Choice of μ_R, μ_f and μ_F and systematic uncertainty of JETPHOX

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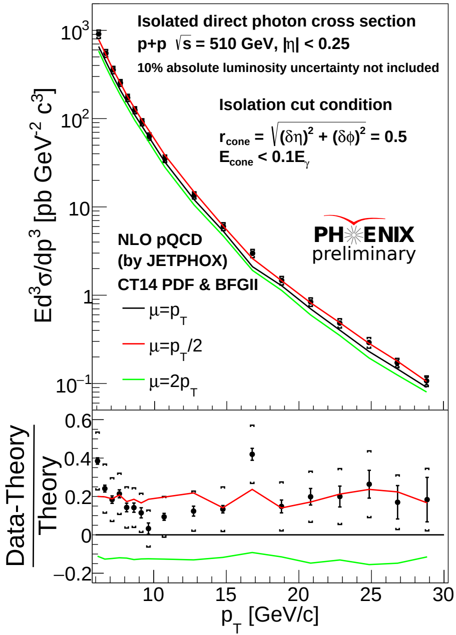


Problems from preliminary plot:

- Why $\mu_R = \mu_f = \mu_F = p_T/2$ agrees best with data?
- Why there is a kink at $p_T = 17$ GeV?
- Is $\mu_R = \mu_f = \mu_F$ a good choice to explore JETPHOX systematic uncertainty?

Motivations

- Most of direct photon measurements agree best with $\mu_R = \mu_f = \mu_F = p_T/2$.
- There is large systematic uncertainty in JETPHOX from choice of μ_R.
- ATLAS direct photon papers vary μ_R, μ_f and μ_F independently to study JETPHOX systematic uncertainty.
- Good news: there is well established method to set optimal μ_R and it is very easy to use.
- There is also similar method to choose μ_f and μ_F, but it is hard to calculate.
- Systematic uncertainty from μ_F is much smaller than that from μ_R or μ_f.

Rusummation by renormalization group equation (RGE)

- The β function of the running coupling $\alpha_s(\mu^2)$:

$$\frac{d\alpha_s(\mu^2)}{d\ln\mu^2} = \beta(\alpha_s) = -b_0\alpha_s^2(\mu^2)$$

$$\Rightarrow \alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + b_0\alpha_s(\mu_0)\ln(\mu^2/\mu_0^2)} = \alpha_s \sum_n \left(-b_0\alpha_s \ln(\mu^2/\mu_0^2)\right)^n$$

  where $b_0 = \frac{1}{4\pi}(\frac{11}{3}N_c - \frac{2}{3}N_f)$.
- The RGE can sum all terms associated with β function into running coupling $\alpha_s(\mu^2)$.

Optimal choice of μ_R: principle of maximum conformality (PMC)

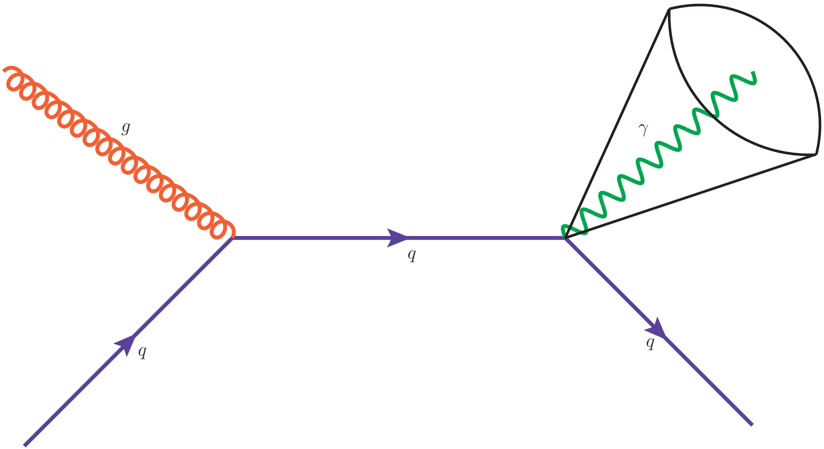
- Use RGE to remove β terms in cross section [PRD 86, 085026 (2012)]:

$$
\begin{aligned}
\sigma &\sim \alpha_s(\mu^2)\left(1 + b_0\alpha_s(\mu^2)\left(\ln\frac{\mu^2}{P^2} + C_{\overline{MS}}(x)\right) + \dots\right) \\
&= \alpha_s(\mu_0^2)\left(1 - b_0\alpha_s(\mu_0^2)\ln\frac{\mu^2}{\mu_0^2} + \mathcal{O}(\alpha_s^2)\right)\left(1 + b_0\alpha_s(\mu_0^2)\left(\ln\frac{\mu^2}{P^2} + C_{\overline{MS}}(x)\right) + \mathcal{O}(\alpha_s^2) + \dots\right) \\
&= \alpha_s(\mu_0^2)\left(1 + b_0\alpha_s(\mu_0^2)\left(\ln\frac{\mu_0^2}{P^2} + C_{\overline{MS}}(x)\right) + \dots\right) + \mathcal{O}(\alpha_s^3)
\end{aligned}
$$

