



THEORETICAL STUDIES ON THE 3D STRUCTURE OF THE NUCLEON

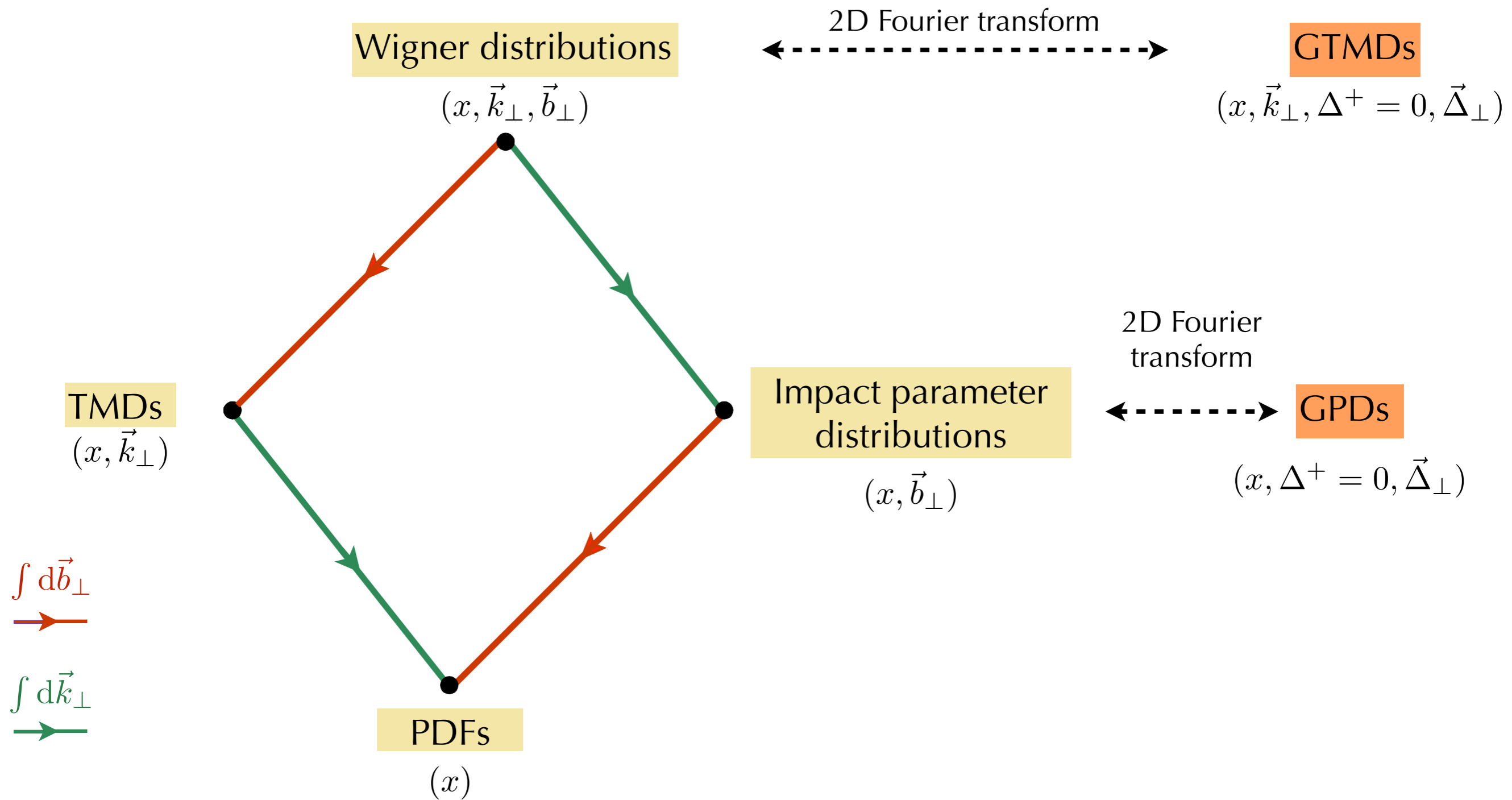
BARBARA PASQUINI

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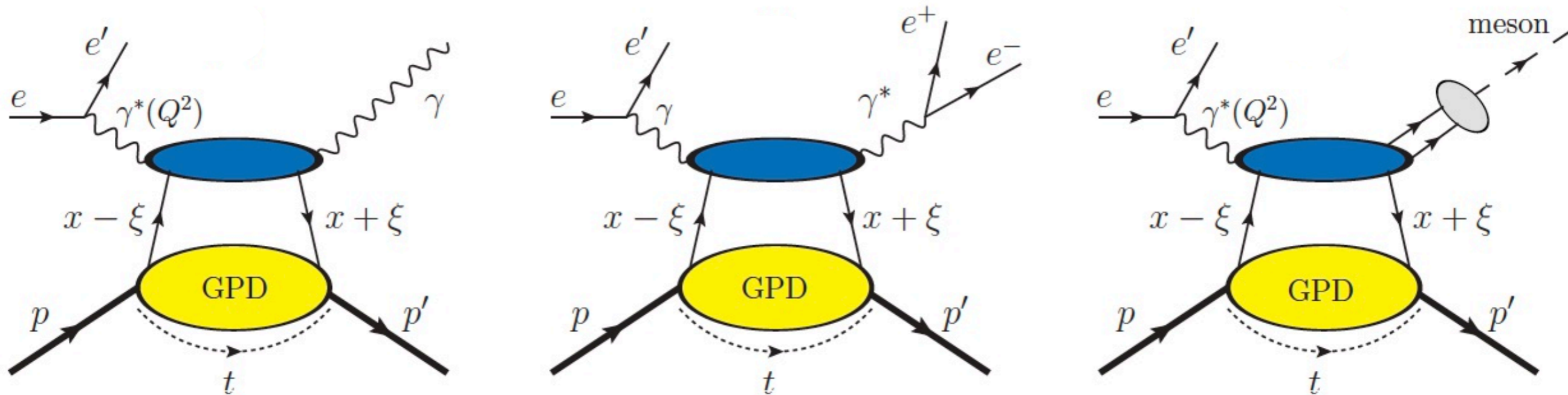
UNIVERSITÀ
DI PAVIA





- map the nucleon's constituent both in space and in momentum
- study the interplay of quark and gluon dynamics
- assessments of spin and orbital angular momentum content of the nucleon
- how the proton spin is correlated with the motion of quarks and gluons

How to measure GPDs



► accessible in exclusive reactions: universality of GPDs

► factorization for large Q^2 , $|t| \ll Q^2, W^2$

► depend on 3 variables: $x, \xi, t = \Delta^2$

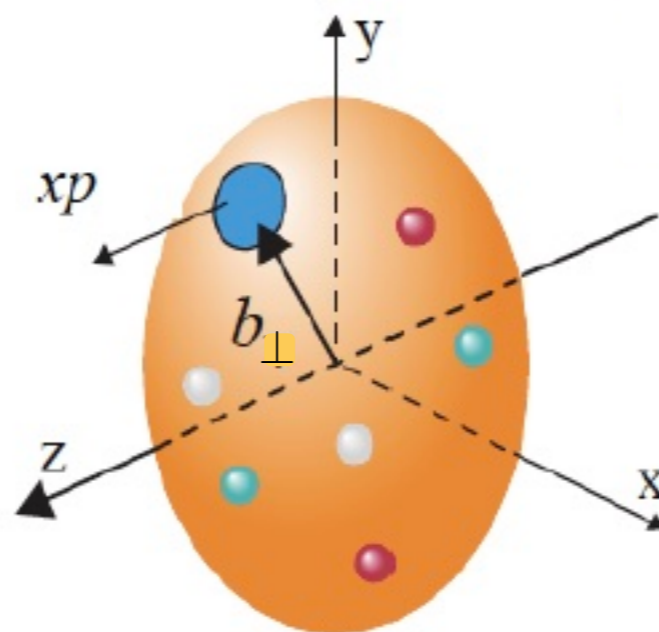
► Compton form factors $\text{Im}\mathcal{H} \stackrel{\text{LO}}{=} H(\xi, \xi, t)$ $\text{Re}\mathcal{H} \stackrel{\text{LO}}{=} \mathcal{P} \int_{-1}^1 dx \frac{H(x, \xi, t)}{x - \xi}$

	quark polarization			
nucleon polarization	GPD	U	L	T
	U	H		\mathcal{E}_T
	L		\tilde{H}	$\tilde{\mathcal{E}}_T$
	T	E	\tilde{E}	H_T, \tilde{H}_T

the distributions in **red** vanish if there is no quark orbital angular momentum

the distributions in **black** survive in the collinear limit

(at $\xi = 0$) $\vec{\Delta}_\perp \xleftrightarrow{\text{FT}} \vec{b}_\perp$ Impact Parameter Distributions

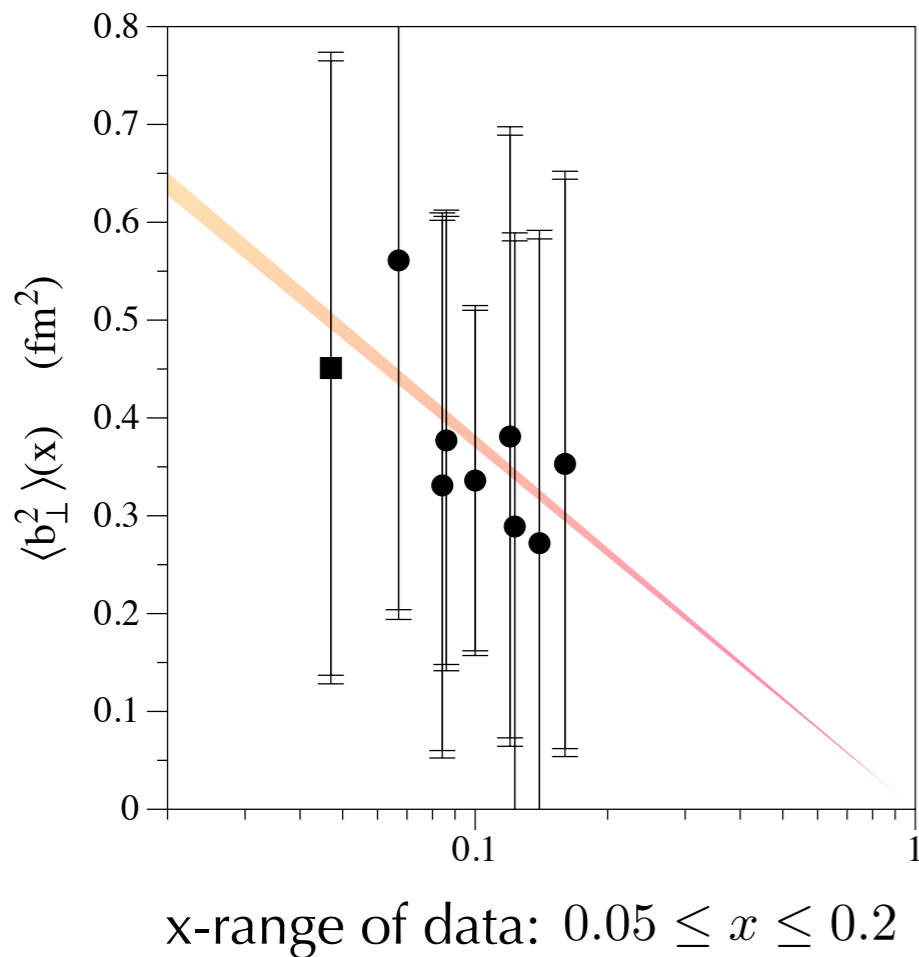


x-dependent transverse squared charge radius

$$H(x, 0, \vec{b}_\perp) = \int_{-\infty}^{+\infty} d^2 \vec{\Delta}_\perp H(x, 0, t) e^{-i \vec{\Delta}_\perp \cdot \vec{b}_\perp} \longrightarrow \langle \vec{b}_\perp^2(x) \rangle = \frac{\int d^2 \vec{b}_\perp \vec{b}_\perp^2 H(x, 0, b_\perp)}{\int d^2 \vec{b}_\perp H(x, 0, b_\perp)}$$

$(t = -\vec{\Delta}_\perp^2)$ $\xi = 0$ extrapolation from data x-dependent transverse squared radius

