^{6}He+p反応を通した共鳴状態の解析

小川翔也　松本琢磨

九州大学

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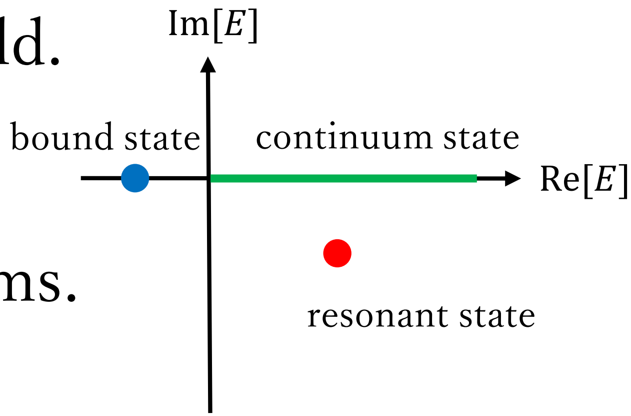
“理研-九大ジョイントワークショップ「数理が紡ぐ素粒子・原子核・宇宙」” @ 九大

Resonant state

- ✓ Resonant states locate above the threshold.
- ✓ The pole of S matrix

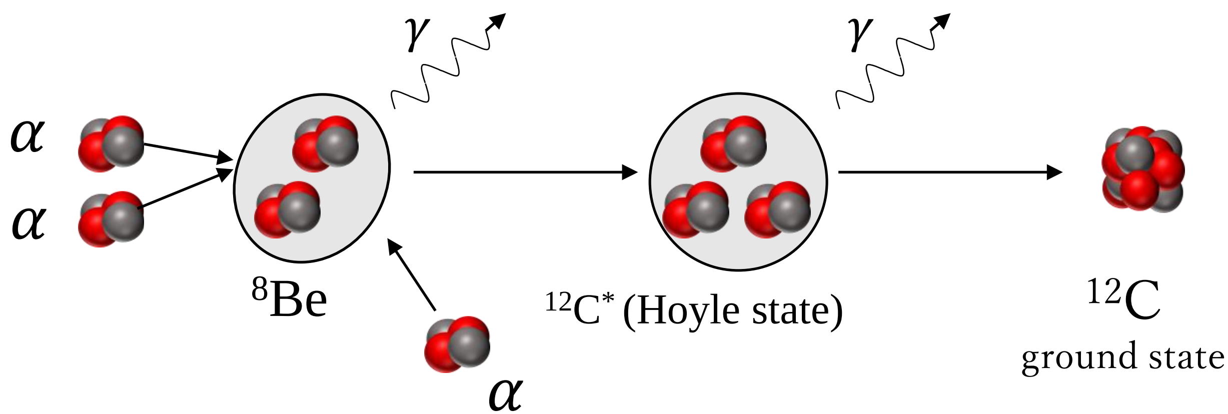


Resonant states appear in many-body systems.

ex) nucleons, quaks, atomics ...

Relationship with nucleosynthesis

ex) The resonant states of ^{12}C, 0_2^+ (Hoyle state)



triple-alpha process

Nuclear physics

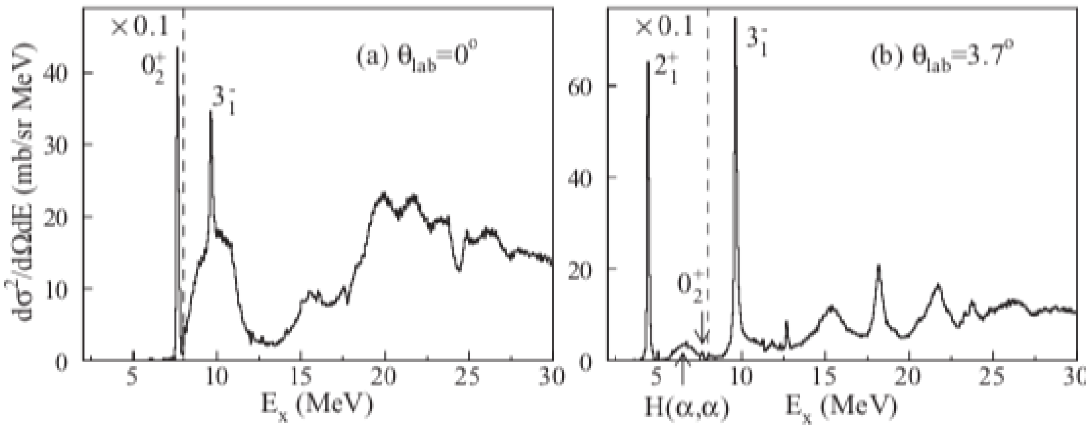
ex) Resonant states of ^{12}C

Reaction

- (p, p') or (α, α') reactions
- Energy spectrum

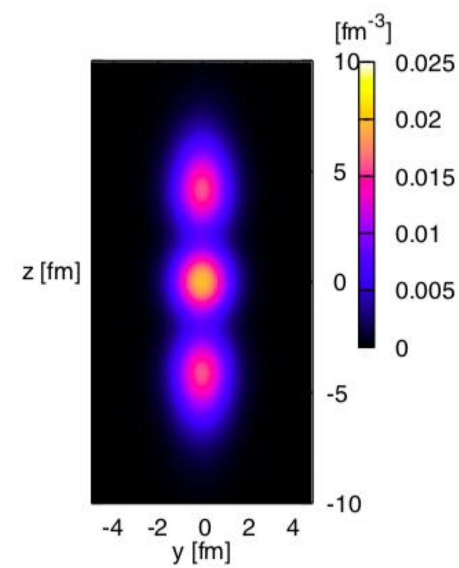
α-^{12}C散乱 @ $E_\alpha = 386$ MeV

M. Itoh *et al.*, Phys. Rev. C **84**, 054308 (2011).



Structure

- The 0_4^+ is considered as a nucleus having the linear-chain configuration of 3α.



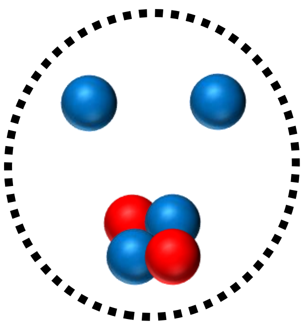
Y. Funaki,
Phys. Rev. C **94**, 024344 (2016)

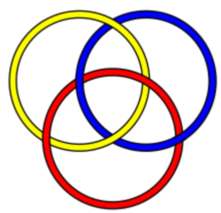
Unstable nuclei

Recently, resonant states of nuclei near neutron dripline have been investigated intensively.

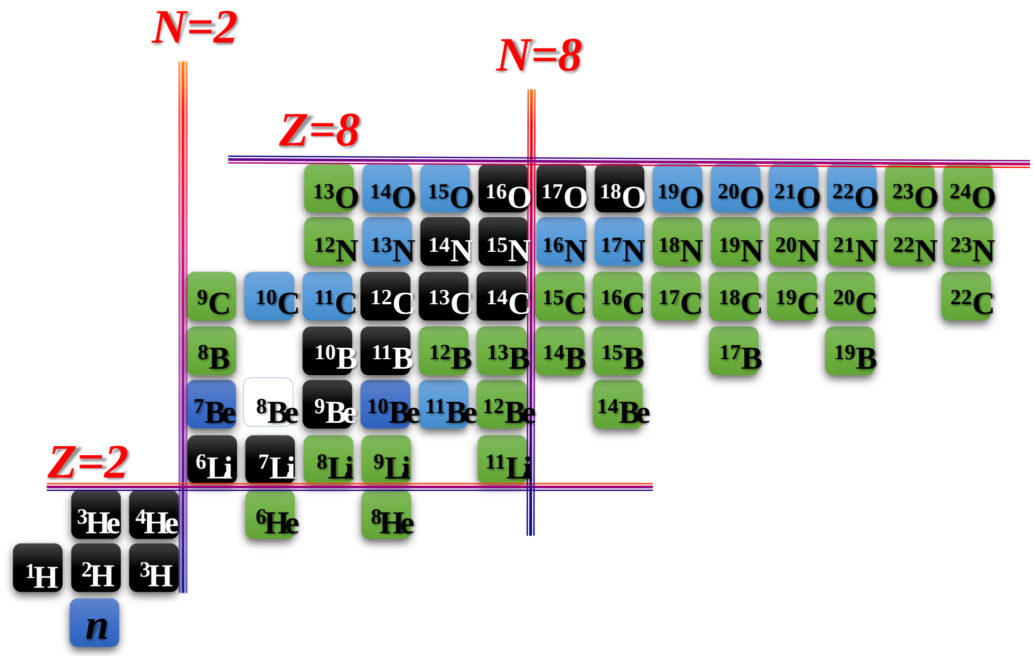
^{6}He

- two neutron halo nuclei
- Borromean nuclei





(from Wikipedia)
Borromean rings



The resonant states of ^{6}He is analyzed via (p, p') reactions with inverse kinematics.

