NEW APPLICATION OF A SONA TRANSITION UNIT:

Observation of direct transitions between quantum states with energy differences of 10 neV and below

19.10.2021 I Chrysovalantis S. Kannis

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What is a Sona transition?

Theoretical framework

Sona transition unit

Static B-field (parallel to the beam direction) that reverses its direction along the quantization axis.

Applied to polarized H/D ion sources with beam direction parallel to the quantization axis.

P. G. Sona, Energia Nucleare **14**, 295 (1967).

Theory

Metastable H atom $(I = J = 1/2)$ in external static B-field

$$H = A\,\boldsymbol{I} \cdot \boldsymbol{J} - \left(g_J \mu_B \boldsymbol{J} + g_I \mu_N \boldsymbol{I}\right) \cdot \boldsymbol{B},$$

where $A = 177.56\ MHz$ (hyperfine-structure constant), $g_J \approx -2$ (electron g-factor), $g_I = 5.59$ (proton g-factor), $\mu_B = 9.27 \times 10^{-24}\ J/T$ (Bohr magneton), and $\mu_N = 5.05 \times 10^{-27}\ J/T$ (nuclear magneton).

What is a Sona transition?

Theoretical framework

The eigensystem in the $|m_J, m_I\rangle$ basis:

$$|\alpha_1\rangle = |\Uparrow, \uparrow\rangle$$

$$|\alpha_2\rangle = \frac{n_1(B)}{h_1(B)}|\Uparrow, \downarrow\rangle + \frac{A}{h_1(B)}|\Downarrow, \uparrow\rangle$$

$$|\beta_3\rangle = |\Downarrow, \downarrow\rangle$$

$$|\beta_4\rangle = \frac{n_2(B)}{h_2(B)}|\Uparrow, \downarrow\rangle + \frac{A}{h_2(B)}|\Downarrow, \uparrow\rangle$$

$$E_1 = \frac{A}{4} - \frac{1}{2}(g_J\mu_B + g_I\mu_N)B$$

$$E_2 = -\frac{A}{4} + \frac{1}{2}\sqrt{A^2 + \left(g_J\mu_B - g_I\mu_N\right)^2 B^2}$$

$$E_3 = \frac{A}{4} + \frac{1}{2}(g_J\mu_B + g_I\mu_N)B$$

$$E_4 = -\frac{A}{4} - \frac{1}{2}\sqrt{A^2 + \left(g_J\mu_B - g_I\mu_N\right)^2 B^2}$$

$$n_{1,2}(B) = -g_J\mu_B B + g_I\mu_N B \pm \sqrt{A^2 + \left(g_J\mu_B - g_I\mu_N\right)^2 B^2}\,, \quad h_{1,2}(B) = \sqrt{A^2 + n_{1,2}^2}$$

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The eigensystem in the $|m_J, m_I\rangle$ basis:

$|\alpha_1\rangle = |\Uparrow, \uparrow\rangle$ **Pure state**

$$|\alpha_2\rangle = \frac{n_1(B)}{h_1(B)}|\Uparrow, \downarrow\rangle + \frac{A}{h_1(B)}|\Downarrow, \uparrow\rangle$$

- $B \to -\infty$, $|\alpha_2\rangle \to |\Downarrow, \uparrow\rangle$
- $B = 0$, $|\alpha_2\rangle = \frac{1}{\sqrt{2}}(|\Uparrow, \downarrow\rangle + |\Downarrow, \uparrow\rangle)$ **Mixed state**
- $B \to +\infty$, $|\alpha_2\rangle \to |\Uparrow, \downarrow\rangle$

$|\beta_3\rangle = |\Downarrow, \downarrow\rangle$ **Pure state**

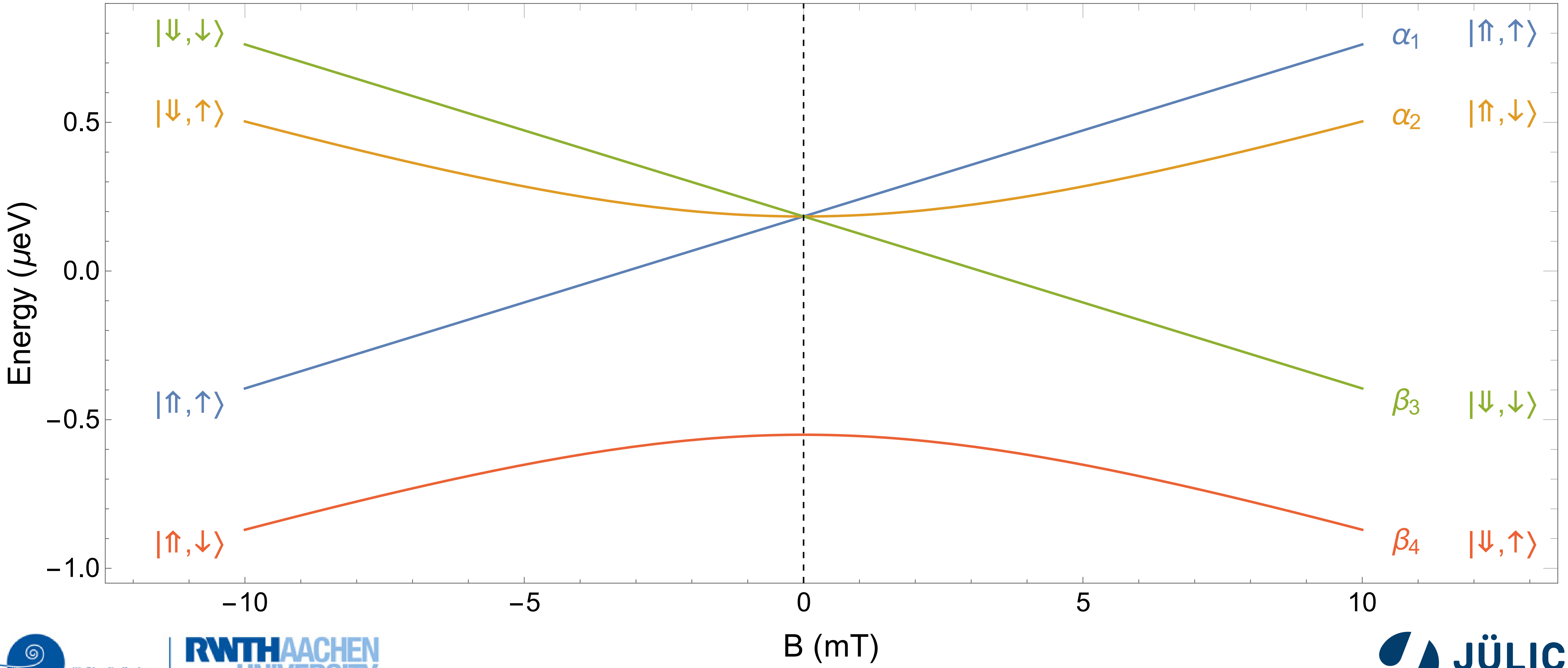
$$|\beta_4\rangle = \frac{n_2(B)}{h_2(B)}|\Uparrow, \downarrow\rangle + \frac{A}{h_2(B)}|\Downarrow, \uparrow\rangle$$

- $B \to -\infty$, $|\beta_4\rangle \to -|\Uparrow, \downarrow\rangle$
- $B = 0$, $|\beta_4\rangle = \frac{1}{\sqrt{2}}(|\Downarrow, \uparrow\rangle - |\Uparrow, \downarrow\rangle)$ **Mixed state**
- $B \to +\infty$, $|\beta_4\rangle \to |\Downarrow, \uparrow\rangle$

$$n_{1,2}(B) = -g_J\mu_B B + g_I\mu_N B \pm \sqrt{A^2 + \left(g_J\mu_B - g_I\mu_N\right)^2 B^2}\,, \quad h_{1,2}(B) = \sqrt{A^2 + n_{1,2}^2}$$

What is a Sona transition?

Theoretical framework



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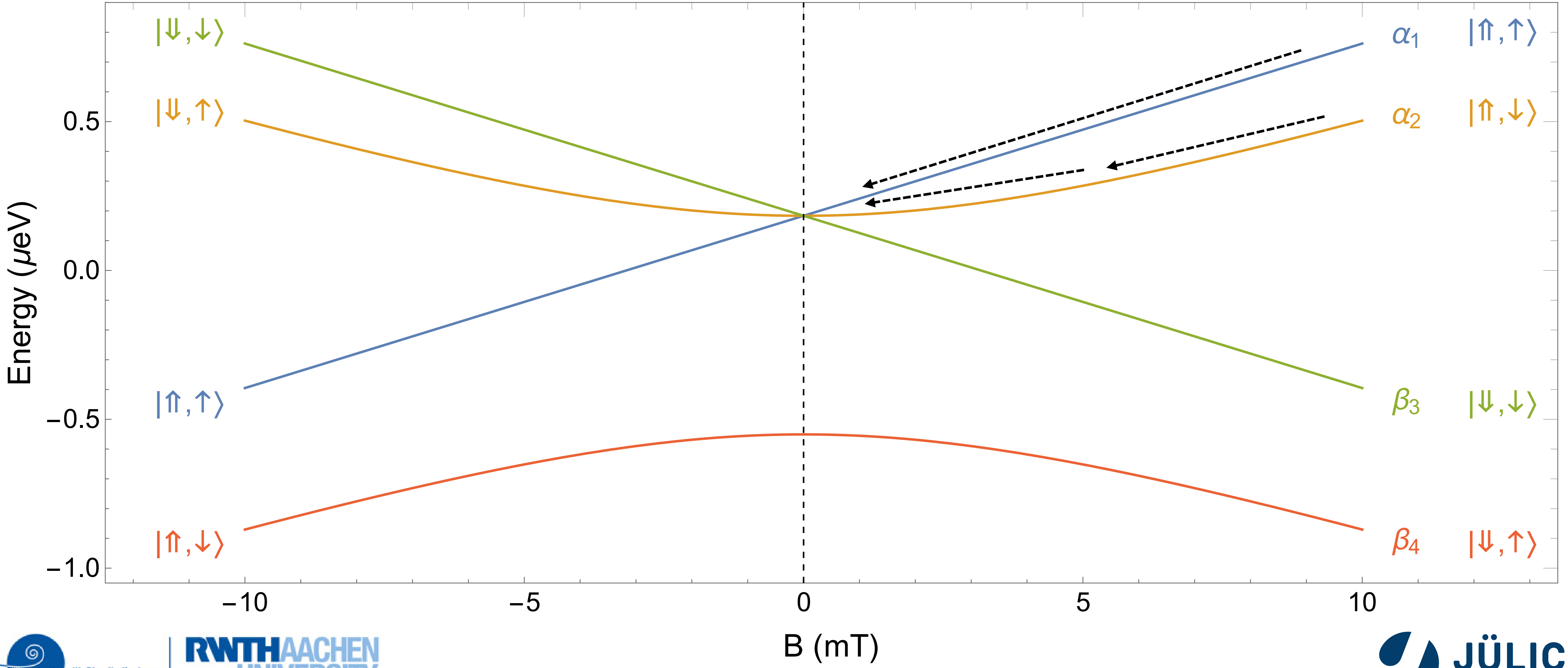
What is a Sona transition?

