



Istituto Nazionale di Fisica Nucleare



UNIVERSITÀ
DI PAVIA

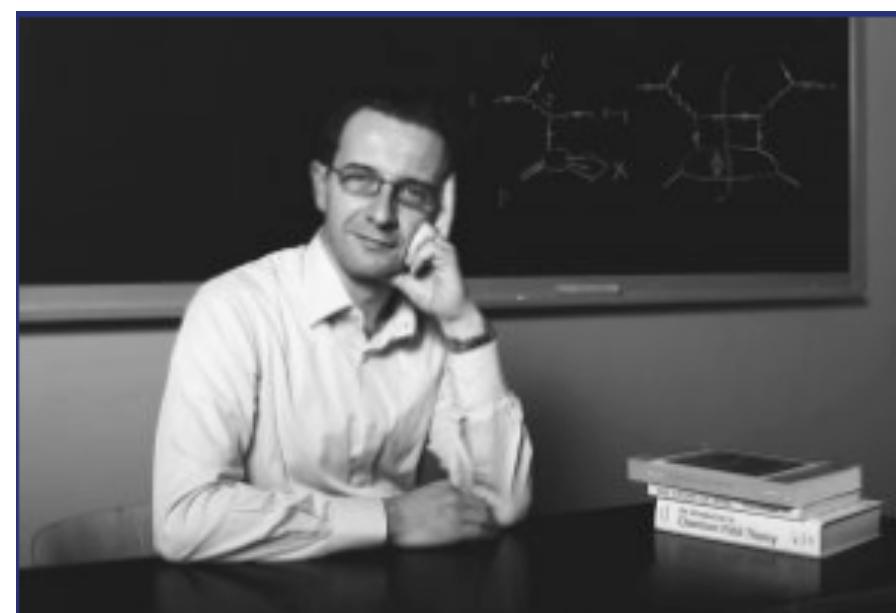
PROGRESS IN THE EXTRACTION OF UNPOLARIZED TMDS FROM GLOBAL DATA SETS

Matteo Cerutti

MAP Collaboration

RESULTS OBTAINED WITH CONTRIBUTIONS FROM

Alessandro Bacchetta



Marco Radici



Andrea Signori



Valerio Bertone



Chiara Bissolotti



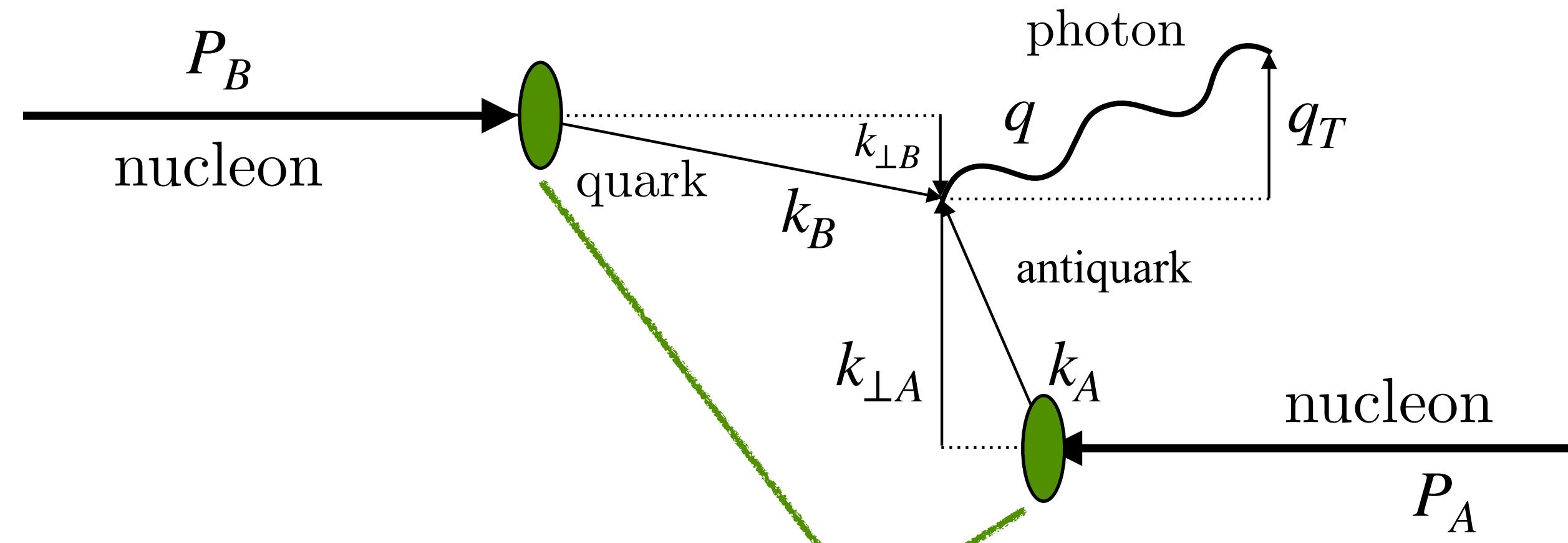
Giuseppe Bozzi



Fulvio Piacenza



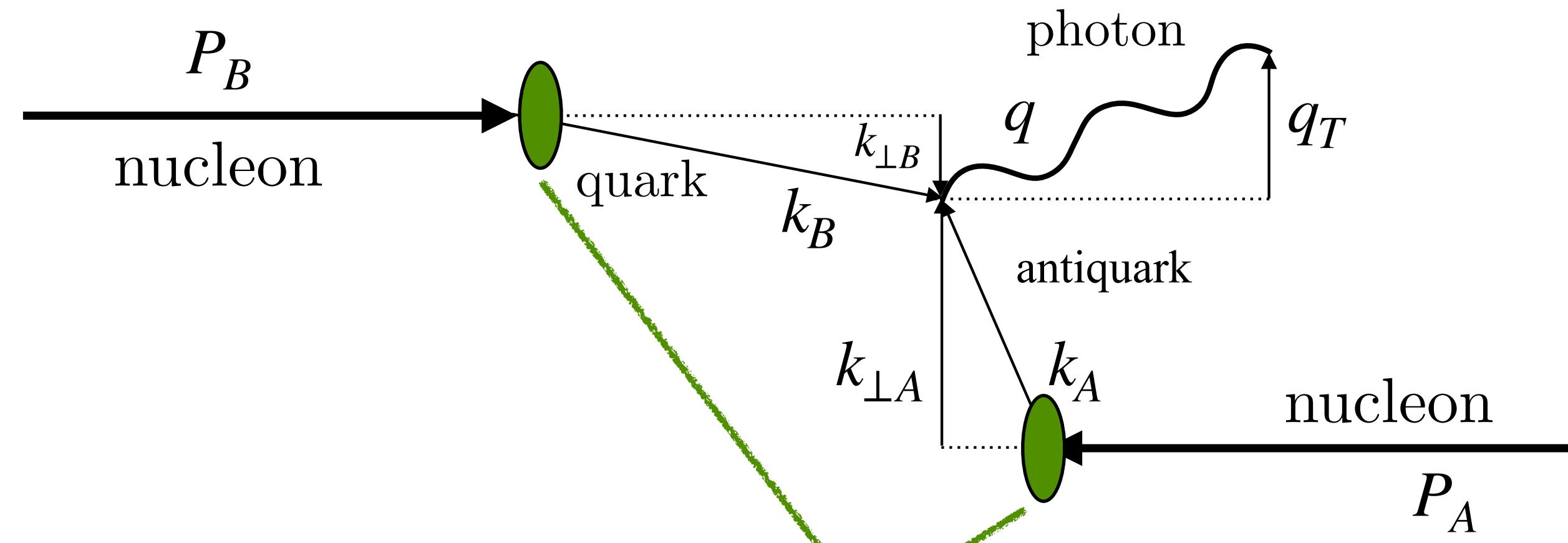
TMDS IN DRELL-YAN PROCESSES



$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$\begin{aligned}
 &= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2 \mathbf{k}_{\perp A} d^2 \mathbf{k}_{\perp B} f_1^a(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{a}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B}) \\
 &\quad + Y_{UU}^1(Q^2, \mathbf{q}_T^2) + \mathcal{O}(M^2/Q^2)
 \end{aligned}$$

TMDS IN DRELL-YAN PROCESSES

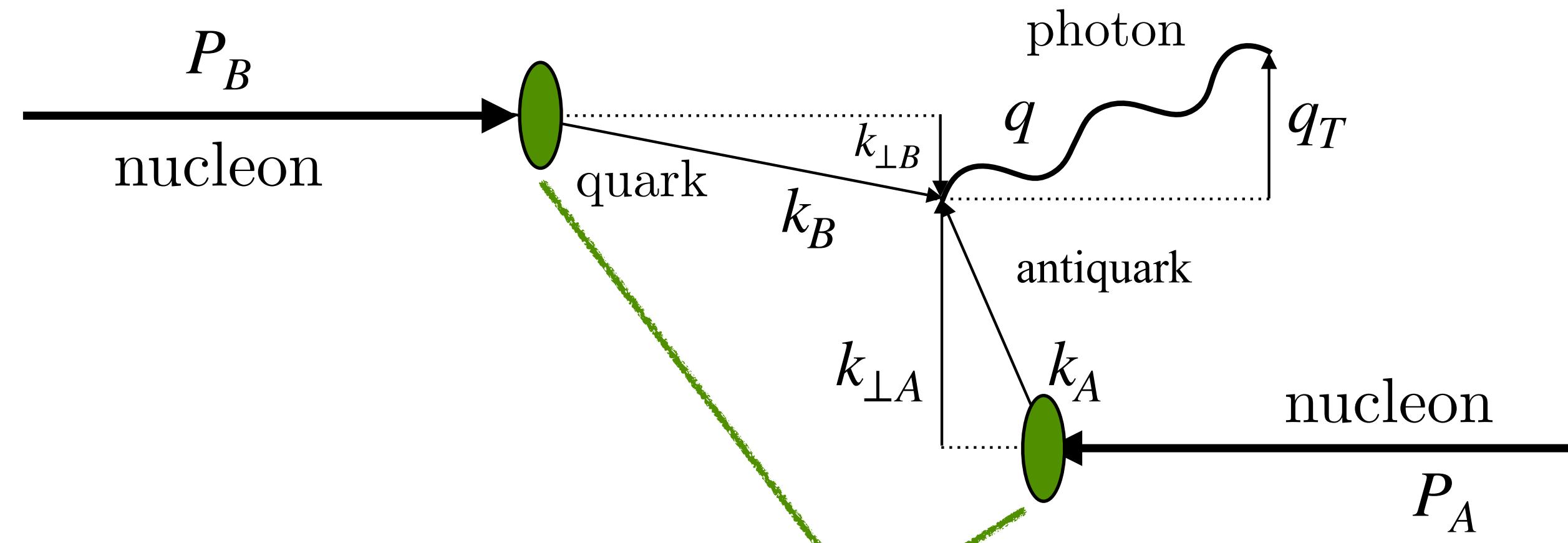


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- The W term dominates in the region where $q_T \ll Q$

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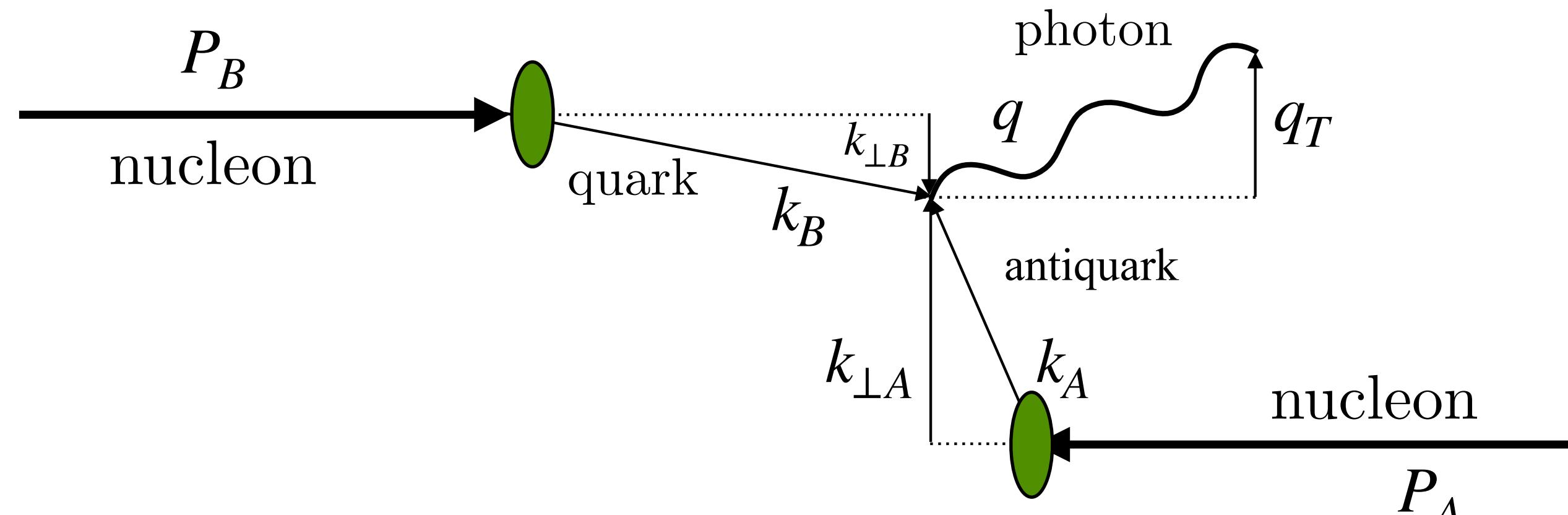


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- The W term dominates in the region where $q_T \ll Q$
- Y term has been excluded in the Pavia analyses

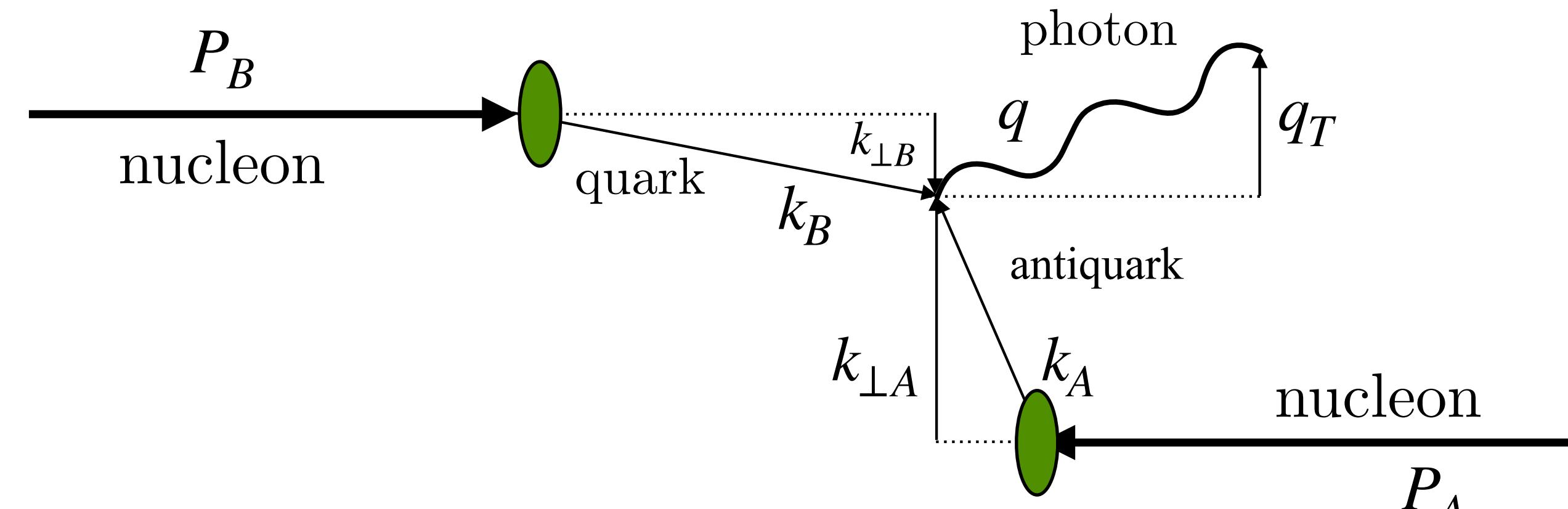
TMDS IN DRELL-YAN PROCESSES



$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$\begin{aligned} &\approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int d^2 \mathbf{k}_{\perp A} d^2 \mathbf{k}_{\perp B} f_1^q(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{q}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B}) \\ &= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2) \end{aligned}$$

TMDS IN DRELL-YAN PROCESSES

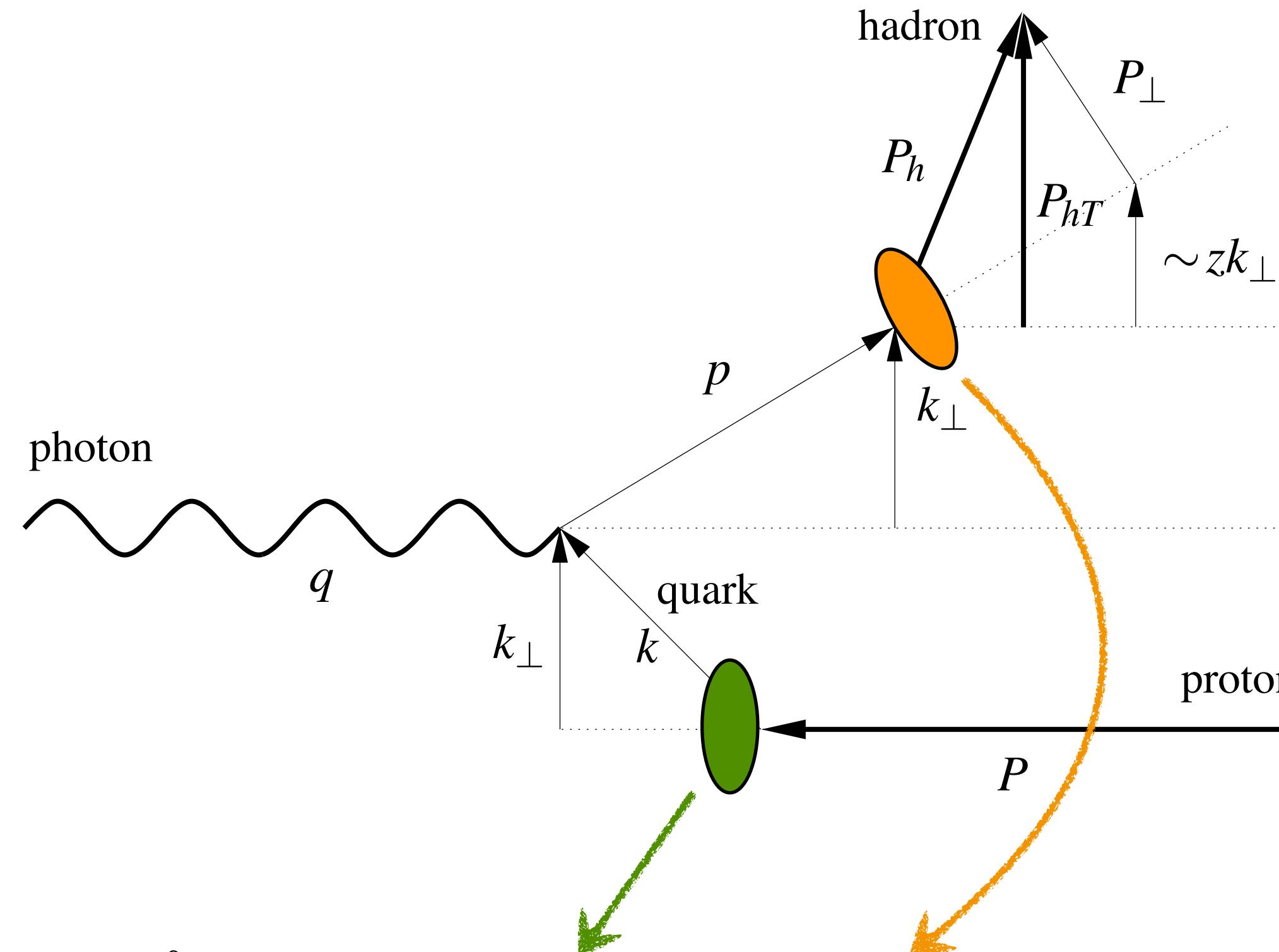


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- At small q_T the dominant part is given by TMDs
- Fourier-transformed space to avoid convolutions
- TMDs formally depend on two scales, but we set them equal.