A. Lagoyannis *et al.*, Phys. Lett. B **518** (2001), 27.
B. S. V. Stepantsov, *et al.*, Phys. Lett. B **542** (2002), 35.

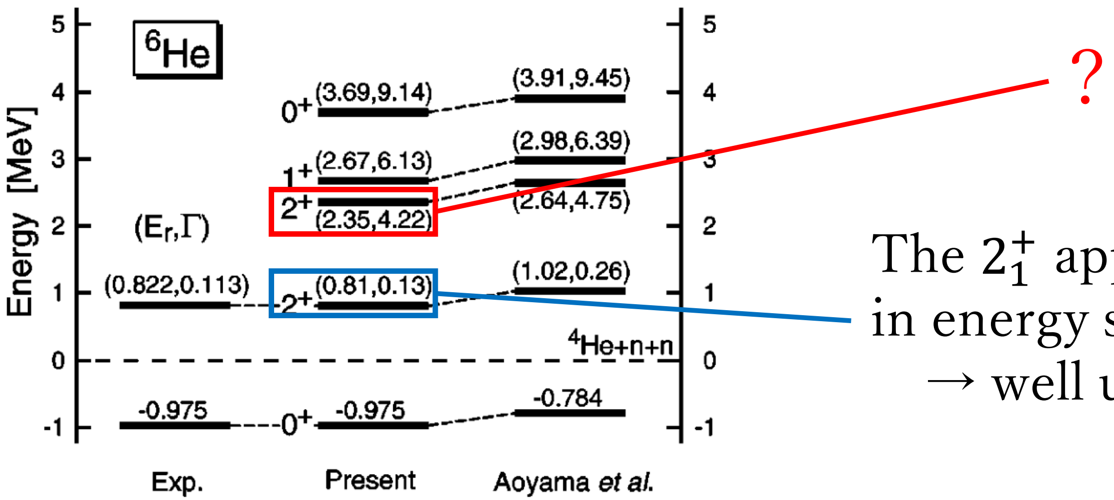
Previous study on ^{6}He

✓ The experimental data of (p, p') are analyzed by a calculation without coupling effects of between various states.

✓ The 2_2^+ state have been discussed by structural calculation.

- This states is considered as the next lower state to the 2_1^+.

T. Myo, *et al.*, Phys. Rev. C **63** (2001), 054313



The 2_1^+ appears as a sharp peak in energy spectrum.
→ well understood.

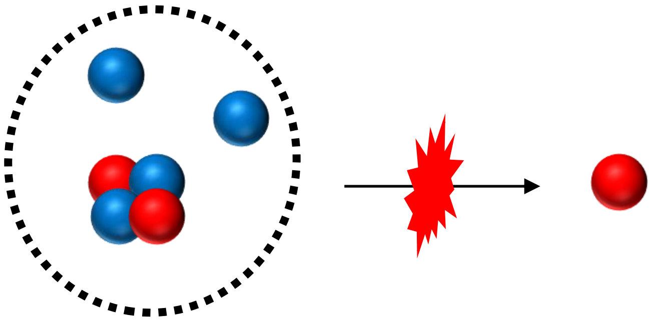
It is not clear that how the 2_2^+ effects on cross sections.

Purpose and Strategy

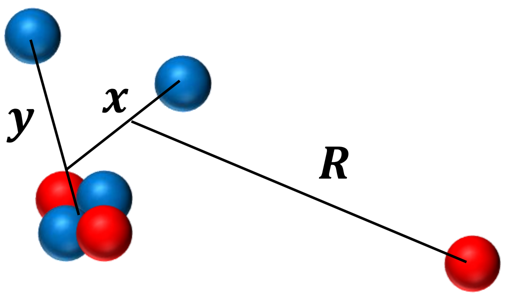
We analyze the effects from 2_2^+ on the observables via ^{6}He(p, p') reactions.

- Resonant state
  - ✓ ^{6}He is described as $\alpha+n+n$ system.
  - ✓ Complex Scaling Method (CSM)

- Analysis of ^{6}He(p, p') reactions
  - ✓ The ^{6}He$+p$ reaction is the $\alpha+n+n+p$ four-body reaction.
  - ✓ Continuum-Discretized Coupled Channels method



Continuum-Discretized Coupled Channels method (CDCC)



$\xi = \{x, y\}$

Schrödinger equation

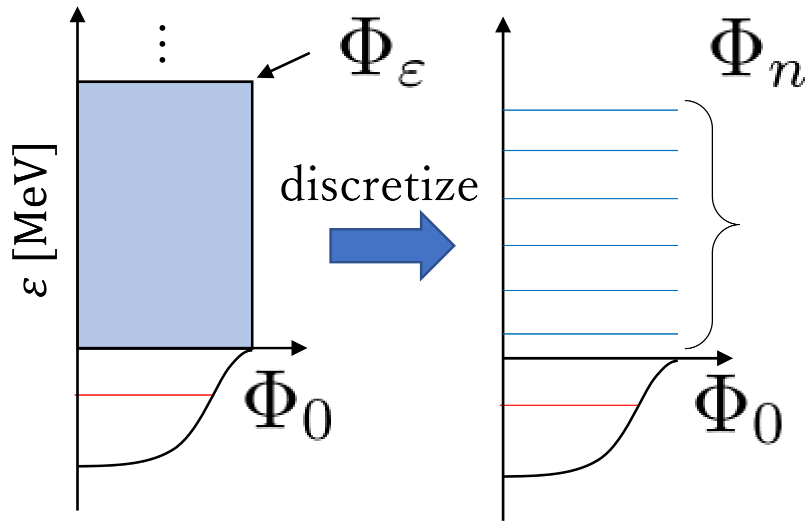
$$\left[K_R + \sum_i v_i + V_C + h_{^6\mathrm{He}} - E\right]\Psi(\boldsymbol{\xi}, \boldsymbol{R}) = 0$$

Continuum states of ^{6}He Φ_ε are discretized.

$$\Psi(\boldsymbol{\xi}, \boldsymbol{R}) = \Phi_0(\boldsymbol{\xi})\psi_0(\boldsymbol{R}) + \int d\epsilon \Phi_\epsilon(\boldsymbol{\xi})\psi_\epsilon(\boldsymbol{R})$$

bound discretized-continuum

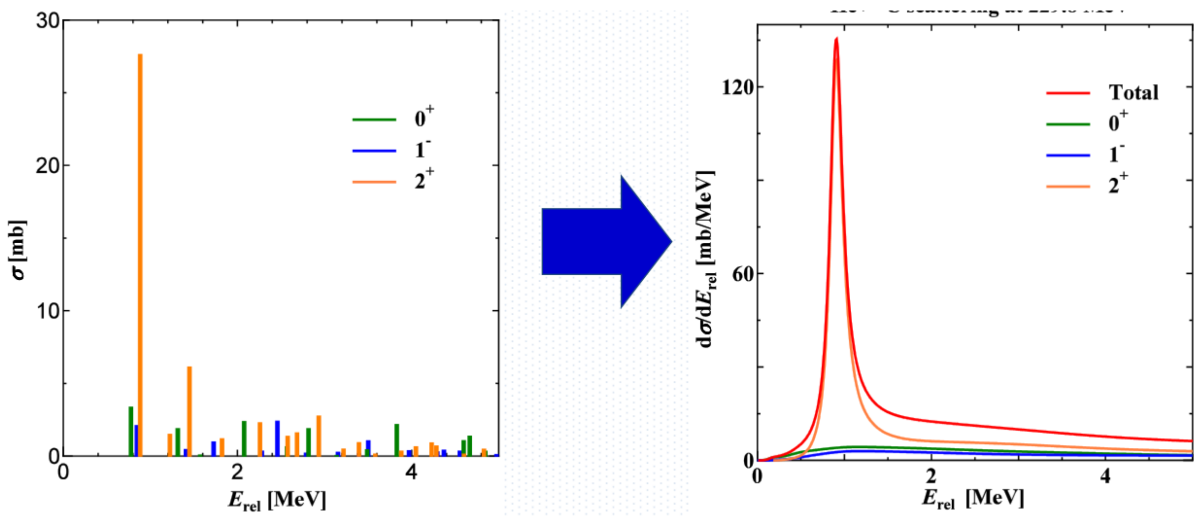
$$\Psi(\boldsymbol{\xi}, \boldsymbol{R}) \approx \boxed{\Phi_0(\boldsymbol{\xi})}\psi_0(\boldsymbol{R}) + \sum_i^{i_{\max}} \boxed{\Phi_n(\boldsymbol{\xi})}\psi_n(\boldsymbol{R})$$



In the CDCC, we solve the coupled-channels equation considering the couplings between the discretized states.