CLAS and Hall A data



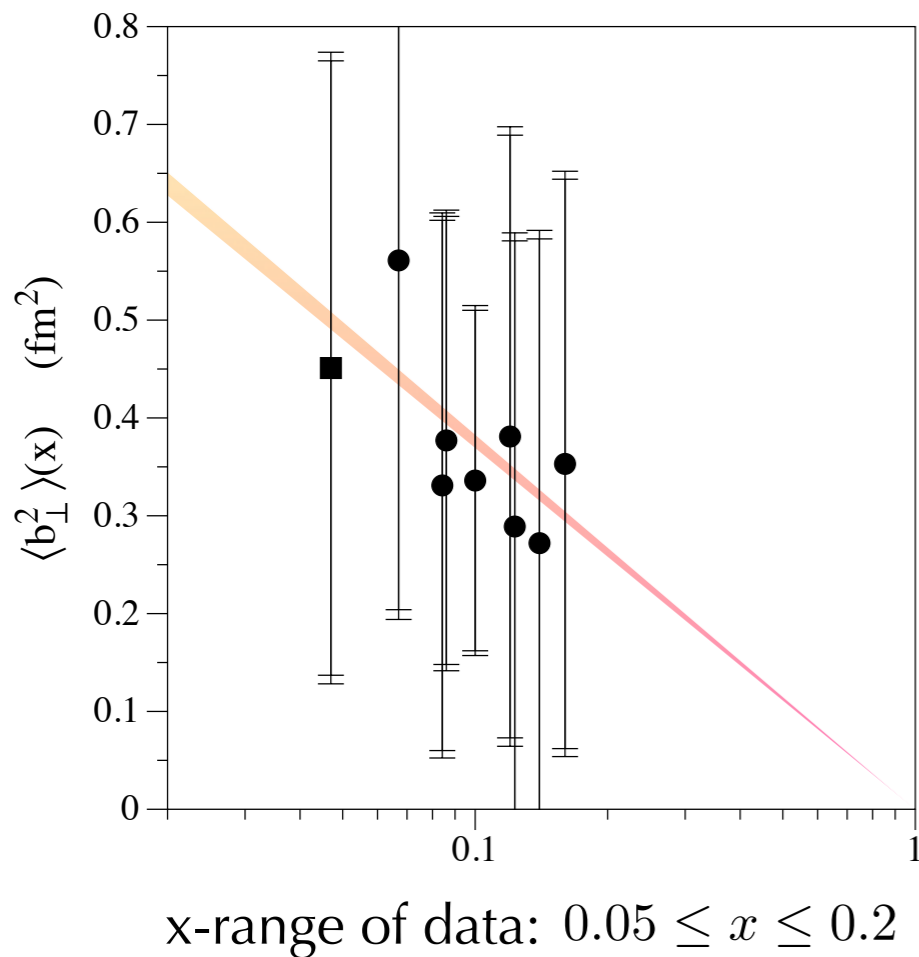
The errors are large,
but slowly we are getting some 3D information

x-dependent transverse squared charge radius

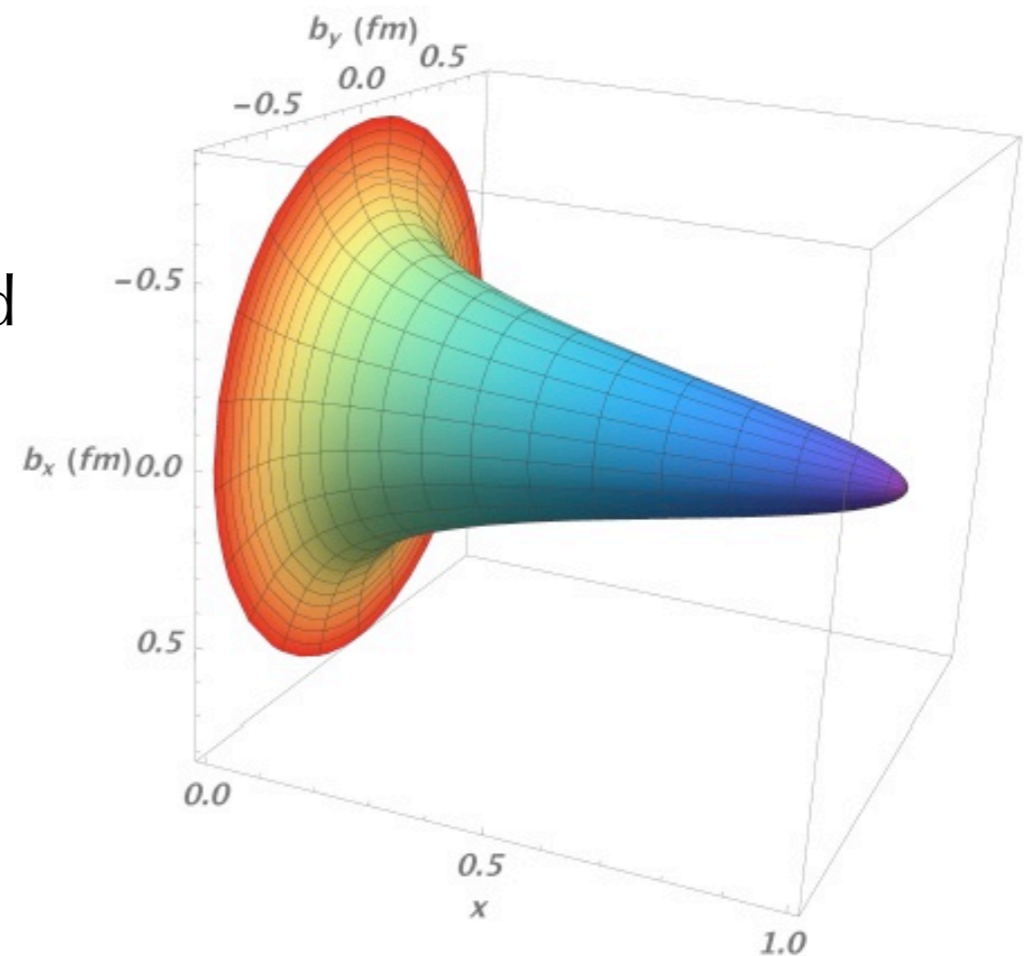
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$(t = -\vec{\Delta}_\perp^2) \quad \xi = 0$ extrapolation from data x-dependent transverse squared radius

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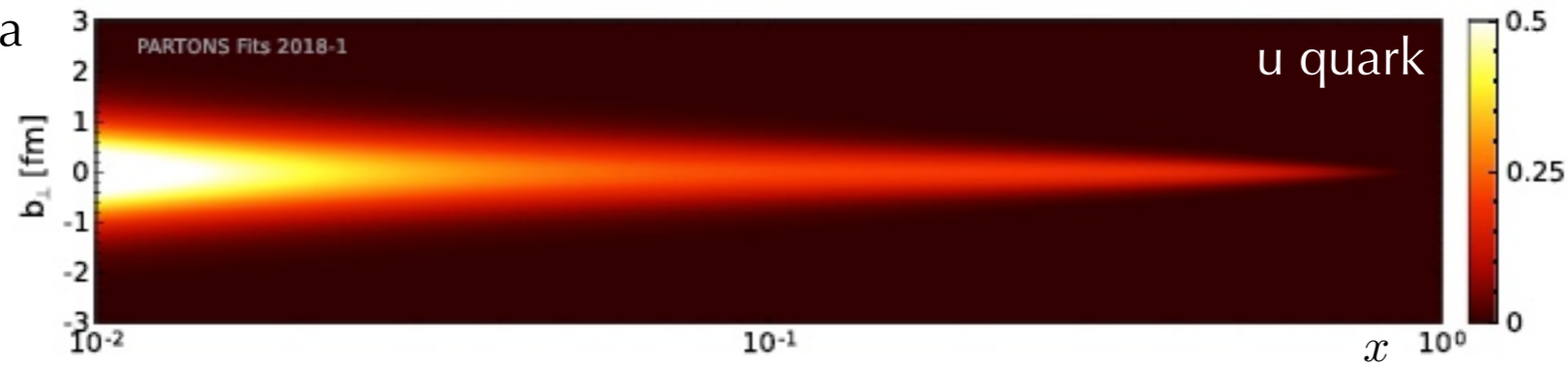
extrapolating
in the unmeasured
x-range



As $x \rightarrow 1$, the active parton carries all the momentum and represents the centre of momentum

New parametrization based on DRs: reduce problems related to the extrapolation to $\xi = 0$

CLAS and HERMES data



Moutarde et al., EPJC78(2018)890

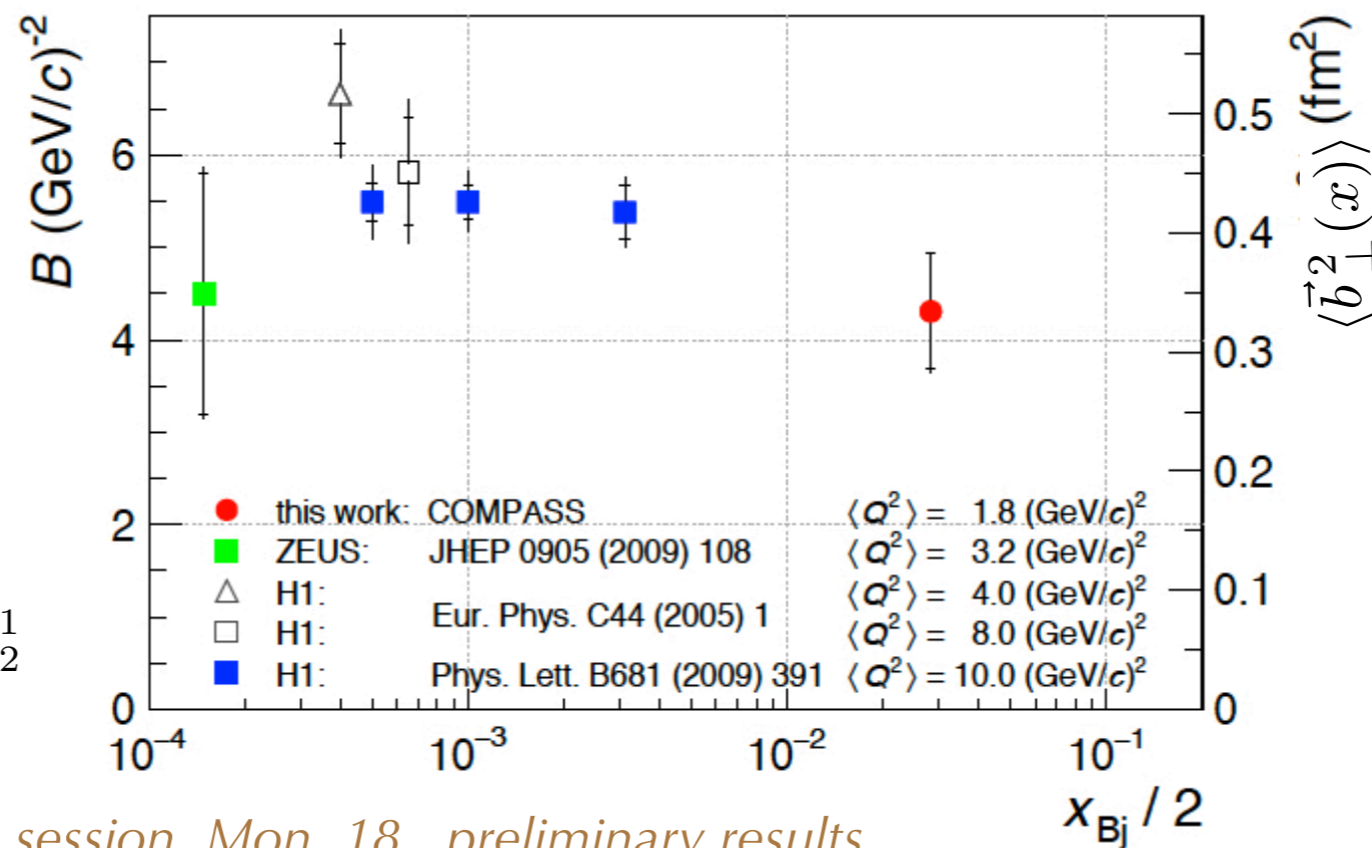
Results from COMPASS Coll.: Phys. Lett. B793 (2019) 188

$$\frac{d\sigma}{dt} \approx e^{-B(x)|t|}$$

$$\langle \vec{b}_\perp^2(x) \rangle = 2 \langle B(x) \rangle$$

at $x = 0.056$:

$$\sqrt{\langle \vec{b}_\perp^2 \rangle} = 0.58 \pm 0.04^{+0.01}_{-0.02}$$



→ Talk of J. Giarra, GPD session, Mon. 18., preliminary results from 2016/2017 Compass run

Model dependence can not be avoided, but different fit methods and parametrizations can help to constraint the theoretical uncertainties

Form Factors of Energy Momentum Tensor

$$T^{\mu\nu} = \begin{array}{c|ccc} \text{Energy Density} & & \text{Momentum Density} & \\ \hline T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \\ \hline \text{Energy Flux} & \text{Momentum Flux} & & \end{array}$$

— shear forces
— pressure

$$\langle p | T_{\mu\nu}^{Q,G} | p' \rangle = \bar{u}(p') \left[M_2^{Q,G}(t) \frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}) \Delta^\rho}{2M_N} + d_1^{Q,G}(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{5M_N} \pm \bar{c}(t) g_{\mu\nu} \right] u(p)$$

Relation with second-moments of GPDs:

$$\sum_q \int dx x H^q(x, \xi, t) = M_2^Q(t) + \frac{4}{5} d_1^Q(t) \xi^2$$

$$\sum_q \int dx x E^q(x, \xi, t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5} d_1^Q(t) \xi^2$$

“Charges” of the EMT Form Factors at t=0

$M_2(0)$ nucleon momentum carried by parton

$J(0)$ angular momentum of partons

$d_1(0)$ D-term (“stability” of the nucleon)

Form Factors of Energy Momentum Tensor

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— shear forces
— pressure

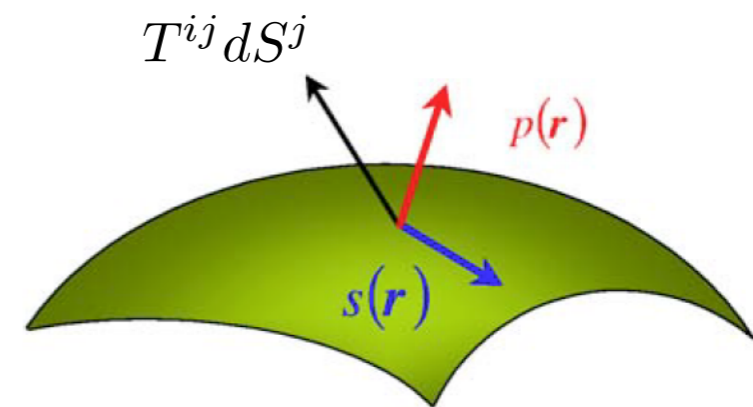
Fourier transform in coordinate space

$$T_{ij}^Q(\vec{r}) = s(\vec{r}) \left(\frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(\vec{r}) \delta_{ij}$$

