

3次元パートン分布(TMD,GPD) とQCDスピン物理

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BNL & 理研BNL

Future DIS experiments worldwide

Planned DIS Colliders around the world

1812.08110

Facility	Years	E_{cm} (GeV)	Luminosity ($10^{33} cm^{-2} s^{-1}$)	Ions	Polarization
EIC in US	> 2028	20 - 100 → 140	2 - 30	p → U	e, p, d, 3He , Li
EIC in China	> 2028	16 - 34	1 → 100	p → Pb	e, p, light nuclei
LHeC (HE-LHeC)	> 2030	200 - 1300 (1800)	10	depends on LHC	e possible
PEPIC	> 2025	530 → 1400	$< 10^{-3}$	depends on LHC	e possible
VHEeP	> 2030	1000 - 9000	$10^{-5} - 10^{-4}$	depends on LHC	e possible
FCC-eh	> 2044	3500	15	depends on FCC-hh	e possible

EPPSU DIS Input

United States

LHeC

FCC-eh

EIC

China

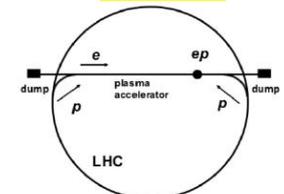
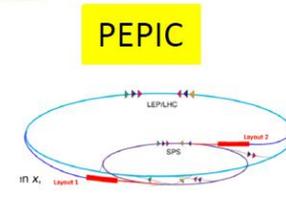
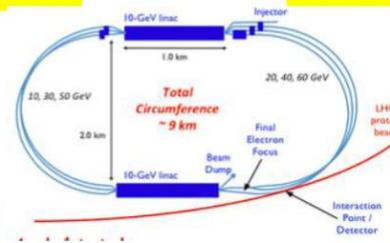
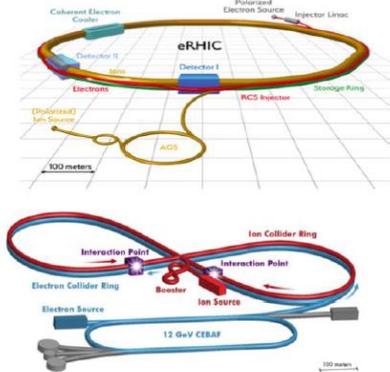
Europe (CERN)

EicC

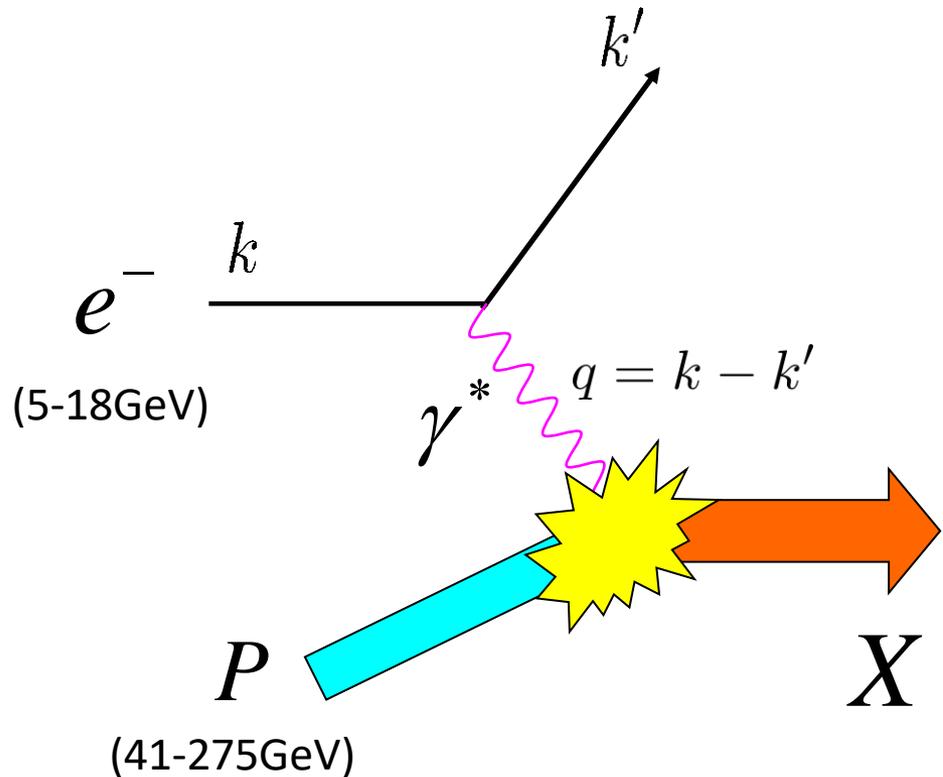
VHEeP

VLEeP

PEPIC



Experiment at EIC: Deep Inelastic Scattering (DIS)



Two most important kinematic variables

$$Q^2 = -q^2 \quad \text{photon virtuality (resolution)}$$

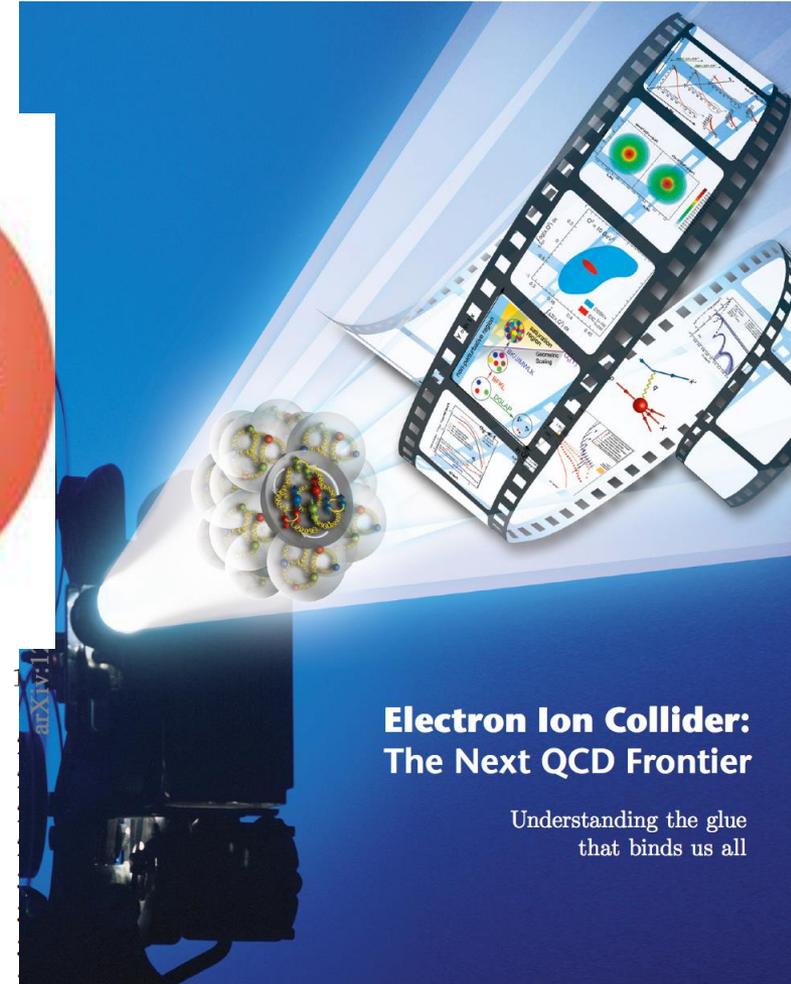
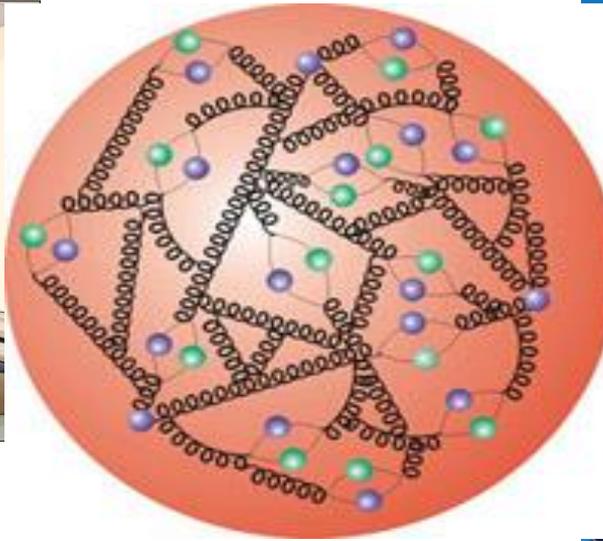
$$x = \frac{Q^2}{2P \cdot q} \quad \text{Bjorken variable (inverse energy)}$$

$$\approx \frac{E_{parton}}{E_{proton}}$$

Proton, deuteron, helium, gold...any nucleus of your choice!