TMDS IN SEMI-INCLUSIVE DIS PROCESS



$$\begin{aligned}
 F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2) &= x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2 \mathbf{k}_{\perp} d^2 \mathbf{P}_{\perp} f_1^a(x, \mathbf{k}_{\perp}^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_{\perp}^2; \mu^2) \delta(z \mathbf{k}_{\perp} - \mathbf{P}_{hT} + \mathbf{P}_{\perp}) \\
 &= x \sum_a \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_{\perp}^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_{\perp}^2; \mu^2)
 \end{aligned}$$

TMD STRUCTURE

$$\hat{f}_1^q(x, b_T; \mu^2) = \int d^2 \mathbf{k}_\perp e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^q(x, \mathbf{k}_\perp^2; \mu^2)$$

see, e.g.,
Collins, “Foundations of Perturbative QCD” (11)

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matching coefficients
(perturbative)

see, e.g.,
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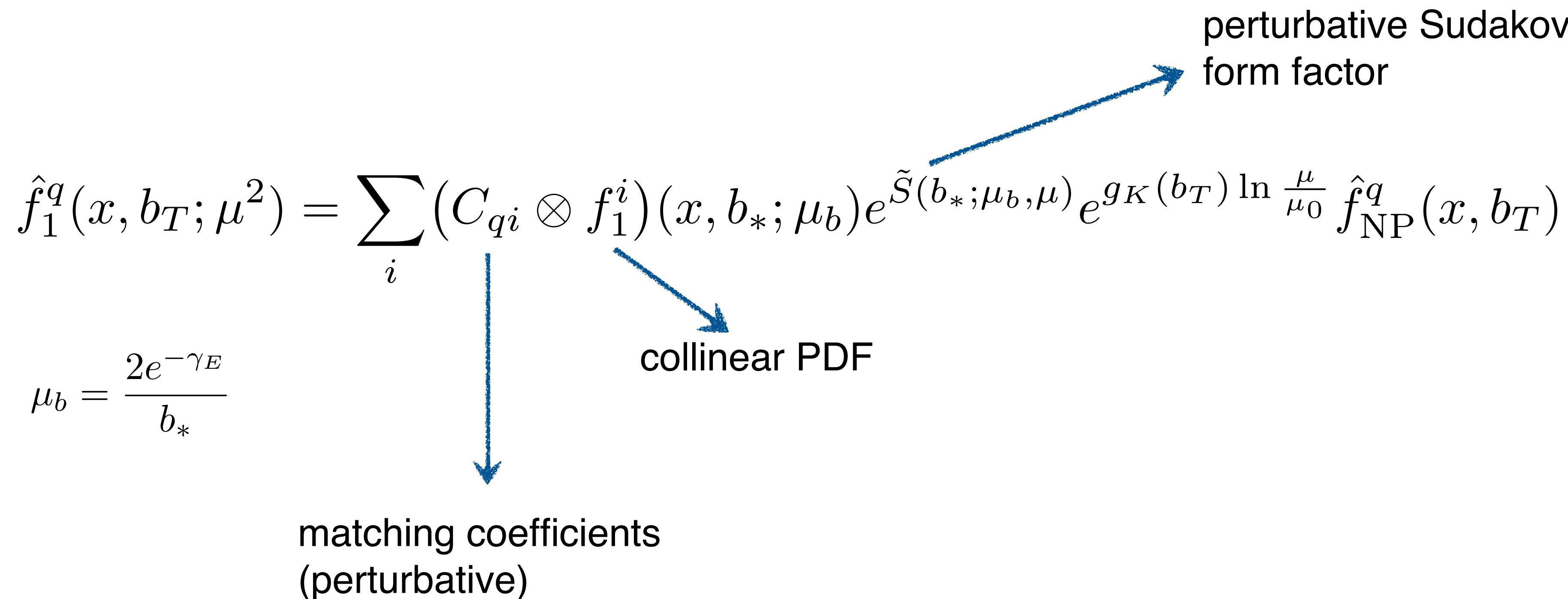
collinear PDF

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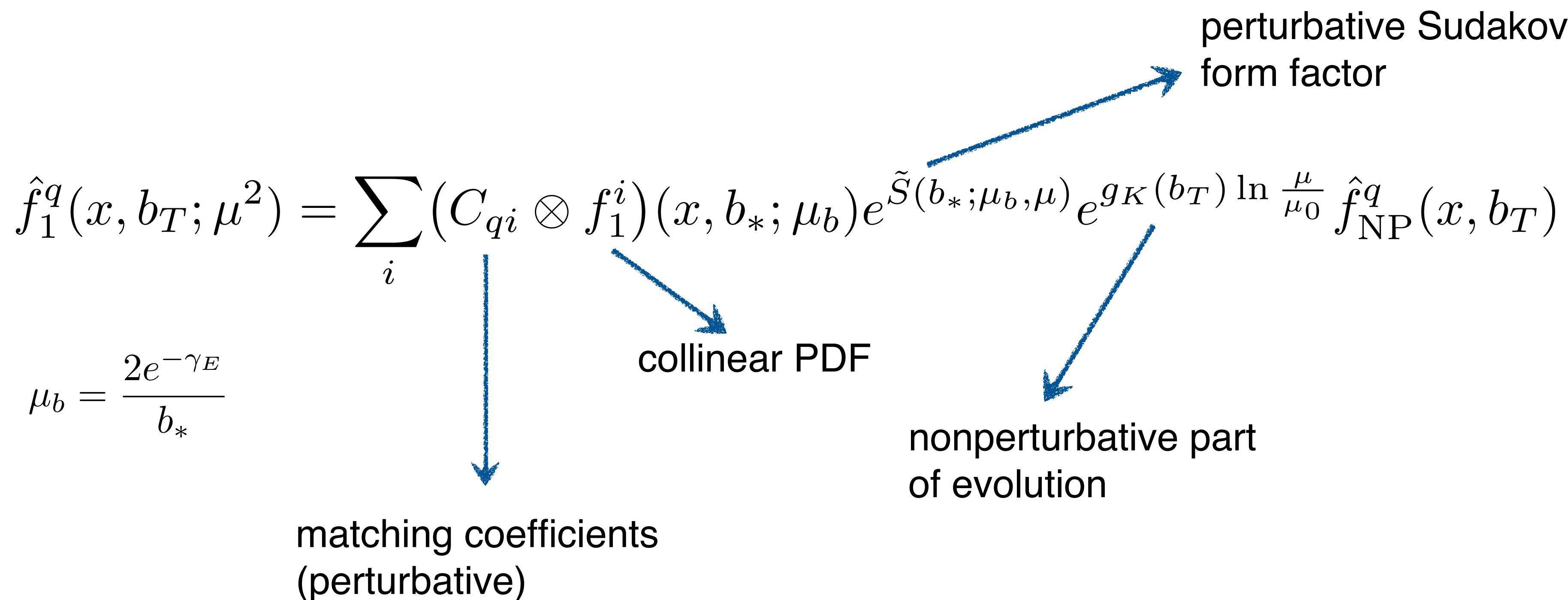
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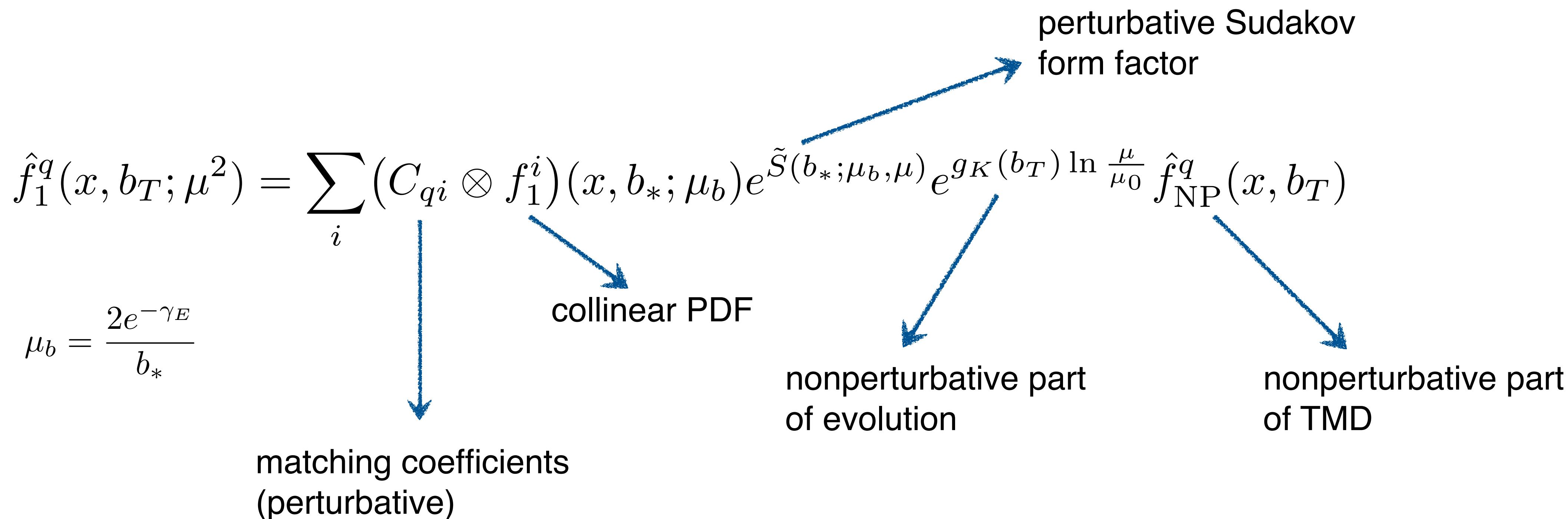
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PERTURBATIVE ORDER OF EACH INGREDIENT

Orders in powers of α_S

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Orders in powers of α_S

| Accuracy | Hard factor and matching coefficient | | Ingredients in perturbative Sudakov form factor | | PDF and a_S evol. |
|----------|--------------------------------------|--------------------|---|-----|---------------------|
| | H and C | K and γ_F | γ_K | | |
| LL | 0 | - | 1 | - | |
| NLL | 0 | 1 | 2 | LO | |
| NLL' | 1 | 1 | 2 | NLO | |
| NNLL | 1 | 2 | 3 | NLO | |

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| NNLL | 1 | 2 | 3 | NLO | |
| NNLL' | 2 | 2 | 3 | NNLO | |
| N^3LL | 2 | 3 | 4 | NNLO | |
| N^3LL' | 3 | 3 | 4 | N^3LO | |

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Collinear fragmentation functions not available beyond NLO!!

RECENT GLOBAL FITS OF UNPOLARIZED TMD DATA

| | Framework | HERMES | COMPASS | DY | Z production | N of points | χ^2/N_{points} |
|---|-----------|--------|---------|----|--------------|-------------|----------------------------|
| Pavia 2017 arXiv:1703.10157 | NLL | ✓ | ✓ | ✓ | ✓ | 8059 | 1.55 |
| SV 2017 arXiv:1706.01473 | NNLL' | ✗ | ✗ | ✓ | ✓ | 309 | 1.23 |
| BSV 2019 arXiv:1902.08474 | NNLL' | ✗ | ✗ | ✓ | ✓ | 457 | 1.17 |
| SV 2019 arXiv:1912.06532 | N^3LL^- | ✓ | ✓ | ✓ | ✓ | 1039 | 1.06 |
| Pavia 2019 arXiv:1912.07550 | N^3LL | ✗ | ✗ | ✓ | ✓ | 353 | 1.02 |

OUR WORK IN THE LAST TWO YEARS

New Global Fit

Simultaneously extraction of unpolarized TMD PDFs and FFs

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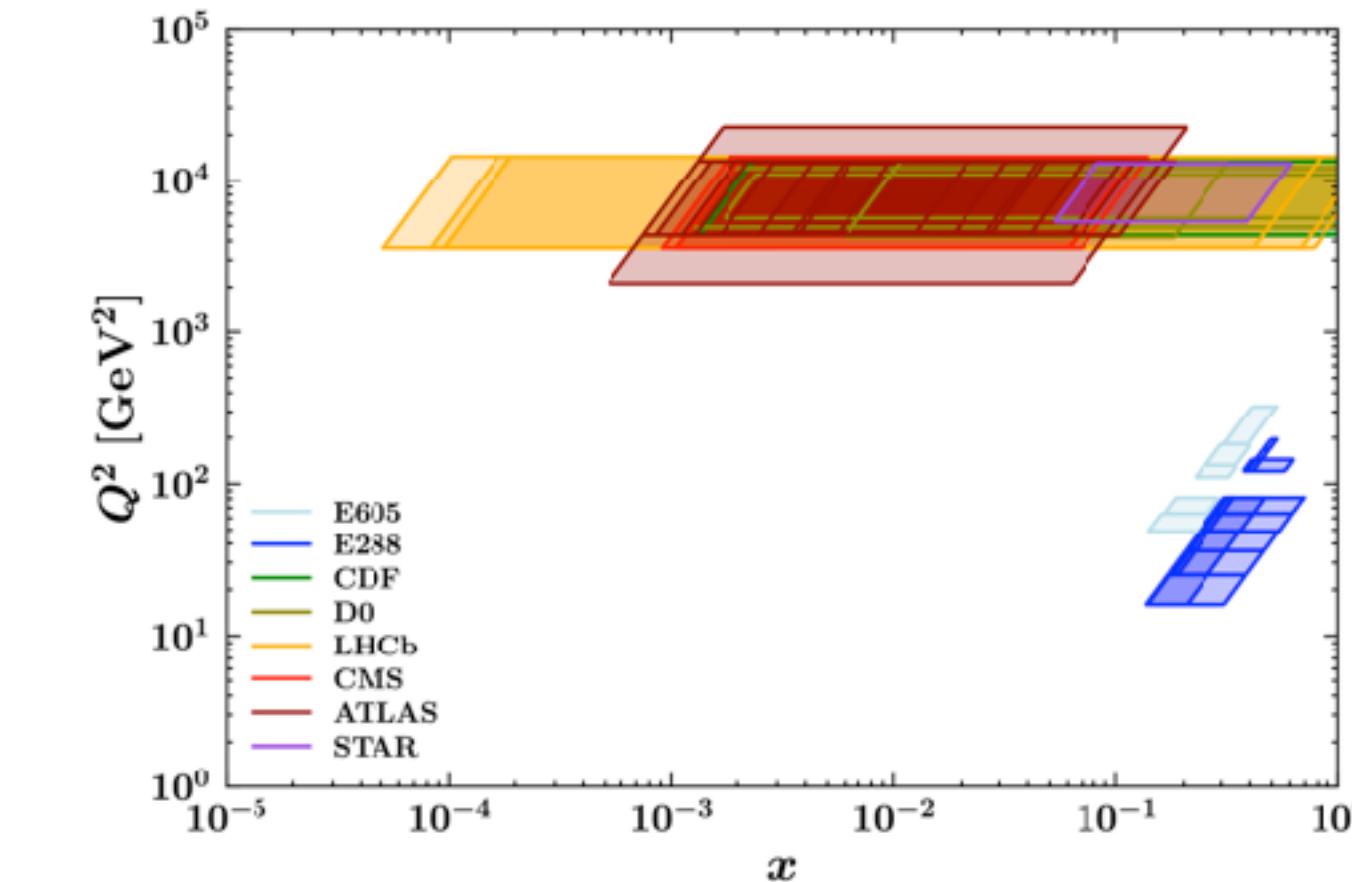
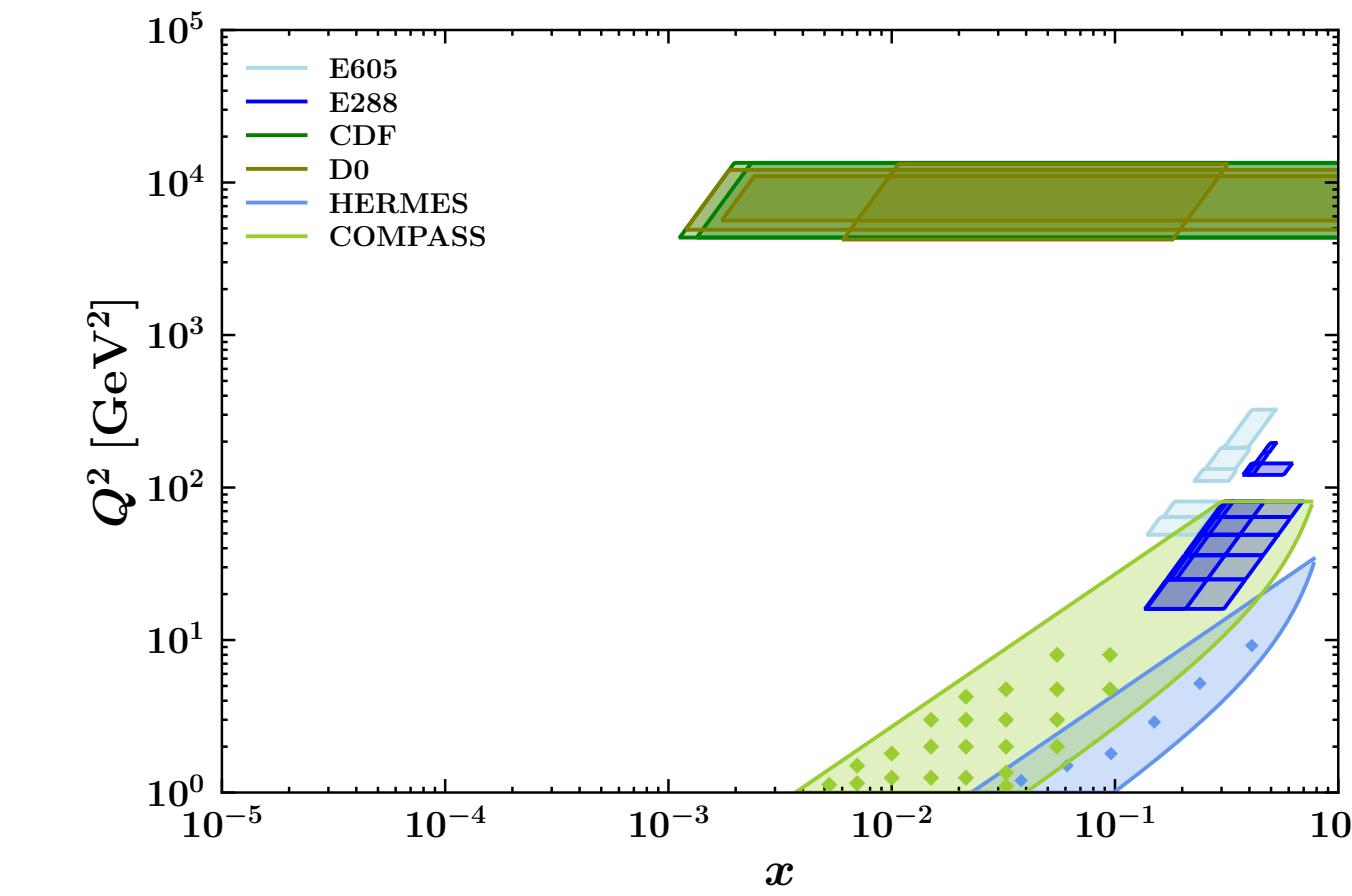
- SIDIS + Drell Yan

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New Global Fit

✓ SIDIS + Drell Yan

Simultaneously extraction of unpolarized TMD PDFs and FFs



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○ Integrated variables

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Simultaneously extraction of unpolarized TMD PDFs and FFs



Nanga Parbat: a TMD fitting framework

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

<https://github.com/vbertone/NangaParbat/releases>

For the last development branch you can clone the master code:

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git clone git@github.com:vbertone/NangaParbat.git
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If you instead want to download a specific tag:

