高密度物質の状態方程式と高エネルギー天体現象

富樫 甫 (九州大学)

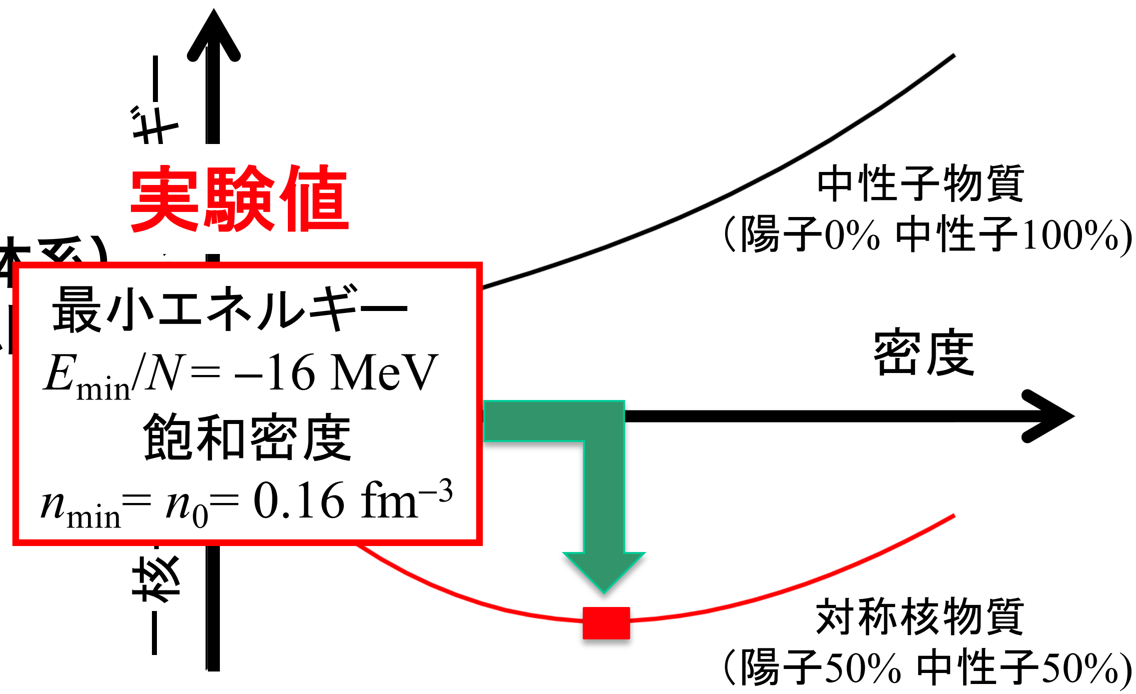
理研-九大ジョイントワークショップ「数理が紡ぐ素粒子・原子核・宇宙」@九州大学　2019年12月24日