↓ shear forces ↓ pressure

$$d_1^Q(0) = 5\pi M_N \int_0^\infty dr r^4 p(r)$$

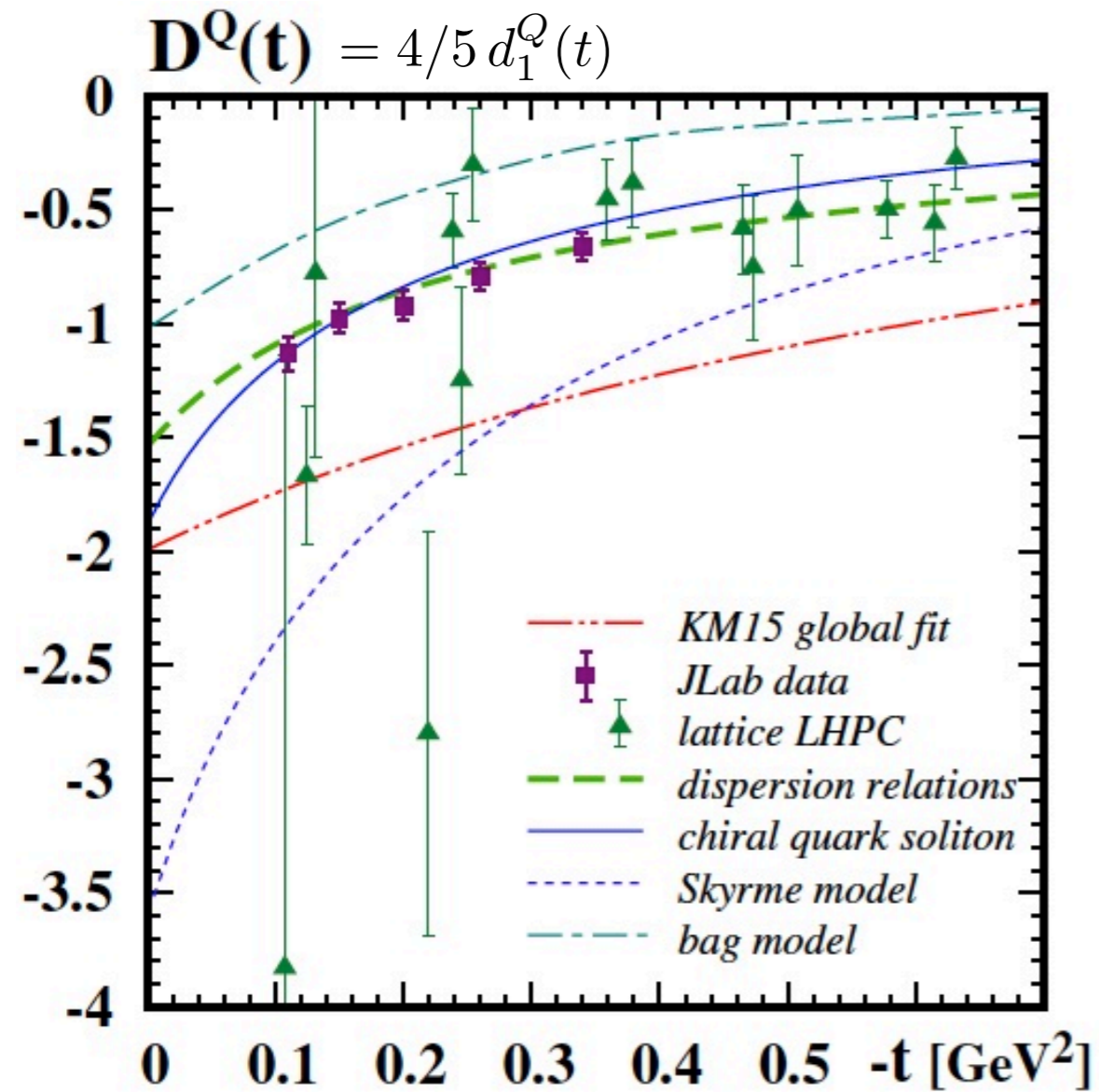
“mechanical properties” of nucleon



M. Polyakov, PLB 555 (2003) 57

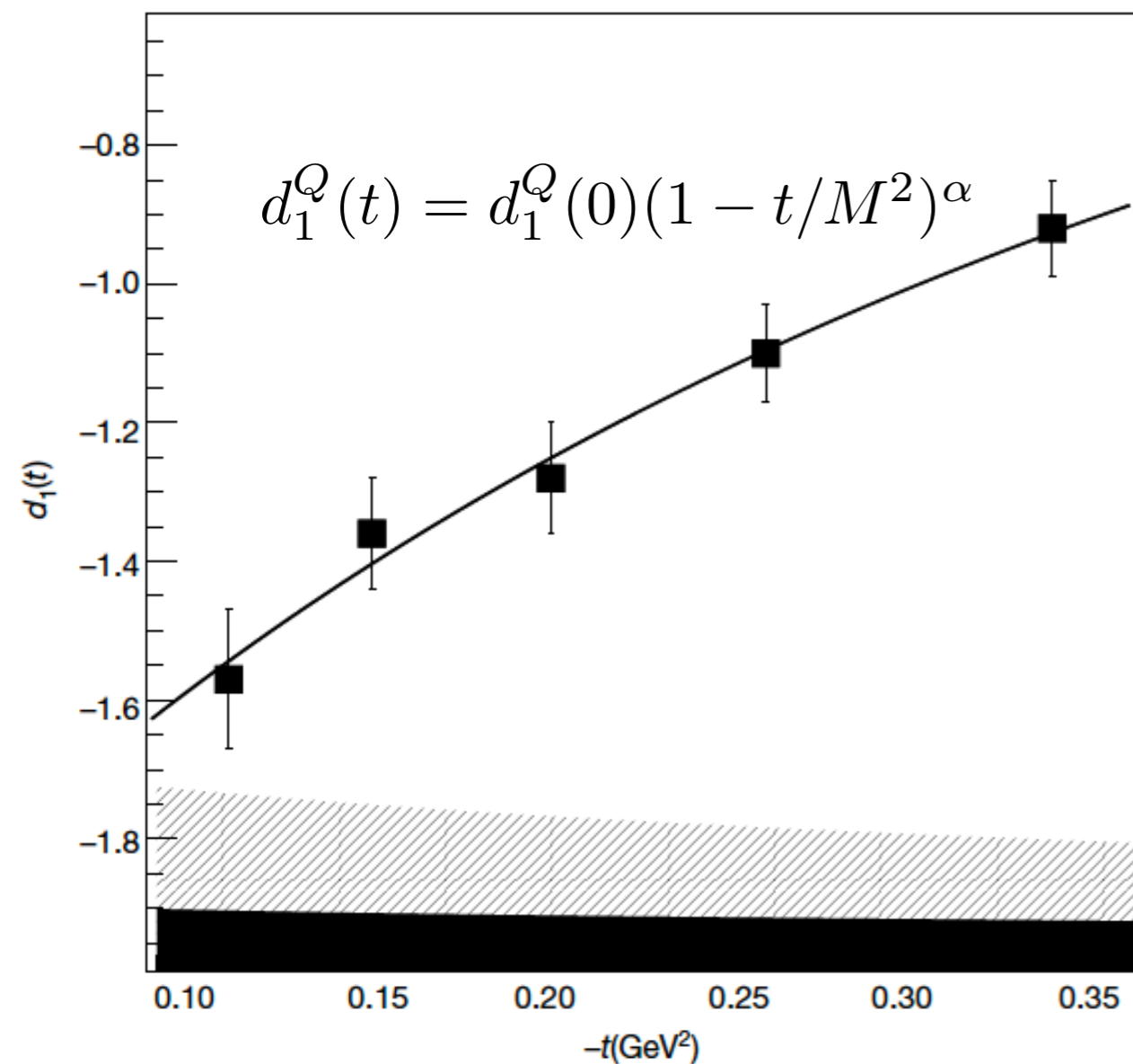
D(t) form factor from data

*Girod, Elouadrhiri, Burkert, Nature 557 (2018) 7705
and arXiv: 2104.02031;
CLAS 6GeV data*



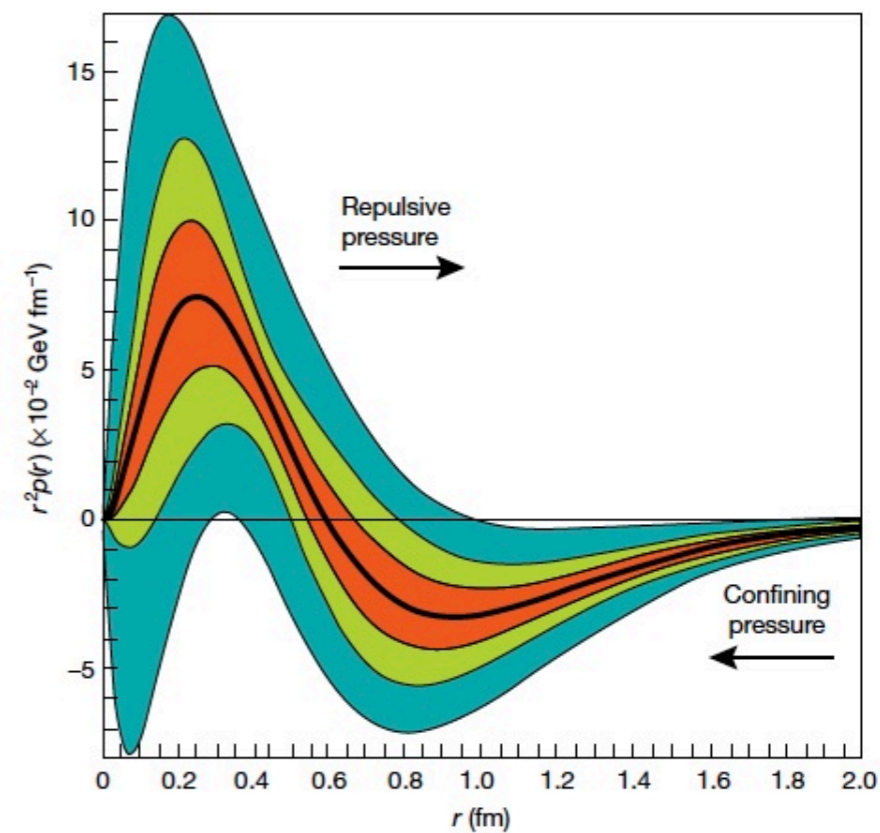
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Girod, Elouadrhiri, Burkert, *Nature* 557 (2018) 7705
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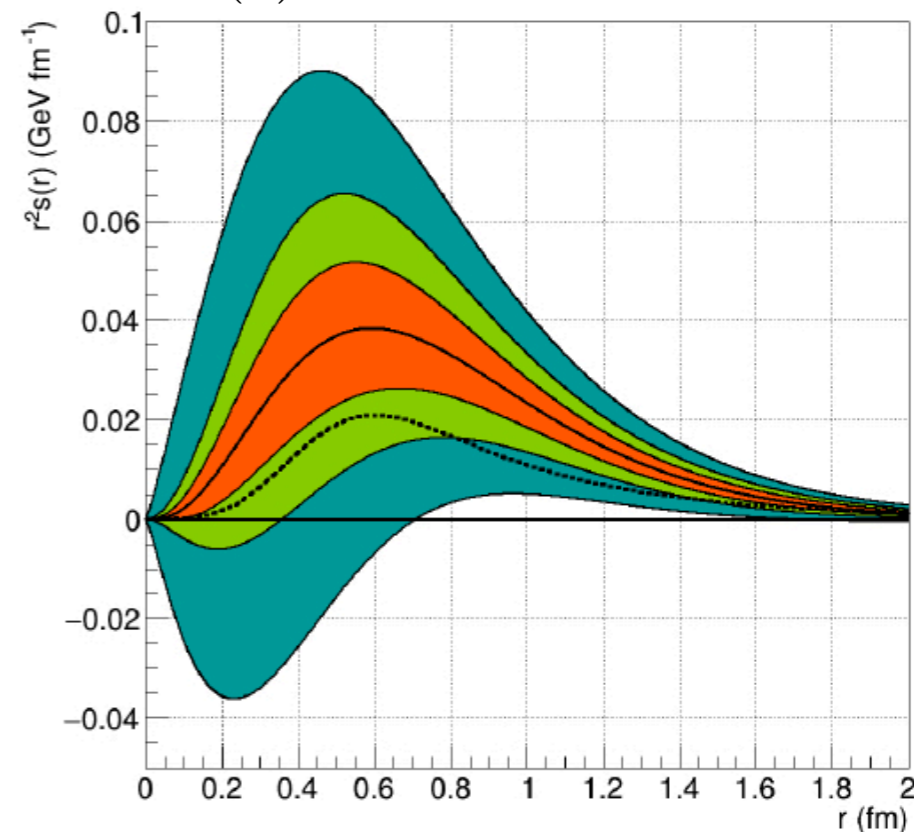