<https://github.com/MapCollaboration>

OUR WORK IN THE LAST TWO YEARS

New Global Fit

✓ SIDIS + Drell Yan

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○ Up to N^2LL/N^3LL

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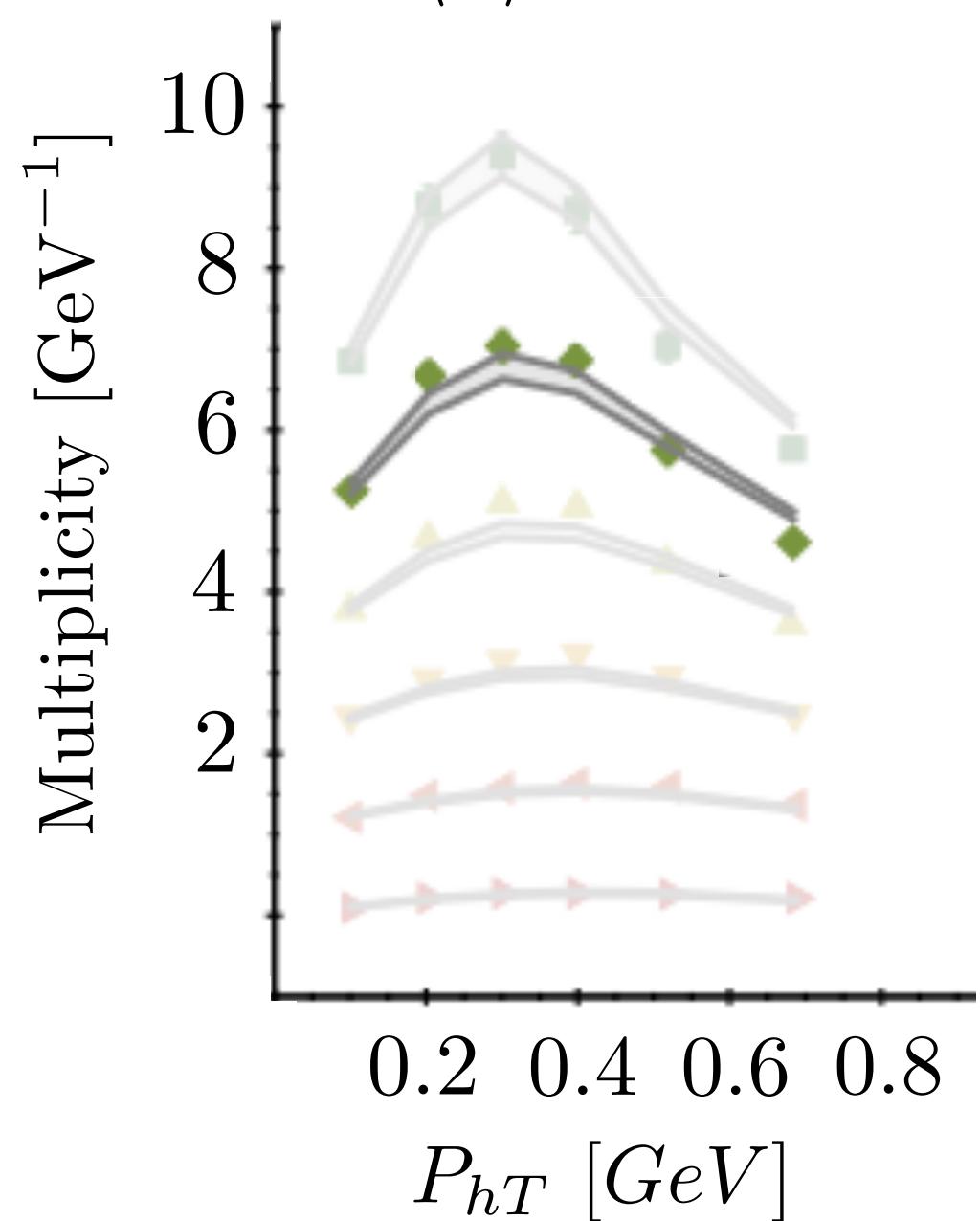
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RESULTS AT NLL: SIDIS (MULTIPLICITIES)

What we expected

$$\langle Q^2 \rangle = 2.9 \text{ GeV}^2$$

$$\langle x \rangle = 0.15$$



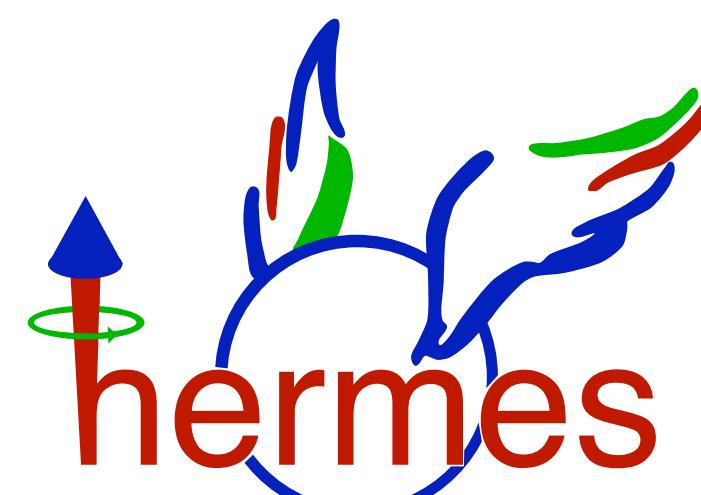
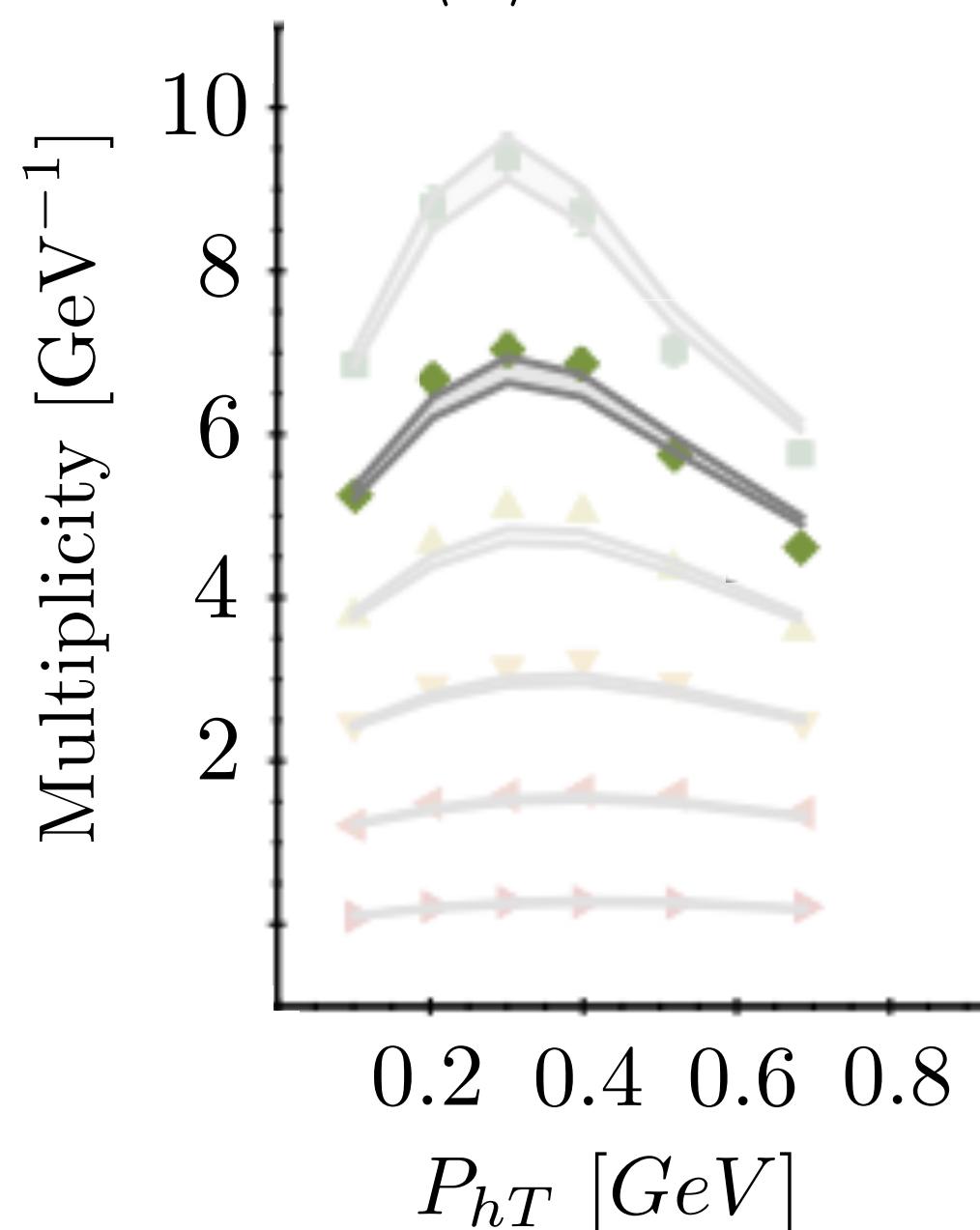
Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

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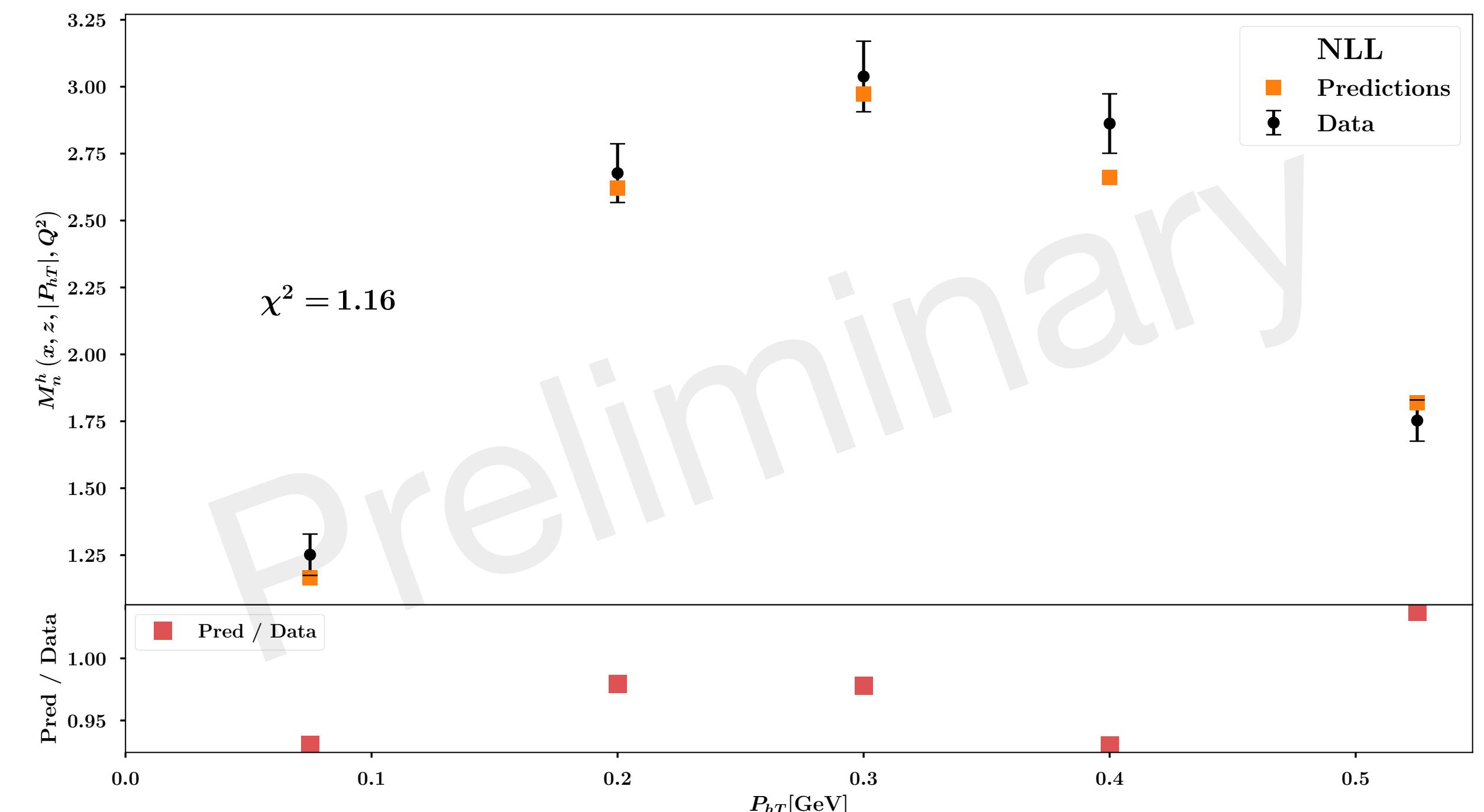
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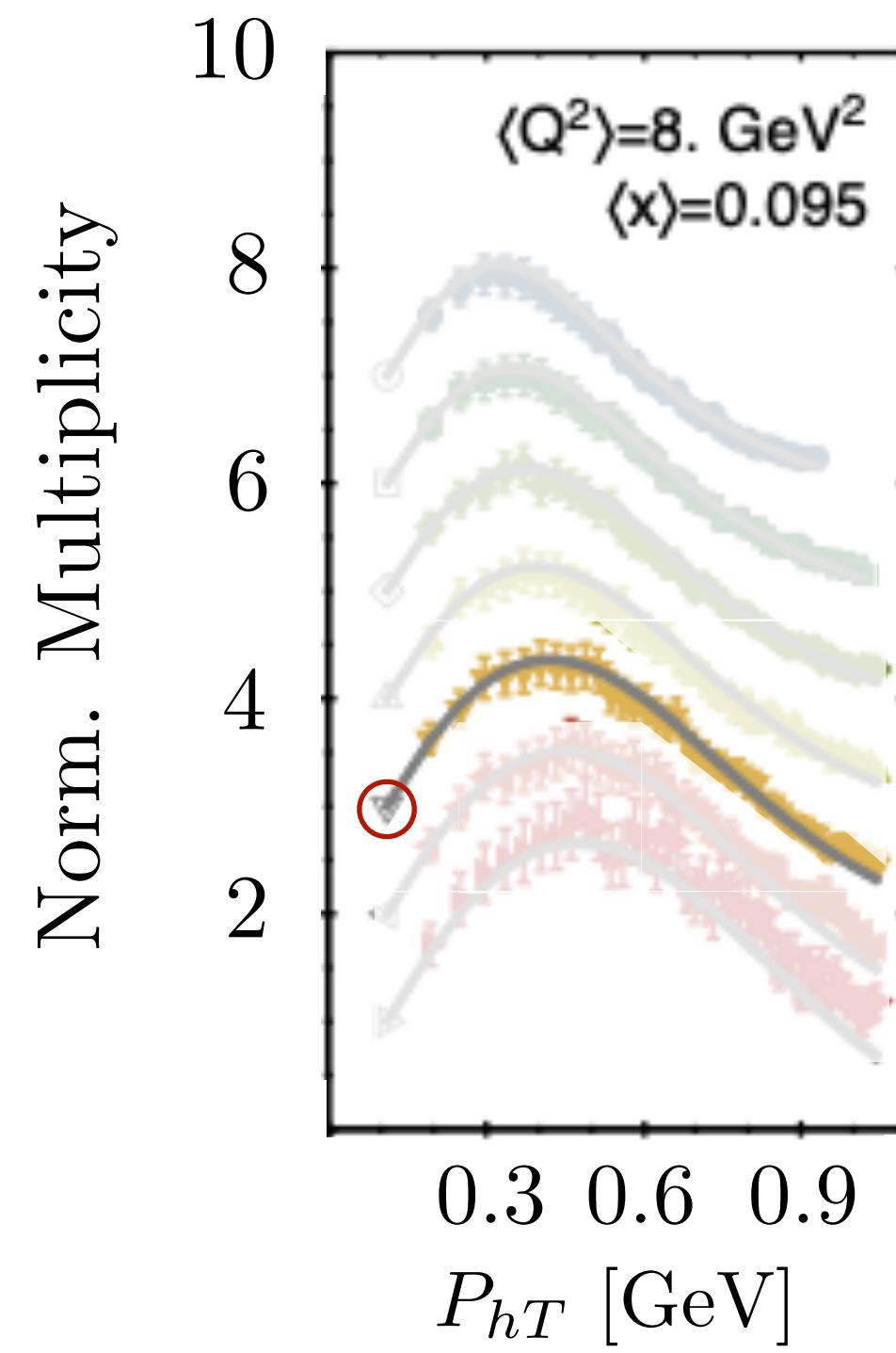
What we found



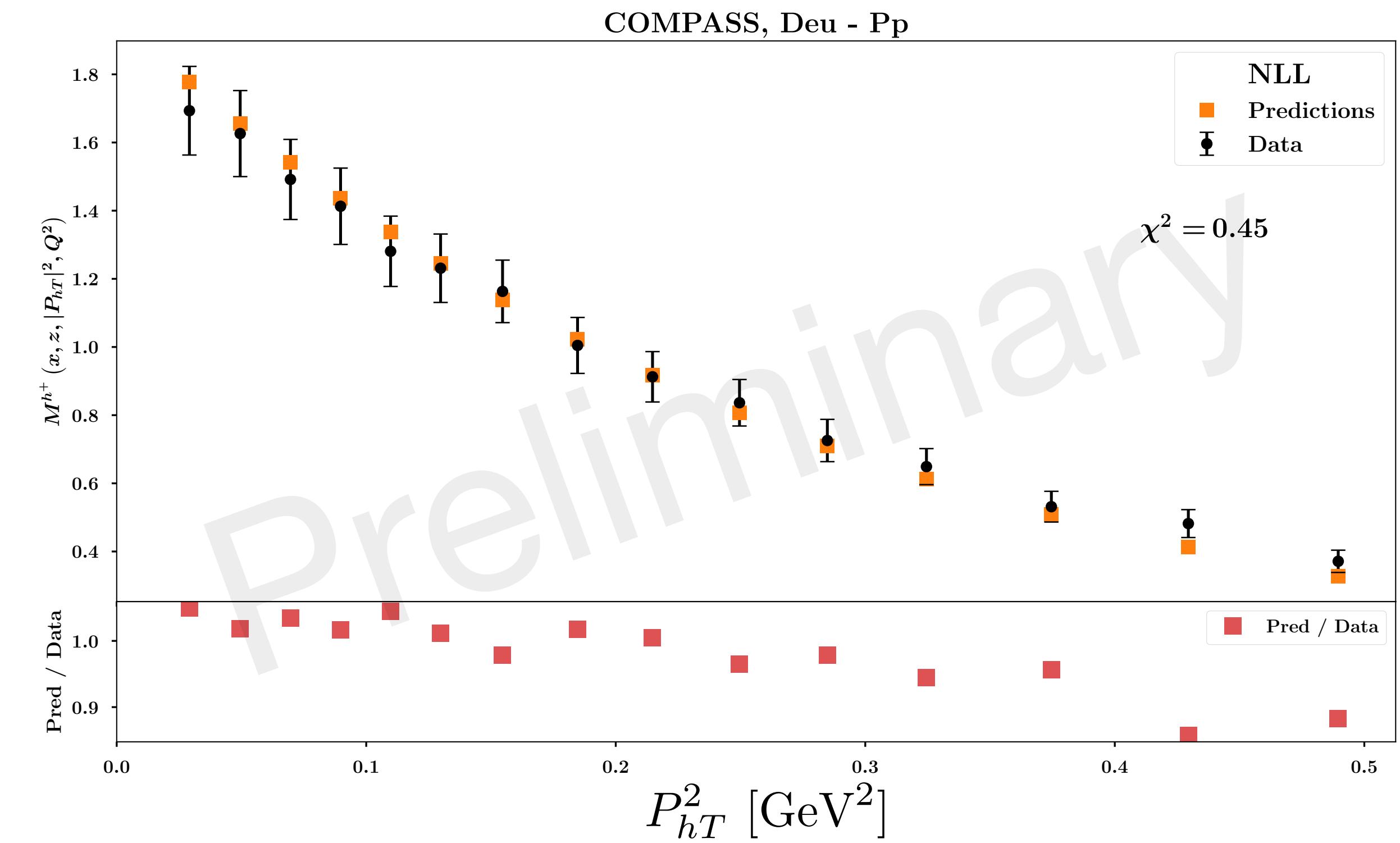
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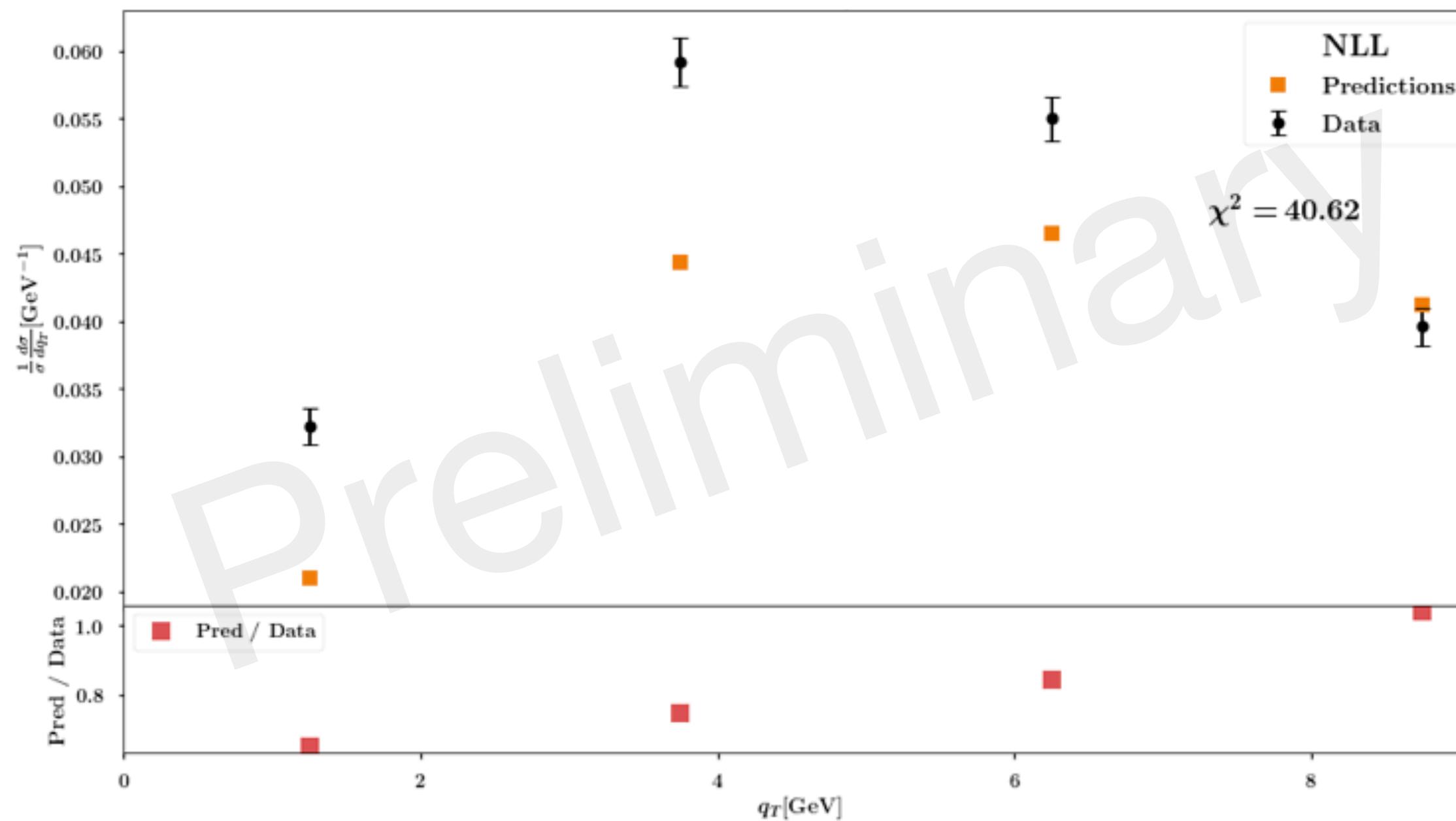
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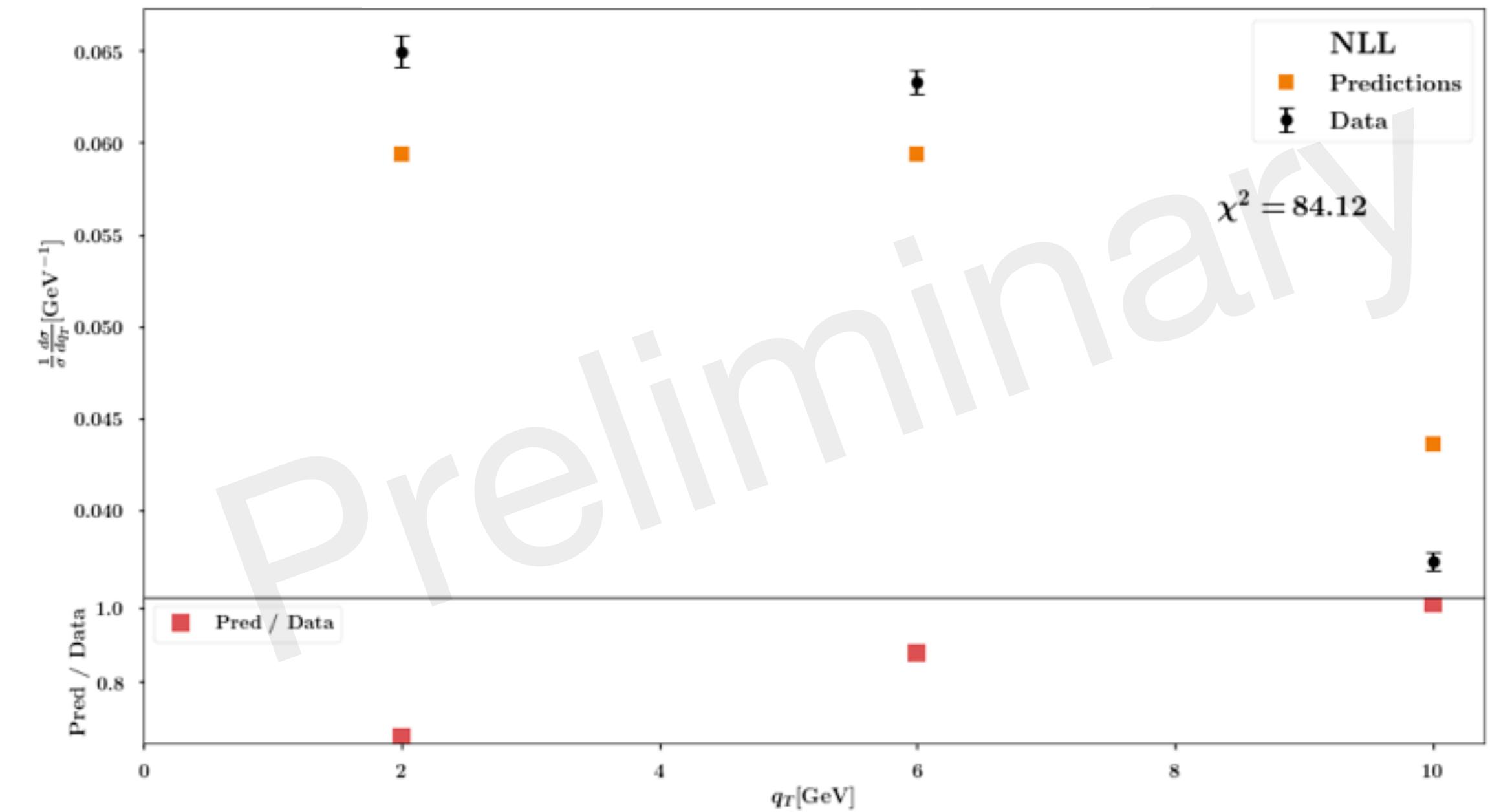
RESULTS AT NLL: DRELL YAN

