SPIN 2021

TMDs and EIC impact studies

Matteo Cerutti, Chiara Bissolotti,
Alessandro Bacchetta and Marco Radici

MAP Collaboration
(Multi-dimensional Analyses of Partonic distributions)

Istituto Nazionale di Fisica Nucleare

3DSPIN MAPPING THE PROTON IN 3D

erc European Research Council

UNIVERSITÀ DI PAVIA

Unpolarized quark TMDs

scales $\mu = Q$

$\zeta = Q^2$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_*}$$

TMD PDF

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left(C_{q/j} \otimes f_1^j\right)(x, b_*; \mu_b) e^{S(b_*;\mu_b,\mu)} e^{g_K(b) ln \frac{\mu}{\mu_0}} f_{\rm NP}^q(x, b, \zeta)$$

Unpolarized quark TMDs

TMD PDF

scales $\mu_b = \frac{2e^{-\gamma_E}}{b_*}$

$\mu = Q$

$\zeta = Q^2$

collinear PDFs

Sudakov factor
perturbative evolution

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left(C_{q/j} \otimes f_1^j \right)(x, b_*; \mu_b) e^{S(b_*;\mu_b,\mu)} e^{g_K(b) ln \frac{\mu}{\mu_0}} f_{\mathrm{NP}}^q(x, b, \zeta)$$

matching coefficients

non perturbative
evolution

intrinsic non perturbative
transverse content

Unpolarized quark TMDs

TMD PDF

collinear PDFs

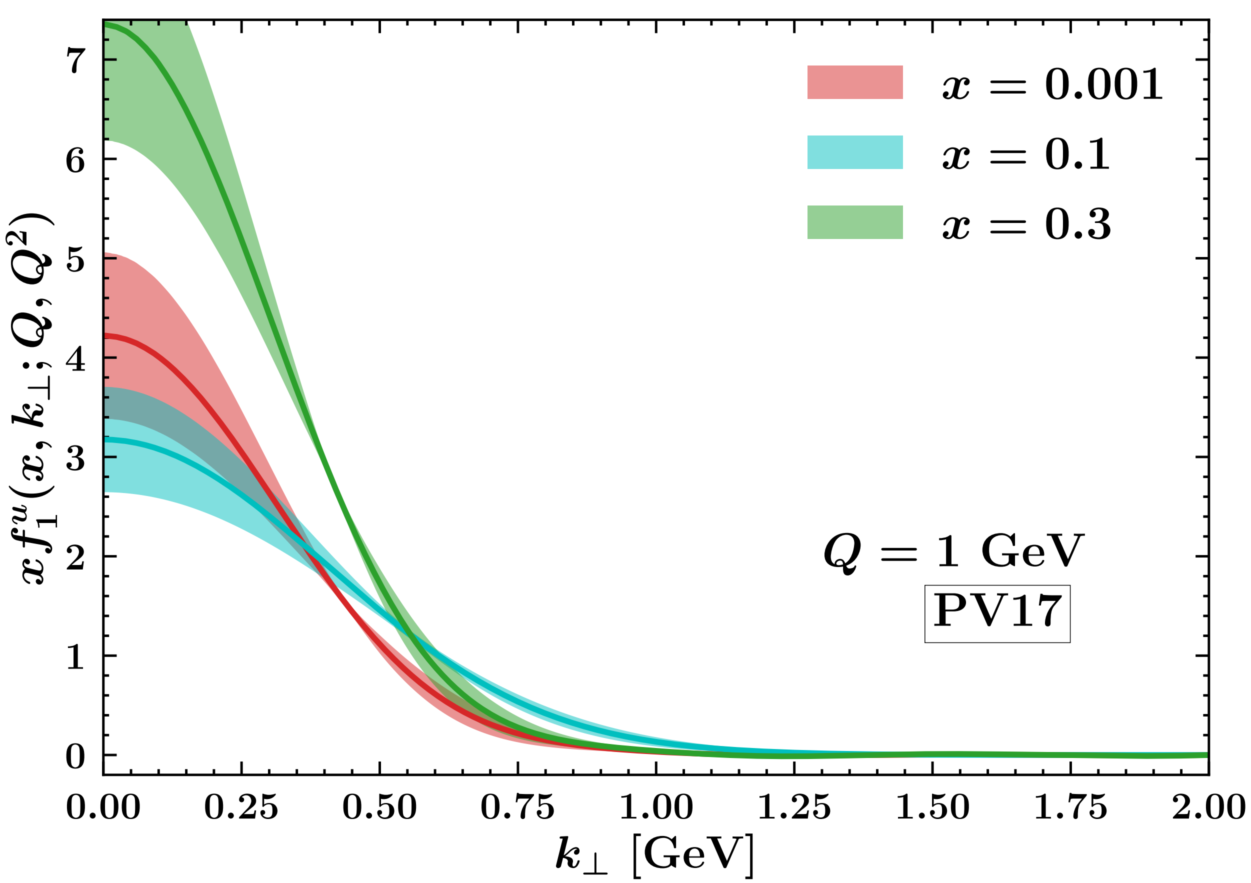
Sudakov factor
perturbative evolution

$$f_1^q(x,b;\mu,\zeta) = \sum_j \left(C_{q/j} \otimes f_1^j \right)(x,b_*;\mu_b) e^{S(b_*;\mu_b,\mu)} e^{g_K(b) ln \frac{\mu}{\mu_0}} f_{\rm NP}^q(x,b,\zeta)$$

matching coefficients

non perturbative
evolution

intrinsic non perturbative
transverse content



non perturbative

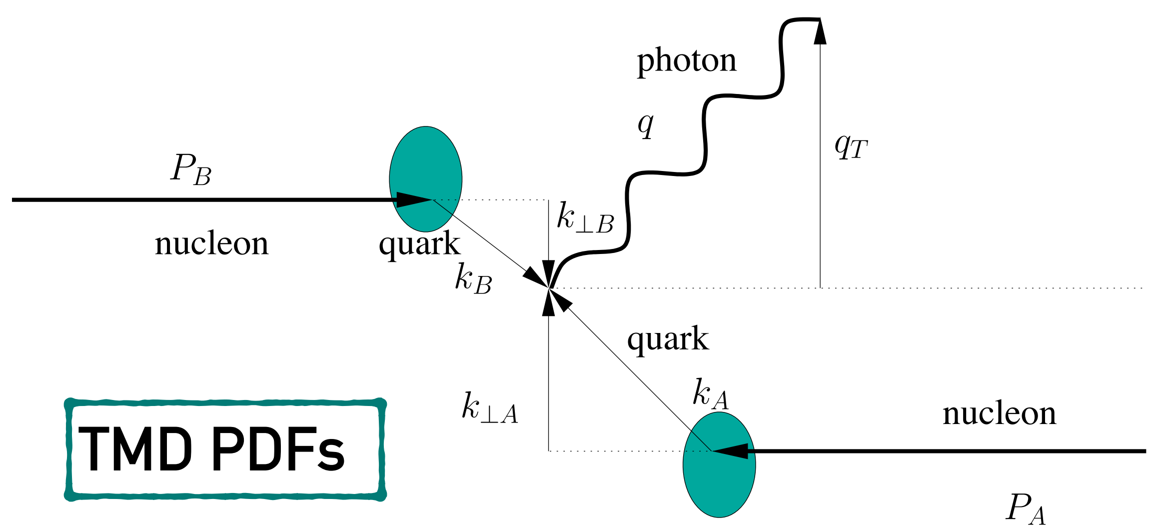
parametrized and fitted to data

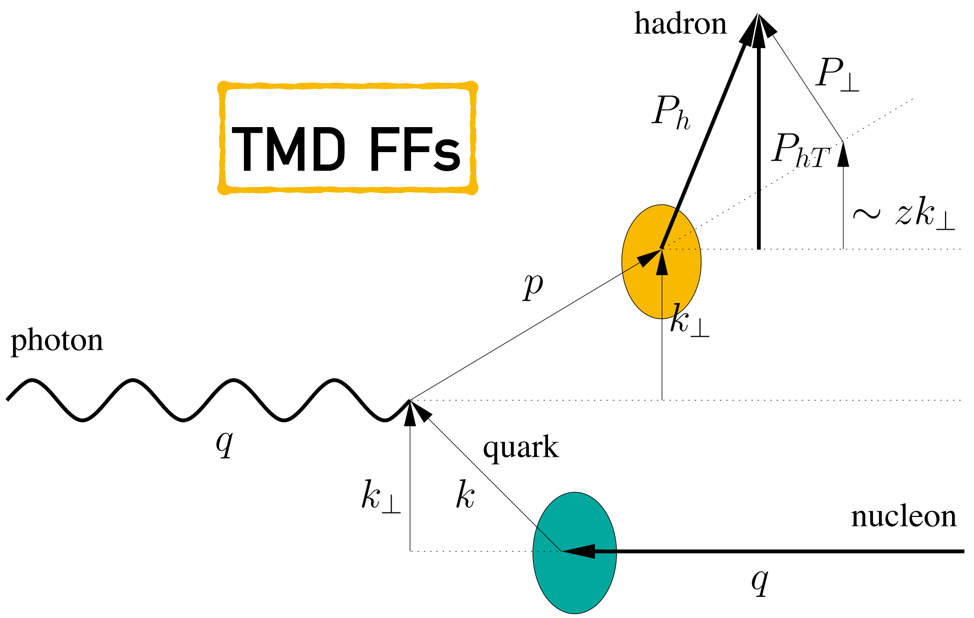
A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori
JHEP06 (2017) 081, arXiv:1703.10157

Drell-Yan and SIDIS

$$N(P_A) + N(P_B) \rightarrow \gamma^*/Z \rightarrow l^+ l^-$$

$$\ell(l) + N(p) \rightarrow \ell(l') + h(P_h) + X$$





$$q_T \ll Q$$

TMD factorization

$$M^2 \ll Q^2 \quad P_{hT}^2 \ll Q^2$$

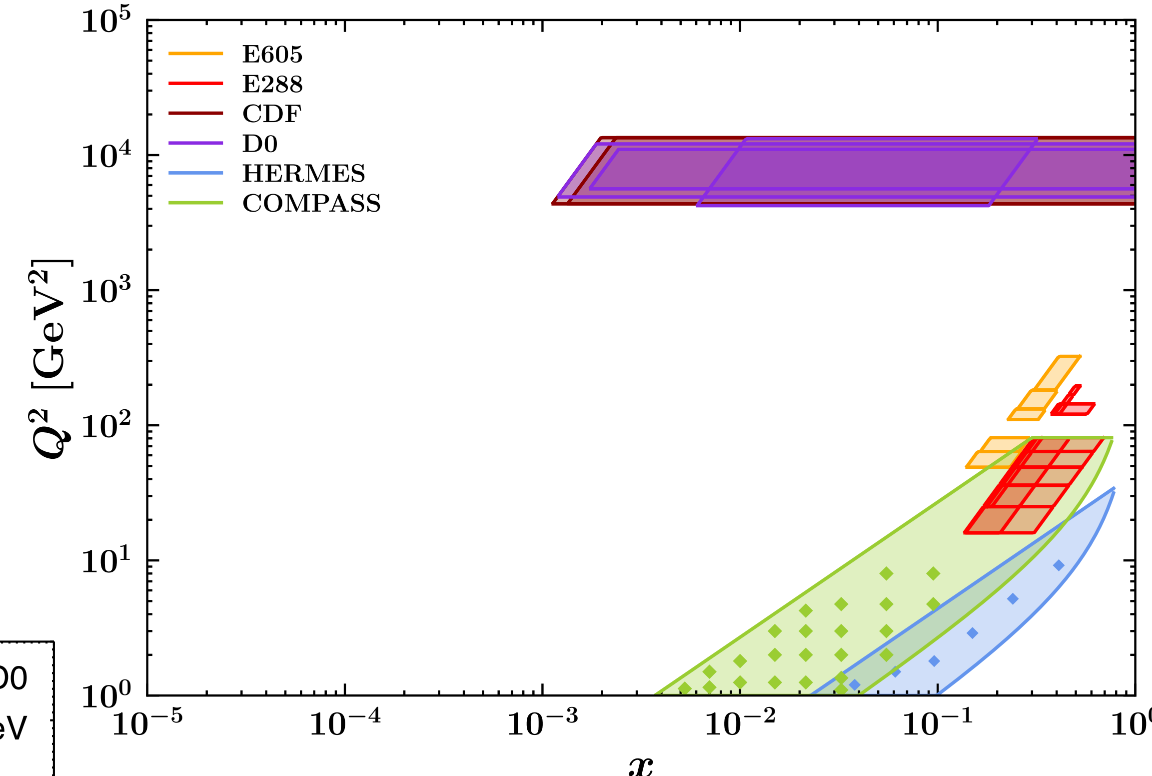
$$\left(\frac{d\sigma}{dq_T}\right) \propto \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} x_1 f_1^q(x_1, \mathbf{b}) x_2 f_1^{\bar{q}}(x_2, \mathbf{b})$$

$$\left(\frac{d\sigma}{dq_T}\right) \propto \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} f_1^q(x, \mathbf{b}) D_1^{q\rightarrow h}(z, \mathbf{b})$$

