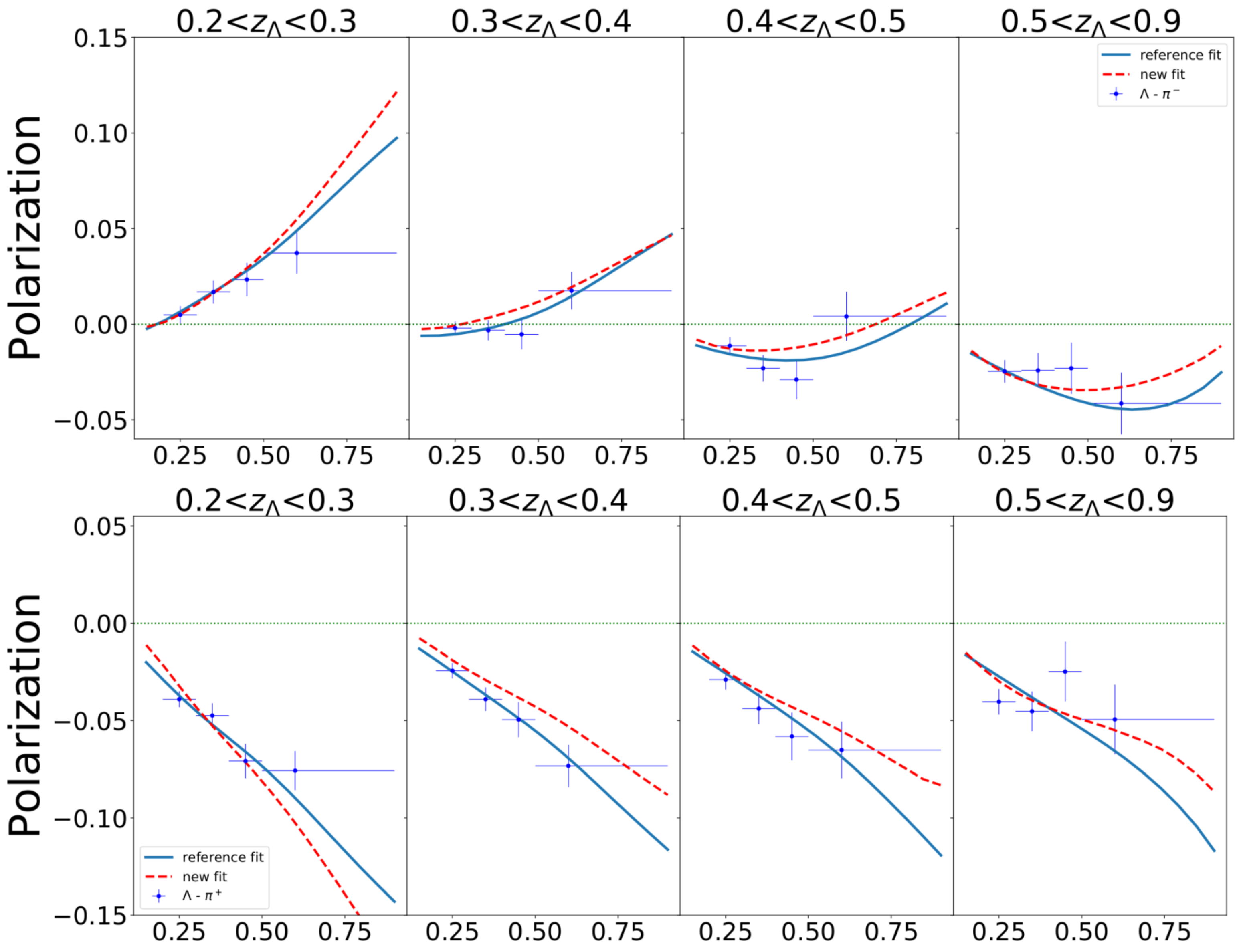
Q=10,58 GeV

Fitted 8 parameters

$\chi^2_{dof} = 1.58$

Lambda – pion: fit comparison



Bin excluded
$z_\pi = [0.5 - 0.9]$