Smoothing factor

Obtained cross sections are discretized in eigen energies.



- CDCC + CSM T. Matsumoto, *et al.,* Phys. Rev. C **82** (2010), 051602(R)

$$\frac{d^2\sigma}{d\varepsilon d\Omega} = \frac{1}{\pi} \sum_{\nu} \sum_{i,j} T_i^{\mathrm{CDCC}\dagger} \underbrace{\frac{\langle \Phi_i | U(-\theta) | \Phi_\nu^\theta \rangle \langle \tilde{\Phi}_\nu^\theta | U(\theta) | \Phi_j \rangle}{\varepsilon - \varepsilon_\nu^\theta}}_{\text{obtained by CSM}} T_j^{\mathrm{CDCC}}$$

T_i^{CDCC} : T matrix calculated with CDCC

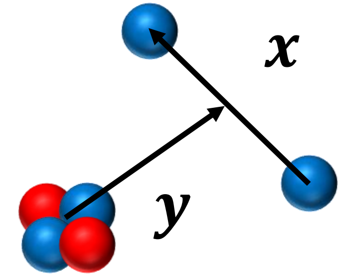
We can separate the part of resonant states from $d^2\sigma/d\varepsilon d\Omega$.

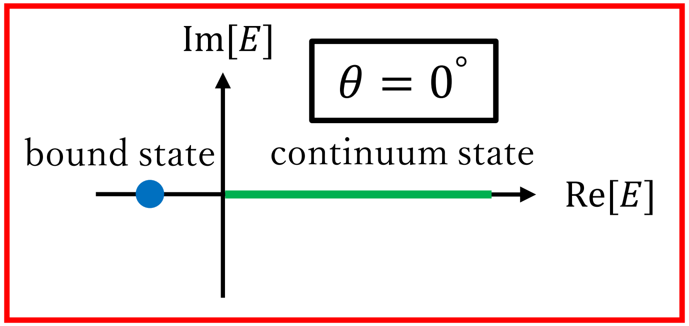
➡ We can evaluate the contribution from resonant states to the cross section.

Complex Scaling Method (CSM)

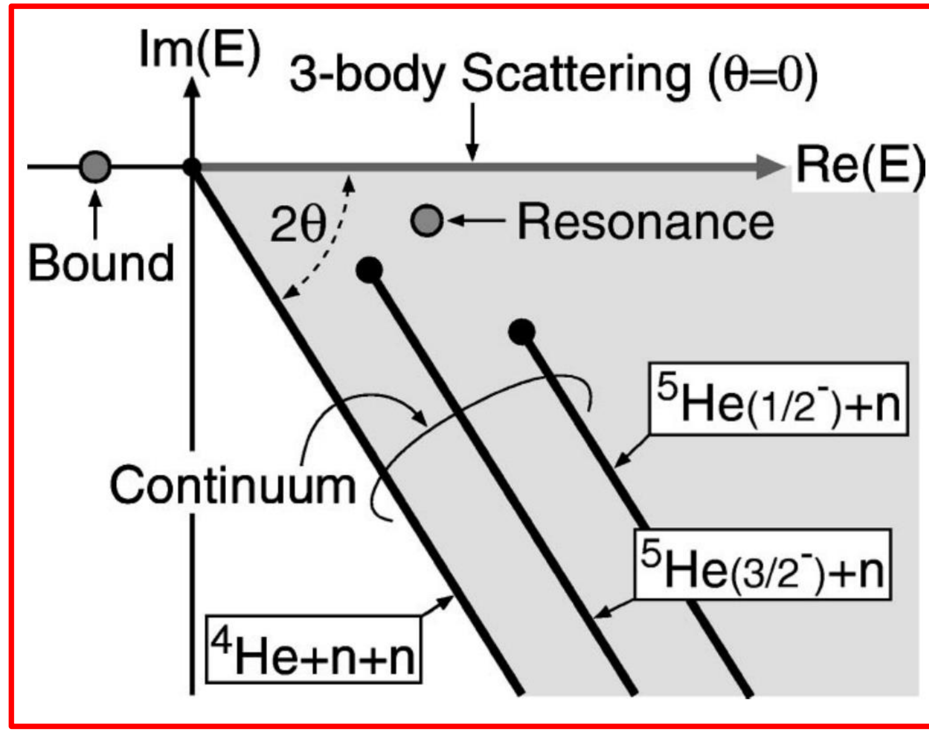
We consider the transformation as followings.

$$U(\theta)\boldsymbol{x}U^{-1}(\theta) = \boldsymbol{x}e^{i\theta}, \quad U(\theta)\boldsymbol{y}U^{-1}(\theta) = \boldsymbol{y}e^{i\theta}$$





$\theta \neq 0^\circ$



T. Myo, *et al.*, Phys Rev C **63** (2001), 054313

Resonant states appear as isolated poles.

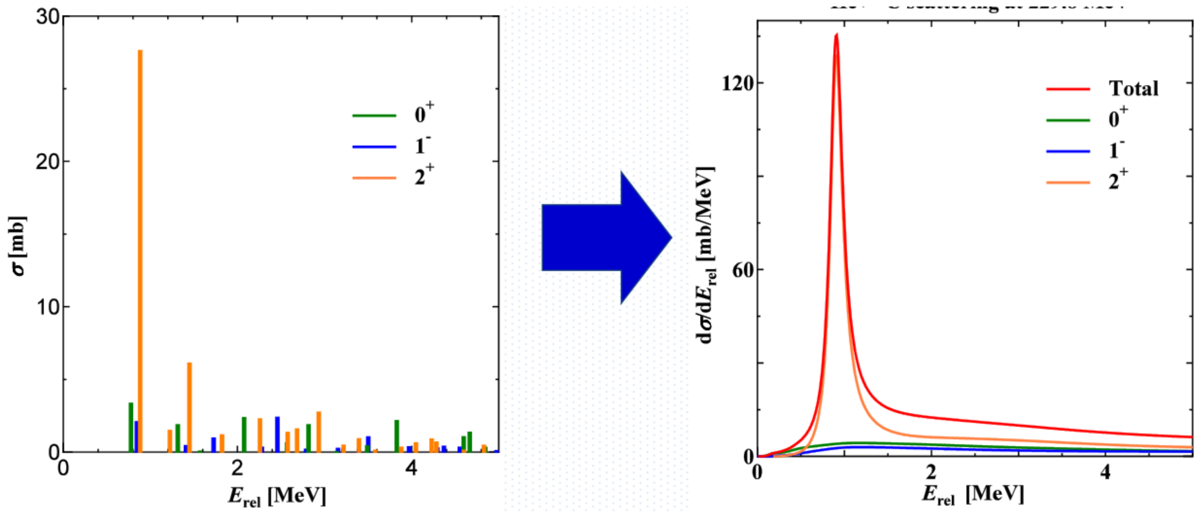
$$E_{\mathrm{res}} = E_r - i\frac{\Gamma}{2}$$

resonance energy

decay width

Smoothing factor

Obtained cross sections are discretized in eigen energies.



- CDCC + CSM T. Matsumoto, *et al.,* Phys. Rev. C **82** (2010), 051602(R)