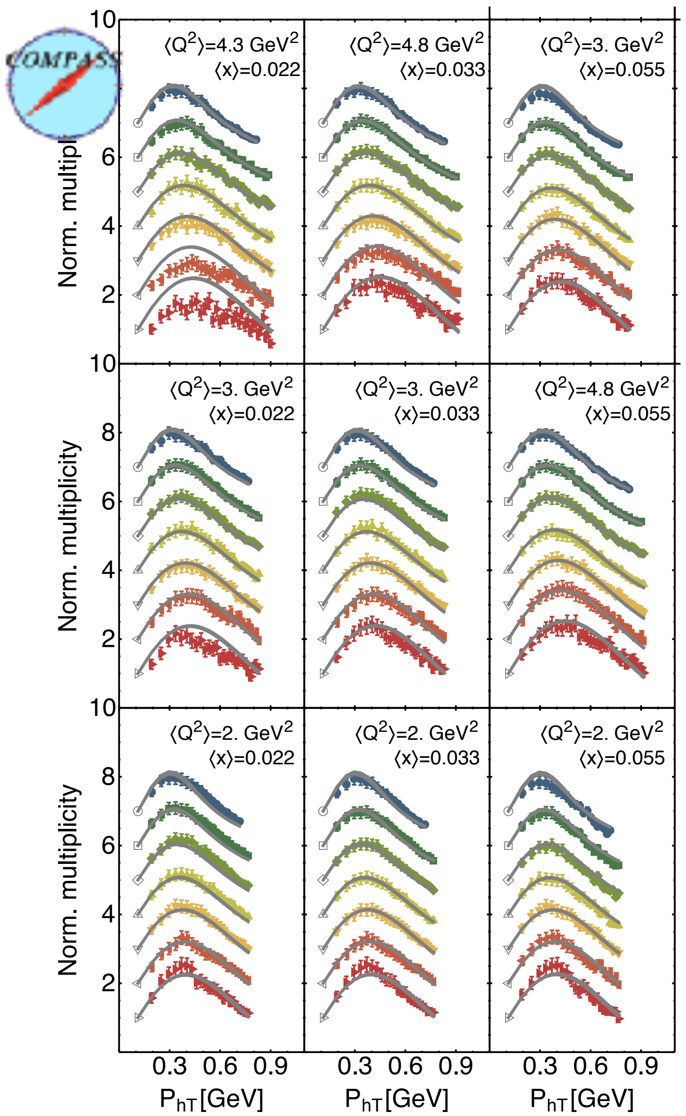
PV17 fit

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori
JHEP06 (2017) 081, arXiv:1703.10157

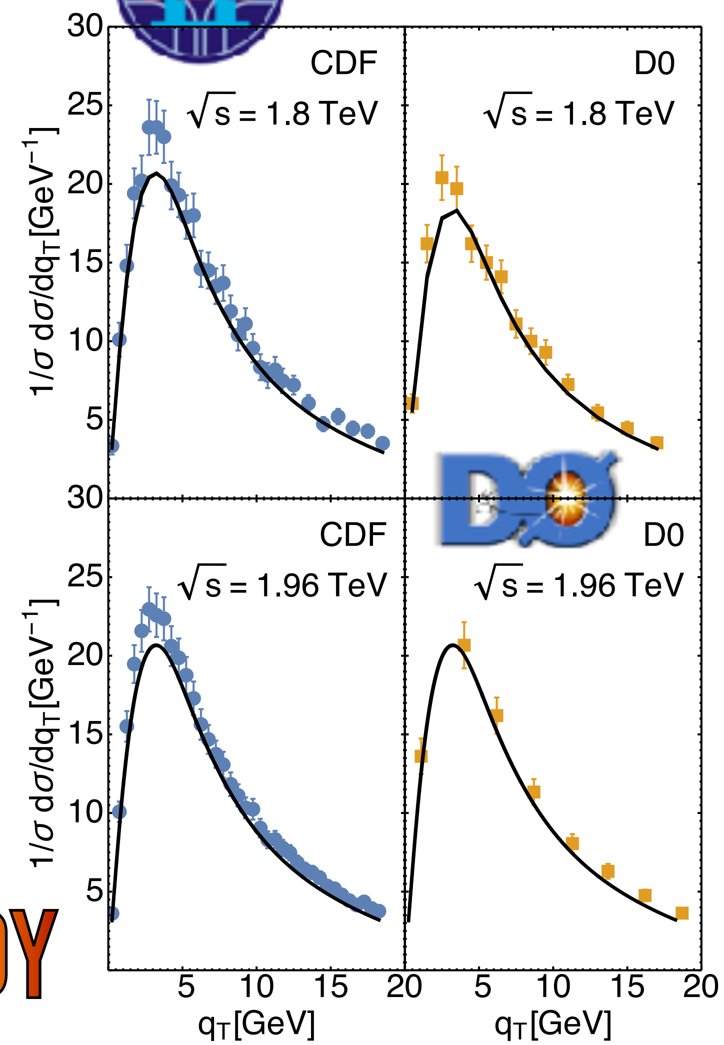
8059 data points



SIDIS



DY



with normalization coefficients

NLL

global

$\chi^2 = 1.55$

PV17 non perturbative functions

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori
JHEP06 (2017) 081, arXiv:1703.10157

$$f_{1\mathrm{NP}}^{a}(x, \mathbf{k}_\perp^2) = \frac{1}{\pi} \frac{\left(1 + \lambda \mathbf{k}_\perp^2\right)}{g_{1a} + \lambda g_{1a}^2} e^{-\frac{\mathbf{k}_\perp^2}{g_{1a}}}$$

$$D_{1\mathrm{NP}}^{a\to h}(z, \mathbf{P}_\perp^2) = \frac{1}{\pi} \frac{1}{g_{3a\to h} + \left(\lambda_F / z^2\right) g_{4a\to h}^2} \left(e^{-\frac{\mathbf{P}_\perp^2}{g_{3a\to h}}} + \lambda_F \frac{\mathbf{P}_\perp^2}{z^2}\, e^{-\frac{\mathbf{P}_\perp^2}{g_{4a\to h}}} \right)$$

x-dependence

$$g_1(x) = N_1 \frac{(1-x)^{\alpha}\, x^{\sigma}}{(1-\hat{x})^{\alpha}\, \hat{x}^{\sigma}}$$

$$g_{3,4}(z) = N_{3,4} \frac{(z^{\beta} + \delta)\, (1-z)^{\gamma}}{(\hat{z}^{\beta} + \delta)\, (1-\hat{z})^{\gamma}}$$

non-perturbative Sudakov factor

$$g_K(b_T) = -g_2 b_T^2 / 2$$

total of 11 parameters

PV17 fit

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori
JHEP06 (2017) 081, arXiv:1703.10157

8059 points
11 parameters
SIDIS + DY
NLL

$$\chi^2_{\mathrm{d.o.f.}} = 1.55 \pm 0.05$$

all standard deviations at 68% c.l.

g_2	N_1 [GeV^2]	β	α	λ_F [GeV^{-2}]
0.13 ± 0.01	0.28 ± 0.06	1.65 ± 0.49	2.95 ± 0.05	5.50 ± 1.23

$\langle x \rangle \pm \Delta x$

fairly well determined *not very constrained*

$g_2 \rightarrow$ non-perturbative evolution

$N_1 \rightarrow$ mid-x width of TMD PDF

$\beta \rightarrow$ low-z width of TMD FF

$\alpha \rightarrow$ high-x width of TMD PDF

$\lambda_F \rightarrow$ weight of second Gaussian in TMD FF

PV17 fit

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori
JHEP06 (2017) 081, arXiv:1703.10157

8059 points
11 parameters
SIDIS + DY
NLL

$$\chi^2_{\mathrm{d.o.f.}} = 1.55 \pm 0.05$$

all standard deviations at 68% c.l.

g_2	N_1 [GeV^2]	β	α	λ_F [GeV^{-2}]
0.13 ± 0.01	0.28 ± 0.06	1.65 ± 0.49	2.95 ± 0.05	5.50 ± 1.23