Power Law Model

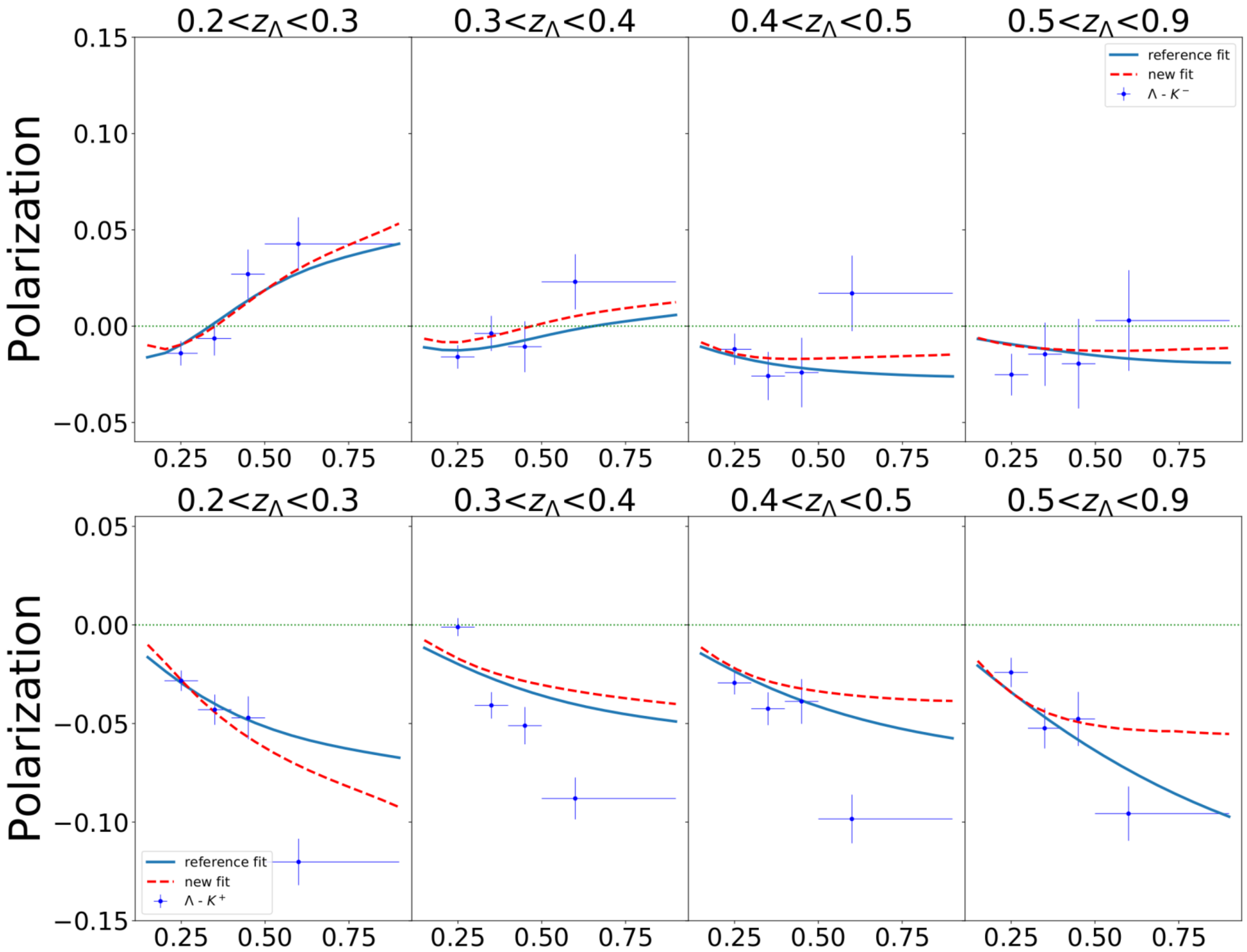
$\chi^2_{dof} = 1.58$

- New fit
- Reference Fit [4]

[4] D'Alesio, Murgia, Zaccheddu, Phys. Rev. D 102, 054001 (2020)

