

# HALQCD 相互作用を用いた $NN\Omega$ の三体構造研究

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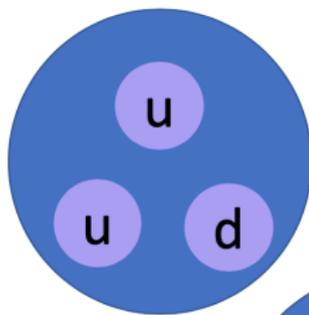
山本 拓実 (M1)<sup>A</sup> 肥山 詠美子<sup>A,B</sup>

December 23, 2019

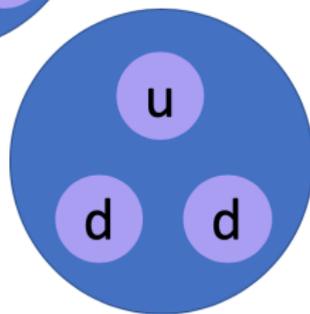
<sup>A</sup> 九州大学      <sup>B</sup> 理研

# Introduction

	I	II	III
mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	<b>u</b> up	<b>c</b> charm	<b>t</b> top
<b>QUARKS</b>	$\approx 4.7 \text{ MeV}/c^2$	$\approx 96 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom



陽子



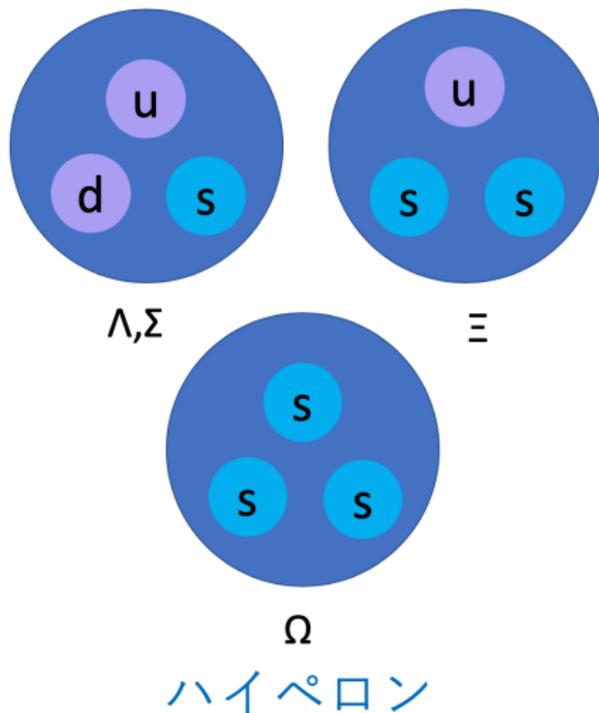
中性子

[https://upload.wikimedia.org/wikipedia/commons/0/00/Standard\\_Model\\_of\\_Elementary\\_Particles.svg](https://upload.wikimedia.org/wikipedia/commons/0/00/Standard_Model_of_Elementary_Particles.svg)

# Introduction

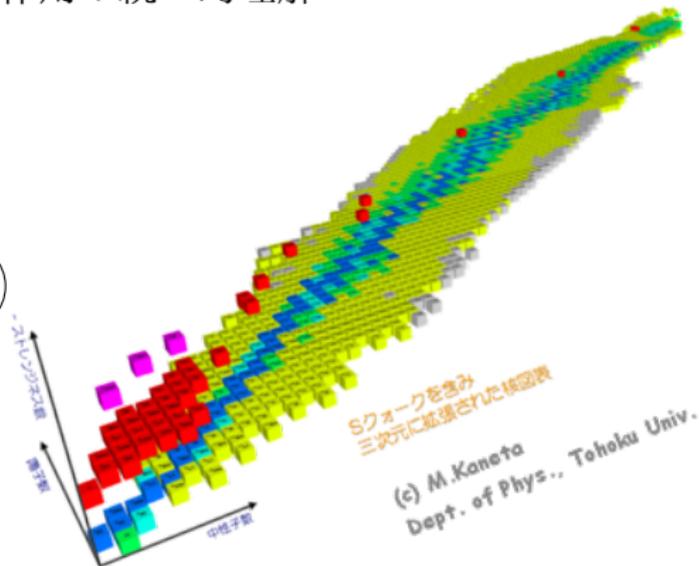
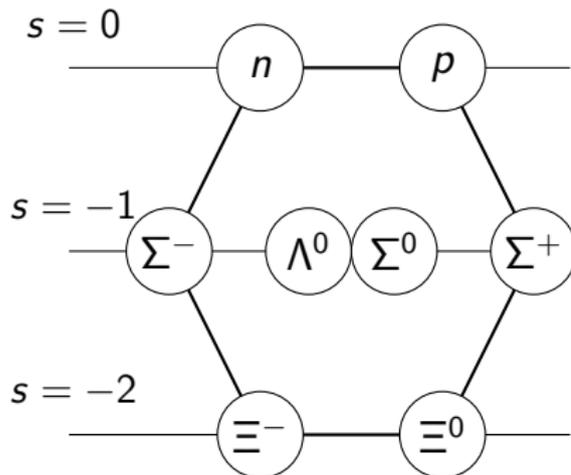
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QUARKS