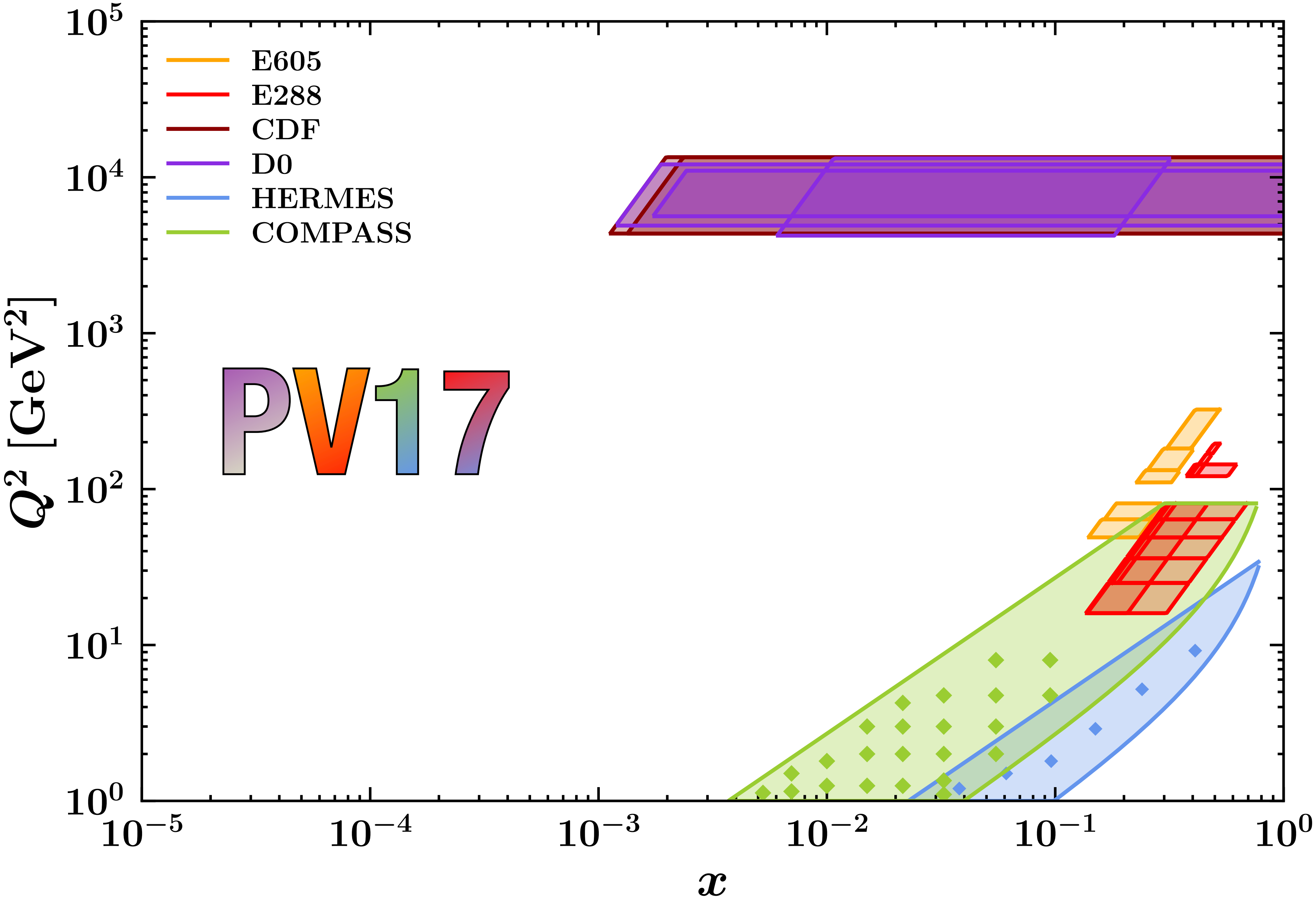
$\langle x \rangle \pm \Delta x$

fairly well determined

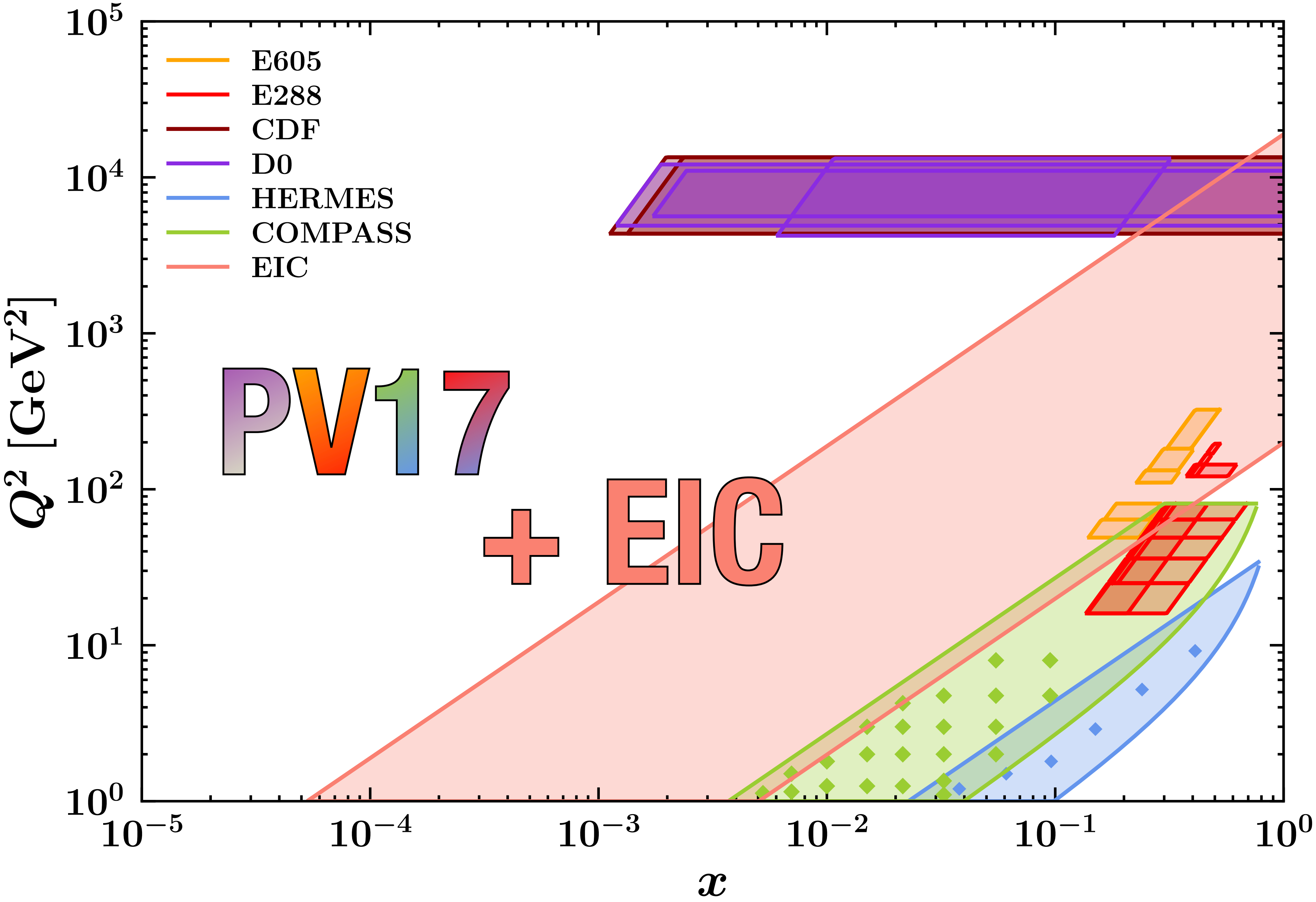
not very constrained

what impact will the EIC have on those uncertainties?

What happens to PV17 TMDs …



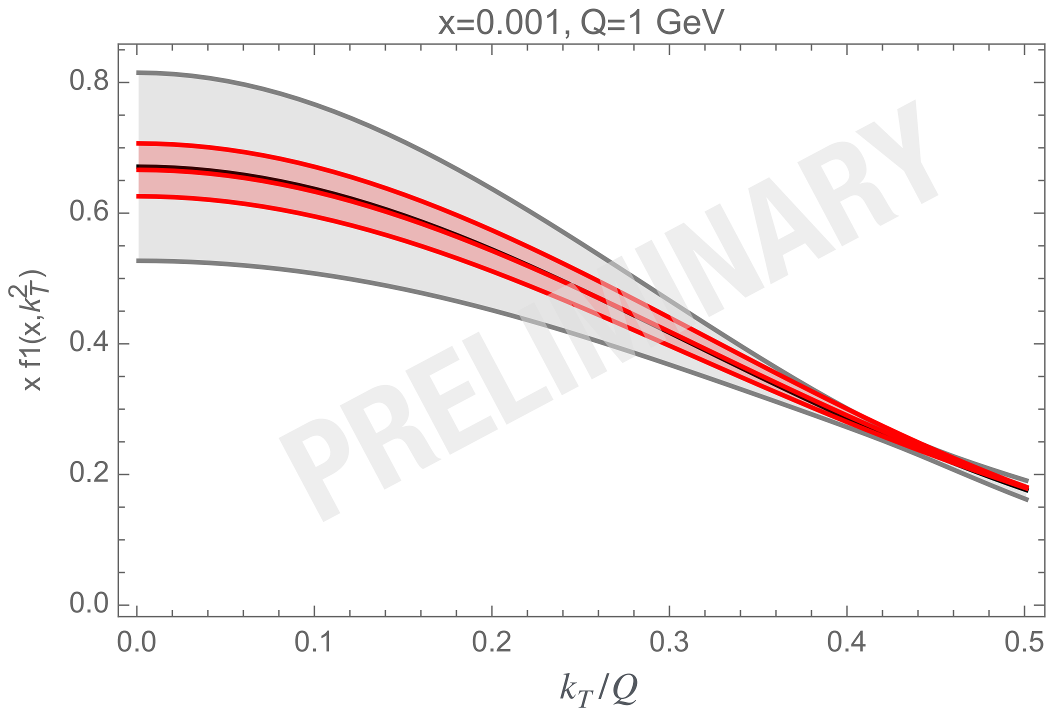
What happens to PV17 TMDs ...

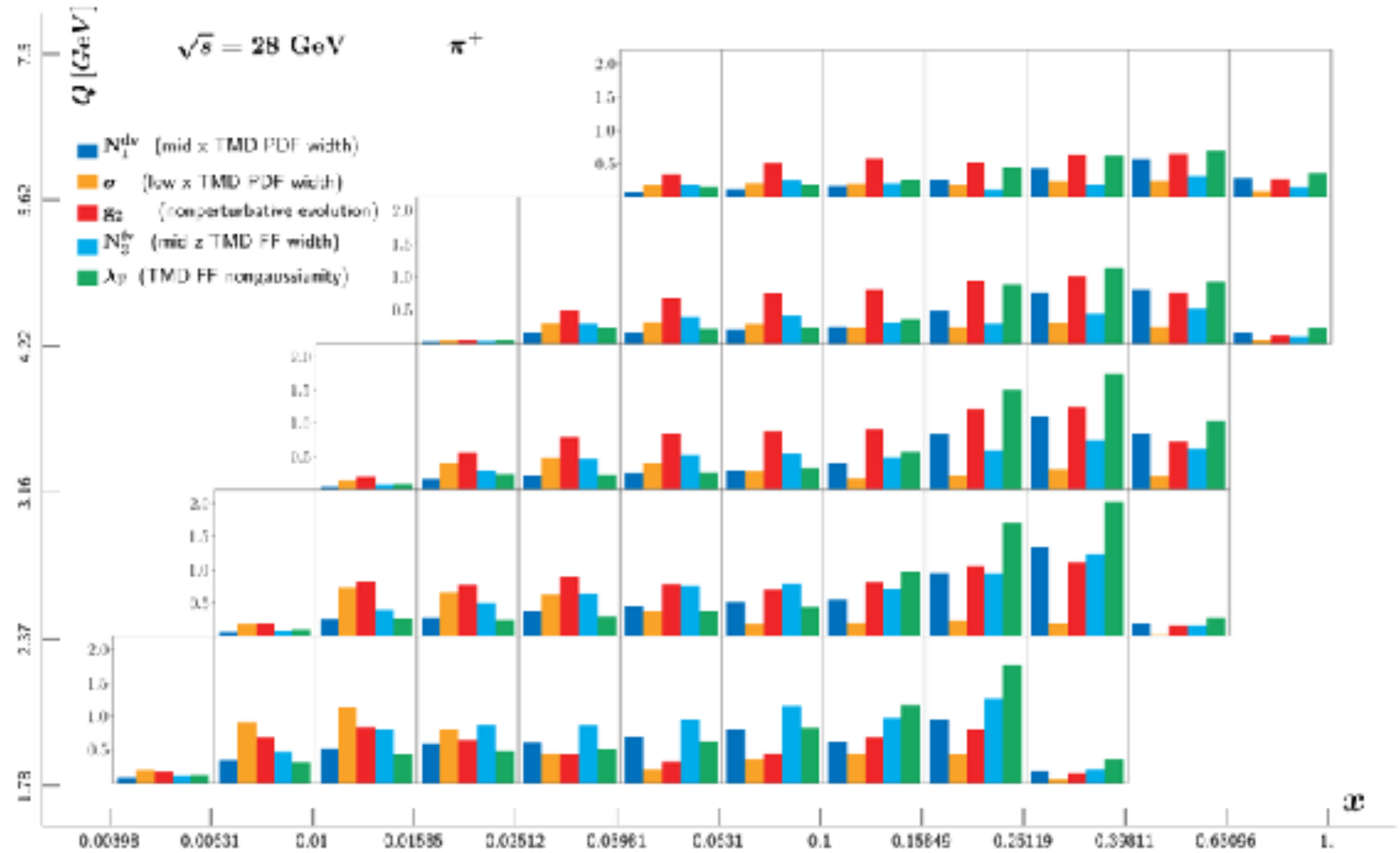


... once we include EIC data?

