Extended Migdal-Watson formula to evaluate background strength in binary breakup reactions

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New formula to describe the background strength in the binary breakup, which is induced by the direct breakup process, is considered on the basis of the Migdal-Watson (MW) formula. The strength of the direct breakup is calculated by the complex scaling method. We have found that the extended MW formula, which is proposed in the present study, nicely reproduces the strength of the direct breakup calculated by Complex Scaling Method.

1 Introduction

Inelastic scattering is a useful tool to explore the intrinsic structure of the excited states of nuclei. In particular, the inelastic excitation of the nucleus to the resonant states above the threshold for the particle decay, which is often called the breakup reaction, is very important because we can pin down the intrinsic nuclear structure in the resonances by controlling the detection of the exit channels, which are the combination of the emitted fragments. A typical and good example about the inelastic scattering to the continuum can be seen in the breakup reaction of ^{12}Be into the $\alpha + {}^8$He and ^{6}He $+ {}^6$He channels [1]. In this experiment, the careful multi-pole decomposition analysis (MDA) was performed, and the MDA analysis elucidates that many resonant states with a sharp width of $\Gamma_R \leq 1$ MeV exist in the spins from $J^\pi = 0^+$ to 8^+ [1]. The resonances in these channels appear in a close energy spacing with $\Delta E \sim 0.5$ MeV in the lower energy region below $E_x \leq 15$ MeV.

In determining the resonance parameters, such as the resonance energies and the decay widths, the evaluation of the non-resonant background strength is indispensable because the resonant enhancements, which have a strong energy dependence, are embedded in the non-resonant background contribution.

In order to extract the information about the resonance parameters precisely, it is quite important to propose the appropriate analytic function for describing the non-resonant background contribution in the breakup reaction. In this article, we investigate the structure of the non-resonant background strength in the binary system, which is generated by the direct breakup process, and propose a new formula to evaluate the background contribution by extending the Migdal-Watson (MW) formula [2, 3, 4]. In the calculation of the strength for the direct breakup, we employ the complex scaling method (CSM) [6], which is a powerful tool to

describe the few-body continuum states. From the CSM calculation, we check the applicability of the extended MW formula to the evaluation of the background contribution for the binary breakup. Here we consider the breakup of ^{20}Ne into $\alpha + {}^{16}$O because the ^{20}Ne nucleus is known to be a typical example of the binary cluster system [7, 8]

The organization of this article is the following. In section 2, theoretical formulation is explained. This section contains the explanation of CSM, the definition of the direct breakup and the details of the computational setting. The original Migdal-Watson theory and its extension are explained in section 3. In section 4, the strength function calculated by CSM is analyzed, and the validity of the extended MW formula is discussed. The final section is devoted to summary.

2 Theoretical framework

2.1 Complex scaling method

In the present study, the direct breakup is defined by the one step transition from the initial ground state to the final excited state embedded in the continuum. The background strength generated by the direct breakup process is evaluated by the complex scaling method (CSM) [6]. In CSM, the transformation of the complex rotation with the rotation angle θ

$$\hat{U}(\theta)f(\mathbf{r}) \ = \ e^{\frac{3}{2}i\theta}f(e^{i\theta}\mathbf{r}) \ = \ f^{\theta} \tag{1}$$

is introduced for the arbitrary function of $f(\mathbf{r})$. Here the rotation on $\mathbf{r}$ should be read as the transformation on the radial part of the coordinate and hence, $r \rightarrow re^{i\theta}$. The Schrödinger equation transformed by this complex rotation becomes

$$\hat{H}^{\theta}\Psi^{\theta} \ = \ E^{\theta}\Psi^{\theta} \ , \tag{2}$$

where Ψ^{θ} is defined by equation (1) and $\hat{H}^{\theta} = \hat{U}(\theta)\hat{H}\hat{U}(\theta)^{-1}$. In the rotated Hamiltonian $\hat{H}_{\theta}$, the dynamical coordinates of $\mathbf{r}$ contained in $\hat{H}$ is complex rotated like $\hat{H}^{\theta} = \hat{H}(e^{i\theta}\mathbf{r})$. The amplitude of the resonant wave function, which originally diverges in the asymptotic region, is damped in the large distant region by this complex rotation and hence, the usual computation technique for the bound state problem, such as the basis expansion method, is possible to apply. The energy eigenvalues calculated from the CSM plus basis expansion technique become the discrete and complex eigenvalue, $E^{\theta} \rightarrow E^{\theta}_{\nu}$ labeled by the eigenvalue number ν. According to the ABC theorem [6], the energy eigenvalues for the bound state are invariant, and the energy eigenvalues for the resonances are clearly separated from the non-resonant continuum states in the complex energy plane [6].