# Introduction

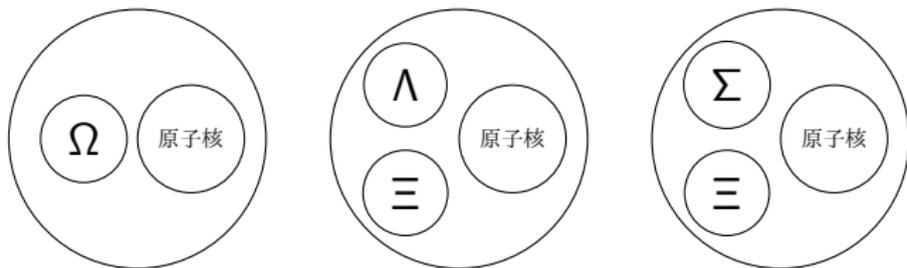
- ハイパー核の分野の研究目的：全てのバリオン間相互作用の情報を得ること
  - 核力,  $\Lambda N$ ,  $\Sigma N$ ,  $\Xi N$  相互作用の統一的理解



[http://lambda.phys.tohoku.ac.jp/img/3d\\_nuclear\\_chart\\_01\\_s.png](http://lambda.phys.tohoku.ac.jp/img/3d_nuclear_chart_01_s.png)

# Introduction

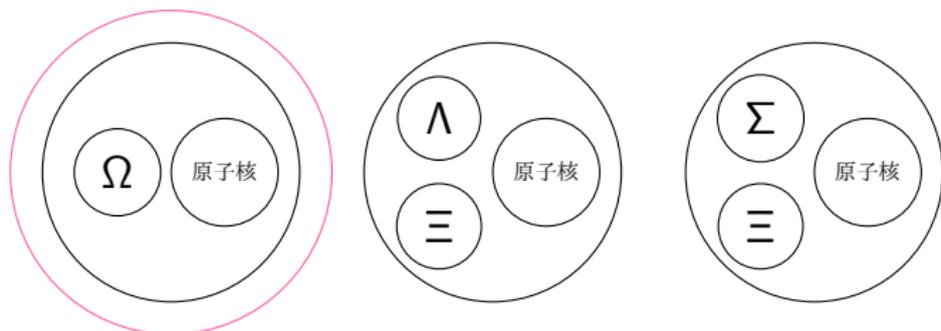
- 次の狙い  $\Rightarrow S = -3$  の場合の情報を得ること



- HAL QCD Collaboration により  $N\Omega$  相互作用についての研究がなされた
- $m_\pi \simeq 146$  MeV,  $m_K \simeq 525$  MeV
  - T. Iritani *et al.* (HAL QCD Collaboration), arXiv:1810.03416 (2019).

# Introduction

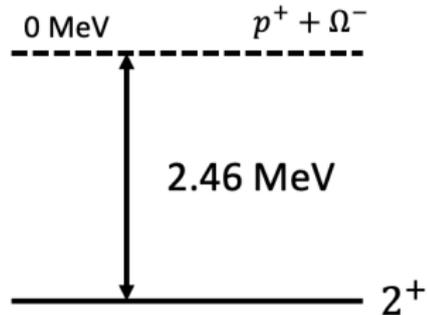
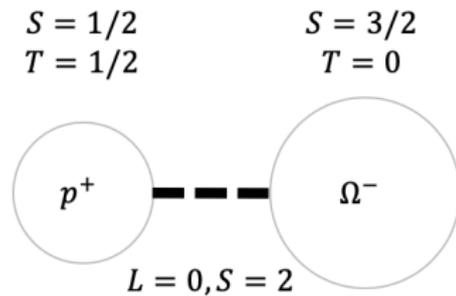
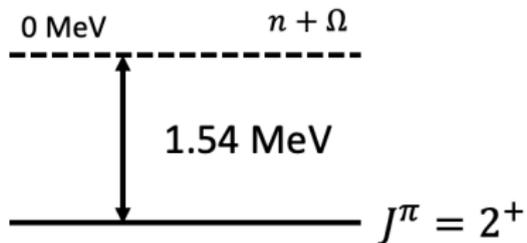
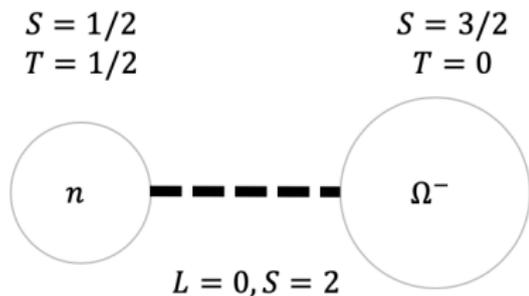
- 次の狙い  $\Rightarrow S = -3$  の場合の情報を得ること



今回のターゲット

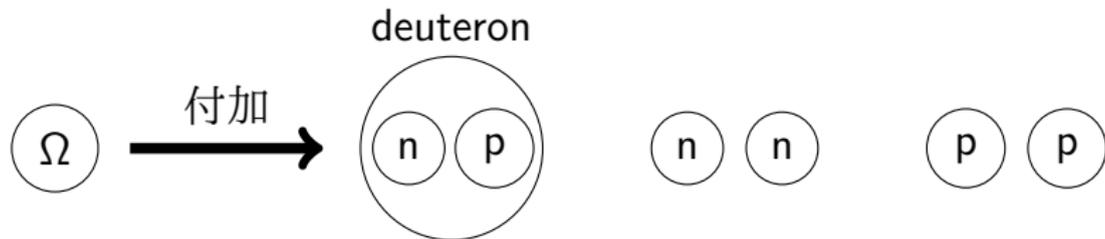
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# Introduction