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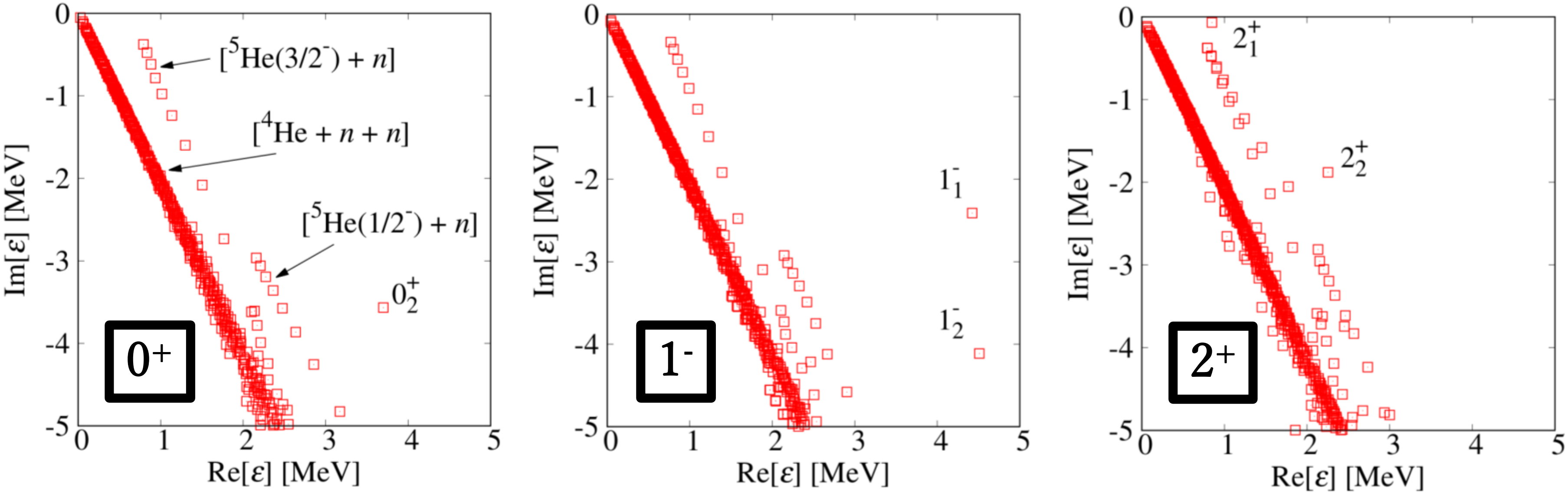
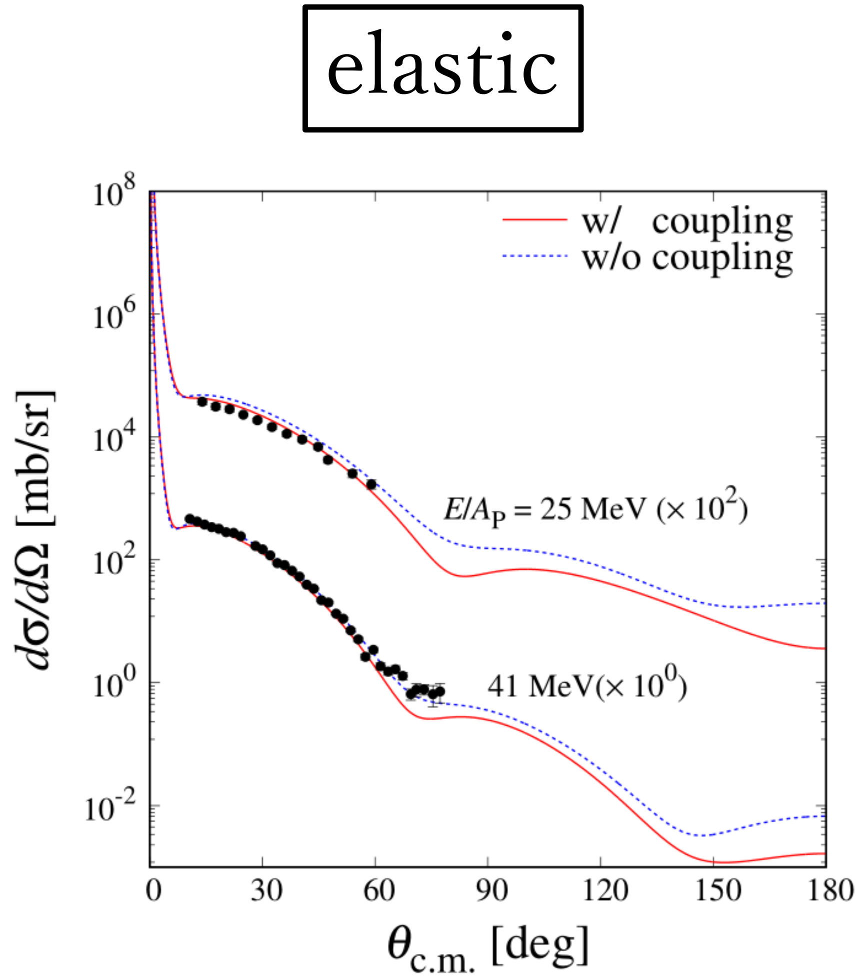
Results of CSM

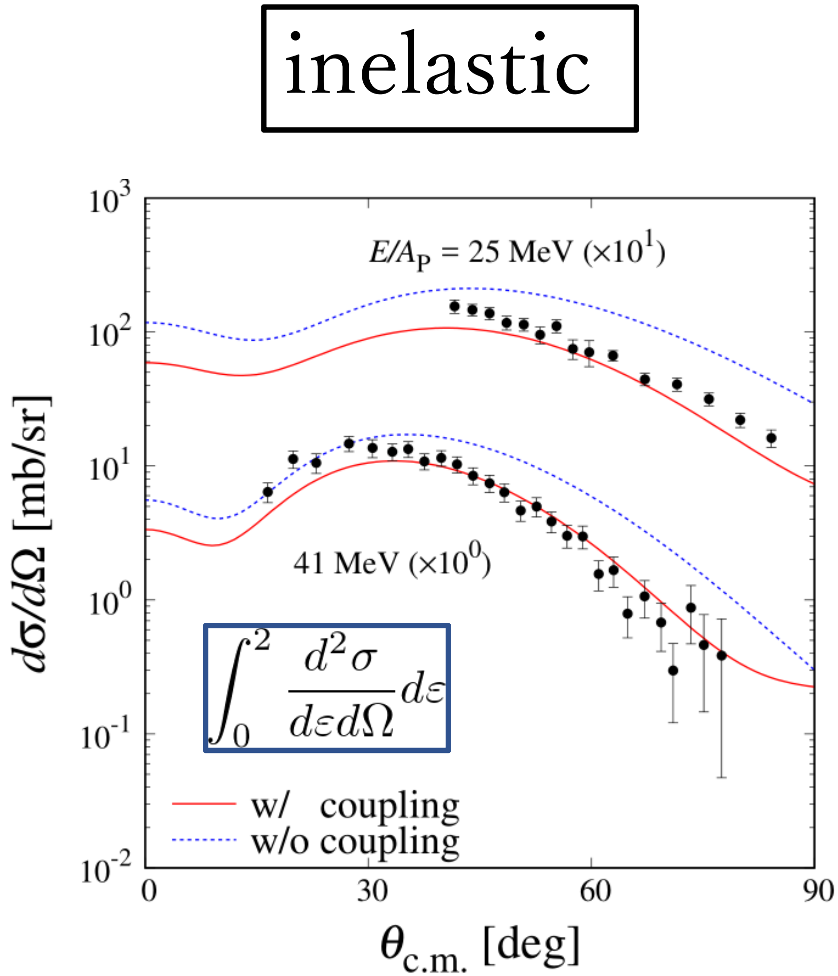


	Calc.		T. Myo, *et al.* [1]	
J^π	E_r	Γ	E_r	Γ
2_1^+	0.85	0.14	0.81	0.13
2_2^+	2.25	3.75	2.35	4.22
0_2^+	3.70	7.13	3.69	9.14
1_1^-	4.42	4.82		
1_2^-	4.51	8.23		