Theoretical framework



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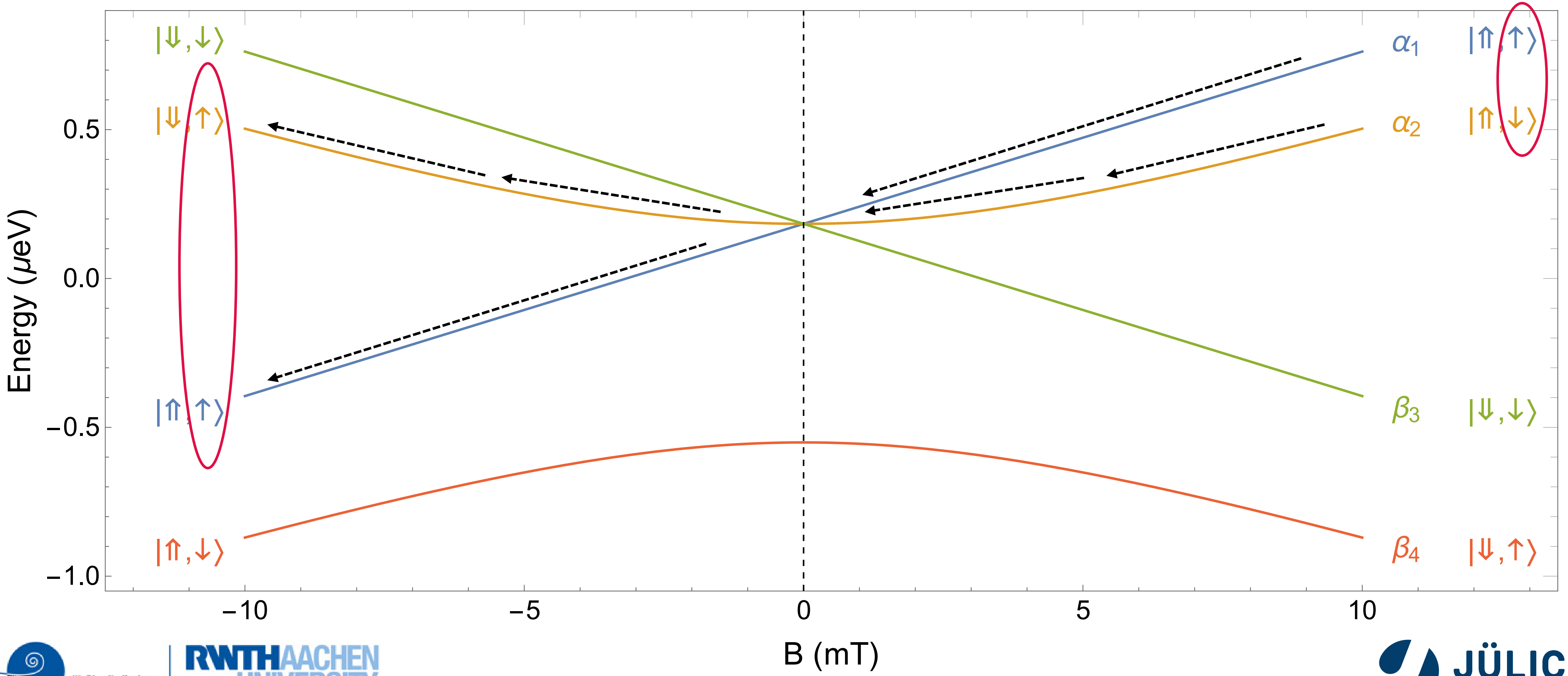
What is a Sona transition?

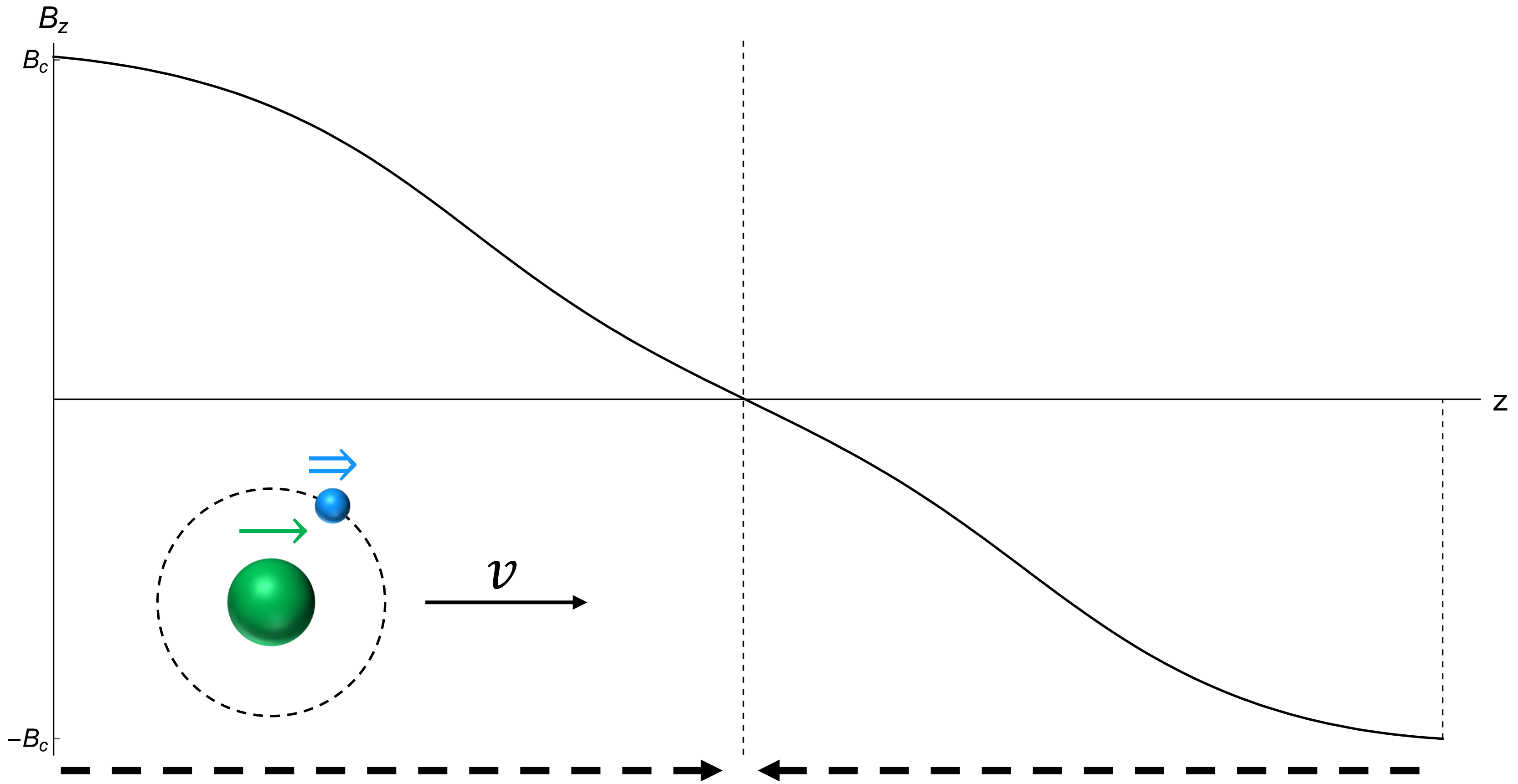
Theoretical framework



What is a Sona transition?

