中性子星と原子核飽和パラメータ

祖谷 元 (RIKEN)

contents

- Neutron stars (NSs) & equation of state (EOS)
  - nuclear saturation parameters
- Low-mass NSs and its mass formula
  - introduction of a new parameter η
  - comment on the possible maximum mass of NSs
  - (M, R) constraint from NICER obs.
- asteroseismology in NSs
  - magnetar QPOs and crustal torsional oscillations
  - fundamental oscillations
  - overtones
  - effect of pasta phases
- summary

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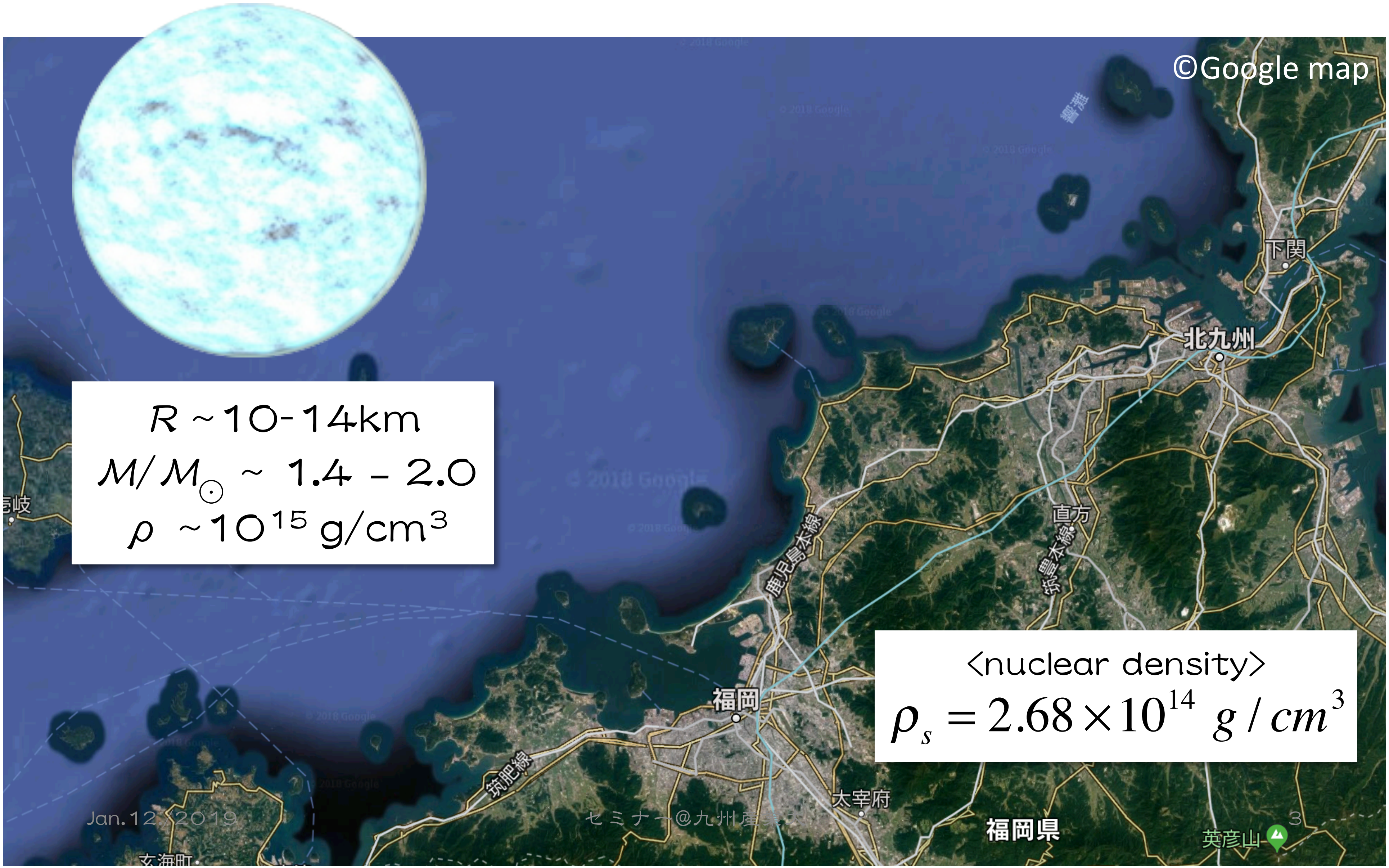
Properties of Neutron Stars (NSs)



$R \sim$ 10-14km

$M/M_\odot \sim 1.4 - 2.0$

$\rho \sim 10^{15}$ g/cm^3

<nuclear density>

$$\rho_s = 2.68 \times 10^{14} \ g/cm^3$$

NSs

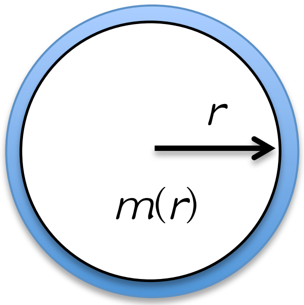
- produce via supernova
- extreme state
  - high density inside the star;
    central density may become ~5 x nuclear density
  - strong magnetic field;
    existence of "magnetars", where $B \sim 10^{14}$ G
    stronger than the critical field strength of the quantum electrodynamics
  - strong gravitational field;
    $GM/Rc^2 \sim 0.2$ (NS), 0.2×10^{-5} (Sun), 0.7×10^{-9} (Earth)
- NSs are a unique environment for understanding the physics under such extreme conditions

How to construct NSs

- Tolman-Oppenheimer-Volkoff (TOV) equation gives density profile of the spherically symmetric equilibrium of cold NSs.

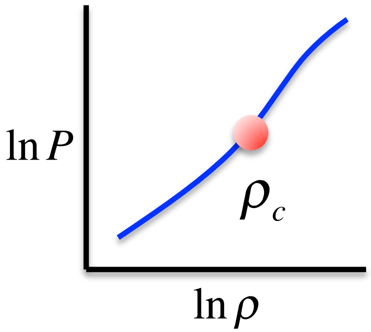
$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2} \underbrace{\left(1 + \frac{4\pi r^3 P}{mc^2}\right)\left(1 + \frac{P}{\rho c^2}\right)\left(1 - \frac{2Gm}{c^2 r}\right)^{-1}}_{\text{relativistic correction}}$$

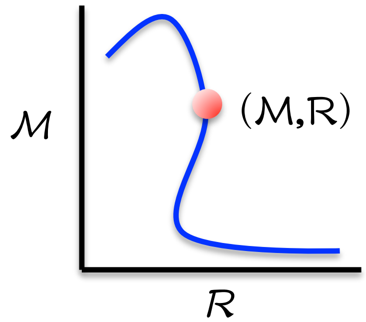
$$\frac{dm}{dr} = 4\pi r^2 \rho$$



$P = P(\rho)$ equation of state (EOS)

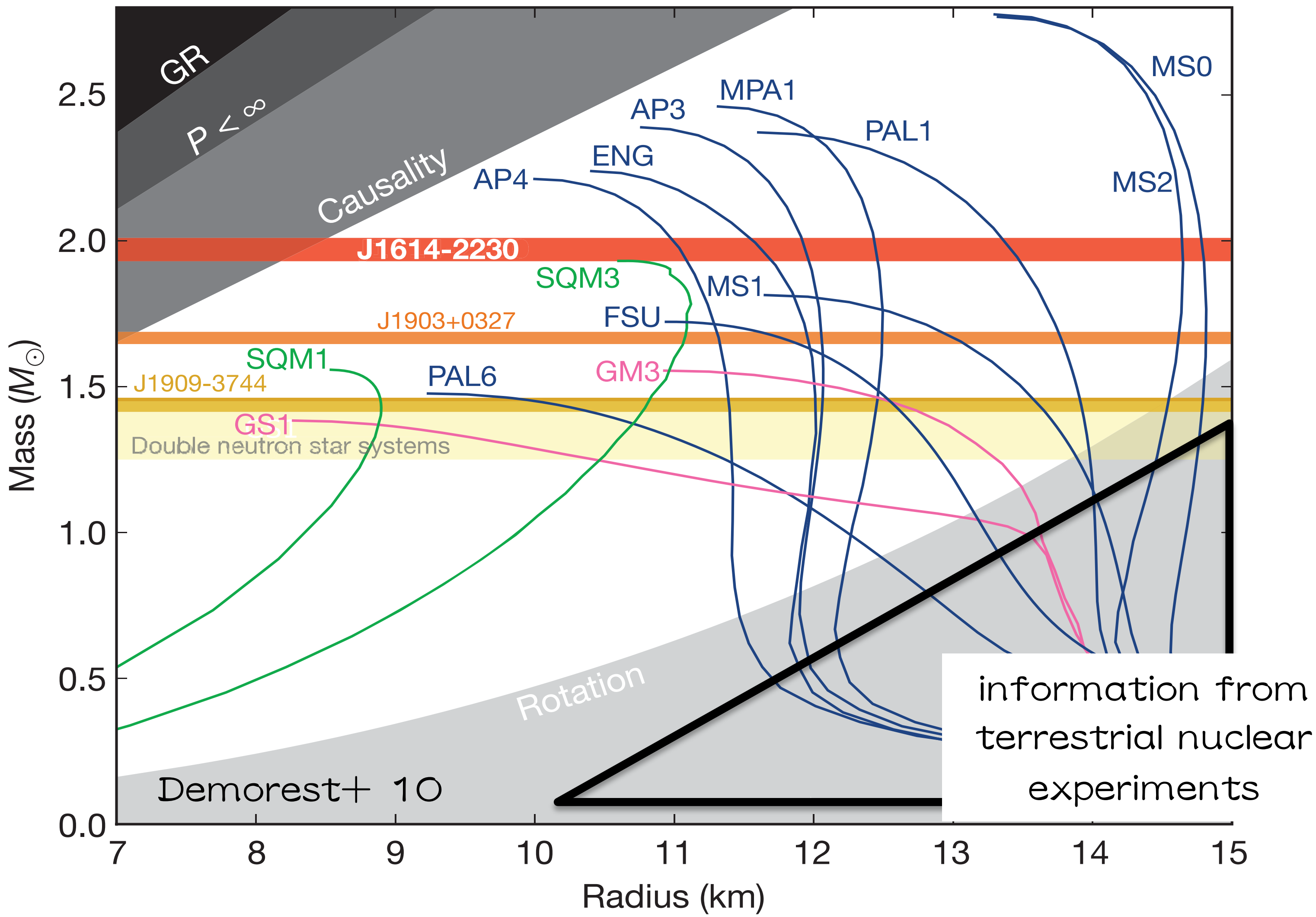
$$\Rightarrow \boxed{\begin{array}{l} R = r \quad @P = 0 \\ M = m(R) \end{array}}$$





EOS is essential to construct the stellar model, but still uncertain.

Cold NS & EOS



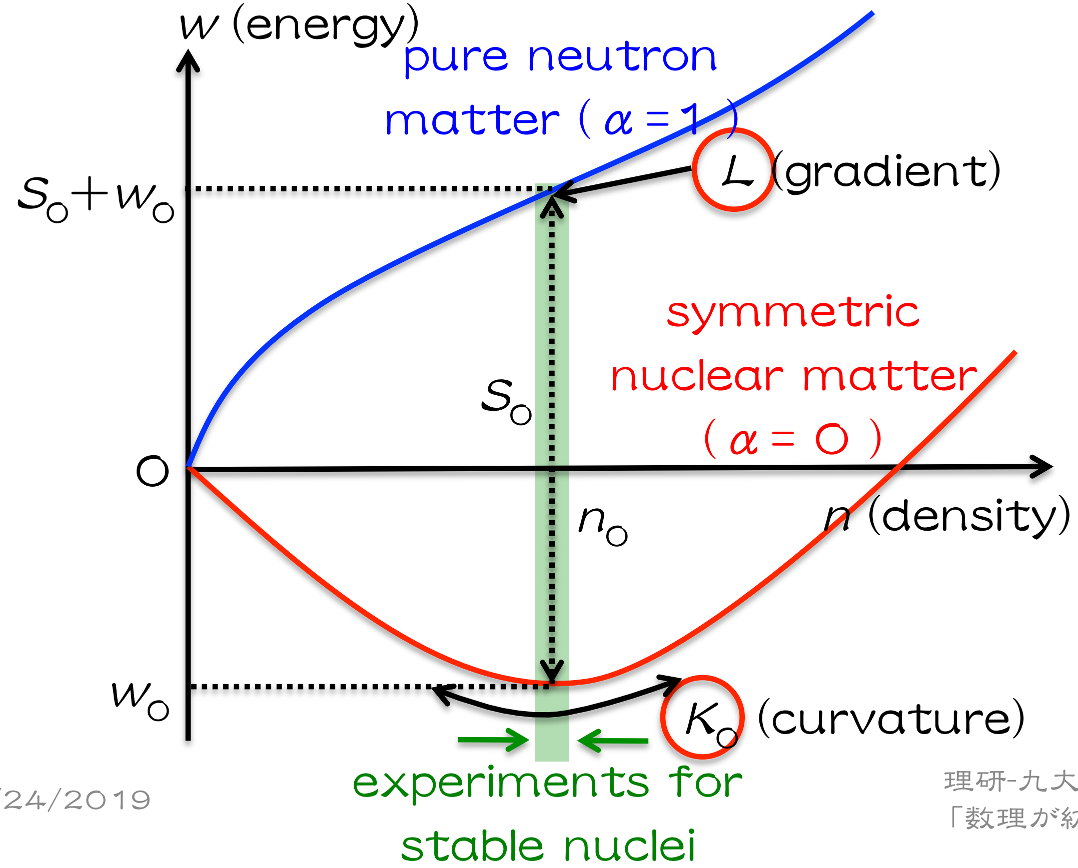
NS observations can make a constraint on EOS!!