[1] T. Myo, *et al.*, Phys. Rev. C **63** (2001), 054313

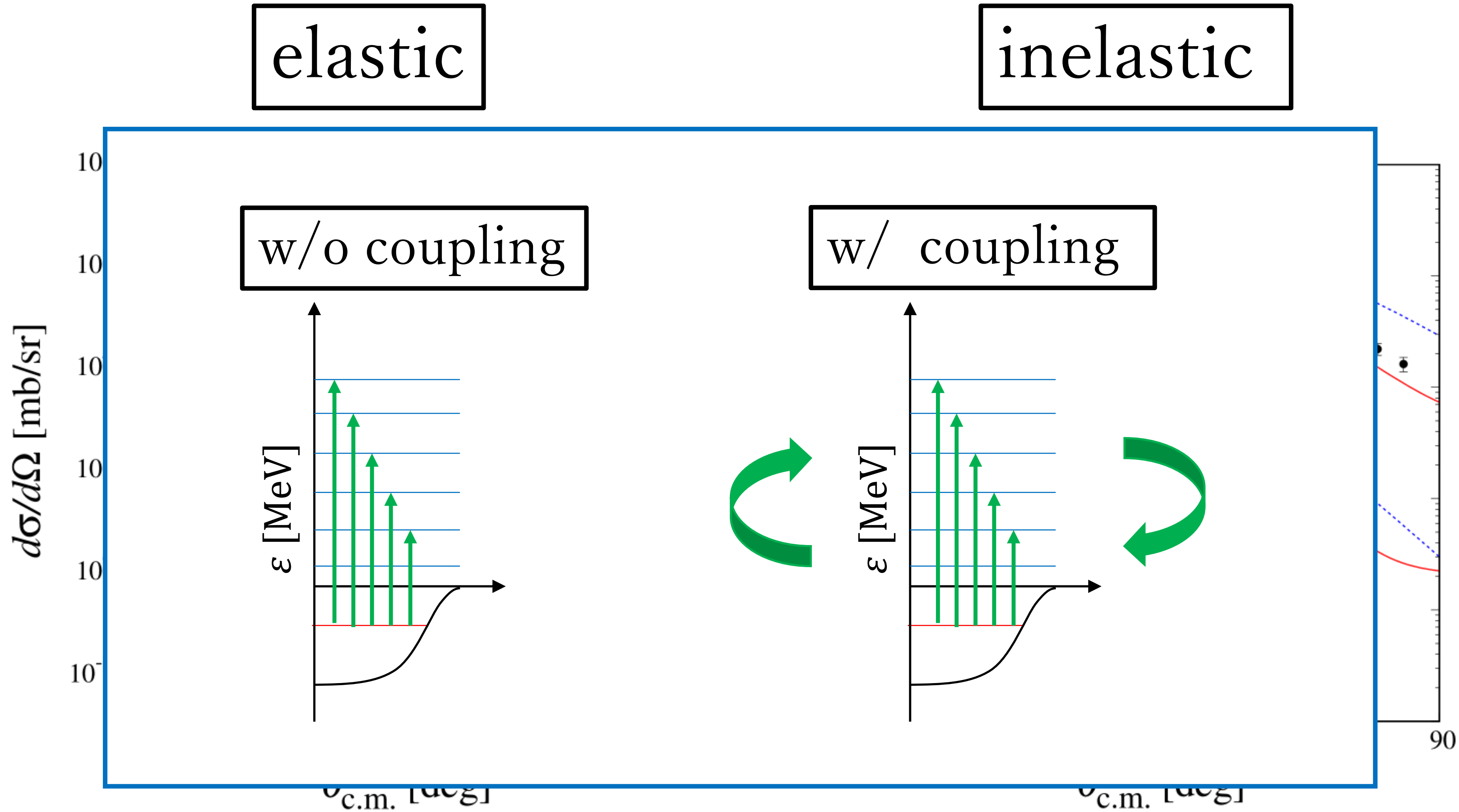
Elastic and Inelastic cross sections





- ✓ The results of CDCC reproduce well the experimental data.
- ✓ Coupling effects are important.

Elastic and Inelastic cross sections



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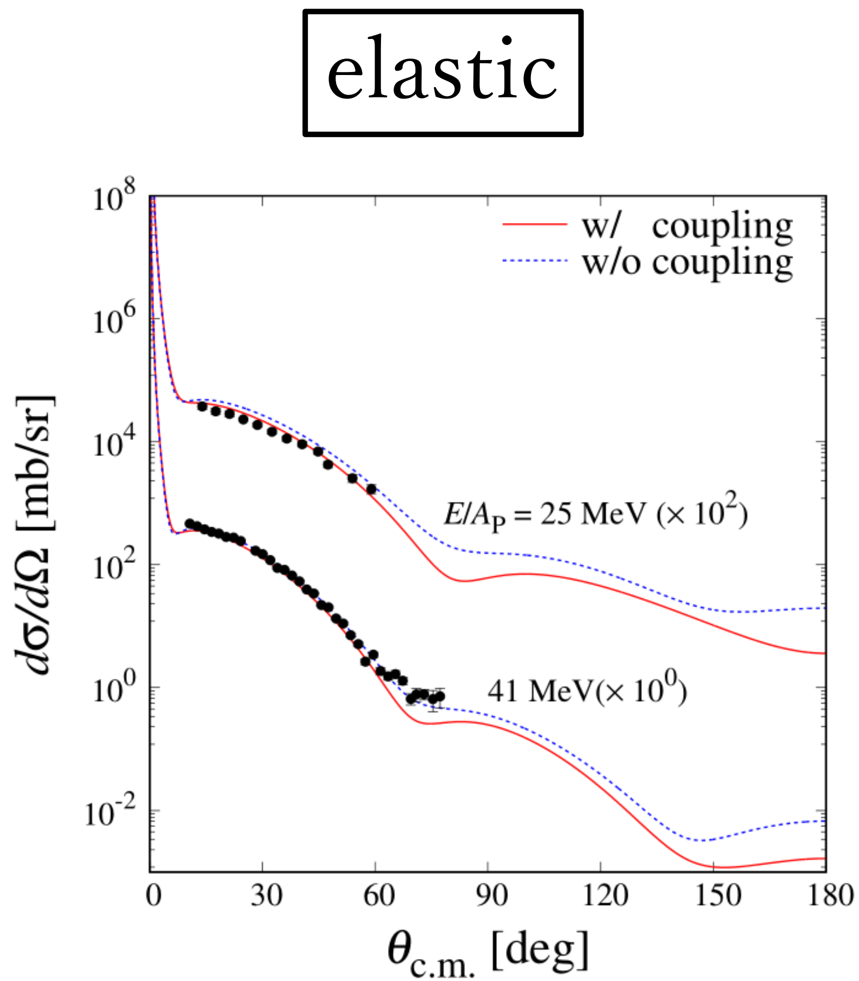
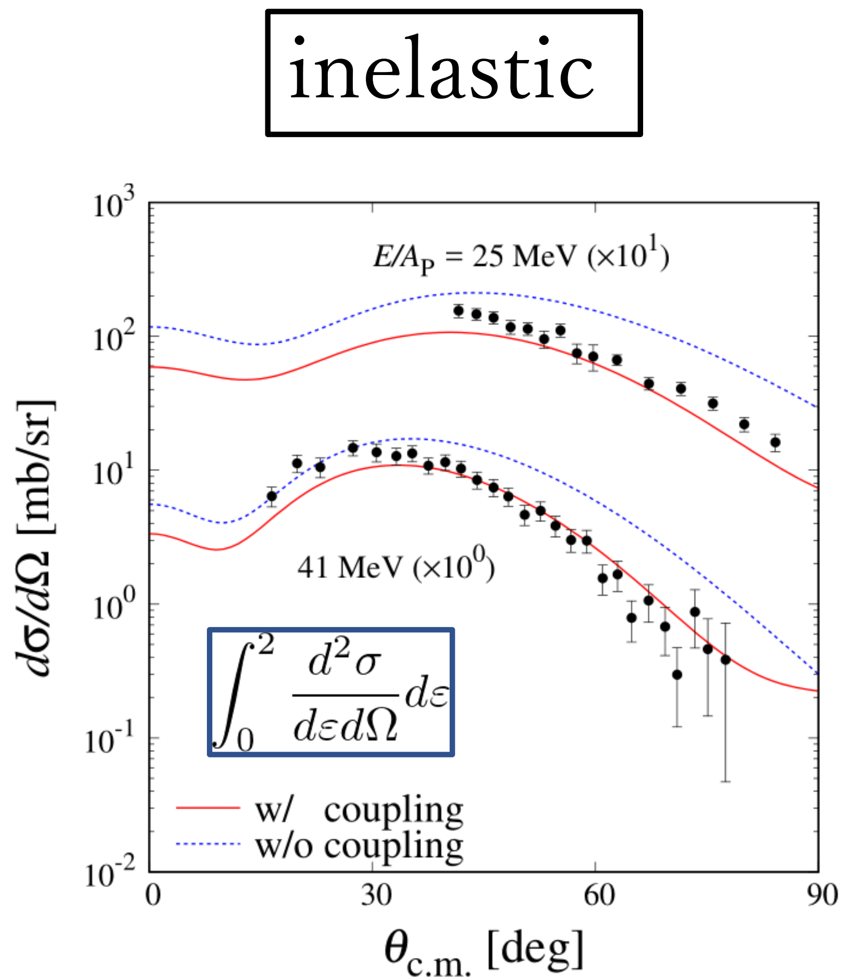
Elastic and Inelastic cross sections