Preliminary Results

Lambda – kaon: fit comparison



Bin excluded
$z_K = [0.5 \; - 0.9]$

Power Law Model

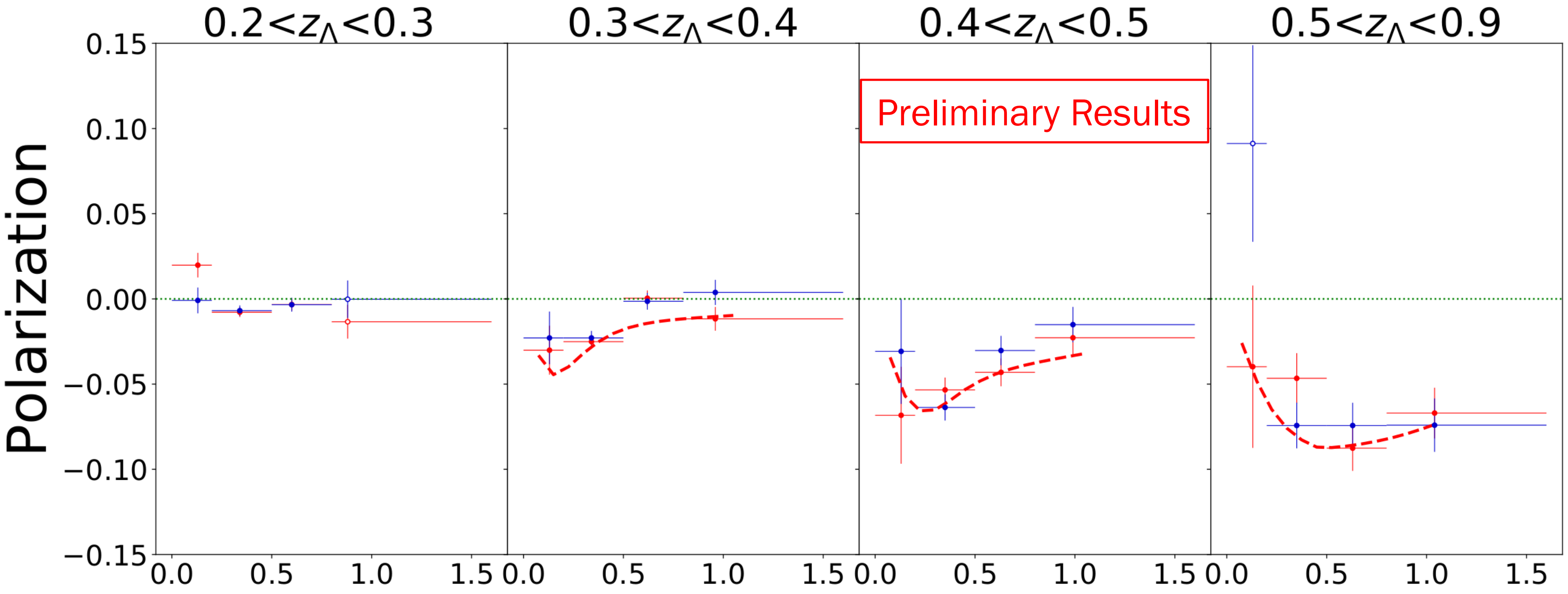
$\chi^2_{dof} = 1.58$

- New fit
- Reference Fit [4]

[4] D'Alesio, Murgia, Zaccheddu, Phys. Rev. D 102, 054001 (2020)