Theoretical framework

Zero-crossing requirements



B_z-field direction

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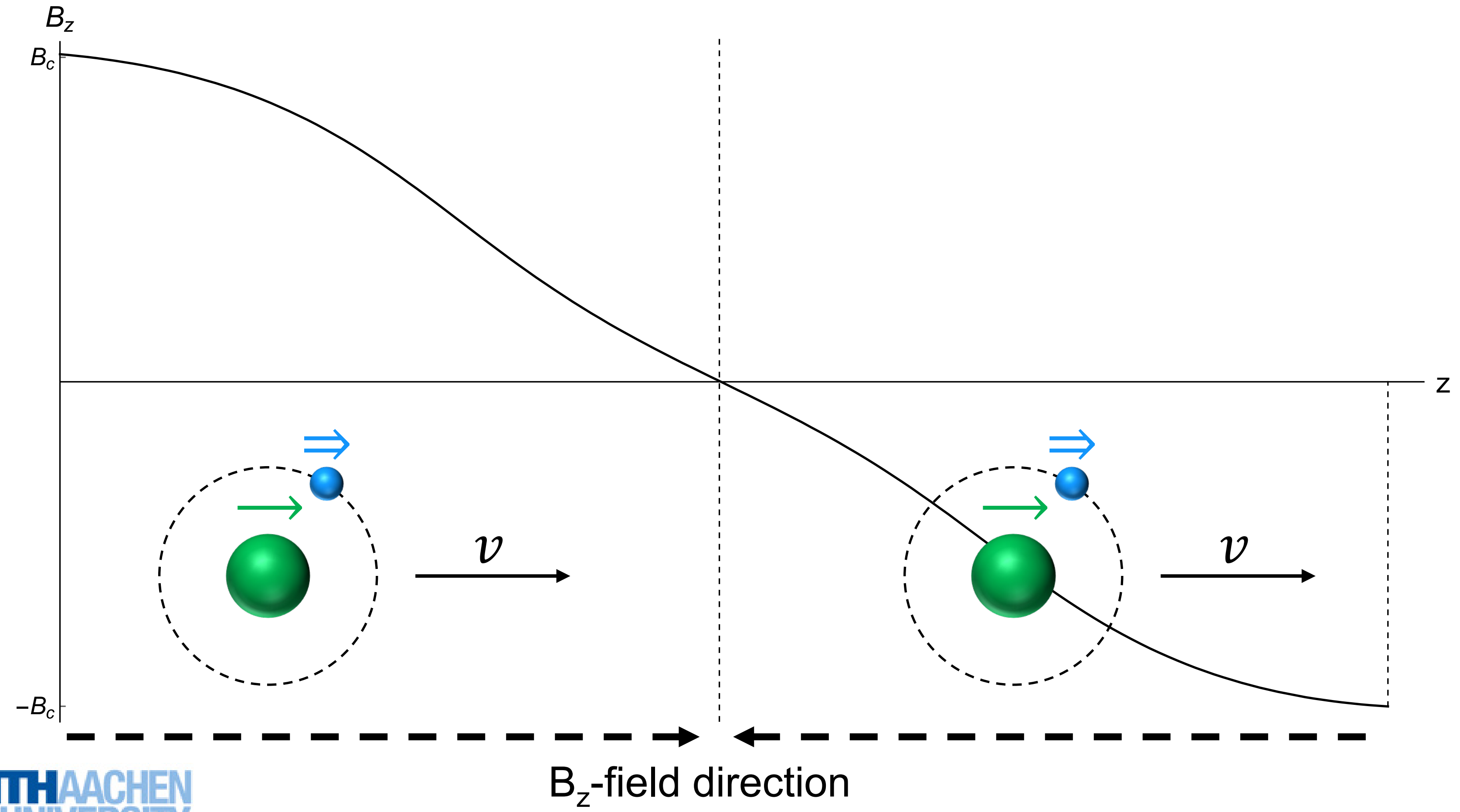
What is a Sona transition?

Theoretical framework

Zero-crossing requirements



What is a Sona transition?

Theoretical framework

Zero-crossing requirements

In the zero-crossing region:

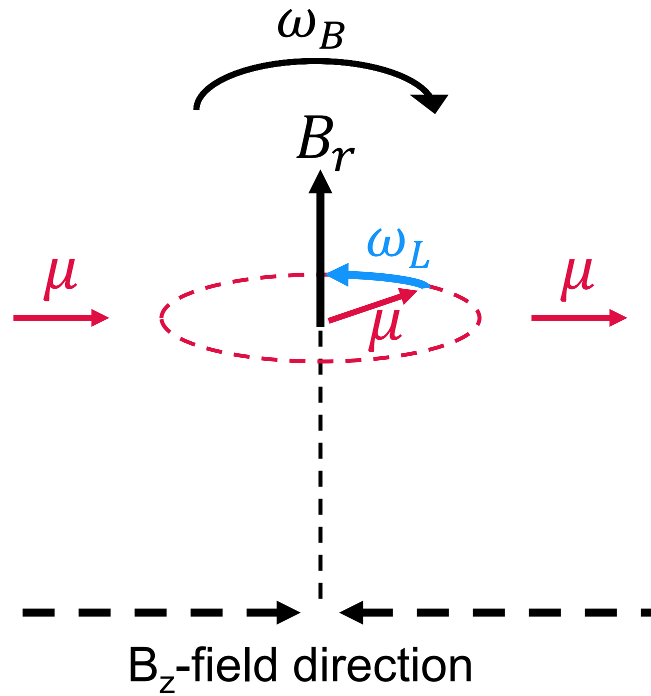
$$\left.\begin{aligned} \nabla \cdot \boldsymbol{B} &= 0 \\ B_z &= B(z) \\ B_\phi &= 0 \end{aligned}\right\} \; B_r = -\frac{r}{2}\frac{dB_z}{dz}$$

$$\boxed{\frac{eB_r^2}{2m_e v} \ll \frac{dB_z}{dz} \ll \frac{8m_e v}{er^2}}$$

Displacement of the zero crossing point

$$\bar{r} = \frac{2B_r}{dB_z/dz}, \qquad \bar{r}^2 \frac{dB_z}{dz} \ll \frac{8m_e v}{e}$$

$$\Rightarrow \frac{eB_r^2}{2m_e v} \ll \frac{dB_z}{dz}$$



"Sudden" zero-field crossing

$$\tau_B \ll \tau_L \Rightarrow \omega_B \gg \omega_L$$

$$\Rightarrow 2\frac{v}{r} \gg \frac{eB_r}{2m_e}$$

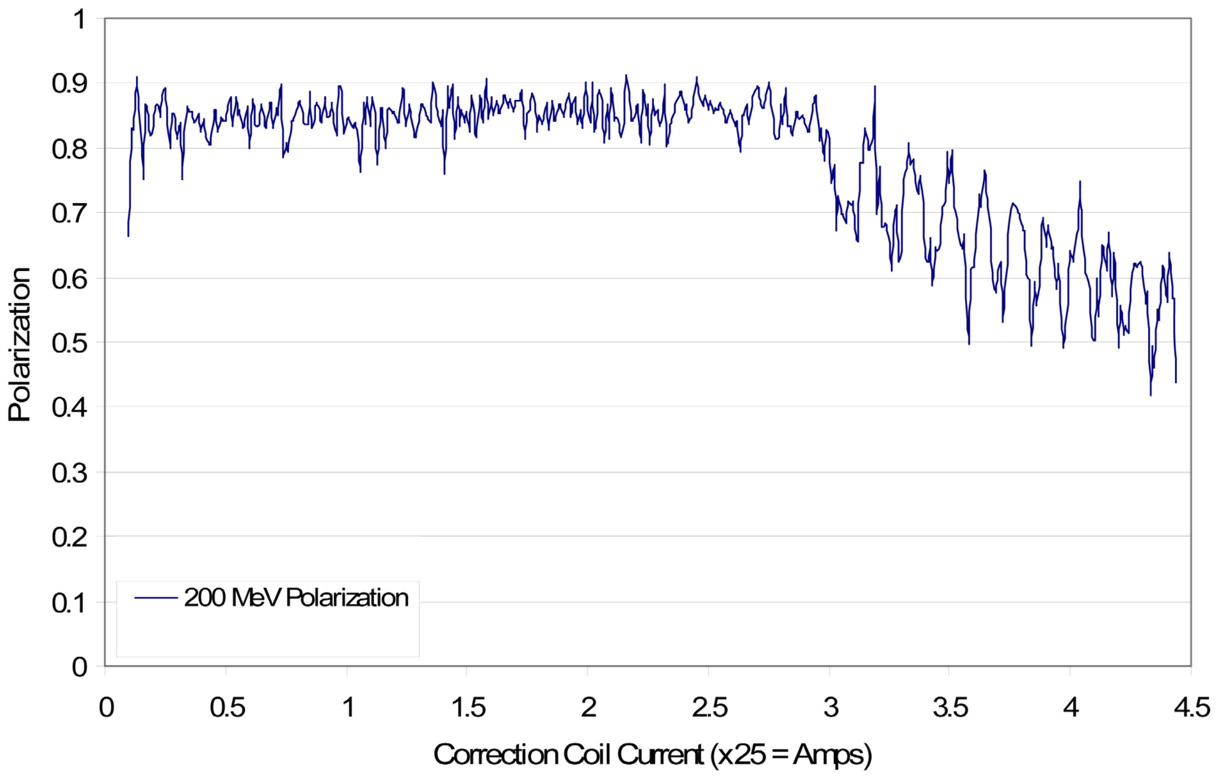
$$\Rightarrow 2\frac{v}{r} \gg \frac{er}{4m_e}\frac{dB_z}{dz}$$

$$\Rightarrow \frac{8m_e v}{er^2} \gg \frac{dB_z}{dz}$$

Sona transition studies

Unexpected polarization oscillations

<u>**O**ptically **P**umped **P**olarized **I**on **S**ource (BNL OPPIS)</u>



A. Kponou et al., AIP Conf. Proc. **980**, 241 (2008).

- An electron-polarized atomic beam passes through a Sona transition unit and gets nuclear-polarized.
- Zero-crossing requirements were fulfilled for H atoms with $\mathrm{E} = 3$ keV.
- Polarization oscillations observed while increasing the coil current.
- First *(qualitative)* explanation attempt: At some values of the coil current, the resulting magnetic field could produce a rotation of the electron spin vector, about the radial field, equal to an even multiple of 2π radians, for peaks in the polarization, and to an odd multiple of π radians for the minima.