EOS near the saturation point

- Bulk energy per nucleon near the saturation point of symmetric nuclear matter at zero temperature;

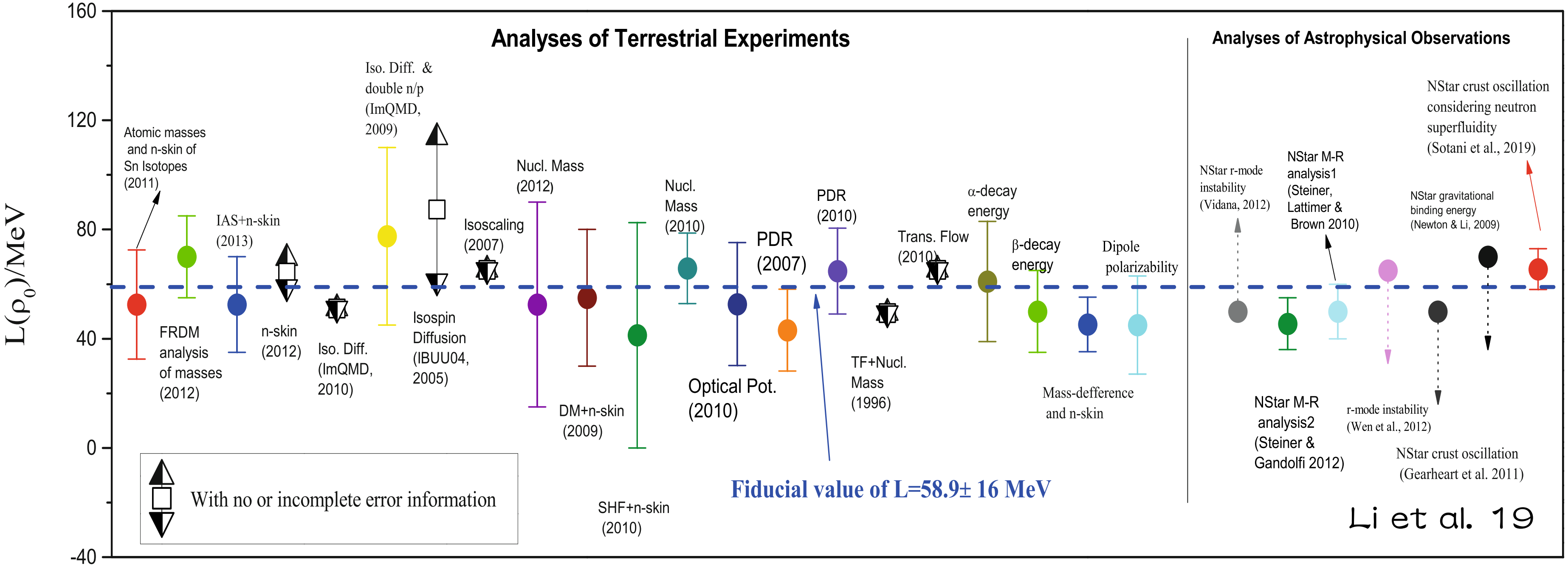
incompressibility　　symmetry parameter

$$w = w_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + \left[S_0 + \frac{L}{3n_0}(n-n_0)\right]\alpha^2$$



Any EOSs can be characterized by the saturation parameters

constraints on L



fiducial values

$L \approx 58.9 \pm 16$ MeV

$S_0 \approx 31.6 \pm 2.7$ MeV

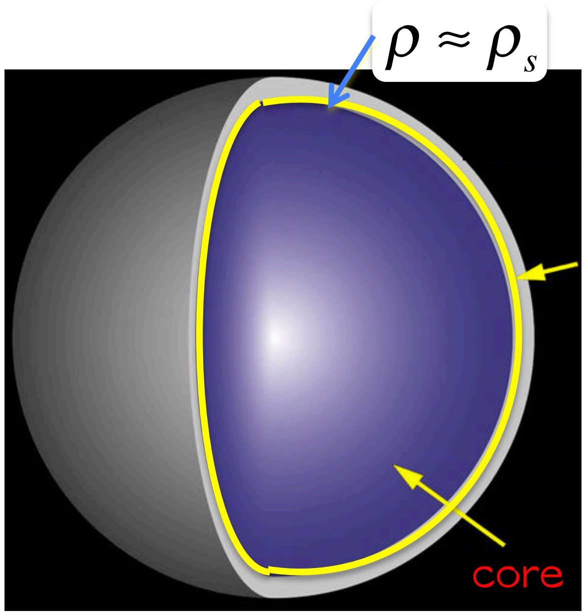
$K_0 \approx 230 \pm 20$ MeV

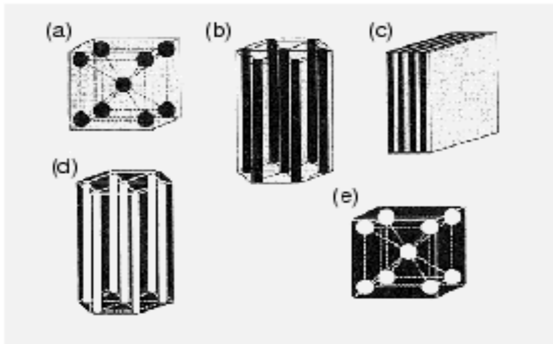
(Shlomo+ 06, Piekarewicz 10)

neutron stars

- Structure of NS
  - solid layer (crust)
  - nonuniform structure (pasta)
  - fluid core (uniform matter)

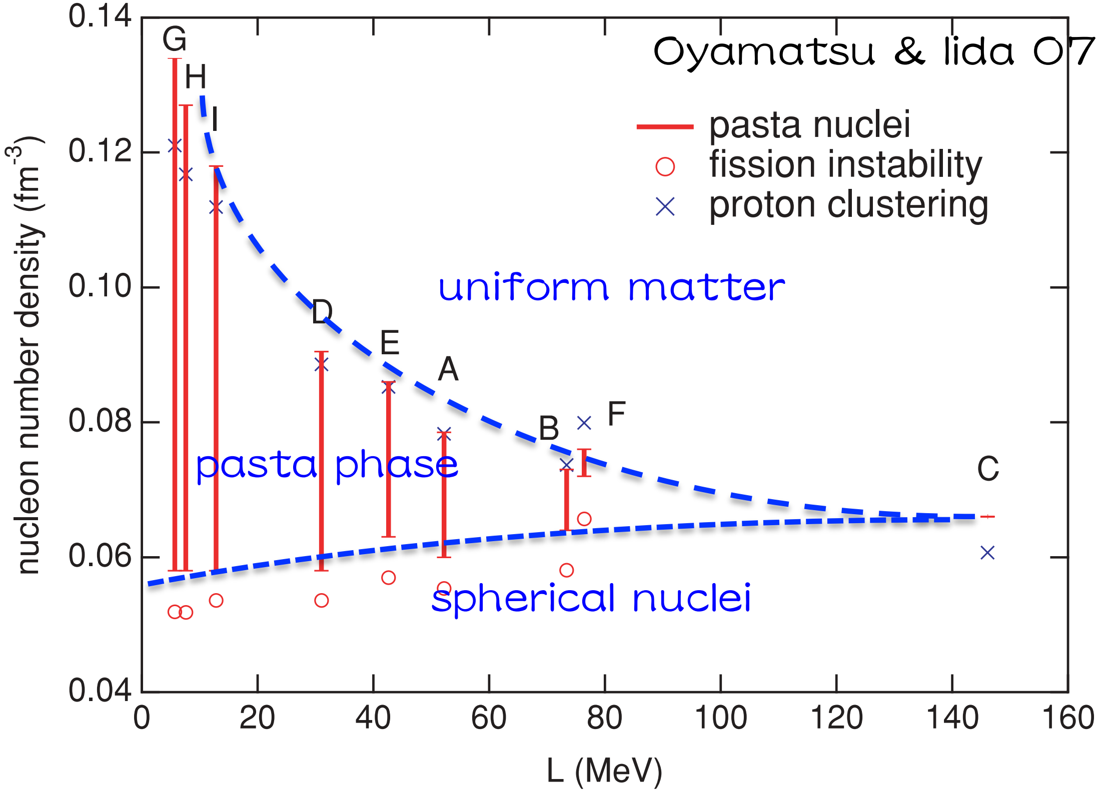
- In particular, the nuclear saturation parameters become important in
  - physics in crust region
  - low-mass NSs itself





Oyamatsu (1993)

pasta phase



- Whether pasta phase exists or not depends strongly on L.
- For $L \gtrsim$ 100MeV, pasta structure almost disappears.

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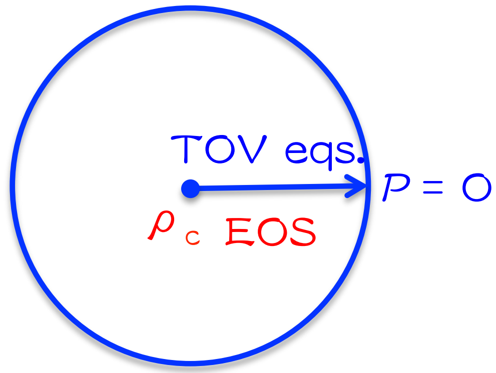
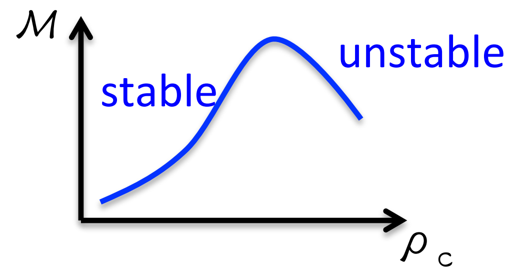
mass formula



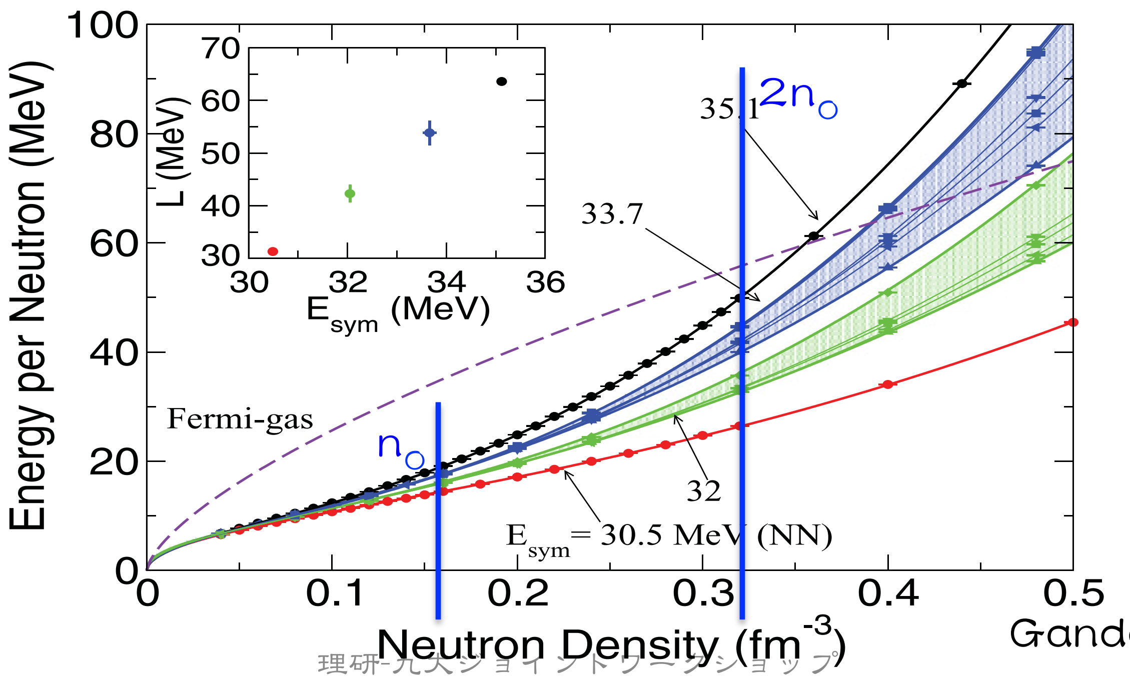
you can get a NS model !!

- well-known mass formula of stable nuclei
  - Bethe-Weizacker mass formula
  - due to the density saturation
- How about NSs ?
  - structure of NS is determined as a result of balance of gravity & pressure gradient.
  - in general, not so simple…
- we are successful to derive a mass formula of low-mass NS
  - as functions of nuclear **saturation parameter η** & central density
  - almost independent of the EOS models

low-mass NS models

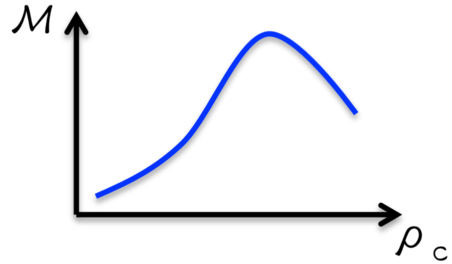


- low-mass NSs
  - low-central density
  - EOS for low-density region plays an important role
- EOS of nuclear matter for $\rho \lesssim \rho_o$ (normal nuclear density) would be determined with reasonable accuracy by terrestrial nuclear experiments.



Gandolfi+12

low-mass NS models



- low-mass NSs
  - low-central density
  - EOS for low-density region plays an important role
- EOS of nuclear matter for $\rho \lesssim \rho_0$ (normal nuclear density) would be determined with reasonable accuracy by terrestrial nuclear experiments.
- For $\rho \lesssim 2\rho_0$, one can almost neglect an uncertainty of three nucleon interaction (Gandolfi+12) and contribution from hyperon (or quark etc...).

→ we especially focus on the NS models for $\rho \lesssim 2\rho_0$

unified EOS modes

- unified-EOS models
  - based on the EOSs of nuclear matter with specific values of K_0 & L
  - consistent with empirical data of masses and radii of stable nuclei
  - describing both the crustal and core regions of NS

- we especially focus on
  - phenomenological EOS with various K_0 & L (Oyamatsu & Iida 03; 07)
  - EOSs based on relativistic mean field models
    - Shen EOS (Shen+ 98)
    - Miyatsu EOS (Miyatsu+ 13)
  - Skyrme-type effective interaction
    - FPS (Pethick+ 95),
    - SLy4 (Douchin & Haensel 01)
    - BSk19, BSk20, BSk21 (Potekhin+ 13)

EOSs based on the different theoretical models

MR relations

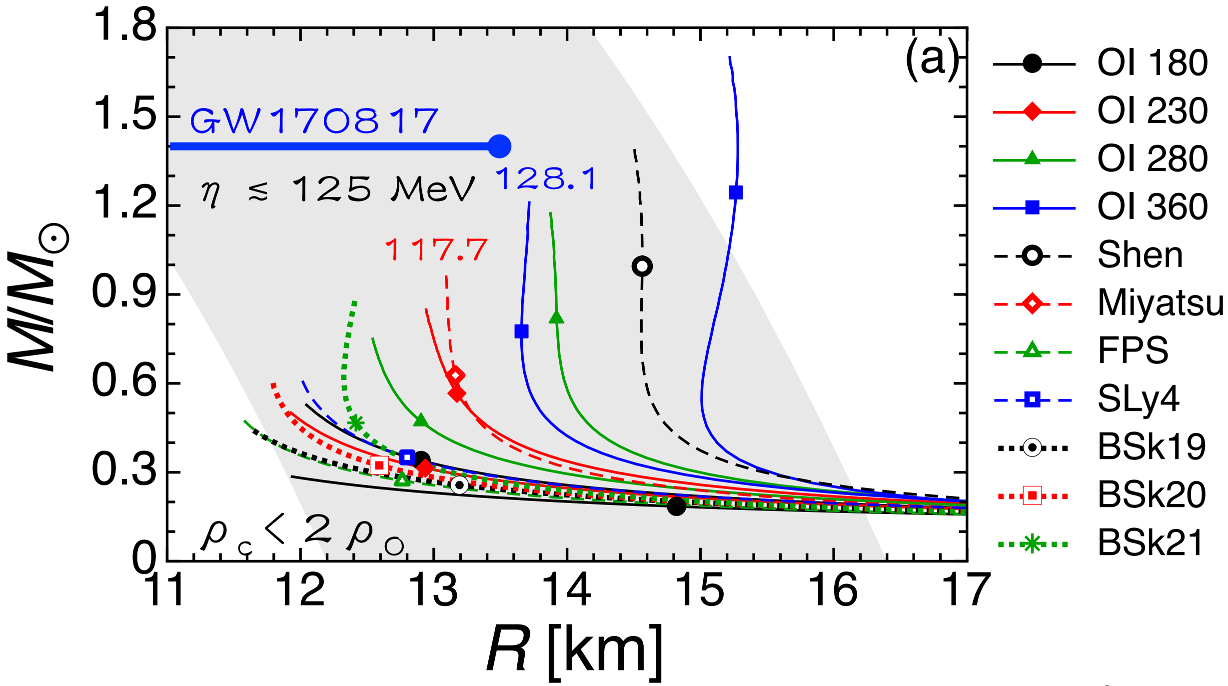
$$55.5 \lesssim \eta \lesssim 120 \text{ MeV.}$$

