A_N vs. P_T Unfolding

RadLab Group Meeting
2020/07/22
9:00 PM (KST/JST)

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Asymmetry and Parameter Scanning – Functional Forms

For different MC samples (OPE, Pythia6, Pythia8, Dpmjet and UPC)

- Spin and P_T dependent weights are scanned over a wide range of linear, quadratic and cubic parts of true A_Ns using random generator.

Three different functional forms have been used for the weights:

❖ 3rd order polynomial (w_{pol3}) based weight:

$$w_{pol3} = 1 + \left(\mathrm{a} + \mathrm{b} * P_{T,T} + c * P_{T,T}^{2} + d * P_{T,T}^{3}\right) * \cos(\Phi_{T,T} + spin * \pi)$$

❖ Power function (w_{pow}) based weight:

$$w_{pow} = 1 + \left(\mathrm{a} + \mathrm{b} * P_{T,T}^{c}\right) * \cos(\Phi_{T,T} + spin * \pi)$$

❖ Exponential function (w_{exp}) based weight:

$$w_{exp} = 1 + \left(\mathrm{a} + \mathrm{b} * \left(1 - \exp\left((\mathrm{c} + \mathrm{d}) * P_{T,T}\right)\right)\right) * \cos(\Phi_{T,T} + spin * \pi)$$

a = constant term (not useful – can not distinguish left-right asymmetry)

b = linear part scanned over a wide range $b_{min} < b < b_{max}$

c = quadratic part scanned over a wide range between $c_{min} < c < c_{max}$

d = cubic part scanned over a wide range between $d_{min} < d < d_{max}$

s = spin (↑↓)

$P_{T,T}$ = true transverse momentum and $\Phi_{T,T}$ = true azimuthal angle distributions

Asymmetry and Parameter Scanning – Chi-Square (Min)

Calculate Chi-Square based on the reconstructed asymmetries and the experimental data asymmetries (Minjung) for the three different functional forms:

1. Third order polynomial (pol3):

$$\chi^2_{pol3} = \sum_i \frac{(A_{N,i}^{Minjung} - A_{N,i}^{w_{pol3},reco})^2}{\Delta A_{N,i}^{2,Minjung} + \Delta A_{N,i}^{2,w_{pol3},reco}}$$

2. Power function (pow)

$$\chi^2_{pow} = \sum_i \frac{(A_{N,i}^{Minjung} - A_{N,i}^{w_{pow},reco})^2}{\Delta A_{N,i}^{2,Minjung} + \Delta A_{N,i}^{2,w_{pow},reco}}$$

3. Exponential function.

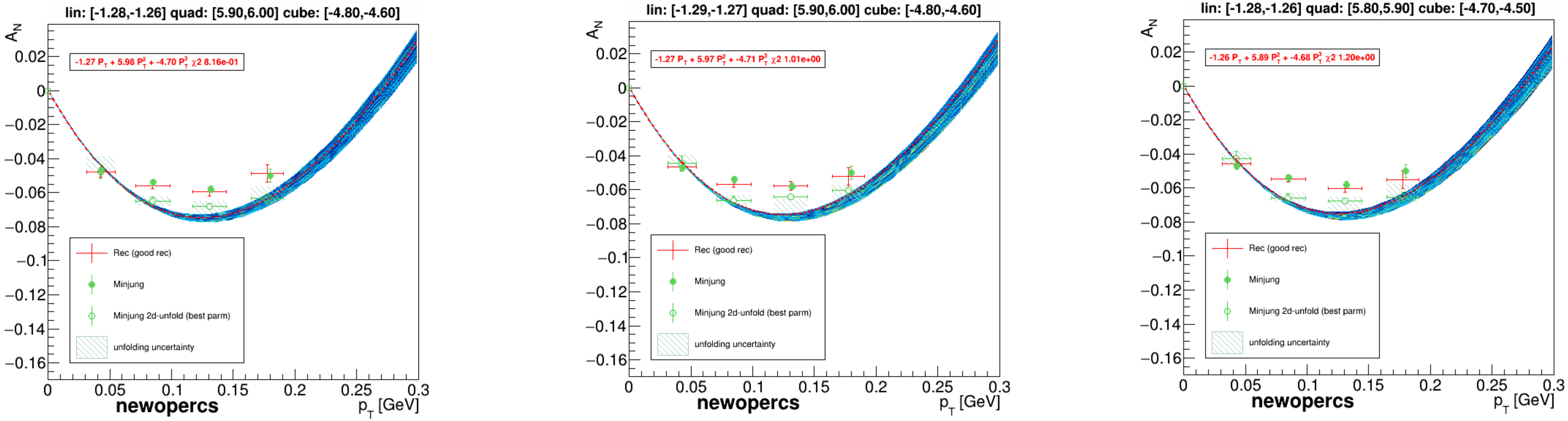
$$\chi^2_{exp} = \sum_i \frac{(A_{N,i}^{Minjung} - A_{N,i}^{w_{exp},reco})^2}{\Delta A_{N,i}^{2,Minjung} + \Delta A_{N,i}^{2,w_{exp},reco}}$$

Best parameter is found (i.e. parameter with lowest Chi-Square for each functional form).

Asymmetry based on 3rd order polynomial function

- Pol3 weights depending on spin and pt are scanned over a wide range of linear, quadratic and cubic parts of true A_N's.

$$w_{pol3} = 1 + \left(a + b * P_{T,T} + c * P_{T,T}^2 + d * P_{T,T}^3\right) * \cos(\Phi_{T,T} + spin * \pi)$$



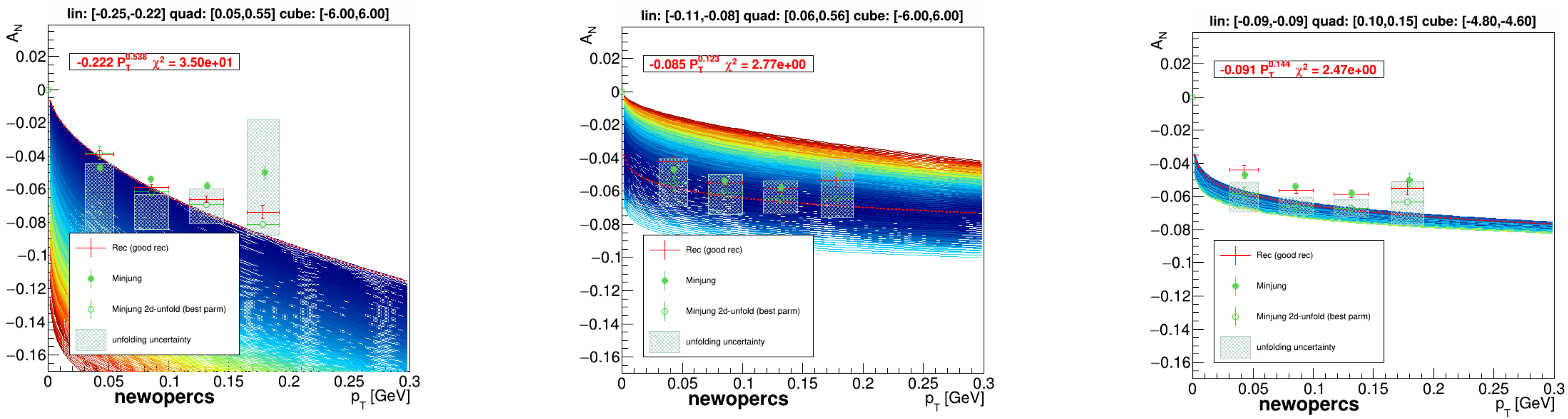
- Based on 3rd degree polynomial weighted reconstructed asymmetries and Minjung's result, Chi-Square is calculated.