The CSM is possible to apply to the calculation of the strength function, which represents the transition strength of the initial ground state (Ψ_i) induced by the external field of $\hat{O}_{\lambda}$ with the multi-polarity λ [8]. The definition of the strength function of $S_{\lambda}(E)$ is given by

$$S_{\lambda}(E) \ = \ \sum_{f} | < \Psi_f|\hat{O}_{\lambda}|\Psi_i > |^2 \delta(E - E_f) \ , \tag{3}$$

where Ψ_f denotes the final state belonging to the f-th eigenstate excited by the external field of $\hat{O}_{\lambda}$. By introducing the complex rotation given by equation (1) and the extended completeness relation [6], the strength function is rewritten like

$$S_{\lambda}(E) \ = \ -\frac{1}{\pi} \Im R_{\lambda}(E) \tag{4}$$

with the response function of $R_{\lambda}(E)$ defined by

$$R_{\lambda}(E) \ = \ \sum_{\nu} \frac{< \tilde{\Psi}^{\theta}_i|(\hat{O}^{\dagger}_{\lambda})^{\theta}|\Psi^{\theta}_{\nu} >< \tilde{\Psi}^{\theta}_{\nu}|\hat{O}^{\theta}_{\lambda}|\Psi^{\theta}_i >}{E - E^{\theta}_{\nu}} \ . \tag{5}$$

Here the Ψ^θ is the solution of CSM, and the tilde in the bra-state means that the complex conjugate is not taken for the radial part of the wave function [6].

2.2 Setting of theoretical calculation

In the present study, the direct breakup of ^{20}Ne into $\alpha + {}^{16}$O is considered because the ^{20}Ne is a typical example of the binary cluster system [9]. In the calculation of the $\alpha + {}^{16}$O system, the computational setting is the same as the setting in reference [7]. The interaction potential V is composed of the nuclear (V_N) and Coulomb (V_C) potentials, and their explicit form is given by

$$V(R) \quad = \quad V_N(R) \ + \ V_C(R) \tag{6}$$

$$V_N(R) \quad = \quad -154 \cdot \exp(-0.1102R^2) \tag{7}$$

$$V_C(R) \quad = \quad 16 \cdot \frac{e^2}{R} \cdot \mathrm{erf}(0.4805R) \tag{8}$$

with the definition of the error function of

$$\mathrm{erf}(x) \ = \ \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt \ . \tag{9}$$

The computational process to prepare the initial wave function (Ψ_i) and the final one (Ψ_f) for the $\alpha - {}^{16}$O relative motion is also the same as those in reference [7]. The pseudo potential with the harmonic oscillator wave function is included to exclude the Pauli's forbidden states in solving the Schrödinger equation with the potentials shown in equations (6), (7) and (8) [7, 8].

2.2.1 Operators for direct breakup

In the present analysis, the standard operator with the multi-polarity of $\lambda = 2$ is used as the external field inducing the direct breakup. Namely, in the calculation of the matrix element in equation (3), we use $\hat{O}_{\lambda=2}$ exciting the relative wave function of two clusters, such as

$$\hat{O}_{\lambda=2} \ = \ \sqrt{4\pi}R^2 Y_{2,0}(\hat{\mathbf{R}}) \ , \tag{10}$$

where $\mathbf{R}$ denotes the relative coordinate of two clusters. In addition, we consider the higher order operator for the monopole excitation, which is given by

$$\hat{O}_{\lambda=0} \quad = \quad \sqrt{4\pi}R^2 Y_{0,0}(\hat{\mathbf{R}}) \ . \tag{11}$$

The importance of the cluster excitation by this higher order operator has recently been pointed out in references [12, 13].

A special treatment is required in the calculation of the monopole transition using equation (11). In the matrix element for the direct breakup in equation (3), the operation of the monopole operator on the initial wave function in the ground state generates the superposition of a series of the wave functions, which contains both of the ground and excited states. Thus, the ground state component must be extracted from the product of the monopole operator and the initial wave function, which is called the initial wave packet [2]. This exclusion can be achieved by the replacement in the radial operator of $R^2 \to R^2 - <R^2>$, in which $<R^2>$ denotes the expectation value of R^2 with the ground wave function.