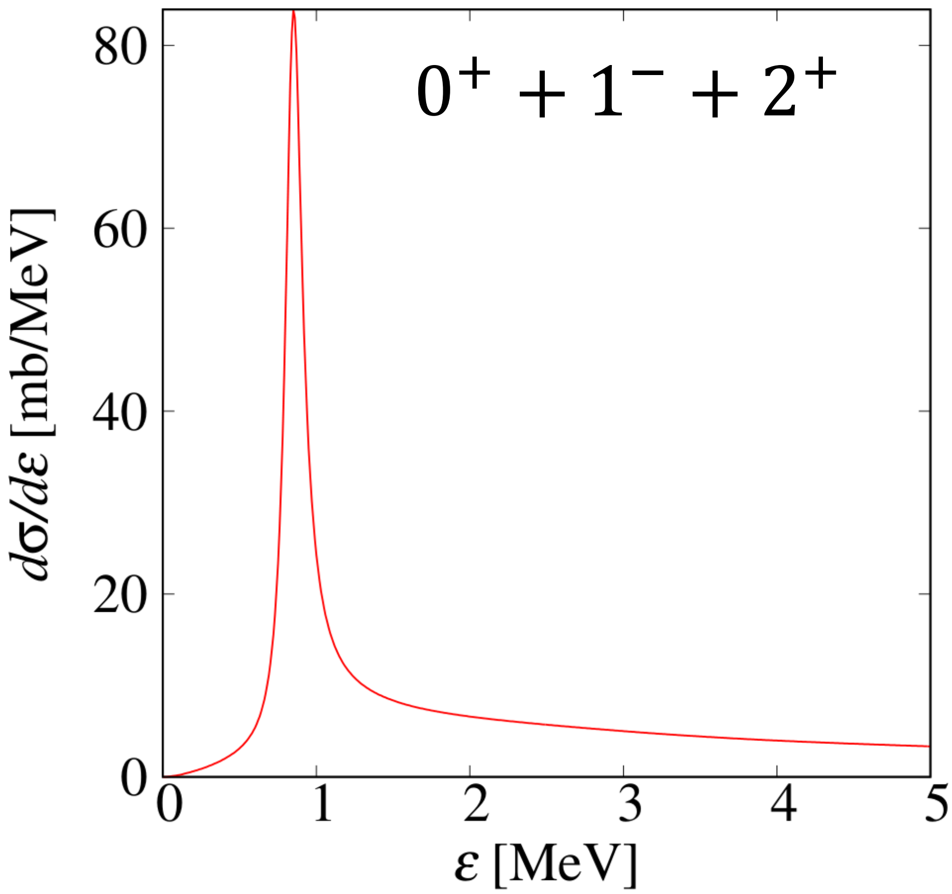
- ✓ The results of CDCC reproduce well the experimental data.
- ✓ Coupling effects are important.

Energy spectrum

$\boxed{E/A{=}41\mathrm{MeV}}$

$$\frac{d\sigma}{d\varepsilon} = \int_{0^\circ \leq \theta \leq 180^\circ} \frac{d^2\sigma}{d\varepsilon d\Omega} d\Omega$$

Calc.		
I^π	E	Γ
2_1^+	0.848	0.136
2_2^+	2.25	3.75
0_2^+	3.70	7.13
1_2^-	4.51	8.23
1_1^-	4.42	4.82



- The 2_1^+ appear as the sharp peak.
- A clear peak from 2_2^+ can not be seen.

Angular distribution

E/A=41MeV

$$\frac{d\sigma}{d\Omega} = \int_{2\lesssim\varepsilon\lesssim 3} \frac{d^2\sigma}{d\varepsilon d\Omega} d\varepsilon$$

Calc.		
I^π	E	Γ
2_1^+	0.848	0.136
$\boxed{2_2^+}$	$\boxed{2.25}$	$\boxed{3.75}$
0_2^+	3.70	7.13
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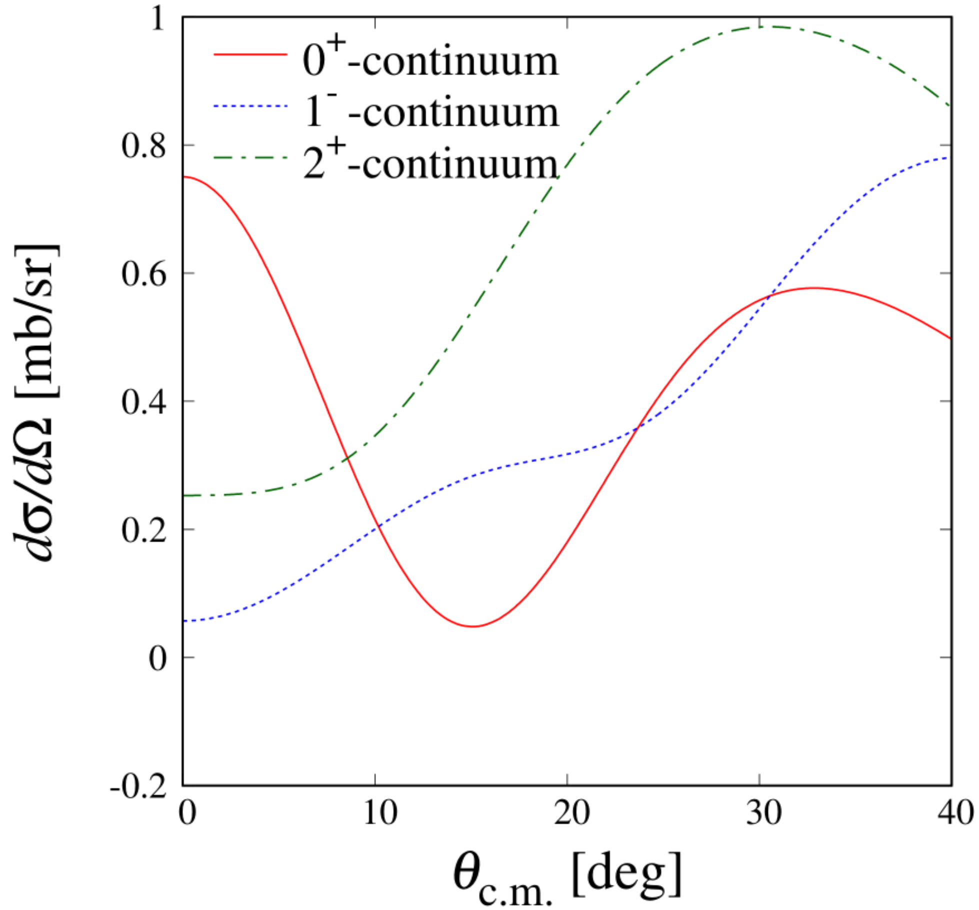

- 2^+-continuum : relatively large in $20^\circ \sim 30^\circ$

Angular distribution

$\boxed{E/A\text{=41MeV}}$

$$\frac{d\sigma}{d\Omega} = \int_{2\leq\varepsilon\leq3} \frac{d^2\sigma}{d\varepsilon d\Omega} d\varepsilon$$

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2_1^+	0.848	0.136
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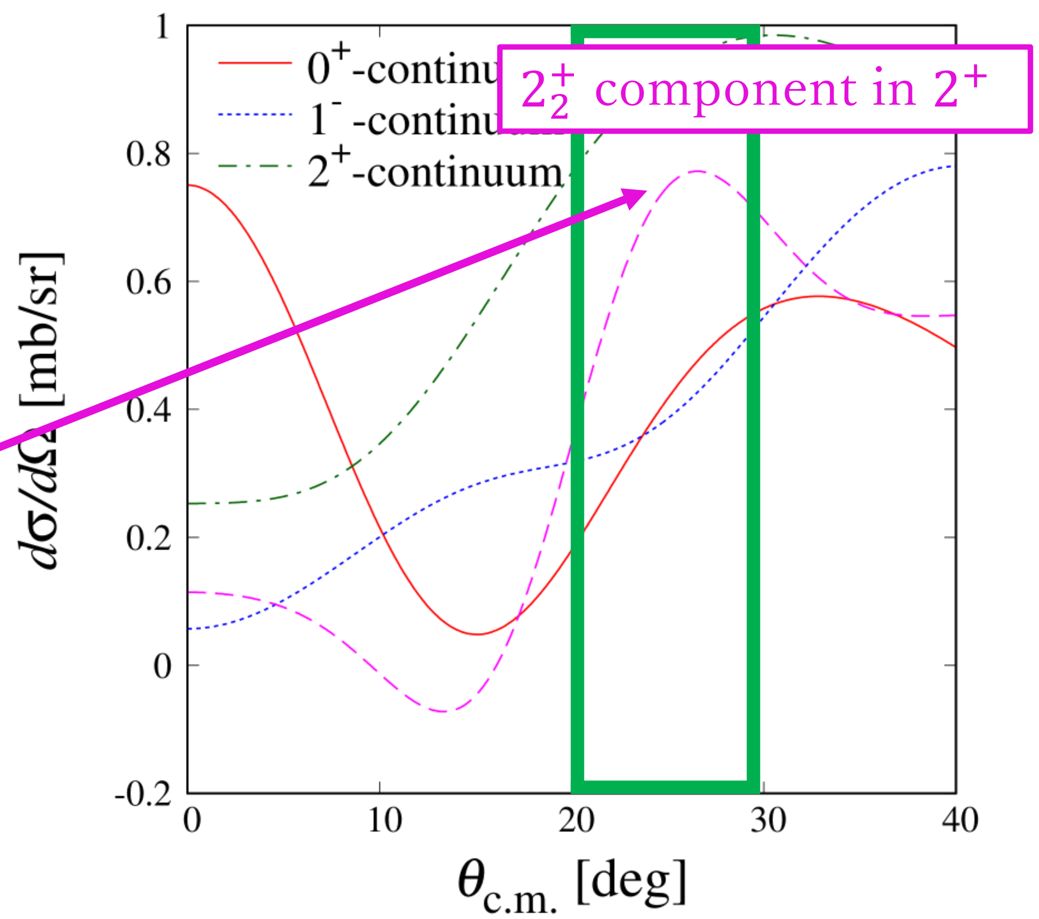


- The effect of 2_2^+ is enhanced in $20^\circ \sim 30^\circ$.