$$\chi^2_{pol3} = \sum_i \frac{\left(A_{N,i}^{Minjung} - A_{N,i}^{w_{pol3},reco}\right)^2}{\Delta A_{N,i}^{2,Minjung} + \Delta A_{N,i}^{2,w_{pol3},reco}}$$

- Best parameter is found (i.e. lowest χ^2).
- Unfold using best parmetrization coresponding to lowest χ^2

χ^2 based on 3rd order polynomial weighted A_N (reco) and Minjung's A_N's

Asymmetry weighted based on the power function



- Power function weights depending on spin and pt are scanned over a wide range of linear, quadratic and cubic parts of true A_N's.

$$w_{pow} = 1 + \left(\mathrm{a} + \mathrm{b} * P_{T,T}^{c}\right) * \cos(\Phi_{T,T} + spin * \pi)$$

Power weight

- Based on exponentially weighted reconstructed asymmetries and Minjung's result, Chi-Square is calculated.

$$\chi^2_{pow} = \sum_i \frac{(A_{N,i}^{Minjung} - A_{N,i}^{w_{pow},reco})^2}{\Delta A_{N,i}^{2,Minjung} + \Delta A_{N,i}^{2,w_{pow},reco}}$$

- Best parameter is found (i.e. lowest χ^2).
- Unfold using best parmetrization coresponding to lowest χ^2.

χ^2 based on power weighted A_N (reco) and Minjung's A_N's

Asymmetry weighted based on the exponential function

- Exponential weights depending on spin and pt are scanned over a wide range of linear, quadratic and cubic parts of true A_N's.

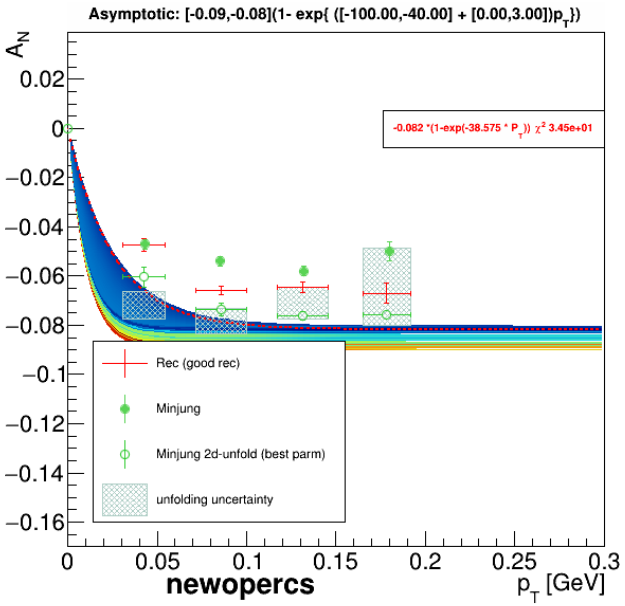
Exponential weight $$w_{exp} = 1 + \left(a + b * \left(1 - \exp\left((c + d) * P_{T,T}\right)\right)\right) * \cos(\Phi_{T,T} + spin * \pi)$$

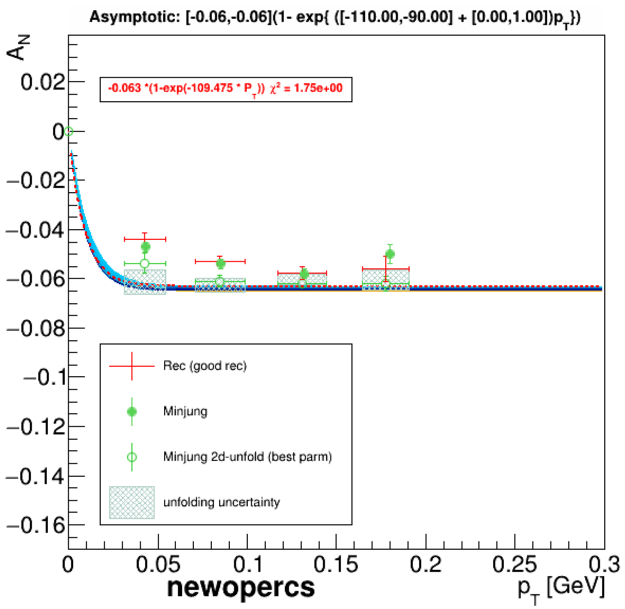
- Based on exponentially weighted reconstructed asymmetries and Minjung's result, Chi-Square is calculated.

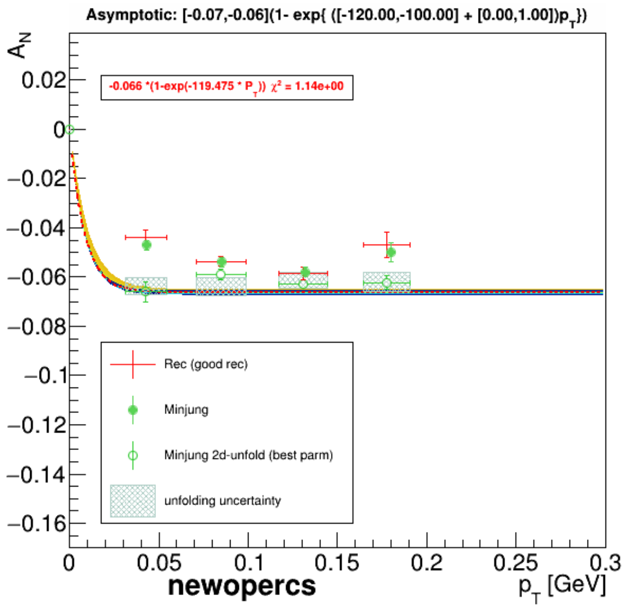
$$\chi^2_{exp} = \sum_i \frac{(A_{N,i}^{Minjung} - A_{N,i}^{w_{exp},reco})^2}{\Delta A_{N,i}^{2,Minjung} + \Delta A_{N,i}^{2,w_{exp},reco}}$$