Electron, proton and light nuclei can be polarized.

Nucleon Tomography



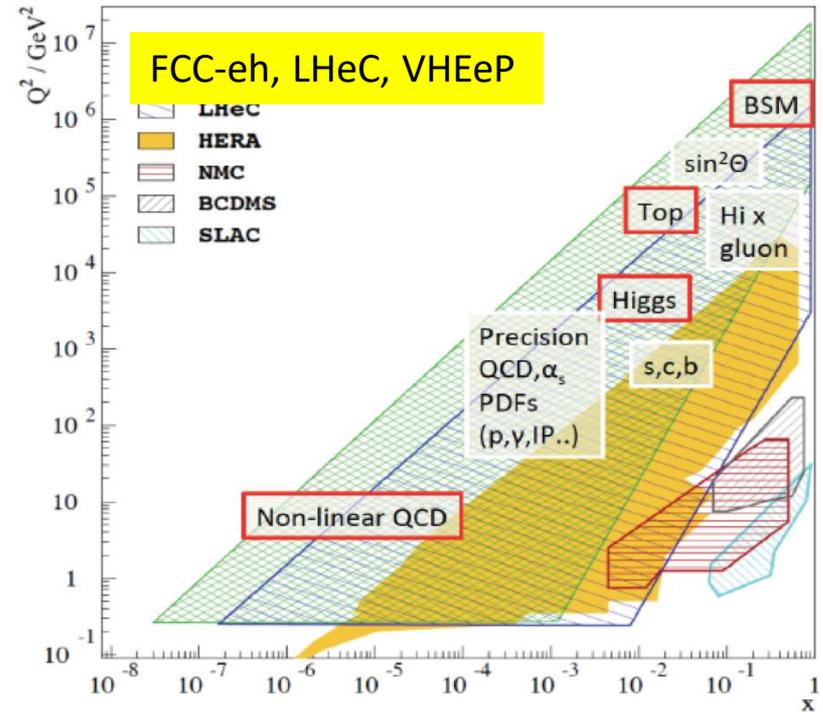
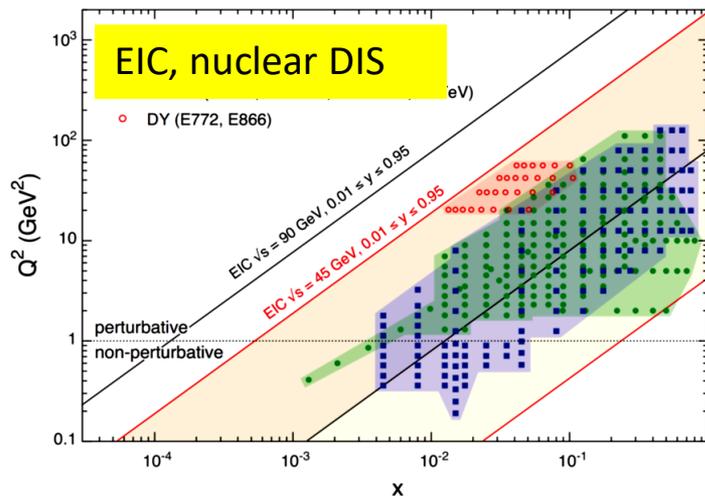
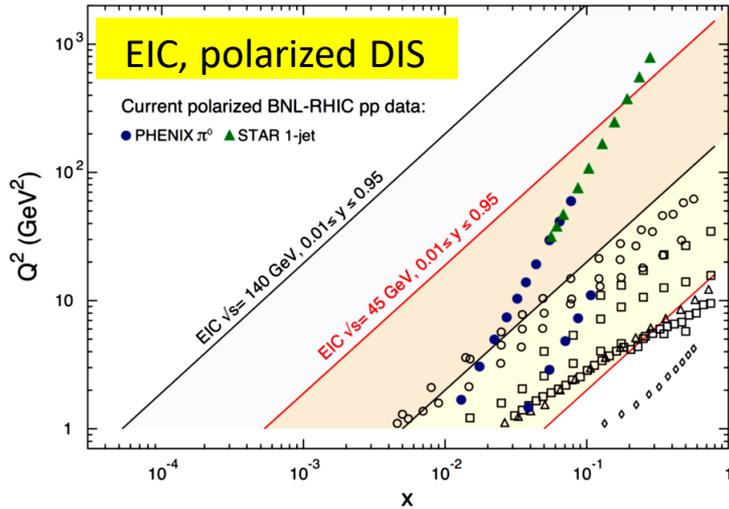
Electron Ion Collider: The Next QCD Frontier

Understanding the glue
that binds us all

2 Spin and Three-Dimensional Structure of the Nucleon

2.1	Introduction	38
2.2	The Longitudinal Spin of the Nucleon	39
2.2.1	Introduction	39
2.2.2	Status and Near Term Prospects	40
2.2.3	Open Questions and the Role of an EIC	41
2.3	Confined Motion of Partons in Nucleons: TMDs	42
2.3.1	Introduction	42
2.3.2	Opportunities for Measurements of TMDs at the EIC Semi-inclusive Deep Inelastic Scattering Access to the Gluon TMDs	43
2.3.3	Summary	44
2.4	Spatial Imaging of Quarks and Gluons	45
2.4.1	Physics Motivations and Measurement Principle	45
2.4.2	Processes and Observables	46
2.4.3	Parton Imaging Now and in the Next Decade	48
2.4.4	Accelerator and Detector Requirements	50
2.4.5	Parton Imaging with the EIC	51
2.4.6	Opportunities with Nuclei	55