A few tools to estimate EIC impact

reweighing





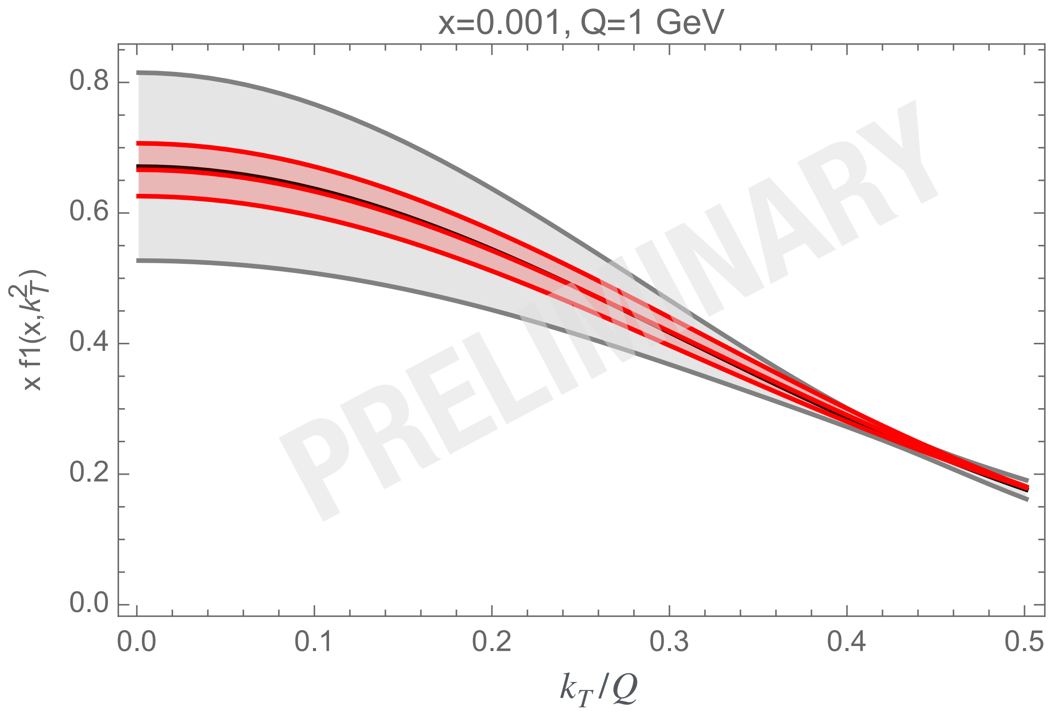
sensitivity coefficients

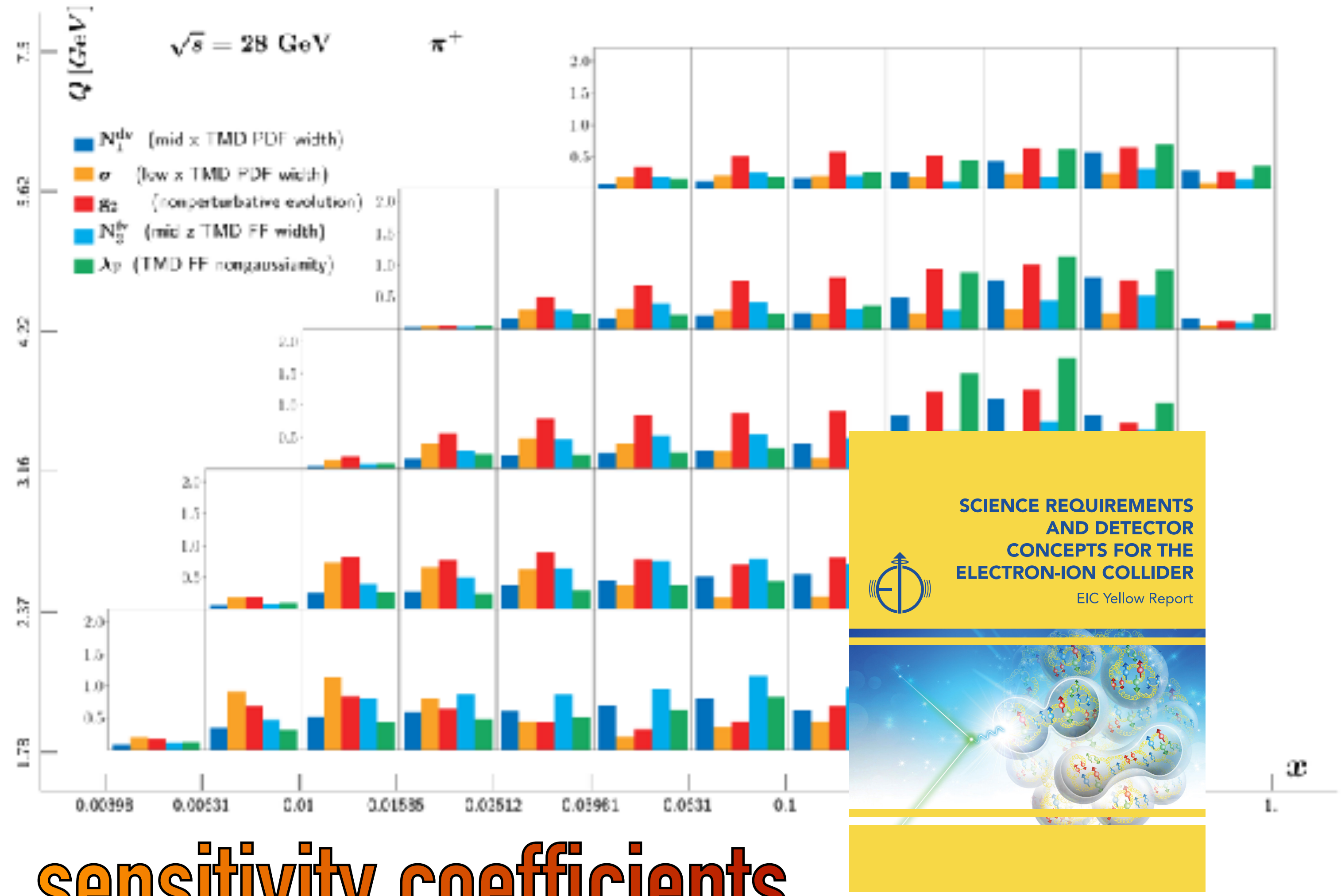
$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\delta \mathcal{O} \Delta f_i}$$

A few tools to estimate EIC impact

reweighing

not satisfactory results





sensitivity coefficients

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\delta \mathcal{O} \Delta f_i}$$

NECESSARY

new fit that includes **EIC** pseudodata

Baseline fit

https://github.com/MapCollaboration/NangaParbat

with **NangaParbat** is not possibile to replicate *exactly* **PV17 results**

APFEL++

NEW FIT

to use as baseline

NLL

with NLO collinear PDFs and FFs

PV17 parameterization

11 parameters

200 replicas

average variables

we do non integrate on any variable

Baseline fit - datasets

NEW FIT

(almost) same datasets as in PV17

global fit

SIDIS

Drell-Yan

2017 COMPASS

hermes

Fermilab low energy

remarks:

- normalization coeff. for Tevatron
- NO LHC data
- uncertainties correlations taken into account

cut in qT/Q

more restrictive than PV17

$$q_T/Q < \min\left[\min[0.2/z, 0.5] + 0.3/zQ, 1\right]$$

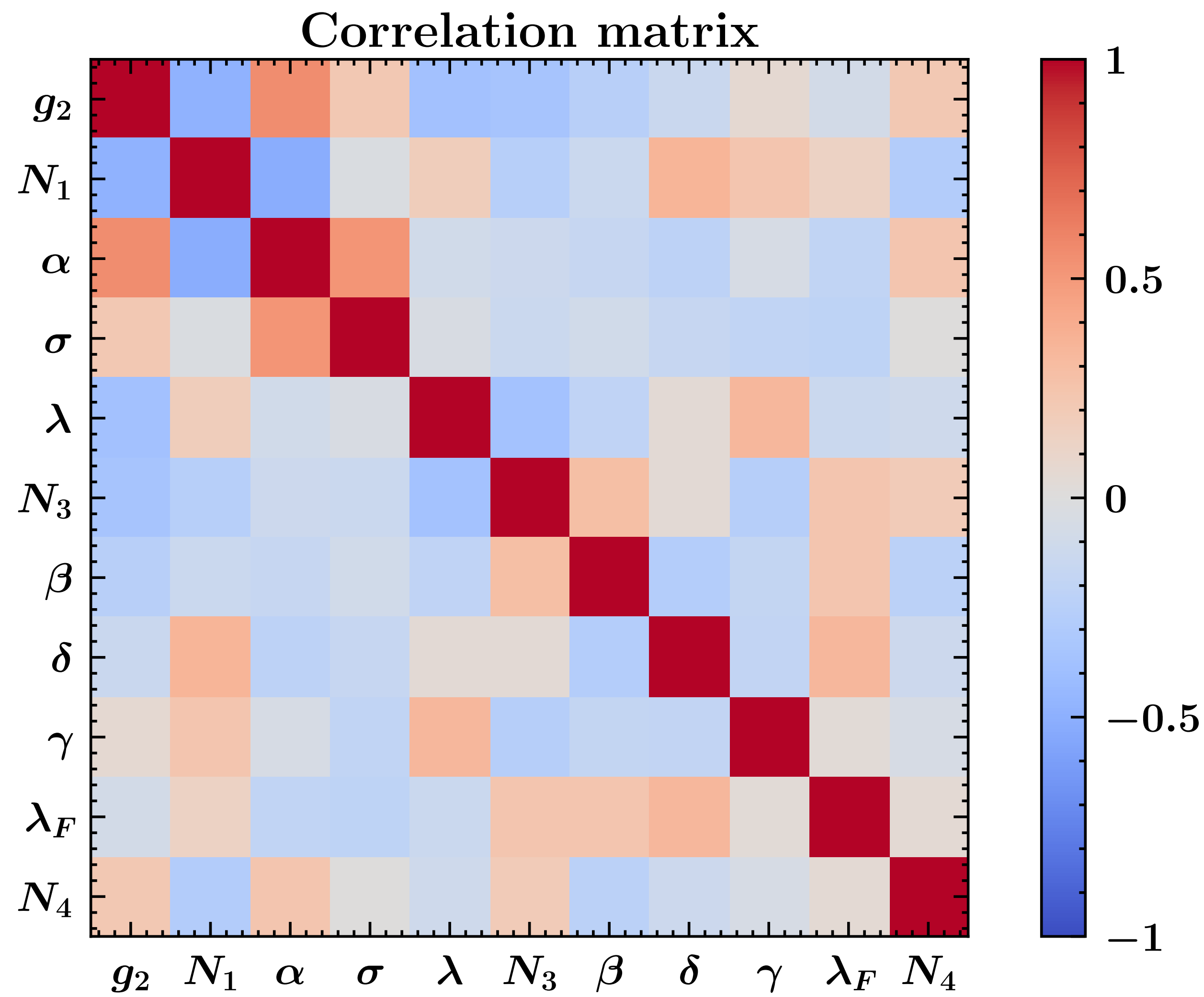
2516

data points

Baseline fit - results

parameters

Parameter	Average over replicas
g_2	0.1171 ± 0.0145
N_1	0.283 ± 0.0368
α	2.2393 ± 1.2967
σ	-0.1416 ± 0.0959
λ	0.2548 ± 0.2549
N_3	0.2203 ± 0.0222
β	2.9304 ± 0.9978
δ	0.1175 ± 0.0506
γ	2.4736 ± 0.1649
λ_F	7.5475 ± 3.2037
N_4	0.0318 ± 0.0068



with **NangaParbat**

a **new fit** with uncertainties

similar to **PV17**

$$\chi^2_{\text{d.o.f.}} = 1.14 \pm 0.06$$

Step towards PV17 + EIC fit

generation of pseudo data

π^+

π^-

final state hadrons

K^+

K^-

5x41

18x275

configurations

5x100

18x100

10x100

$\sim$ **2500** pseudodata points

central value of pseudo data obtained using average parameters of the PV17 baseline fit

uncertainties of pseudo data are given by simulations done by the EIC SIDIS working group

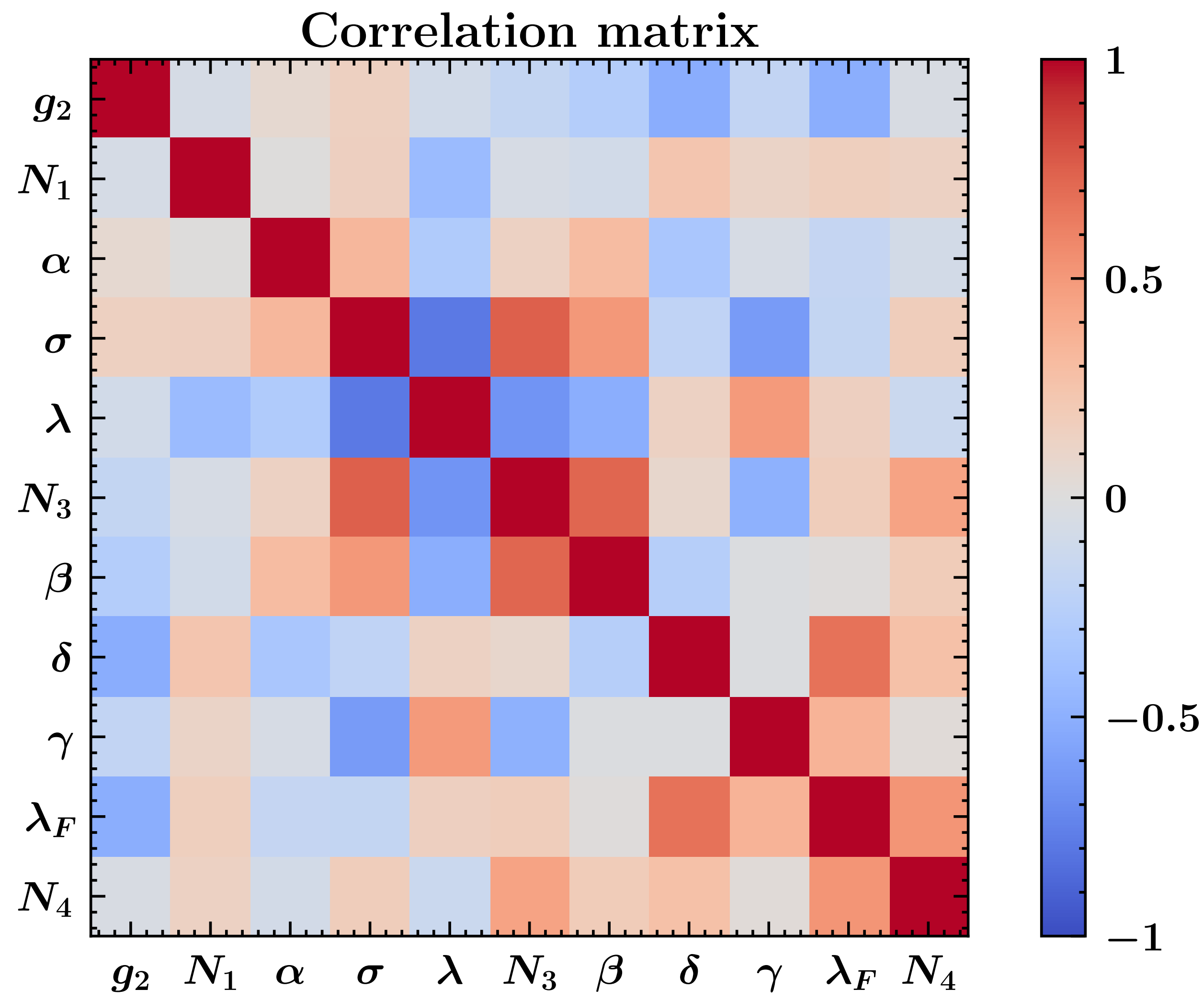
PV17 baseline + EIC fit

impact on parameters

$$g_K(b_T) = -g_2 b_T^2/2$$

$$g_1(x) = N_1 \frac{(1-x)^{\alpha}\, x^{\sigma}}{(1-\hat{x})^{\alpha}\, \hat{x}^{\sigma}}$$