3 Migdal-Watson theory and its extension

In the s-wave breakup of the binary system composed of the charge neutral particles, the strength function can be expressed by the closed formula in the case of the short range limit of the initial state. This formula is called the Migdal-Watson (MW) theory [3, 4], which express the strength function in terms of the effective range theory [5]. The detailed explanation of the MW formula and its application to the breakup of the di-neutron system is reported in reference [2]. Here we explain the essence of the MW formula and extend the formula to the case of the binary breakup reaction including the effects of the the finite charge, the finite spin and the finite size.

In the MW theory, the monopole strength of the direct breakup with the relative energy E for the binary fragments ($S_{\lambda=0}(E)$) is given by

$$S_0(E) \propto \frac{\sqrt{E}}{AE^2 + BE + C} \ , \tag{12}$$

where the constants of A, B and C are function of the scattering length and the effective range [2, 3, 4]. The MW formula in equation (12) must be valid for the breakup from the initial wave packet strongly localized inside of the nuclear interaction.

The MW formula shown in equation (12) is valid for the s-wave breakup from the initial wave packet localized inside of the nuclear interaction, which corresponds to the tightly binding system composed of the charge neutral fragments. Here we try to extend equation (12) so as to describe the direct breakup reaction of the general binary systems, which have the finite charge, the finite spin in the final scattering state ($S_{\lambda\neq0}(E)$) and the finite size of the initial wave packet.

We extend equation (12) to the following function:

$$S_\lambda(E) \propto \frac{P_\lambda(ka)e^{-\beta E}}{AE^2 + BE + C} \ . \tag{13}$$

In this function, $P_\lambda(ka)$ denotes the penetration factor with the momentum k for the binary decaying fragments and the channel radius a. P_λ corresponds to the extension of the factor of $\sqrt{E}$ in equation (12). If we consider the limit of $k \to 0$ and $\lambda = 0$, $P_\lambda(ka)$ is reduced to $\sqrt{E}$.

On the contrary, β in the exponential term simulates the effect of the finite size of the initial packet. β should be small in the case of the limit of the strong binding system, and vice versa. We try to reproduce the background strength generated by the direct breakup by controlling the five fitting parameters: A, B, C, a, β.

4 Results

The fitting result using the extended MW formula in equation (13) is shown in the two panels of figure 1. The quadrupole and monopole strengths for $^{20}\mathrm{Ne} \to \alpha + {}^{16}\mathrm{O}$ are shown in the left and right panels, respectively. The calculated strength functions (solid curves) are nicely reproduced by the extended MW formula (open circles). In both the panels, the solid circle with the error bar means the resonance energy ($\mathcal{E}_R$) with the decay width (Γ_R) of the potential resonance in the final state. The peak structure in the strength function nicely corresponds to the resonance energy and the width and hence, the enhancement is originated from the resonance formation.

In the fitting analysis of the quadrupole transition, we do not use the original resonance parameter ($\mathcal{E}_R = 4.2$ MeV and $\Gamma_R = 2.8$ MeV) but the tuned parameters for the Breit-Wigner part: $A = 0.025$, $B = -0.21$, $C = 0.48$, $a = 5.5$ fm and $\beta = 0.015$ MeV^{-1} in equation (13). In the monopole transition,

the resonance parameter of the 0^+ resonance is $\mathcal{E}_R = 3.3$ MeV and $\Gamma_R = 2.0$ MeV. The parameters used for the fitting to the monopole transition are $A = 0.029$, $B = -0.19$, $C = 0.33$,