FT

$r^2 p(r)$ radial pressure distribution



$r^2 s(r)$ shear forces distribution



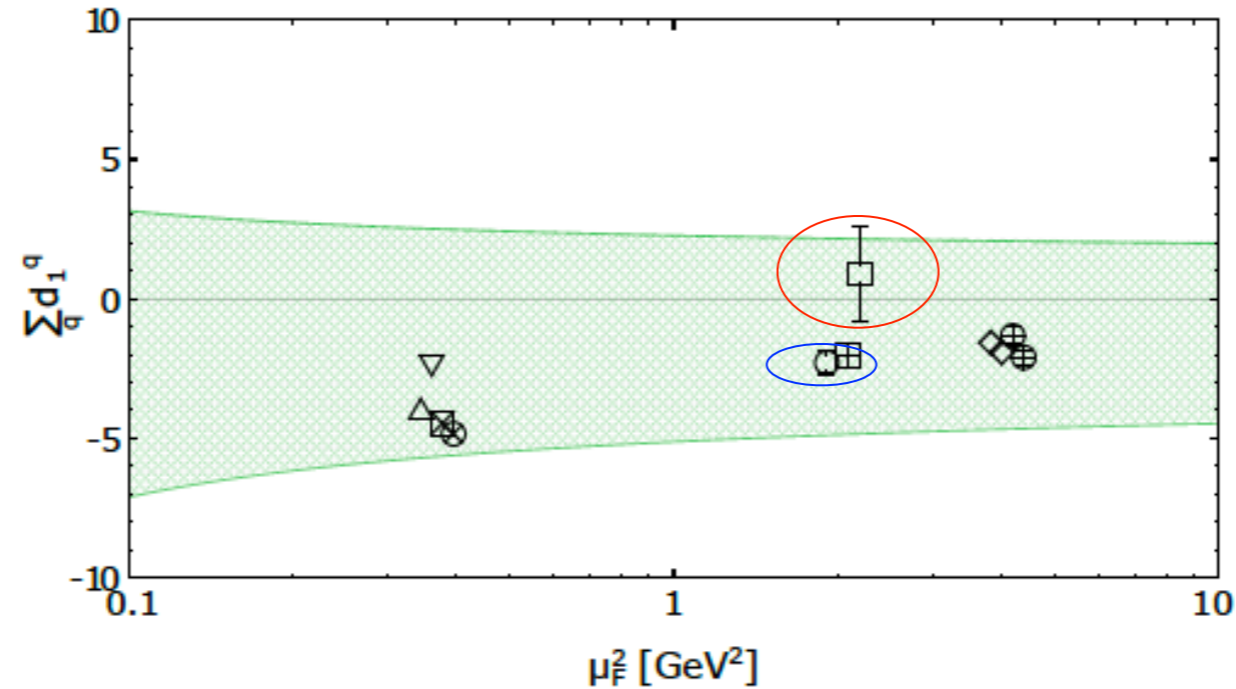
- data before 6 GeV JLab
- 6 GeV JLab data
- projected 12 GeV JLab data

Necessary to verify model assumptions in the exp extraction
with more data coming from JLab, COMPASS and the future EIC, ElcC

Kumericki, Nature 570 (2019) 7759; Dutrieux et al, Eur. Phys. J. C81 (2021) 4

Talk of Dutrieux, Moutarde, Sznajder, GPD sessions, Mon. and Wend.

global fit to DVCS data
with artificial neural networks



CLAS data, with fixed param.,
Girod et al.

CLAS data, with neural networks
Kumericki

$$\sum_q d_1^q < 0$$

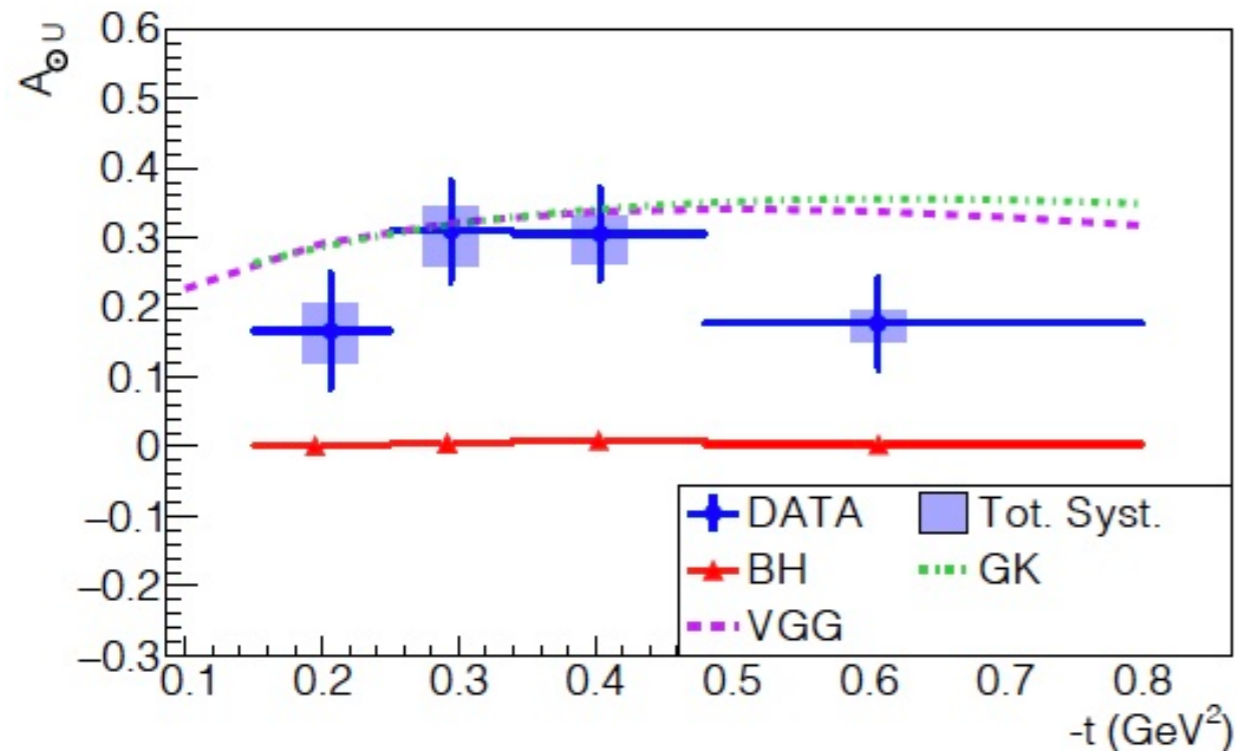
in all model calculations
for a stable proton

Marker in Fig. 3	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV ²	# of flavours	Type
	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data
	0.88 ± 1.69	2.2	2	from experimental data
	-1.59	4	2	<i>t</i> -channel saturated model
	-1.92	4	2	<i>t</i> -channel saturated model
	-4	0.36	3	χ QSM
	-2.35	0.36	2	χ QSM
	-4.48	0.36	2	Skyrme model
	-2.02	2	3	LFWF model
	-4.85	0.36	2	χ QSM
	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\text{MS}}$)
	-2.11 ± 0.27	4	2	lattice QCD (MS)

Timelike Compton scattering

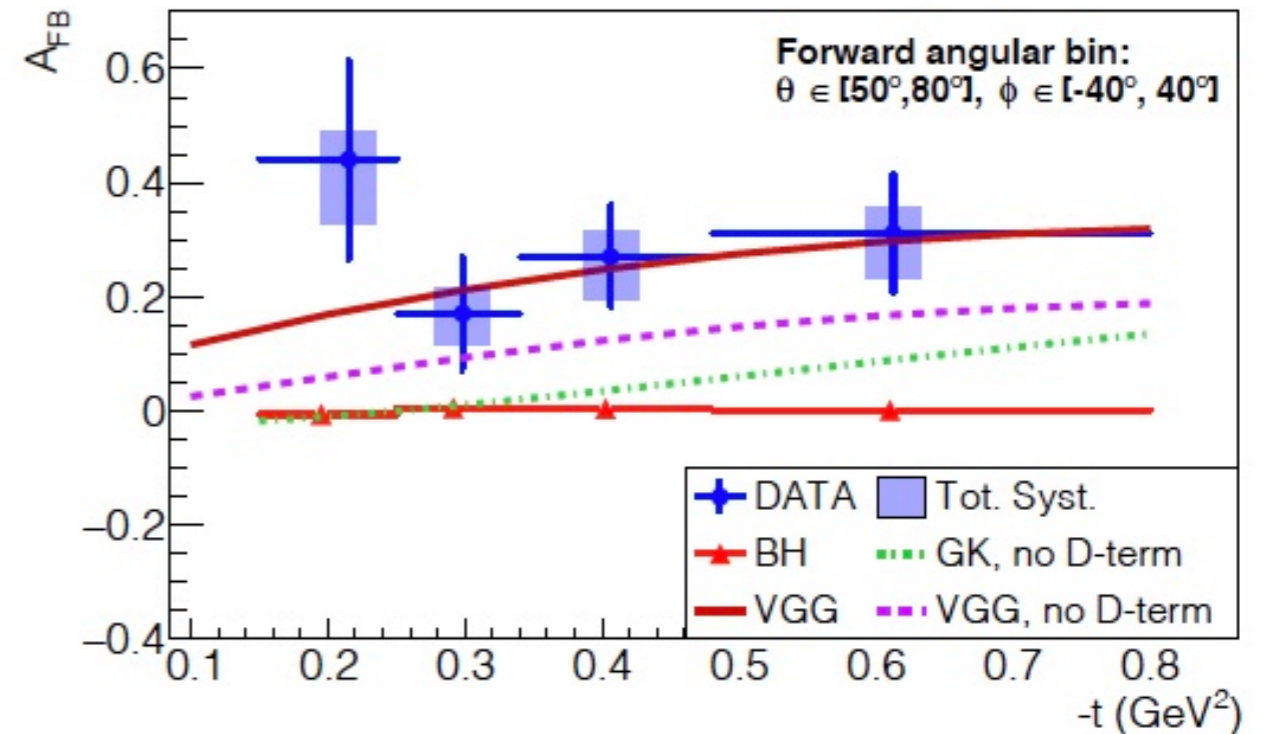
Chatagnon et al. (CLAS12 Coll.), arXiv: 2108.11746

photon polarization asymmetry



access to $\text{Im } \mathcal{H}$

forward-backward asymmetry



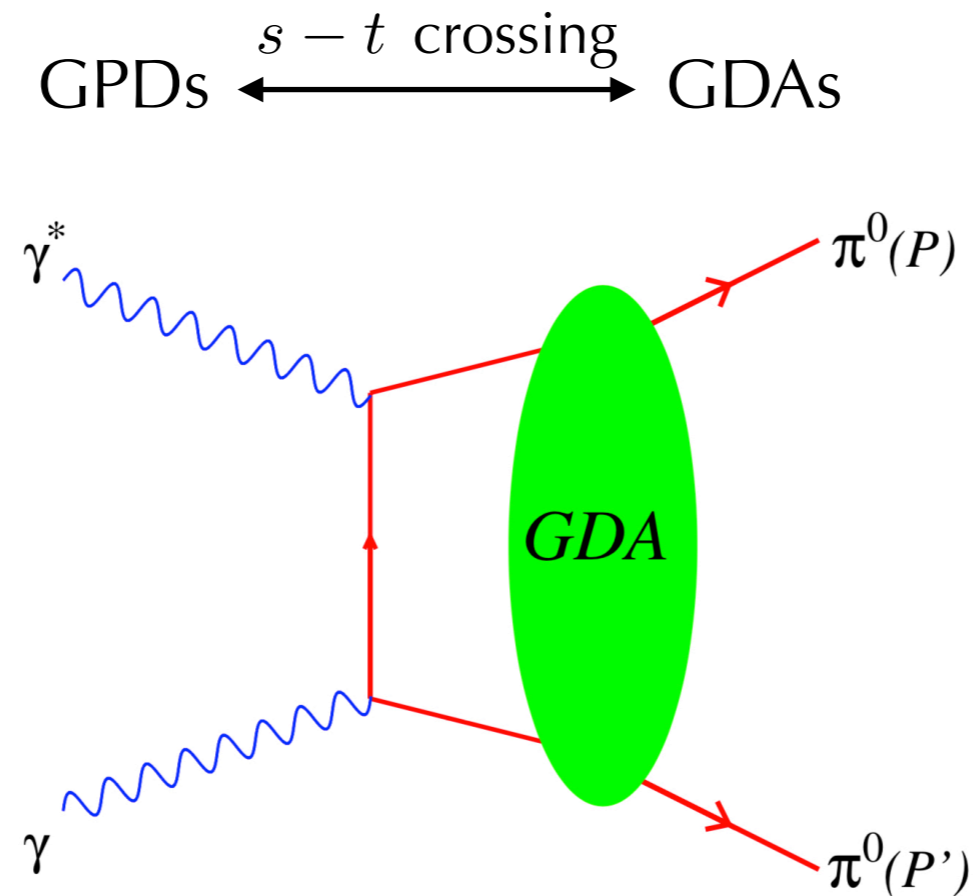
access to $\text{Re } \mathcal{H}$

✓ Test of the universality of GPDs

✓ Further data from JLab12 and future EIC

✓ New promising path towards the extraction of $\text{Re } \mathcal{H}$ and then the D-term

Generalized Distribution Amplitudes (GDAs)



Access form factors of EMT of unstable particle through GDA via $\gamma^*\gamma \rightarrow \pi^0\pi^0$ in e^+e^-

Belle Coll., PRD93 (2016) 032003

Best fit of GDAs to Belle data \rightarrow timelike EMT form factors of the pion

Dispersion relations \rightarrow spacelike EMT form factors

at $\langle Q^2 \rangle = 16.6 \text{ GeV}^2 \rightarrow D_{\pi^0}^Q \approx -0.7$ (compatible with soft pion theorem)

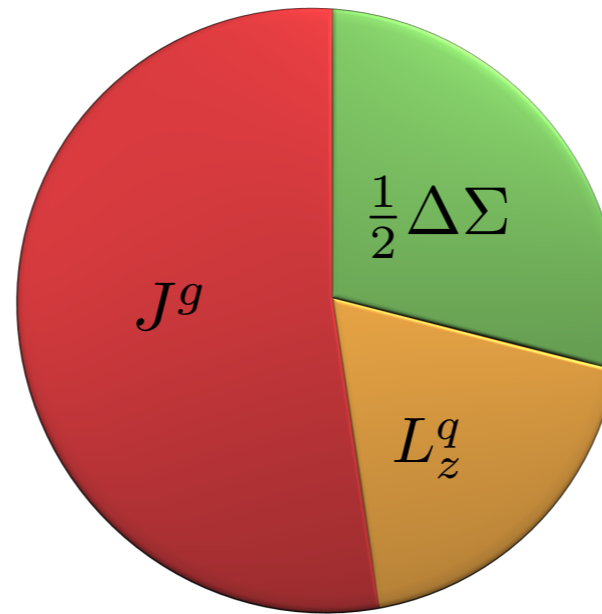
Kumano, Song, Teryaev, PRD97 (2018) 014020