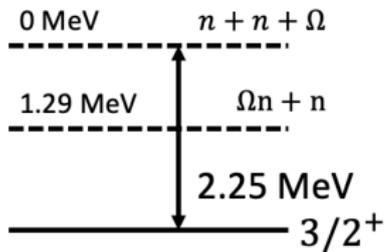
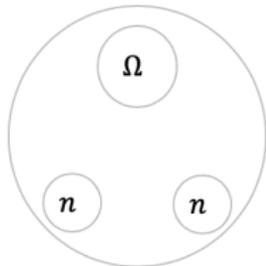
# Introduction

- $\Rightarrow$  Garcilazo らは  $\Omega NN$  三体構造の計算を行った
- H. Garcilazo, and A. Valcarce, Phys. Rev. C **99**, 014001 (2019).

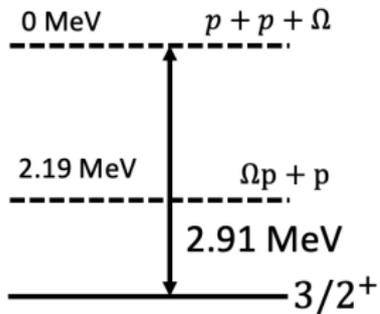
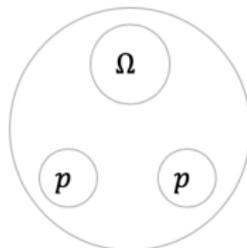


束縛？非束縛？

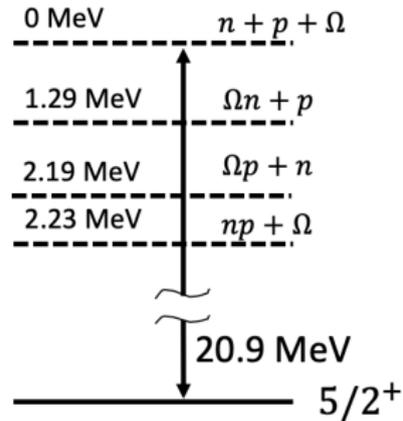
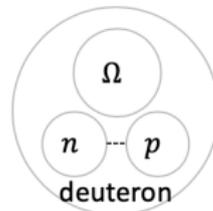
$$J^\pi = 3/2^+$$



$$J^\pi = 3/2^+$$



$$J^\pi = 5/2^+$$



# Motivation

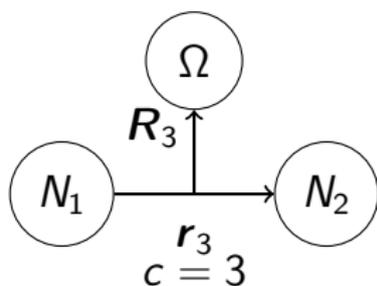
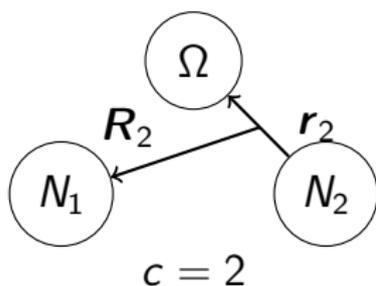
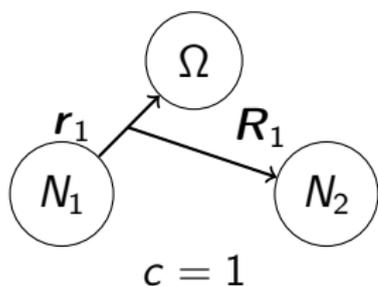
- $\Omega d$  について構造が全体的にコンパクトになっている
- $\Rightarrow NN$  相互作用の核力の違いが  $\Omega NN$  の束縛エネルギーに影響する可能性がある
  - Malfliet-Tjon potential(Garcilazo が使用したもの)
    - J. L. Friar *et al.* Phys. Rev. C **42**, 1838 (1990).
  - Minnesota potential
    - D. R. Thompson *et al.* Nucl. Phys. A **286**, 53 (1977).
  - AV8 potential
    - B. S. Pudliner *et al.* Phys. Rev. C **56**, 1720 (1997).

# Method

## Gaussian Expansion Method

### Schrödinger equation

$$\left[ T + V_{\Omega N}^{(1)}(r_1) + V_{\Omega N}^{(2)}(r_2) + V_{NN}^{(3)}(r_3) - E \right] \Psi_{JM,TT_z} = 0$$



E. Hiyama *et al.* Progress in Particle and Nuclear Physics Volume **51**, 223 (2003).

# Method

$$\Psi_{JM,TT_z} = \Phi_{JM,TT_z}^{(c=1)}(\mathbf{r}_1, \mathbf{R}_1) + \Phi_{JM,TT_z}^{(c=2)}(\mathbf{r}_2, \mathbf{R}_2) + \Phi_{JM,TT_z}^{(c=3)}(\mathbf{r}_3, \mathbf{R}_3)$$

$$\Phi_{JM,TT_z}^{(c)}(\mathbf{r}_c, \mathbf{R}_c) = \sum_{\alpha} A_{\alpha}^{(c)} [ [ [\chi_{s1} \otimes \chi_{s2}]_s \otimes \chi_{s3} ]_{\Sigma} \otimes [ \phi_{n_c l_c}^G(\mathbf{r}_c) \otimes \psi_{N_c L_c}^G(\mathbf{R}_c) ]_{\Lambda} ]_{JM} [ [\eta_{t1} \otimes \eta_{t2}]_{T'} \otimes \eta_{t3} ]_{TT_z}$$