Exploring *terra incognita*



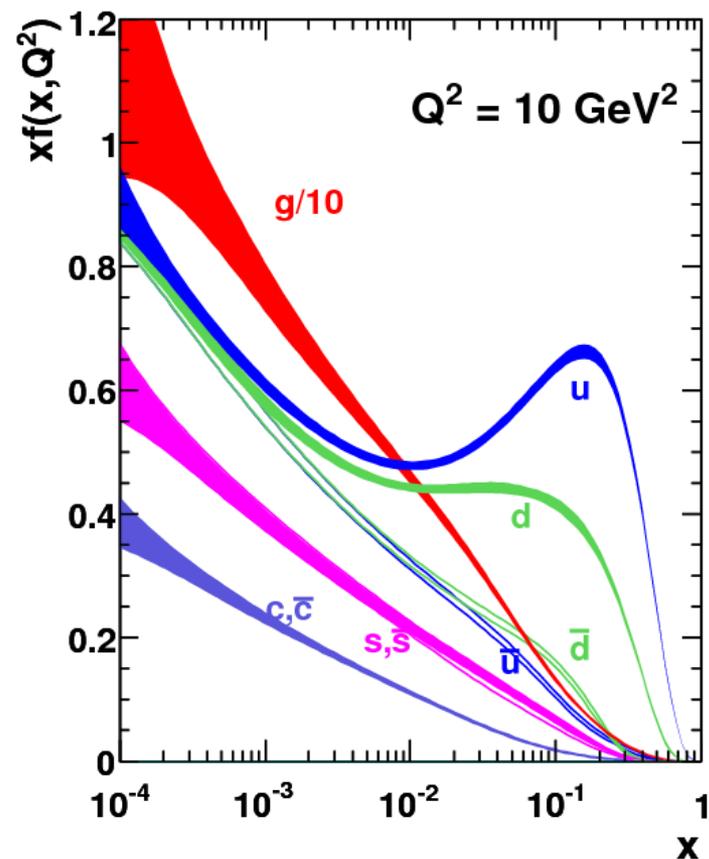
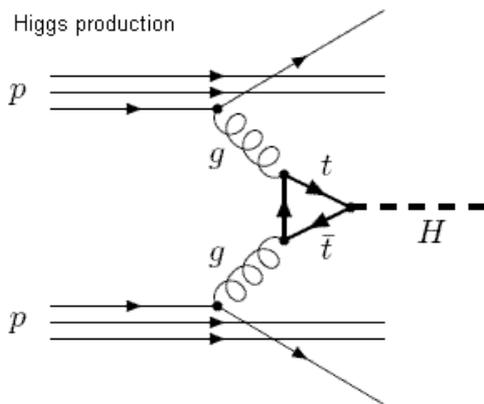
Unprecedented coverage in kinematics.
 Tremendous physics opportunities.

Parton distribution function

$$u(x) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle P | \bar{u}(-z^-/2) \gamma^+ u(z^-/2) | P \rangle$$

Number distribution of up quarks
with momentum fraction x inside the proton

QCD factorization $\sigma = \sigma_0 \otimes g(x_1) \otimes g(x_2)$



Universality of PDF—the **same** function can be used for different processes.
Fundamental to the predictive power of pQCD

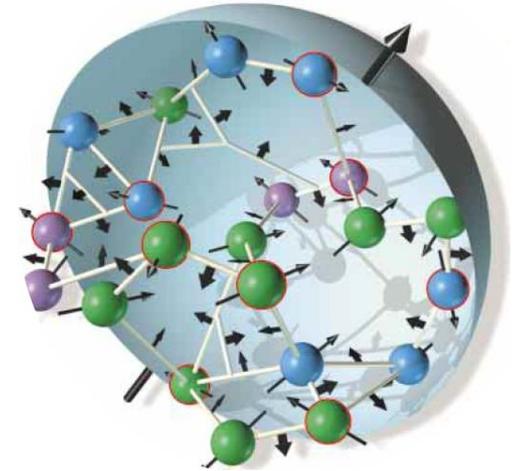
Polarized parton distributions

Nucleon spin sum rule

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L^q + L^g$$

↑
↑
↑
↑

Quarks' helicity Gluons' helicity Orbital angular Momentum (OAM)



$$\Delta\Sigma = \sum_q \int_0^1 dx (\Delta q(x) + \Delta \bar{q}(x))$$

Relatively well-constrained

$$\Delta G = \int_0^1 dx \Delta G(x)$$

Huge uncertainty at small-x

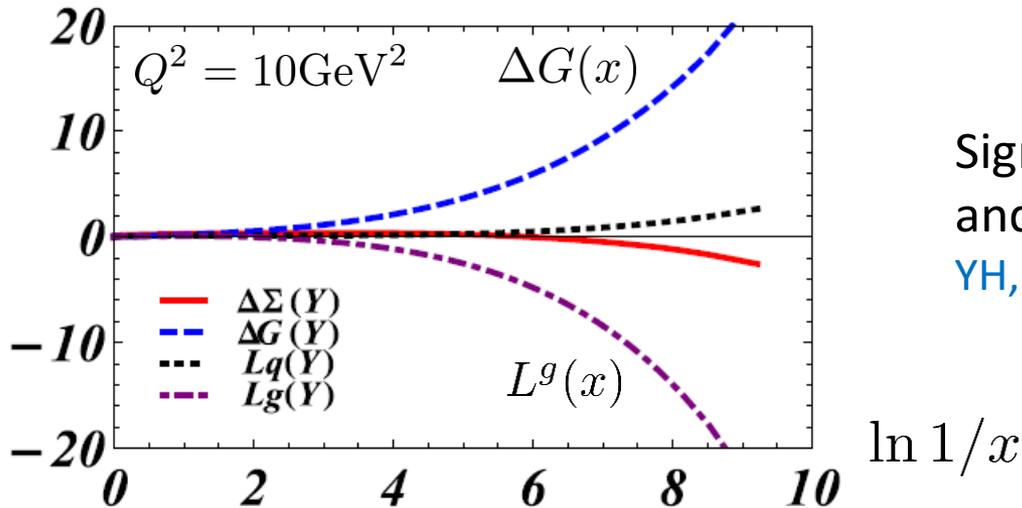
$$L^{q,g} = \int_0^1 dx L^{q,g}(x)$$

twist-3 PDF
YH, Yoshida (2013)

WW part related to $\Delta q(x), \Delta G(x)$

Helicity and OAM at small-x

Huge uncertainties in ΔG at small-x \rightarrow spin at small-x will be an important topic at EIC



Significant cancellation between helicity and OAM at small-x from 1-loop DGLAP
 YH, Yang (2018)

All-order result [Boussarie, YH, Yuan \(2019\)](#)

$$\text{If } \Delta G(x) \sim \frac{1}{x^\alpha}, \text{ then } L_g(x) \approx -\frac{2}{1+\alpha} \Delta G(x)$$

Nucleon EDM and spin physics

CP-violating Weinberg operator →
important source of nucleon electric dipole moment (EDM)

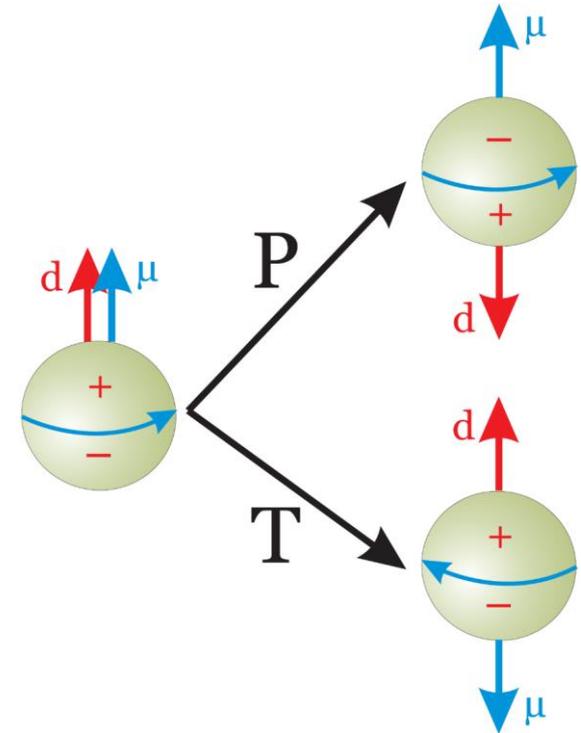
$$\mathcal{O}_W = g f_{abc} \tilde{F}_{\mu\nu}^a F_b^{\mu\alpha} F_{c\alpha}^\nu.$$