χ^2 based on exponentially weighted A_N (reco) and Minjung's A_N's

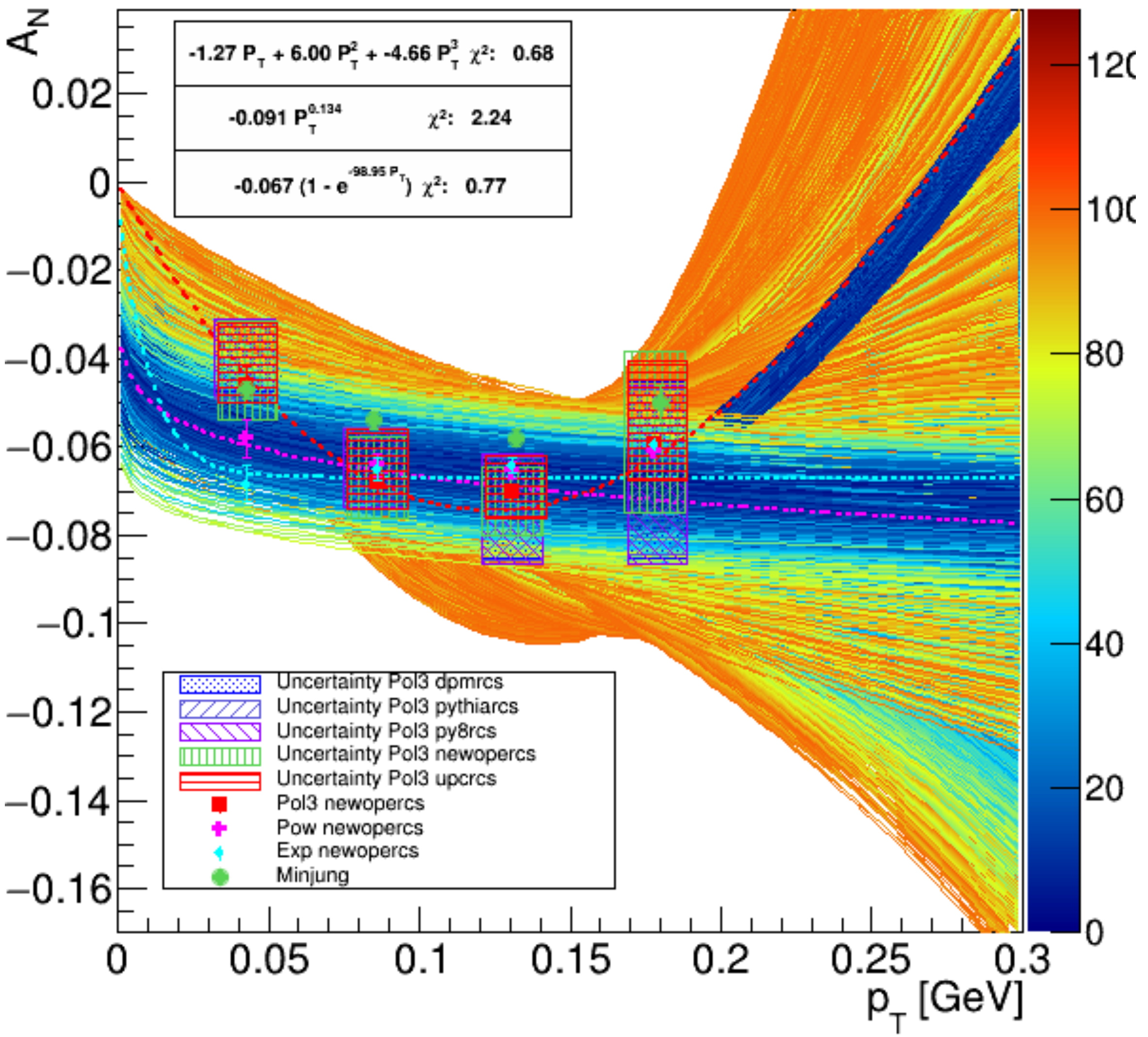
- Best parameter is found (i.e. lowest χ^2).
- Unfold using best parmetrization coresponding to lowest χ^2







Asymmetry based on Polynomial, Power and Exponential Functions



Files range: 0 and 100 (different parameters & Chi2)

Best chosen parameters:

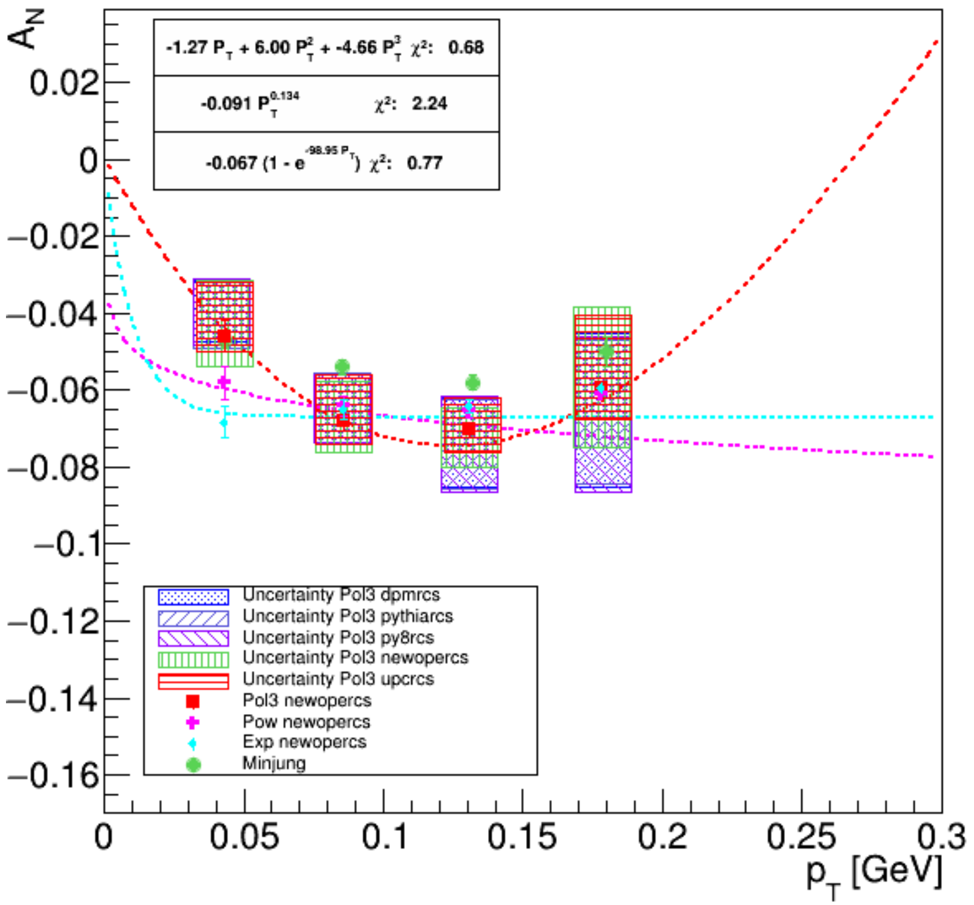
- Pol3 Chi2 = 0.68
- Powr Chi2 = 2.24
- Expo Chi2 = 0.77

All show reasonable χ^2 with respect to Minjung's asymmetry results.