$(\alpha \equiv \{s_1, s_2, s_3, s, \Sigma, n_c l_c, N_c L_c, \Lambda, JM, t_1, t_2, t_3, T', TT_z\})$

where

$$\phi_{nlm}^G(\mathbf{r}) = \phi_{nl}^G(r) Y_{lm}(\hat{\mathbf{r}}), \quad \phi_{nl}^G(r) = N_{nl} r^l e^{-\nu_n r^2} \quad (n = 1 - n_{max})$$

$$\psi_{NLM}^G(\mathbf{R}) = \psi_{NL}^G(R) Y_{LM}(\hat{\mathbf{R}}), \quad \psi_{NL}^G(R) = N_{NL} R^L e^{-\lambda_N R^2} \quad (N = 1 - N_{max})$$

$$\nu_n = 1/r_n^2, \quad r_n = r_1 a^{n-1} \quad (n = 1 - n_{max})$$

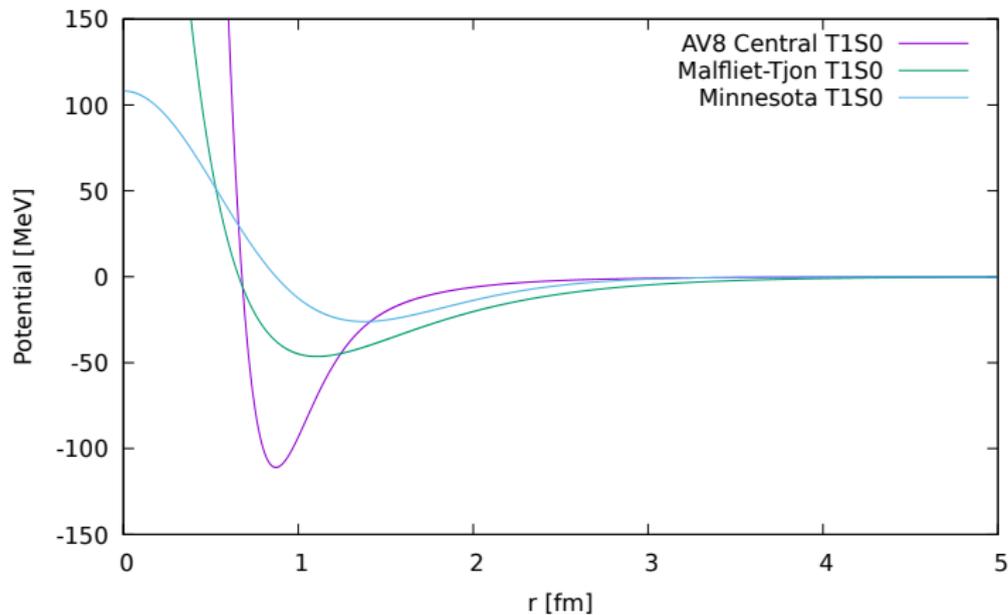
$$\lambda_N = 1/R_N^2, \quad R_N = R_1 A^{N-1} \quad (N = 1 - N_{max})$$

## Method

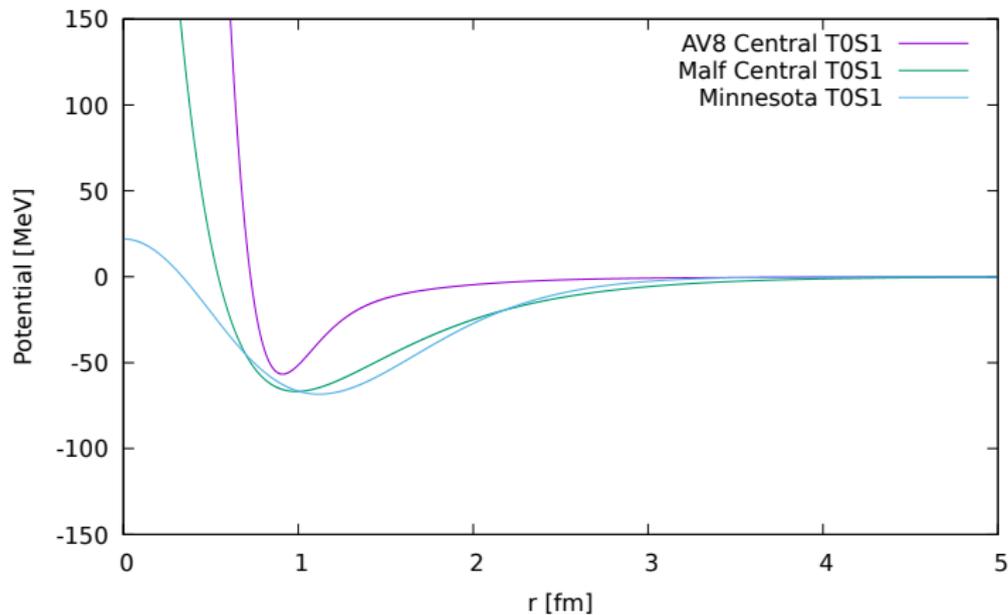
According to Rayleigh-Ritz variational principle

$$\left( \begin{array}{c} \mathbf{H} - E\mathbf{N} \end{array} \right) \left( \begin{array}{c} \mathbf{A}_{\alpha}^{(c)} \end{array} \right) = \mathbf{0}$$