Matrix element related to part of twist-4 corrections
in polarized DIS [YH \(2020\)](#)

$$\int_0^1 g_1^{p,n}(x, Q^2) dx$$

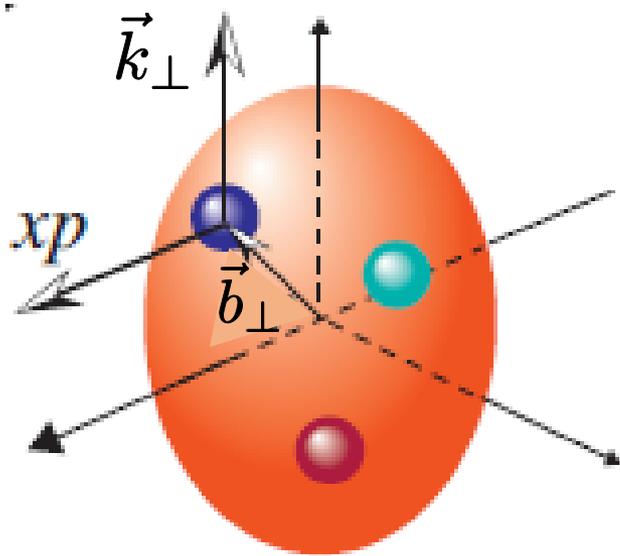
$$= \left(\pm \frac{1}{12} g_A + \frac{1}{36} a_8 \right) \left(1 - \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right) + \frac{1}{9} \Delta\Sigma \left(1 - \frac{33 - 8N_f}{33 - 2N_f} \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right)$$

$$- \frac{8}{9Q^2} \left[\left\{ \pm \frac{1}{12} f_3 + \frac{1}{36} f_8 \right\} \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{-\frac{\gamma_{NS}^0}{2\beta_0}} + \frac{1}{9} f_0 \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{-\frac{1}{2\beta_0} (\gamma_{NS}^0 + \frac{4}{3} N_f)} \right],$$

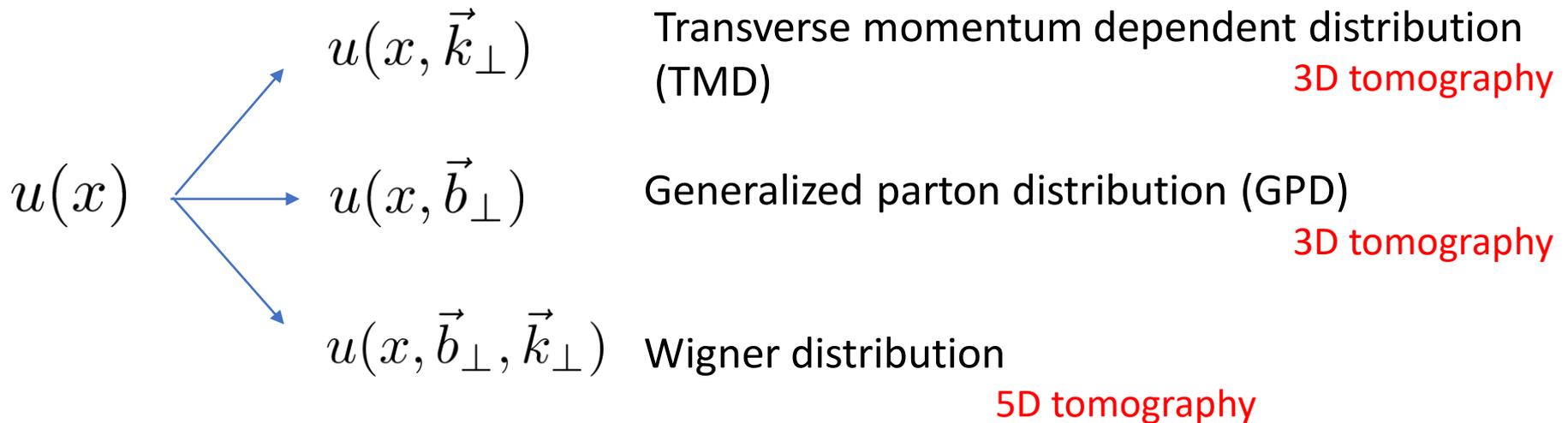


New connection between QCD spin and BSM physics

Multi-dimensional tomography



The nucleon is much more complicated!
Partons also have transverse momentum \vec{k}_\perp
and are spread in impact parameter space \vec{b}_\perp

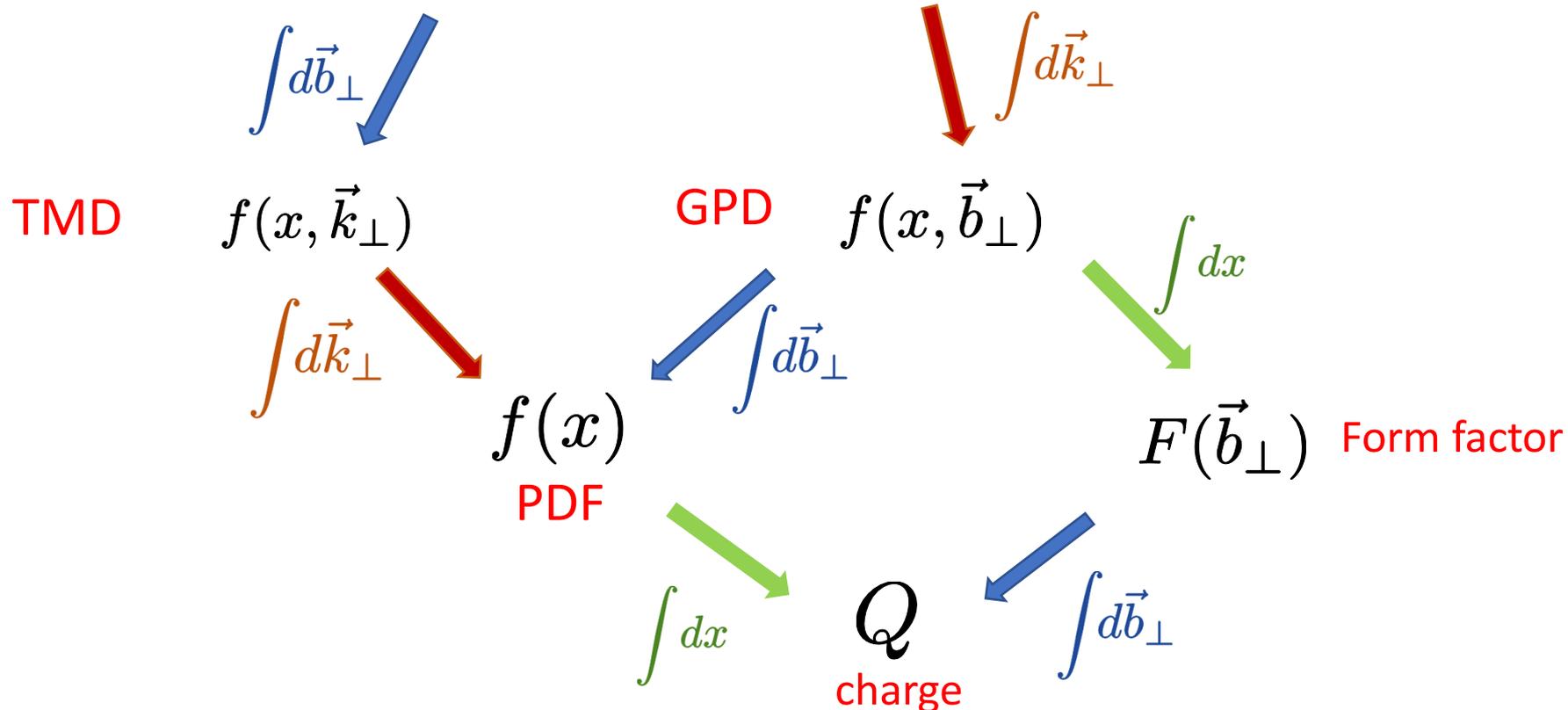


PDF family tree

Wigner distribution—the ‘mother’ distribution

Belitsky, Ji, Yuan (2003)

$$W(x, \vec{k}_\perp, \vec{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+ z^- - i\vec{k}_\perp \cdot \vec{z}_\perp} \langle P - \frac{\Delta}{2} | \bar{q}(-z/2) \gamma^+ q(z/2) | P + \frac{\Delta}{2} \rangle$$