All MC uncertainties:

- Shown for Pol3 Dpmjet, Pythia6, Pythia8, Ope & Upc.

Inclusive Result with Statistical Uncertainties from Monte Carlo only



Files range: 0 and 100 (different parameters & Chi2)

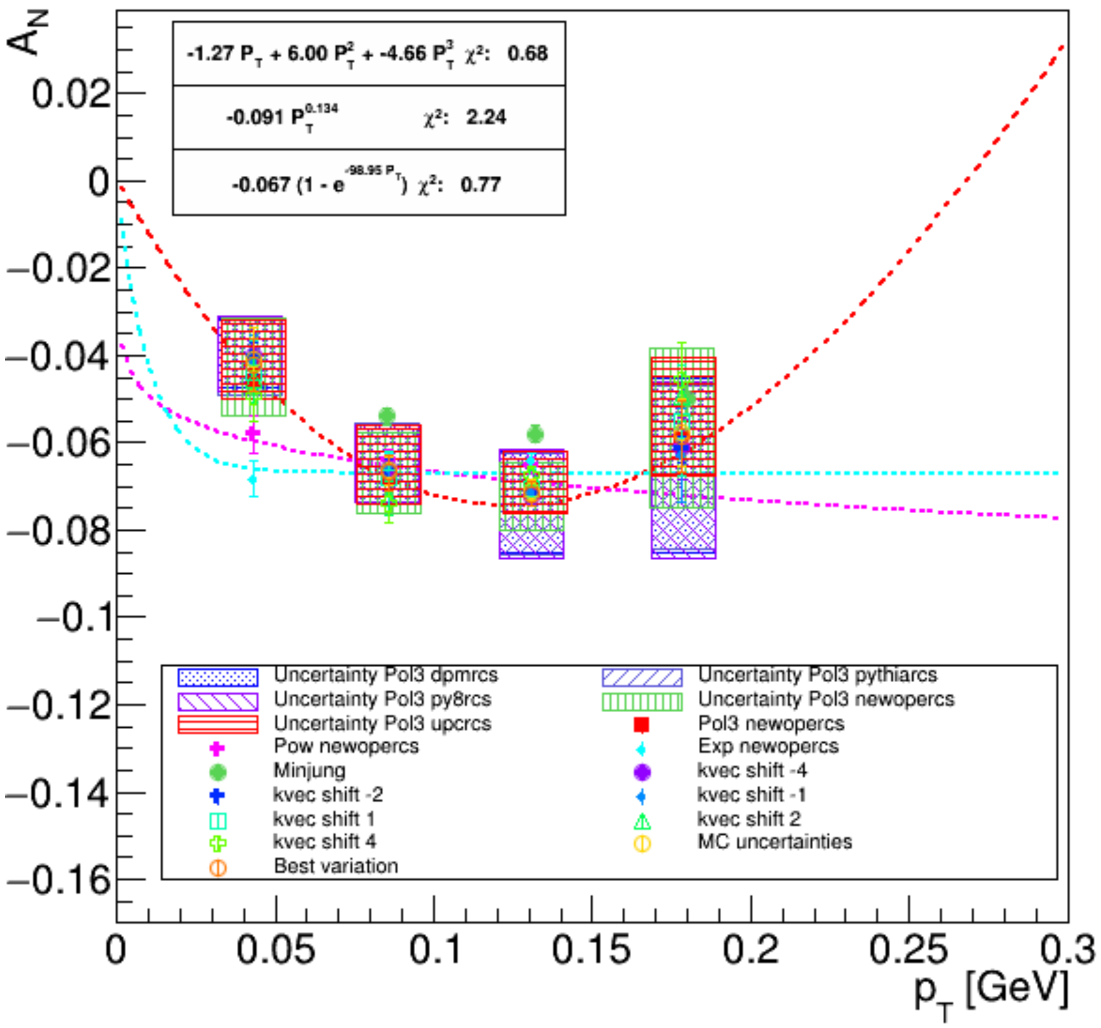
Best chosen parameters:

- Pol3 Chi2 = 0.68
- Powr Chi2 = 2.24
- Expo Chi2 = 0.77

MC (Poly3) systematic uncertainties:

- For Dpmjet, Pythia6, Pythia8, Ope & Upc using Pol3.

Inclusive Result with Statistical Uncertainties from MC and Unfolding



Files range: 0 and 100 (different parameters & Chi2)

Best parameters:

- Pol3 Chi2 = 0.68
- Powr Chi2 = 2.24
- Expo Chi2 = 0.77

All systematic uncertainties:

1. MC (Pol3): Dpmjet, Pythia6, Pythia8, Ope & Upc.
2. Unfolding: (Regularization: kvec shifts, $\pm 1, \pm 2, \pm 4$)
3. Uncertainties on response matrix (MC uncertainties)
4. Best variation

Summary:

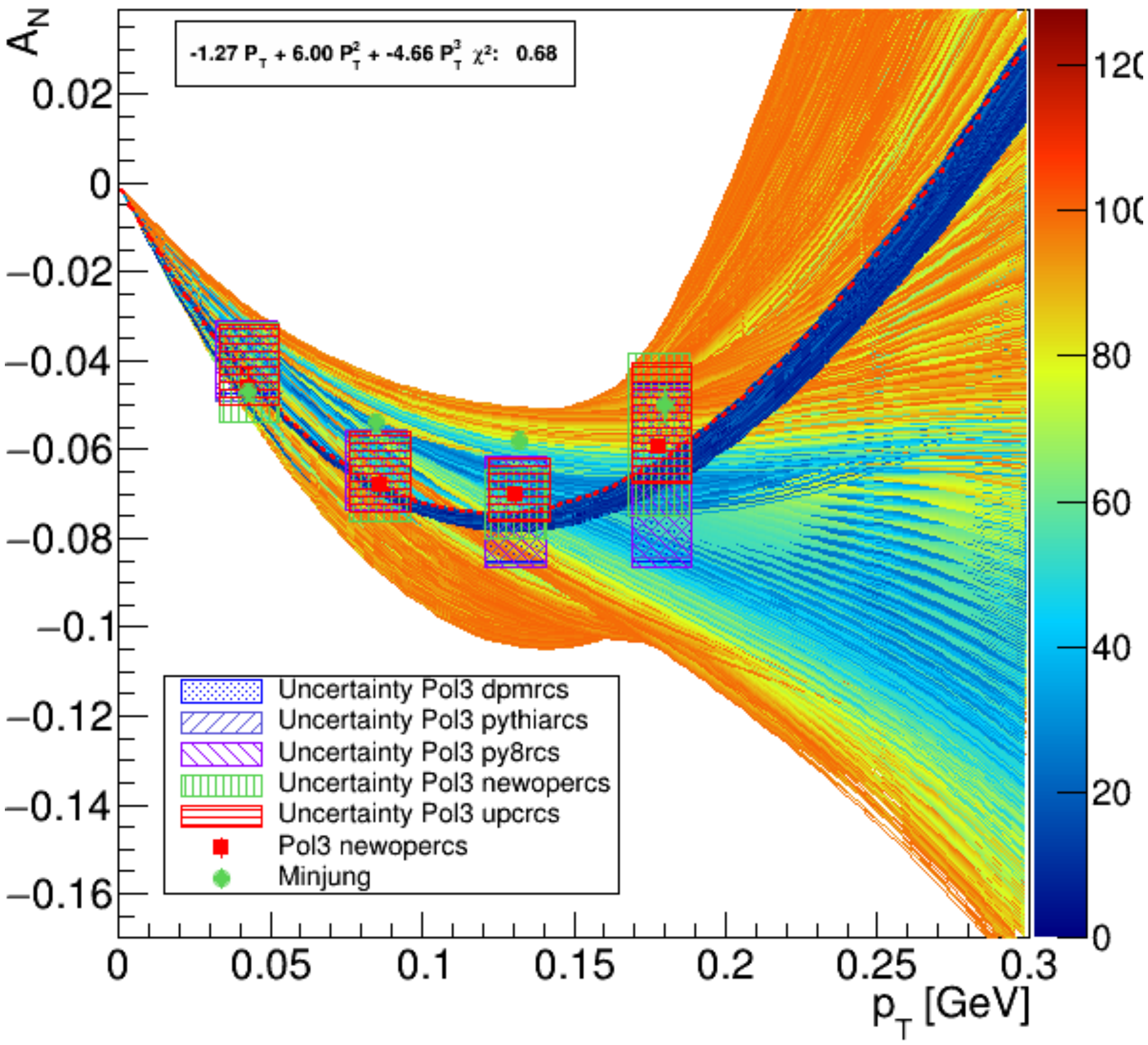
- Unfolding is reasonable with best parametrization.