# $NN$ interactions $T=1$ $S=0$



# $NN$ interactions $T=0$ $S=1$



# Eigenenergy ( $\Omega nn$ & $\Omega pp$ $J^\pi = 3/2^+$ )

$\Omega nn$

Malfliet-Tjon

Minnesota

AV8

0 MeV ( $\Omega n$ ) +  $n$

0 MeV ( $\Omega n$ ) +  $n$

0 MeV ( $\Omega n$ ) +  $n$

1.01 MeV  $\underline{\hspace{1.5cm}}$   $3/2^+$

0.86 MeV  $\underline{\hspace{1.5cm}}$   $3/2^+$

1.06 MeV  $\underline{\hspace{1.5cm}}$   $3/2^+$

$\Omega pp$

Malfliet-Tjon

Minnesota

AV8

0 MeV ( $\Omega p$ ) +  $p$

0 MeV ( $\Omega p$ ) +  $p$

0 MeV ( $\Omega p$ ) +  $p$

1.14 MeV  $\underline{\hspace{1.5cm}}$   $3/2^+$

0.98 MeV  $\underline{\hspace{1.5cm}}$   $3/2^+$

1.20 MeV  $\underline{\hspace{1.5cm}}$   $3/2^+$

# Eigenenergy ( $\Omega d$ $J^\pi = 5/2^+$ )

Malfliet-Tjon

0 MeV  $d + \Omega$

Minnesota

0 MeV  $d + \Omega$

AV8

0 MeV  $d + \Omega$

19.3 MeV  $5/2^+$

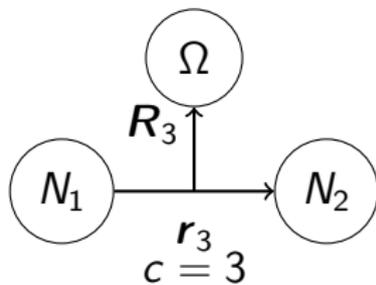
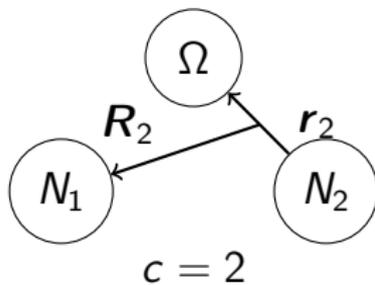
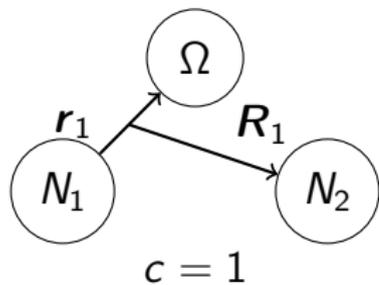
19.0 MeV  $5/2^+$

20.6 MeV  $5/2^+$

# Root mean square radius ( $\Omega nn$ )

System	Target	Malf	Minne
$\Omega n$	$\sqrt{\langle r^2 \rangle}$	4.11	
$\Omega nn$	$\sqrt{\langle r_1^2 \rangle}, \sqrt{\langle r_2^2 \rangle}$	4.39	4.62
	$\sqrt{\langle R_1^2 \rangle}, \sqrt{\langle R_2^2 \rangle}$	4.30	4.55
	$\sqrt{\langle r_3^2 \rangle}$	5.42	5.76
	$\sqrt{\langle R_3^2 \rangle}$	3.46	3.61

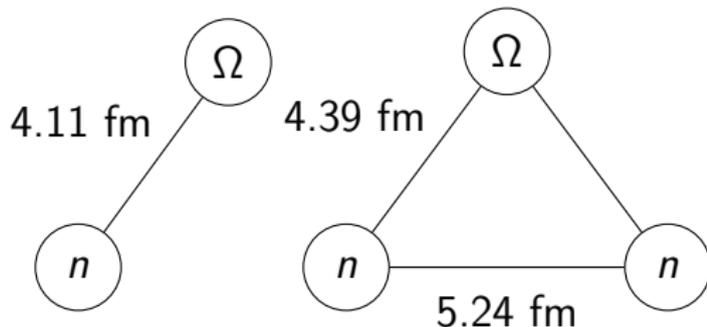
Table 1: Root mean square radius [fm]



# Root mean square radius ( $\Omega nn$ )

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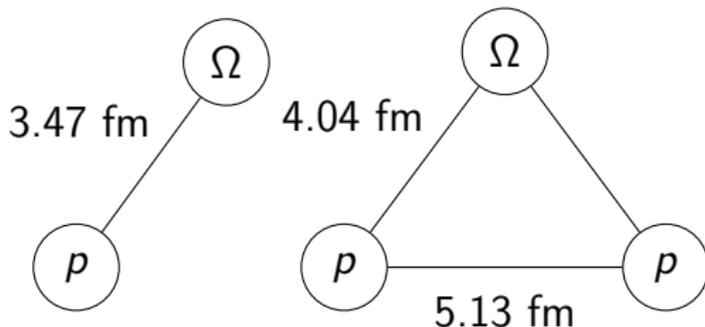
Table 1: Root mean square radius [fm]