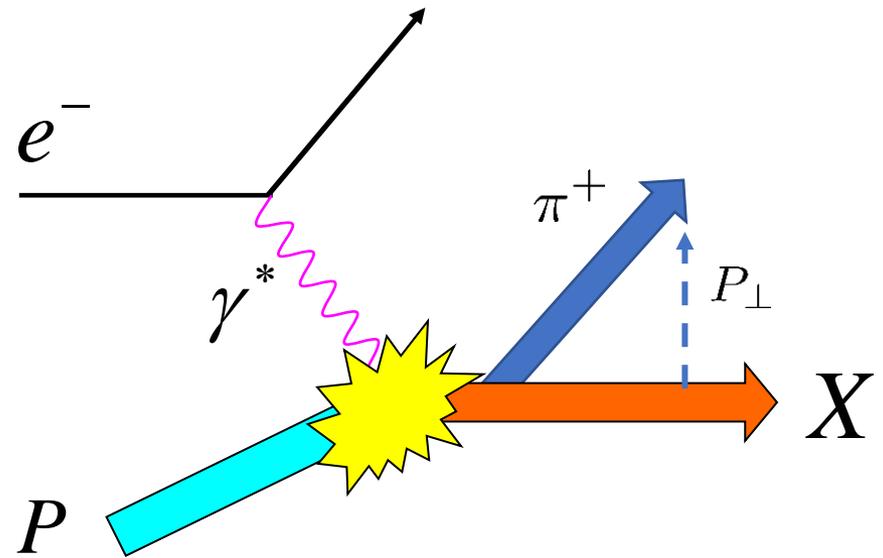


Semi-inclusive DIS

Tag one hadron species
with fixed transverse momentum P_{\perp}

When P_{\perp} is small, **TMD factorization**

Collins, Soper, Sterman; Ji, Ma, Yuan,...



$$\frac{d\sigma}{dP_{\perp}} = H(\mu) \int d^2q_{\perp} d^2k_{\perp} \underbrace{f(x, k_{\perp}, \mu, \zeta)}_{\text{TMD PDF}} \underbrace{D(z, q_{\perp}, \mu, Q^2/\zeta)}_{\text{TMD FF}} \delta^{(2)}(zk_{\perp} + q_{\perp} - P_{\perp}) + \dots$$

$$f(x, \vec{k}_{\perp}) = \int \frac{dz^- d^2z_{\perp}}{16\pi^3} e^{ixP^+ z^- - i\vec{k}_{\perp} \cdot \vec{z}_{\perp}} \langle P | \bar{q}(-z/2) \gamma^+ W q(z/2) | P \rangle$$

Open up a new class of observables where perturbative QCD is applicable!

TMD is becoming precision physics

Define Fourier transform $\int d^2k_\perp e^{ik_\perp r_\perp} f(k_\perp \dots) = f(r_\perp \dots)$

RG equation $\frac{\partial}{\partial \ln \mu} f(x, r_\perp, \mu, \zeta) = \gamma_F(\alpha_s(\mu), \zeta/\mu^2) f(x, r_\perp, \mu, \zeta)$

$$\gamma_F = -\frac{\gamma_K}{2} \ln \frac{\zeta}{\mu^2} + \gamma_F^0$$

Related to DGLAP splitting function

$$P(z) = \frac{\gamma_K}{2} \frac{z}{(1-z)_+} + \frac{\gamma_F^0}{2} \delta(1-z) + \mathcal{O}(1-z) \quad \text{Known to three loops}$$

Collins-Soper equation

$$\frac{\partial}{\partial \ln \zeta} f(x, r_\perp, \mu, \zeta) = -\mathcal{D}(r_\perp, \mu) f(x, r_\perp, \mu, \zeta)$$

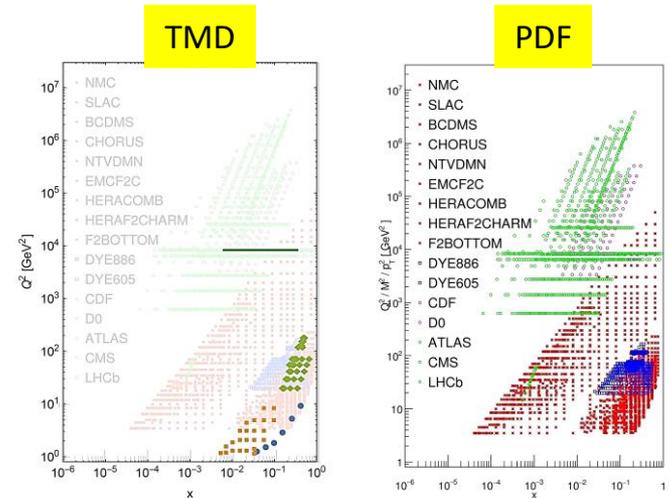
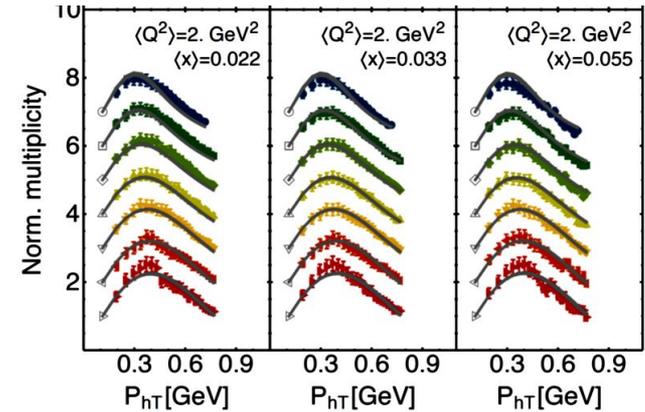
Recently computed to three loops [Li, Zhu \(2017\); Vladimirov \(2017\)](#)

Computable from lattice QCD at large r_\perp [Ebert, Stewart, Zhao \(2018\)](#)

TMD global analysis

	Framework	W+Y	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	W	✗	✗	✓	✓	98
QZ 2001 hep-ph/0506225	NLO-NLL	W+Y	✗	✗	✓	✓	28 (?)
RESBOS resbos@msu	NLO-NNLL	W+Y	✗	✗	✓	✓	>100 (?)
Pavia 2013 arXiv:1309.3507	LO	W	✓	✗	✗	✗	1538
Torino 2014 arXiv:1312.6261	LO	W	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	W	✗	✗	✓	✓	223
EIKV 2014 arXiv:1401.5078	LO-NLL	W	1 (x, Q ²) bin	1 (x, Q ²) bin	✓	✓	500 (?)
SIYY 2014 arXiv:1406.3073	NLO-NLL	W+Y	✗	✓	✓	✓	200 (?)
Pavia 2017 arXiv:1703.10157	LO-NLL	W	✓	✓	✓	✓	8059
SV 2017 arXiv:1706.01473	NNLO-NNLL	W	✗	✗	✓	✓	309
BSV 2019 arXiv:1902.08474	NNLO-NNLL	W	✗	✗	✓	✓	457

Still in its infancy. Fully blossoms in the EIC era!