\rightarrow Talk of Teryaev, GPD, Form factors session, Mon. 18;

pion GPDs from Sullivan process at EIC, talk of Morgado Chávex, GPD session, Mon. 18

Ji's Relation

X. Ji, PRL 78 (1997) 610



$$\frac{1}{2} = J^q + J^g$$

$$L_z^q = J^q - \frac{1}{2} \Delta \Sigma$$

$$J^{q,g} = \frac{1}{2} \int_{-1}^1 dx x (H^{q,g}(x, \xi, 0) + E^{q,g}(x, \xi, 0))$$

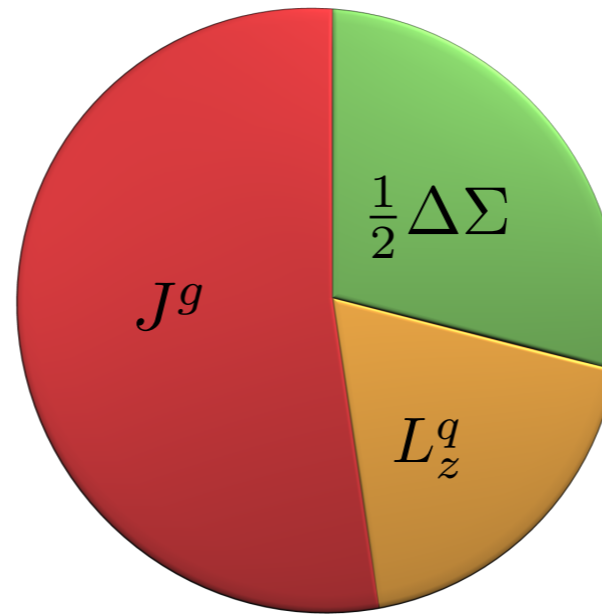
at $\xi = 0$ unpolarized PDF

not directly accessible

- $H(x, \xi, t), E(x, \xi, t)$: twist-2 GPDs

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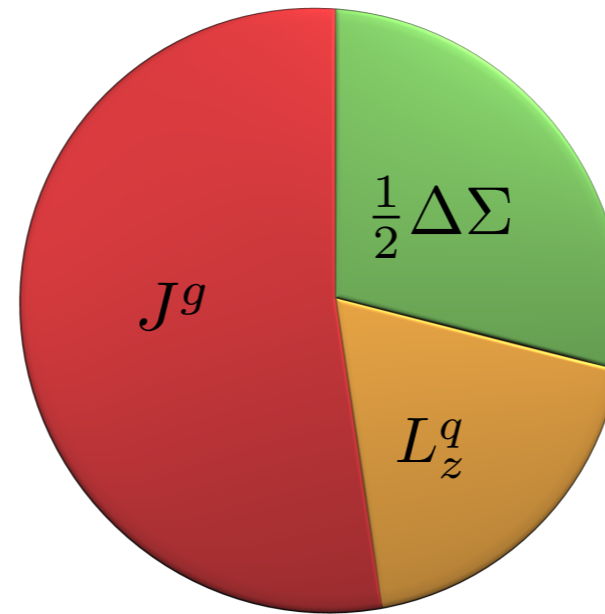
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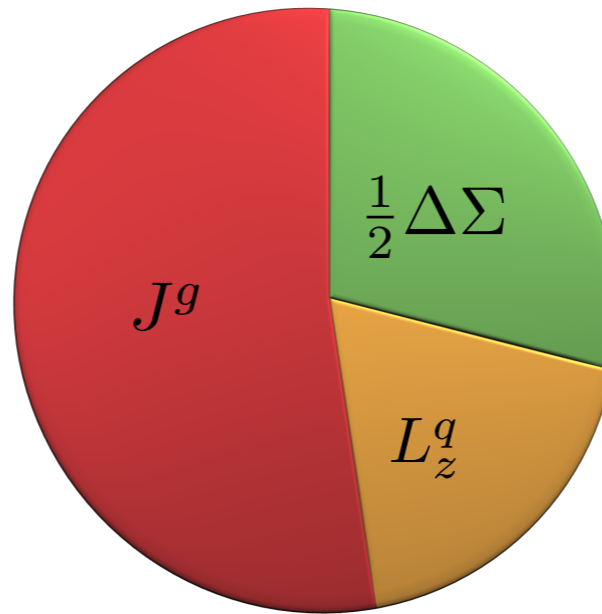
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- $H(x, \xi, t), E(x, \xi, t)$: twist-2 GPDs
- Requires extrapolation to $t=0$
- Requires spanning x at fixed values of ξ ($\xi = 0$ is the most convenient)

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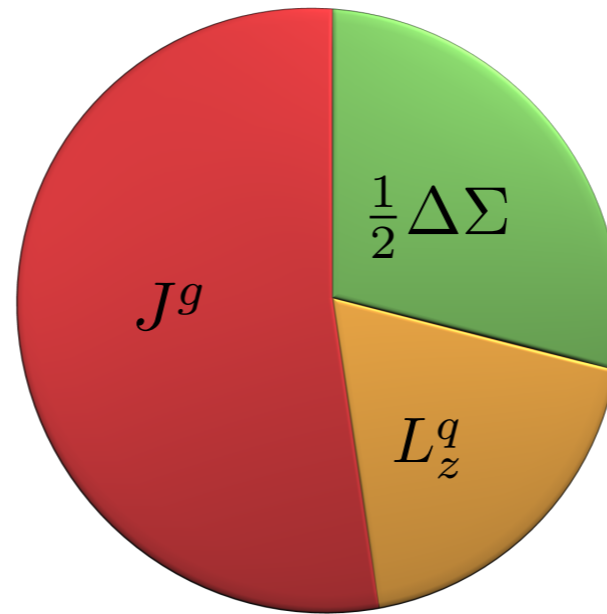
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- Requires extrapolation to $t=0$
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- $J^{q,g}(x) \neq \frac{1}{2}[xH^{q,g}(x, 0, 0) + E^{q,g}(x, 0, 0)] \longrightarrow$ not angular momentum density

Ji's Relation

X. Ji, PRL 78 (1997) 610



$$\frac{1}{2} = J^q + J^g$$

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at $\xi = 0$ unpolarized PDF

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- $H(x, \xi, t), E(x, \xi, t)$: twist-2 GPDs
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- $J^{q,g}(x) \neq \frac{1}{2}[xH^{q,g}(x, 0, 0) + E^{q,g}(x, 0, 0)] \longrightarrow$ not angular momentum density
- OAM can be related to twist-3 GPDs: not simple partonic interpretation, but definition of a gauge-invariant covariant OAM density

Orbital Angular momentum of the proton from available GPD measurements

$$J^{q,g} = \frac{1}{2} \int_{-1}^1 dx x (H^{q,g}(x, \xi, 0) + E^{q,g}(x, \xi, 0)) \quad L^q = J^q - \frac{1}{2} \Delta \Sigma$$

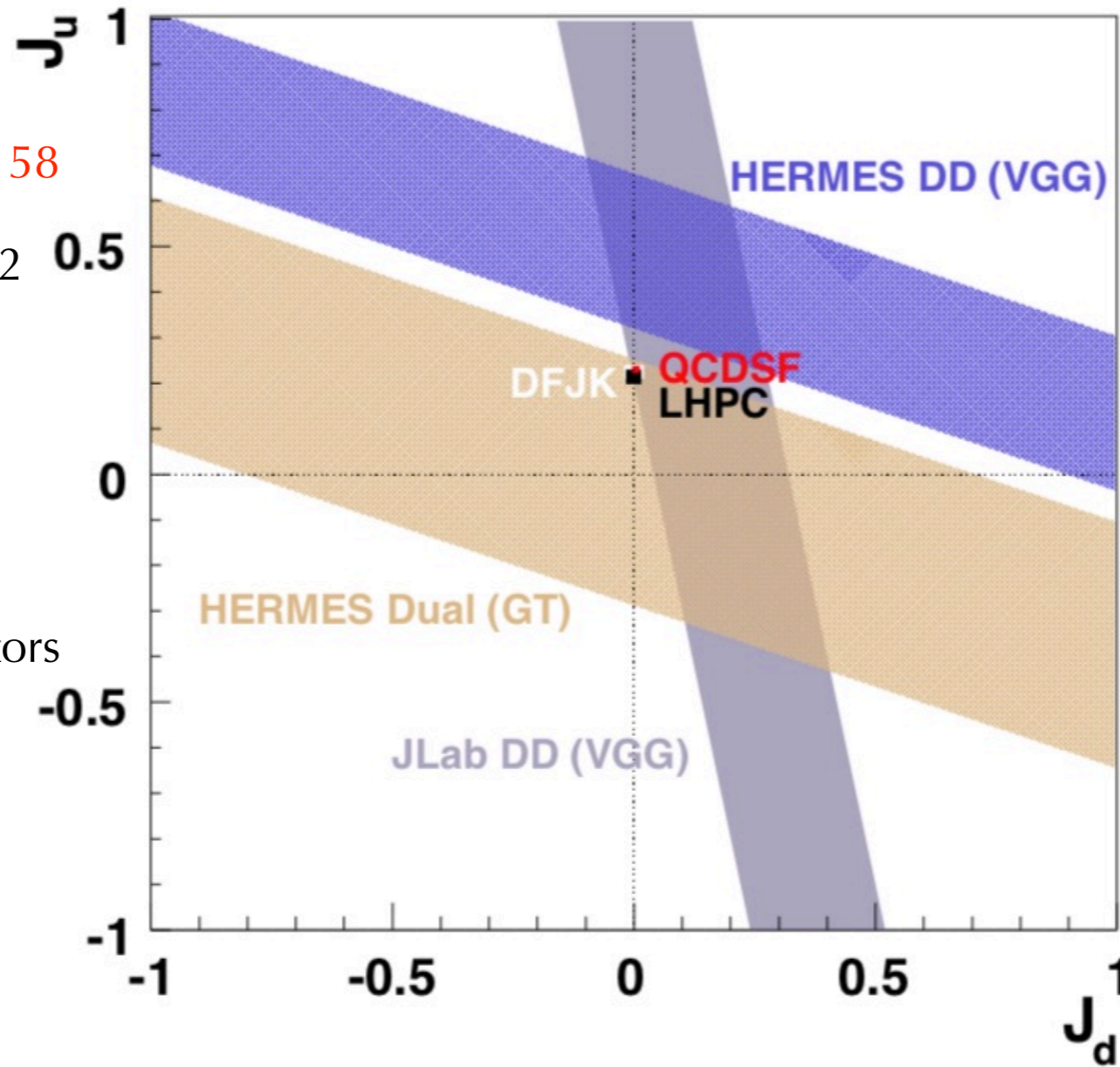
Lattice results

QCDSF: PoS (Lattice 2007) 158

LHPC: PRD77 (2008) 094502

GPDs extracted from form factors

DFJK, EPJC39 (2005) 1



extractions from HERMES data using two different models

JLab Hall A, Phys. Rev. Lett. 99 (2007) 242501

Hermes Coll., JHEP 06 (2008) 066

Improved accuracy with JLab12 and future EIC measurements!

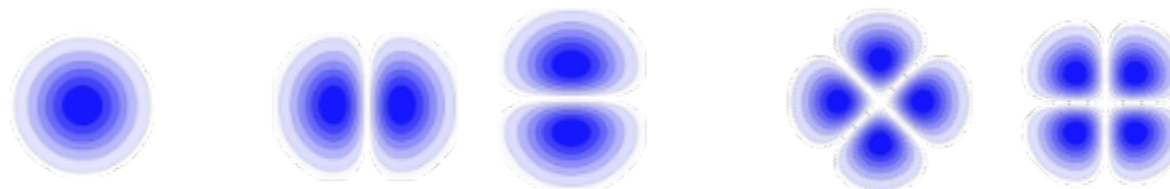
Key information from Transverse Momentum Dependent PDFs

- Complete momentum spectrum of single particle
- Transverse momentum size as function of x (3D map)
- Spin-Spin and Spin-Orbit Correlations of partons
- Information on parton orbital angular momentum (no direct model-independent relation)

quark polarization

nucleon polariz.	TMD	U	L	T
	U	f_1		h_1^\perp
	L		g_{1L}	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