Introduction

状態方程式（Equation of State: EOS）

- 圧力（または一核子あたりのエネルギー）と密度、温度、粒子組成との関係式

一様核物質（無限核子多体系）
状態方程式の理論予想

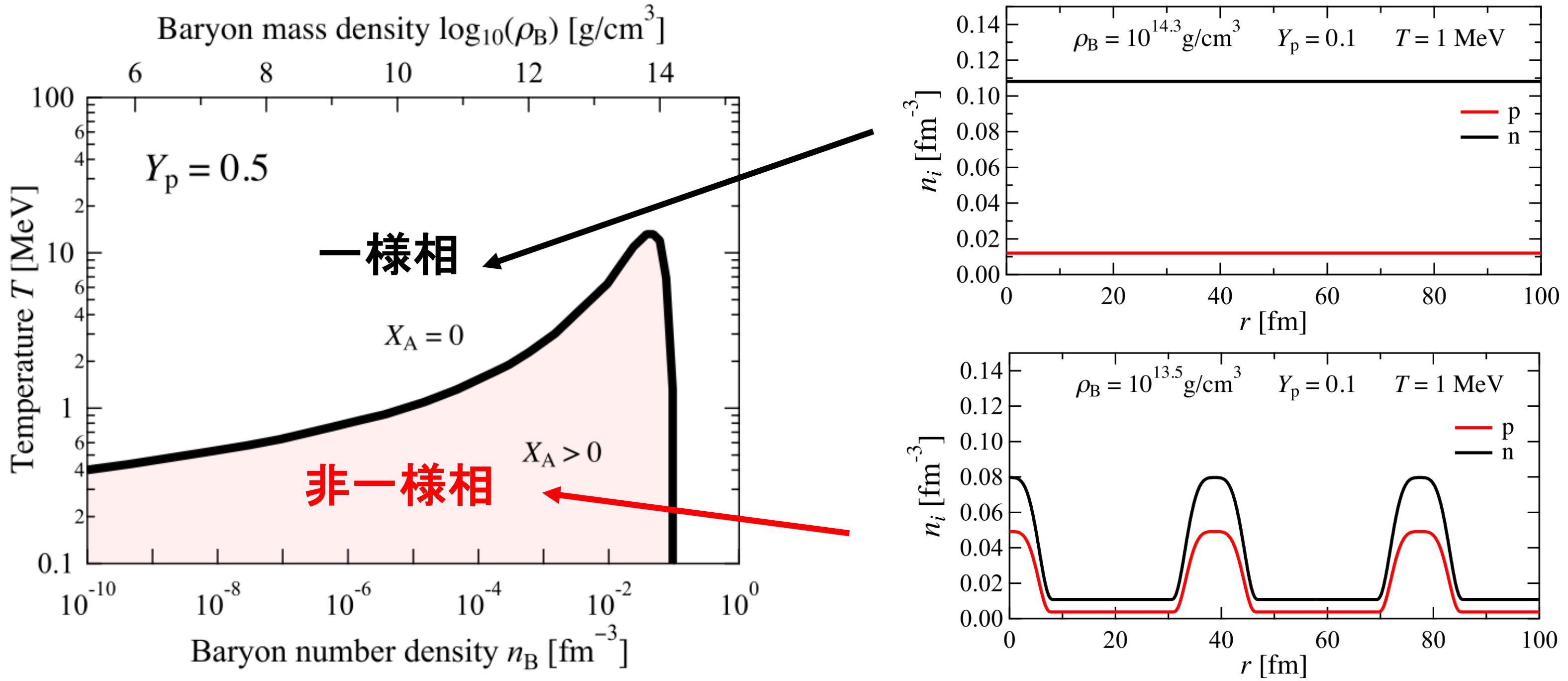


状態方程式の求め方

① 解析的な関数形で表す（Toy model）
② 現象論的模型を作る（有効理論）
③ 生の核力から量子多体計算を行う（微視的理論）

超新星爆発計算のための核物質状態方程式

- 一様相と非一様相を無矛盾に取り扱う必要がある



Phase diagram of nuclear matter [based on HT *et al*., NPA 961 (2017) 78]

- 温度・密度・陽子混在度の幅広い領域をカバーする必要がある

- 温度 T : $0 \le T \le 100$ MeV
- 密度 ρ : $10^{5.1} \le \rho_B \le 10^{16.0}$ g/cm^3
- 陽子混在度 Y_p : $0 \le Y_p \le 0.65$

超新星爆発計算に適用可能な核物質状態方程式

(Rev. Mod. Phys. 89 (2017) 015007)

Model	Nuclear Interaction	Degrees of Freedom	$M_{\rm max}$ ($M_\odot$)	$R_{1.4M_\odot}$ (km)	Ξ	publ. avail.	References
H&W	SKa	$n, p, \alpha, \{(A_i, Z_i)\}$	2.21[a]	13.9 [a]		n	El Eid and Hillebrandt (1980); Hillebrandt *et al.* (1984)
LS180	LS180	$n, p, \alpha, (A, Z)$	1.84	12.2	0.27	y	Lattimer and Swesty (1991)
LS220	LS220	$n, p, \alpha, (A, Z)$	2.06	12.7	0.28	y	Lattimer and Swesty (1991)
LS375	LS375	$n, p, \alpha, (A, Z)$	2.72	14.5	0.32	y	Lattimer and Swesty (1991)
STOS	TM1	$n, p, \alpha, (A, Z)$	2.23	14.5	0.26	y	Shen *et al.* (1998); Shen *et al.* (1998, 2011)
FYSS	TM1	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.22	14.4	0.26	n	Furusawa *et al.* (2013b)
HS(TM1)	TM1*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.21	14.5	0.26	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(TMA)	TMA*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.02	13.9	0.25	y	Hempel and Schaffner-Bielich (2010)
HS(FSU)	FSUgold*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.74	12.6	0.23	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(NL3)	NL3*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.79	14.8	0.31	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(DD2)	DD2	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.42	13.2	0.30	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(IUFSU)	IUFSU*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.95	12.7	0.25	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
SFHo	SFHo	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.06	11.9	0.30	y	Steiner *et al.* (2013a)
SFHx	SFHx	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.13	12.0	0.29	y	Steiner *et al.* (2013a)
SHT(NL3)	NL3	$n, p, \alpha, \{(A_i, Z_i)\}$	2.78	14.9	0.31	y	Shen *et al.* (2011b)
SHO(FSU)	FSUgold	$n, p, \alpha, \{(A_i, Z_i)\}$	1.75	12.8	0.23	y	Shen *et al.* (2011a)
SHO(FSU2.1)	FSUgold2.1	$n, p, \alpha, \{(A_i, Z_i)\}$	2.12	13.6	0.26	y	Shen *et al.* (2011a)

+ Nuclear EOS tables based on the Liquid drop model with Skyrme interaction by A. S. Schneider (2017)

超新星爆発計算に適用可能な核物質状態方程式

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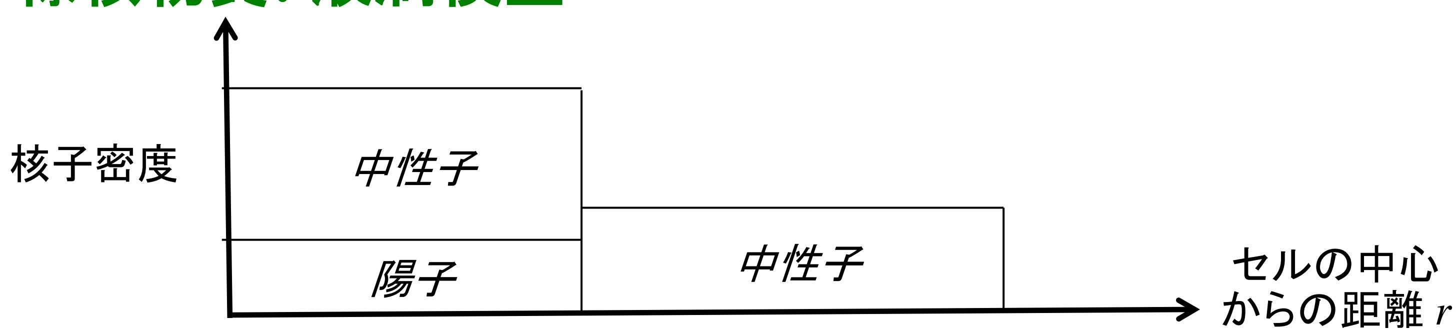
Lattimer-Swesty EOS