Uncertainties studied through:

- Varying best regularization parameter of the unfolding.
- MC uncertainties through response matrix
- Comparison of different MC samples
- Repeating of best parametrization

BACKUP

Asymmetry based on Polynomial, Power and Exponential Functions



Files range: 0 and 100 (different parameters & Chi2)

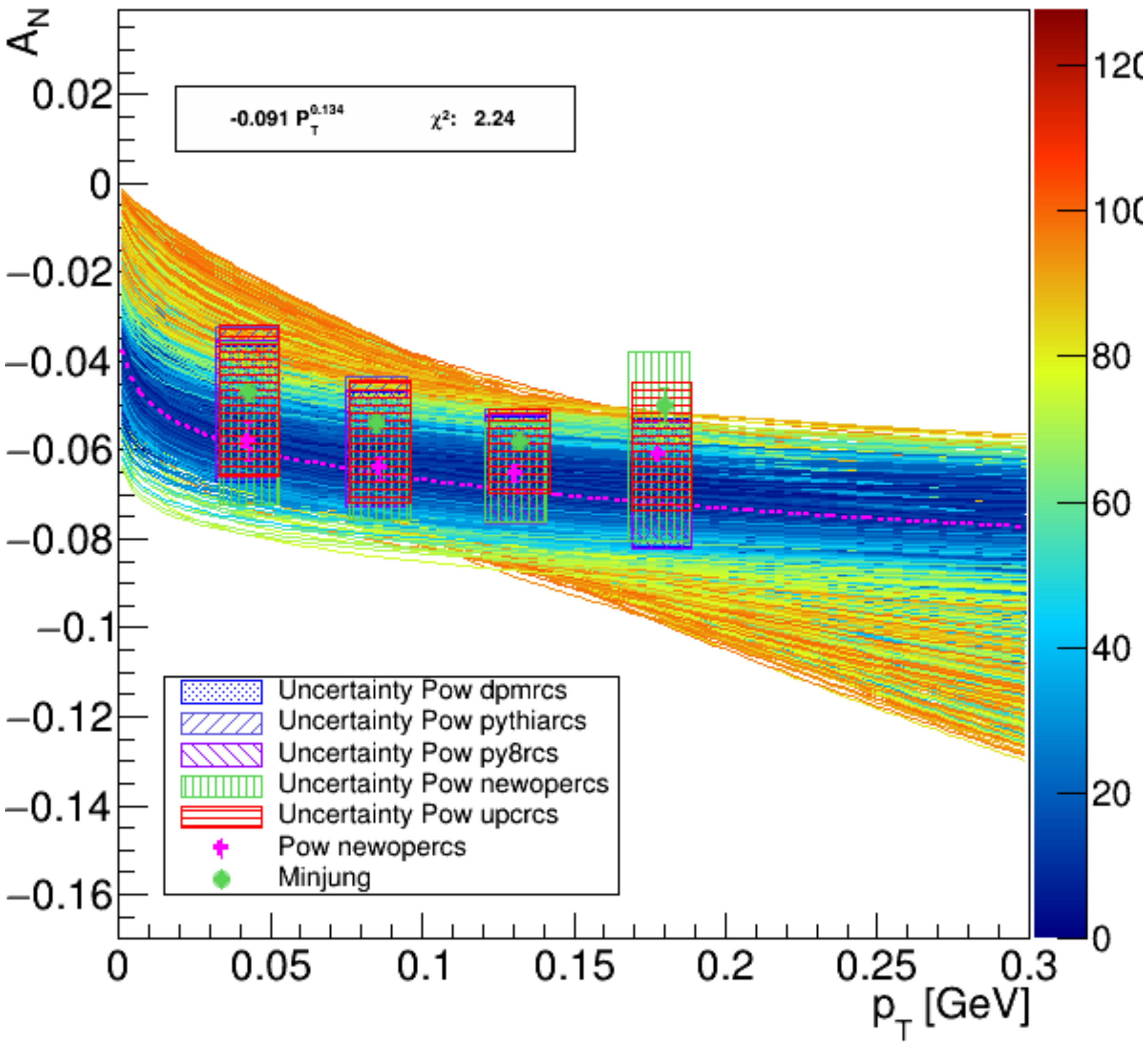
Best parameters:

- Linear = -1.27
- Quadratic = +6.00
- Cubic = -4.66
- Chi-Square = 0.68

Uncertainties:

- Shown for Pol3 Dpmjet, Pythia6, Pythia8, Ope & Upc.