*similar classification for gluon TMDs



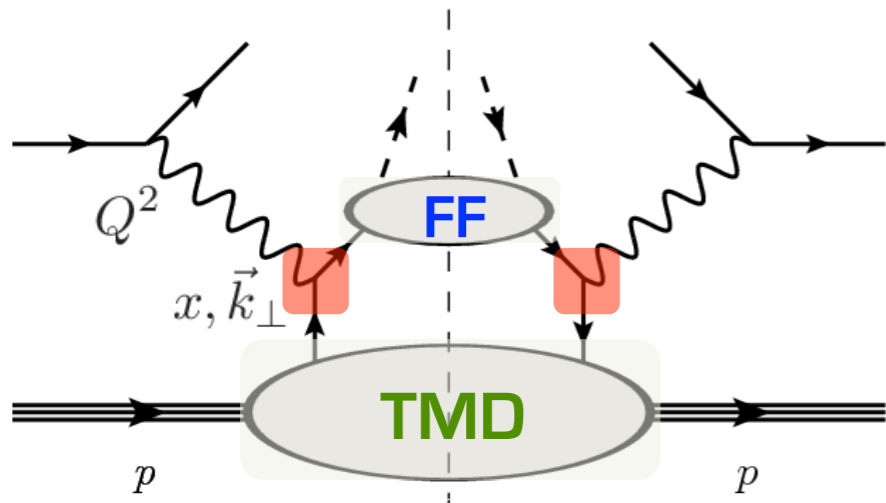
unpolarized
target and partons

deformation due to
spin-spin and spin-orbit correlations

How to measure TMDs

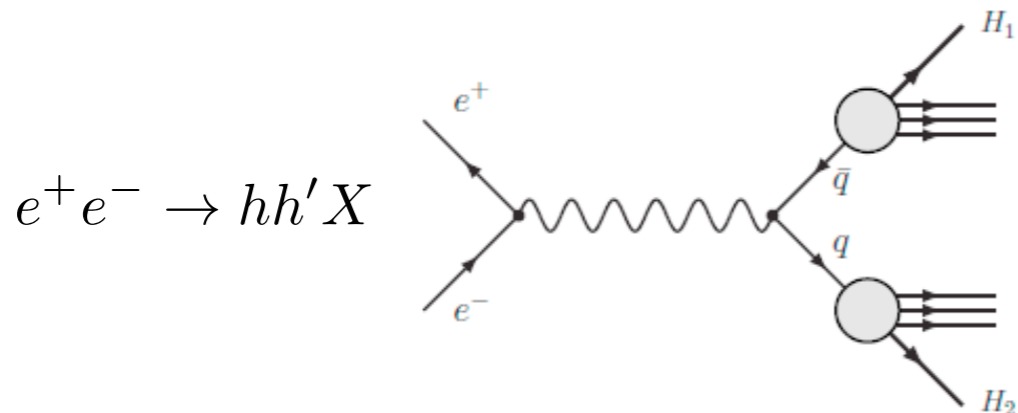
SIDIS

$$\ell(l) + N(P) \rightarrow \ell(l') + h(P_h) + X$$



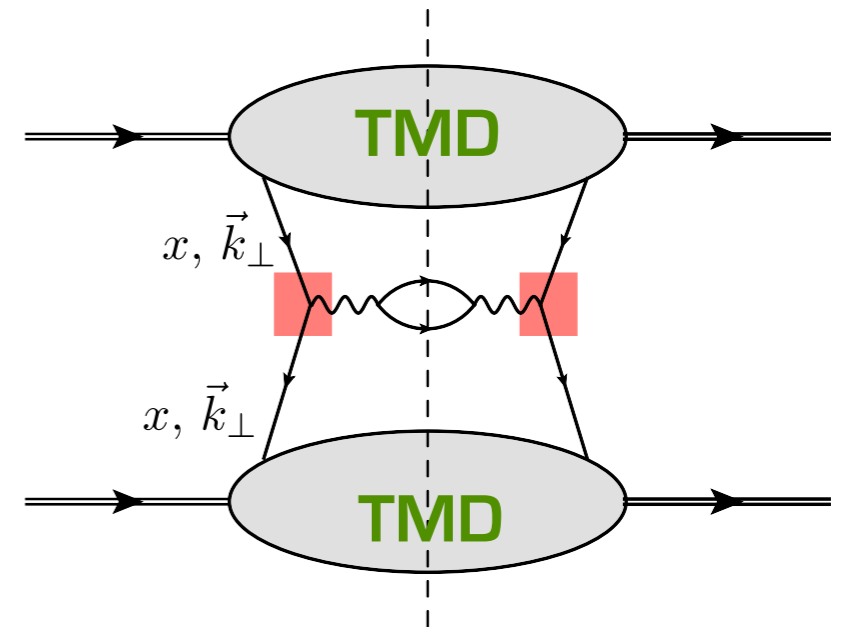
$$d\sigma \sim \sum \text{TMD}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard} \otimes \text{FF}(z, \vec{p}_\perp) + \mathcal{O}\left(\frac{P_T}{Q}\right)$$

Fragmentation Functions



Drell-Yan

$$h(P_1) + h(P_2) \rightarrow \ell^+(l) + \ell^-(l')$$



$$d\sigma \sim \sum \text{TMD}(x, \vec{k}_\perp) \otimes \overline{\text{TMD}}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard}$$

✓ Factorization

✓ Universality

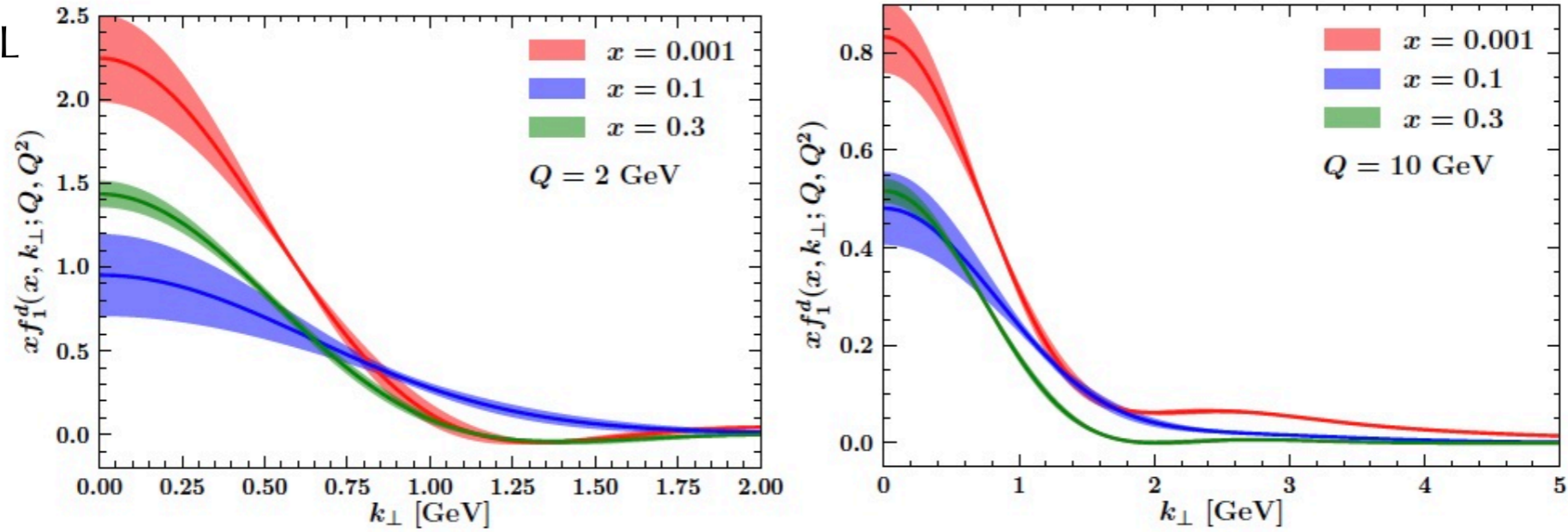
Quark unpolarized TMD extractions

	Framework	HERMES	COMPASS	DY	Z Production	N of points
Pavia 2016 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059
SV 2017 arXiv:1706.01473	NNLL	✗	✗	✓	✓	309
BSV 2019 arXiv:1902.08474	NNLL	✗	✗	✓	✓	457
Pavia 2019 arXiv:1912.07550	NNNLL	✗	✗	✓	✓	353
SV 2020 arXiv:1912.06532	NNNLL	✓	✓	✓	✓	1039
MAP 2022 <i>in progress</i>	NNNLL	✓	✓	✓	✓	>1500

→ *Talk of M. Cerutti, TMD session, Tue. 19.*

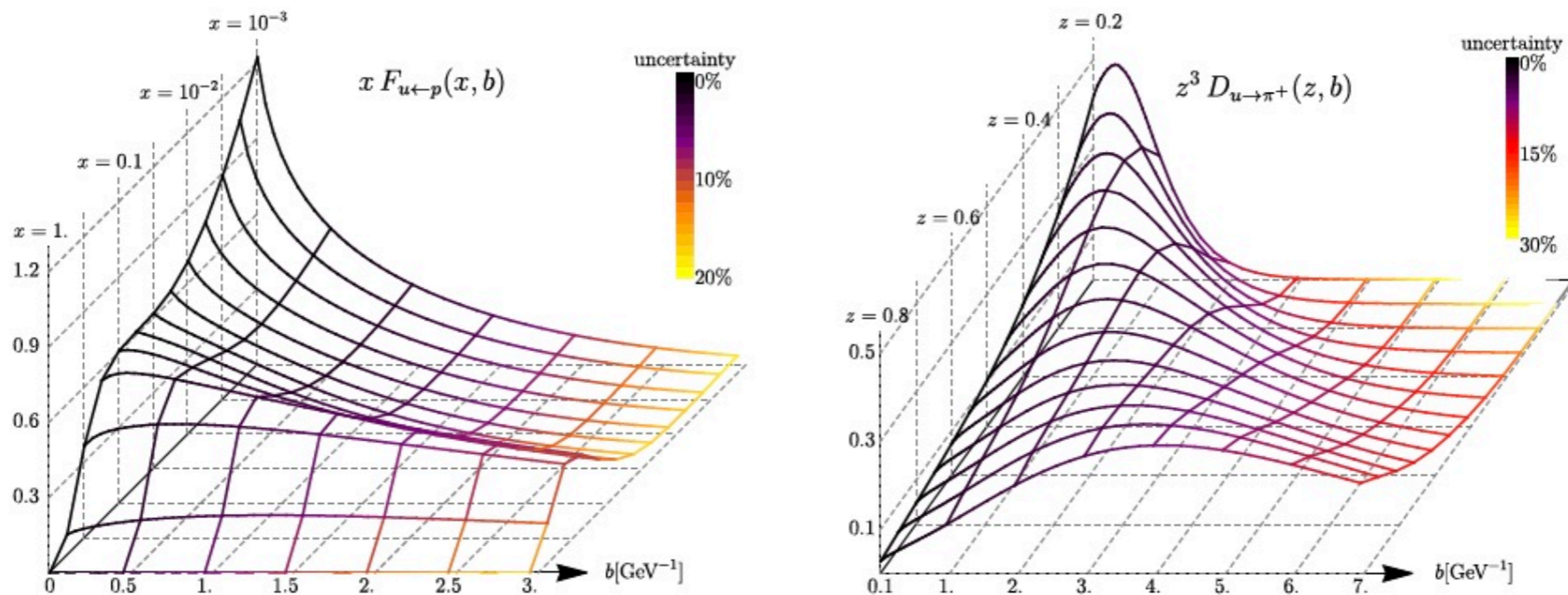
Quark unpolarized TMD extractions $f_1(x, \vec{k}_\perp)$