一様核物質: 解析的な関数形(①の手法)

Based on Skyrme type effective interaction

$$E_{\rm bulk}(n, x, T) = \sum_i \frac{\hbar^2 \tau_i}{2m_i^*} + [a + 4bx(1-x)]n^2 + cn^{1+\delta} - xn\Delta$$

非一様核物質: 液滴模型



超新星爆発計算に適用可能な核物質状態方程式

(Rev. Mod. Phys. 89 (2017) 015007)

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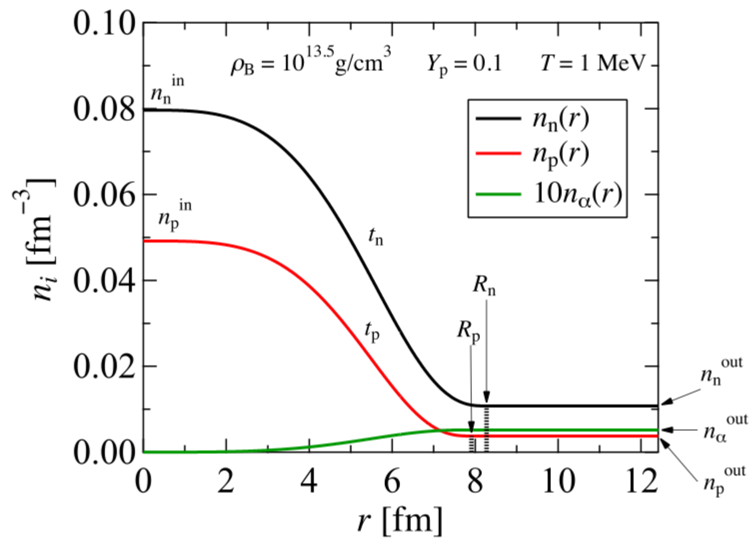
Shen EOS

一様核物質: 現象論的模型（②の手法）

$$\mathcal{L}_{\mathrm{RMF}} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu} - e\gamma_\mu\frac{1-\tau_3}{2}A^\mu\right]\psi$$
$$+\tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \tfrac{1}{2}m_\sigma^2\sigma^2 - \tfrac{1}{3}g_2\sigma^3 - \tfrac{1}{4}g_3\sigma^4$$
$$-\tfrac{1}{4}W_{\mu\nu}W^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \tfrac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2$$
$$-\tfrac{1}{4}R^a_{\mu\nu}R^{a\mu\nu} + \tfrac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu} - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

非一様核物質: Thomas-Fermi 近似



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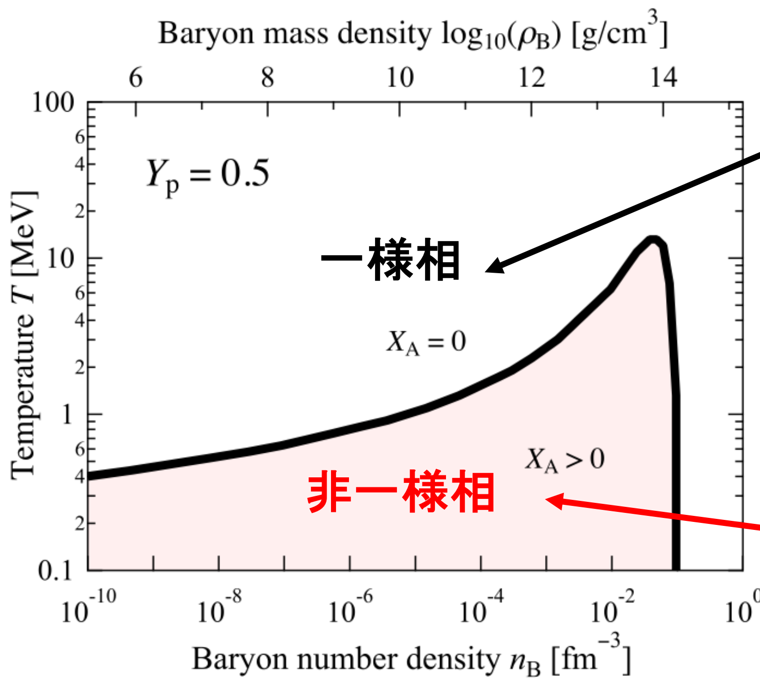
現実的核力に基づく超新星爆発計算用の核物質状態方程式がない!