RESULTS AT NLL: DRELL YAN

CMS 7 TeV

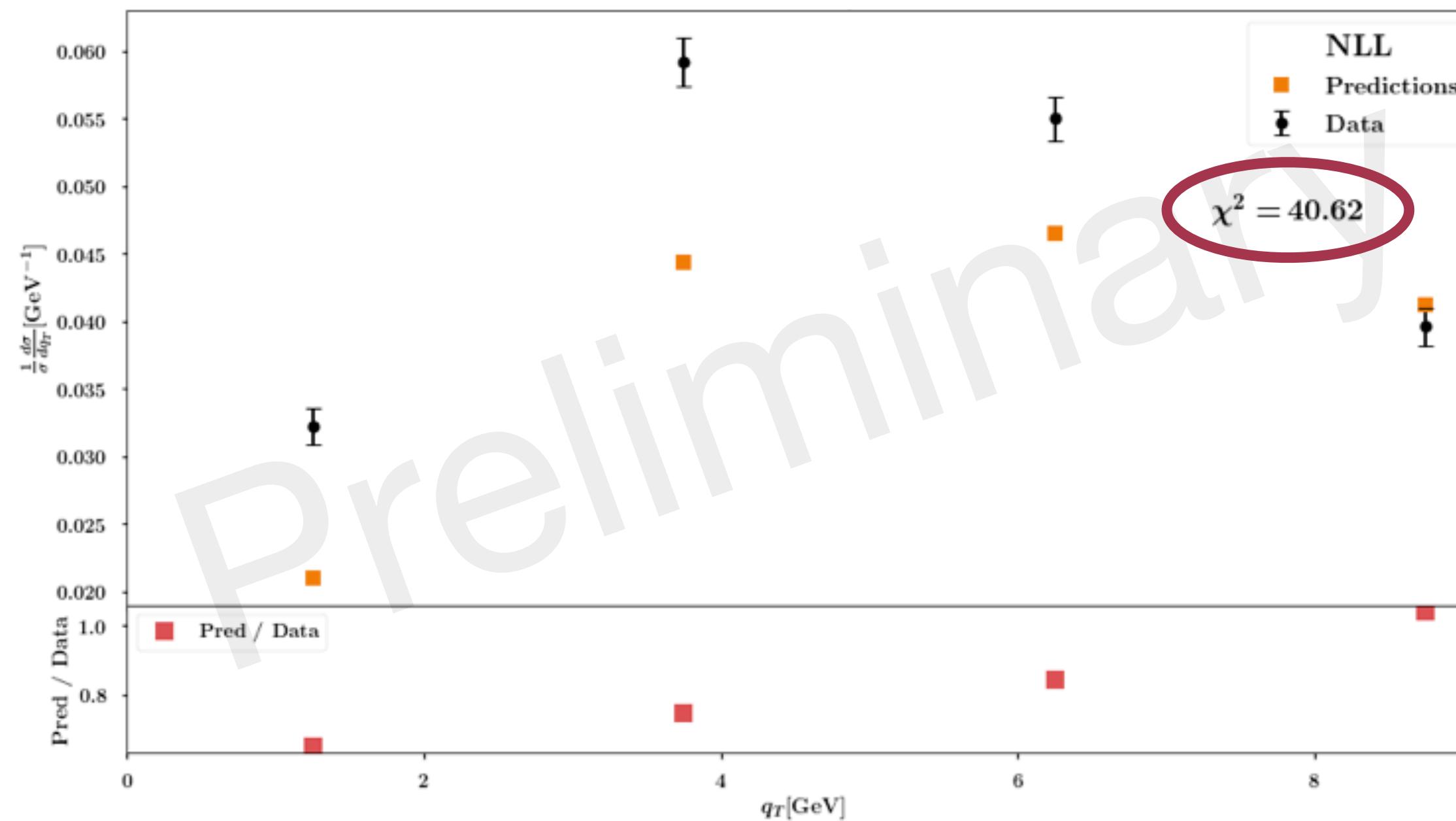


D0 Run II muons

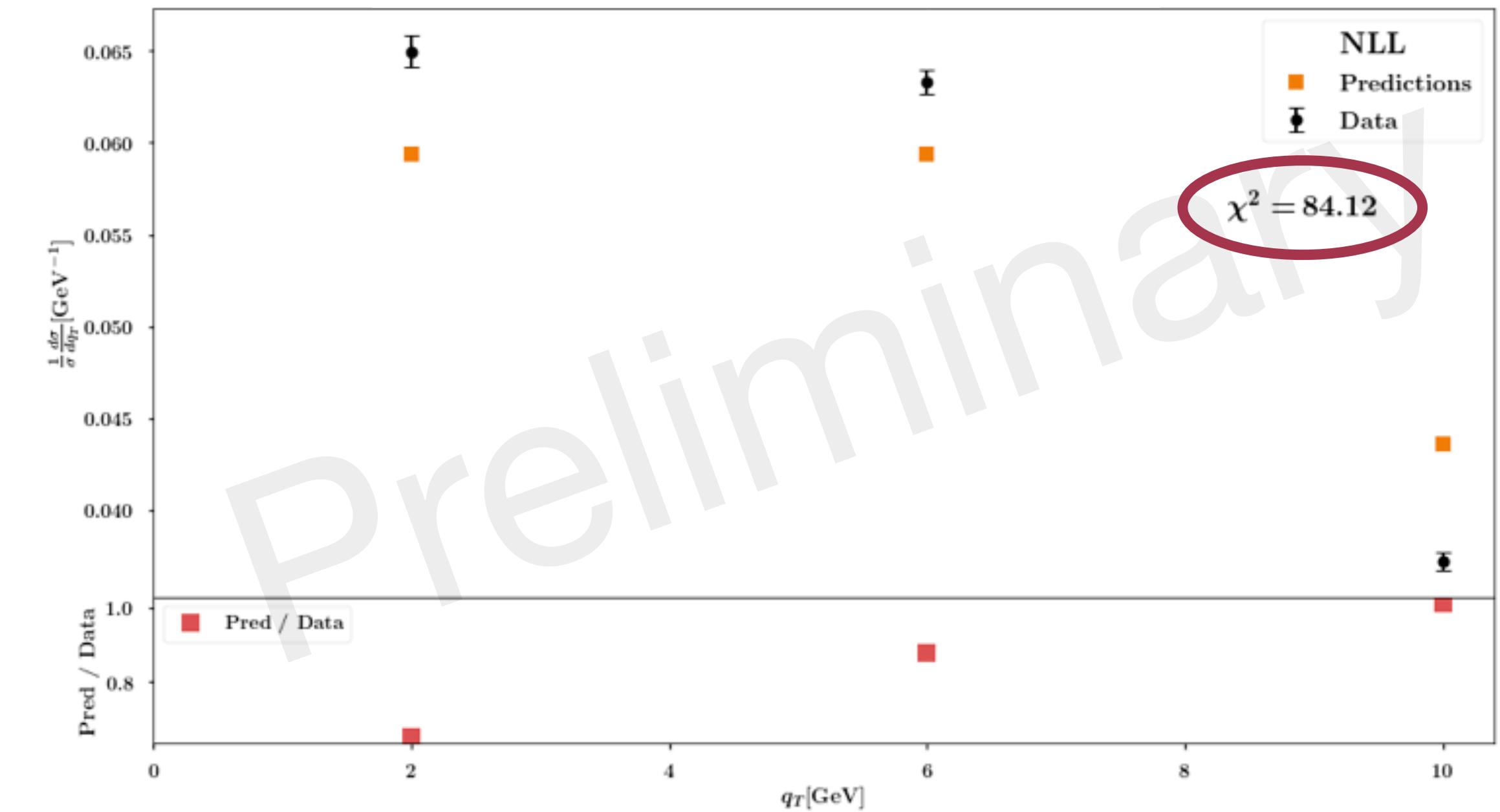


RESULTS AT NLL: DRELL YAN

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D0 Run II muons



We need to increase the accuracy!!

COMPARISON OF DIFFERENT ORDERS

Accuracy at N²LL and N³LL

COMPARISON OF DIFFERENT ORDERS

Accuracy at N²LL and N³LL

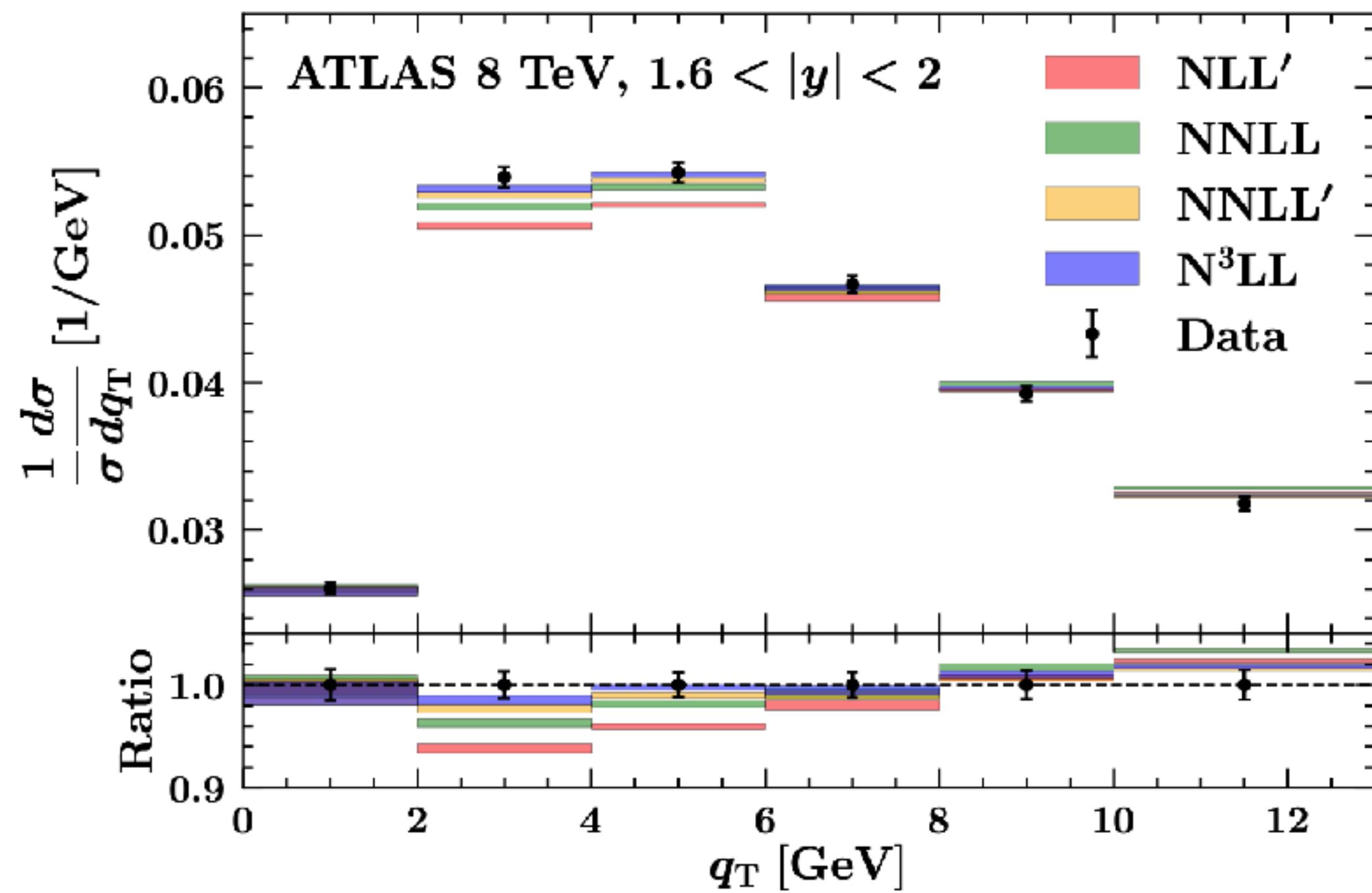
What we expected

COMPARISON OF DIFFERENT ORDERS

Accuracy at N²LL and N³LL

What we expected

$Q \sim 100$ GeV



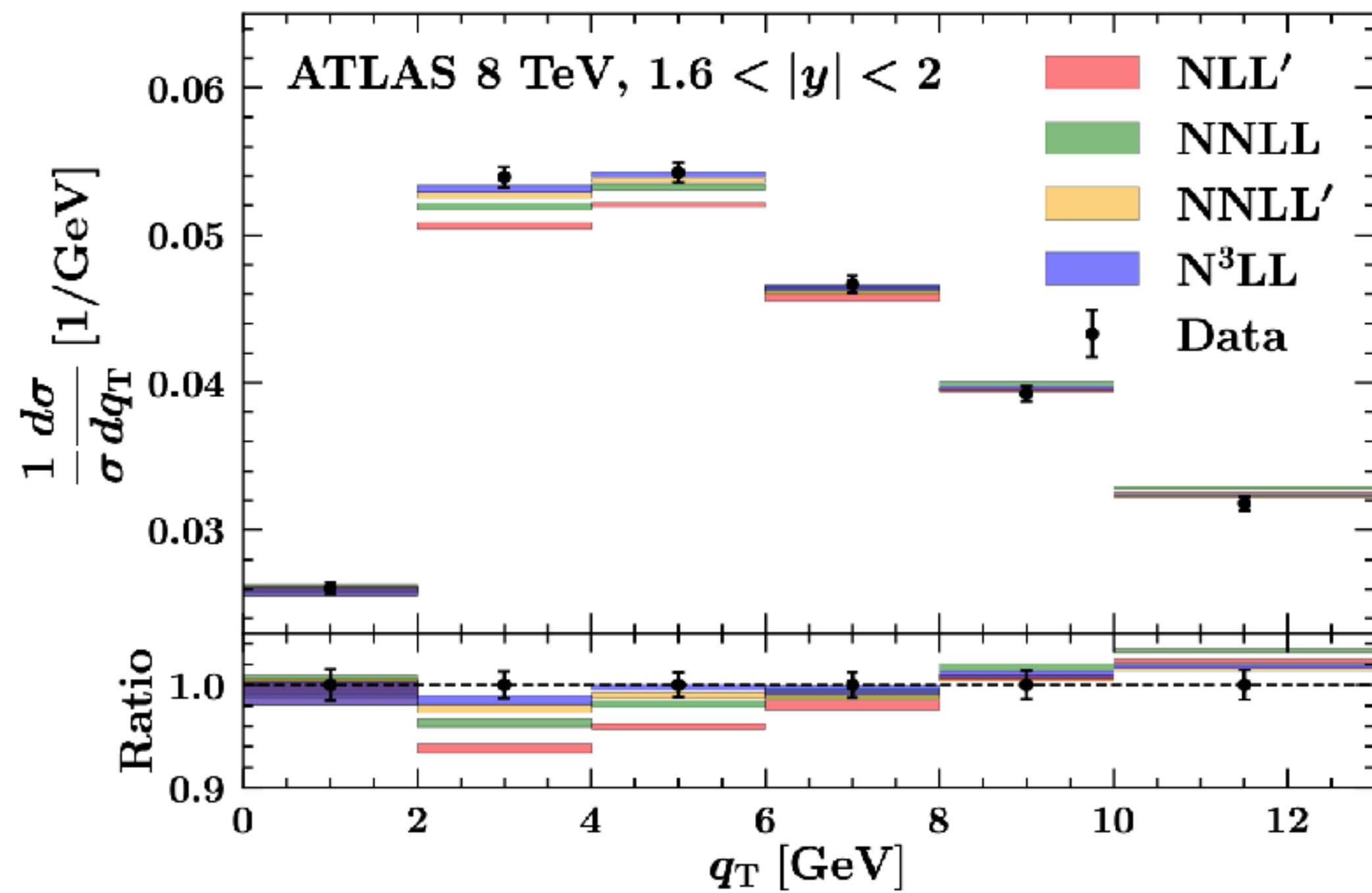
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What we get