Asymmetry based on Polynomial, Power and Exponential Functions



Files range: 0 and 100 (different parameters & Chi2)

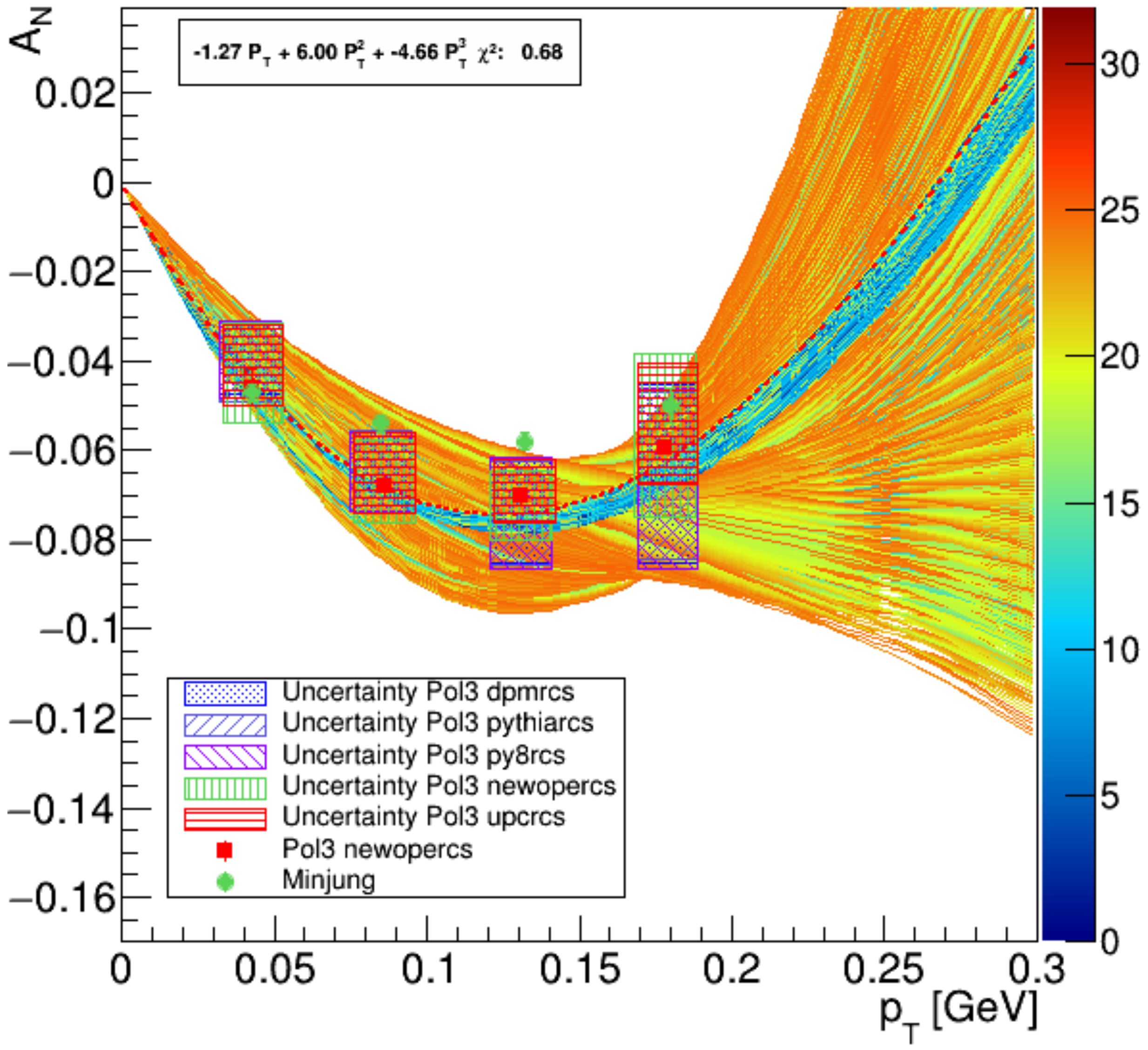
Best chosen parameters:

- Linear = -0.091
- Quadratic = +0.134
- Chi-Square = 2.24

Uncertainties:

- Shown for Pol3 Dpmjet, Pythia6, Pythia8, Ope & Upc.

Asymmetry based on Polynomial, Power and Exponential Functions



Files range: 0 and 25 (different parameters & Chi2)

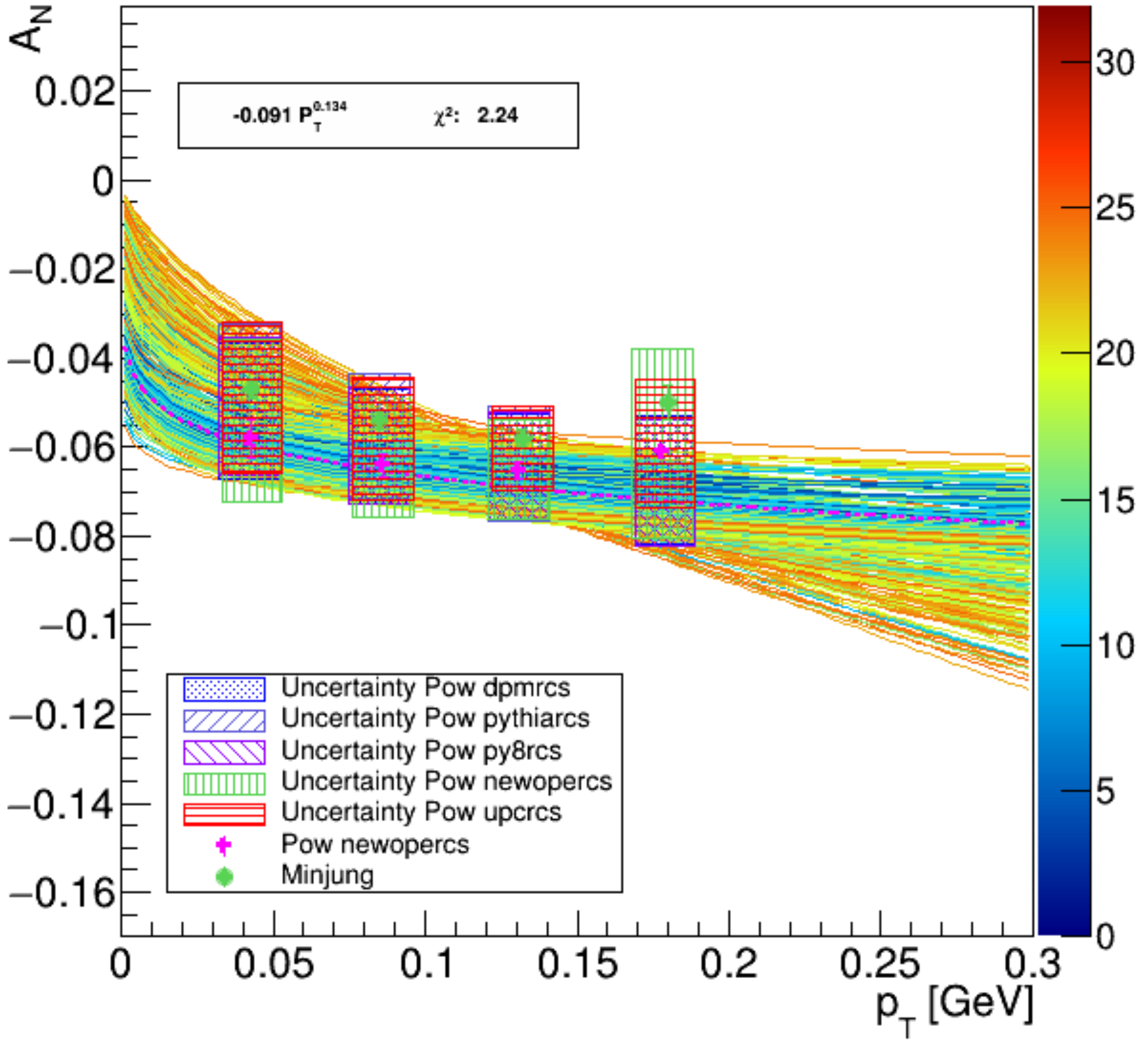
Best parameters:

- Linear = -1.27
- Quadratic = +6.00
- Cubic = -4.66
- Chi-Square = 0.68

Uncertainties:

- Shown for Pol3 Dpmjet, Pythia6, Pythia8, Ope & Upc.