現実的核力から出発した量子多体変分計算に基づいて
新しい超新星爆発計算用の核物質状態方程式を完成(③の手法)

(HT, K. Nakazato, Y. Takehara, S. Yamamuro, H. Suzuki, M. Takano, NPA961 (2017) 78)

超新星爆発計算のための核物質状態方程式



Phase diagram of nuclear matter [based on HT *et al*., NPA 961 (2017) 78]

現実的核力から出発した
クラスター変分法

Thomas-Fermi計算により
非一様核物質EOSへと拡張

- 温度 T : $0 \leq T \leq 100$ MeV
- 密度 ρ : $10^{5.1} \leq \rho_B \leq 10^{16.0}$ g/cm^3
- 陽子混在度 Y_p : $0 \leq Y_p \leq 0.65$

2. 現実的核力に基づく核物質状態方程式

Nuclear Hamiltonian

$$H = -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{i<j}^{N} V_{ij} + \sum_{i<j<k}^{N} V_{ijk}$$

Argonne v18 (AV18) two-body potential (PRC 51 (1995) 38)

$$\begin{aligned} V_{ij} &= \sum_{t=0}^{1}\sum_{s=0}^{1} [V_{\mathrm{C}ts}(r_{ij}) + sV_{\mathrm{T}t}(r_{ij})S_{\mathrm{T}ij} + sV_{\mathrm{SO}t}(r_{ij})(\boldsymbol{L}_{ij}\cdot\mathbf{s}) \\ &\quad + V_{\mathrm{qL}ts}(r_{ij})\left|\boldsymbol{L}_{ij}\right|^2 + sV_{\mathrm{qSO}t}(r_{ij})(\boldsymbol{L}_{ij}\cdot\mathbf{s})^2] P_{tsij} \end{aligned}$$

Urbana IX (UIX) three-body potential (PRL 74 (1995) 4396)

$$\begin{aligned} V_{ijk} &= U \sum_{\mathrm{cyc}} [T(r_{ij})]^2 [T(r_{ik})]^2 \\ &+ A \sum_{\mathrm{cyc}} \left[\{x_{ij}, x_{ik}\}\{\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j, \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k\} + \frac{1}{4}[x_{ij}, x_{ik}][\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j, \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k] \right] \end{aligned}$$

$$x_{ij} = Y(r_{ij})\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + T(r_{ij})S_{\mathrm{T}ij}$$

一様核物質の１核子あたりのエネルギー

Jastrow wave function

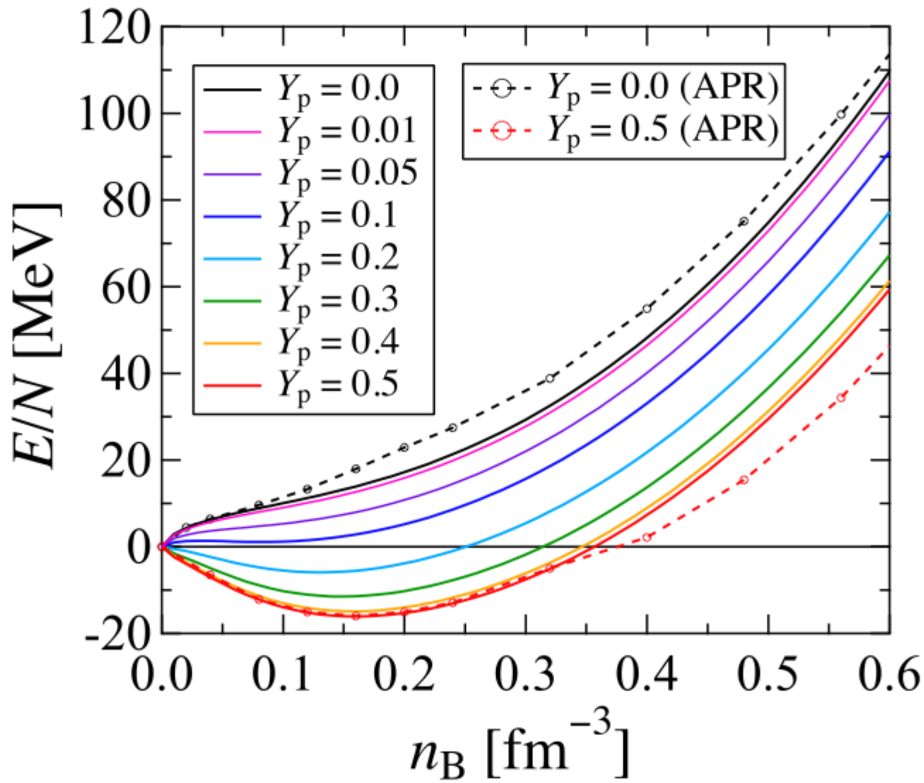
$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right]\Phi_{\mathrm{F}}$$

Φ_{F}: The Fermi-gas wave function

Correlation function

$$f_{ij} = \sum_{t=0}^{1}\sum_{\mu}\sum_{s=0}^{1}\left[f^{\mu}_{\mathrm{C}ts}(r_{ij}) + s f^{\mu}_{\mathrm{T}t}(r_{ij}) S_{\mathrm{T}ij} + s f^{\mu}_{\mathrm{SO}t}(r_{ij})(\boldsymbol{L}_{ij}\cdot \boldsymbol{s})\right] P^{\mu}_{tsij}$$

The two-body cluster approximation is adopted to obtain the energy per nucleon.