L = 30-80 MeV, K0 = 190-270 MeV

(η = 92.75 MeV; L = 58.9 MeV, K0 = 230 MeV)

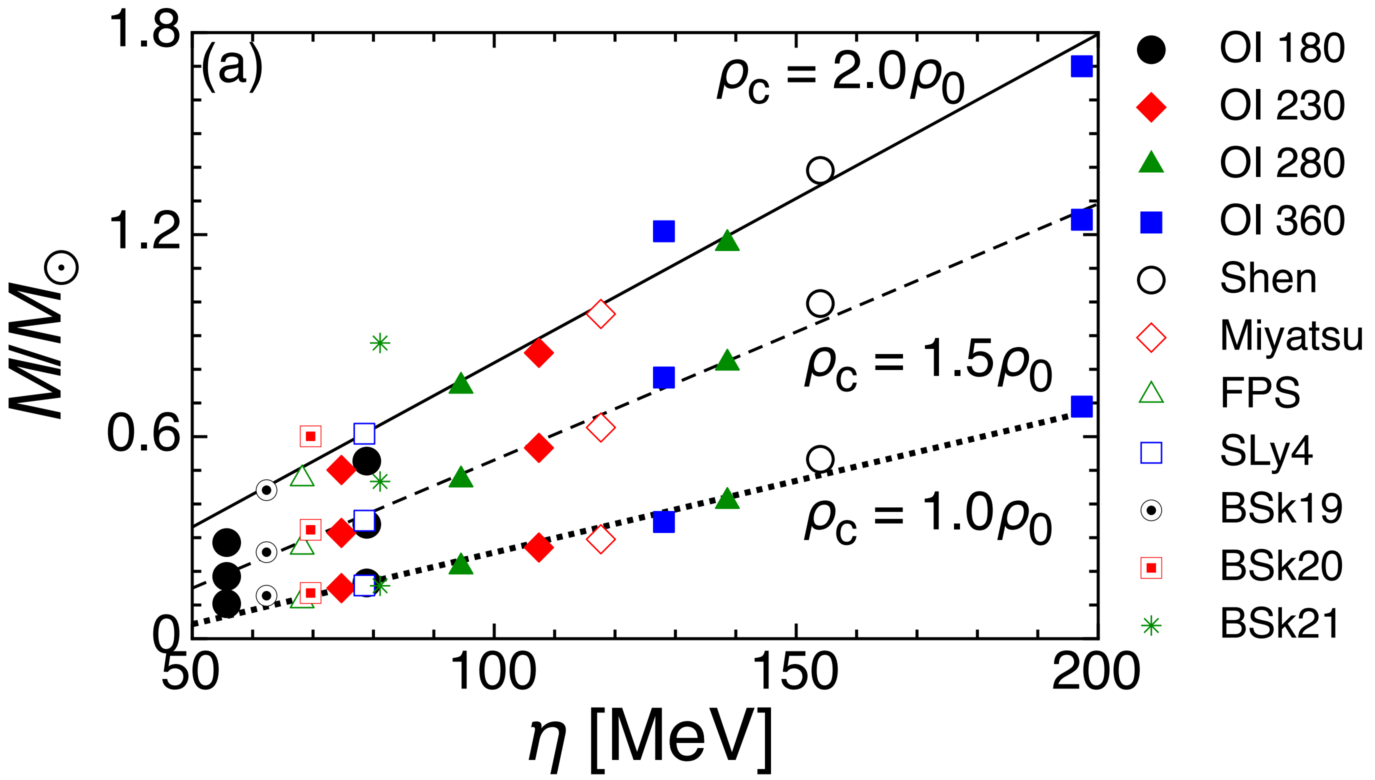
- NS models are constructed with various sets of K_0 & L
- We can find the specific combination of K_0 & L describing the low-mass NSs,

$$\eta = (K_0 L^2)^{1/3}$$

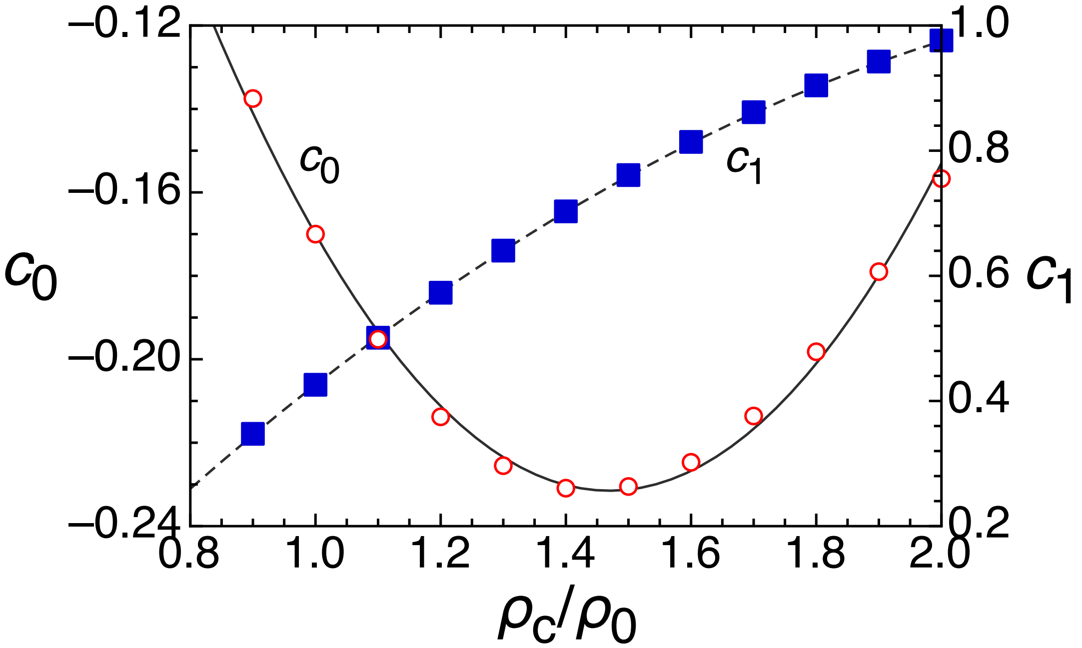


EOS	K_0 (MeV)	L (MeV)	η (MeV)
OI-EOS	180	31.0	55.7
	180	52.2	78.9
	230	42.6	74.7
	230	73.4	107.4
	280	54.9	94.5
	280	97.5	138.6
	360	76.4	128.1
	360	146.1	197.3
Shen	281	114	154.0
Miyatsu	274	77.1	117.7
FPS	261	34.9	68.2
SLy4	230	45.9	78.5
BSk19	237	31.9	62.3
BSk20	241	37.4	69.6
BSk21	246	46.6	81.1

mass formula



$$\frac{M}{M_\odot} = c_0 + c_1 \left(\frac{\eta}{100\,\mathrm{MeV}} \right)$$



$$u_c \equiv \rho_c/\rho_0$$

$$\boxed{\frac{M}{M_\odot} = 0.371 - 0.820u_c + 0.279u_c^2 - (0.593 - 1.254u_c + 0.235u_c^2)\left(\frac{\eta}{100\,\mathrm{MeV}}\right)}$$

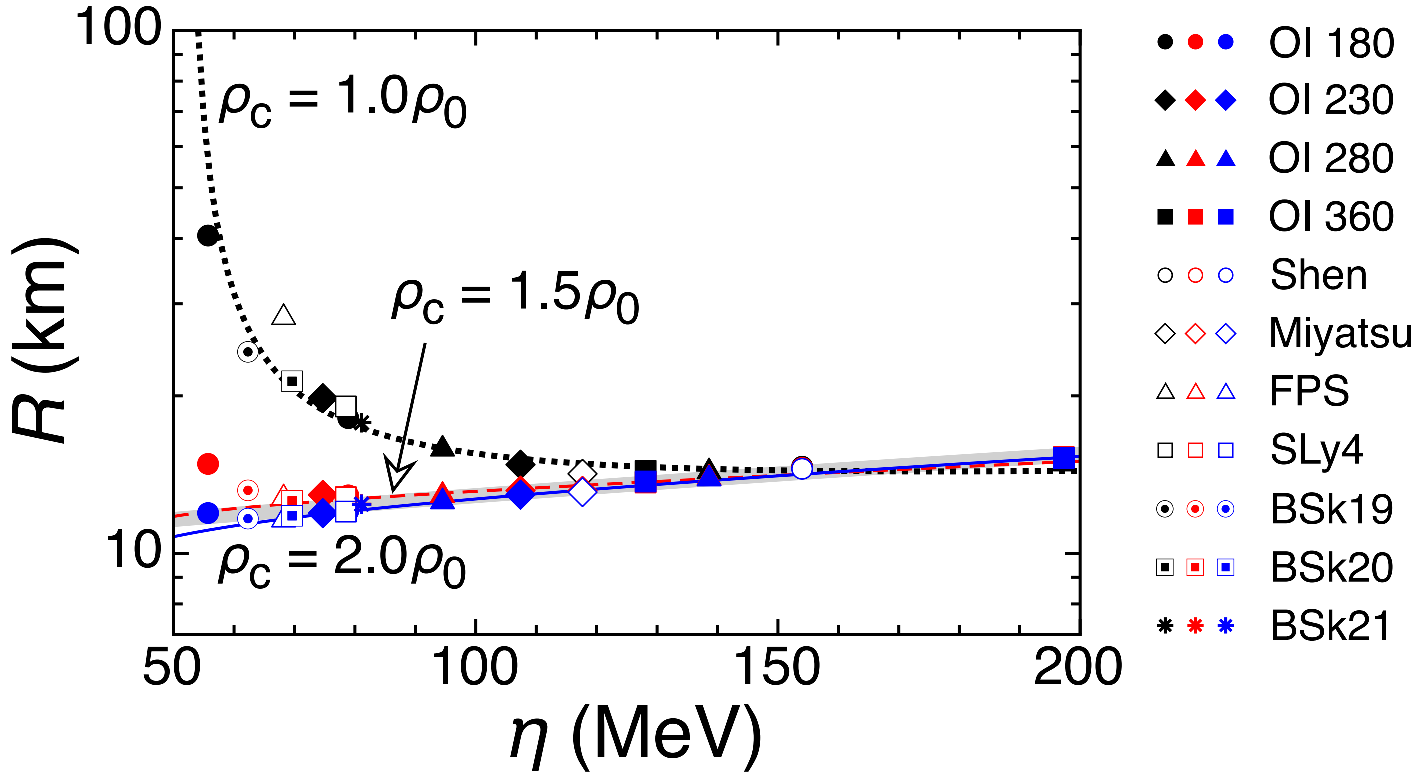
$$\boxed{z = 0.00859 - 0.0619u_c + 0.0255u_c^2 - (0.0429 - 0.108u_c + 0.0120u_c^2)\left(\frac{\eta}{100\,\mathrm{MeV}}\right)}$$

$$z = 1/\sqrt{1 - 2GM/Rc^2} - 1$$

- via the simultaneous observations of M & z (or R or R_∞), one could extract the values of η & ρ_c !!

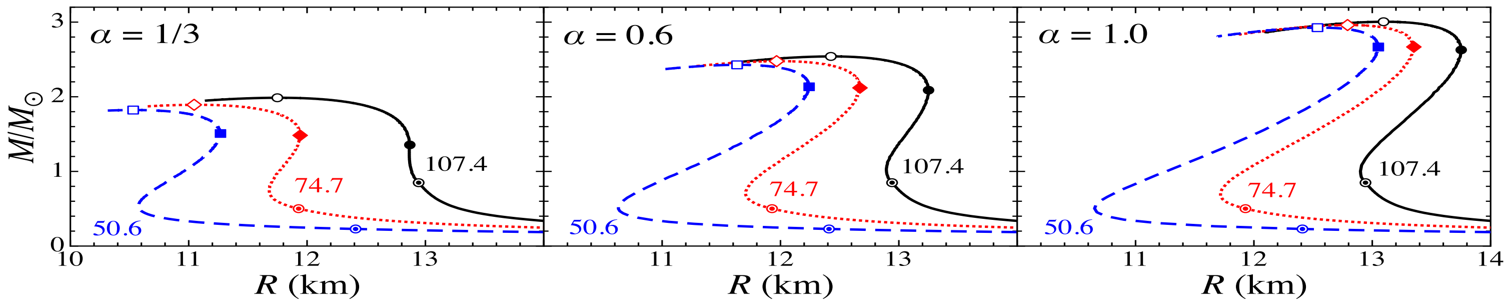
radii of low-mass NSs

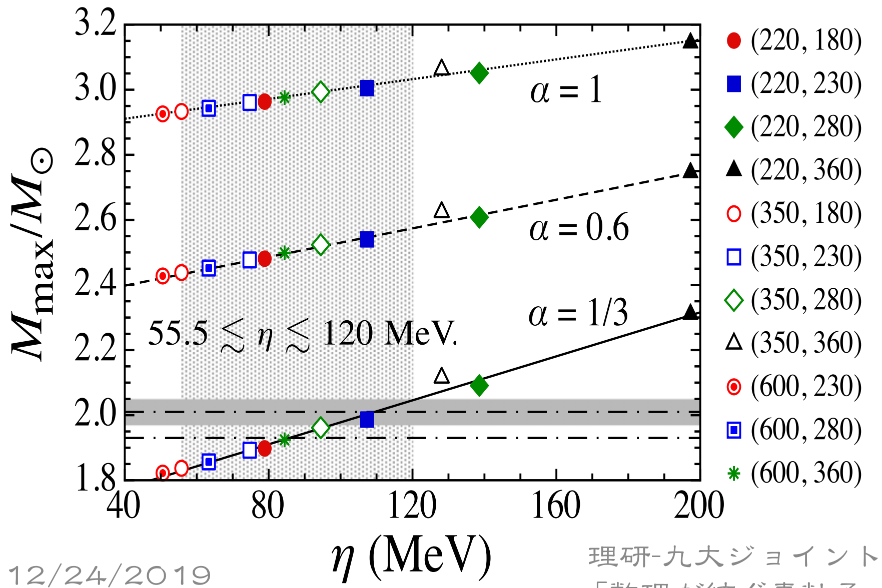
- with using the formulas of mass and gravitational redshift, one can also predict the radius of NS.