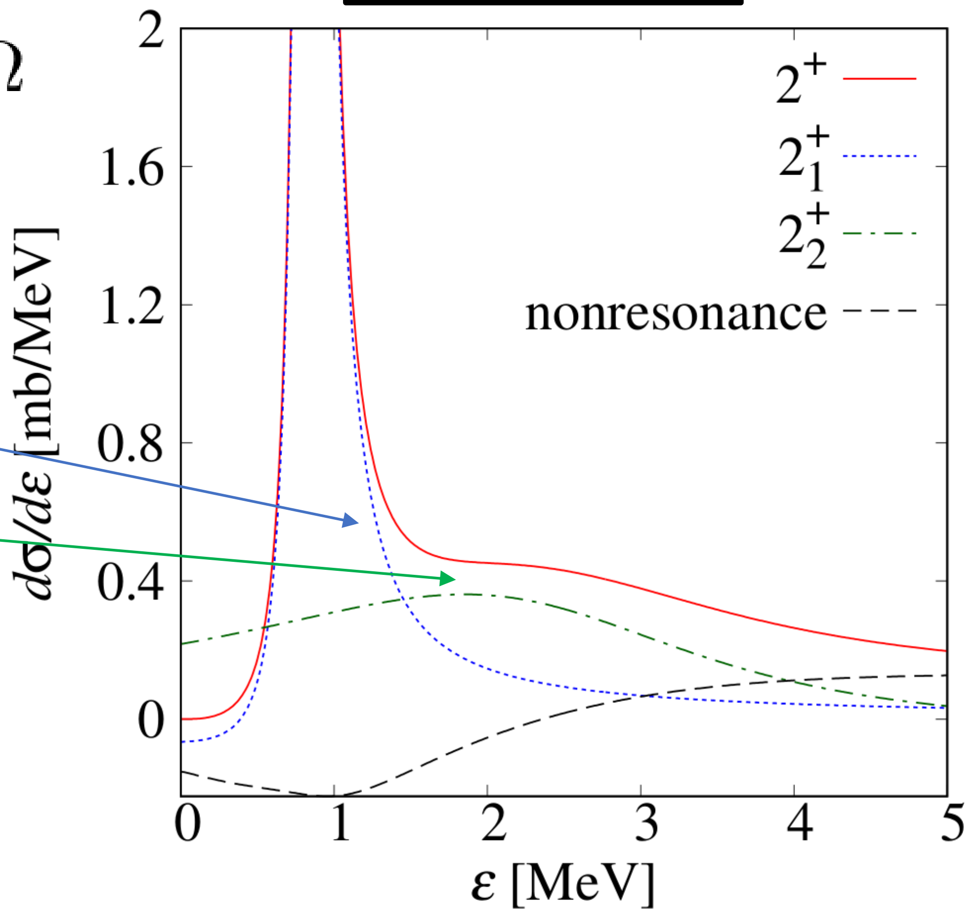
Energy spectrum (2^+)

E/A=41MeV

$$\frac{d\sigma}{d\varepsilon} = \int_{20^\circ \leq \theta \leq 30^\circ} \frac{d^2\sigma}{d\varepsilon d\Omega} d\Omega$$

	Calc.	
I^π	E	Γ
2_1^+	0.848	0.136
2_2^+	2.25	3.75
0_2^+	3.70	7.13
1_2^-	4.51	8.23
1_1^-	4.42	4.82

2^+-continuum



- The shoulder peak comes from 2_2^+ in 2~3 MeV.

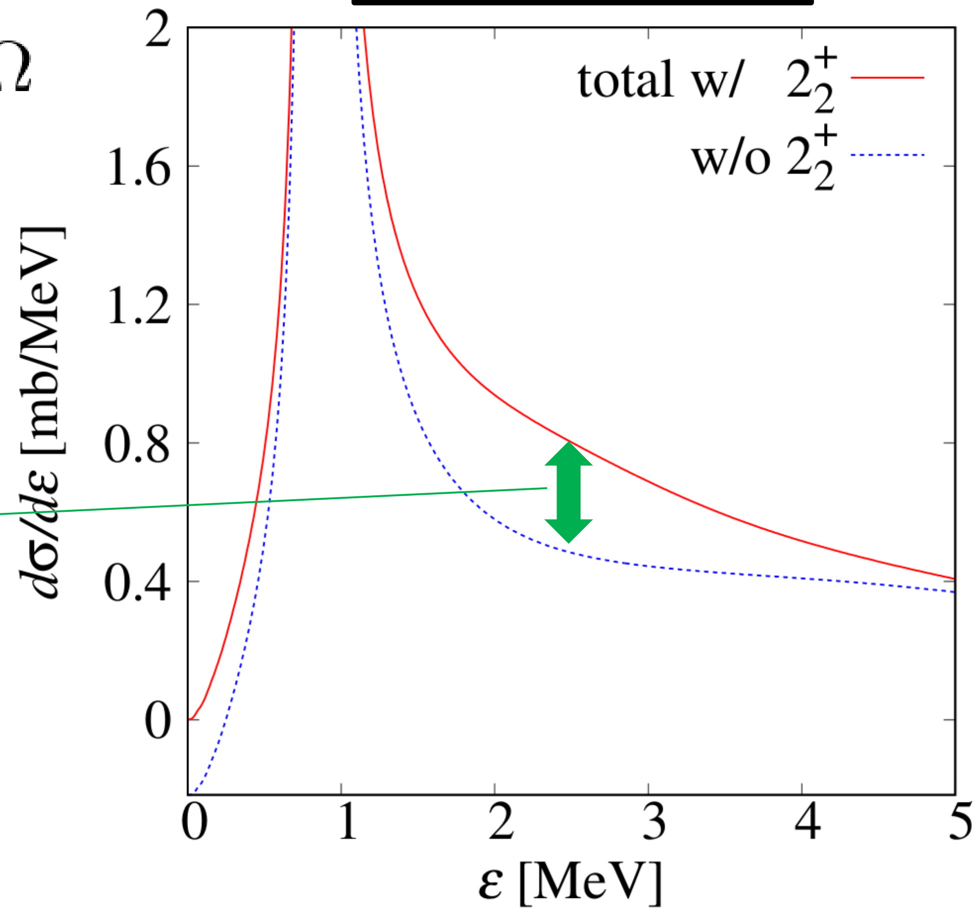
Energy spectrum (total)

E/A=41MeV

$$\frac{d\sigma}{d\varepsilon} = \int_{20^\circ \leq \theta \leq 30^\circ} \frac{d^2\sigma}{d\varepsilon d\Omega} d\Omega$$

Calc.		
I^π	E	Γ
2_1^+	0.848	0.136
2_2^+	2.25	3.75
0_2^+	3.70	7.13
1_2^-	4.51	8.23
1_1^-	4.42	4.82

total components



- The difference between w/ and w/o 2_2^+ appears in 2~3 MeV.

Summary

We analyze the effects from 2_2^+ on the observables via $^6\mathrm{He}(p, p')$ reactions with CDCC and CSM.

CSM

- The result of 2_2^+ is similar to one of previous study which calculated by CSM.

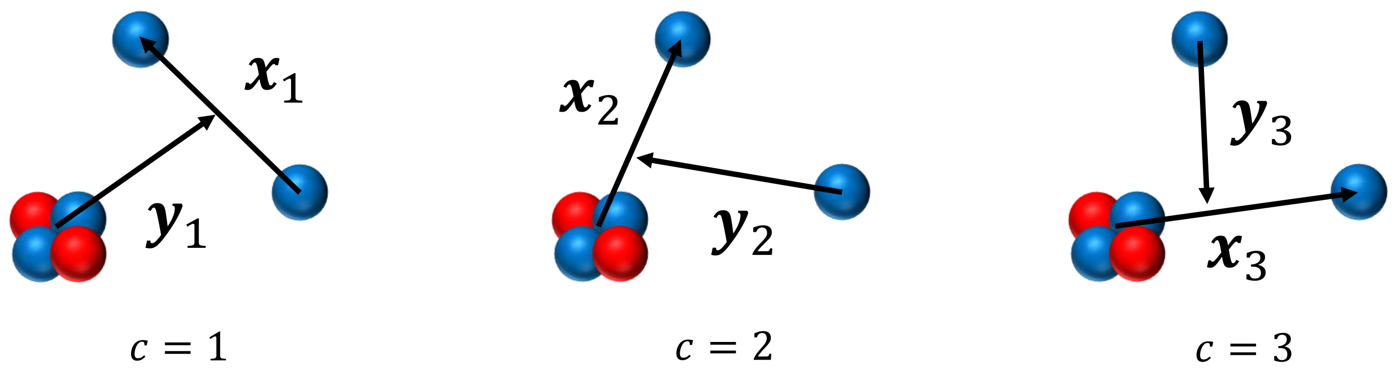
$^6\mathrm{He}(p, p')$ reactions

- The coupling effects are important.
- In 2^+ component, 2_2^+ appears as like a shoulder peak.