$$g_{3,4}(z) = N_{3,4} \frac{(z^{\beta}+\delta)\,(1-z)^{\gamma}}{(\hat{z}^{\beta}+\delta)\,(1-\hat{z})^{\gamma}}$$



$$f_{1NP}^{a}(x,\mathbf{k}_\perp^2) = \frac{1}{\pi}\frac{(1+\lambda \mathbf{k}_\perp^2)}{g_{1a}+\lambda g_{1a}^2} e^{-\frac{\mathbf{k}_\perp^2}{g_{1a}}}$$

$$\chi^2_{\text{d.o.f.}} = 0.88 \pm 0.32$$

$$D_{1NP}^{a\to h}(z,\mathbf{P}_\perp^2) = \frac{1}{\pi}\frac{1}{g_{3a\to h}+(\lambda_F/z^2)g_{4a\to h}^2}\left(e^{-\frac{\mathbf{P}_\perp^2}{g_{3a\to h}}} + \lambda_F \frac{\mathbf{P}_\perp^2}{z^2}\, e^{-\frac{\mathbf{P}_\perp^2}{g_{4a\to h}}}\right)$$

Impact of EIC

reduction by factor 10

non - perturbative evolution

Parameter	PV17 baseline Average over replicas	PV17 baseline + EIC Average over replicas
g_2	0.1171 ± 0.0145	0.119 ± 0.0025
	0.283 ± 0.0368	0.2814 ± 0.0362
	2.2393 ± 1.2967	2.3882 ± 0.5448
	-0.1416 ± 0.0959	-0.1445 ± 0.0134
λ	0.2548 ± 0.2549	0.3061 ± 0.4085
N_3	0.2203 ± 0.0222	0.2122 ± 0.0157
β	2.9304 ± 0.9978	2.6773 ± 0.3861
δ	0.1175 ± 0.0506	0.1099 ± 0.0358
γ	2.4736 ± 0.1649	2.4643 ± 0.12
λ_F	7.5475 ± 3.2037	5.3198 ± 2.0531
N_4	0.0318 ± 0.0068	0.0346 ± 0.0048

Impact of EIC

reduction by factor 10

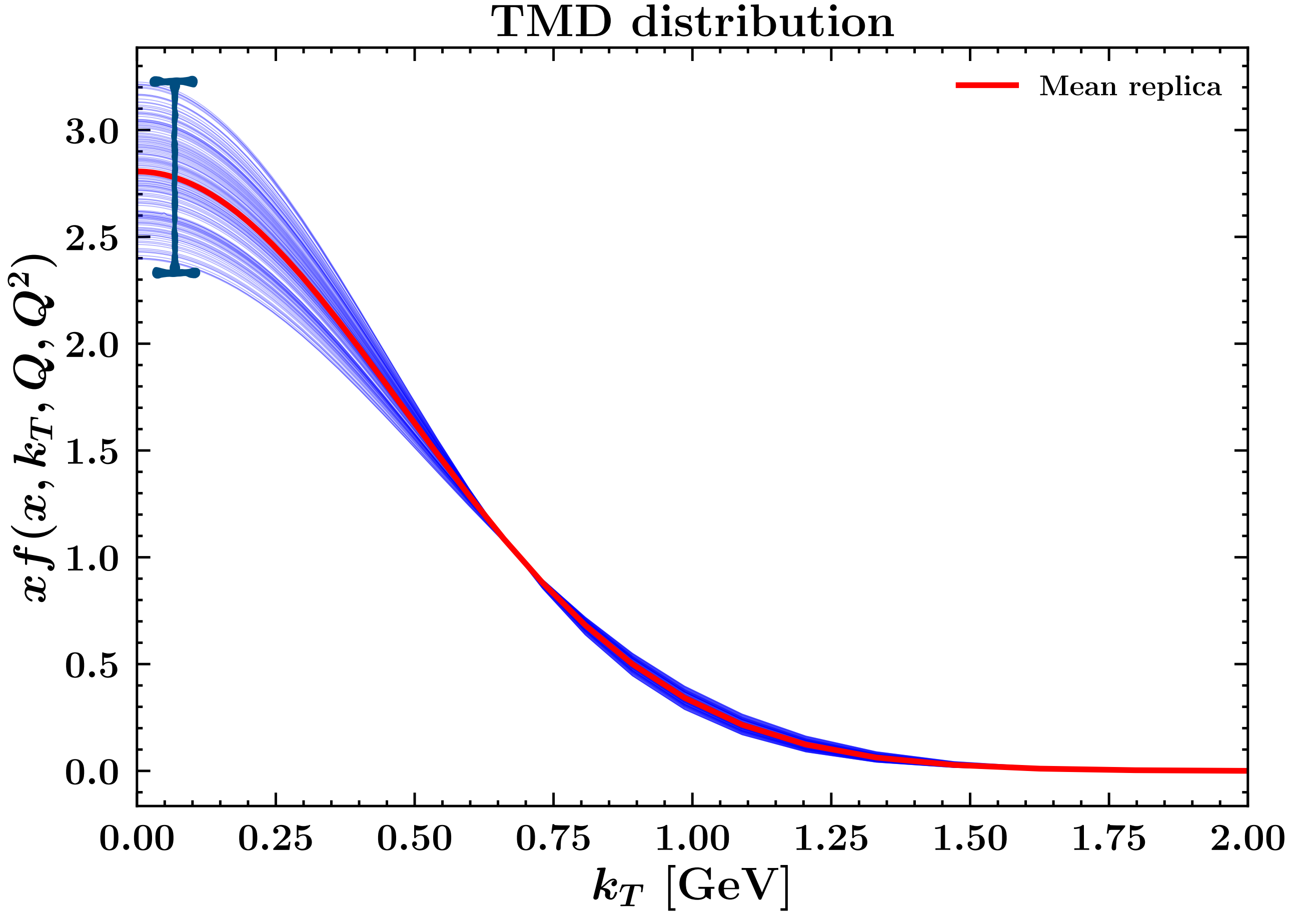
Average over replicas (PV17 baseline)	Parameter	Average over replicas (PV17 baseline + EIC)
0.1171 ± 0.0145	g_2	0.119 ± 0.0025
0.283 ± 0.0368	N_1	0.2814 ± 0.0362
2.2393 ± 1.2967	α	2.3882 ± 0.5448
-0.1416 ± 0.0959	σ	-0.1445 ± 0.0134
0.2548 ± 0.2549	λ	0.3061 ± 0.4085
0.2203 ± 0.0222	N_3	0.2122 ± 0.0157
2.9304 ± 0.9978	β	2.6773 ± 0.3861
0.1175 ± 0.0506	δ	0.1099 ± 0.0358
2.4736 ± 0.1649	γ	2.4643 ± 0.12
7.5475 ± 3.2037	λ_F	5.3198 ± 2.0531
0.0318 ± 0.0068	N_4	0.0346 ± 0.0048