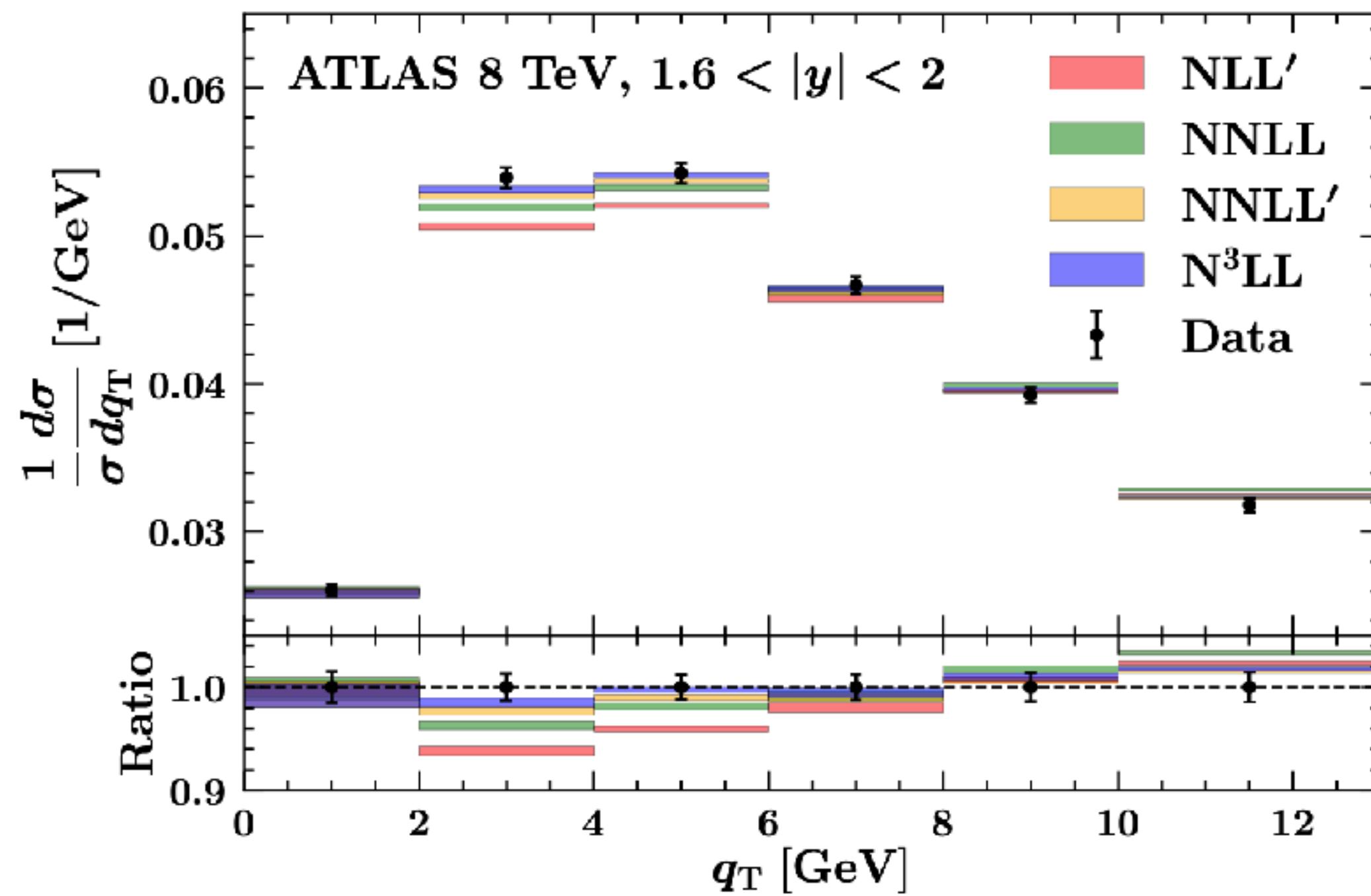


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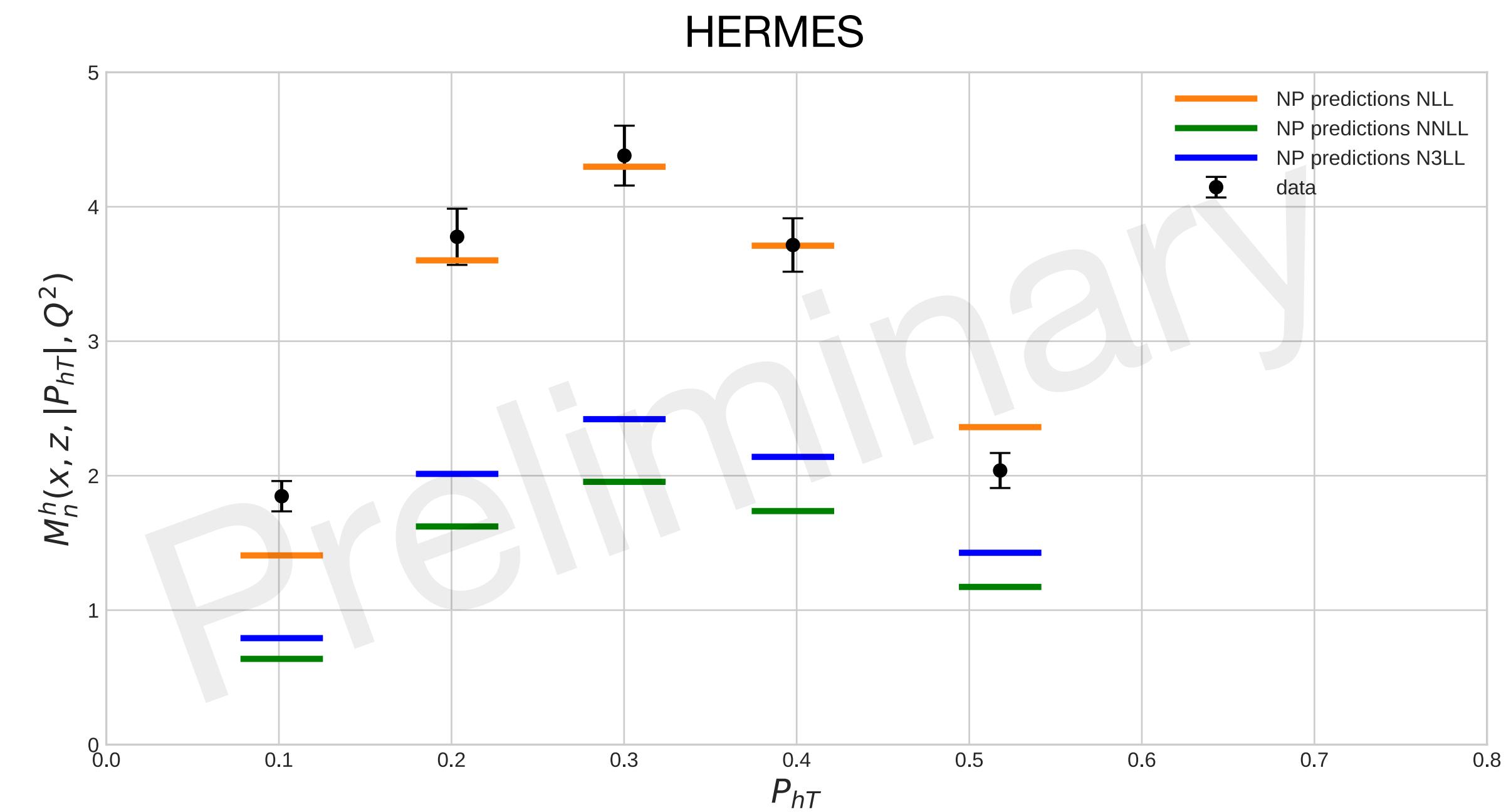
What we expected

$Q \sim 100$ GeV



What we get

$Q \sim 2$ GeV

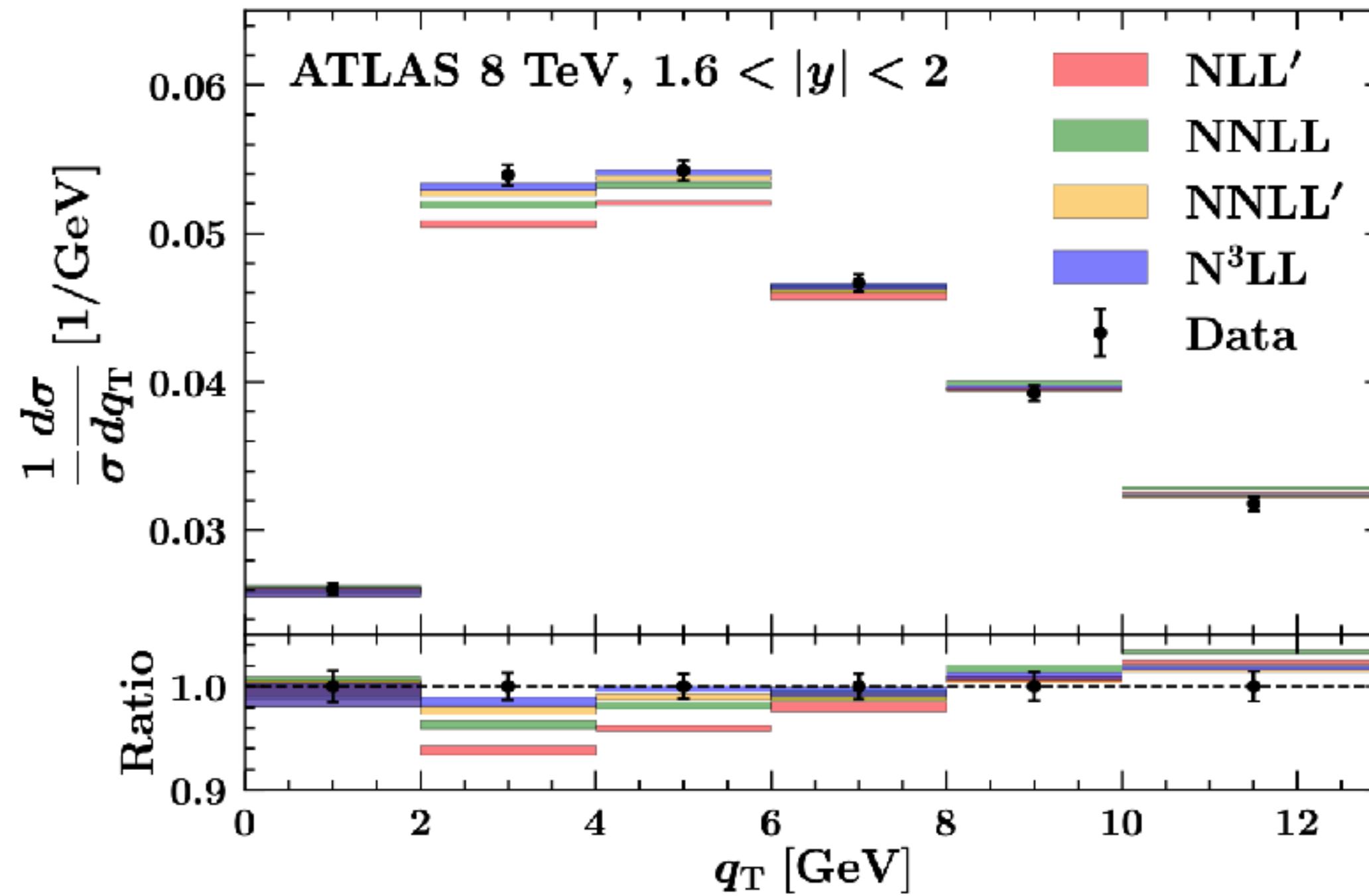


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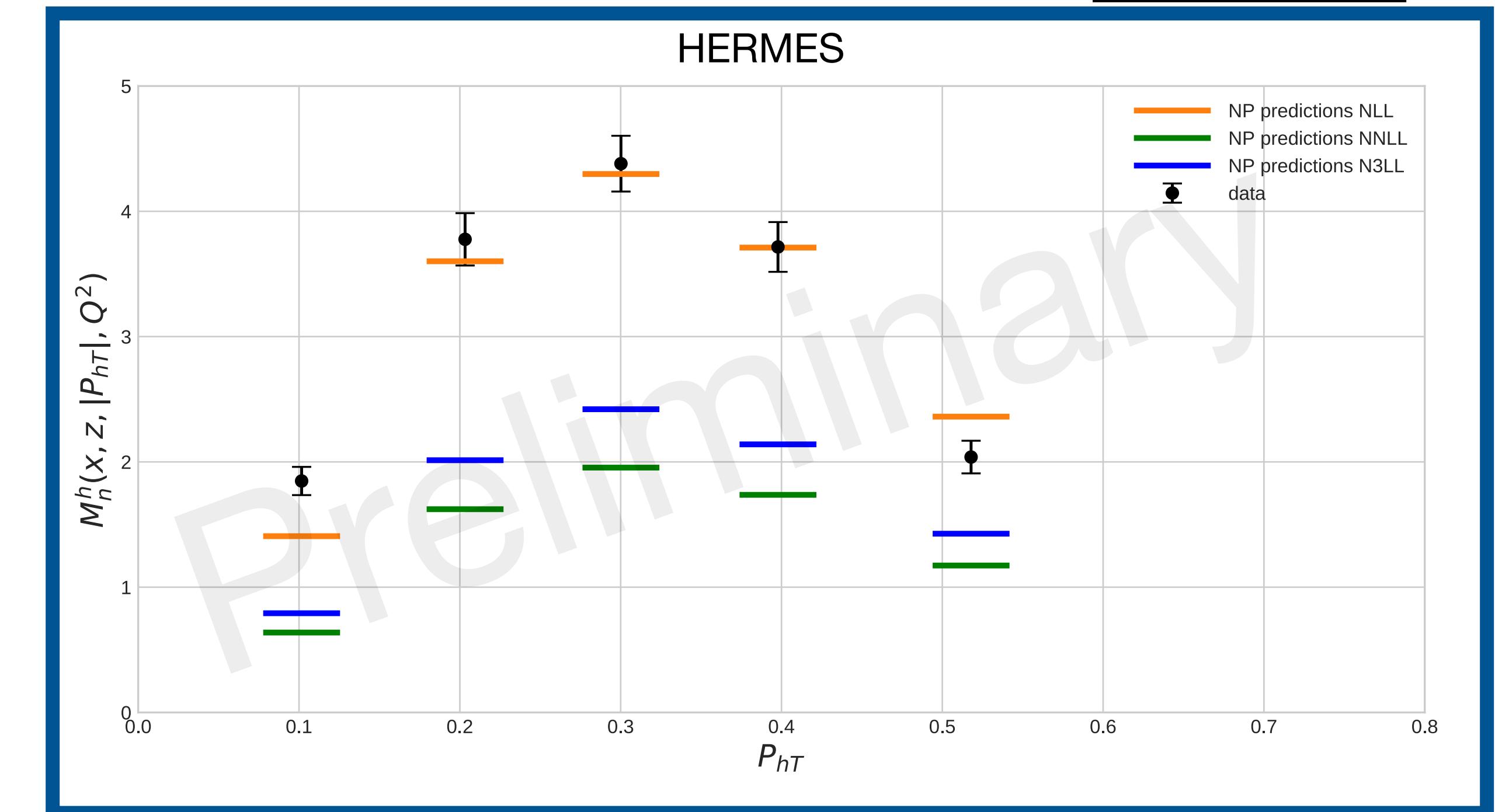
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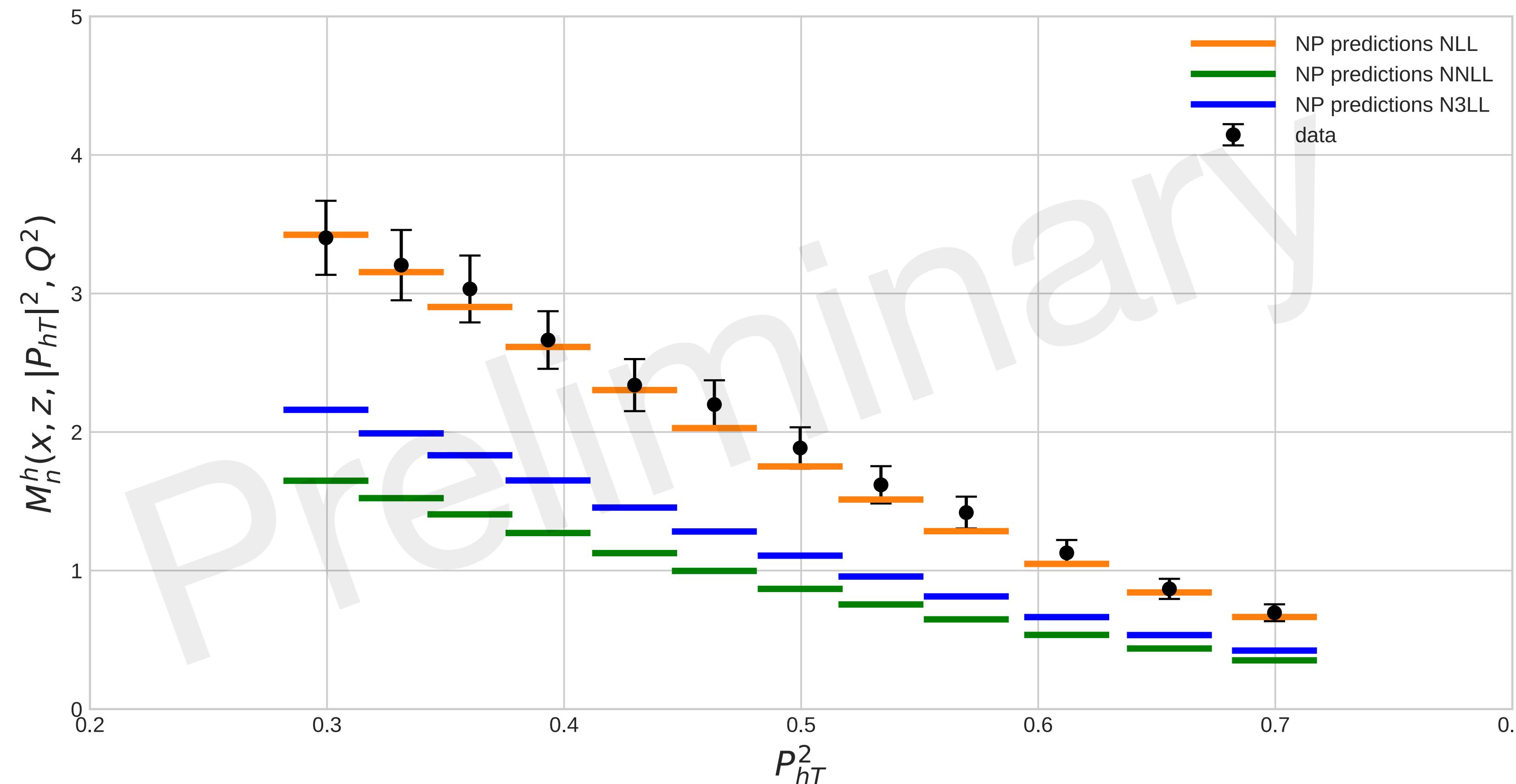
What we get

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COMPARISON OF DIFFERENT ORDERS - SIDIS

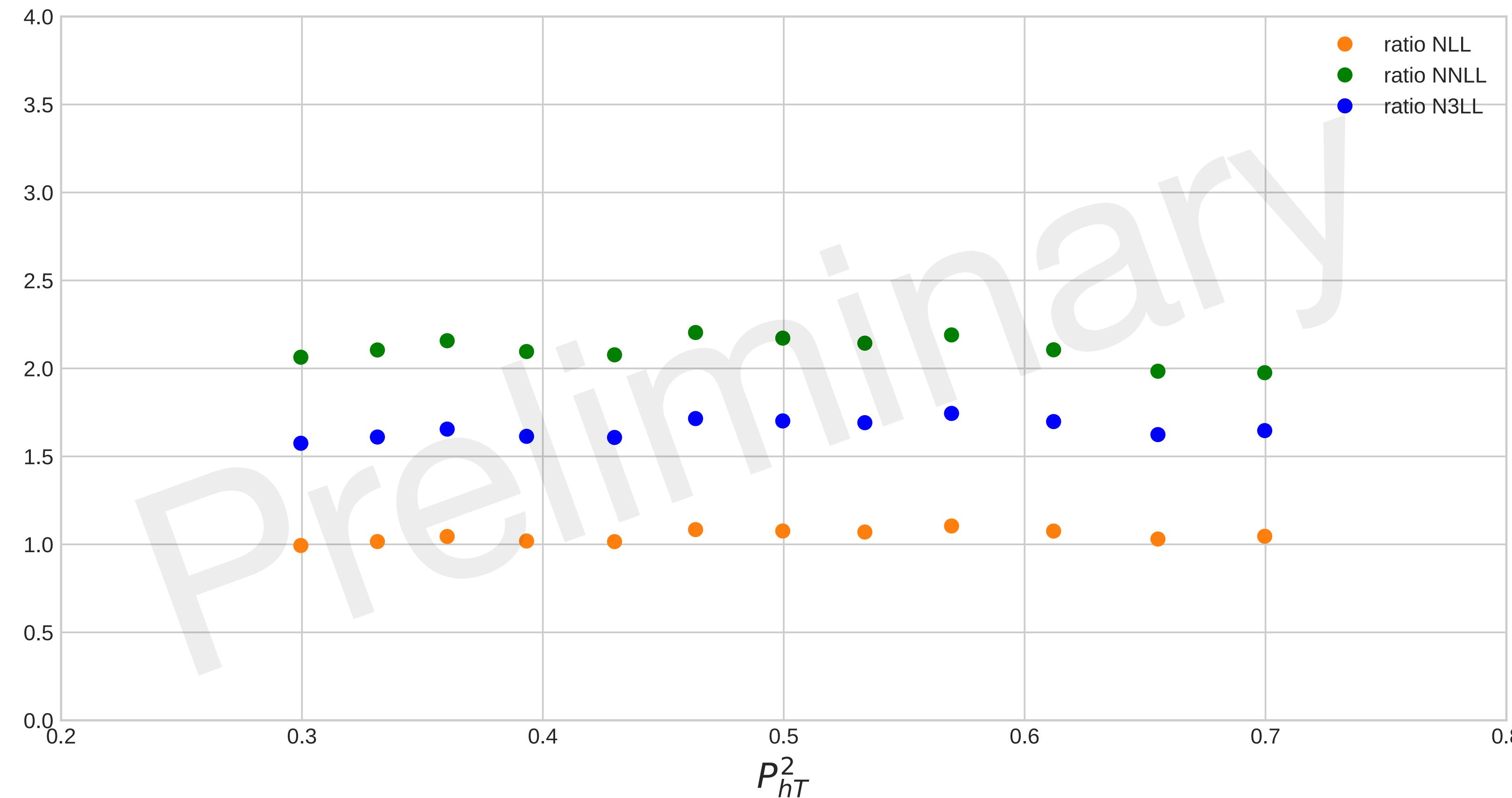
COMPASS multiplicities (one of many bins)



The description considerably worsens at higher orders!!