Uncertainties in W -boson mass partly coming from TMD

$$\begin{aligned}
 m_W &= 80370 \pm 7 \text{ (stat.)} \pm 11 \text{ (exp. syst.)} \pm \underline{14 \text{ (mod. syst.)}} \text{ MeV} \\
 &= 80370 \pm 19 \text{ MeV,}
 \end{aligned}$$

$$-6 \leq M_{W^+} \leq 9 \text{ MeV}$$

$$-4 \leq M_{W^-} \leq 7 \text{ MeV}$$

Bacchetta, et al. (2018)

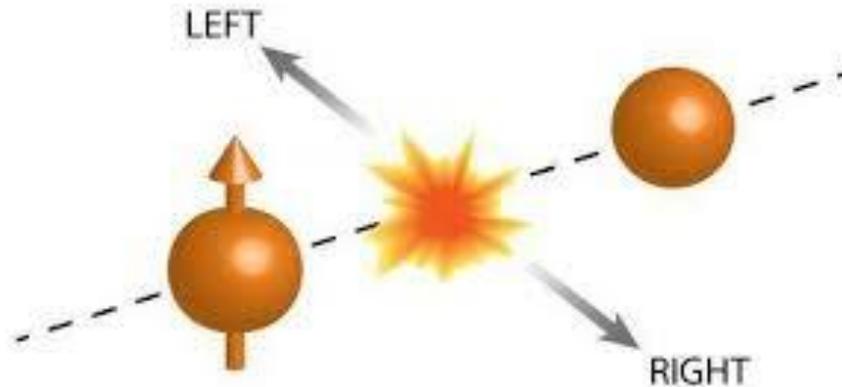
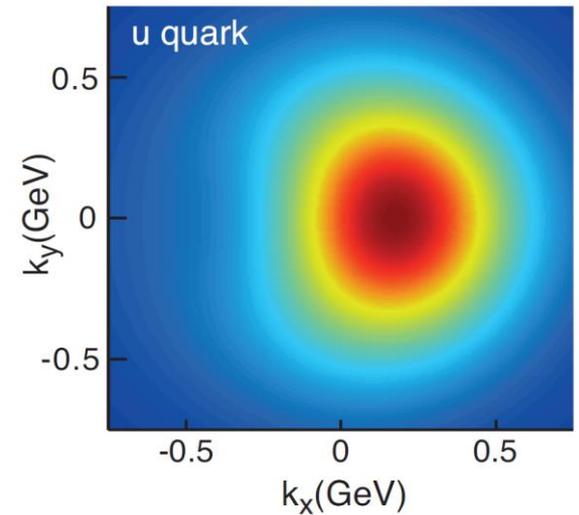
Sivers function

Spin-momentum correlation in a **transversely** polarized proton

$$f(x, k_{\perp}) = f_0(x, k_{\perp}) + (\vec{S}_{\perp} \times \vec{k}_{\perp}) \cdot \hat{p} f_{1T}^{\perp}(x, k_{\perp})$$

Classic example of TMD.

One of the origins of observed large **single spin asymmetry (SSA)**



EIC will pin down the origin(s) of SSA.

Gluon Sivers can also be studied at EIC → SSA of open charm, J/ψ , dijet,...

Zheng, Aschenauer, Lee, Xiao, Yin (2018)

Sivers function: a different look

$$f(x, k_{\perp}) = f_0(x, k_{\perp}) + \underbrace{(\vec{S}_{\perp} \times \vec{k}_{\perp}) \cdot \hat{p}}_{\text{More properly, use the nucleon spinor}} f_{1T}^{\perp}(x, k_{\perp})$$

More properly,
use the nucleon spinor

$$k_{\perp}^i \bar{u}(PS_{\perp}) \sigma^{+i} u(PS_{\perp}) f_{1T}^{\perp}(x, k_{\perp})$$

non-forward generalization (GTMD)



$$k_{\perp}^i \bar{u}(P'S') \sigma^{+i} u(PS) F_{12}(x, k_{\perp}, \Delta_{\perp})$$

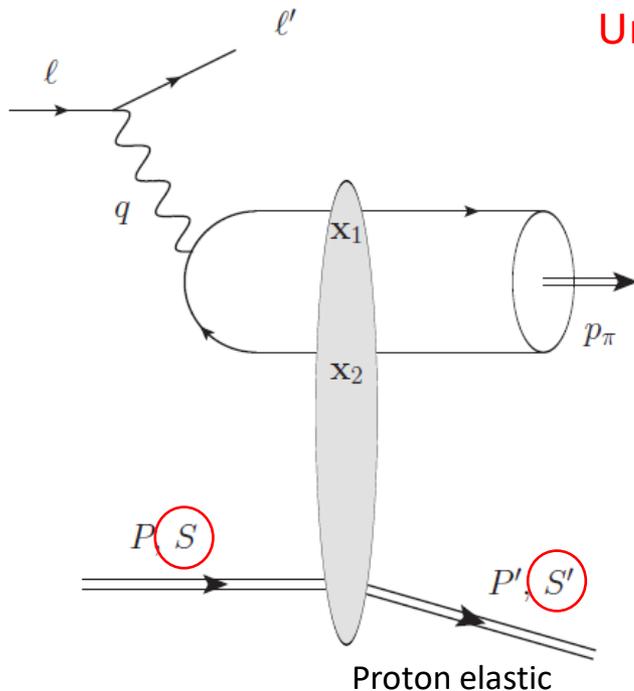
Reduces to Sivers in the
forward limit

This spinor product is also nonvanishing if longitudinally polarized
and if **helicity flips** $S^{\mu} = -S'^{\mu}$

$$\bar{u}(P, -S_L) \sigma^{+i} u(PS_L) = (\pm i, -1)$$

Exclusive π^0 production at EIC

Boussarie, YH, Szymanowski, Wallon (2020)



Unpolarized cross section at small-x, $t \approx 0$

$$\frac{d\sigma}{dt} = \sum_{SS'=\pm} \frac{d\sigma_{SS'}}{dt} \approx \frac{d\sigma_{+-}}{dt} + \frac{d\sigma_{-+}}{dt}$$

$$\frac{d\sigma}{dx_B dQ^2 d|t|} = \frac{\pi^5 \alpha_{\text{em}}^2 \alpha_s^2 f_\pi^2}{2^3 x_B N_c^2 M^2 Q^6} \left(1 - y + \frac{y^2}{2}\right) \times \left[\int_0^1 dz \frac{\phi_\pi(z)}{z\bar{z}} \int dk^2 \frac{k^2}{k^2 + z\bar{z}Q^2} x f_{1T}^{\perp g}(x, k^2) \right]^2$$

Cross section in the forward limit is dominated by gluon Sivers at small-x!

Odderon = gluon Sivers at small-x

Zhou 1308.5912

Odderon: Predicted in the 70s as a C-odd counterpart of Pomeron

Experimentally elusive for decades. Finally found at the LHC? (TOTEM collaboration)

POPULAR MECHANICS

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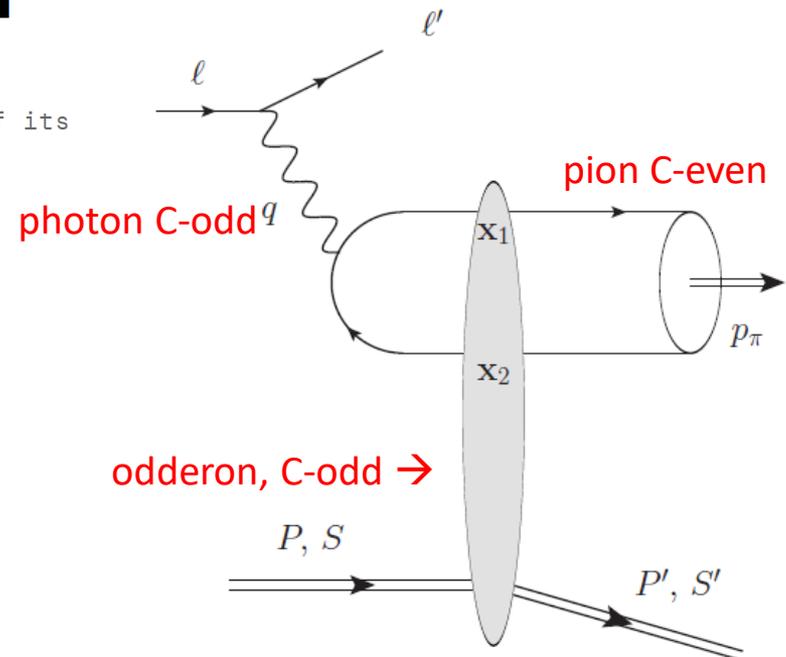
LHC Scientists Discover First Evidence of Particle Proposed Nearly 50 Years Ago

The odderon was first proposed in 1973, but actual evidence of its existence eluded scientists until now.