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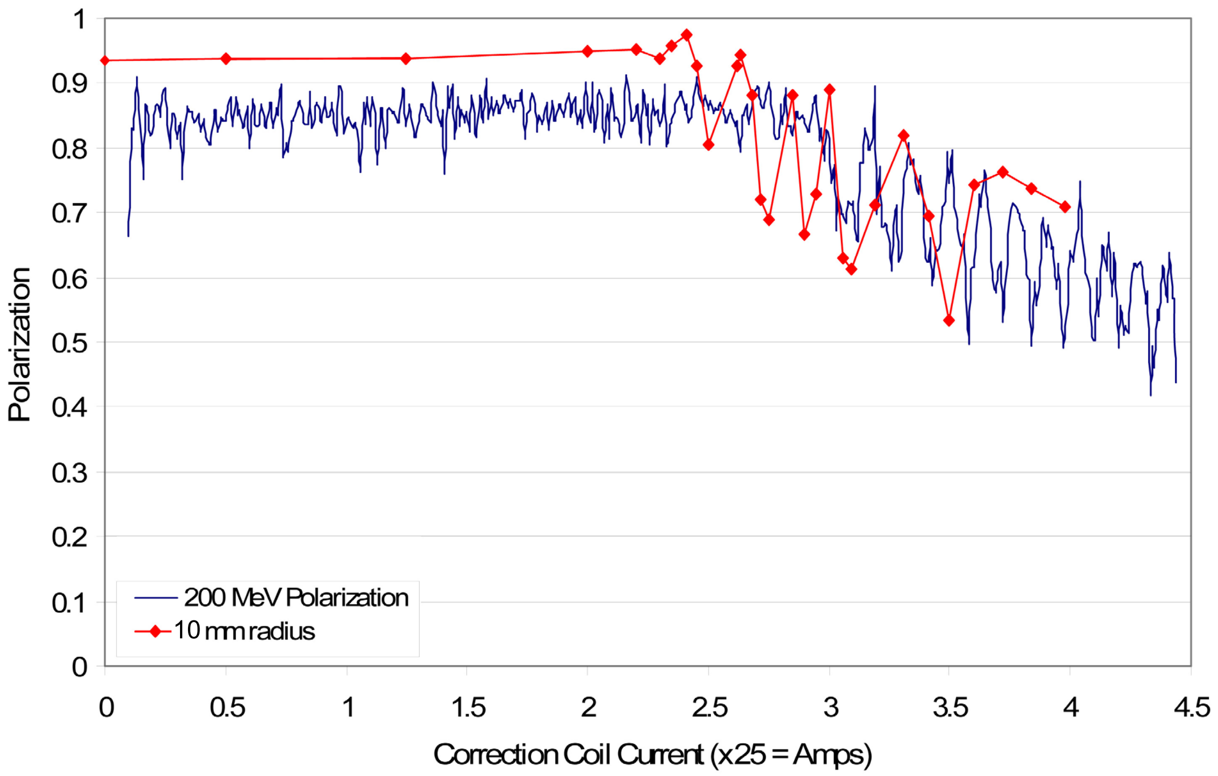
Sona transition studies

Unexpected polarization oscillations

<u>**O**ptically **P**umped **P**olarized **I**on **S**ource (BNL OPPIS)</u>



A. Zelenski et al., EPAC (2008).

- E. P. Antishev and A. S. Belov, AIP Conf. Proc. **980**, 263 (2008).
- Non-adiabatic passage through the Sona transition unit for the corresponding B-field configuration.
- The simulated beam polarization has been calculated by averaging over an effective beam diameter of 10 mm.
- The loss of polarization is well-described, but the number of peaks (and dips) and their positions are not sufficiently well determined.

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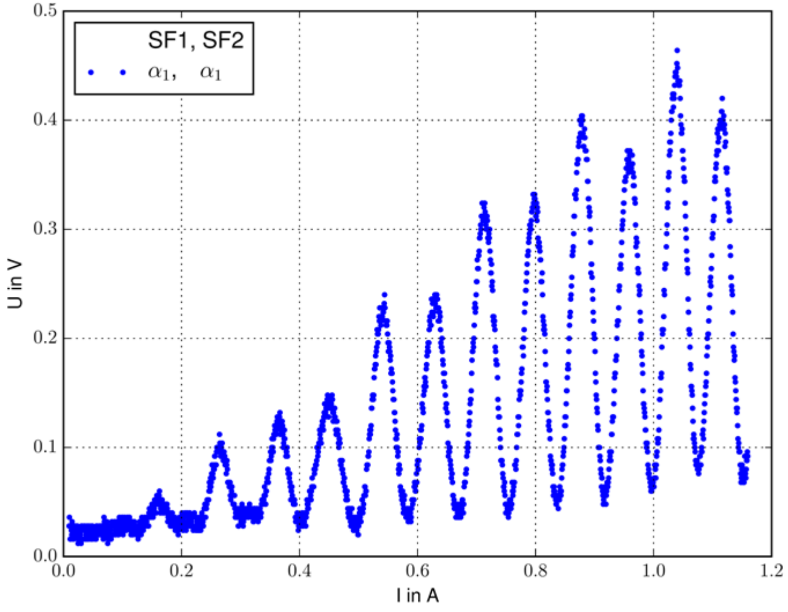
Sona transition studies

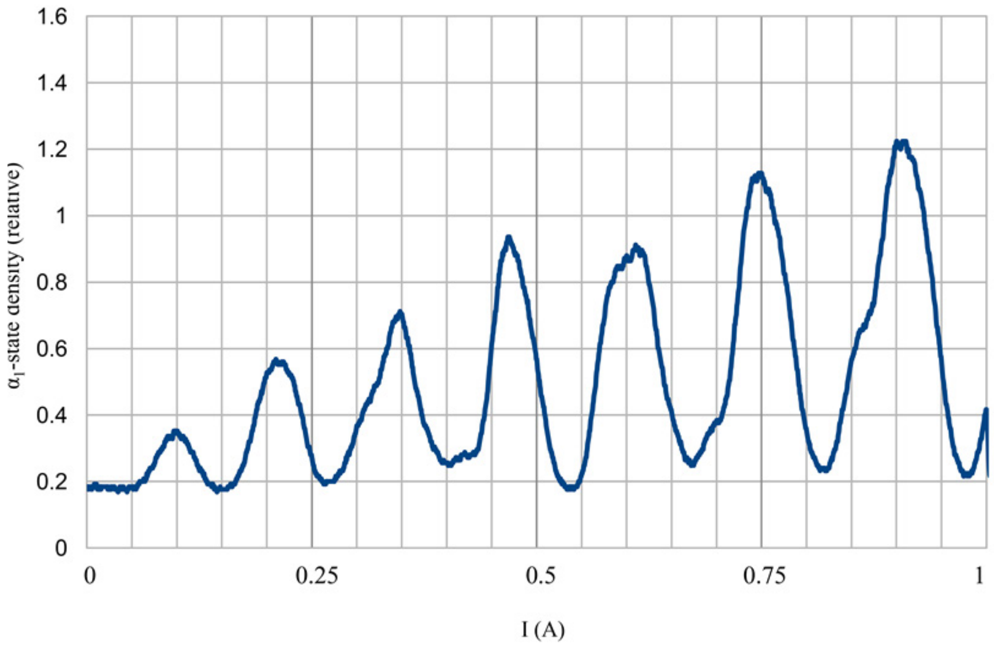
Unexpected polarization oscillations

<u>Neutron **Bo**und **B**eta Decay (BoB)</u>

- For the detection of metastable H atoms in the β_3-state with a **L**amb-**s**hift **p**olarimeter (LSP), a Sona transition unit is necessary to transfer the atoms from the β_3-state to the α_1-state.



P. Buske, B.S. Thesis (2016).

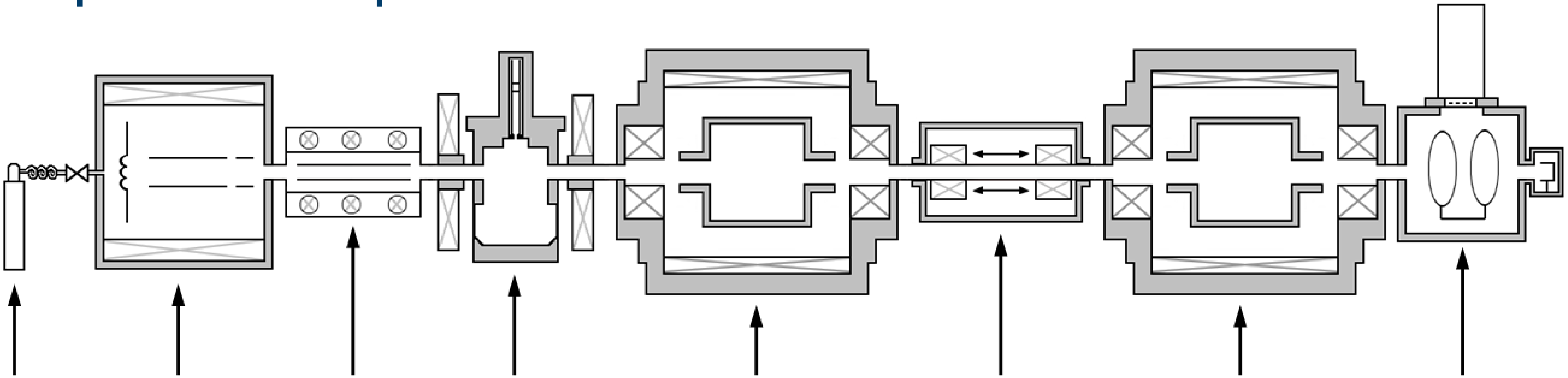


Y. Gan, B.S. Thesis (2017).

- Zero-crossing requirements were fulfilled for metastable H atoms with energy in the keV range.
- Polarization oscillations observed while increasing the coil current.

Current developments

Experimental setup



$$\mathrm{H}_2 \longrightarrow \mathrm{H}_2^+, \mathrm{p} \longrightarrow \mathrm{p} \longrightarrow \mathrm{H}(2\mathrm{S}_{1/2}) \longrightarrow \alpha_1\text{-state} \longrightarrow \beta_3\text{-}state \longrightarrow \alpha_1/\alpha_2\text{-}state \longrightarrow \text{Lyman-}\alpha$$