RATIO DATA/PREDICTIONS: SIDIS

COMPASS multiplicities (one of many bins)



The discrepancy amounts to an almost constant factor!!

OUR TENTATIVE SOLUTION

Introduction of a normalization prefactor

$$\text{PREFACTOR}(x, z, Q) = \frac{\frac{d\sigma^h}{dxdQ^2dz} \Big|_{\text{nonmix.}}}{\int W d^2q_T}$$

OUR TENTATIVE SOLUTION

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$$\frac{d\sigma^h}{dxdQ^2dz} \Big|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} \left\{ [D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N}](x, z, Q) \right\} \Big|_{\text{nonmix.}}$$

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OUR TENTATIVE SOLUTION

Introduction of a normalization prefactor

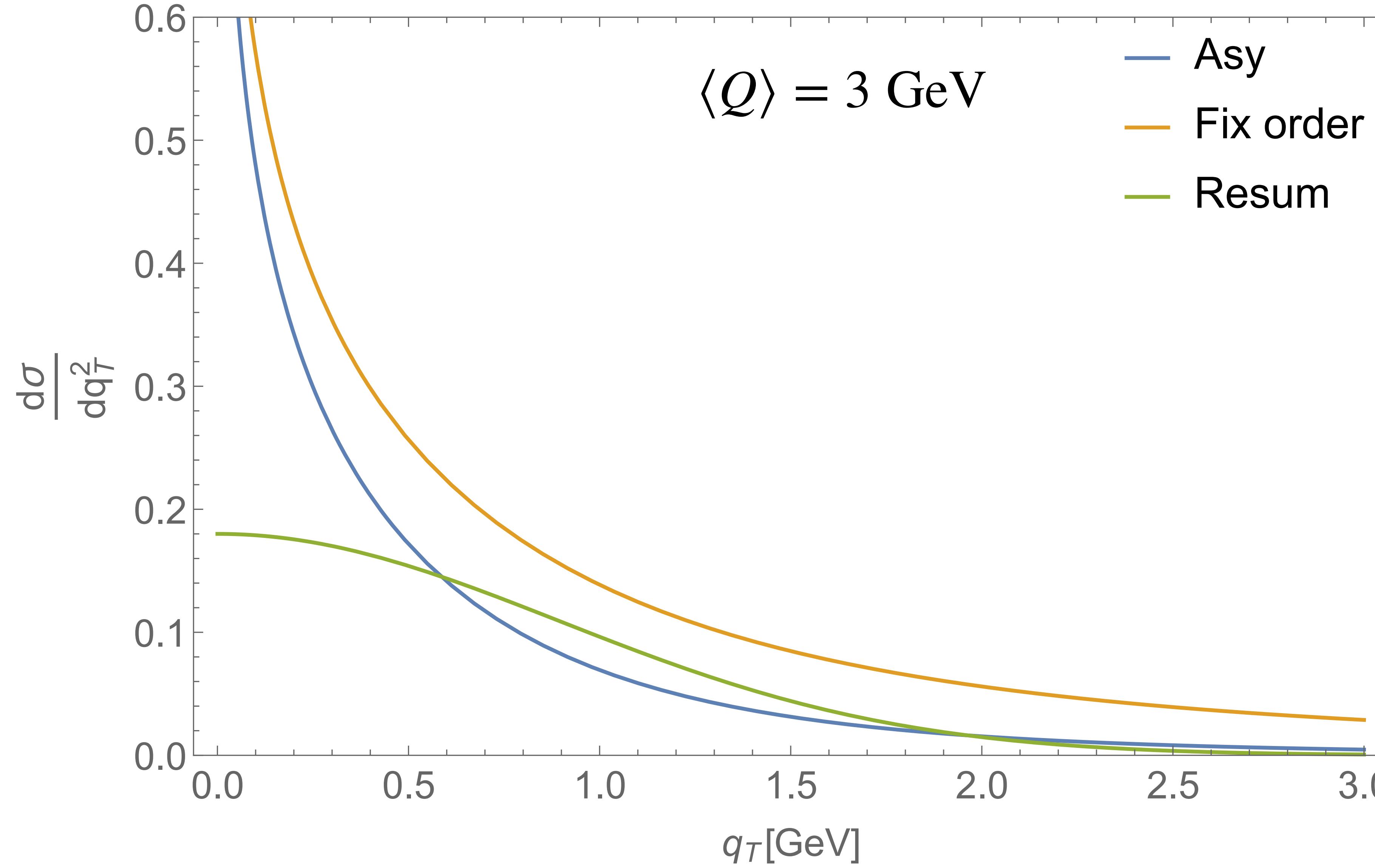
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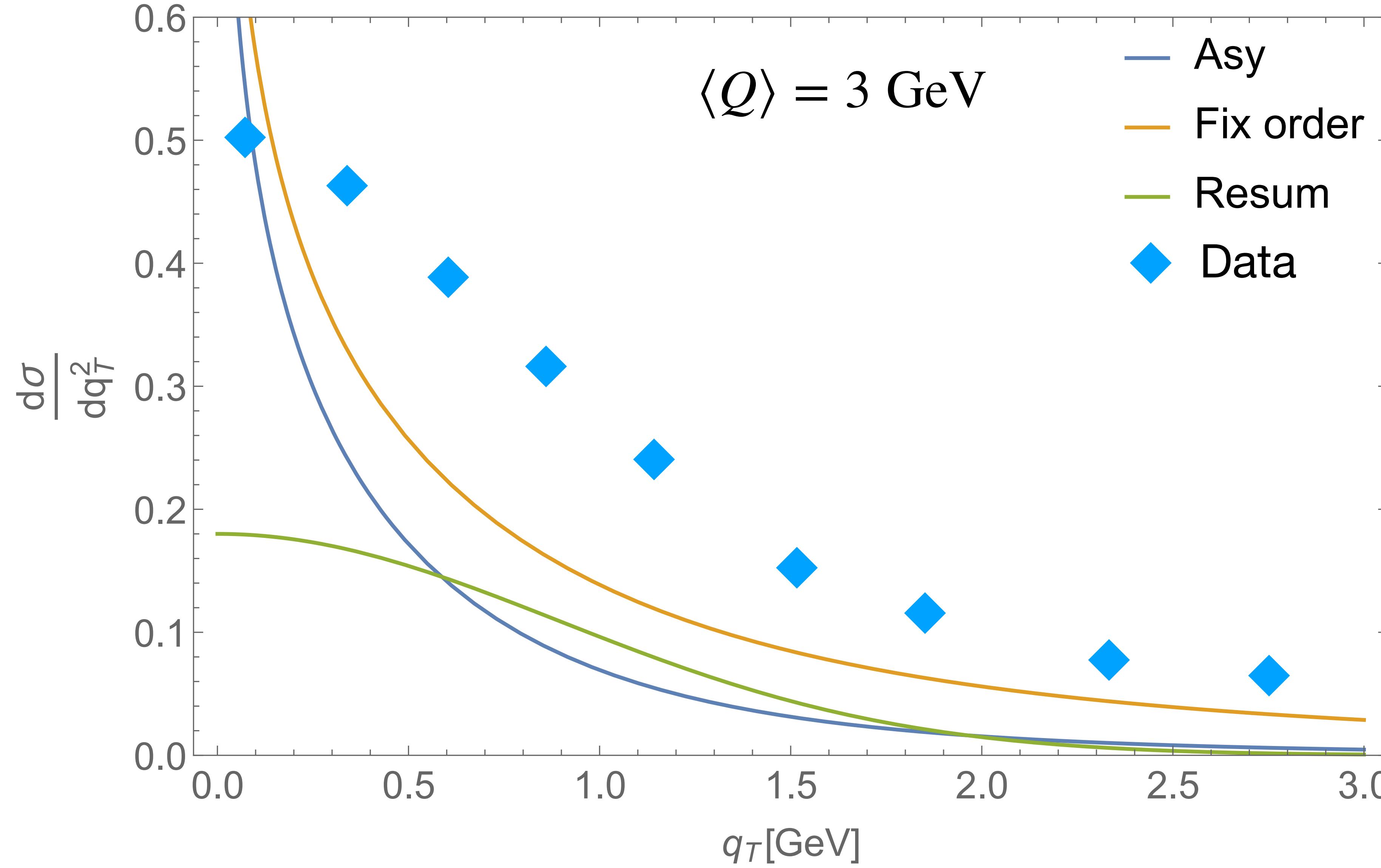
$$\int W \Big|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} [D_1^{h/f'} \otimes C_{\text{TMD}}^{f'f} \otimes f_1^{f/N}](x, z, Q)$$

Independent of the fitting parameters!!

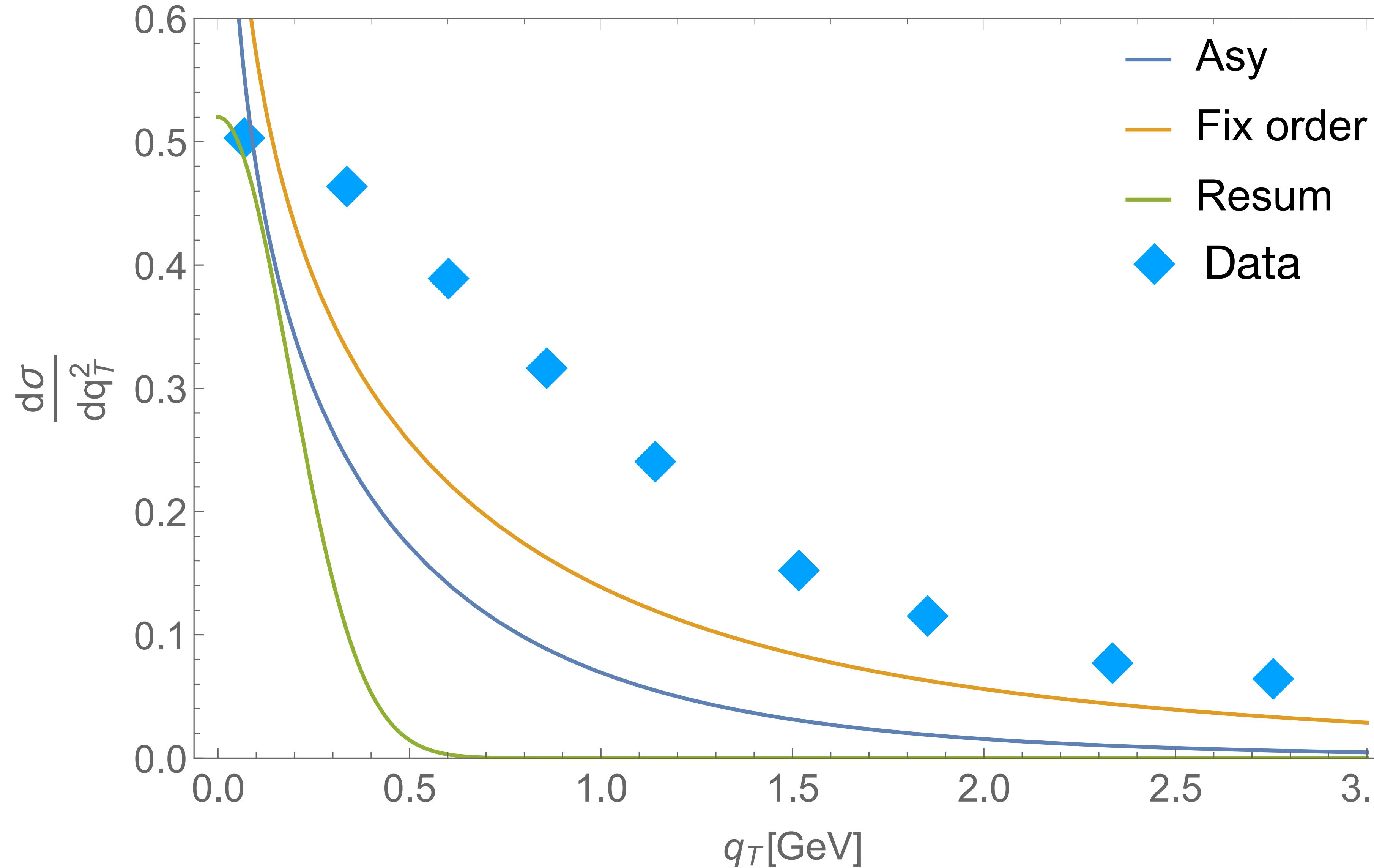
SOME JUSTIFICATION: INITIAL SITUATION



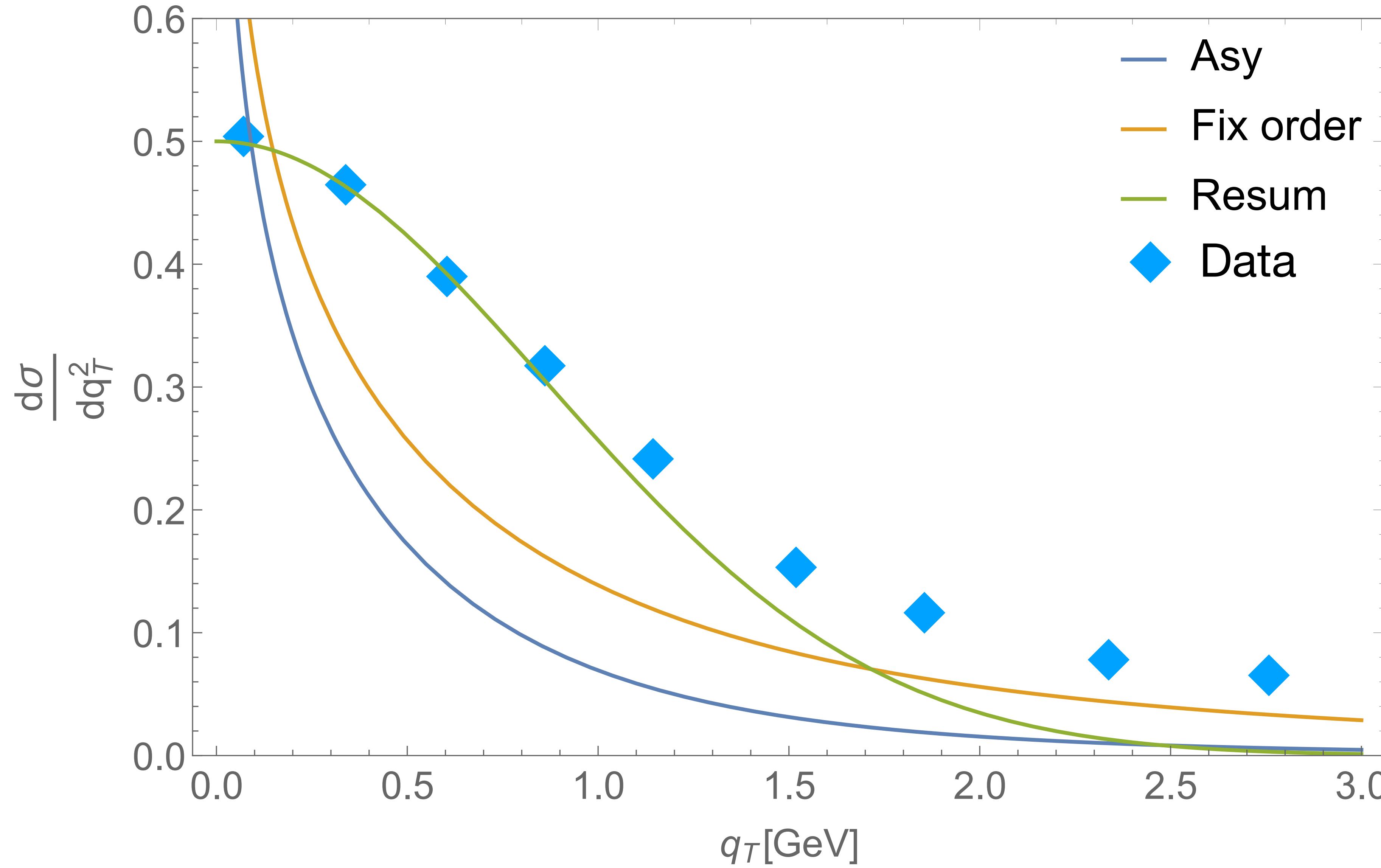
SOME JUSTIFICATION: INITIAL SITUATION



SOLUTION1: RESTRICT TMD REGION



SOLUTION2: ENHANCE TMD CONTRIBUTIONS



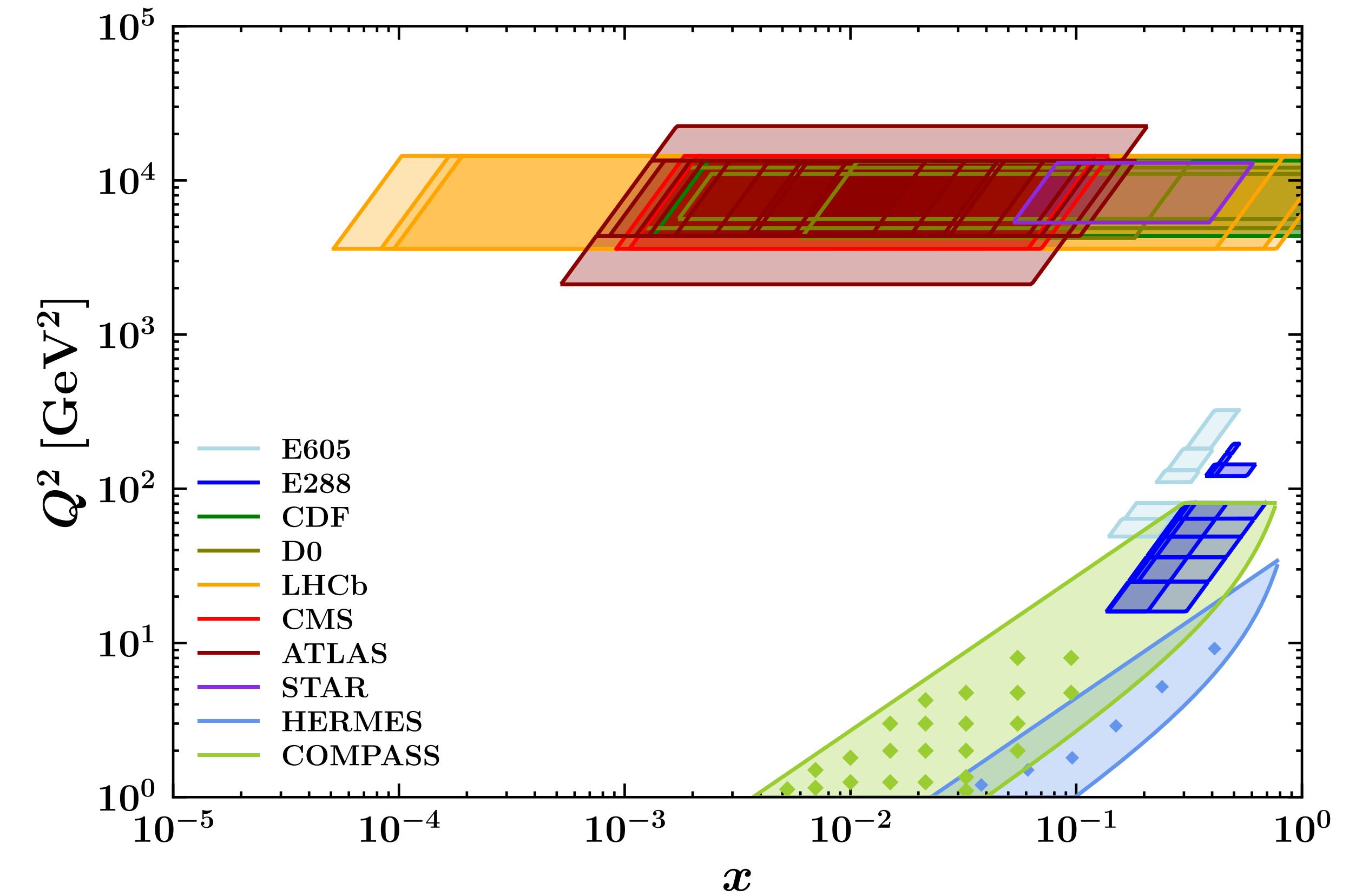
MAP21 TMD FIT CHOICES (PRELIMINARY)