PRELIMINARY

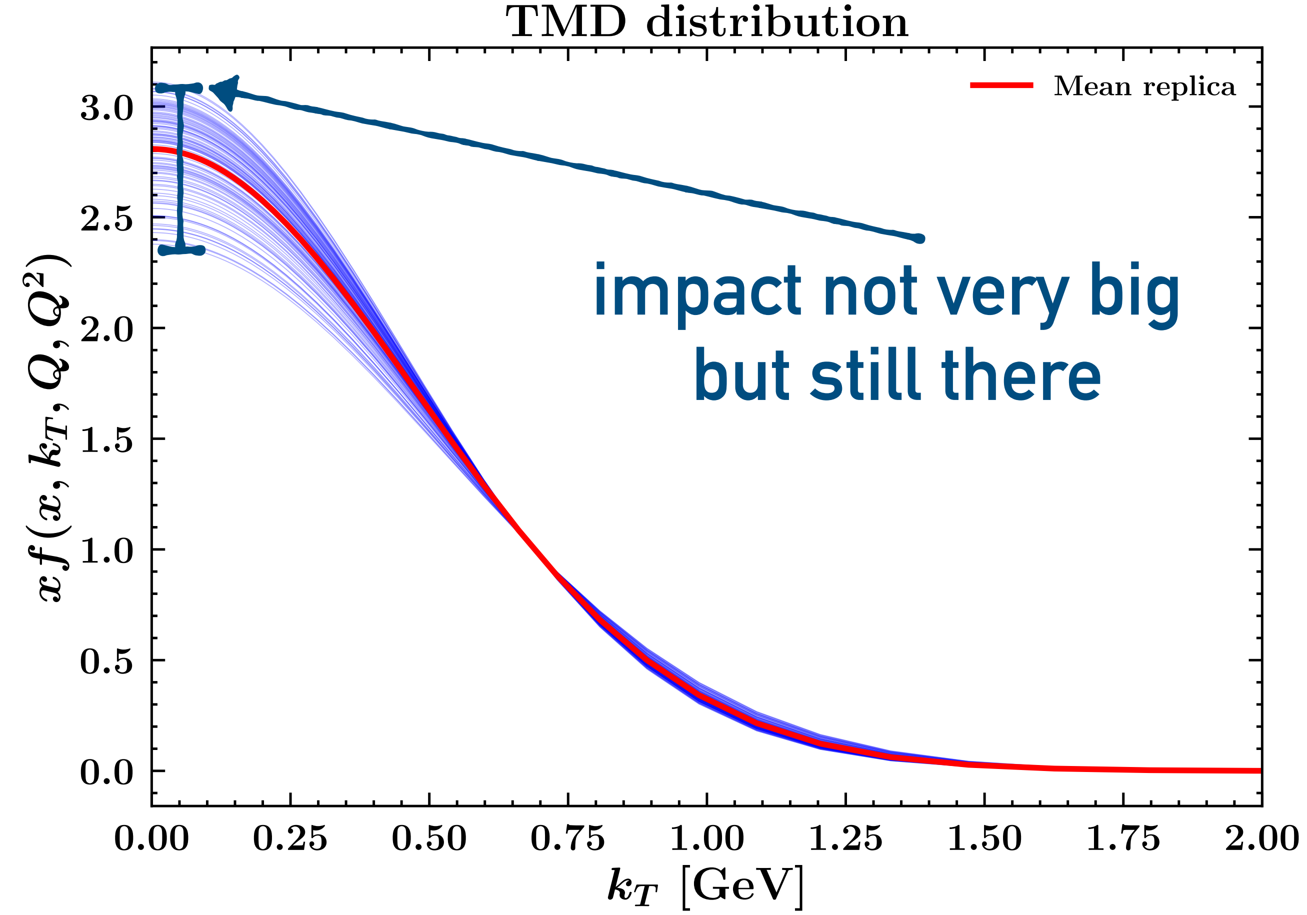
general improvement of uncertainties

EIC impact on TMD PDFs

PV17 baseline



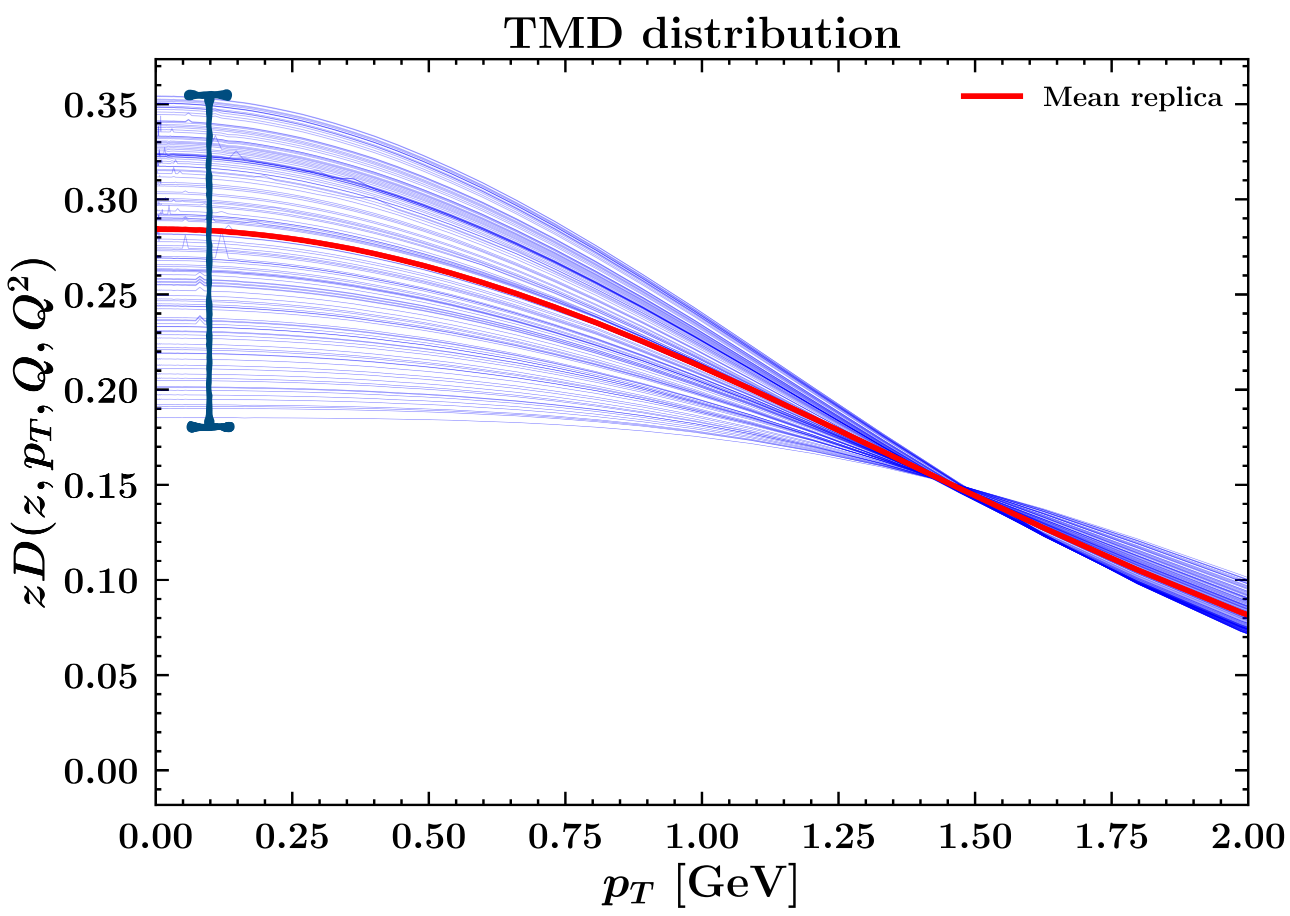
PV17baseline + EIC



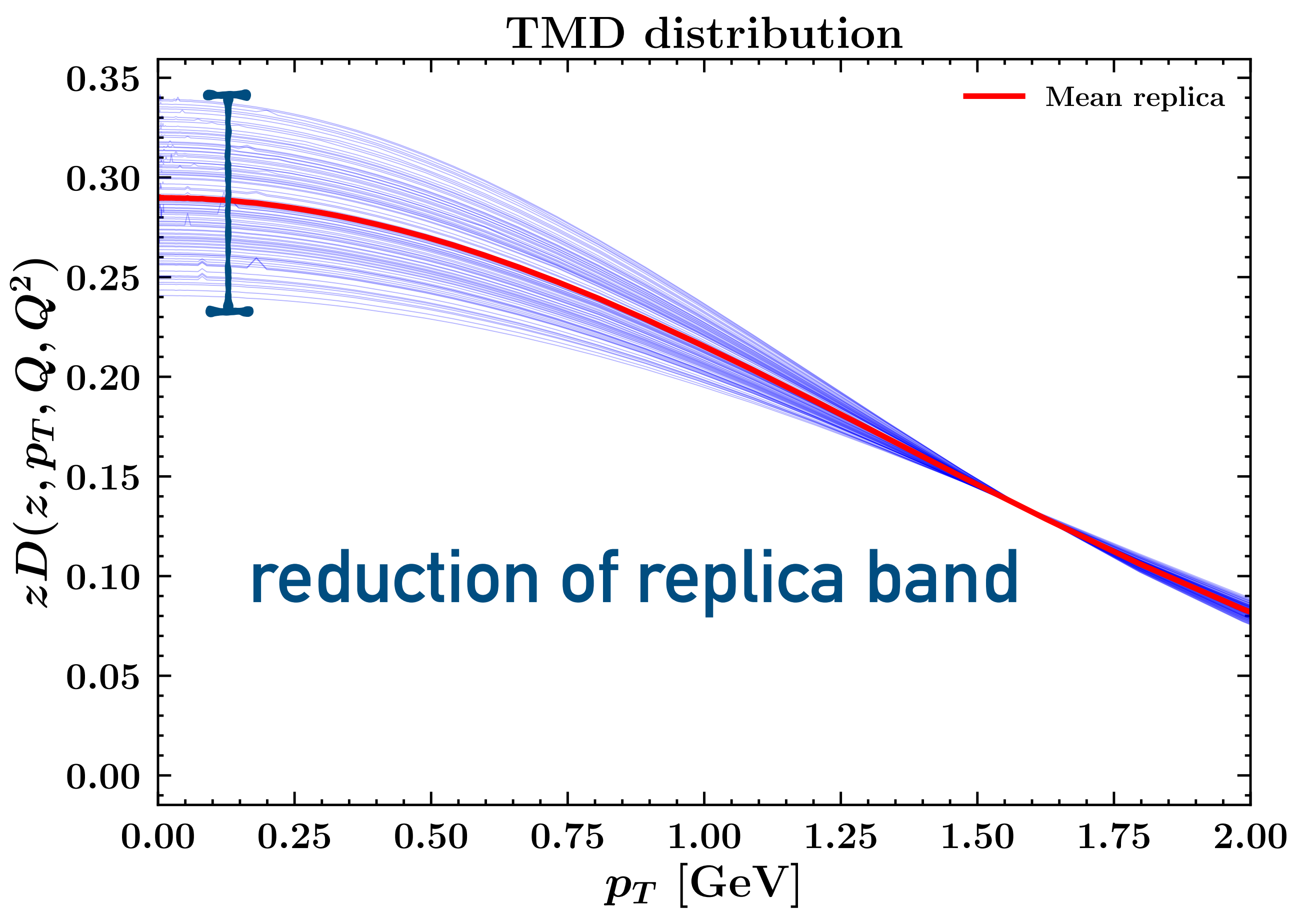
quark up, $Q = 2$ GeV, $x = 0.1$

EIC impact on TMD FFs

PV17 baseline



PV17baseline + EIC



quark up, $Q = 2$ **GeV**, $z = 0.25$

Conclusions

EIC will have a significant impact on TMDs

will cover a large region
not covered by present data

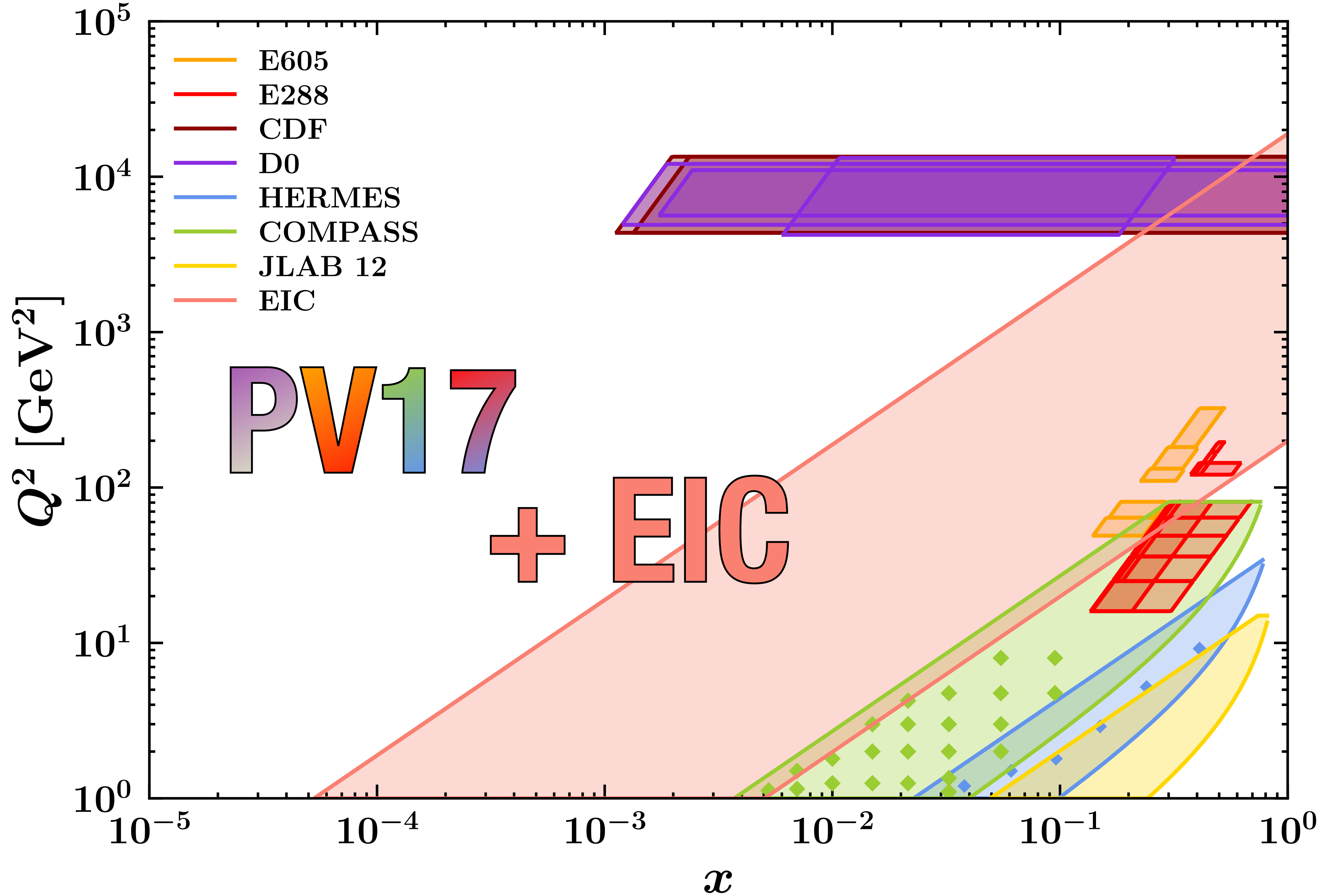
from impact studies,
and in particular from EIC PSEUDO DATA FITS
we have encouraging results on uncertainties reduction

CAVEAT
results depend on
the chosen parameterization

preliminary work
need to include also
other EIC energy configurations

Backup

Electron-Ion Collider



little side-note:

impact studies can be also done for **JLAB 12**

EIC pseudodata

generated by Ralf Seidl and available on
https://github.com/VladimirovAlexey/EIC_YR_TMD