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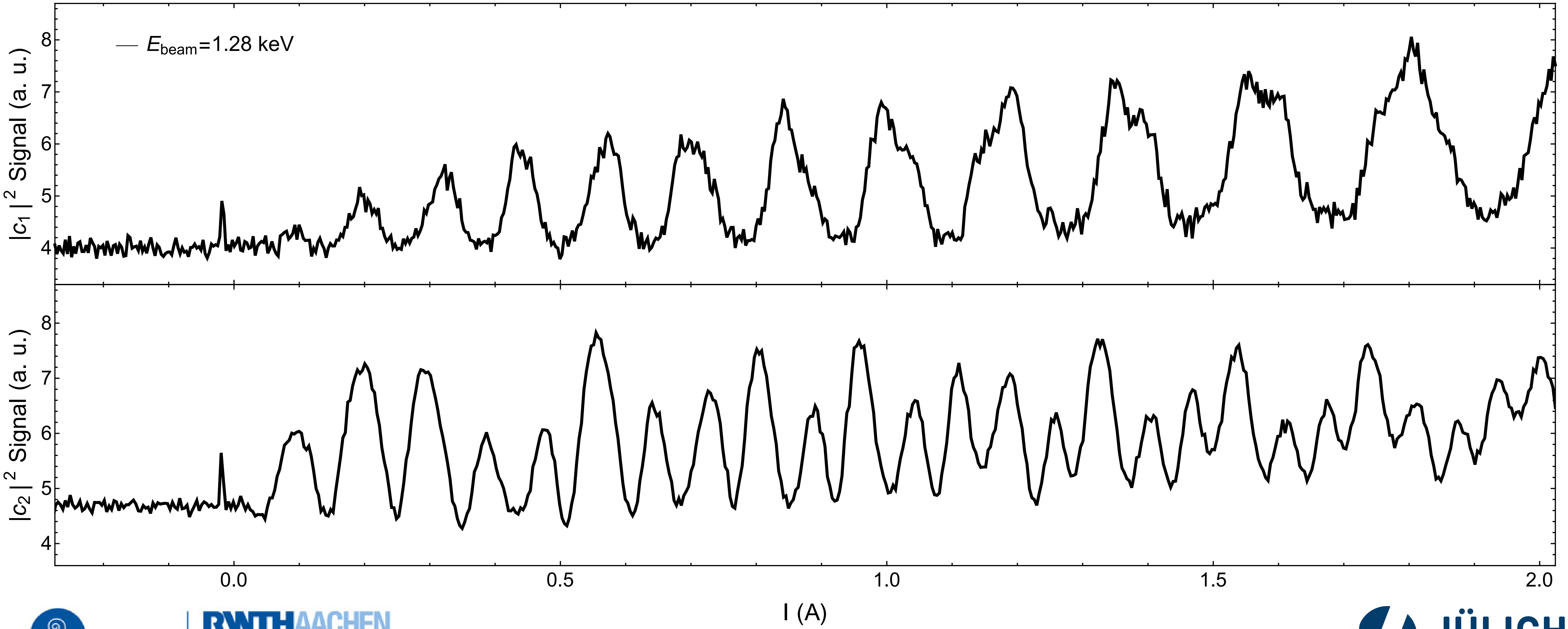
Current developments

Measured signals



Current developments

Numerical simulations

Time-dependent perturbation theory

- In the rest frame: $H(t) = A\,\boldsymbol{I}\cdot\boldsymbol{J} - \left(g_J\mu_B\boldsymbol{J} + g_I\mu_N\boldsymbol{I}\right)\cdot\boldsymbol{B}(\boldsymbol{t}) = H_0 + V(t)$
- Cylindrically symmetric magnetic field: $\boldsymbol{B}(\boldsymbol{t}) = B_r(t)\,cos\phi\,\hat{x} + B_r(t)\,sin\phi\,\hat{y} + B_z(t)\,\hat{z}$
- Evaluate $H_0 = A\,I_z\,J_z + \frac{1}{2}\,A\,(I_+J_- + I_-J_+)$

and $V(t) = -g_J\mu_B\left(\frac{1}{2}B_r(t)\left(J_+e^{-i\phi} + J_-e^{i\phi}\right) + B_z(t)\,J_z\right) - g_I\mu_N\left(\frac{1}{2}B_r(t)\left(I_+e^{-i\phi} + I_-e^{i\phi}\right) + B_z(t)I_z\right)$

- Solve the eigenvalue equation $H_0|n\rangle = E_n|n\rangle$ to calculate the eigenenergies and eigenstates

$\|1\rangle = \|\Uparrow,\uparrow\rangle$	$E_1 = \frac{A}{4}$	$\|3\rangle = \|\Downarrow,\downarrow\rangle$	$E_3 = \frac{A}{4}$
$\|2\rangle = \frac{1}{\sqrt{2}}(\|\Uparrow,\downarrow\rangle + \|\Downarrow,\uparrow\rangle)$	$E_2 = \frac{A}{4}$	$\|4\rangle = \frac{1}{\sqrt{2}}(\|\Downarrow,\uparrow\rangle - \|\Uparrow,\downarrow\rangle)$	$E_4 = -\frac{3A}{4}$

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Current developments

Numerical simulations

Time-dependent perturbation theory

- Solve the unperturbed Schrödinger equation $i\hbar\frac{\partial|n\rangle}{\partial t} = H_0|n\rangle$ to obtain the unperturbed time evolution of the system: $|n(t)\rangle = e^{-\frac{iE_n t}{\hbar}}|n\rangle, \;\; \text{for} \;\; n = 1,..,4$
- Any quantum state can be expressed in this basis

$$|\psi(t)\rangle = \sum_{n=1}^{4} c_n(t)\,|n(t)\rangle = \sum_{n=1}^{4} c_n(t)e^{-iE_n t/\hbar}\,|n\rangle$$

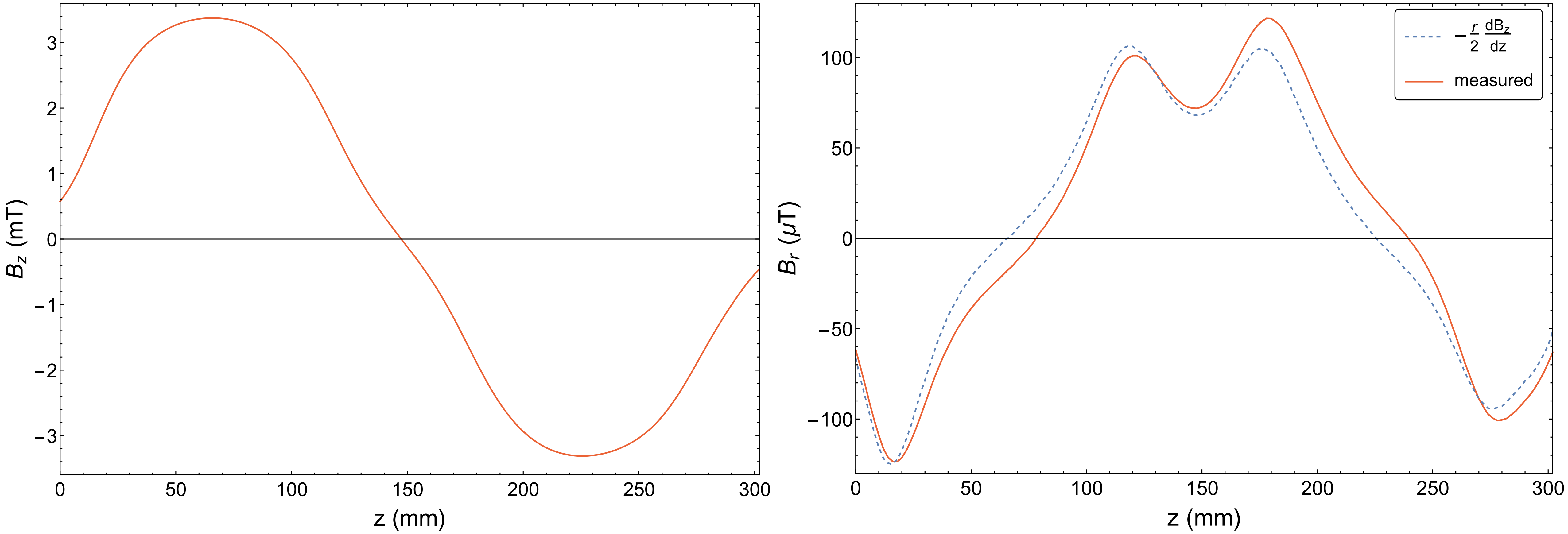
- Evaluate the Schrödinger equation of the system

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H(t)|\psi(t)\rangle = \big(H_0 + V(t)\big)|\psi(t)\rangle \quad \Rightarrow \quad \ldots \quad \Rightarrow \quad \boxed{i\hbar\dot{c}_m(t) = \sum_{l=1}^{4} c_l(t)e^{-i(E_l-E_m)t/\hbar}\,\langle m|V(t)|l\rangle}$$