$\langle Q \rangle > 1.3$ GeV

$0.2 < z < 0.6$

$q_T < 0.2 Q$ (DY)

$P_{hT} < \min[\min[0.2Q, 0.5zQ] + 0.3$ GeV, $zQ]$ (SIDIS)



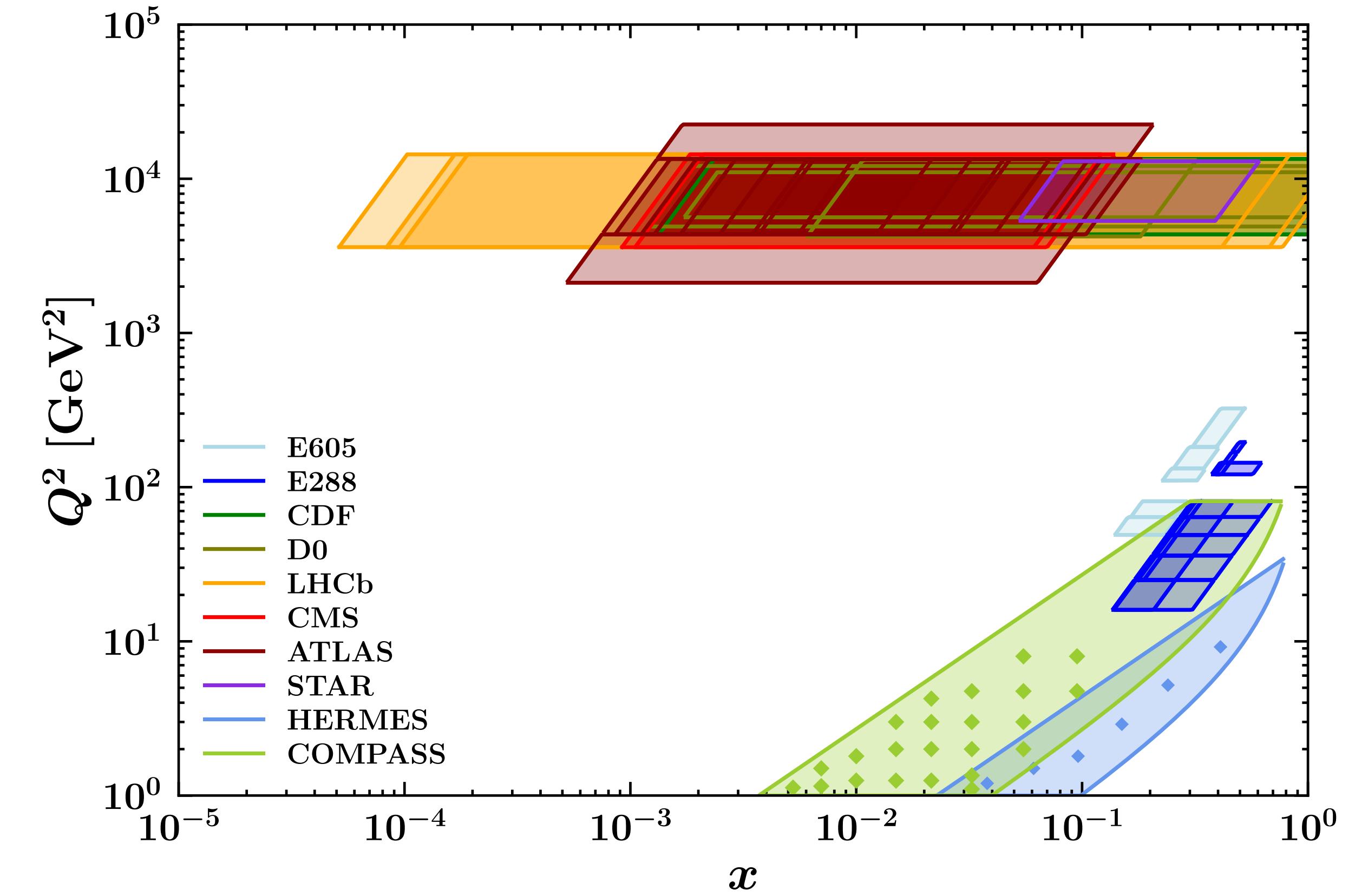
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Number of points > 1500

FUNCTIONAL FORM (PRELIMINARY)

$$f_{1NP}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$

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Still working on the flexibility
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$$g_1(x) = N_1 \frac{(1-x)^\alpha}{(1-\hat{x})^\alpha} \frac{x^\sigma}{\hat{x}^\sigma}$$

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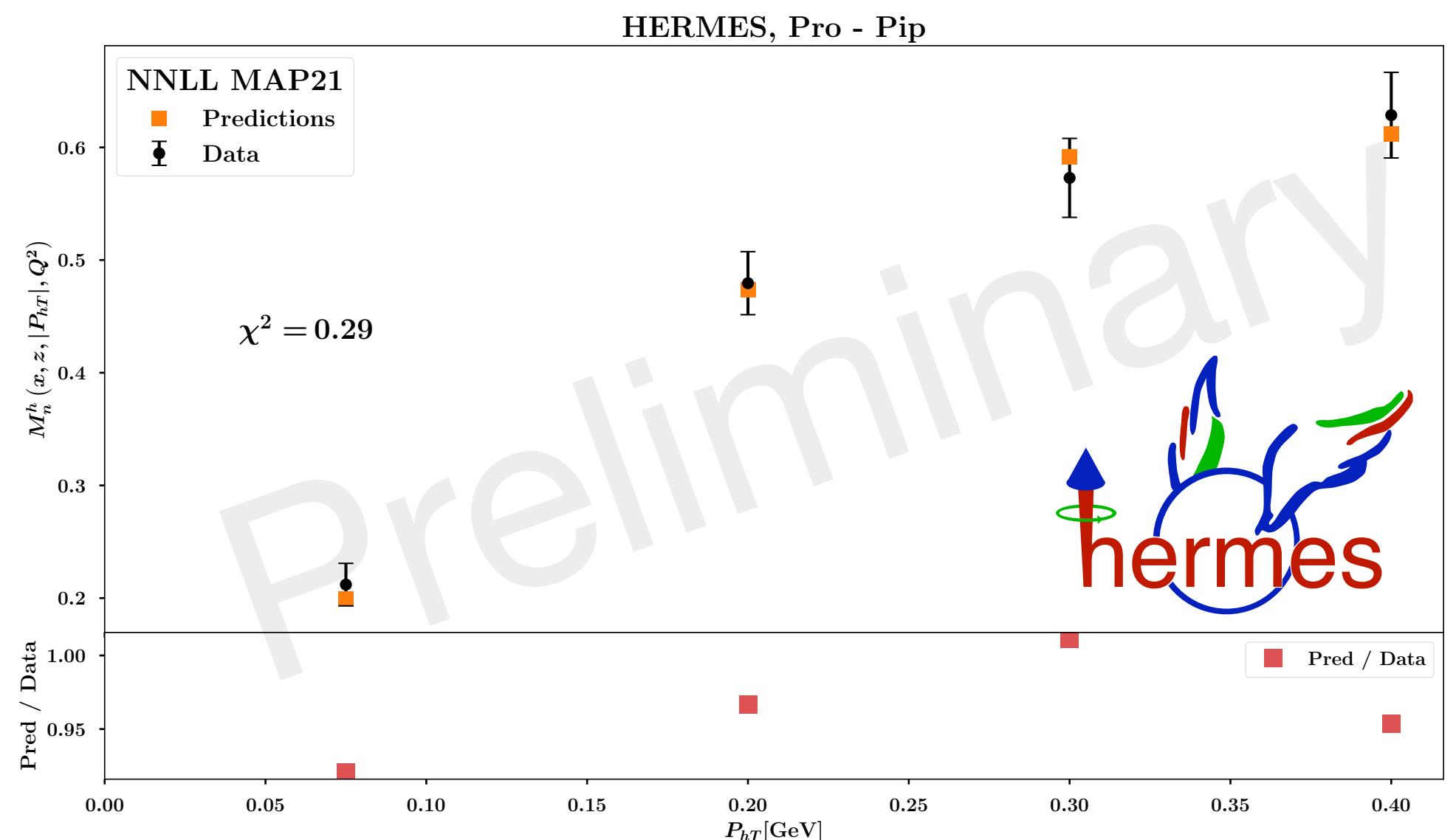
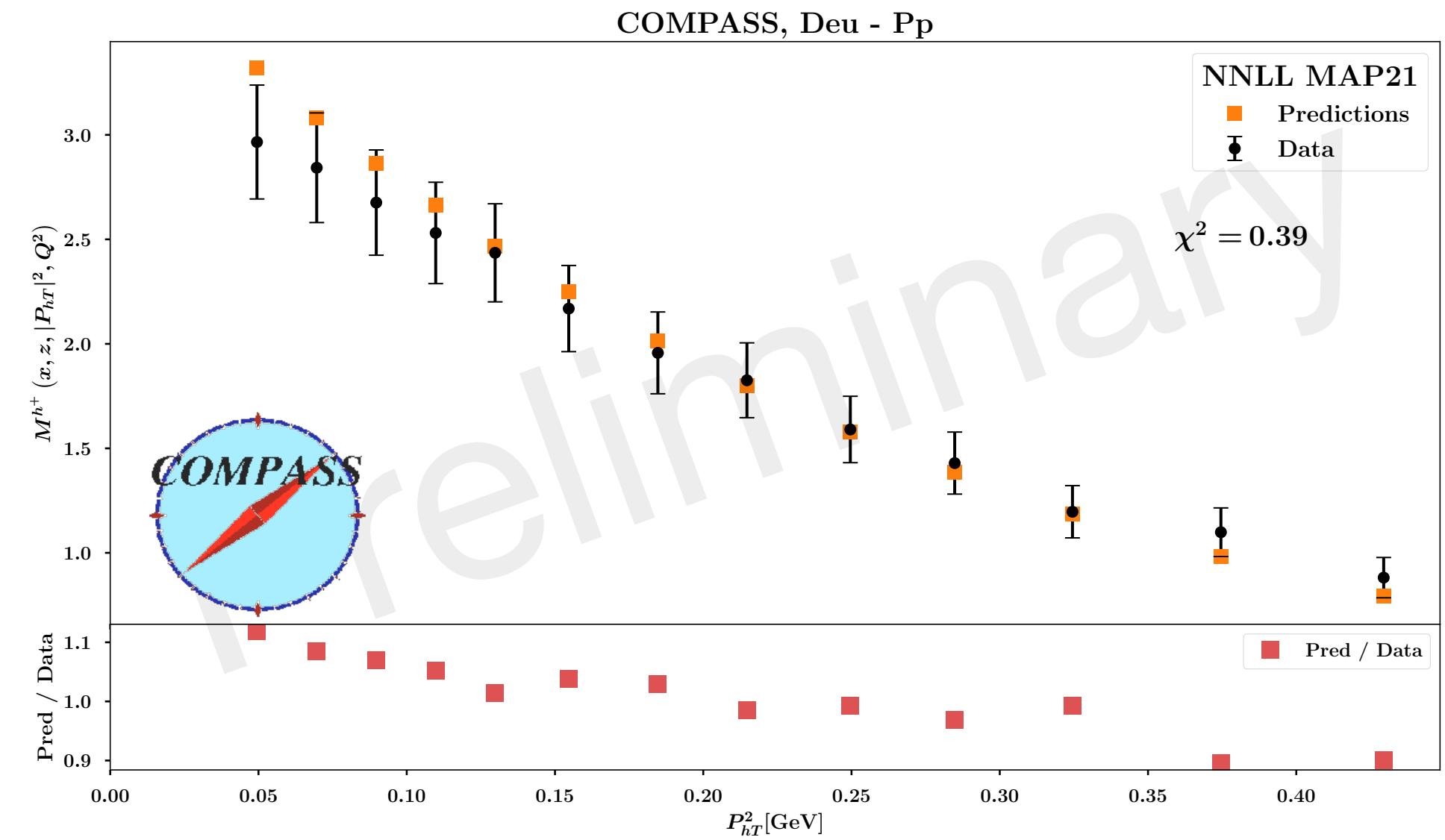
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Still working on the flexibility
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11 parameters for TMD PDF
+ 2 for NP evolution + 14 for FF
= 27 free parameters