DY data at NNNLL



Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, JHEP 07 (2020) 117

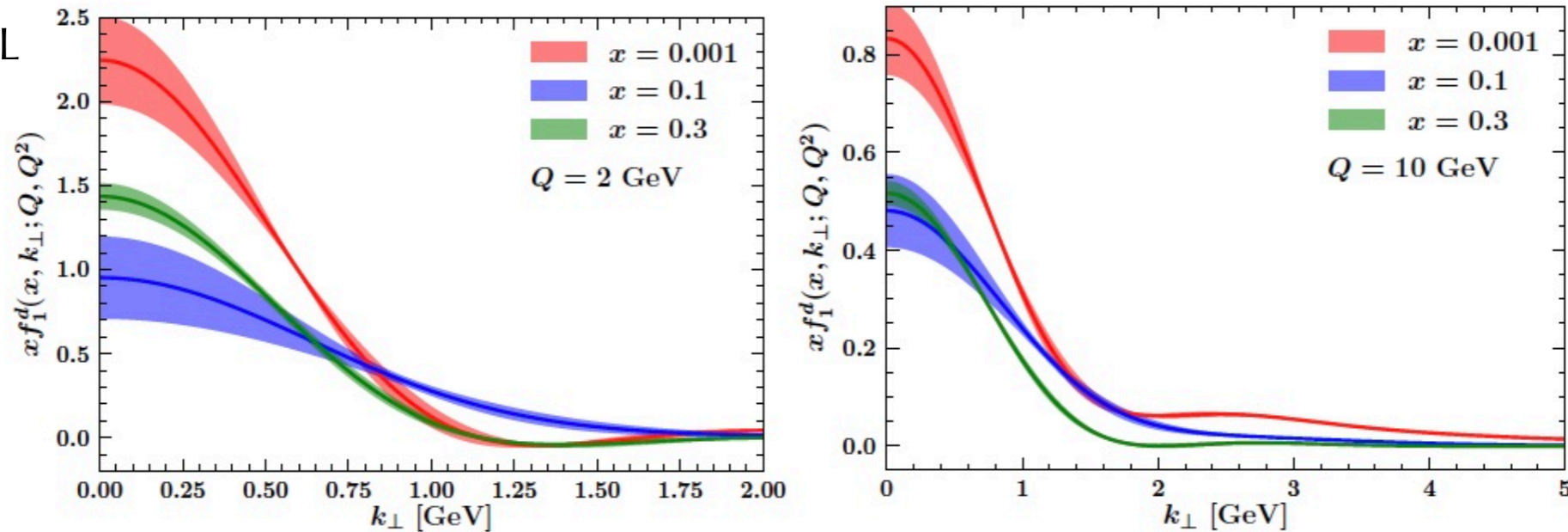
DY+ SIDIS data at NNNLL



Scimemi, Vladimirov, JHEP 06 (2020) 137

Quark unpolarized TMD extractions $f_1(x, \vec{k}_\perp)$

DY data at NNNLL

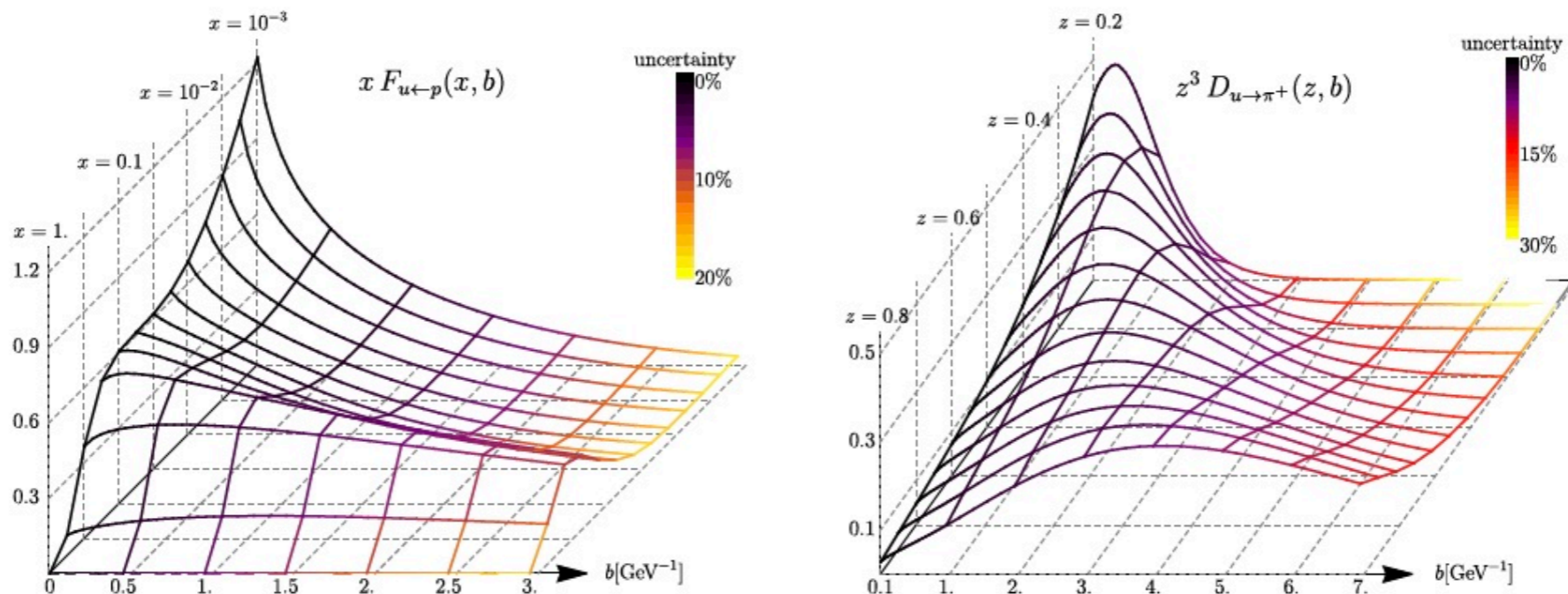


Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, JHEP 07 (2020) 117

Open issues:

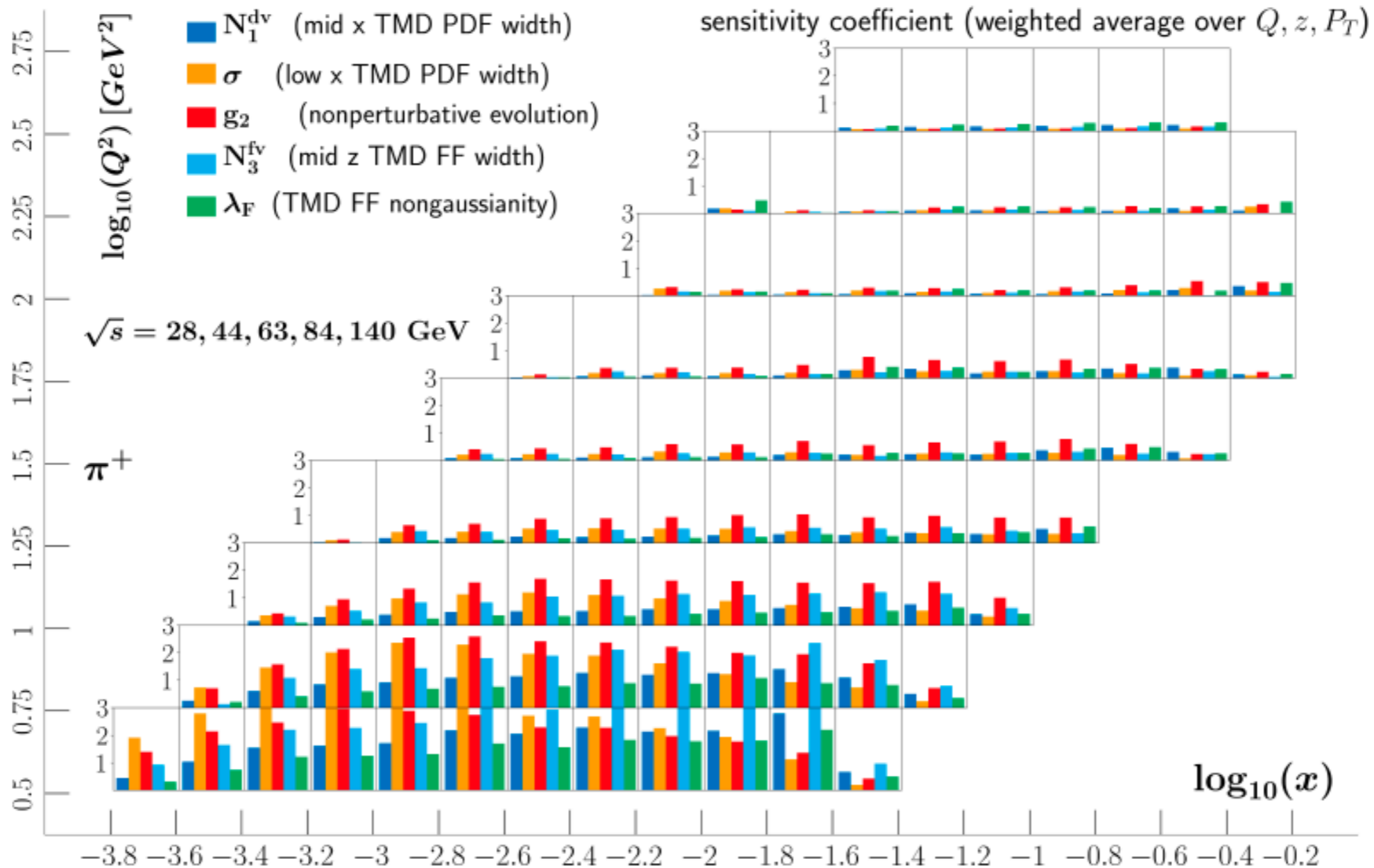
- Flavor dependence and more flexible functional forms
- Different choices in implementation of TMD formalism
- More data needed to test the formalism and functional form of parametrizations
- Improvements on the knowledge of the fragmentation functions

DY+ SIDIS data at NNNLL



Scimemi, Vladimirov, JHEP 06 (2020) 137

Foreseen EIC impact on unp. TMDs: SIDIS measurements

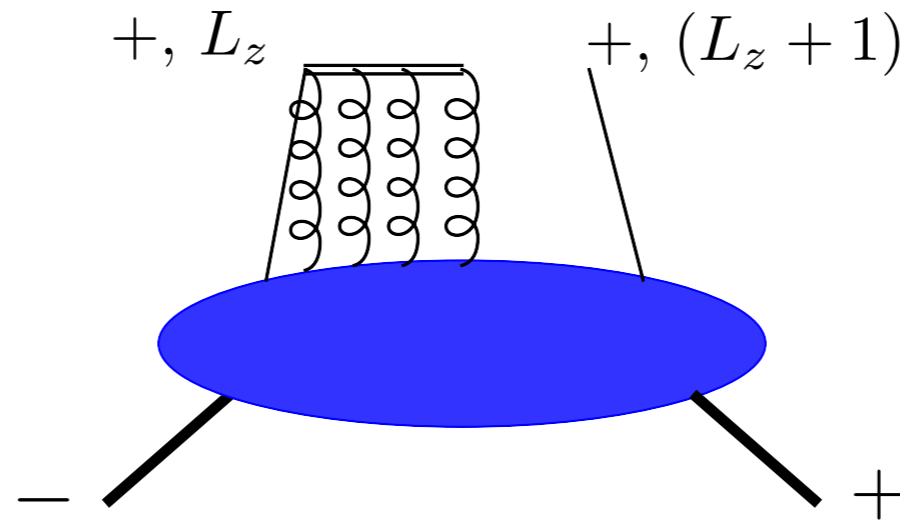


Sensitivity coefficients: measure of the correlation between fit parameters and measurable quantities at EIC

Sivers function

$$f_{1T}^\perp = \text{---} \left(\begin{array}{c} \circ \\ \downarrow \\ \circ \end{array} \right) \text{---} - \text{---} \left(\begin{array}{c} \circ \\ \uparrow \\ \circ \end{array} \right) \text{---}$$

unpolarized quarks in \perp pol. nucleon



the helicity mismatch requires orbital angular momentum (OAM)

non trivial correlation between quark OAM and nucleon transverse spin

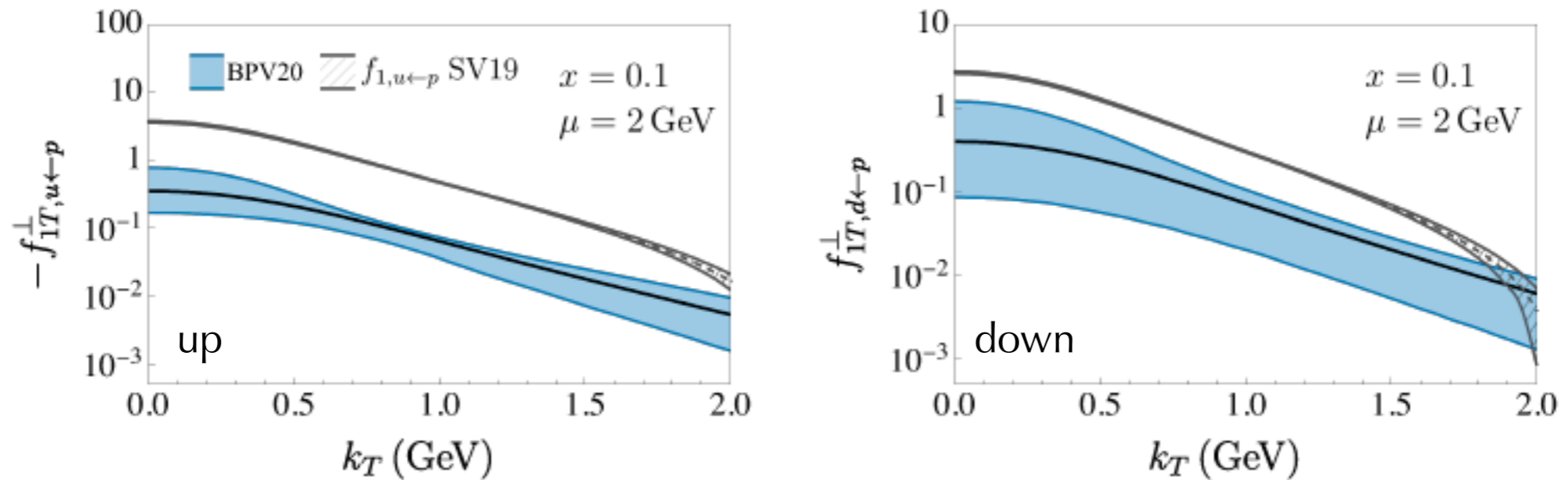
no counterpart in IPD and PDF case

non-zero ONLY with final-state interaction

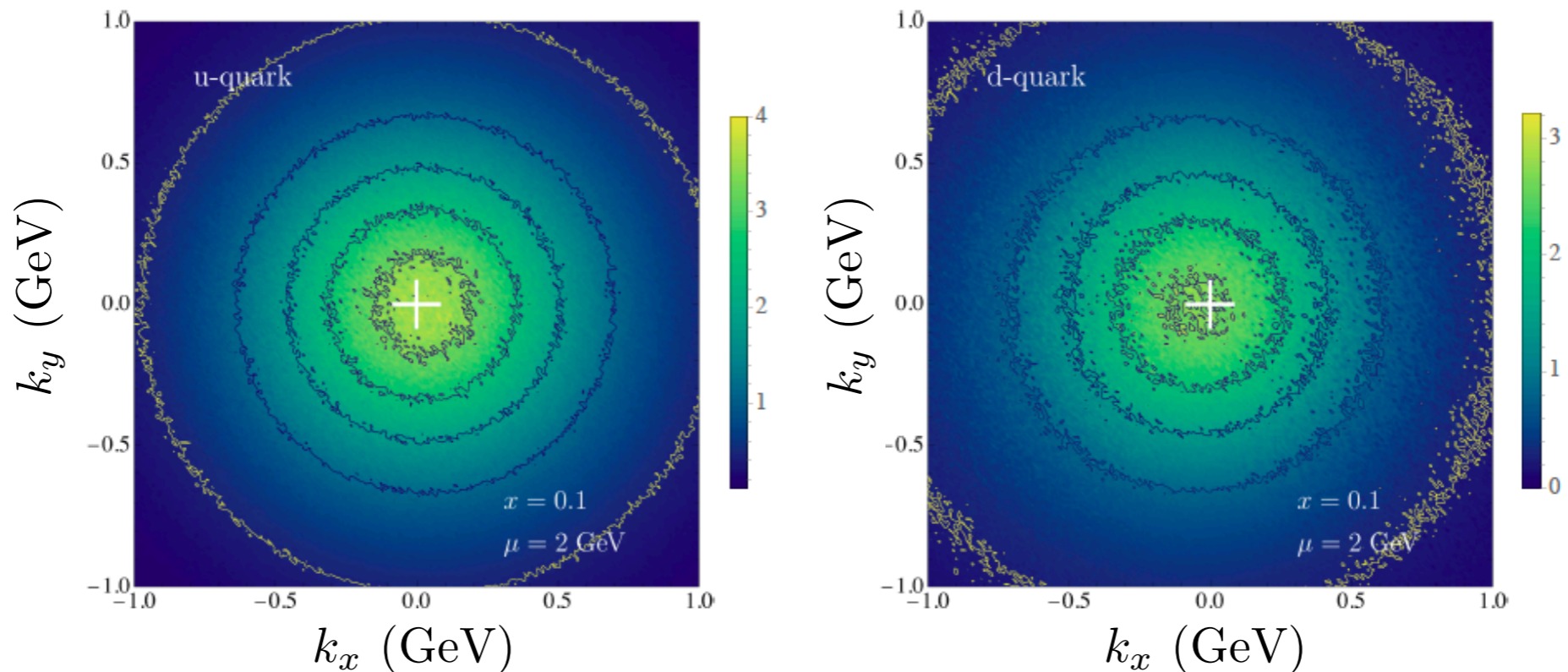
$$f_{1T}^{\text{SIDIS}}(x, k_\perp) = -f_{1T}^{\text{DY}}(x, k_\perp)$$

first hints of sign change from STAR and COMPASS data

Global fit to SIDIS, DY, W^\pm/Z boson production



$$\rho_{UT_y}(x, \vec{k}_\perp, S_y) = f_1(x, k_\perp) - \frac{k_x}{M} f_{1T}^\perp(x, k_\perp)$$