Asymmetry based on Polynomial, Power and Exponential Functions



Files range: 0 and 25 (different parameters & Chi2)

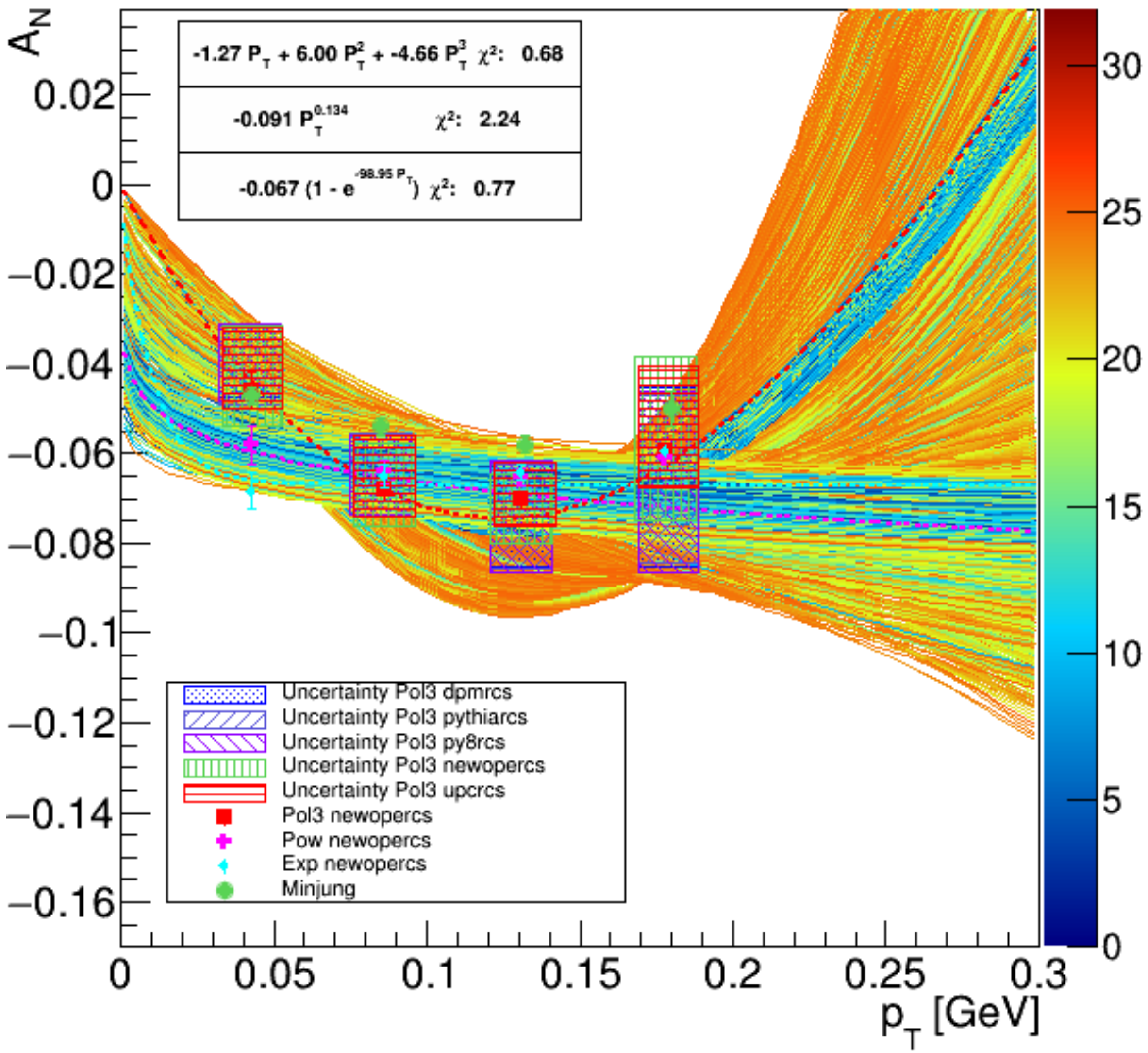
Best chosen parameters:

- Linear = -0.091
- Quadratic = +0.134
- Chi-Square = 2.24

Uncertainties:

- Shown for Pol3 Dpmjet, Pythia6, Pythia8, Ope & Upc.

Asymmetry based on Polynomial, Power and Exponential Functions



Files range: 0 and 25 (different parameters & Chi2)