Exclusive π^0 production...classic discovery channel for Odderon.

Null result at HERA (H1 collaboration), but observable not optimal.

New measurement at EIC armed with improved theory.



Gluon Sivers=Odderon in pp at the LHC

Elastic pp scattering, unpolarized

$$\frac{d\sigma}{dt} = \sum_{S_1 S_2 S_3 S_4 = \pm} \frac{d\sigma_{S_1 S_2 \rightarrow S_3 S_4}}{dt}$$

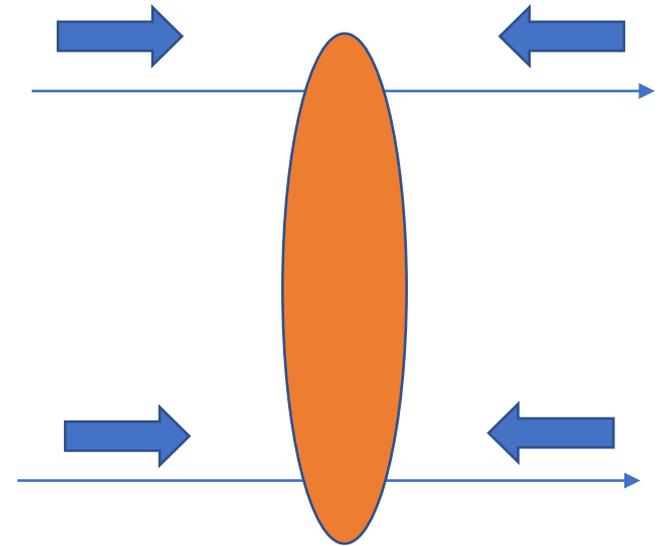
Hagiwara, YH, Pasechnik, Zhou (2020)

Double helicity-flip from
gluon Sivers=Odderon

$$\left. \frac{d\sigma}{dt} \right|_{t=0} = \frac{\sigma_{\text{tot}}^2}{16\pi} (1 + \rho^2 + 2|r_2|^2)$$

rho-parameter (spin non-flip)

$$\rho(s, t) = \frac{\text{Re}T(s, t)}{\text{Im}T(s, t)}$$



Generalized parton distributions (GPD)

$$P^+ \int \frac{dy^-}{2\pi} e^{ixP^+y^-} \langle P' S' | \bar{\psi}(0) \gamma^\mu \psi(y^-) | PS \rangle$$
$$= H_q(x, \Delta) \bar{u}(P' S') \gamma^\mu u(PS) + E_q(x, \Delta) \bar{u}(P' S') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2m} u(PS)$$

Non-forward $\Delta = P' - P$ generalization of PDF

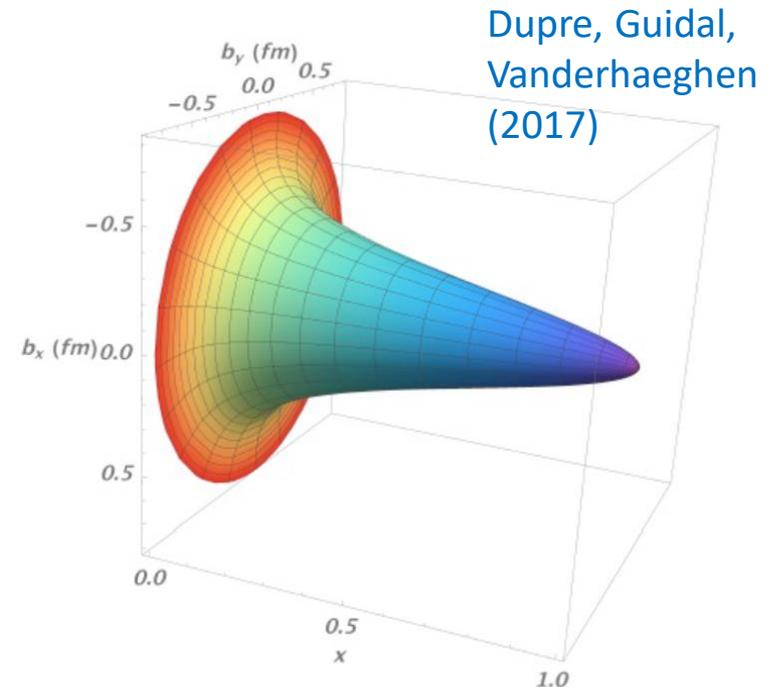
Fourier transform $\Delta_\perp \rightarrow b_\perp$



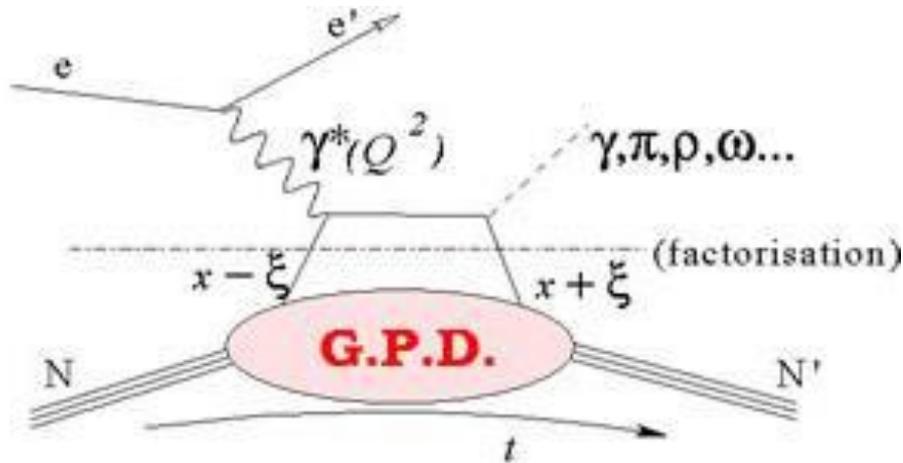
Distribution of partons in **impact parameter** space

First moment \rightarrow elemag form factors

$$\int dx H_q(x, \Delta) = F_1(\Delta^2)$$



Deeply Virtual Compton Scattering (DVCS)



Experimentally probe GPDs.
HERA, JLab, Compass, EIC,...