possible maximum mass of NSs

- parameterize EOS (Hartle 78, Kalogera+ 96)
  - for $\rho \lesssim 2\rho_0$: stellar properties strongly depend on η
  - for $\rho > 2\rho_0$: $p = \alpha(\rho - \rho_t) + p_t$, where $\rho_t = 2\rho_0$





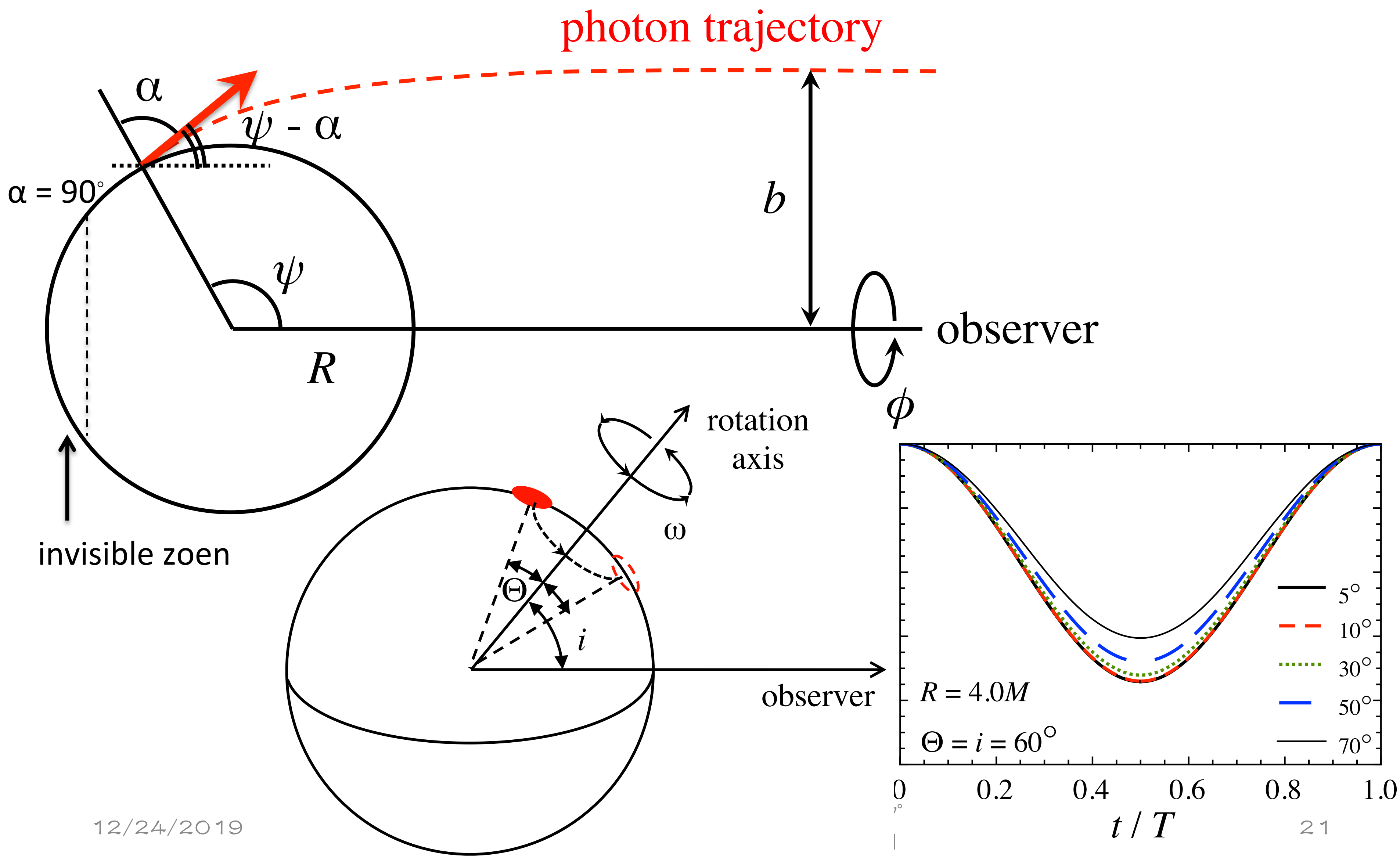
$$\boxed{\frac{M_{\rm max}}{M_\odot} = a_1 + a_2\left(\frac{\eta}{1\ {\rm MeV}}\right)}$$

if α=1,

$M_{max}/M_\odot = 2.86 + 1.51\times10^{-3}(\eta/\mathrm{MeV})$

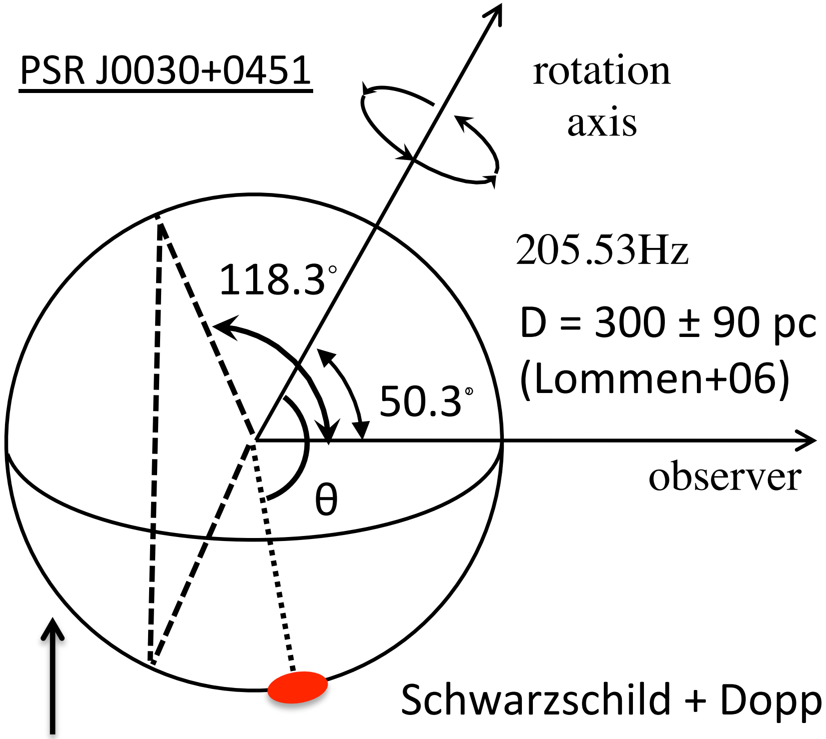
$$\boxed{\begin{array}{c} 2.94 \lesssim M_{max}/M_\odot \lesssim 3.04 \\ 3.00M_\odot \text{ with } \eta = 92.75 \text{ MeV} \end{array}}$$

constraining M/R with light curve



from NICER ob.

PSR J0030+0451



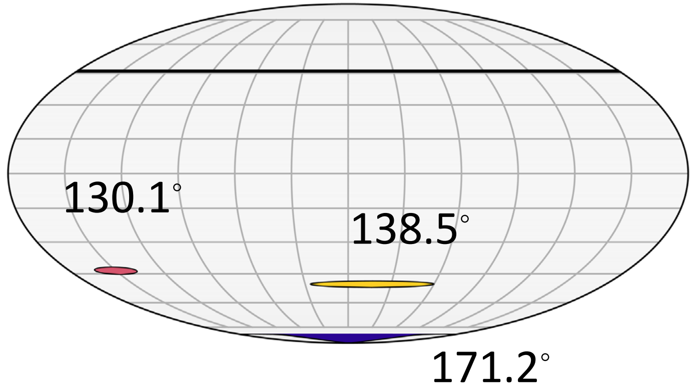
205.53Hz

D = 300 ± 90 pc
(Lommen+06)

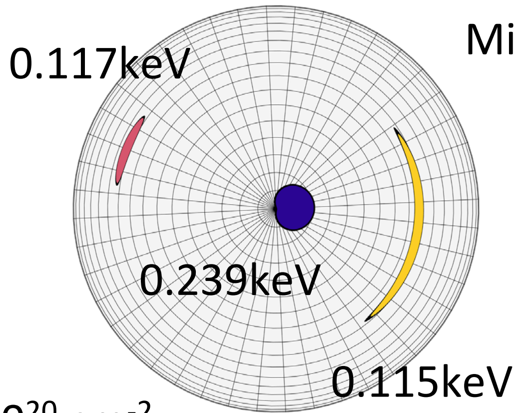
invisible zoen

Schwarzschild + Doppler

D = 327 pc, $N_H = 0.244 \times 10^{20}$ cm^{-2}



Miller+19

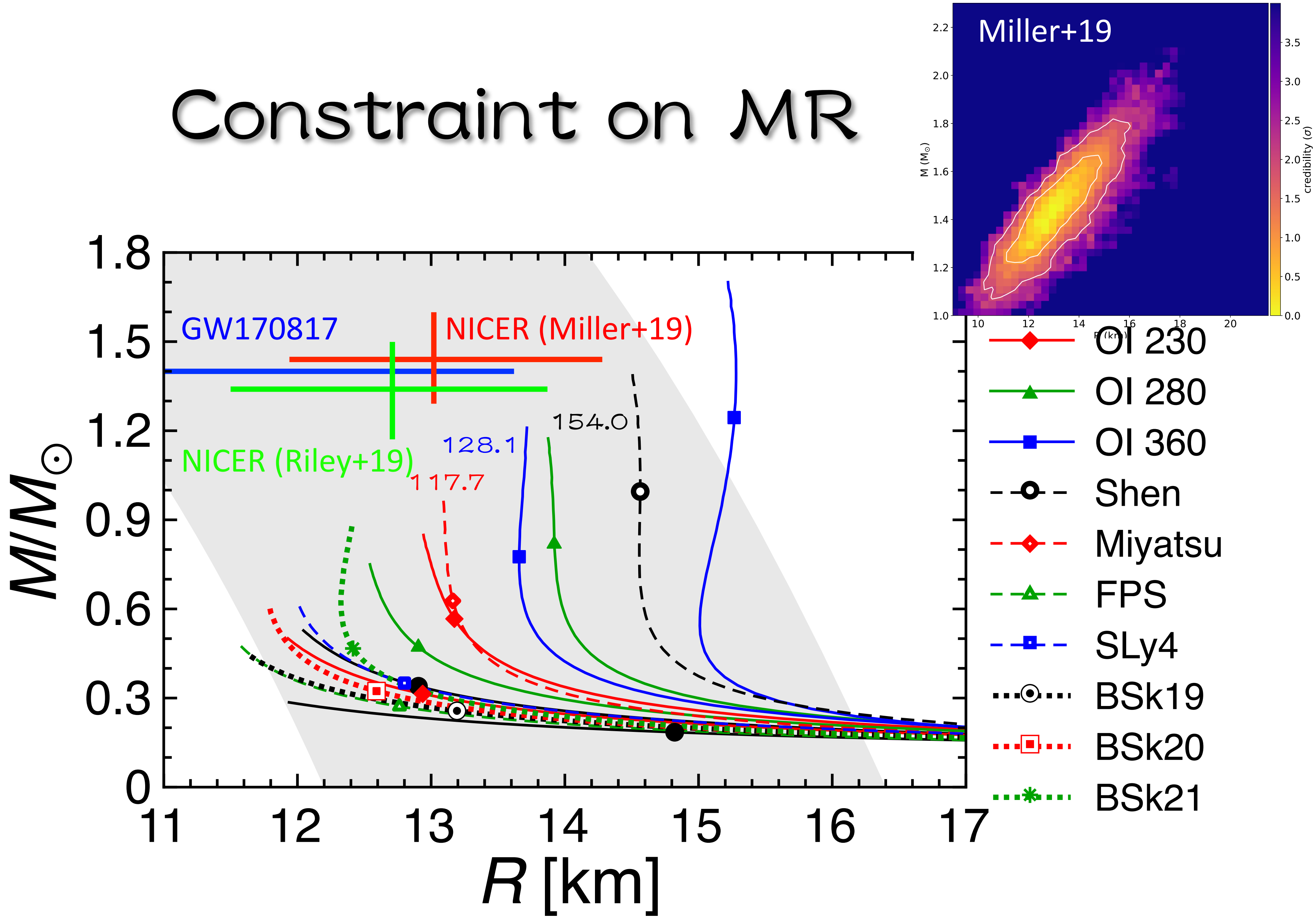


Parameter
R_e(km)
$GM/(c^2R_e)$
M ($M_\odot$)
θ_{c1} (rad)
$\Delta\theta_1$ (rad)
f_1
$kT_{\mathrm{eff},1}$ (keV)
θ_{c2} (rad)
$\Delta\theta_2$ (rad)
f_2
$kT_{\mathrm{eff},2}$ (keV)
$\Delta\phi_2$ (cycles)
θ_{c3}(rad)
$\Delta\theta_3$ (rad)
f_3
$kT_{\mathrm{eff},3}$ (keV)
$\Delta\phi_3$ (cycles)
θ_{obs} (rad)
N_{H} (10^{20} cm^{-2})
D (kpc)

$R = 13.02^{+1.24}_{-1.06}$ km, $M = 1.44^{+0.15}_{-0.14} M_\odot$ (Miller+19)

$R = 12.71^{+1.14}_{-1.19}$ km, $M = 1.34^{+0.15}_{-0.16} M_\odot$ (Riley+19)

Constraint on MR



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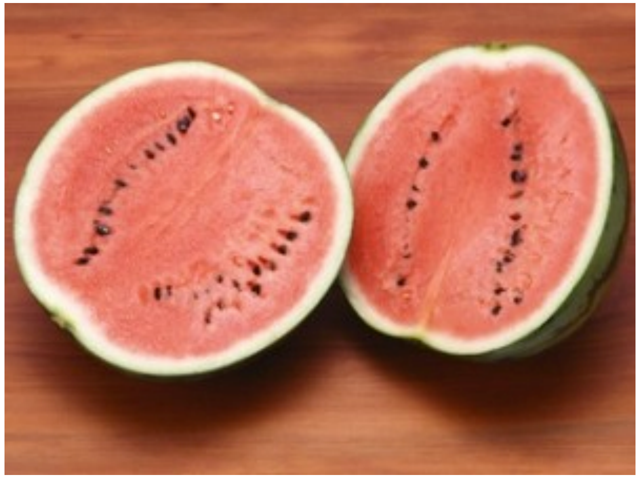
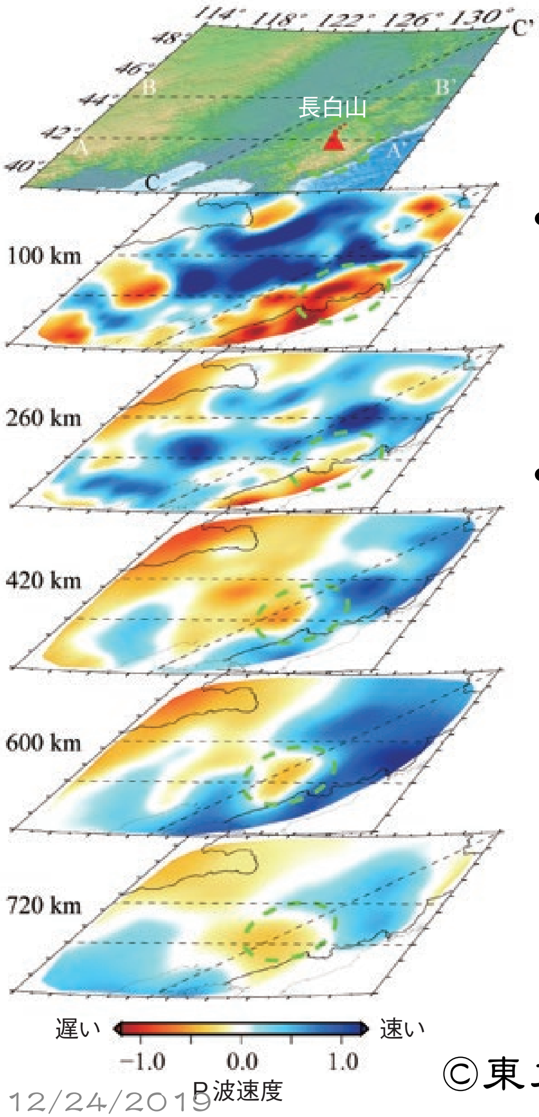
watermelon

- how to know the best time to eat a watermelon ?
  - inside can not be checked before cutting
- “empirical rule”
  - to check the best time, knock a watermelon
    - high frequency “KIN-KIN” ; too young
    - “BAN-BAN” ; best time !
    - low frequency “BON-BON” ; too old
  - need many years to get this ability
- one could see the interior with specific sound from object.
  - asteroseismology !!