$a = 5.0$ fm and $\beta = 0.015$ MeV^{-1}, in which (A, B, C) are modified from the original resonance parameters.
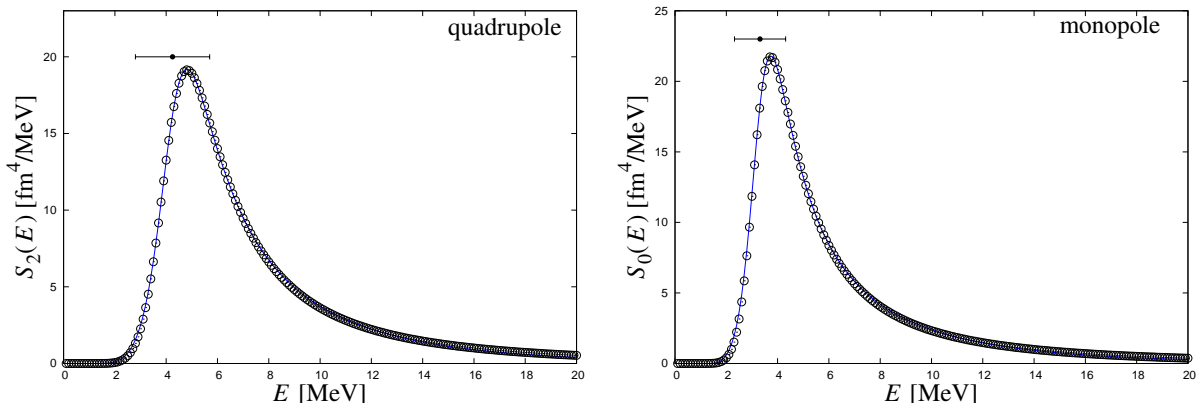

Figure 1: Comparison of the strength function of ^{20}Ne $\to \alpha + {}^{16}$O with fitting results by the extended Migdal-Watson formula in equation (13). The left panel shows the quadrupole $(S_2(E))$ transition, while the right one shows the monopole $(S_0(E))$ transitions. The α threshold energy is set to the zero point in the abscissa. In both of the panels, the solid curve and the open circles represent the strength function calculated from CSM and fitting results by equation (13), respectively. The solid circle with the error bar shows the resonance energy with the decay width.

In the monopole transition, the excitation energy of the peak position corresponds to about 10 MeV, and this energy is much lower than the excitation energy of the monopole single particle excitation, which reaches about 30 MeV in this mass region [10]. One of the characteristic feature in the α cluster excitation is that the strong monopole strength appears at the lower excitation energy region as pointed out in the previous calculations [10, 11]. Therefore, the extended MW formula, which is developed by the present analysis, is important in the evaluation of the low-lying strength of the monopole transition induced by the α cluster excitation.

5 Summary

In summary, we have investigated the feature of the strength function for the direct breakup process, in which a bound nucleus dissociates into the binary fragments. The direct breakup process is defined by the direct and one-step transition from the specific component of the binary channel in the many-body bound state to the distorted wave for this channel, which is generated by the nuclear and Coulomb potentials in the final scattering state. The transition strength of the direct breakup is considered to be the main component of the non-resonant background strength in the realistic breakup experiment.

In order to describe the background strength in the simple manner, we have considered the analytic function by extending the Migdal-Watson (MW) formula [3, 4, 5], which has recently been discussed in the s-wave breakup of the di-neutron system [2]. The MW formula gives the function of the quadratic energy denominator times the square root of energy, such as $\sqrt{E}/(AE^2 + BE + C)$. This formula is valid for the s-wave binary breakup of the charge neutral system, and the initial wave function is assumed to be strongly confined inside of the short range nuclear interaction.

We have extended the MW formula so as to include the effects of the finite spins, the finite charge and the finite size of the initial wave packet. First, the numerator of $\sqrt{E}$ is replaced by

the penetration factor of $P_\lambda(ka)$ to take into account the effects of the finite spin (λ) and finite charge.

Furthermore, we have introduced the exponential damping factor, $\exp(-\beta E)$, which is originated from the tunneling tail of the initial wave packet outside of the nuclear interaction.

The extended MW formula proposed for the direct breakup is

$$\frac{P_\lambda(ka) \cdot \exp(-\beta E)}{AE^2 + BE + C} \quad . \tag{14}$$

We have tested the extended MW formula by fitting the strength function obtained from the theoretical calculation of the direct breakup. Here we have evaluated the strength by the direct breakup in $^{20}\text{Ne} \to \alpha + {}^{16}\text{O}$ [7] on the basis of the formulation of the complex scaling method (CSM) [6]. In the CSM calculation of the monopole and quadrupole transitions, we have confirmed that the resonant peak appears around $E \sim 5$ MeV above the α threshold energy, and the strength functions are nicely reproduced by the extended MW formula. Since we have confirmed the availability of the MW formula in the direct breakup in the binary system, it is important to apply the formula to other binary breakup reactions, such as $^{12}\text{Be} \to \alpha + {}^8\text{He}$ [1]. The application to ^{12}Be is now under progress.

Acknowledgments

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