5x41 18x275

configurations

5x100 18x100

10x100

π^+

π^- final state

hadrons K^+

K^-

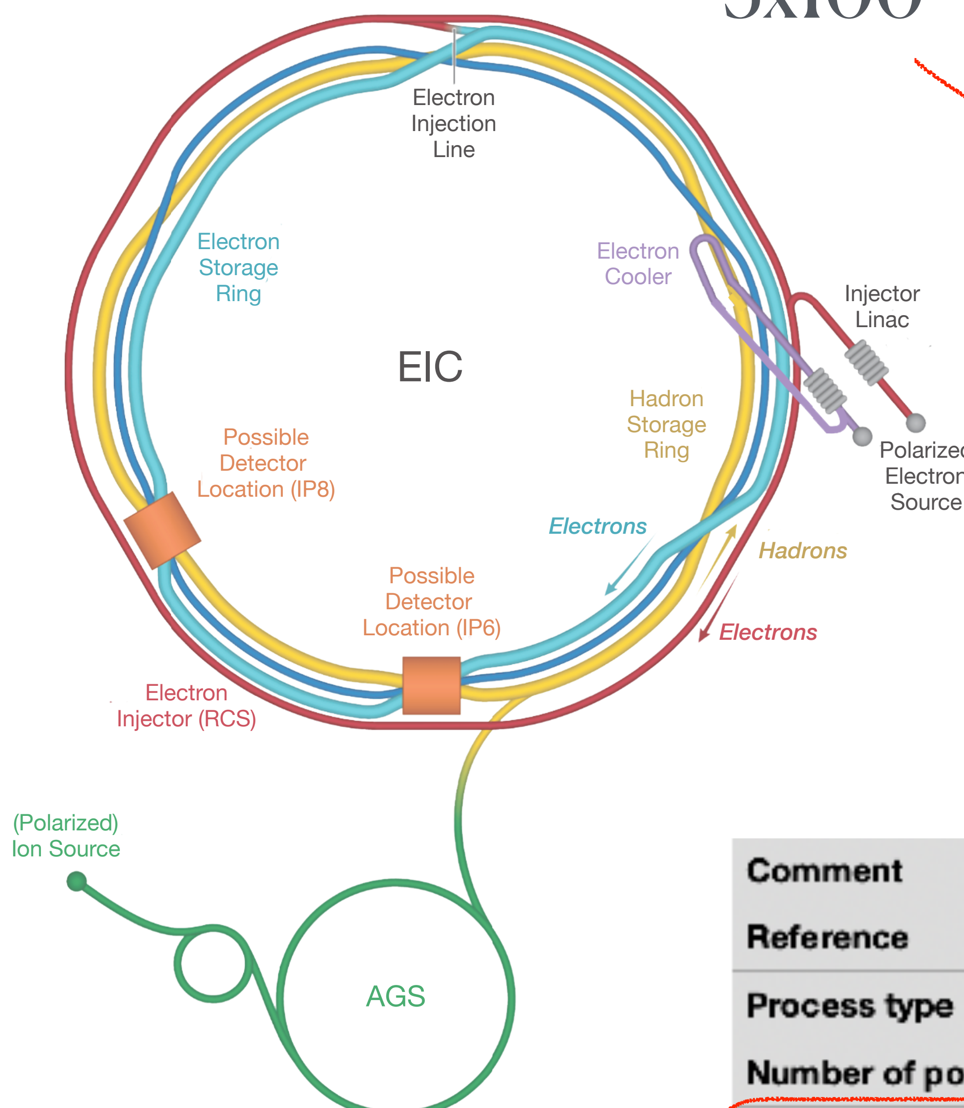


image from EIC YR
arXiv:2103.05419

Comment	Ralf's pseudo data for EIC.
Reference	Ralf
Process type	SIDIS
Number of points	2958
Number of uncorr.errors	2
Number of corr.errors	0
Number of norm.errors	1
List of norm.errors (relative)	0.03
Total cross-section normalized	False

uncertainties

eight options for EIC settings:
we choose option 8

EIC impact studies
sensitivity coefficients

E. Aschenauer, I. Borsa, G. Lucero, A. S. Nunes, R. Sassot
arXiv:2007.08300

$F_{UU,T}(x, z, q_T; Q^2)$ — observable

distribution — TMD parameters

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\xi \Delta \mathcal{O} \Delta f_i}$$

experimental uncertainty
(from pseudodata)

$$\xi \equiv \frac{\delta \mathcal{O}}{\Delta \mathcal{O}}$$

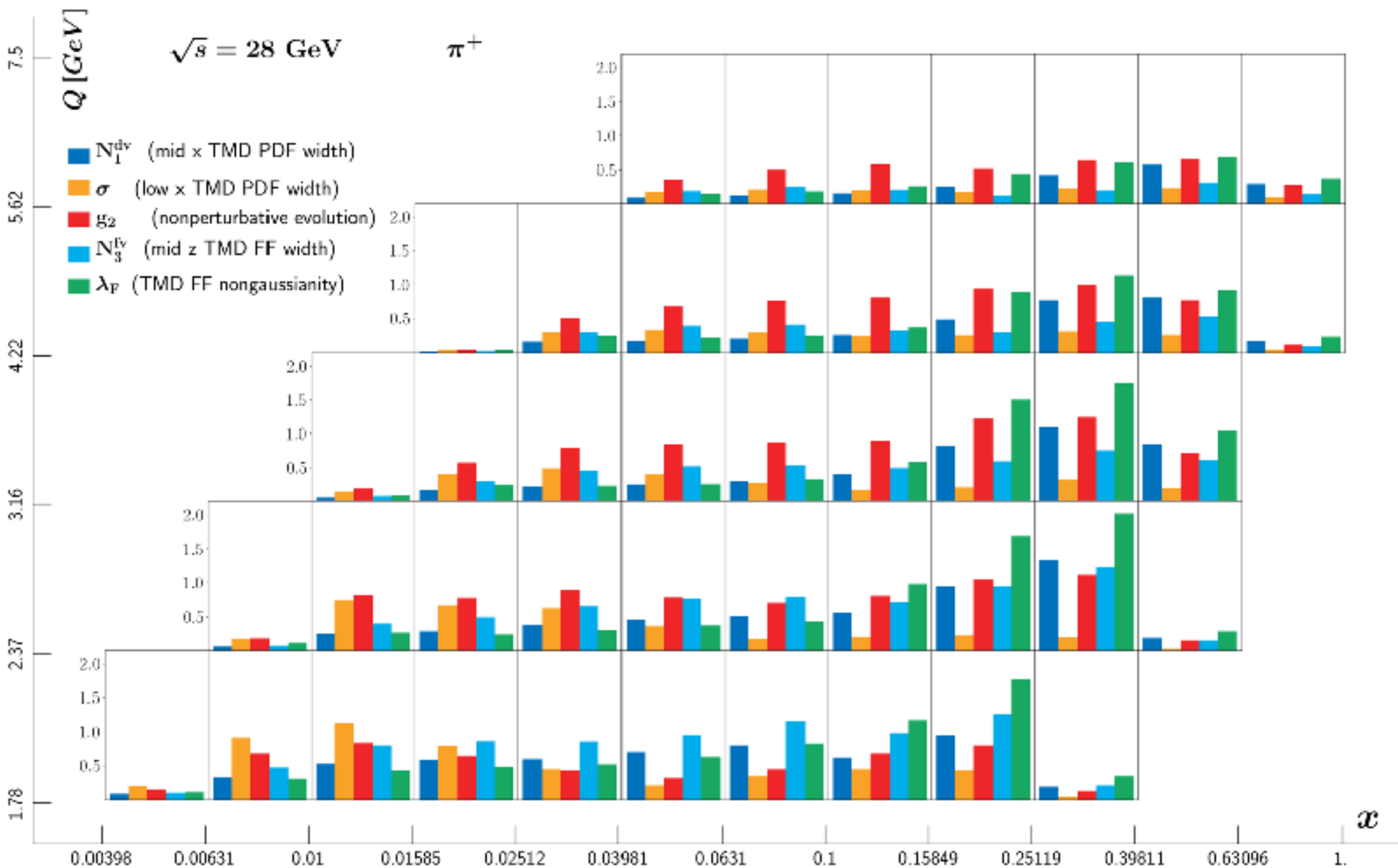
theoretical uncertainty

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{k=1}^{N} \mathcal{O}[f_i^{(k)}]$$

EIC impact studies

sensitivity coefficients

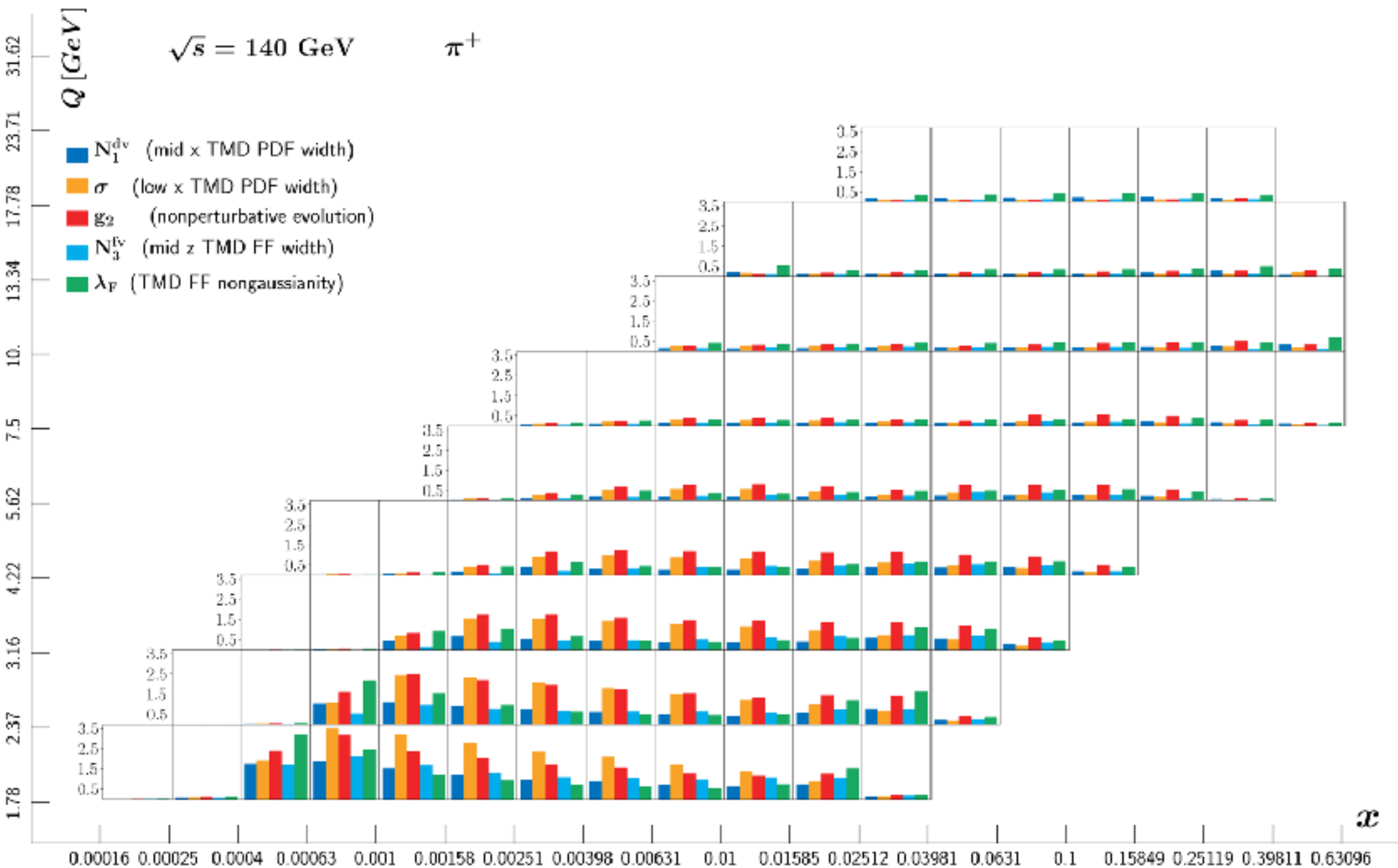
$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\delta \mathcal{O} \Delta f_i}$$



EIC impact studies

sensitivity coefficients

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\delta \mathcal{O} \Delta f_i}$$



Sensitivity coefficient and standard deviation

how to estimate uncertainties reduction?

$$S[\mathcal{O}, f] = \frac{\langle \mathcal{O} f \rangle - \langle \mathcal{O} \rangle \langle f \rangle}{\delta \mathcal{O}\, \Delta f}$$

from **PV17**
we know a parameter A with error ΔA

- if we perform a new measurement that produces on A an error equal to its initial standard deviation, $\delta A = \Delta A$

  ⟶ this corresponds to $S(A) = 1$

  in fact, if A can be ideally considered as parameter and observable, then

$$S(A, A) = \frac{\langle A\, A \rangle - \langle A \rangle \langle A \rangle}{\delta A\, \Delta A} = \frac{(\Delta A)^2}{\Delta A\, \Delta A} = 1$$

- the error on A scales as

$$1/\sqrt{2} = 1/\sqrt{1 + (S = 1)}$$

if the new measurement is more precise, then S >1 and the error is further reduced

for n measurements, the error on A should scale as

$$1/\sqrt{1 + S_1 + .. + S_n}$$

Non perturbative evolution g_2

which energy configuration has the highest impact?

summing over all (x, Q^2) bins

PV17 fit

$\Delta g_2 = 0.01$

$$R(A) = \frac{(\Delta A)_{\text{prev}}}{(\delta A)_{\text{EIC}}}$$

run at $\sqrt{s} = 28$ GeV π+	$\longrightarrow$ 0.00155	$R(g_2) = 6.45$
run at $\sqrt{s} = 44$ GeV π+	$\longrightarrow$ 0.00120	$R(g_2) = 8.33$
run at $\sqrt{s} = 63$ GeV π+	$\longrightarrow$ 0.00108	$R(g_2) = 9.26$
run at $\sqrt{s} = 84$ GeV π+	$\longrightarrow$ 0.00105	$R(g_2) = 9.52$
run at $\sqrt{s} = 140$ GeV π+	$\longrightarrow$ 0.00096	$R(g_2) = 10.36$