Seismology, Helioseismology



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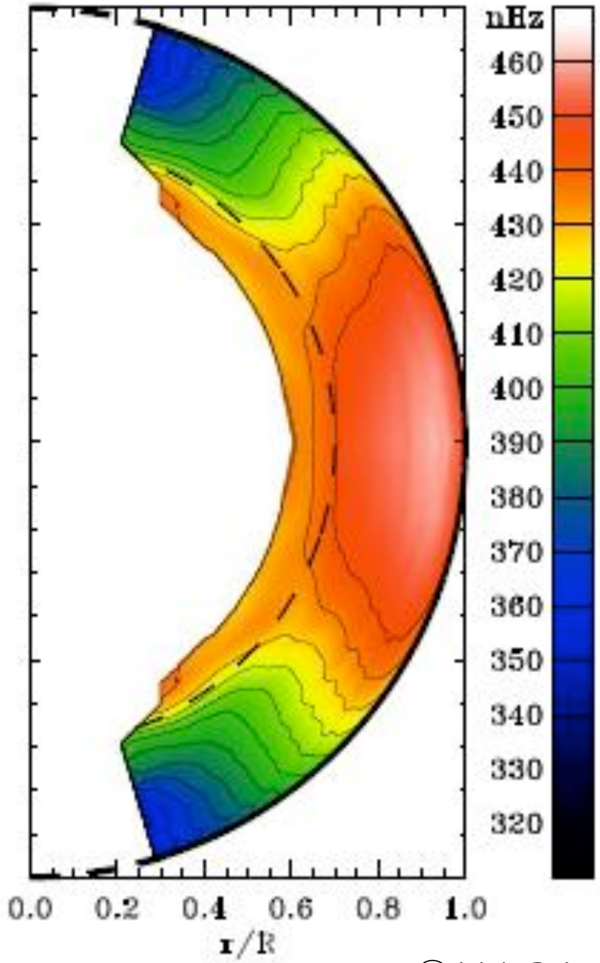
- Seismic waves tell us information inside the Earth (seismology)
- The interior of the Sun can be probed through the wave pattern on the surface (helioseismology)



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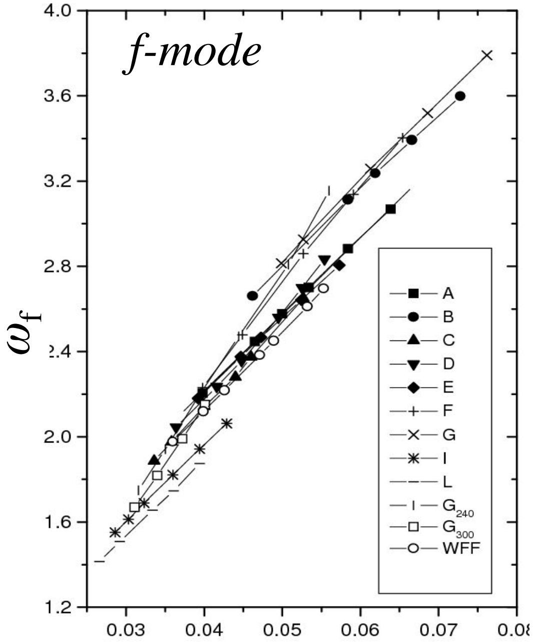
non-radial Oscillations in neutron stars

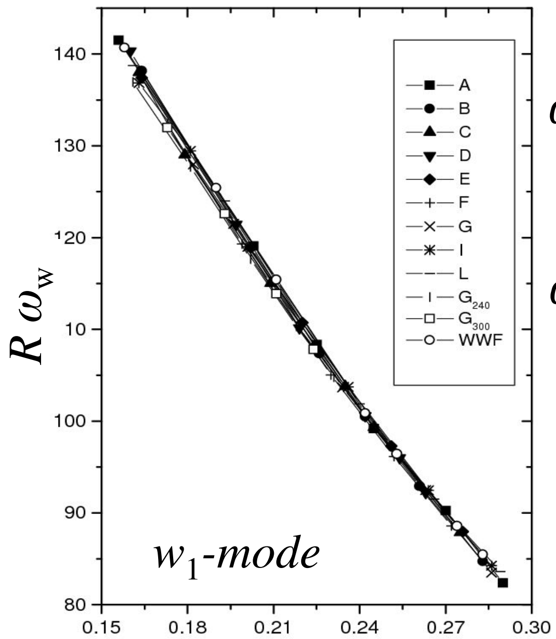
- axial type oscillations
  - no stellar deformation, no density variation
    - t-modes (torsional oscillations) : oscillations due to the elasticity
    - w-modes (spacetime) : oscillations of specetime itself ~ M/R
- polar type oscillations
  - with density variation & stellar deformation
  - important for considering the GWs emission
    - f-mode (fundamental) ~ $(M/R^3)^{1/2}$
    - p-modes (pressure) : sound speed crossing ~ $(M/R^3)^{1/2}$
    - g-modes (gravity) : thermal/composition gradients ~ B-V frequency
    - s-modes (shear) : oscillations due to the elasticity
    - Alfven modes
    - inertial modes (effect of rotation)
    - w-modes (spactime) : oscillations of specetime itself ~ M/R

GW asteroseismology in NSs

- via the observations of GW frequencies, one might be able to see the properties of NSs



average density



compactness

Andersson & Kokkotas (1998)

$$\omega_f \approx 0.78 + 1.64\left[\left(\frac{M}{1.4M_\odot}\right)\left(\frac{10\text{km}}{R}\right)^3\right]^{1/2}$$

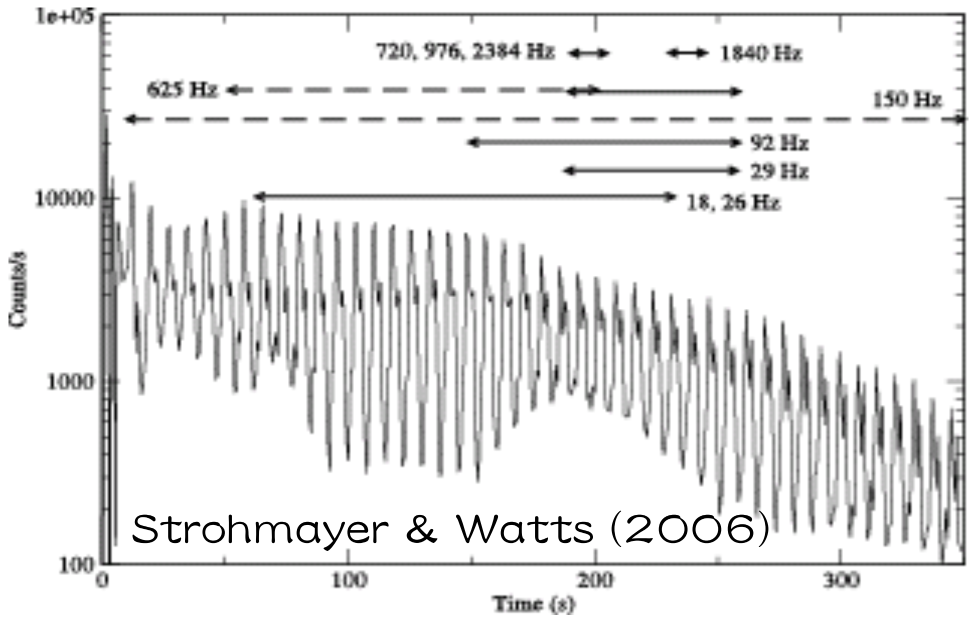
$$\omega_w \approx \left(\frac{10\text{km}}{R}\right)\left[20.92 - 9.14\left(\frac{M}{1.4M_\odot}\right)\left(\frac{10\text{km}}{R}\right)\right]$$

determination of (M, R)

QPOs in SGRs

- Quasi-periodic oscillations (QPOs) in afterglow of giant flares from soft-gamma repeaters (SGRs)
  - SGR 0526-66 (5^{th}/3/1979) : 43 Hz
  - SGR 1900+14 (27^{th}/8/1998) : 28, 54, 84, 155 Hz
  - SGR 1806-20 (27^{th}/12/2004) : 18, 26, 30, 92.5, 150, 626.5, 1837 Hz

  (Barat+ 1983, Israel+ 05, Strohmayer & Watts 05, Watts & Strohmayer 06)
  - additional QPO in SGR 1806-20 is found : 57Hz (Huppenkothen + 2014)



Strohmayer & Watts (2006)

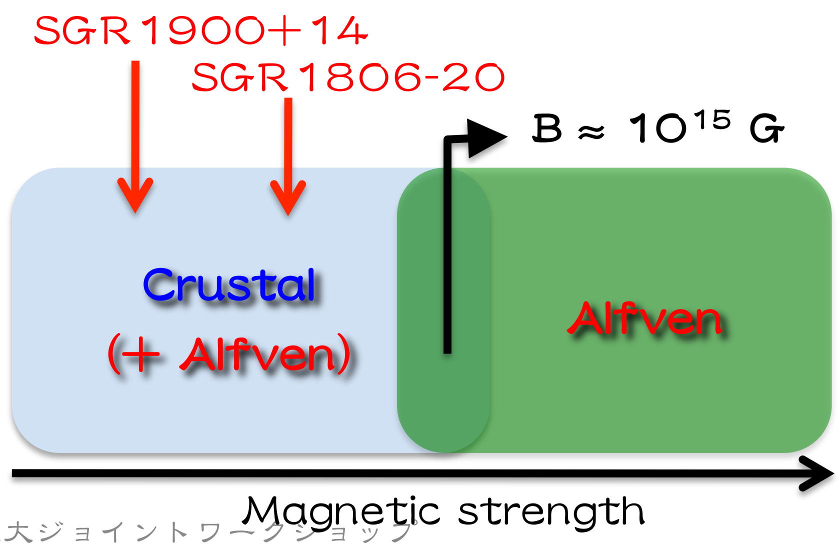
- Crustal torsional oscillation ?
- Magnetic oscillations ?

advantage for crustal oscillations

- magnetic configuration inside NSs are still uncertain
- EOSs for core region are unfixed yet

- **to avoid such uncertainties, we focus on the crustal torsional oscillations without effects of magnetic field**
  - fluid core: zero shear modulus ---> no shear oscillations
  - torsional oscillations localize only in crust region

- magnetic effect on torsional oscillations



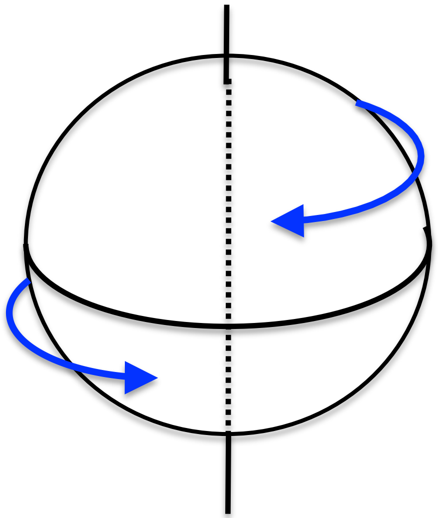
torsional oscillations 1

- axial parity oscillations
  - incompressible
  - no density perturbations (less associated with GWs)
- in Newtonian case (Hansen & Cioff 1980)

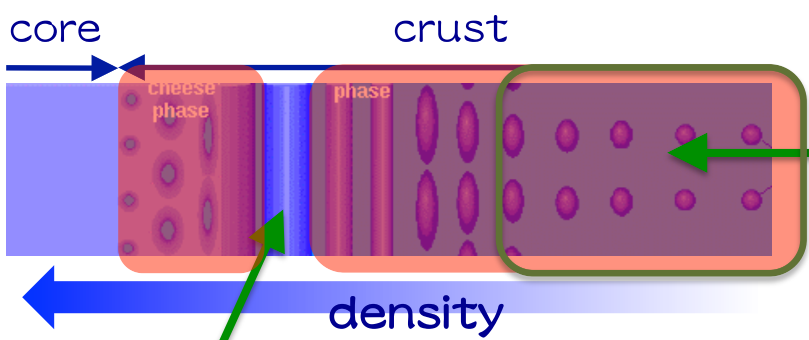
$$ {}_\ell t_0 \sim \frac{\sqrt{\ell(\ell+1)\mu/\rho}}{2\pi R} \sim 16\sqrt{\ell(\ell+1)}\ \mathrm{Hz} \qquad {}_\ell t_n \sim \frac{\sqrt{\mu/\rho}}{2\Delta r} \sim 500 \times n\ \mathrm{Hz} $$

  - μ : shear modulus
  - frequencies ∝ shear velocity $v_s = \sqrt{\mu/\rho}$
  - overtones depend on crust thickness
- one can consider torsional oscillations independently of core EOS
- effect of magnetic field
  - frequencies can become larger