# Root mean square radius ( $\Omega pp$ )

System	Target	Malf	Minne
$\Omega p$	$\sqrt{\langle r^2 \rangle}$	3.47	
$\Omega pp$	$\sqrt{\langle r_1^2 \rangle}, \sqrt{\langle r_2^2 \rangle}$	4.08	4.29
	$\sqrt{\langle R_1^2 \rangle}, \sqrt{\langle R_2^2 \rangle}$	4.04	4.26
	$\sqrt{\langle r_3^2 \rangle}$	5.13	5.44
	$\sqrt{\langle R_3^2 \rangle}$	3.18	3.31

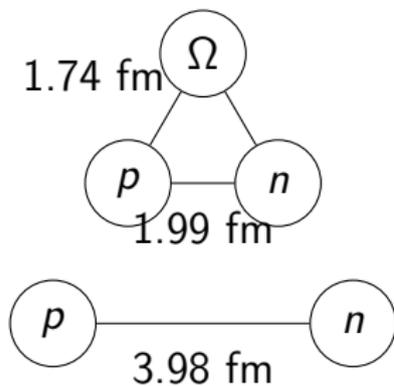
Table 2: Root mean square radius [fm]



# Root mean square radius ( $\Omega d$ )

System	Target	Malf	Minne
$\Omega d$	$\sqrt{\langle r_1^2 \rangle}, \sqrt{\langle r_2^2 \rangle}$	1.74	1.68
	$\sqrt{\langle R_1^2 \rangle}, \sqrt{\langle R_2^2 \rangle}$	1.64	1.57
	$\sqrt{\langle r_3^2 \rangle}$	1.99	1.90
	$\sqrt{\langle R_3^2 \rangle}$	1.43	1.39

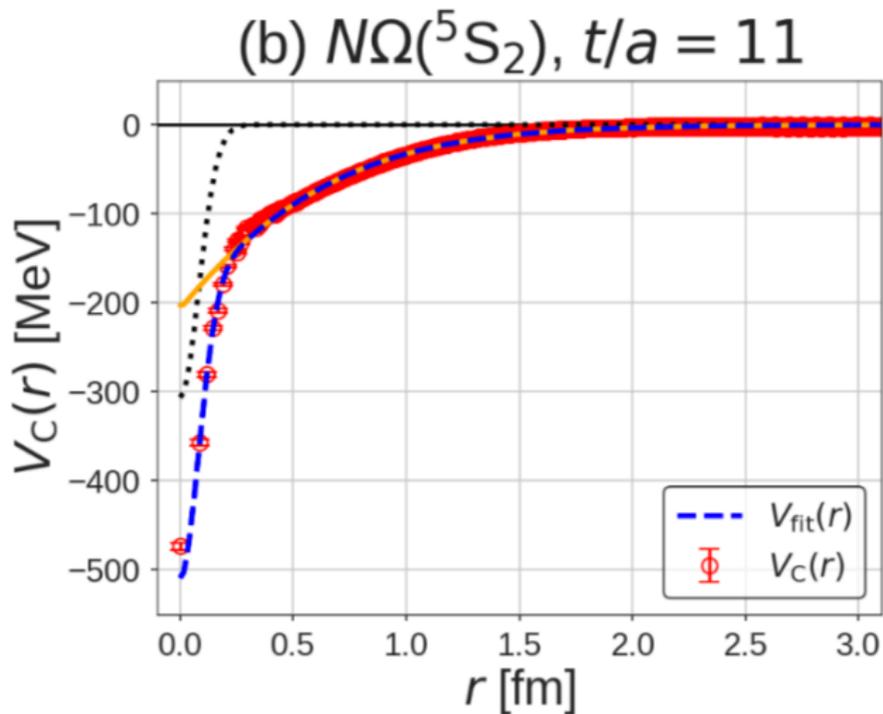
Table 3: Root mean square radius [fm]



# Summary

- Gaussian Expansion Method を用いて、 $\Omega NN$  三体構造を計算した
- 3つの異なる  $NN$  相互作用を用いて計算した
- Lowest threshold から  $\Omega d$  は 20 MeV ほど束縛し、強く縮んだ構造であった
- $\Rightarrow NN$  相互作用の斥力芯が束縛状態に影響を及ぼしていた
- $\Omega NN$  の三体構造の原子核の大きさを計算した
- その中でも  $\Omega d$  は強く縮んでいた

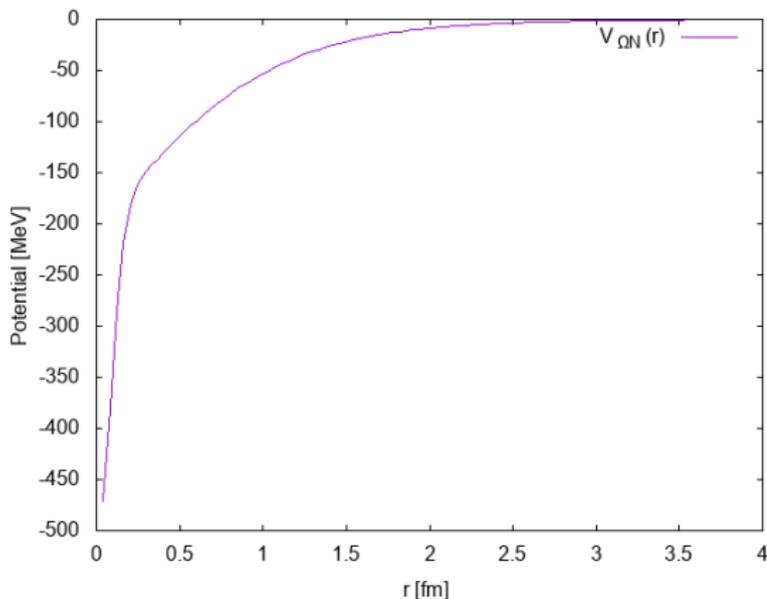
$\Rightarrow \Omega NN$  三体系について  $NN$  相互作用の核力の違いが、どのように構造に影響を及ぼすのかを研究した