(no correlation between measurements in different bins)

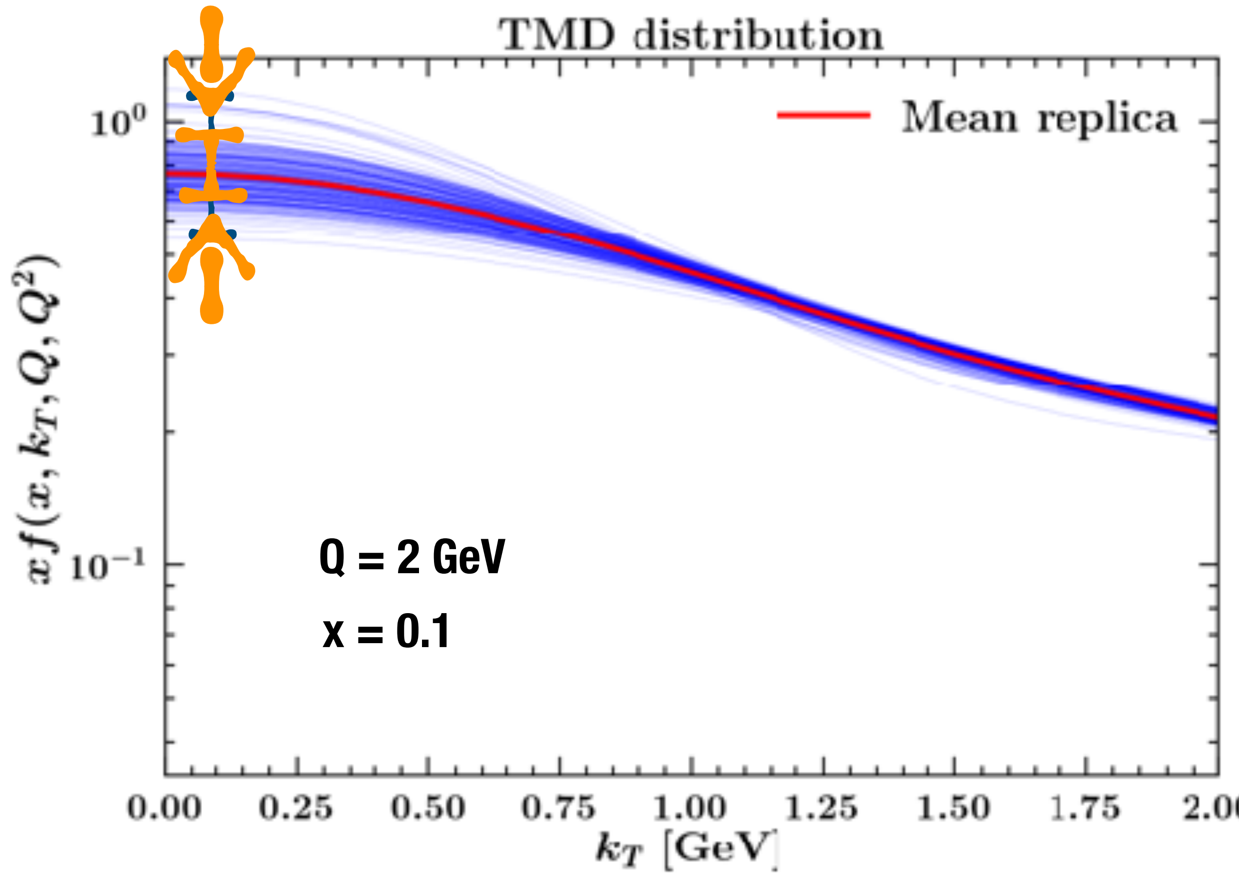
consistent trend:
evolution parameter better constrained by larger covered (x, Q^2)
$\longrightarrow$ larger $\sqrt{s}$ configuration

Attempts at reweighing

200 replicas are compared
with pseudodata

$$\chi^2_k = \chi^2_{k,\mathrm{EIC}} + \chi^2_{k,\mathrm{PV17}}$$

'original' χ^2
with respect to PV17 data



weights

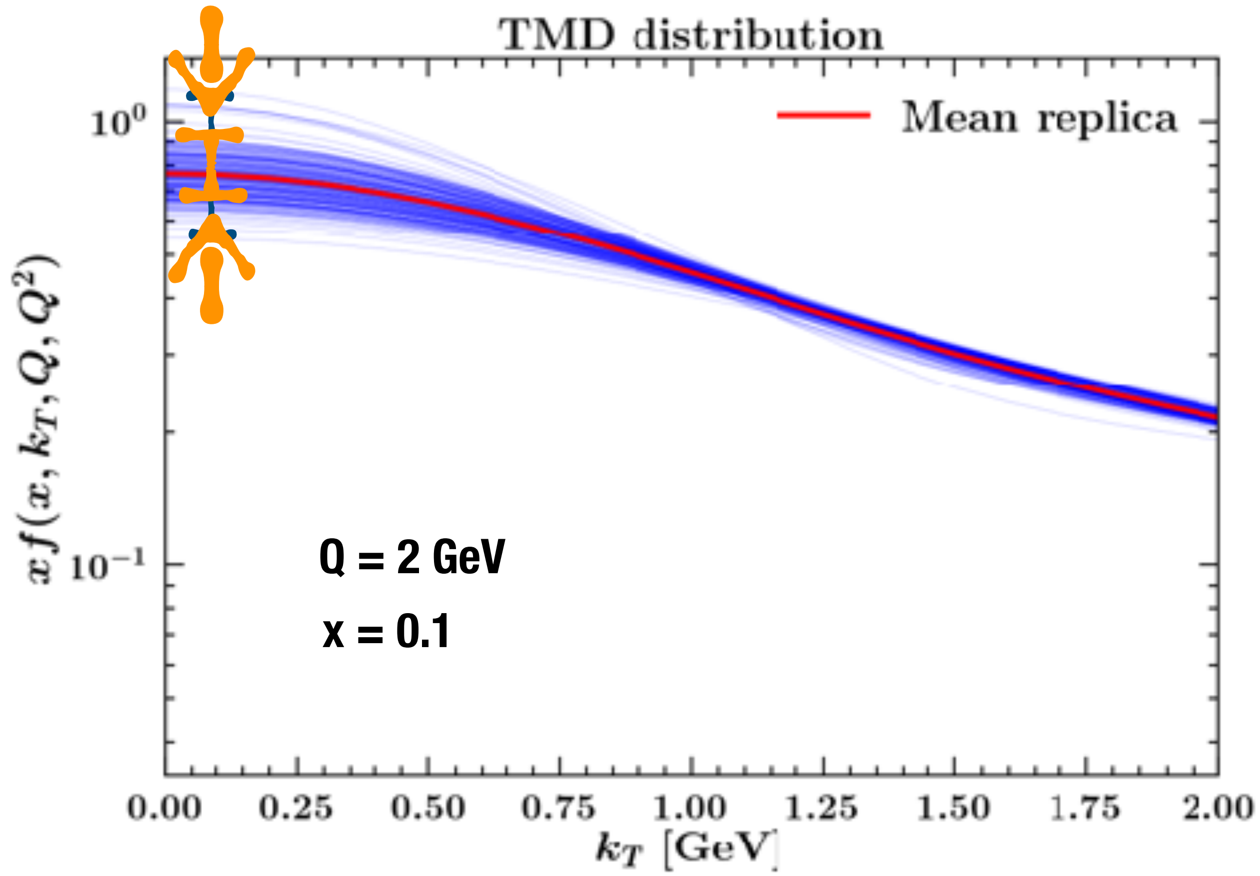
$$w_k \propto \mathcal{P}(f_k|\chi_k) \propto \chi_k^{n-1} e^{-\frac{1}{2}\chi_k^2}$$

used to select replicas

from NNPDF Collaboration
arXiv:1108.1758

reflect the impact of EIC data on extracted TMDs

Attempts at reweighing



different mathematical formulas to compute the weights

N. Sato, J. Owens, H. Prosper, PRD 89 (2014) 114020;
H. Paukkunen, P. Zurita, JHEP 12 (2014) 100

$$w_k \propto \mathcal{P}(f_k|\chi_k) \propto e^{-\frac{1}{2}\chi_k^2}$$

selects replicas with very low χ^2

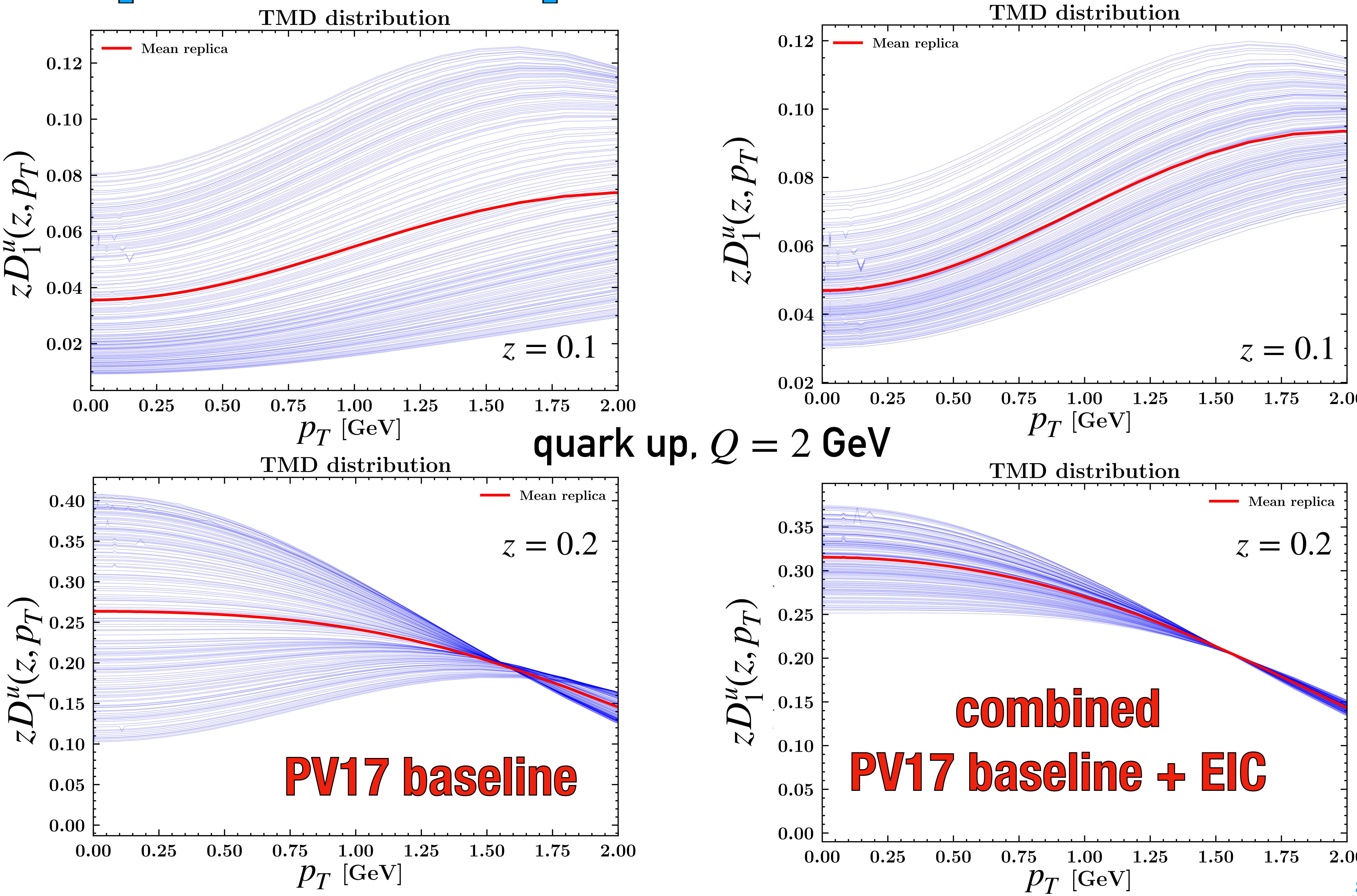
NNPDF Collaboration
arXiv:1108.1758

total number of points

$$w_k \propto \mathcal{P}(f_k|\chi_k) \propto \chi_k^{n-1} e^{-\frac{1}{2}\chi_k^2}$$

suppresses replicas with very high AND very low χ^2

Impact on unpolarized TMD FFs



quark up, $Q = 2$ GeV