backup

Gaussian Expansion Method

Wavefunctions of ^{6}He are spanned with Gaussian basis functions.



$$\Phi_{Im}(\boldsymbol{x},\boldsymbol{y}) = \sum_{c=1}^{3} \sum_{i,j,l,\lambda} \phi_{i,l}(x_c)\varphi_{j,\lambda}(y_c) \left[\left[Y_l(\hat{\boldsymbol{x}}_c) \otimes Y_\lambda(\hat{\boldsymbol{y}}_c) \right]_\Lambda \otimes \left[\eta_{1/2} \otimes \eta_{1/2} \right]_S \right]_{Im}$$

$$\phi_{i,l}(x_c) = x_c^l e^{-(x_c/x_i)^2} \qquad \varphi_{j,\lambda}(y_c) = y_c^\lambda e^{-(y_c/y_j)^2}$$

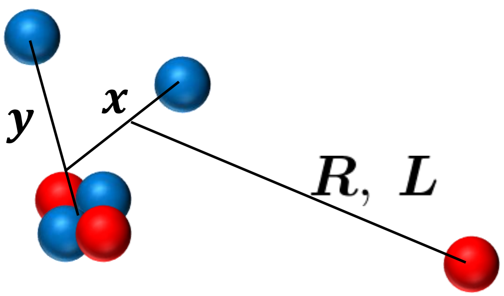
$$x_i = x_1 \left(\frac{x_{\max}}{x_1} \right)^{\frac{i-1}{i_{\max}-1}} \qquad y_i = y_1 \left(\frac{y_{\max}}{r_1} \right)^{\frac{j-1}{j_{\max}-1}}$$

✓n- n interaction : Minnesota   D. R. Thompson, *et al.*, Nucl. Phys. A 286 (1977), 53.

✓n- α interaction : KKNN   H. Kanada, et al., Prog. Theor. Phys. 61 (1979), 1327.

CDCC equation

CDCC wavefunction



$$\Psi^{\mathrm{CDCC}}_{JM}(\boldsymbol{\xi}, \boldsymbol{R}) = \sum_{I,n} \sum_{L=|J-I|}^{J+I} \frac{\chi_{nIL}(K_n, R)}{R} \mathcal{Y}^{nIL}_{JM}(\boldsymbol{\xi}, \hat{\boldsymbol{R}})$$

$$K_n = \frac{\sqrt{2\mu(E-\epsilon^I_n)}}{\hbar} \qquad \mathcal{Y}^{nIL}_{JM}(\boldsymbol{\xi}, \hat{\boldsymbol{R}}) = \left[\Phi^I_n(\boldsymbol{\xi}) \otimes i^L Y_L(\hat{\boldsymbol{R}})\right]_{JM}$$

$\boldsymbol{\xi} = \{\boldsymbol{x}, \boldsymbol{y}\}$ $\quad J, M$: total spin and its projection on z-axis, $\quad I$: spin of ^{6}He

Schrödinger equation of radial direction

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + \frac{\hbar^2}{2\mu}\frac{L(L+1)}{R^2} + U_{\gamma\gamma} + \frac{Z_p Z_{^6\mathrm{He}} e^2}{R} - (E-\epsilon^I_n)\right]\chi_\gamma(K_n, R) = -\sum_{\gamma'\neq\gamma} U_{\gamma\gamma'}\chi_{\gamma'}(K_{n'}, R)$$

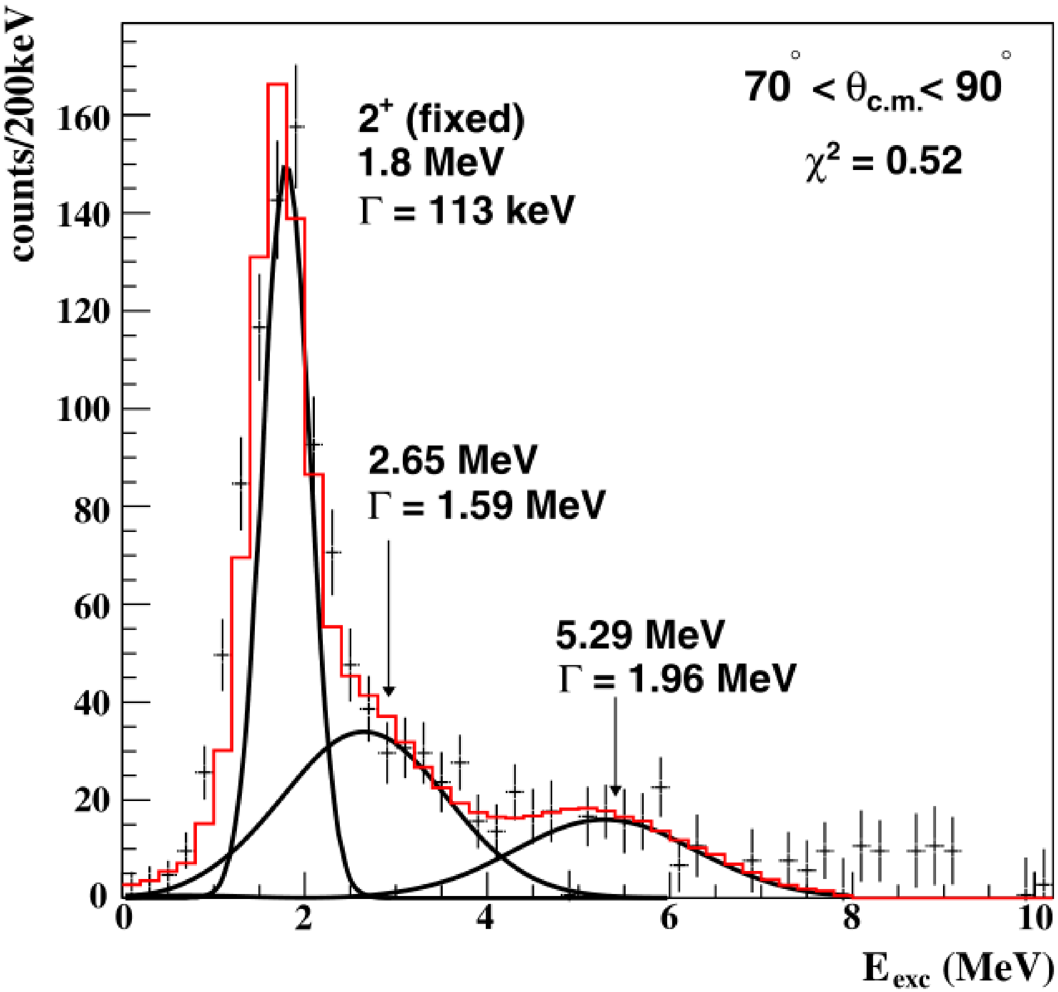
$$U_{\gamma\gamma'}(R) = \frac{A_{\mathrm{P}}-1}{A_{\mathrm{P}}} \int \rho_{\gamma\gamma'}(\boldsymbol{s}, \hat{\boldsymbol{R}}) \boxed{g_i(E, \bar{\rho}, \boldsymbol{R}_j)} d\boldsymbol{s} d\hat{\boldsymbol{R}} \qquad \gamma = \{n, I, L\}$$

JLM effective NN interaction

$$\boxed{\rho_{\gamma\gamma'}(\boldsymbol{s}, \hat{\boldsymbol{R}})} = \langle \mathcal{Y}^{\gamma}_{JM} | \sum_{i\in{}^6\mathrm{He}} \delta(\boldsymbol{s}-\boldsymbol{s}_i) | \mathcal{Y}^{\gamma'}_{JM} \rangle_{\boldsymbol{\xi}} \qquad \bar{\rho}(s) = \frac{1}{2}\int \{\rho_{\gamma\gamma}(\boldsymbol{s}, \hat{\boldsymbol{R}}) + \rho_{\gamma'\gamma'}(\boldsymbol{s}, \hat{\boldsymbol{R}})\} d\hat{\boldsymbol{s}} d\hat{\boldsymbol{R}}$$

This is calculated with ^{6}He wavefunction.

$p(^8\mathrm{He}, t)$



X. Mougeot, et al., Phys. Lett. B 718 (2012), 441.