  where P^2 is some physical scale and $C_{\overline{MS}}(x)$ is a function depends on kinematic variables x and renormalization scheme $\overline{MS}$.
- Sum β terms into $\alpha_s(\mu_{PMC}^2)$: $\ln\frac{\mu_{PMC}^2}{P^2} + C_{\overline{MS}}(x) = 0 \Rightarrow \mu_{PMC} = Pe^{-C_{\overline{MS}}(x)/2}$.
- Changing $\mu \to \mu_{PMC}$, σ does not change at $\mathcal{O}(\alpha_s^2)$ (NLO), but RGE can sum all terms associated with β function.

Process for direct photon production

- ~85% direct photons are produced by $q + g \to q + \gamma$ in pp collisions.
- $q + g \to q + \gamma$ and $q + \bar{q} \to g + \gamma$ are related by crossing symmetry.



μ_{PMC} for direct photon production

- The physical scale in both processes is p_T, so we can use NLO $q\bar{q} \to g\gamma$ results from [Nucl. Phys. B 297, 661 (1988)]. Only the terms associated with $b_0 = \frac{1}{4\pi}(\frac{11}{3}N_c - \frac{2}{3}N_f)$ are interested:

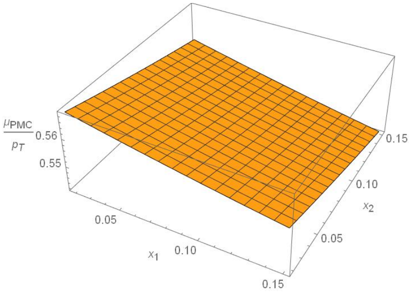
$$\sigma_{NLO}^{q\bar{q}\to g\gamma} \sim b_0 \left(\int_{\frac{x_1}{2}}^{1-\frac{x_2}{2}} dv \left(v^2 + (1-v)^2\right) \ln \frac{\mu^2}{\hat{s}} - 2 \int_{\frac{x_1}{2}}^{1-\frac{x_2}{2}} dv (v(1-v) - 1) \right) + \dots$$

$$\Rightarrow \mu_{PMC} = \sqrt{\hat{s}} \cdot \exp \left(\frac{\int_{\frac{x_1}{2}}^{1-\frac{x_2}{2}} dv (v(1-v)-1)}{\int_{\frac{x_1}{2}}^{1-\frac{x_2}{2}} dv \left(v^2 + (1-v)^2\right)} \right)$$

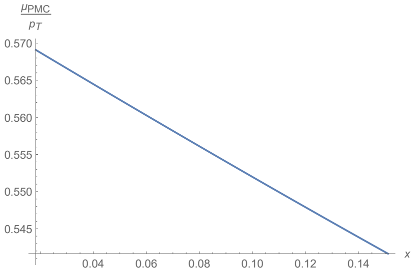
$$= p_T \sqrt{2 + e^{y_\gamma - y_J} + e^{y_J - y_\gamma}} \cdot \exp \left(\frac{-\frac{5}{6} + \frac{x_1}{2} - \frac{x_1^2}{8} + \frac{x_1^3}{24} + \frac{x_2}{2} - \frac{x_2^2}{8} + \frac{x_2^3}{24}}{\frac{2}{3} - \frac{x_1}{2} + \frac{x_1^2}{4} - \frac{x_1^3}{12} - \frac{x_2}{2} + \frac{x_2^2}{4} - \frac{x_2^3}{12}} \right)$$

by using kinematics in run 6 paper [PRD 86, 072008 (2012)].

p_T factor in μ_{PMC}



Vary x_1 and x_2 independently



Set $x_1 = x_2$

p_T factor vs (x_1, x_2) with $y_\gamma = y_J = 0$ for 6 GeV $< p_T <$ 30 GeV and $|\eta| < 0.25$

- After considering $y_\gamma \neq 0$ and $y_J \neq 0$ but limited by central arm, the p_T factor varies from 0.54 to 0.58.