Current developments

Numerical simulations

Experimental conditions



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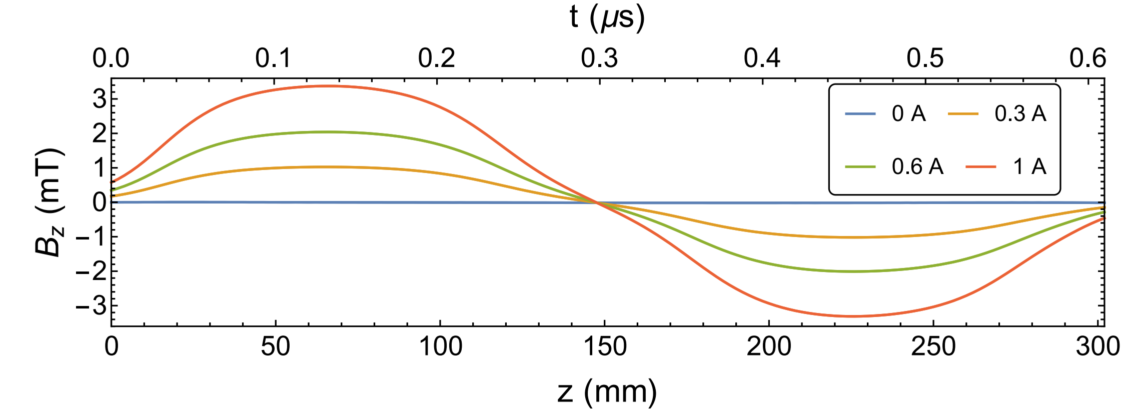
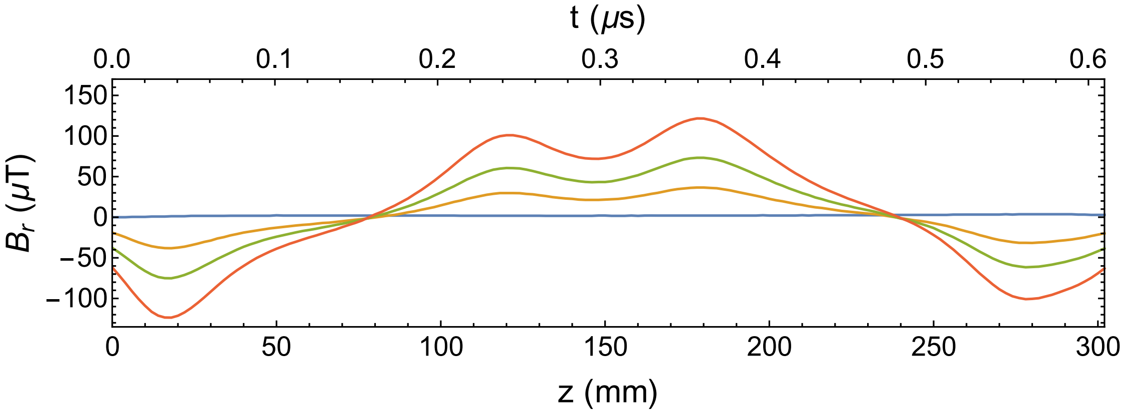
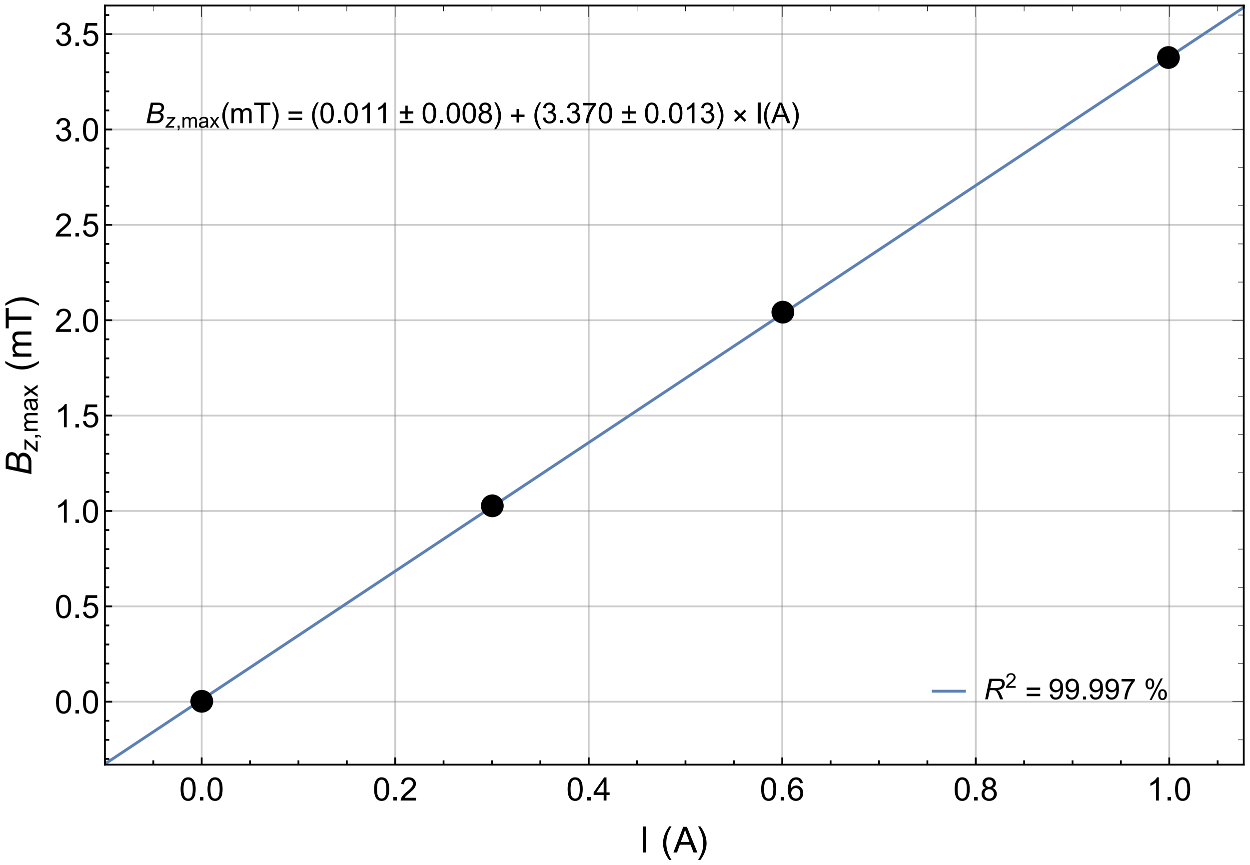
Current developments

Numerical simulations

Experimental conditions







Current developments

Numerical simulations

Experimental conditions

- Integration over a Gaussian beam profile.

$$f(x,y) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{1}{2}(x/\sigma_x)^2} \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{1}{2}(y/\sigma_y)^2},$$

where $\sigma_x = \sigma_y = \sigma = 0.5$ cm.

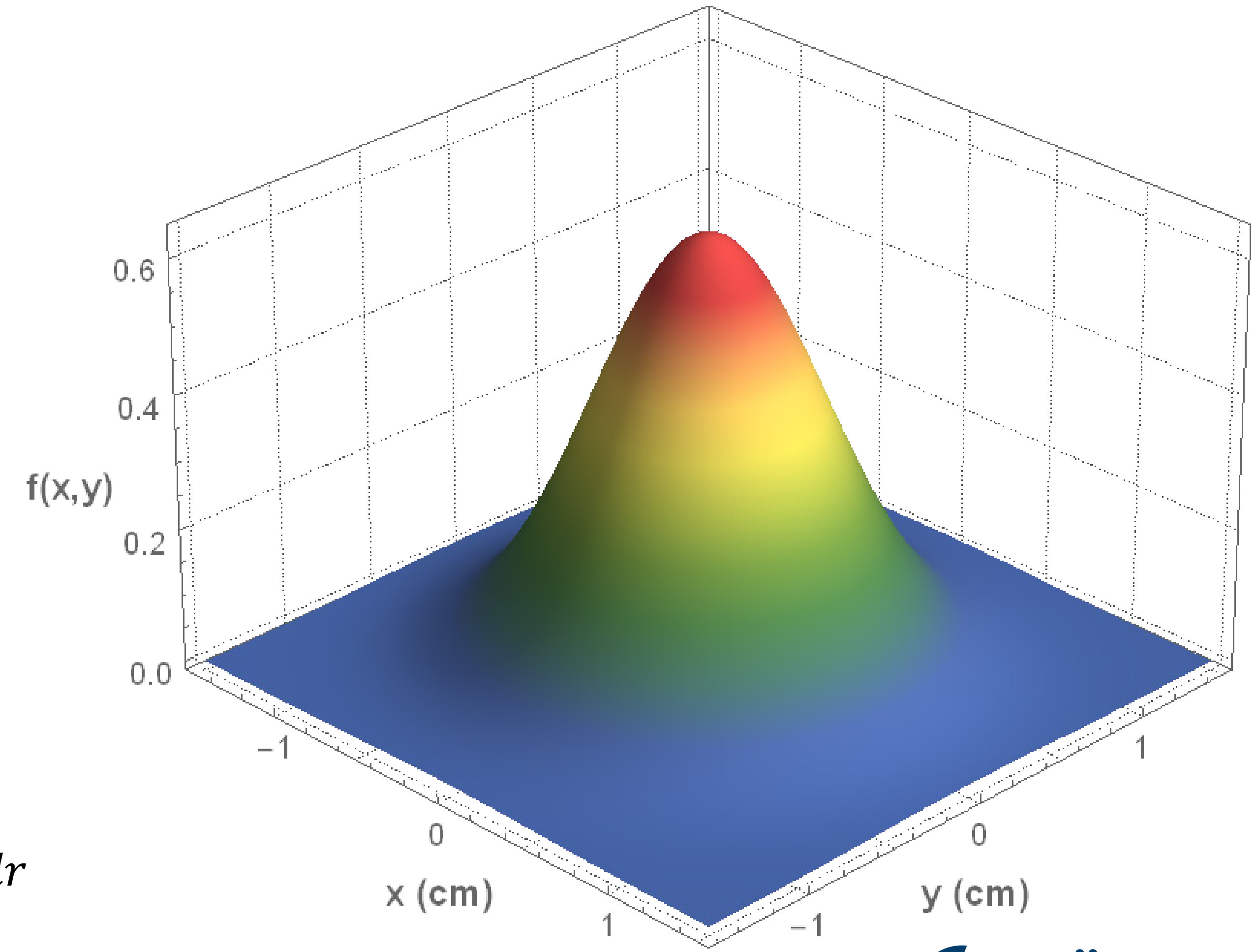
In polar coordinates ($x = rcos\phi$ and $y = rsin\phi$)

$$f(r,\phi) = f(r) = \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}(r/\sigma)^2},$$

where $\int_0^{2\pi} \int_0^{6\sigma} f(r) r\, dr\, d\phi = 1$.

$|c_m|^2$ are independent of ϕ.