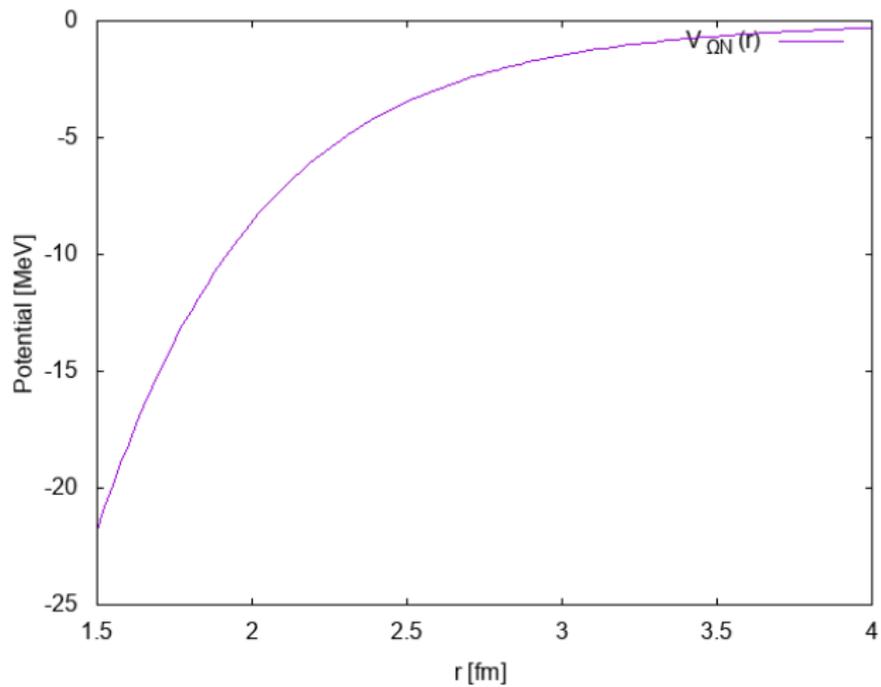
# $\Omega N$ interaction

The plots are fitted by

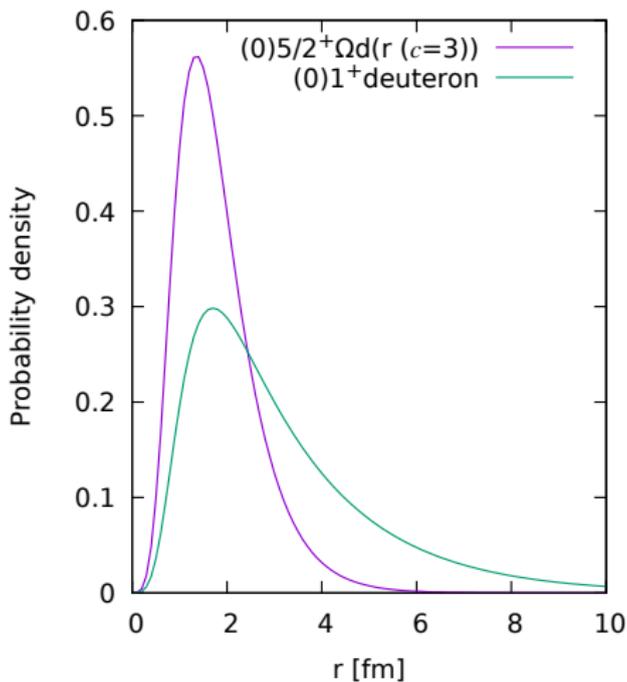
$$V_{\Omega N}(r) = b_1 e^{-b_2 r^2} + b_3 (1 - e^{b_4 r^2}) \left( \frac{e^{-m_\pi r}}{r} \right)^2$$