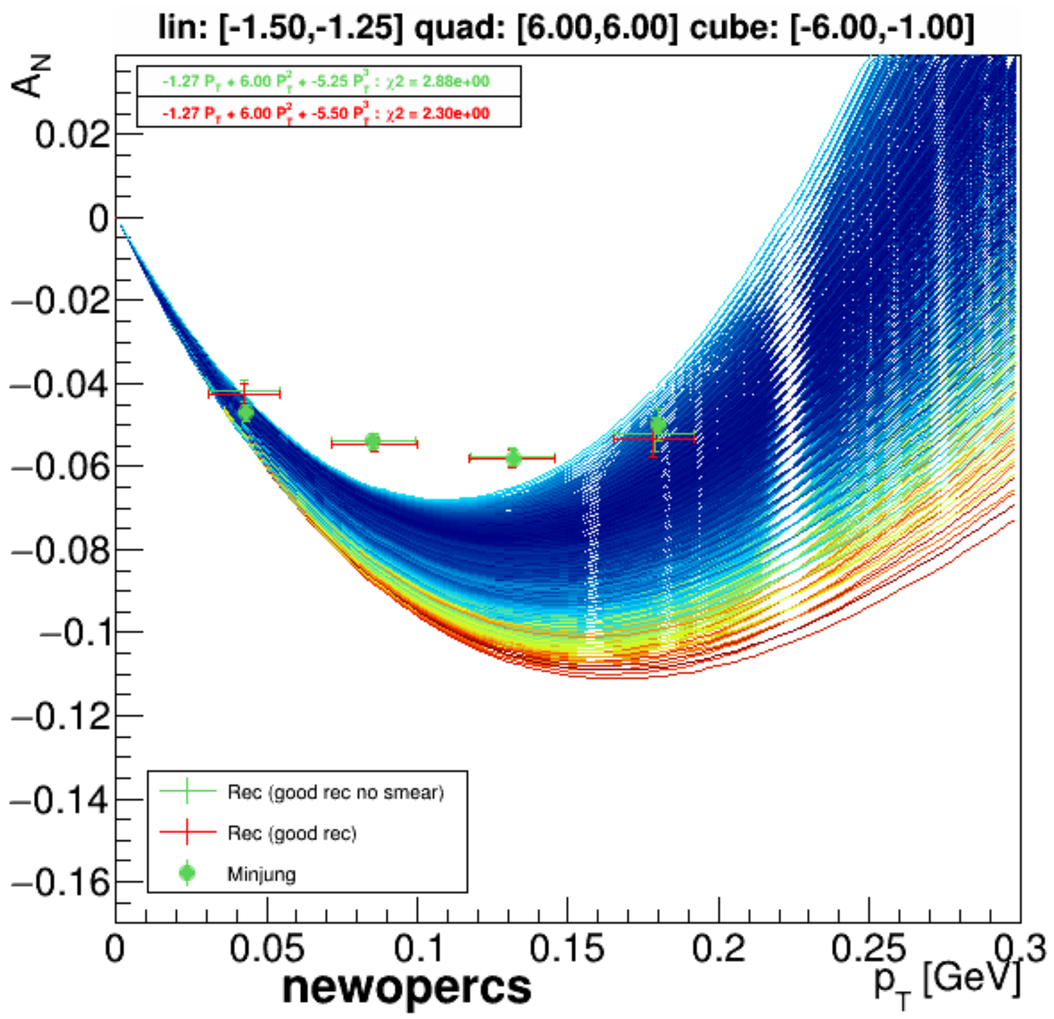
Best chosen parameters:

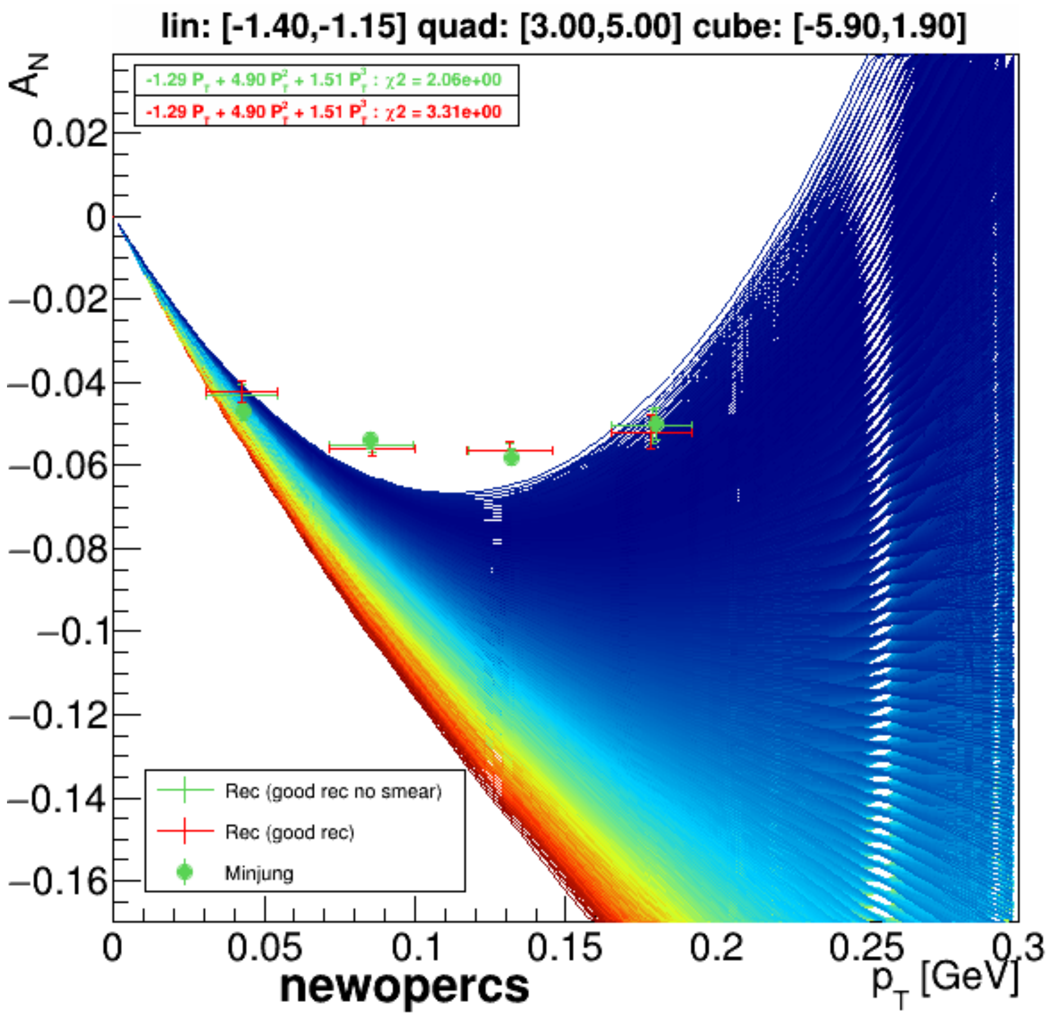
- Linear = -0.067
- Quad + Cube = 98.95
- Chi-Square = 0.77

Uncertainties:

- Shown for Dpmjet, Pythia6, Pythia8, Ope & Upc.

Scan of Linear, Quadratic and Cubic Parameter Values





Need to zoomin by scanning a finer range of parameters for linear, quadratic and cubic and unfold

Algorithm

1. Create two spin states using TRandom Number Generator:
   Spin up (0)
   Spin down (1)

2. Create spin depended weight according to Taylor series of a polynomial in the form:

$$w = 1 + \left(a + b * P_{T,T} + c * P_{T,T}^{2} + d * P_{T,T}^{3}\right) cos(\varphi_T + spin * \pi)$$

the parameters are:
a = constant
b = linear
c = quadratic
d = cube
spin * pi = phase shift
spin = 0 (up)
1 (down)

Note: Other functional forms can also be scanned and tried to describe data asymmetries.

Algorithm…

3. Scan parameters for different functional forms over a wide range using chi-square based on the reconstructed asymmetries from pp collision monte carlo samples and run 15 pp asymmetry results (Minjung's result) to find the best parameter, i.e. parameter with lowest,

$$\chi^2 = \sum_i \frac{\left(A_{N,i}^{Minjung} - A_{N,i}^{w,reco}\right)^2}{\left(\Delta A_{N,i}^{2,Minjung} + \Delta A_{N,i}^{2,w,reco}\right)}$$

4. Extract the asymmetry using the best Chi-squared parameters,

$$A_N = \frac{N_{\Phi\uparrow} - N_{\Phi\downarrow}}{N_{\Phi\uparrow} + N_{\Phi\downarrow}}$$