TMDs and OAM

TMDs are sensitive to various aspects of OAM

NO model-independent relations between TMDs and OAM

→ any quantitative statement must rely on model assumptions

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Pretzelosity and OAM

$$\mathcal{L}_z = - \int dx d^2\vec{k}_\perp \frac{k_\perp^2}{2M^2} h_{1T}^\perp(x, k_\perp^2)$$

- valid in quark models with spherical symmetry in the rest frame (bag model, light-front quark models)

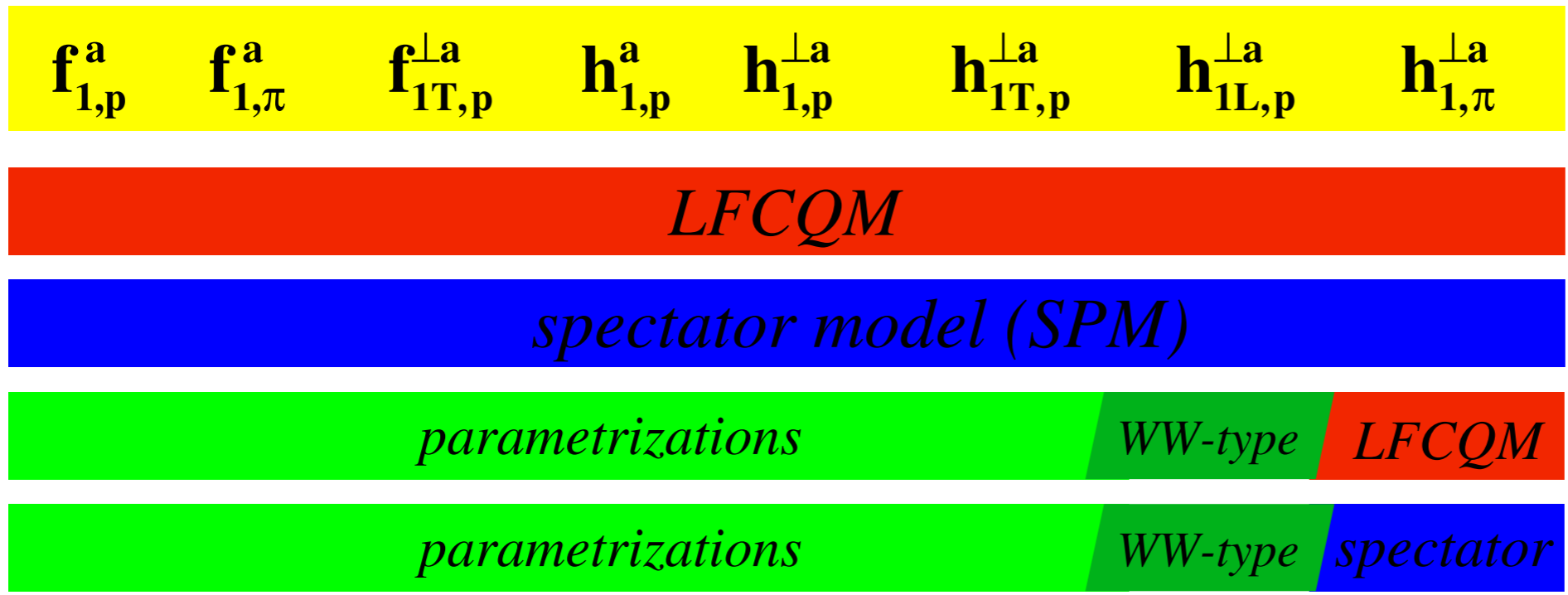
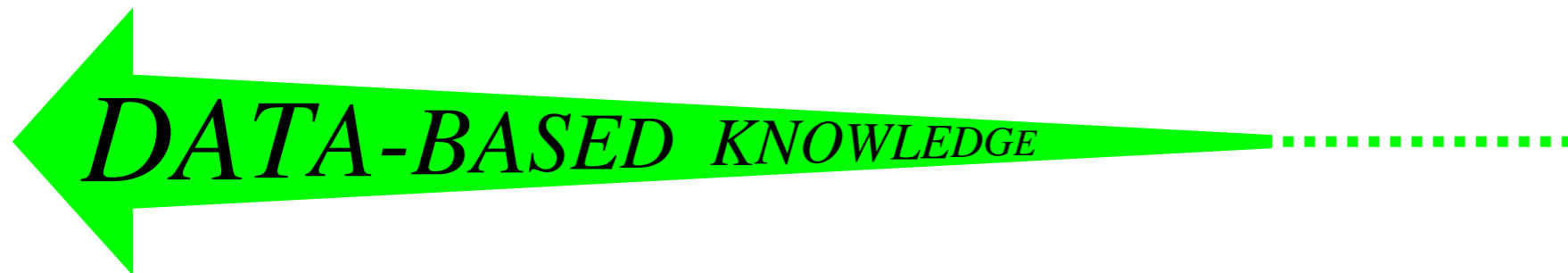
She, Zhu, Ma, PRD79, 2009; Avakian, Efremov, Schweitzer, Yuan, PRD81, 2010; Lorcé and Pasquini, PLB710, 2012

Pion induced Drell-Yan

$$\pi^- + \vec{p} \rightarrow \ell^+ \ell^- + X$$

New insights in the partonic structure of **both** the pion and the nucleon

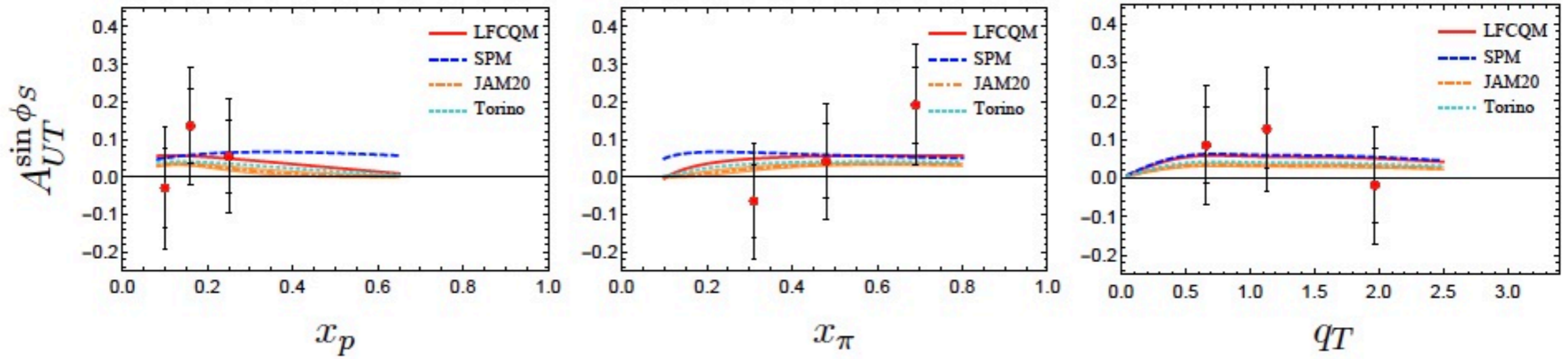
Ongoing analysis of COMPASS DY data \longrightarrow *Talk of A. Chumakov, TMD session, Mon. 18*



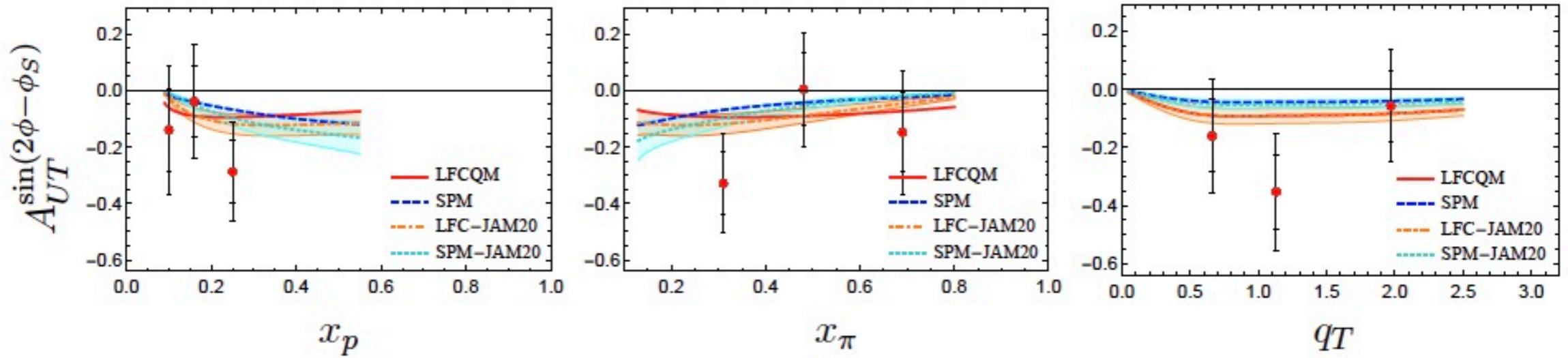
1. *LFCQM all*
2. *spectator all*
3. *LFCQM hybrid*
4. *spectator hybrid*

Complete description of polarized DY at leading twist using TMD evolution at NLL accuracy

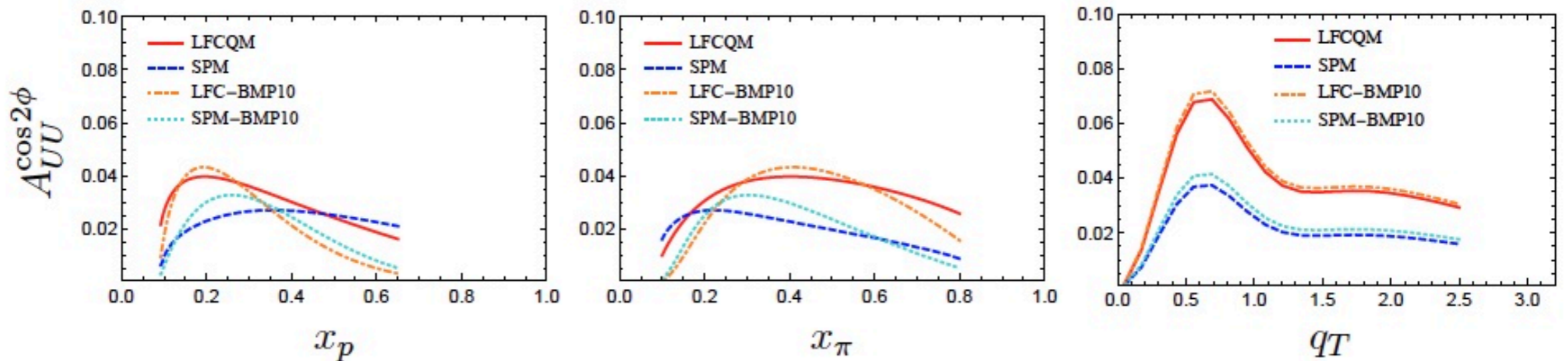
$$A_{UT}^{\sin \phi_s} \propto f_{1T}^{\perp u}(x_p) > 0 \implies f_{1T}^{\perp u}(x_p) > 0$$



$$A_{UT}^{\sin(2\phi - \phi_s)} \propto -h_{1,\pi^-}^{\perp(1)\bar{u}}(x_\pi) h_{1,p}^u(x_p) < 0 \implies h_{1,\pi^-}^{\perp\bar{u}} > 0$$



$$A_{UU}^{\cos 2\phi} \propto h_{1,\pi^-}^{\perp(1)\bar{u}}(x_\pi) h_{1,p}^{\perp(1)u}(x_p) > 0 \implies h_{1,p}^{\perp u}(x_p) > 0$$



→ Consistent with ongoing analysis of COMPASS data, Talk of A. Chumakov, TMD session, Mon. 18

Library and Plotting tools for collinear parton distributions

LHAPDF

lhpdf.hepforge.org



APFEL++

github.com/vbertone/apfelxx
apfel.mi.infn.it

Dedicated Softwares to study GPDs



PARtonic
Tomography
Of
Nucleon
Software



GeParD

not yet public

Dedicated software to study and fit TMDs

arTeMiDe

teorica.fis.ucm.es/artemide

TMD lib and TMD Plotter

tmdlib.hepforge.org

NangaParbat

MapCollaboration/NangaParbat

**Efforts to combine different inputs to understand
PDFs, TMDs and GPDs in an unified framework**