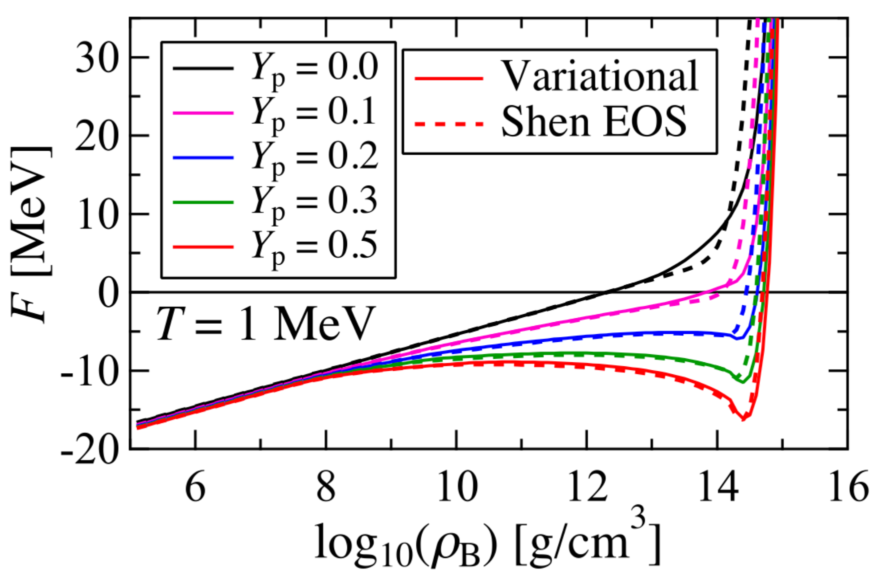
n_0[fm^{-3}]	E_0[MeV]	K [MeV]	S [MeV]	L [MeV]
0.16	−16.1	245	30.0	38.7

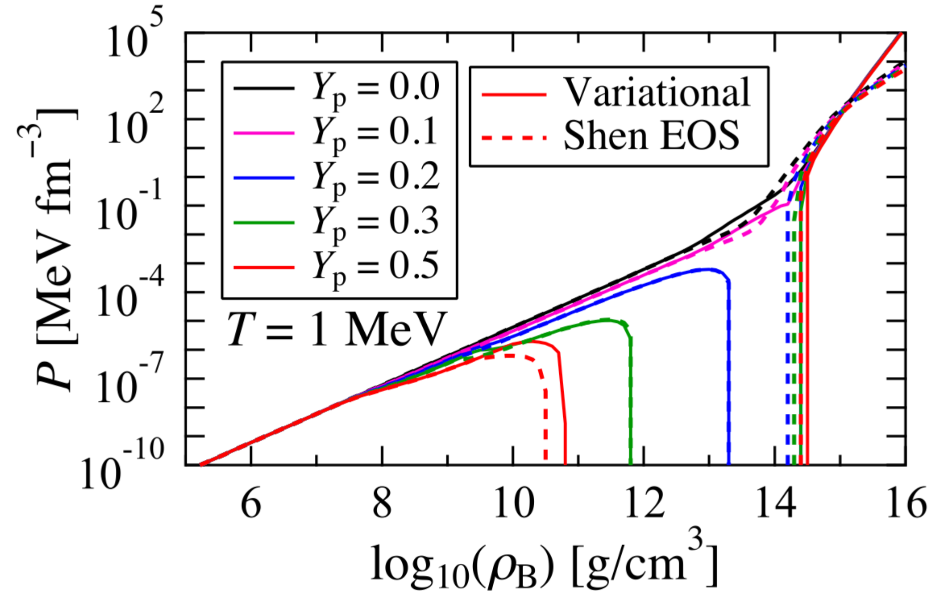
Our EOS : HT and M. Takano, NPA 902 (2013) 53

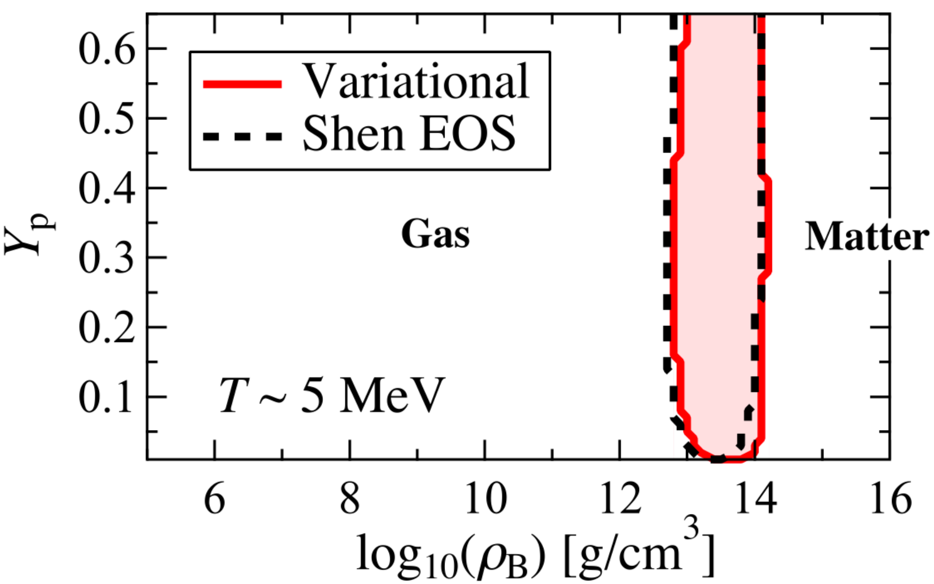
APR : A. **A**kmal, V. R. **P**andharipande, D. G. **R**avenhall, PRC 58 (1998) 1804