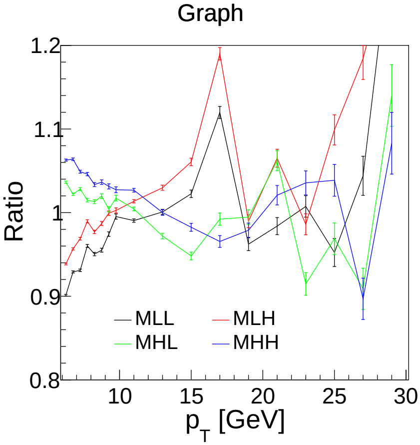
Optimal choice of μ_f and μ_F

- Idea: sum parton multiple splittings into PDF by DGLAP equations.
- Method: use NLO splitting process to decide μ_f for LO PDF [Eur. Phys. J. C 77, 218 (2017)]:

$$\sigma_{NLO}^{splitting}(\mu_0) = |\mathcal{M}^{LO}|^2 \otimes PDF(\mu_0) \otimes \frac{\alpha_s}{\pi} P^{real}(z) \ln \frac{\mu_f}{\mu_0}$$

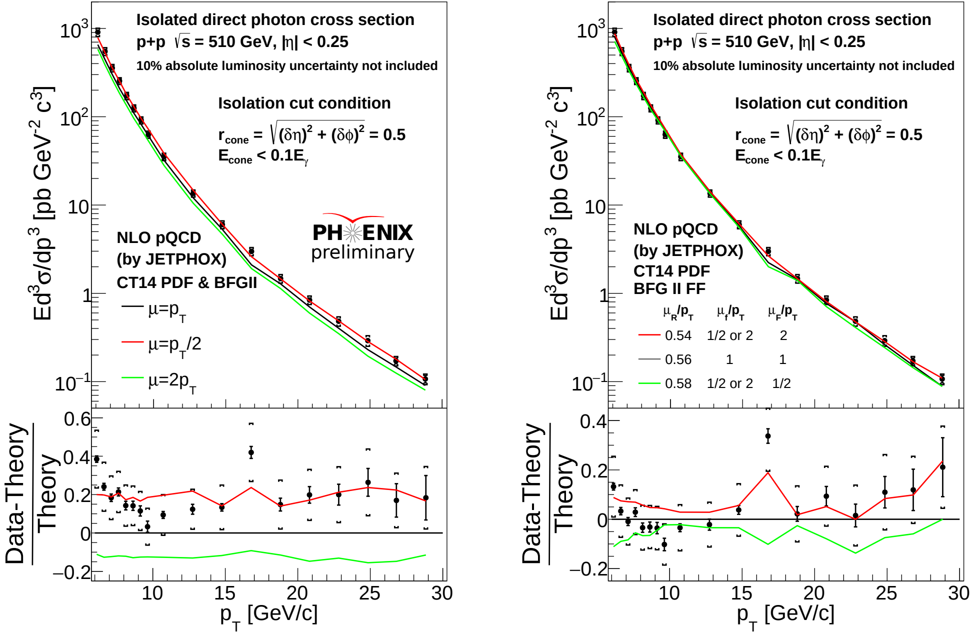
  where $P^{real}(z)$ are the splitting functions belonging to real emission.
- Difficult: need convolutions with PDF and iterations.
- Physical meaning of μ_f: process with energy lower than μ_f included in $PDF(\mu_f)$, higher than μ_f included in hard matrix element $|\mathcal{M}^{NLO}(\mu_f)|^2$.
- Conclusion: still use μ_f and μ_F as p_T, $p_T/2$ and $2p_T$, but vary them independently to explore the systematic uncertainty.

Systematic uncertainty from μ_f and μ_F



Cross section ratios with same μ_R (first letter) but different μ_f (second letter) and μ_F (third letter). M for $0.56p_T$, L for $p_T/2$, H for $2p_T$.

- Red and black are similar, green and blue are similar, so the differences mainly come from μ_f.
- Compare red with black, as well as green with blue, we see $\sigma(\mu_F = 2p_T) > \sigma(\mu_F = p_T/2)$.
- We also know $\sigma(\mu_R = p_T/2) > \sigma(\mu_R = 2p_T)$.
- We choose LLH, LHH, LLL, HLL, HHL, HHH and use their maxima and minima as the bound of systematic uncertainties.



Isolated direct photon cross section
p+p √s = 510 GeV, |η| < 0.25
10% absolute luminosity uncertainty not included
Isolation cut condition
r_cone = √((δη)^2 + (δφ)^2) = 0.5
E_cone < 0.1E_γ
NLO pQCD
(by JETPHOX)
CT14 PDF & BFGII
μ=p_T
μ=p_T/2
μ=2p_T
PHENIX
preliminary
Ed^3σ/dp^3 [pb GeV^-2 c^3]
Data-Theory / Theory
p_T [GeV/c]
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CT14 PDF
BFG II FF
μ_R/p_T μ_f/p_T μ_F/p_T
0.54 1/2 or 2 2
0.56 1 1
0.58 1/2 or 2 1/2
Ed^3σ/dp^3 [pb GeV^-2 c^3]
Data-Theory / Theory
p_T [GeV/c]

Conclusions and next step

- Central value of JETPHOX is shifted to the measurement and its systematic uncertainty is much reduced by using PMC.
- By varying μ_f and μ_F independently, its systematic uncertainty is well explored.
- After tuning, JETPHOX shows better agreement with data.
- Next step is using PYTHIA and PISA to study background from charged pions in inclusive direct photon yield.