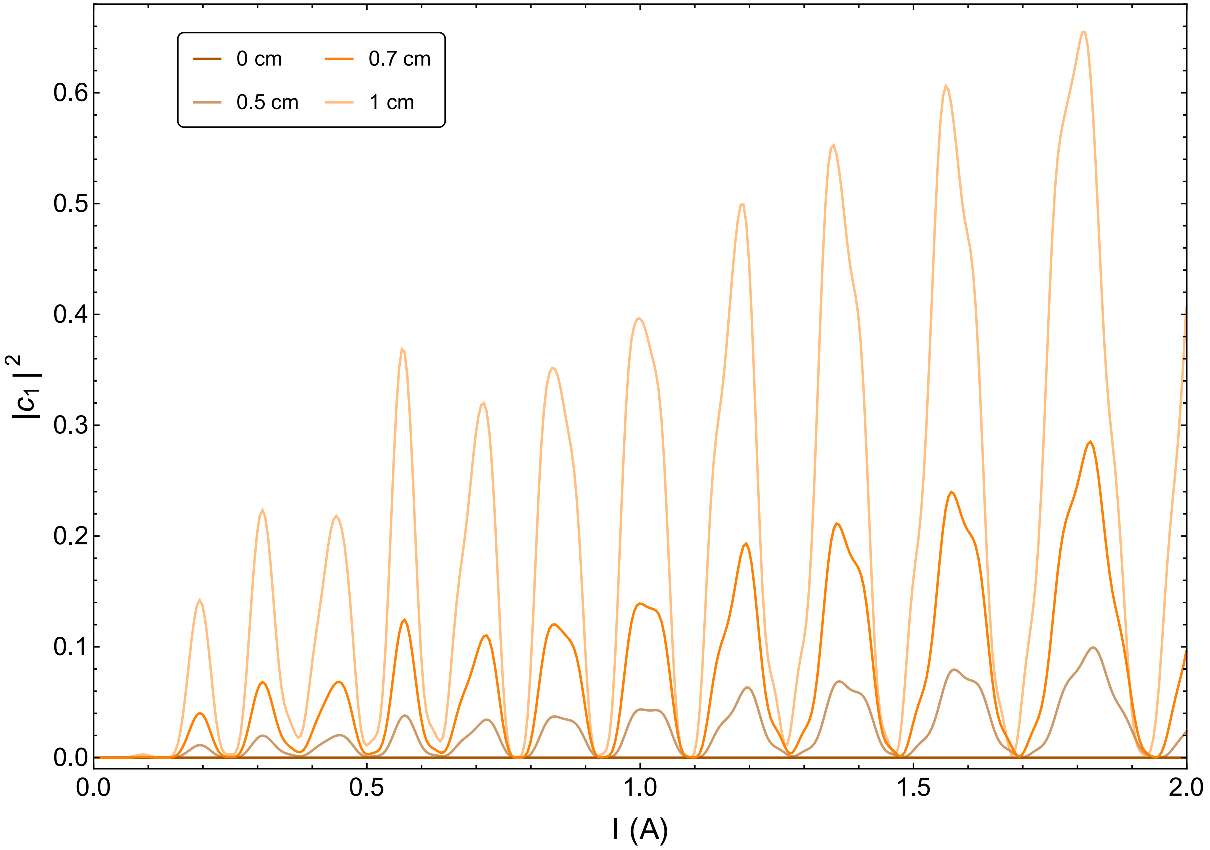
$$2\pi \int_0^{6\sigma} |c_m|^2 f(r) r\, dr$$

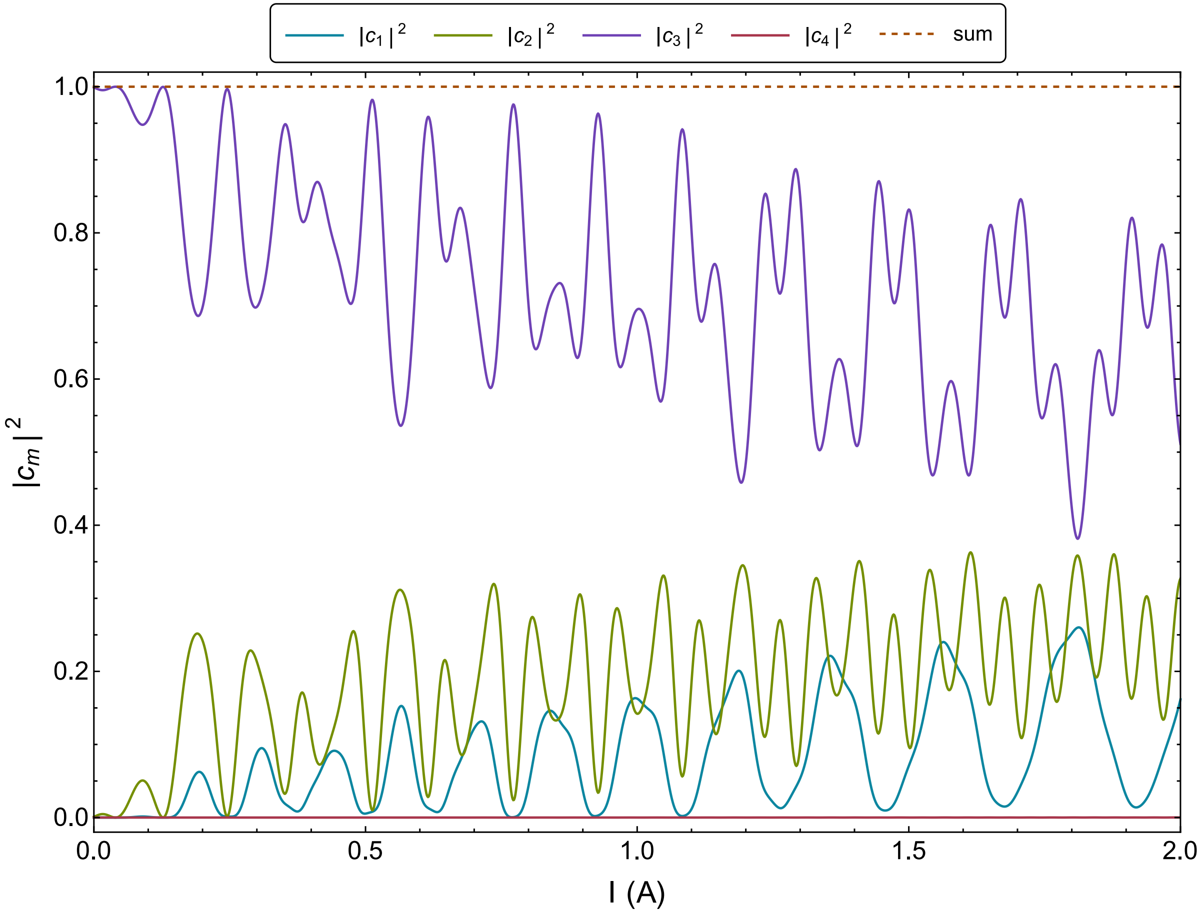


Current developments

Numerical simulations

Simulated occupation numbers





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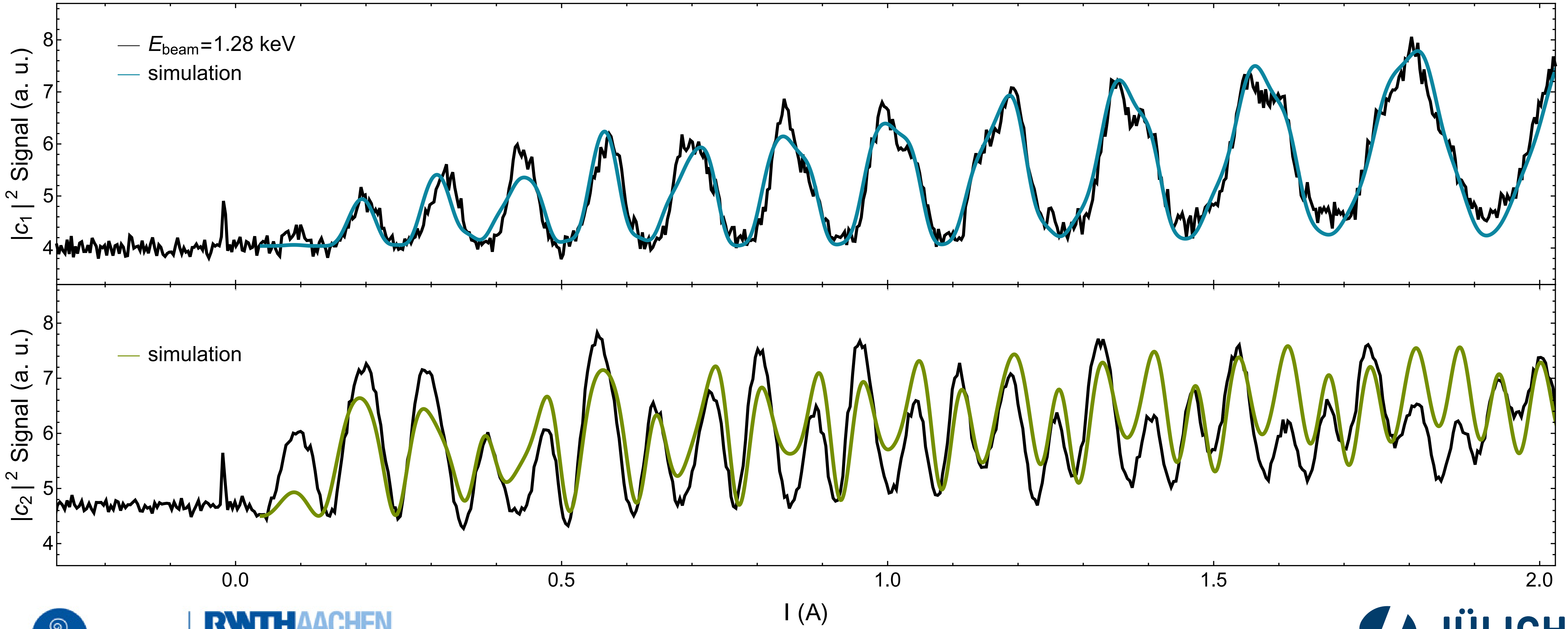
Current developments

Simulated and measured signals



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Current developments

Spectroscopy

Interpretation of measurements

- The atoms are transferred from state α_1 to β_3, during their passage from one Sona solenoid to the other (the direction of quantization axis is reversed).
- The atoms experience a time-varying radial magnetic field with a definite shape. The magnetic field amplitude depends on the coil current and the distance of the atoms from the center of the coils.
- This induces transitions from state β_3 to α_2 and from α_2 to α_1.
- Equivalently, multiple photons of frequency f are absorbed:

$$\Delta E = n \cdot h \cdot f$$

n denotes an integer number.