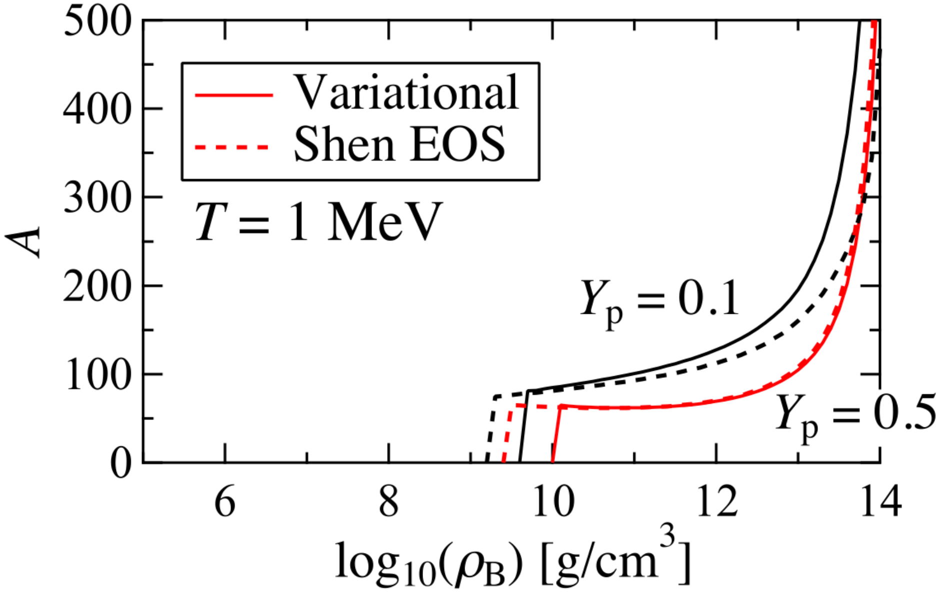
有限温度核物質の様々な熱力学量

非一様核物質：Thomas-Fermi 計算 (PTP 100 (1998) 1013, APJS 197(2011) 20)









HT *et al.*, NPA 961 (2017) 78

Home Page of Variational EOS Table

http://www.np.phys.waseda.ac.jp/EOS/

Equation of state for nuclear matter w

Equation of state (EOS) based on the variational man
matter, the EOS is constructed with the cluster variat
potential and the Urbana IX three-body nuclear poter
approximation. Alpha particle mixing is also taken int
This EOS table is open for general use in any studies
referred to in your publication.

User's Guide (read me first)

guide.pdf

User's Guide

User Note for the Variational EOS Table

H. Togashi[1,2], K. Nakazato[3], H. Suzuki[4], and M. Takano[2]

[1]RIKEN Nishina Center for Accelerator-Based Science, RIKEN, Saitama 351-0198, Japan

[2]Research Institute for Science and Engineering, Waseda University, Tokyo 169-8555, Japan

[3]Faculty of Arts and Science, Kyushu University, Fukuoka 891-0395, Japan

[4]Faculty of Science and Technology, Tokyo University of Science, Chiba 278-8510, Japan

March 8, 2017

Abstract

This is a guide for users of the nuclear equation of state (EOS) table based on the Argonne v18 two-body and Urbana IX three-body potentials. We construct the nuclear EOS using the cluster variational method for uniform matter and the Thomas-Fermi calculation for non-uniform matter.