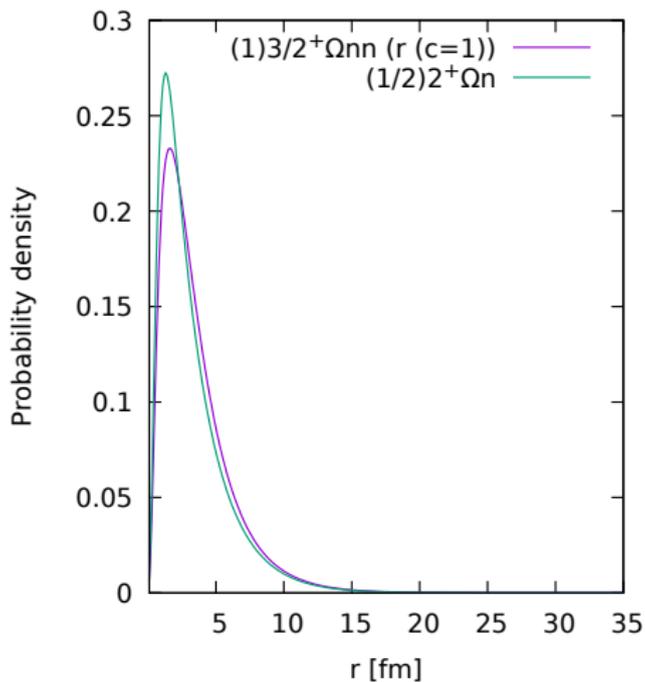
# $\Omega N$ interaction



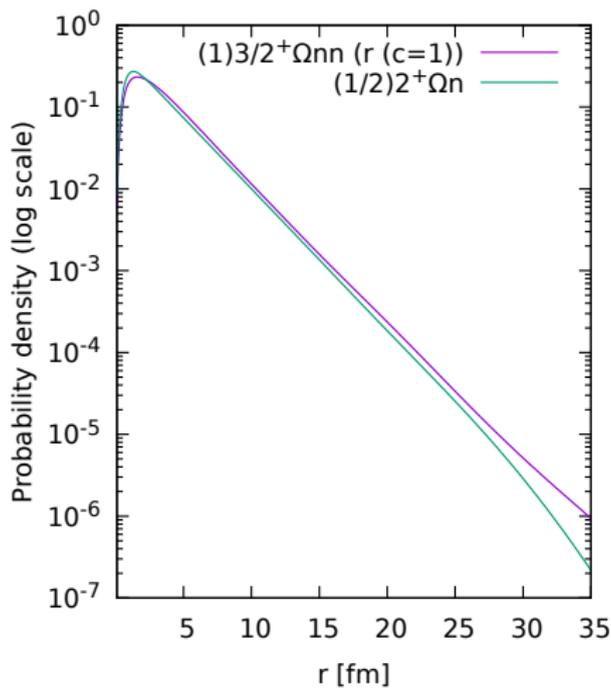
# Density distribution ( $(0)5/2^+ \Omega d$ and $(0)1^+$ deuteron)



# Density distribution $((1)3/2^+\Omega_{nn}$ and $(1/2)2^+\Omega_n$ )



# Density distribution $((1)3/2^+\Omega_{nn}$ and $(1/2)2^+\Omega_n$



## Binding energy

Parameter set			$p_1$	$p_2$	$p_3$	$p_4$
$(I)J^P$	System		Binding energy [MeV]			
Garcilazo	$(1/2)2^+$	$\Omega n$	1.29(1.52)	1.38(1.61)	1.29(1.44)	1.37(1.60)
Ours	$(1/2)2^+$	$\Omega n$	1.29(1.52)	1.38(1.61)	1.22(1.44)	1.37(1.60)
Difference			0.00(0.00)	0.00(0.00)	0.07(0.00)	0.00(0.00)
Garcilazo	$(1/2)2^+$	$\Omega p$	2.19(2.42)	2.30(2.53)	2.19(2.34)	2.28(2.51)
Ours	$(1/2)2^+$	$\Omega p$	2.16(2.42)	2.26(2.54)	2.08(2.35)	2.25(2.52)
Difference			0.03(0.00)	0.04(0.01)	0.11(0.01)	0.03(0.01)
Malfliet-Tjon potential						
Garcilazo	$(1)3/2^+$	$\Omega nn$	2.25(2.60)	2.35(2.72)	2.14(2.50)	2.34(2.71)
Ours	$(1)3/2^+$	$\Omega nn$	2.30(2.67)	2.41(2.79)	2.20(2.57)	2.40(2.77)
Difference			0.05(0.07)	0.06(0.07)	0.06(0.07)	0.06(0.06)
Garcilazo	$(1)3/2^+$	$\Omega pp$	2.91(3.30)	3.04(3.43)	2.81(3.20)	3.02(3.41)
Ours	$(1)3/2^+$	$\Omega pp$	3.30(3.71)	3.44(3.86)	3.20(3.62)	3.41(3.83)
Difference			0.39(0.41)	0.40(0.43)	0.39(0.42)	0.39(0.42)
Garcilazo	$(0)5/2^+$	$\Omega d$	20.9(22.0)	21.3(22.4)	20.7(21.8)	21.2(22.3)
Ours	$(0)5/2^+$	$\Omega d$	21.5(22.6)	22.0(23.1)	21.4(22.5)	21.8(22.9)
Difference			0.6(0.6)	0.7(0.7)	0.7(0.7)	0.6(0.6)
(We newly incorporated different $NN$ int. from previous research.)						
Minnesota potential						
Ours	$(1)3/2^+$	$\Omega nn$	2.15(2.50)	2.26(2.62)	2.05(2.40)	2.25(2.60)
Ours	$(1)3/2^+$	$\Omega pp$	3.14(3.53)	3.27(3.67)	3.04(3.43)	3.25(3.64)
Ours	$(0)5/2^+$	$\Omega d$	22.8(24.0)	23.3(24.5)	22.7(23.9)	23.1(24.3)
AV8 potential						
Ours	$(1)3/2^+$	$\Omega nn$	2.35(2.76)	2.47(2.88)	2.25(2.65)	2.45(2.87)
Ours	$(1)3/2^+$	$\Omega pp$	3.37(3.82)	3.50(3.97)	3.27(3.72)	3.48(3.94)