(Sotani+07, Gabler+12,13)



torsional oscillations 2



spherical (Strohmayer+ 91)

$$\mu = 0.1194 \frac{n_i (Ze)^2}{a}$$

n_i : ion number density
Z : charge of nuclei
a : Wigner-Seitz radius

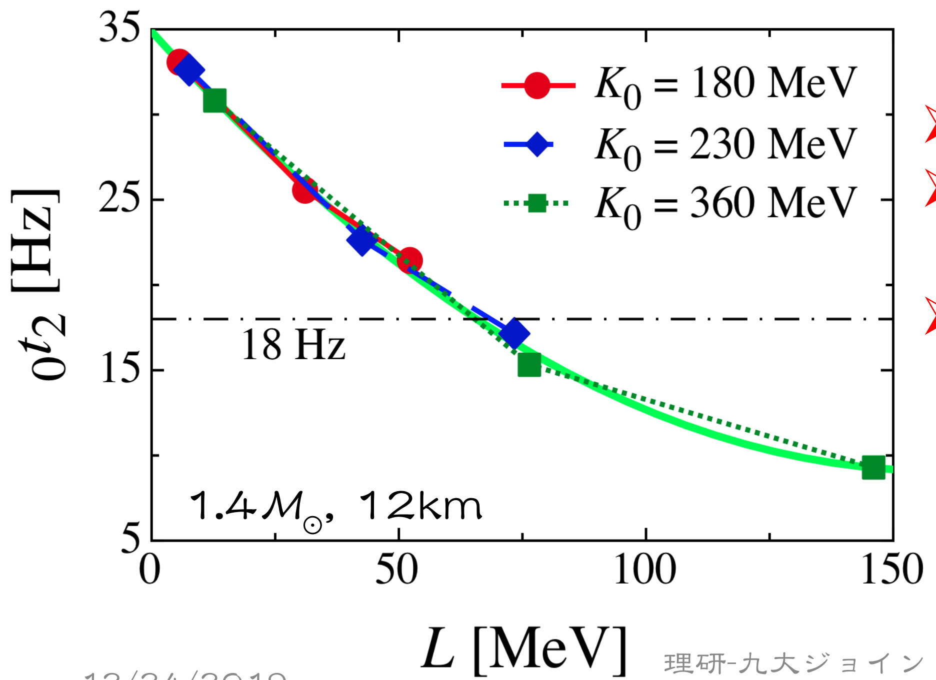
slab-like

linear response : fluid
(Landau)

→ two independent oscillations
(i) spherical + cylindrical
(ii) cylindrical-hole + bubble

torsional oscillations in Sph. nuclei

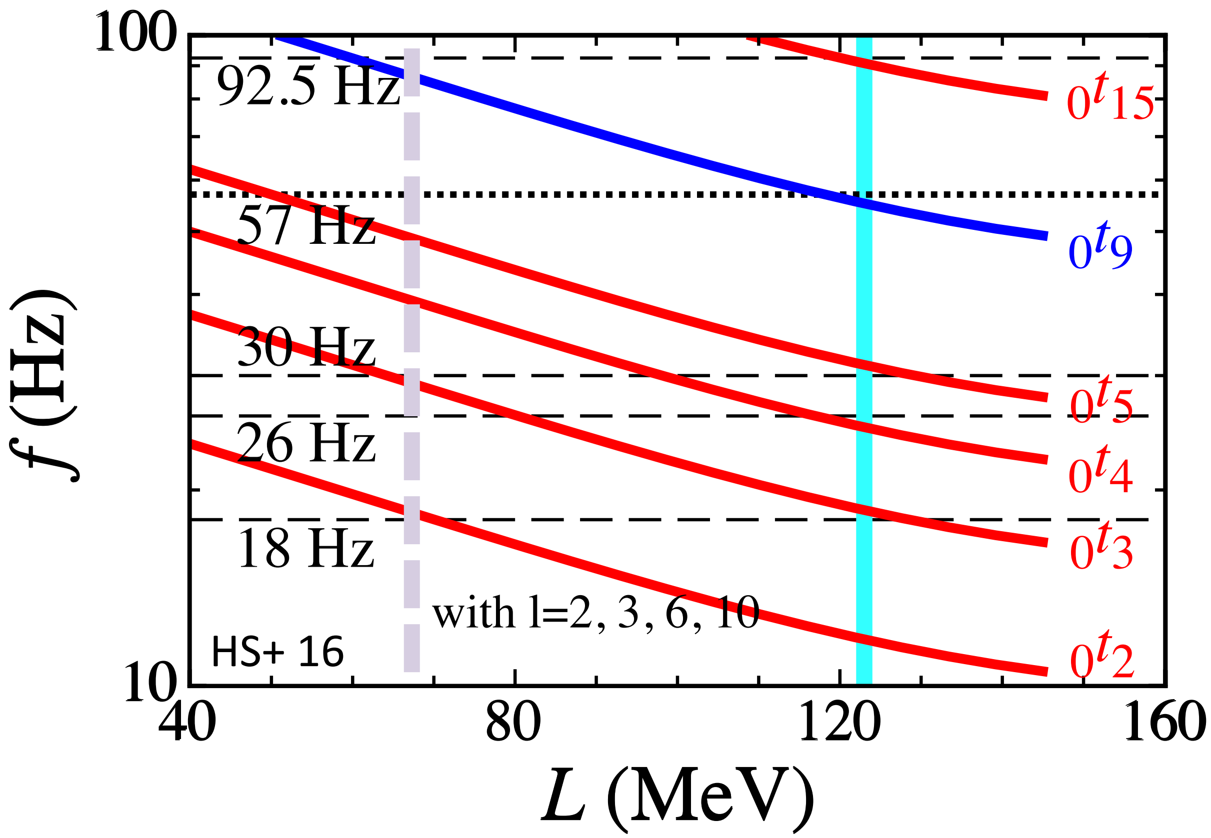
- EOS for core region is still uncertain.
- To prepare the crust region, we integrate from $r=R$.
  - M, R : parameters for stellar properties
  - L, K_0 : parameters for curst EOS (Oyamatsu & Iida 03, 07)
- In crust region, torsional oscillations are calculated.
  - frequency of fundamental oscillation $\propto v_s$ ($v_s^2 \sim \mu/H$)



- independent of the value of K_0
- similar to the cases for R=10-14 km, $M/M_\odot$=1.4-1.8, and various l.
- we focus on the L-dependence on ${}_0t_l$

comparison to SGR 1806-20

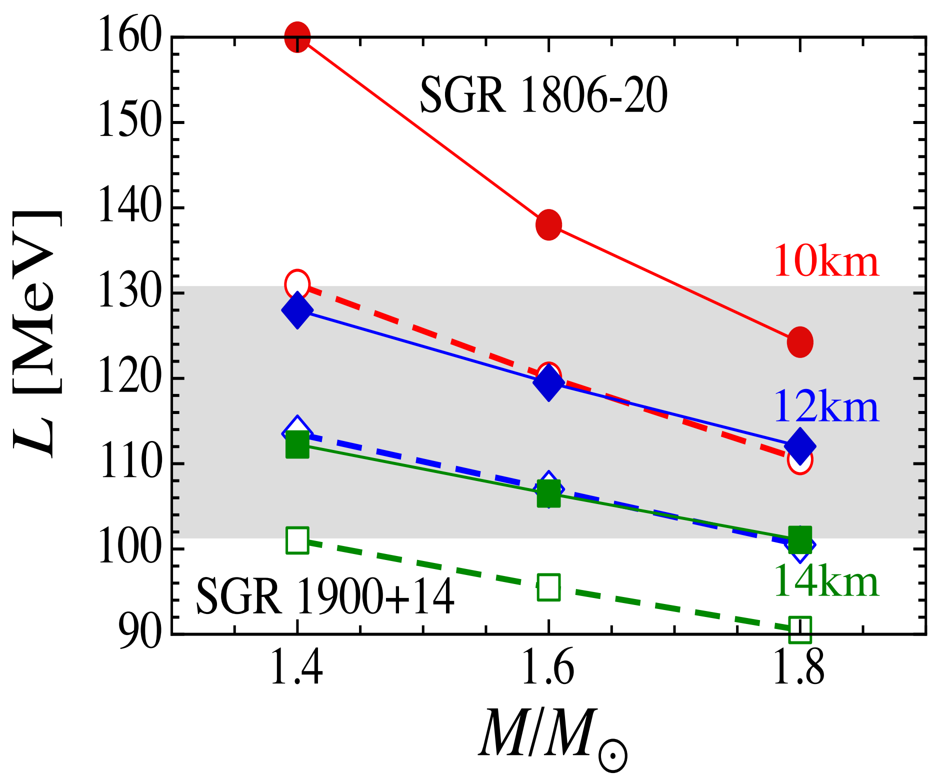
- for R = 12 km and $M = 1.4 M_{\odot}$
- discovery of new QPO from SGR1806-20, which is 57Hz



1) l = 3, 4, 5, 9, 15
   all QPOs can be identified
2) l = 2, 3, 6, 10
   except for 26 Hz, QPOs can be identified

constraint on L via QPO frequencies

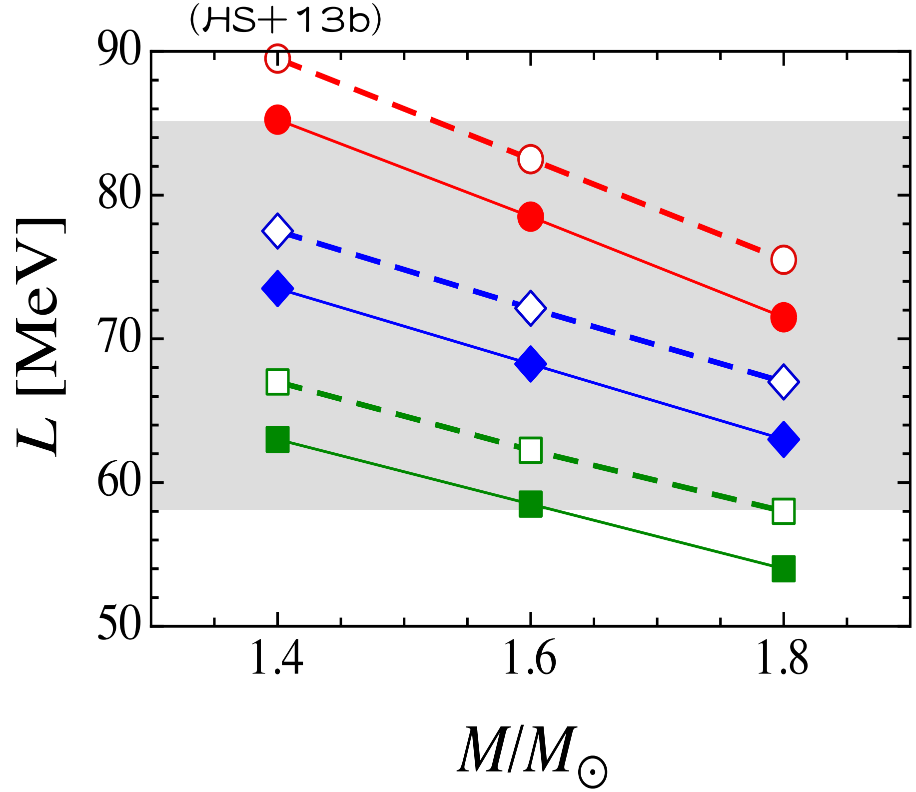
1) all QPOs come from crustal torsional oscillations (HS+13a)



➡ $101.1 \leq L \leq 131.0$ MeV

cf) L = 58.9 ± 16 MeV ??

2) QPOs except for 26Hz come from crustal torsional oscillations (HS+13b)

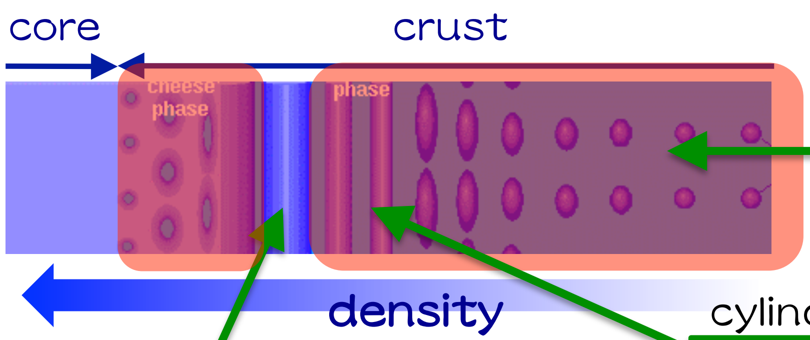


➡ $58.0 \leq L \leq 85.3$ MeV

one needs to consider another oscillation mechanism for 26 Hz

effect of pasta structure

HS+ 18



spherical (Strohmayer+ 91)

$$\mu = 0.1194 \frac{n_i (Ze)^2}{a}$$

slab-like

linear response : fluid
(Landau)

cylindrical (Potekhin+98)

$$\mu_{cy} = \frac{2}{3} E_{Coul} \times 10^{2.1(w_2 - 0.3)}$$

E_{Coul} : Coulomb energy per unit volume
w_2 : volume fraction

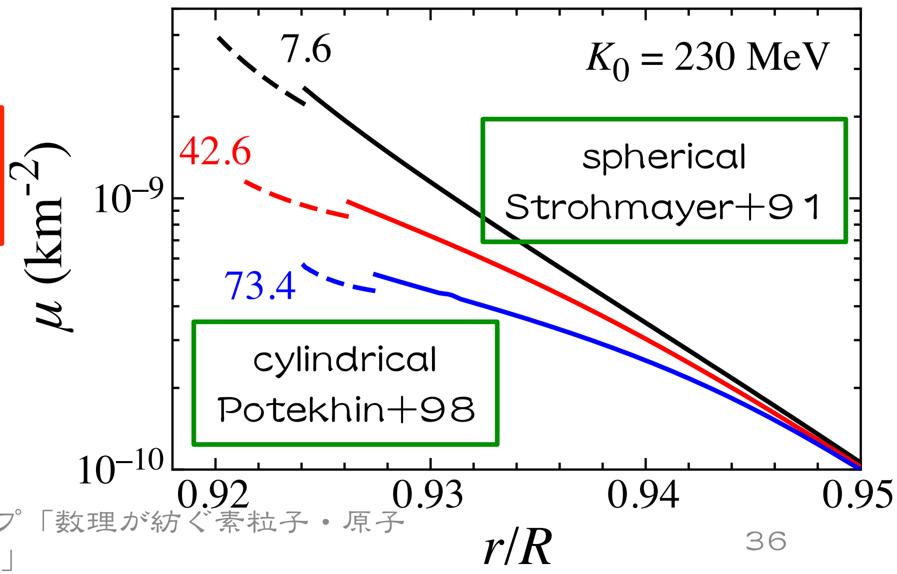
→ two independent oscillations
(i) spherical + cylindrical
(ii) cylindrical-hole + bubble → 26Hz?

identification of fundamental oscillations
→ constraint on L (?)