EOS tables

eoszip

Numerical Data

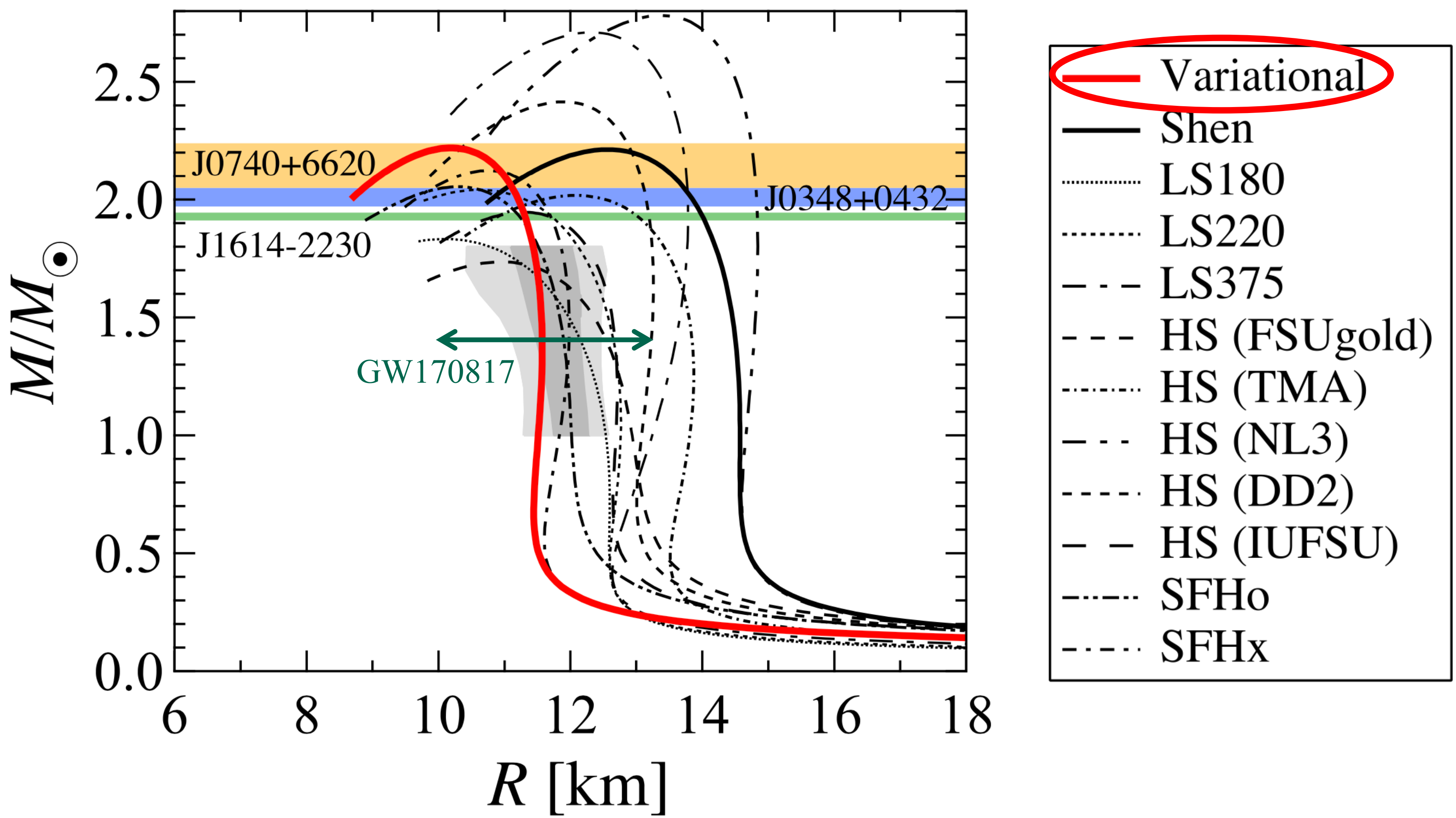
```
cccccccccccc
 Log10(Temp)    Temp
-1.000000E+00  1.000000E-01
```

5.100E+00	7.581427E-11	1.000E-02	-1.516998E+00	7.968970E+00	1.427632E+01	1.004453E+02
5.200E+00	9.544451E-11	1.000E-02	-1.494684E+00	7.968916E+00	1.405264E+01	1.005867E+02
5.300E+00	1.201575E-10	1.000E-02	-1.472371E+00	7.968862E+00	1.382897E+01	1.007294E+02
5.400E+00	1.512693E-10	1.000E-02	-1.450059E+00	7.968809E+00	1.360532E+01	1.008713E+02
5.500E+00	1.904368E-10	1.000E-02	-1.427748E+00	7.968757E+00	1.338169E+01	1.010151E+02
5.600E+00	2.397458E-10	1.000E-02	-1.405439E+00	7.968705E+00	1.315807E+01	1.011550E+02
5.700E+00	3.018220E-10	1.000E-02	-1.383130E+00	7.968653E+00	1.293447E+01	1.013009E+02

Contact

- Hajime Togas
  Nishina Cent
  2-1 Hirosawa, Wako, Saitama 351-0198, Japan

3. 高密度天体現象への適用



中性子星の質量と半径の関係

J1614-2230 (Nature 467 (2010) 1081, APJ 832 (2016) 167)
J0348+0432 (Science 340 (2013) 1233232)
J0740+6620 (Nat. Astron. (2019))

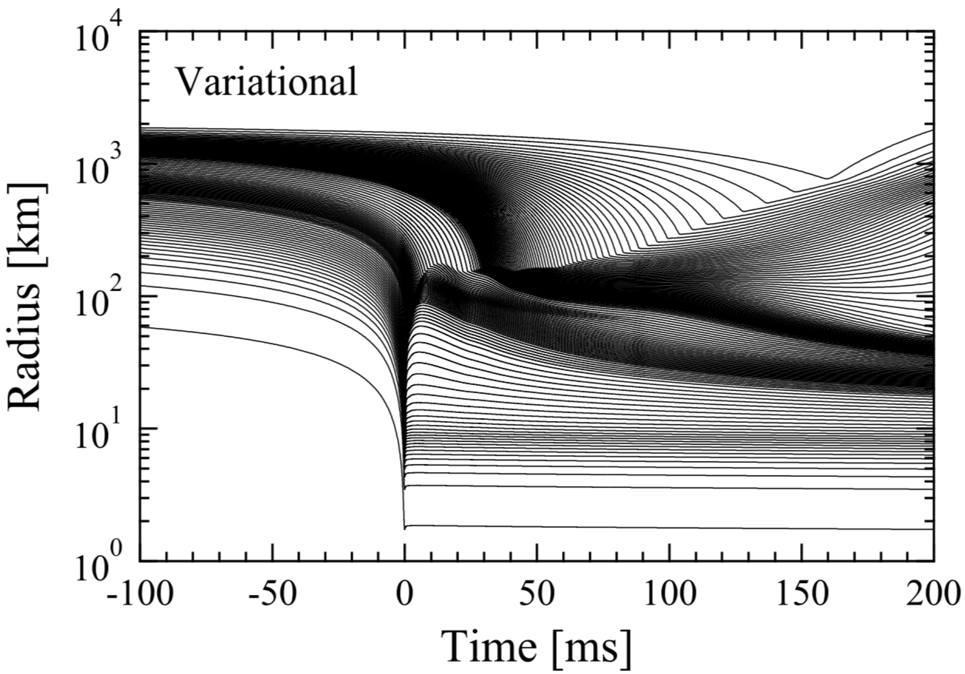
GW170817 (PRL 121 (2018) 161101)
Shaded region (APJ 722 (2010) 33)