- The frequency f depends on:
  - ✓ the shape of the radial magnetic field (i.e. distance between the coils)
  - ✓the velocity of the hydrogen atoms

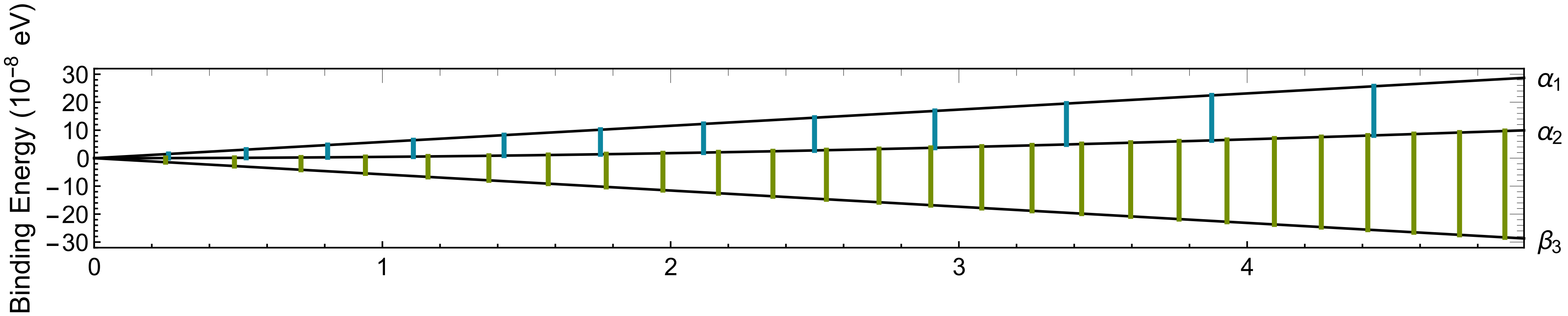
Current developments

Spectroscopy

Interpretation of measurements

- For $E_{beam} = 1.28$ keV (or $v_H = 4.95 \times 10^5$ m/s) and distance of 60 mm between the closest ends of the coils.

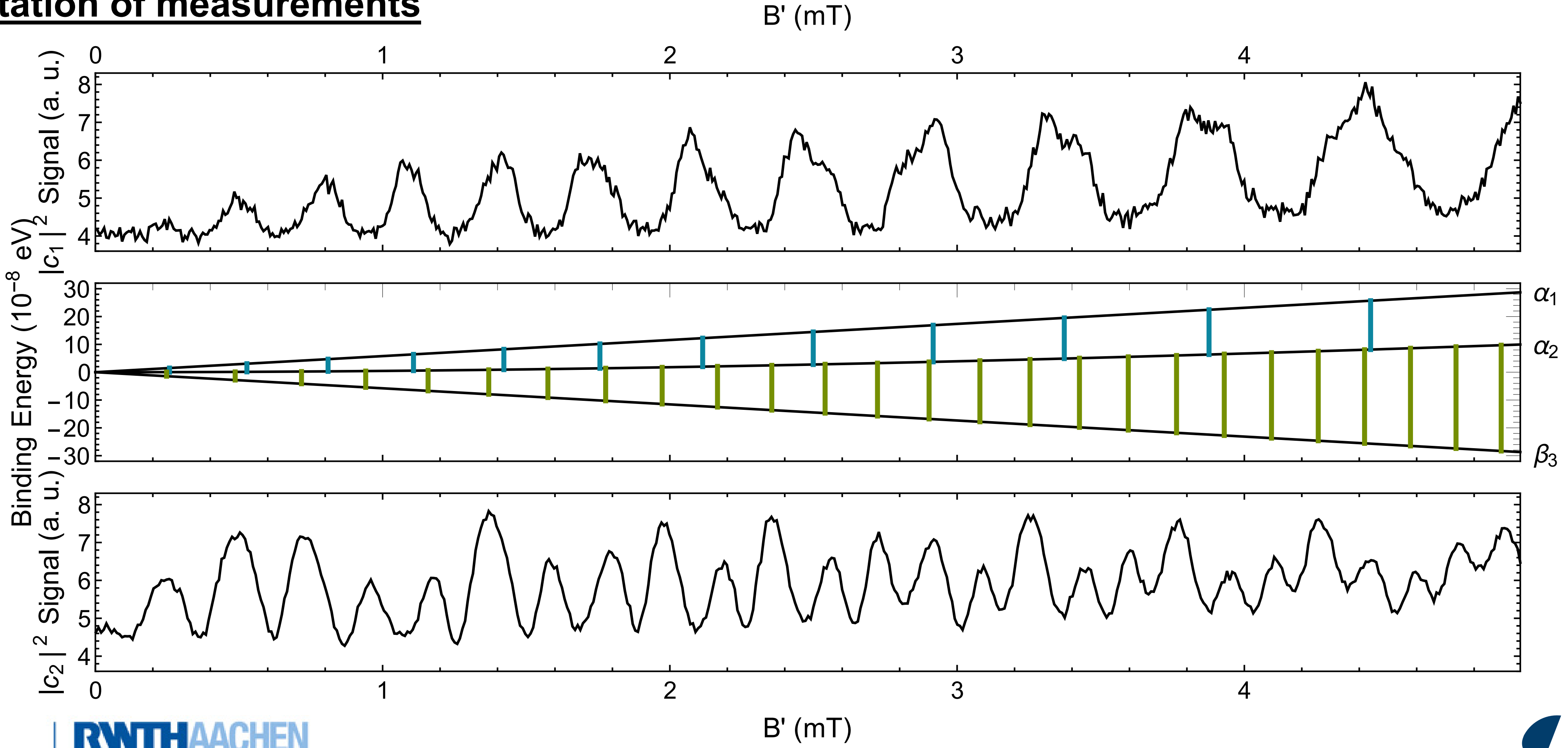
$$\Delta E = n \cdot h \cdot f \Rightarrow \Delta E = n \cdot 3.536\ \mathrm{MHz} \quad \text{or} \quad n \cdot 1.462 \times 10^{-8}\ \mathrm{eV}$$



Current developments

Spectroscopy

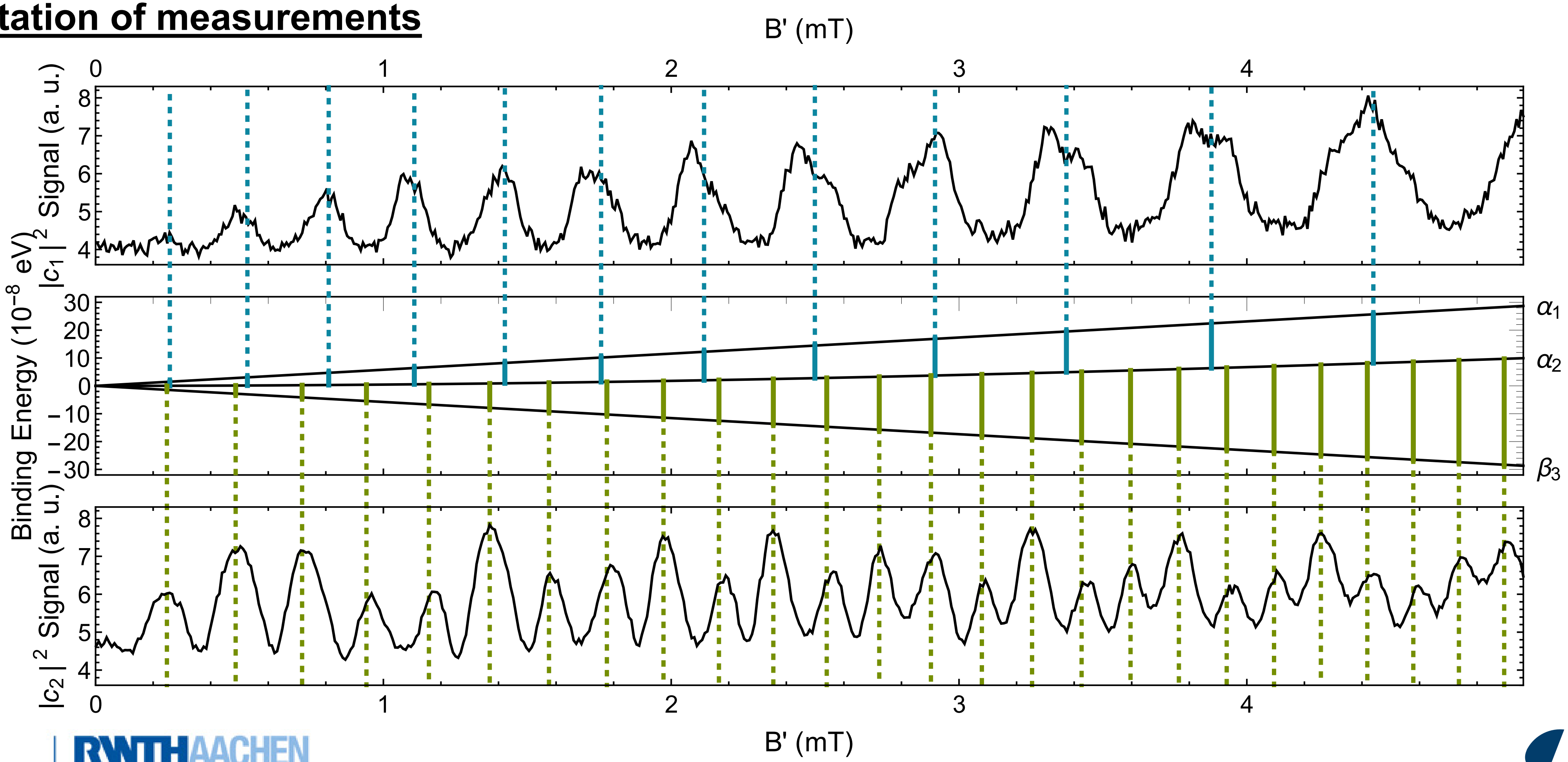
Interpretation of measurements



Current developments

Spectroscopy

Interpretation of measurements



Current developments

Spectroscopy

Interpretation of measurements

- $B' = (0.723 \pm 0.003) \cdot B_{max} + (0.004 \pm 0.020)\ \mathrm{mT}$
- $f = (3.536 \pm 0.007)\ \mathrm{MHz}$ or $\Delta E_{n=1} = (1.462 \pm 0.003) \times 10^{-8}\ \mathrm{eV}$
- Determine transitions with an energy difference of 14.62 neV and an uncertainty of 3×10^{-11} eV or 7 kHz.

Future improvements

- Replace the ionizer with an ECR ion source to increase the beam intensity and decrease the statistical uncertainty down to 10^{-13} eV.
    - ❖Such an uncertainty allows to test the QED corrections in the Breit-Rabi formula.
    - ❖Similar measurements for deuterium atoms.