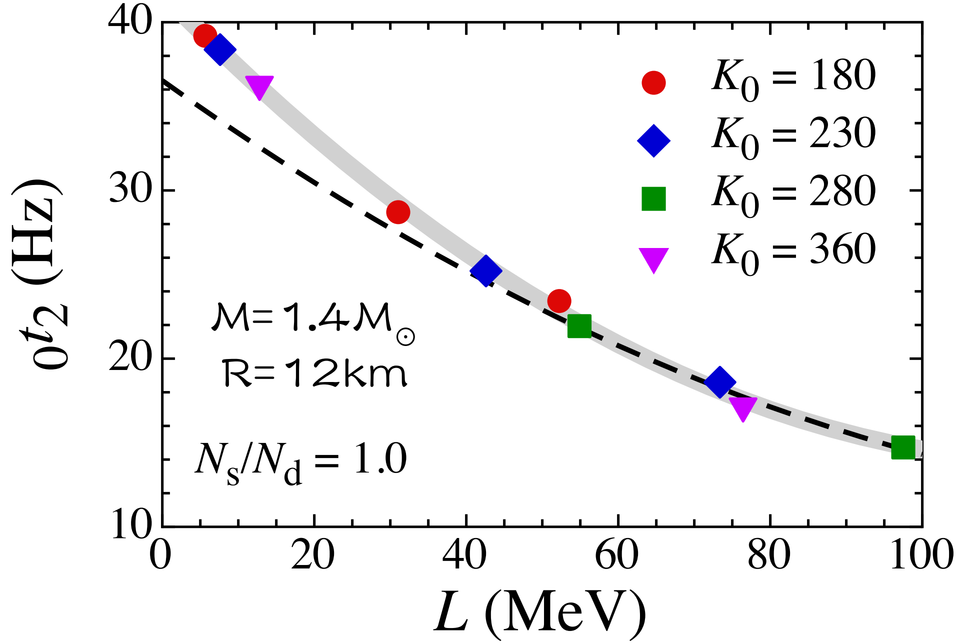
identification of overtone (?)

$$ {}_1t_2 \sim v_s / \Delta R $$

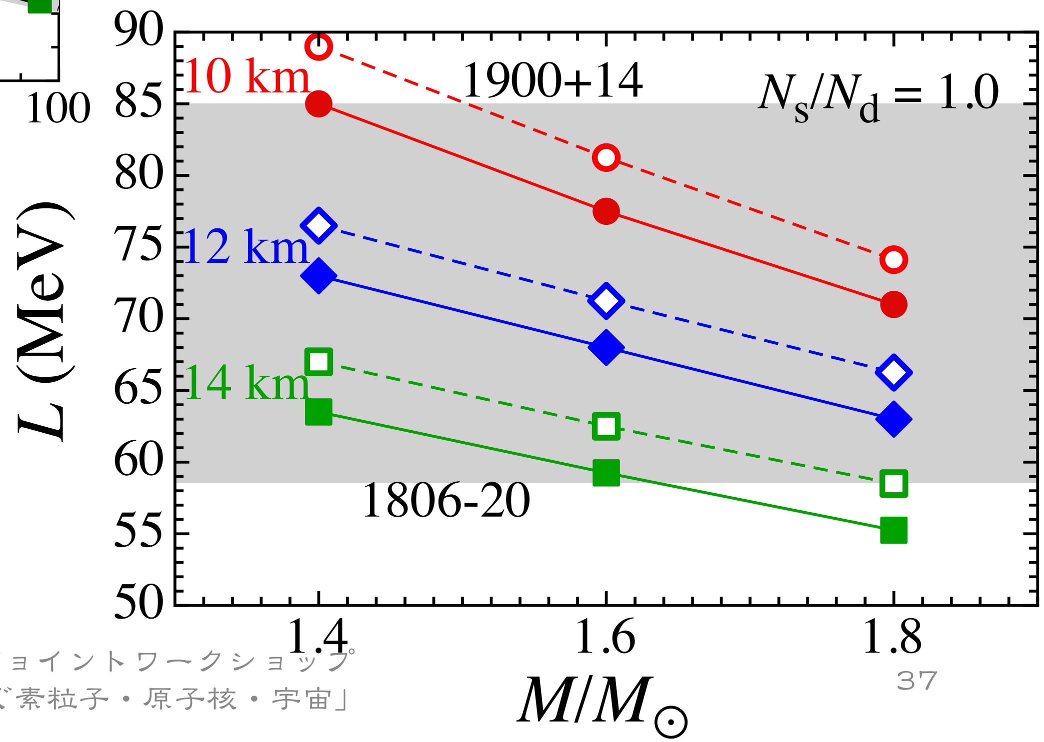
→ additional information



constraint on L



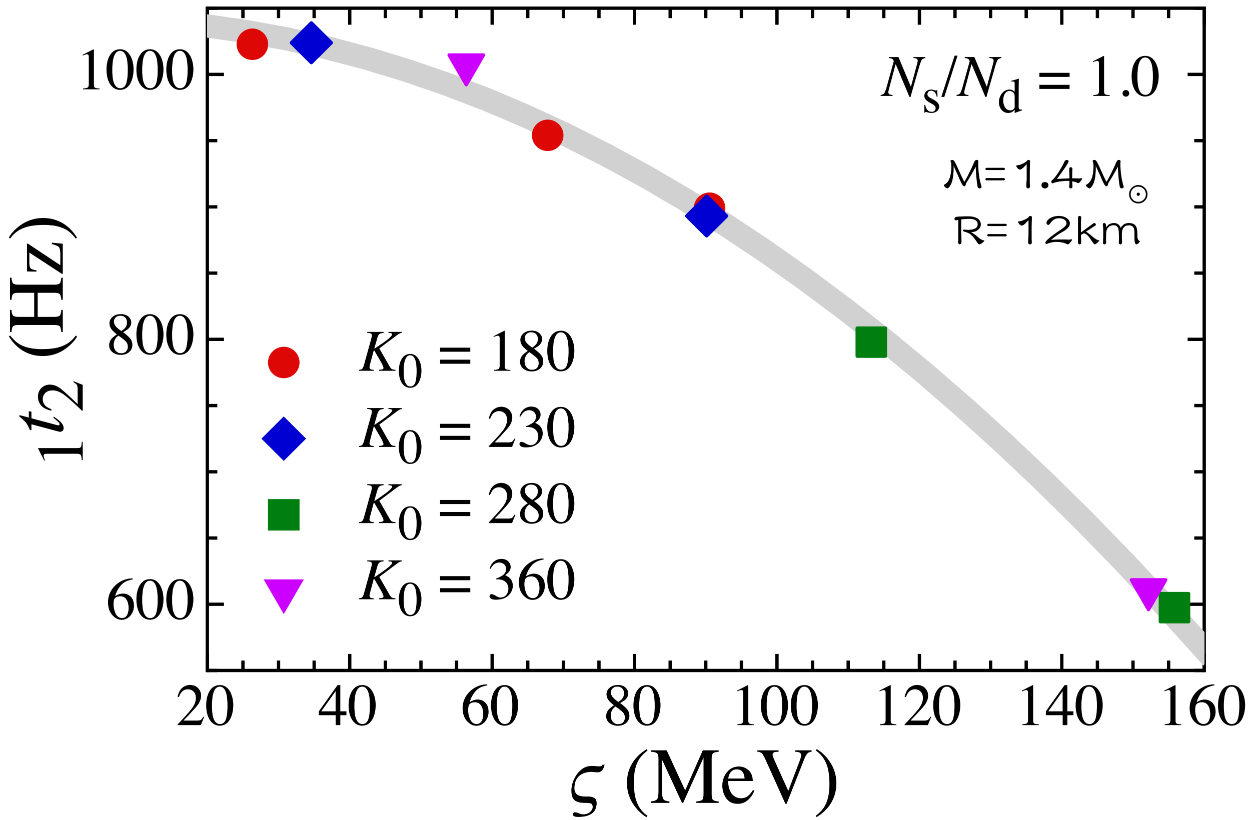
- fundamental oscillations weakly depend on the existence of cylindrical phase.
- all of QPOs except for 26Hz can be identified as same as the previous calculations .



1^{st} overtone HS+ 18

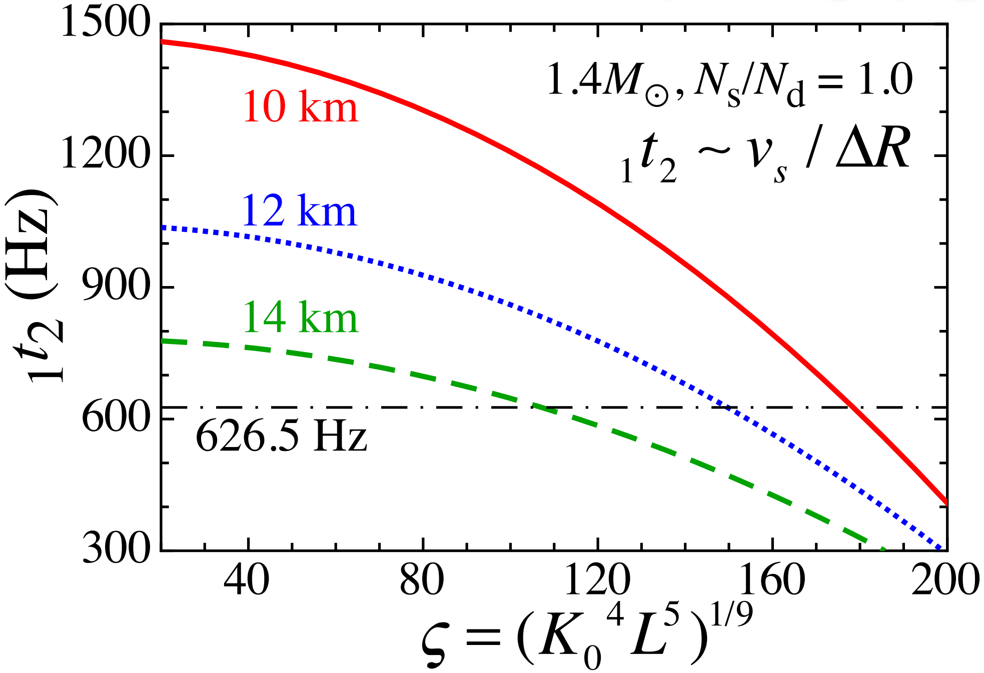
- frequencies of 1^{st} overtone depend on K_0 & L
- we find the good combination of K_0 & L for expressing the 1^{st} overtone frequencies.

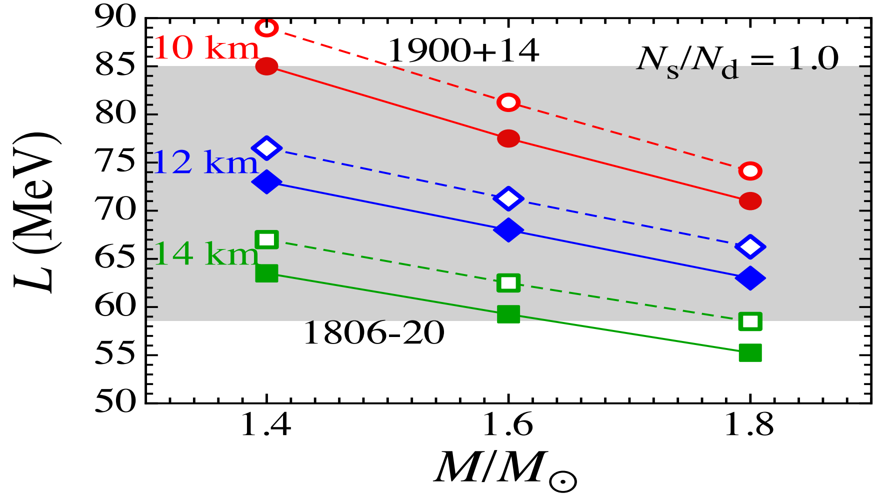
$$\boxed{\varsigma = (K_0{}^4 L^5)^{1/9}}$$