超新星爆発シミュレーションへの適用

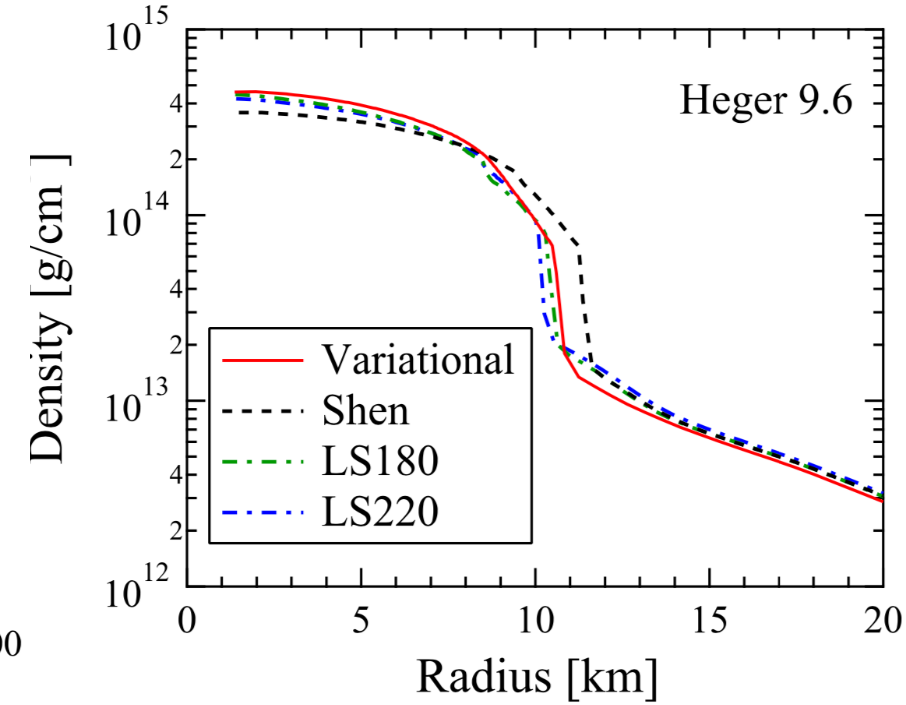
1D neutrino-radiation hydrodynamics simulations

Progenitor model : 9.6 $M_\odot$ star (provided by A. Heger)

SN simulation numerical code: K. Sumiyoshi, et al., APJ 629 (2005) 922



Radial trajectories of mass elements



Variational : 4.59×10^{14} g/cm^3
Shen : 3.55×10^{14} g/cm^3
LS180 : 4.46×10^{14} g/cm^3
LS220 : 4.22×10^{14} g/cm^3

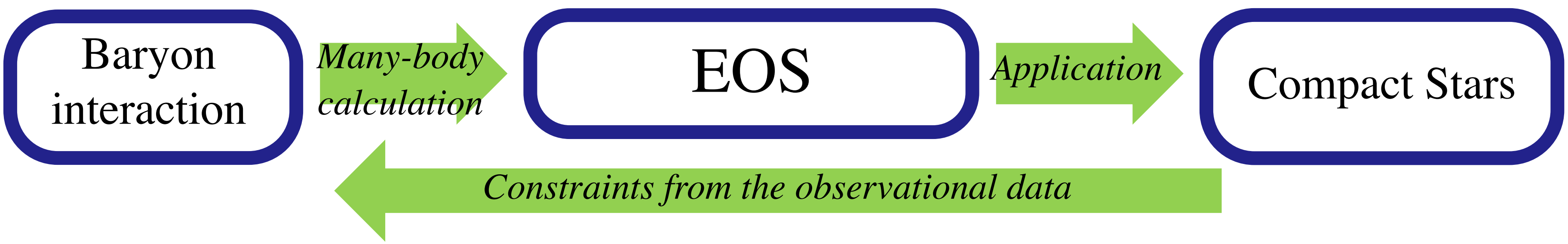
HT *et al.*, in preparation

まとめ

現実的核力に基づく新しい核物質状態方程式を作成し高密度天体現象（中性子星・超新星爆発）に適用した。

一様核物質: クラスター変分法　非一様核物質 : Thomas-Fermi 近似

http://www.np.phys.waseda.ac.jp/EOS/



- *Uncertainty in the three-nucleon force*
- *Hyperon mixing in high-density matter　etc…*