Preliminary Results

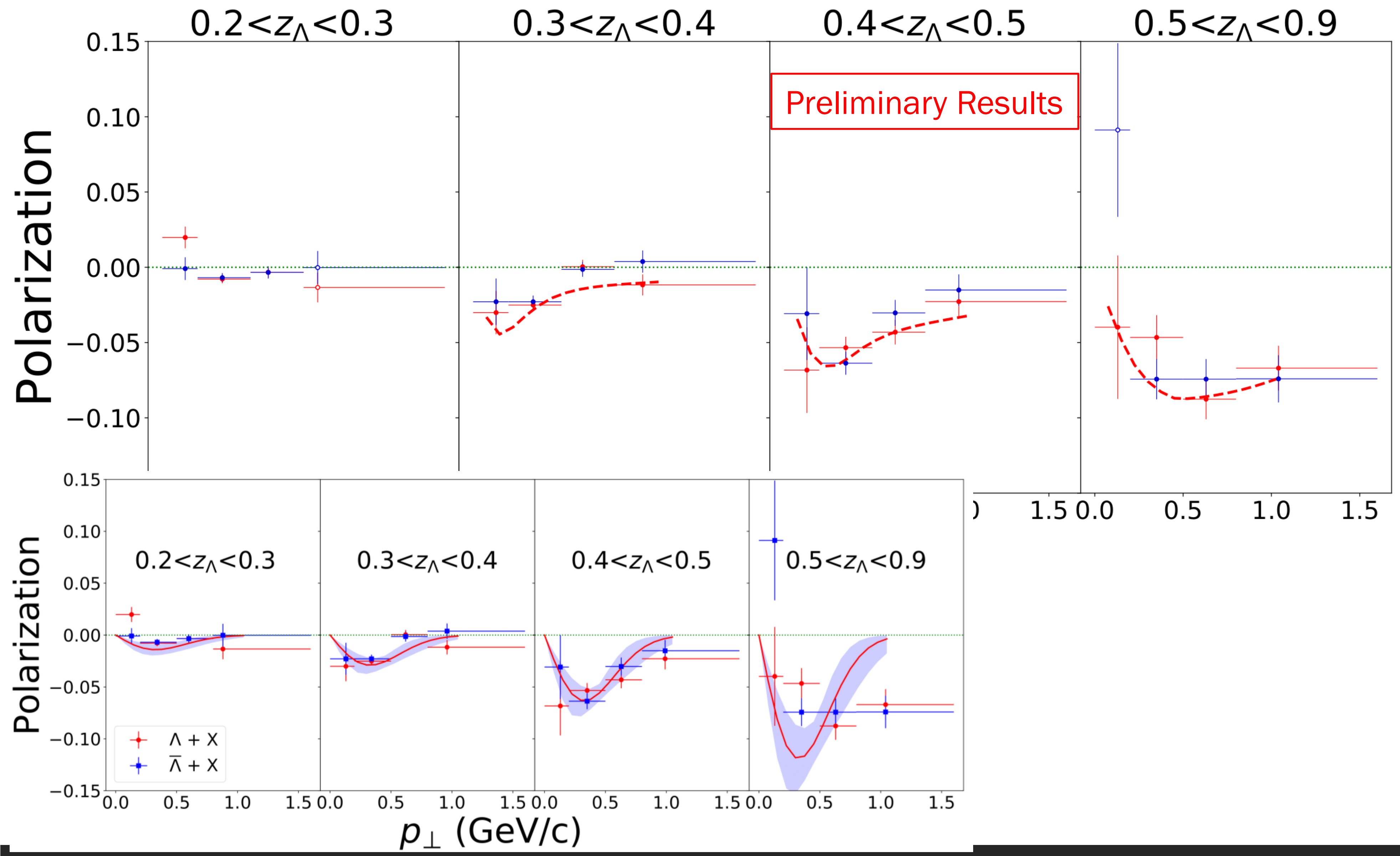
Lambda - thrust: fit comparison



Bin excluded
$z_\Lambda = [0.2 \; - 0.3]$

$\chi^2_{dof} = 1.58$

Lambda - thrust: fit comparison



Bin excluded
$z_{\Lambda} = [0.2 - 0.3]$

$\chi^2_{dof} = 1.58$

Conclusions

- Polarization of $e^+e^- \to h^{\uparrow}{}_1 h_2 X$ and $e^+e^- \to \mathrm{h}_1^{\uparrow}(\mathrm{jet})X$ in terms of convolutions;
- Convolutions using the evolution equations for TMD FFs;
- Fit of 2-h data with different models and g_K;
- Combined Fit of 2-h and 1-h data with Power Law model;
- Comparison with previous fit.