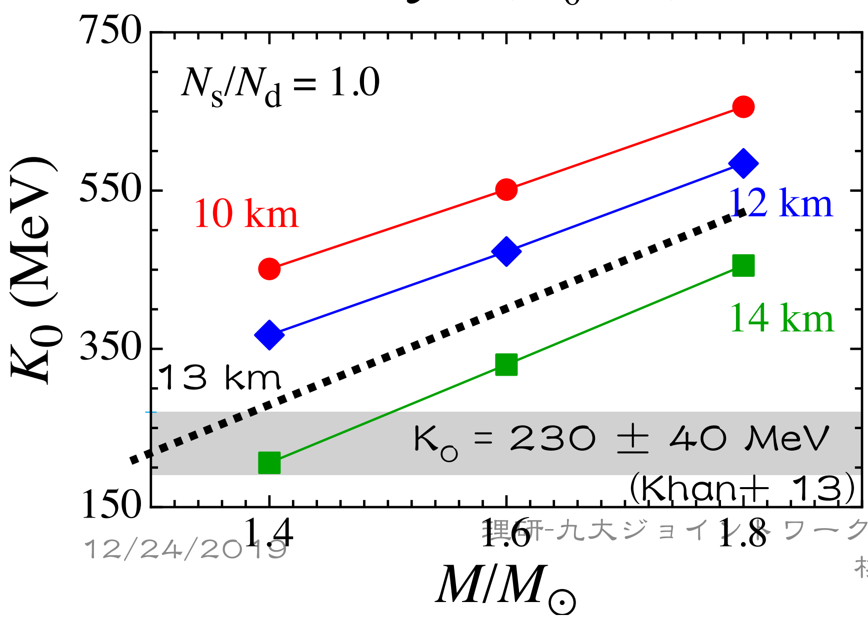
1^{st} overtone

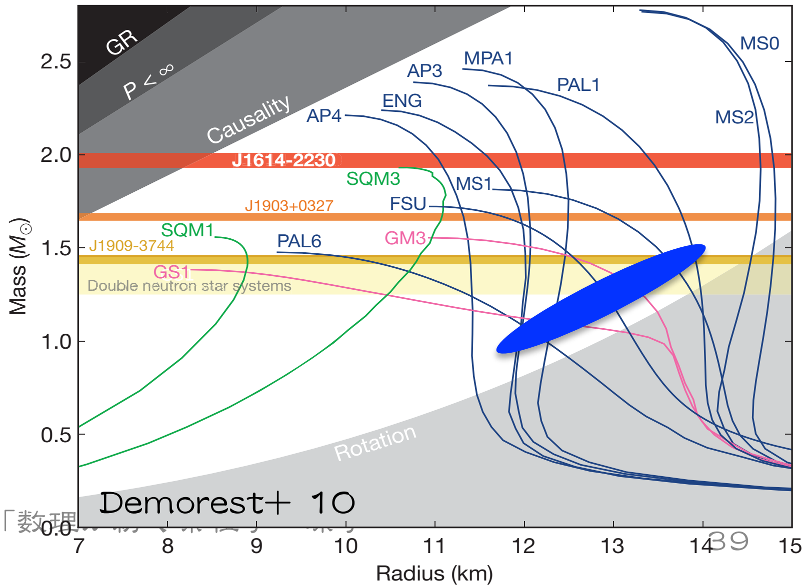
HS+ 18



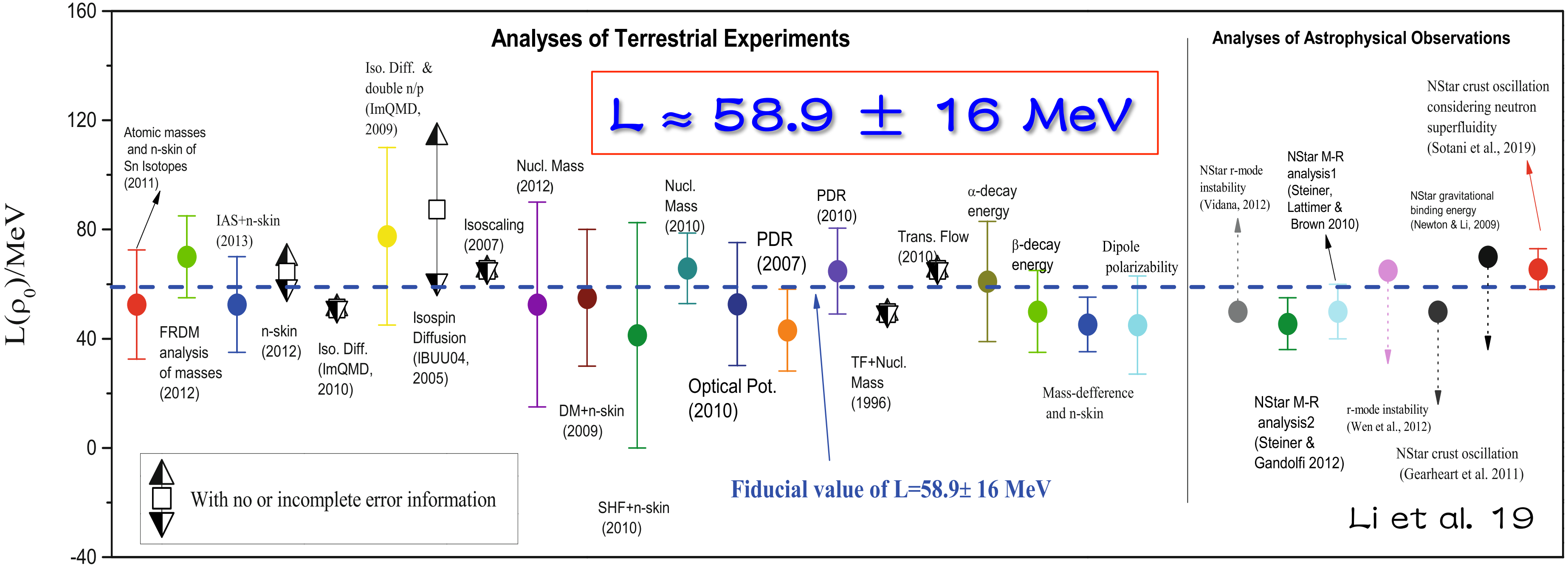


SGR 1806-20 : 13km & 1.2-1.4$M_{\odot}$
→ L ≃ 62-73MeV



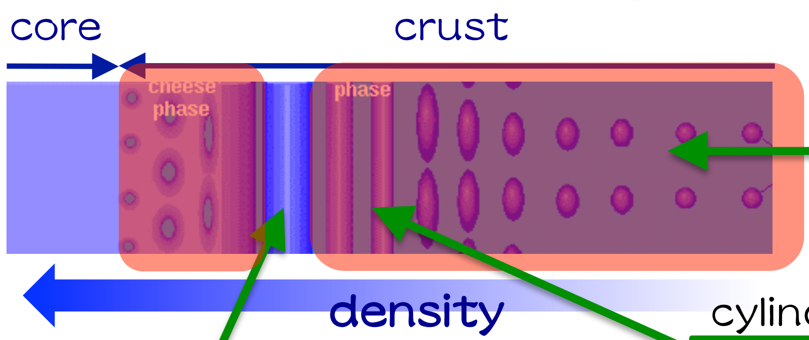


constraints on L



26Hz : inner phase(?), 626.5Hz : spherical + cylindrical ($_1t_2$)
→ SGR1806-20 should be relatively low mass NS ($M \sim 1.2\text{-}1.4 M_\odot$, R~13km??)
→ **L ~ 58-73MeV**

can we identify the 26Hz QPO?



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cylindrical (Potekhin+98)

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E_{Coul} : Coulomb energy per unit volume

w_2 : volume fraction

slab-like

linear response : fluid
(Landau)

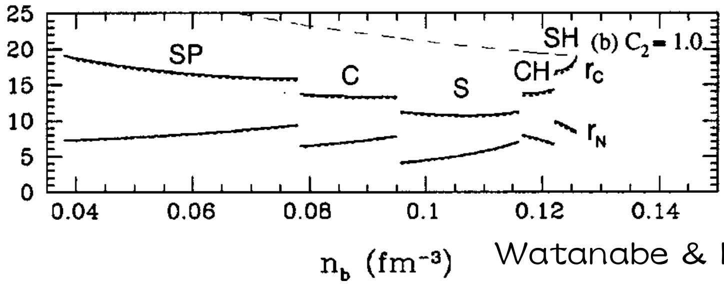
→ two independent oscillations

(i) spherical + cylindrical (sp+cy) → except for 26Hz (OK)

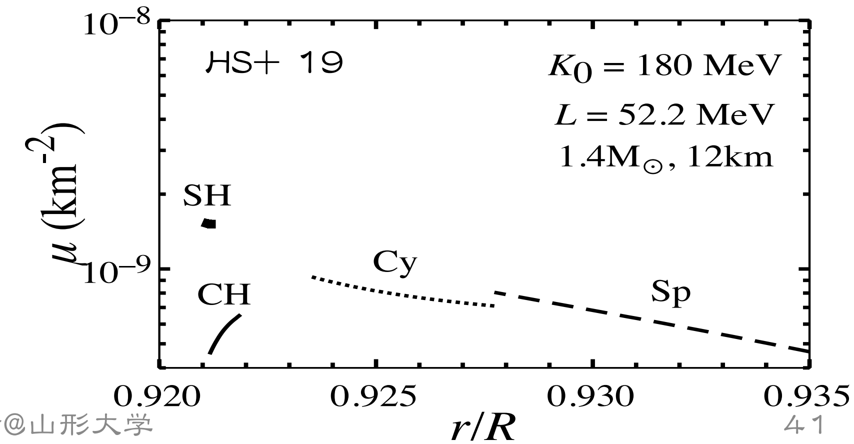
(ii) tube + bubble (tu+bu) → 26Hz? consistently with the L constraint?

→ bubble ~ spherical

tube ~ cylindrical

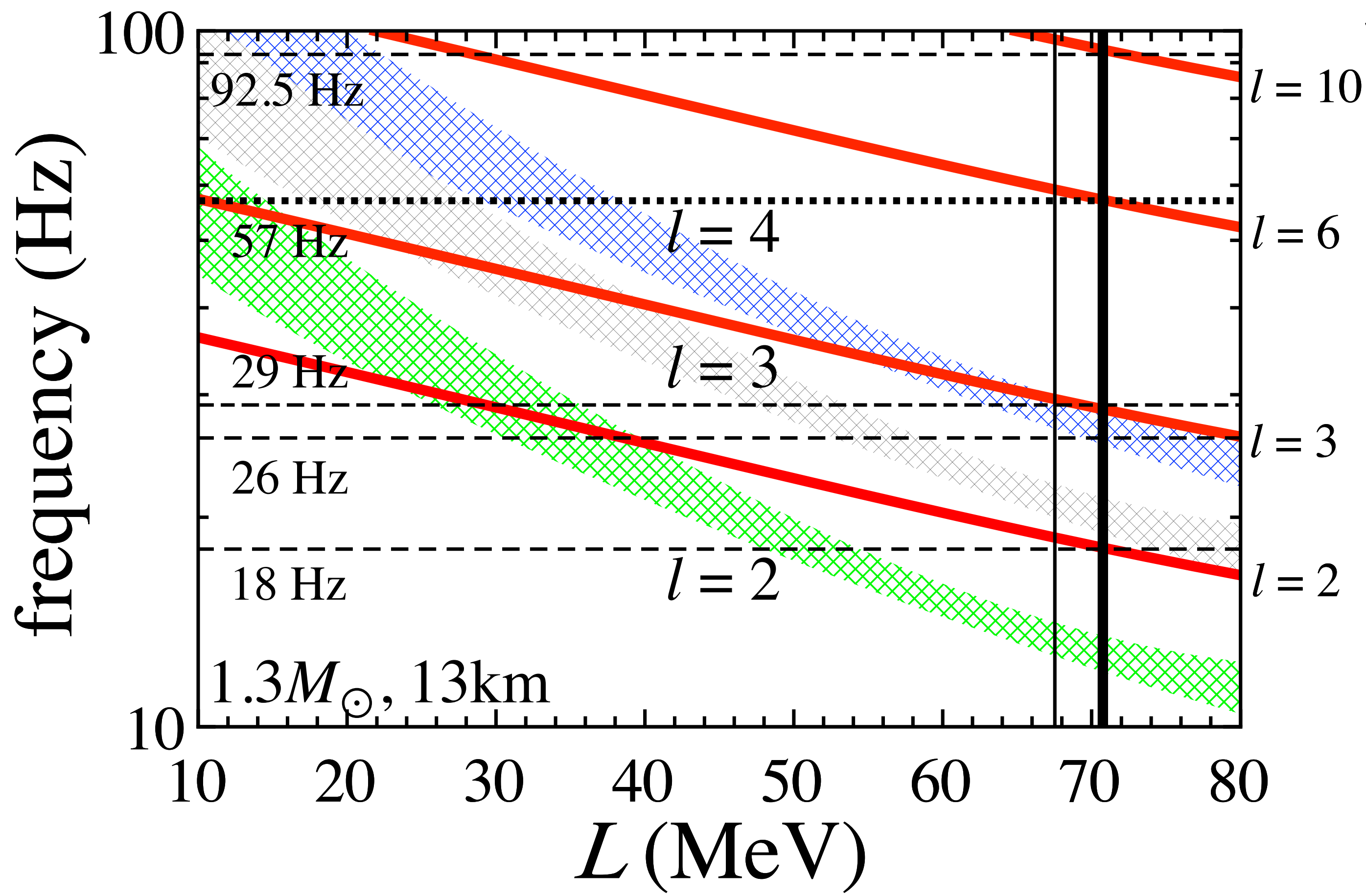


Watanabe & Iida 03



oscillations in tu+bu & in Sp+Cy

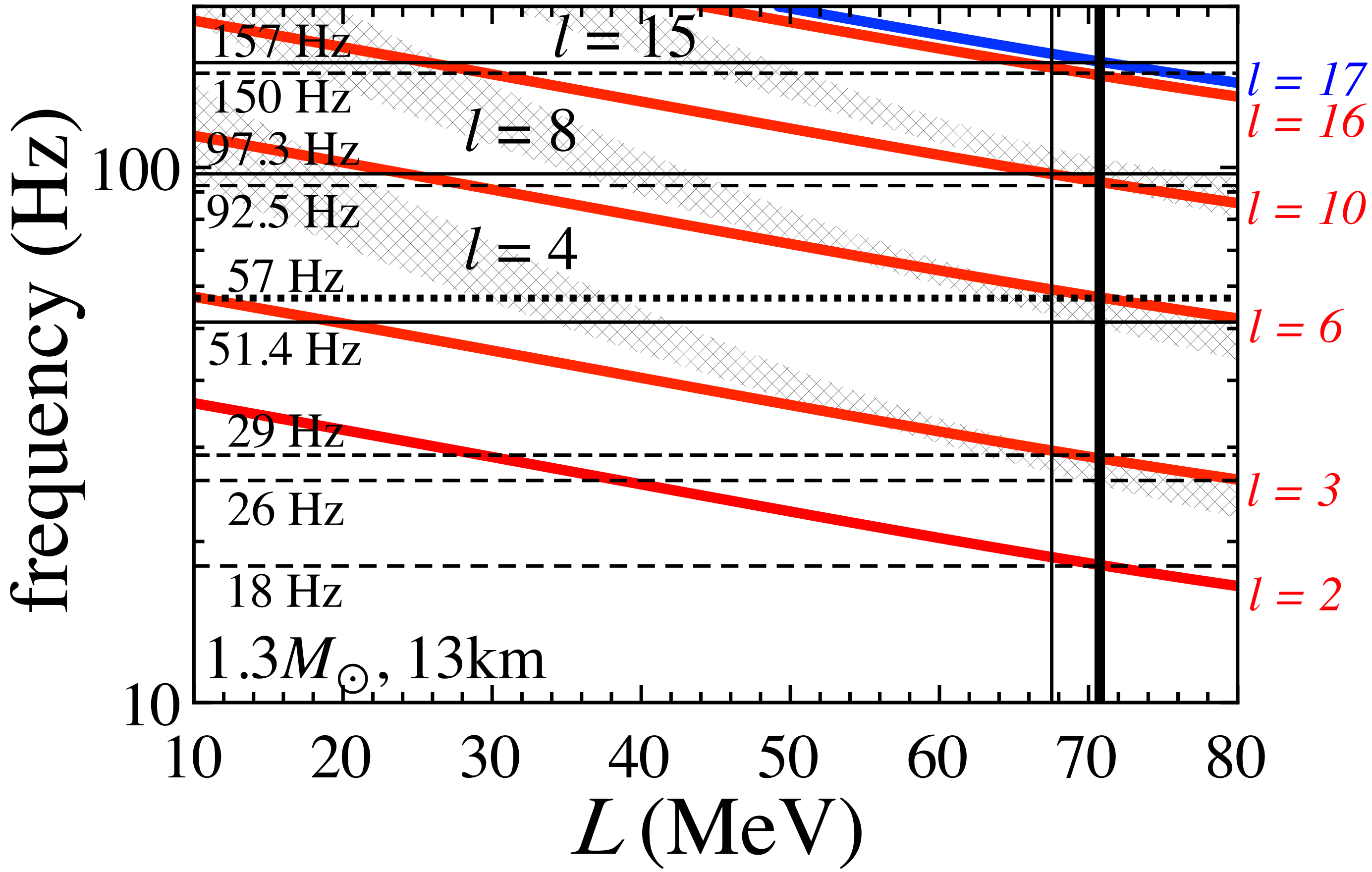
HS+ 19



- one can explain the 26Hz QPO with the torsional oscillations in tu+bu phase

additional QPOs

- recently, the existence of additional QPOs is suggested by Miller et al.
  - 51.4, 97.3, 157Hz



summary

- NSs are a good candidate for probing physics in extreme conditions
- Low-mass NSs directly connect to nuclear saturation parameters
  - mass and gravitational redshift (radius) can be expressed with $\boxed{\eta = (K_0 L^2)^{1/3}}$
  - maximum mass of NSs, $2.94 \leqq M_{max}/M_\odot \leqq 3.04$ with $v_s = 1$
- observational constraints
  - $R \lesssim 13.6$km from GW170817
  - $R = 13.02\,^{+1.24}_{-1.06}$ km, $M = 1.44\,^{+0.15}_{-0.14} M_\odot$ from NICER (Miller+19)
  - $R = 12.71\,^{+1.14}_{-1.19}$ km, $M = 1.34\,^{+0.15}_{-0.16} M_\odot$ from NICER (Riley+19)
- QPOs in SGR could be strongly associated with the NS oscillations.
  - fundamental torsional oscillations are almost independent of K_0
  - overtones depend on K_0 and L, which can be expressed by $\varsigma = (K_0{}^4 L^5)^{1/9}$
  - 26Hz QPO comes from inner phase
- Identifying the QPO observations with the terrestrial constraint on K_0
  - SGR1806-20 should be relatively low mass NS ($M/M_\odot$~1.2-1.3, R~13km??)
  - L ~ 58-73MeV