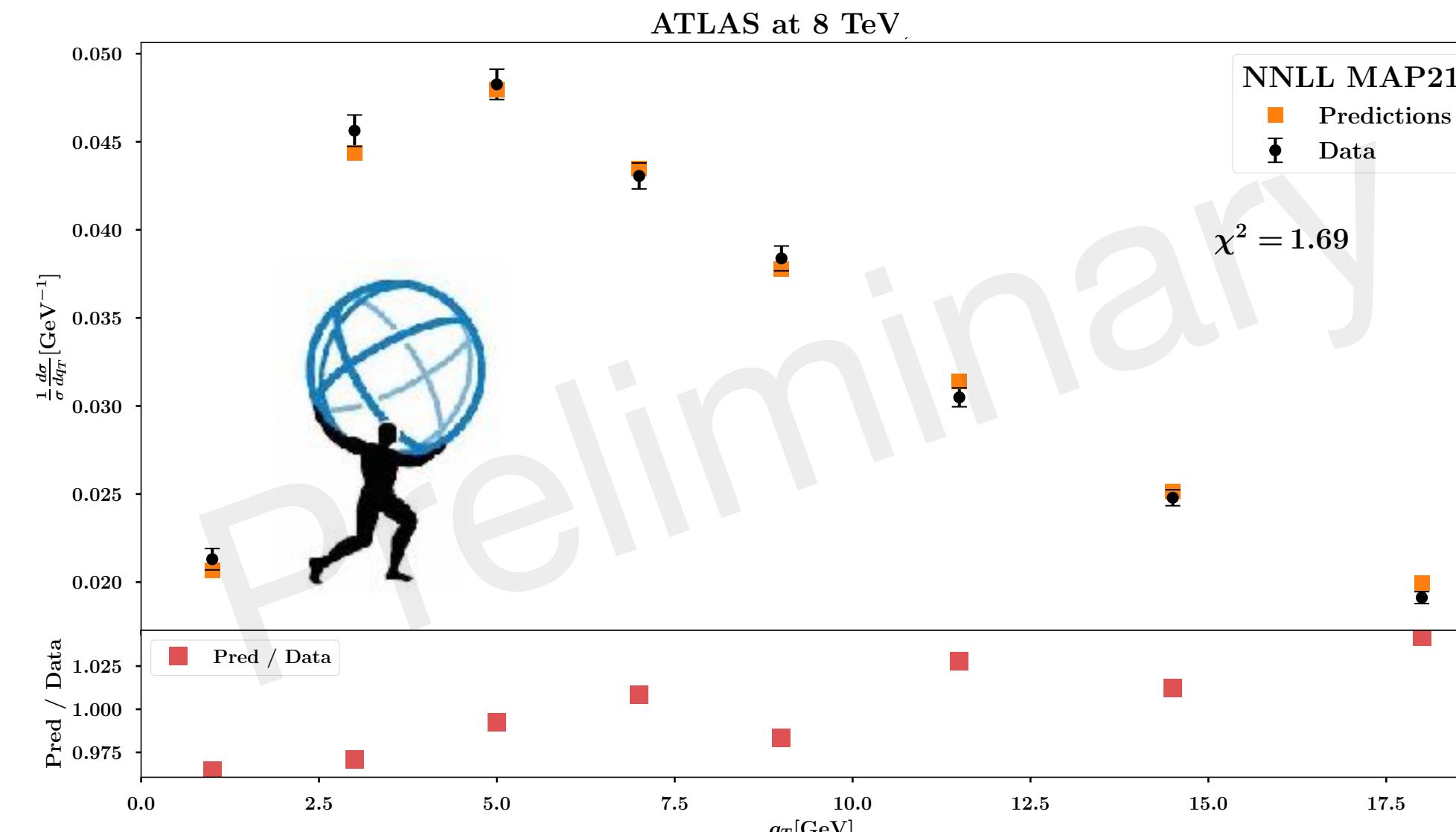
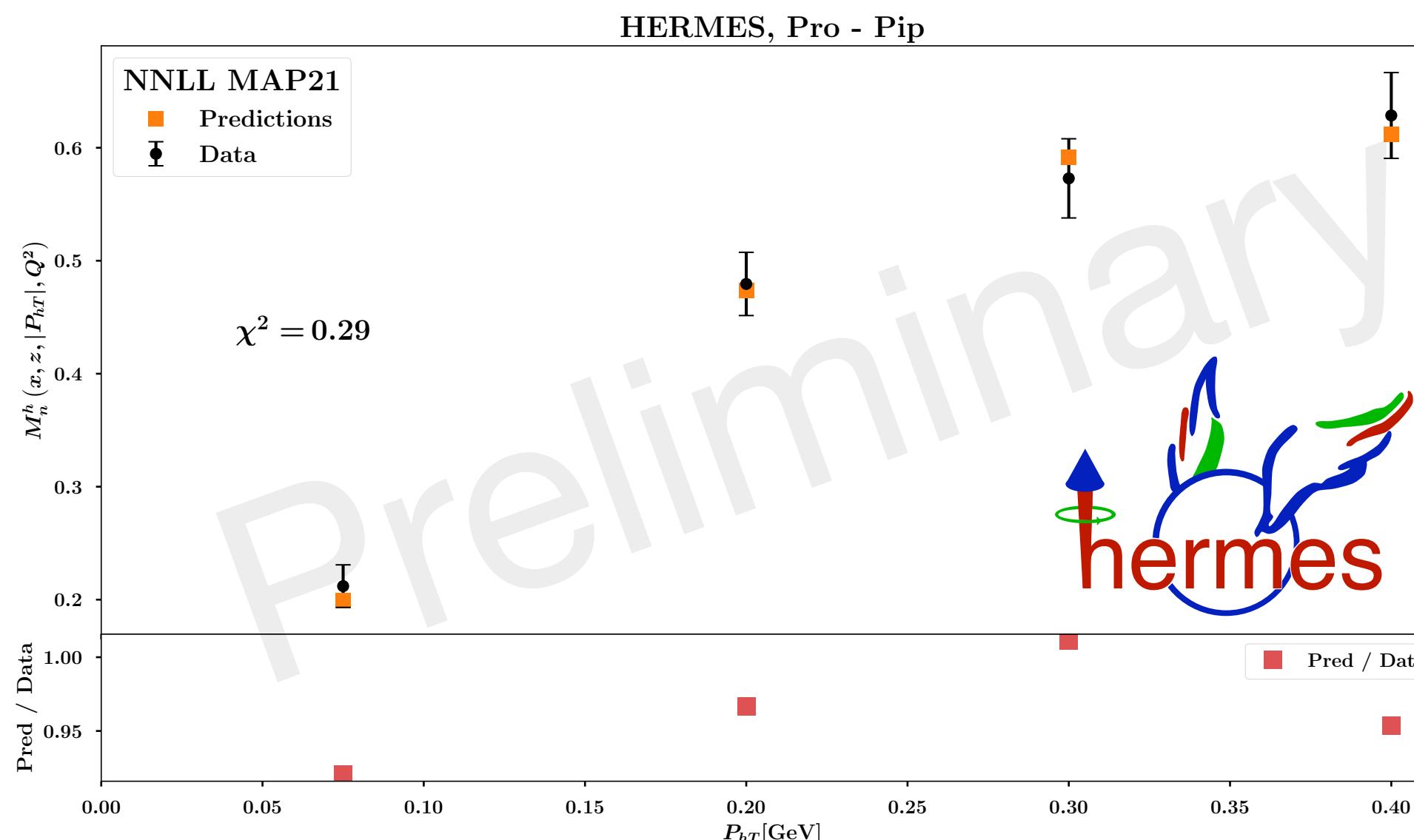
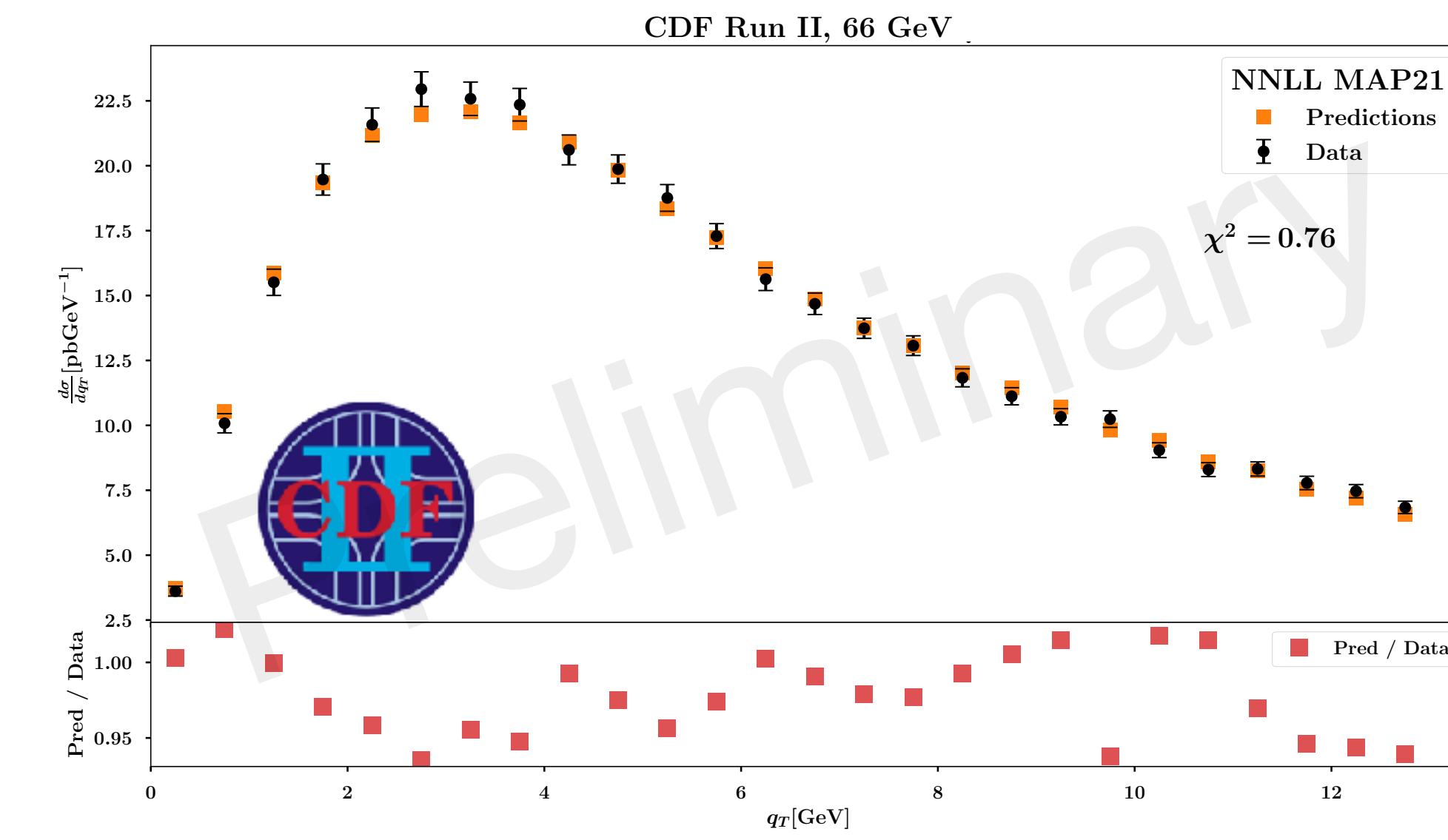
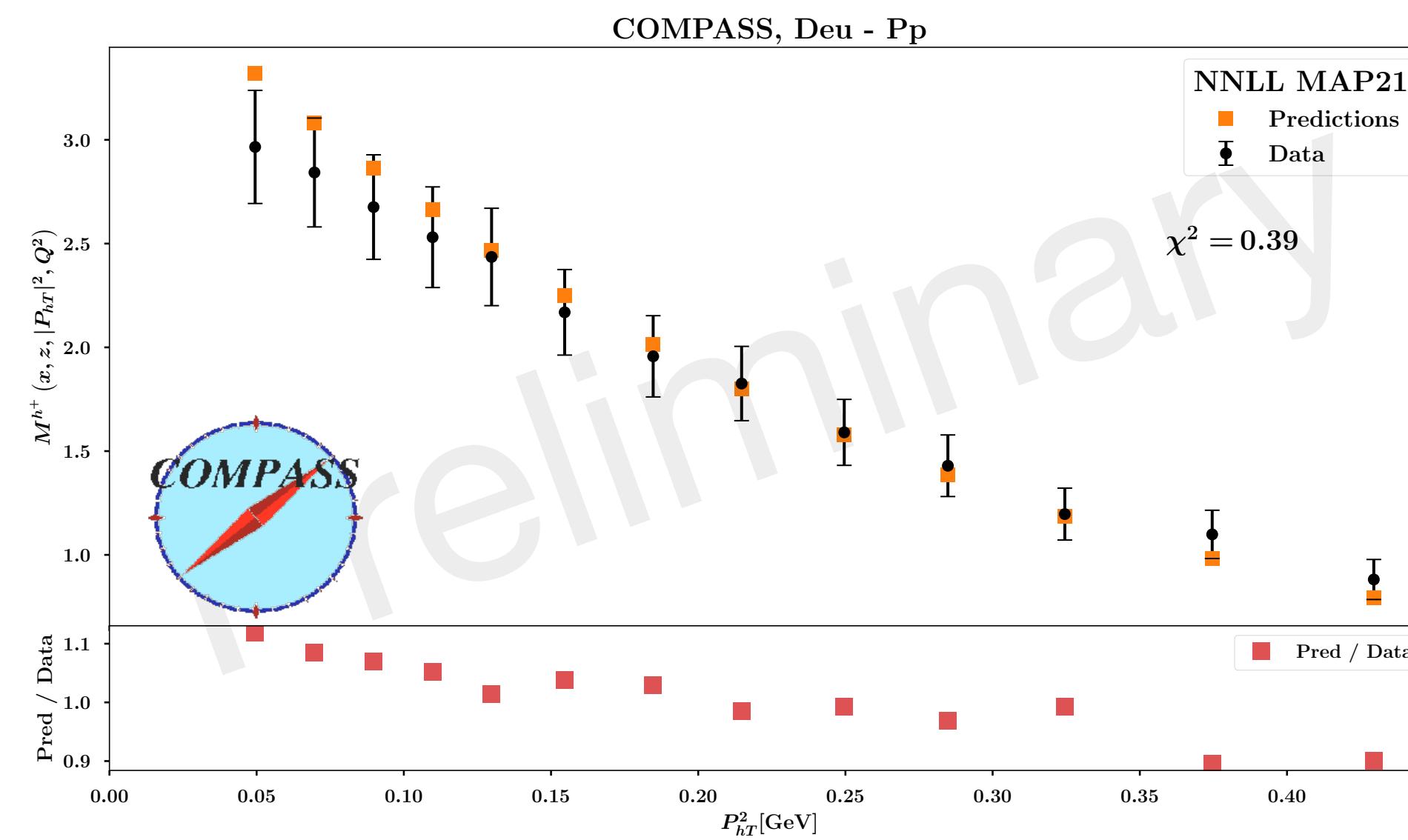
N²LL: EXAMPLE OF GOOD BINS

Global $\chi^2 < 1.1$



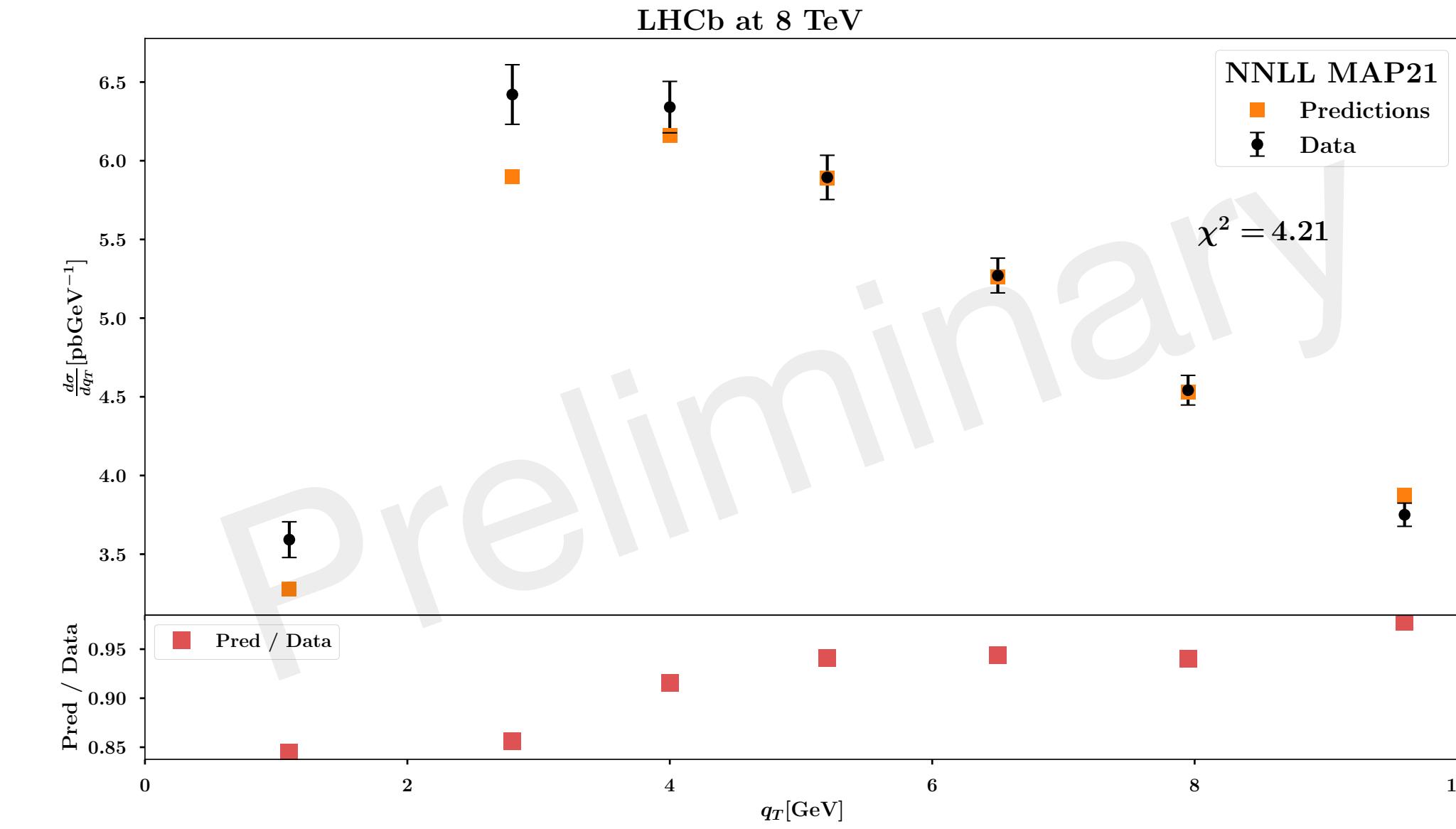
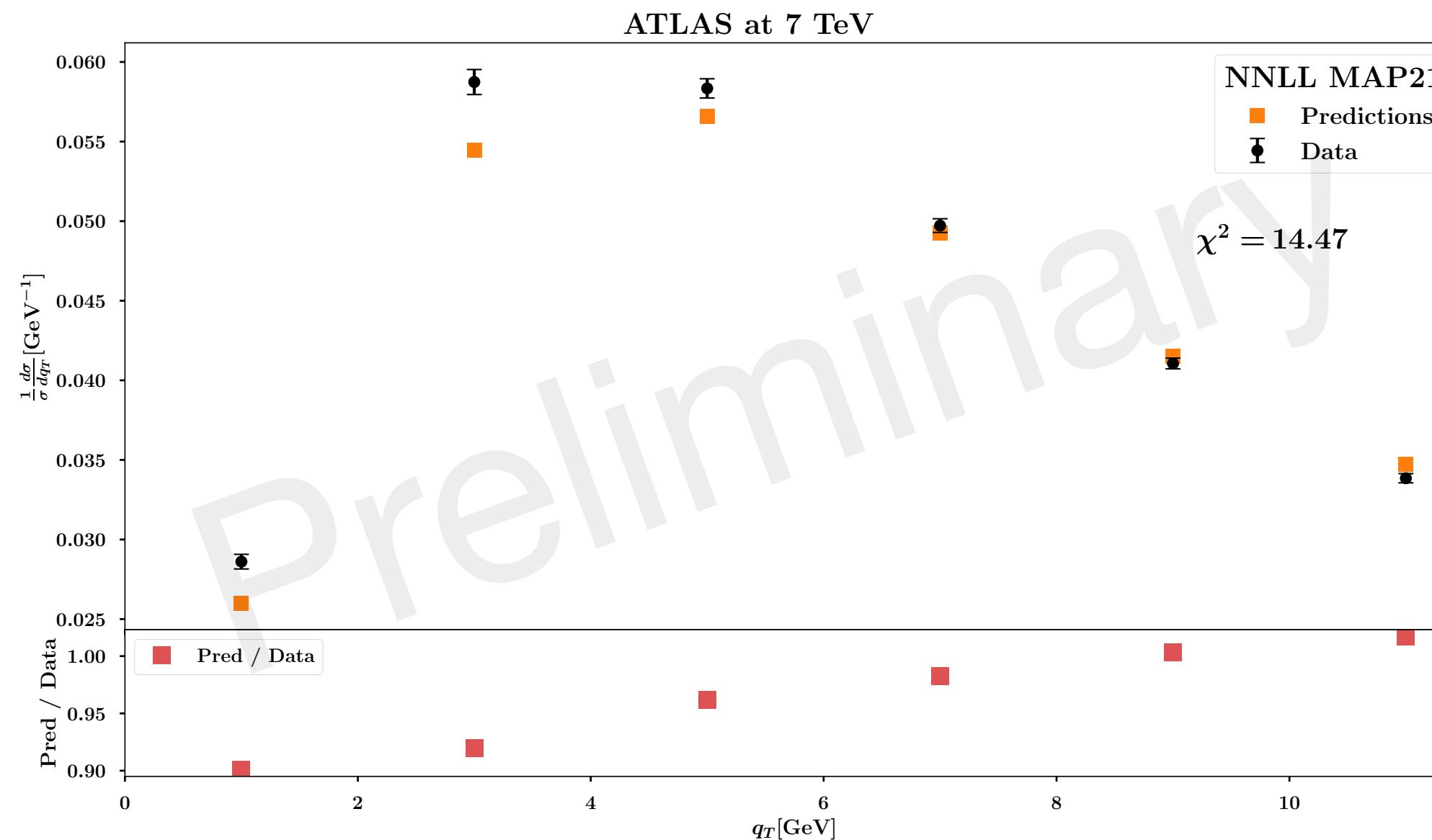
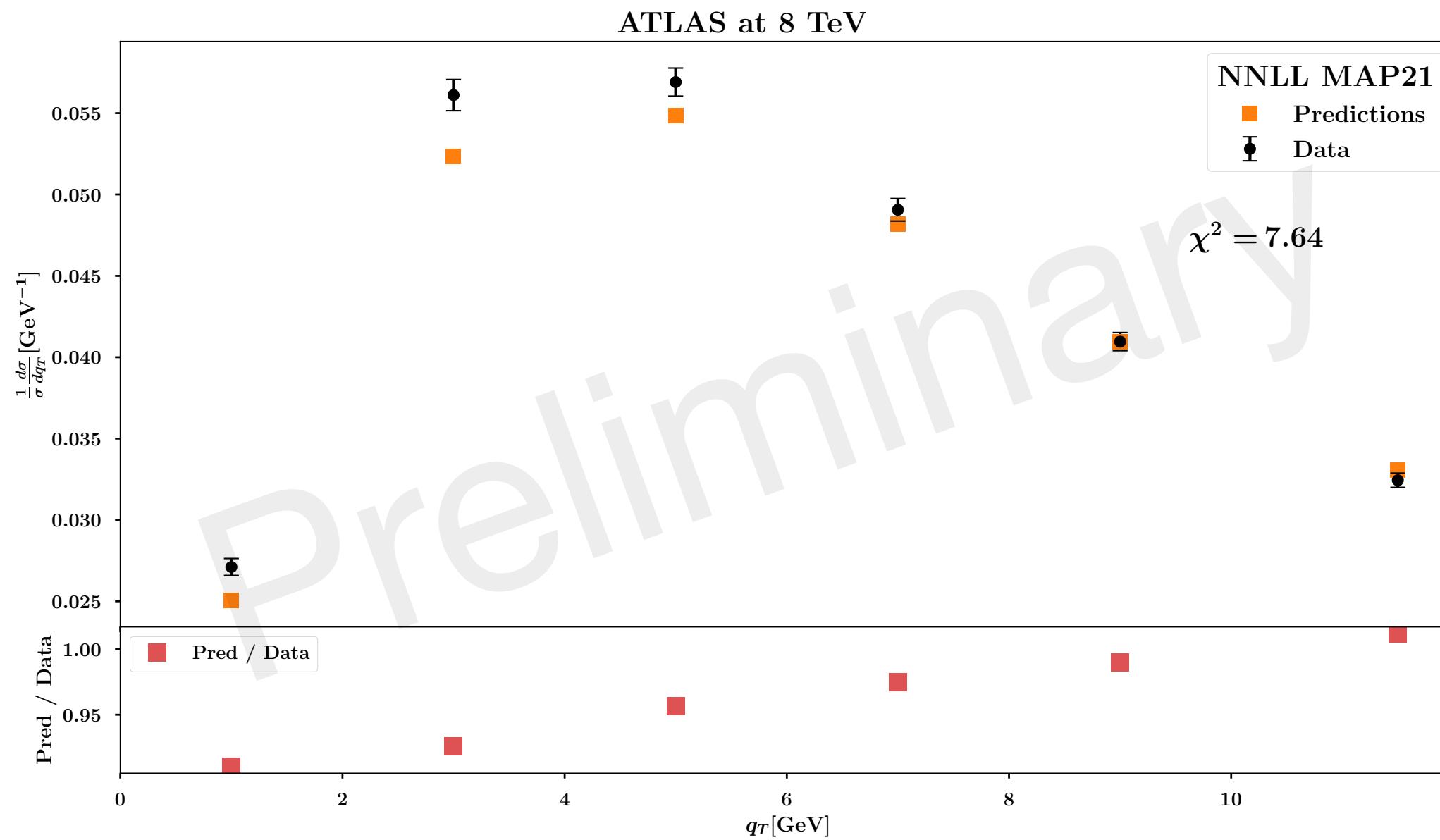
N²LL: EXAMPLE OF GOOD BINS

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N²LL: EXAMPLE OF BAD BINS

Global $\chi^2 < 1.1$



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- ➍ Good global χ^2 can be reached at N^2LL , but some LHC data remain hard to describe

BACKUP SLIDES

LOGARITHMIC ACCURACY

Sudakov form factor

LL

$$\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right)$$

NLL

$$\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left(\frac{Q^2}{\mu_b^2} \right)$$

NLL'

$$\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left(\frac{Q^2}{\mu_b^2} \right)$$

Matching coefficient

$$\tilde{C}^0$$

$$\tilde{C}^0$$

$$(\tilde{C}^0 + \alpha_S \tilde{C}^1)$$

the difference between the two is NNLL:

$$\alpha_S^n \ln^{2n-2} \left(\frac{Q^2}{\mu_b^2} \right)$$

NON-MIXED TERMS IN COLLINEAR SIDIS CROSS SECTION - BACKUP

$$\begin{aligned} \left. \frac{d\sigma^h}{dx dQ^2 dz} \right|_{O(\alpha_s^1)} &= \sigma_0 \sum_{ff'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_s}{\pi} \left\{ \left[D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right. \\ &\quad \left. + \frac{1-y}{1+(1-y)^2} \left[D_1^{h/f'} \otimes C_L^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right\}, \end{aligned}$$

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