Extraction challenging due to
multiple variables and complicated
convolution

Compton form factor

$$\begin{aligned}
 & i \int d^4 y e^{iqy} \langle P' | T \{ J^\mu(y) J^\nu(0) \} | P \rangle \\
 &= -(g^{\mu+} g^{\nu-} + g^{\nu+} g^{\mu-} - g^{\mu\nu}) \int \frac{dx}{2} \left(\frac{1}{x + \xi - i\epsilon} + \frac{1}{x - \xi + i\epsilon} \right) H_q(x, \eta, \Delta) \bar{u}(P') \gamma^+ u(P) + \dots
 \end{aligned}$$

$$\xi = \frac{Q^2}{2P \cdot q}$$

3-loop evolution of GPD (non-singlet)
2-loop coefficient functions in DVCS

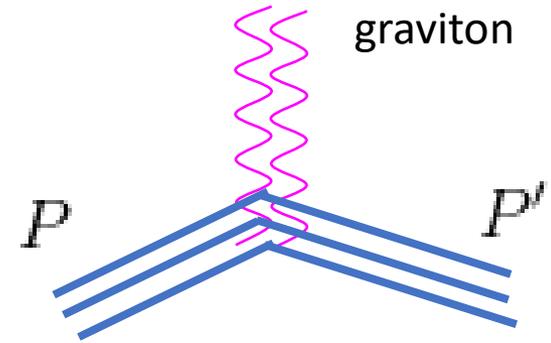
Braun, Manashov, Moch, Strohmaier (2017~)

Global analysis Moutarde, Sznajder, Wagner; Kumericki, ...

Connection to proton spin

Second moment of GPD = **gravitational form factor**

$$\int_{-1}^1 dx x \int \frac{dy^-}{2\pi} e^{ixP^+y^-} \langle \bar{\psi}(0) \gamma^+ \psi(y^-) \rangle = \frac{1}{(P^+)^2} \langle P' | T_q^{++} | P \rangle$$

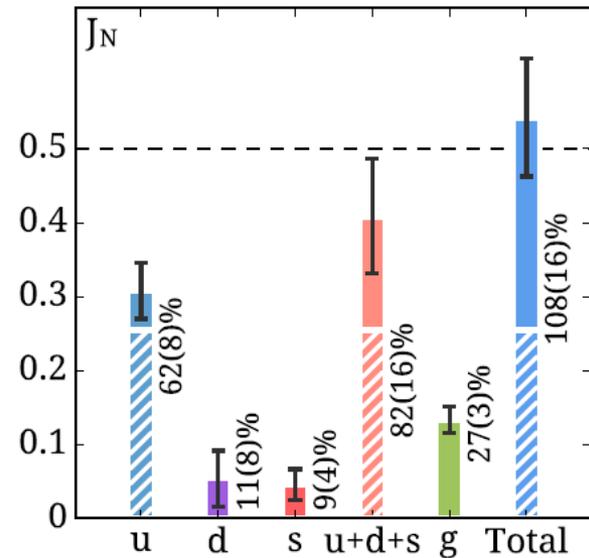


$$\langle P' | T_{q,g}^{\mu\nu} | P \rangle = \bar{u}(P') \left[A_{q,g} \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g} \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M} + D_{q,g} \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} + \bar{C}_{q,g} M g^{\mu\nu} \right] u(P)$$

Ji sum rule
$$\frac{1}{2} = \sum_q J_q + J_g$$

$$J^q = \frac{1}{2} \int dx x (H_q(x) + E_q(x))$$

$$J^g = \frac{1}{4} \int dx (H_g(x) + E_g(x))$$



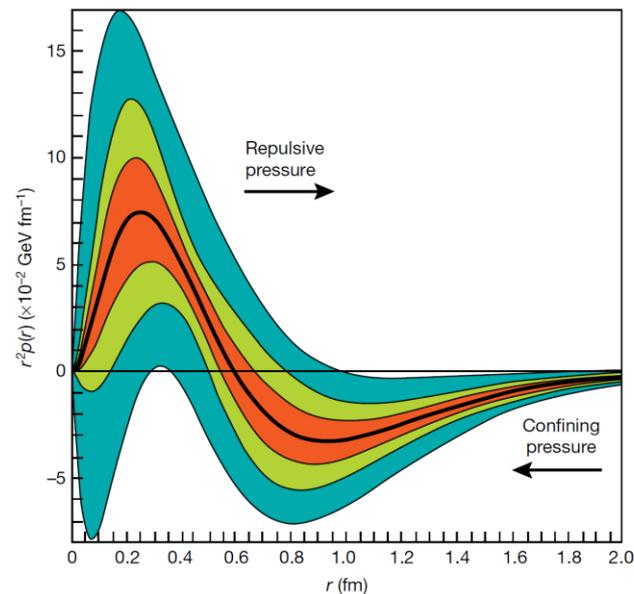
D-term: the last global unknown

$D(t=0)$ is a conserved charge of the nucleon, just like mass and spin!

Related to the radial force ('pressure') distribution inside a nucleon
Polyakov, Schweitzer,...

$$T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) s(r) + \delta^{ij} p(r)$$

Burkert, Elouadrhiri, Girod (2018)



u,d-quark D-term from DVCS, related to the subtraction constant in the dispersion relation for the Compton form factor

Teryaev (2005)

$$\text{Re}\mathcal{H}_q(\xi, t) = \frac{1}{\pi} \int_{-1}^1 dx \text{P} \frac{\text{Im}\mathcal{H}_q(x, t)}{\xi - x} + 2 \int_{-1}^1 dz \frac{D_q(z, t)}{1 - z} \quad \int_{-1}^1 dz z D_q(z, t) = D_q(t)$$

Need a significant lever-arm in Q^2 to disentangle different moments → EIC

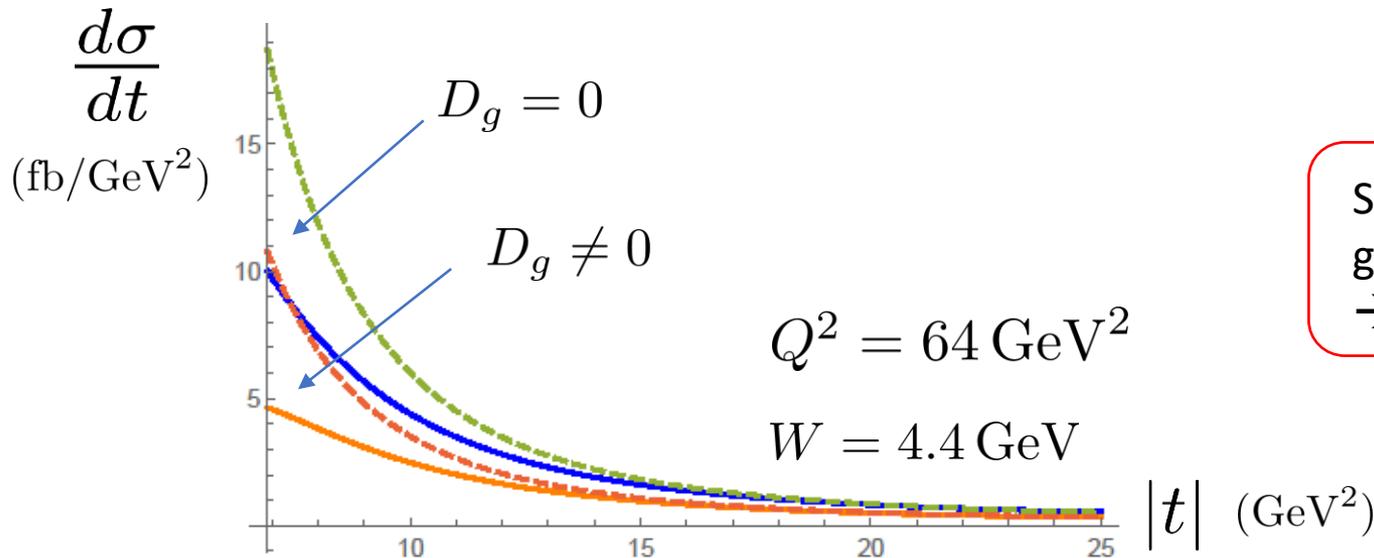
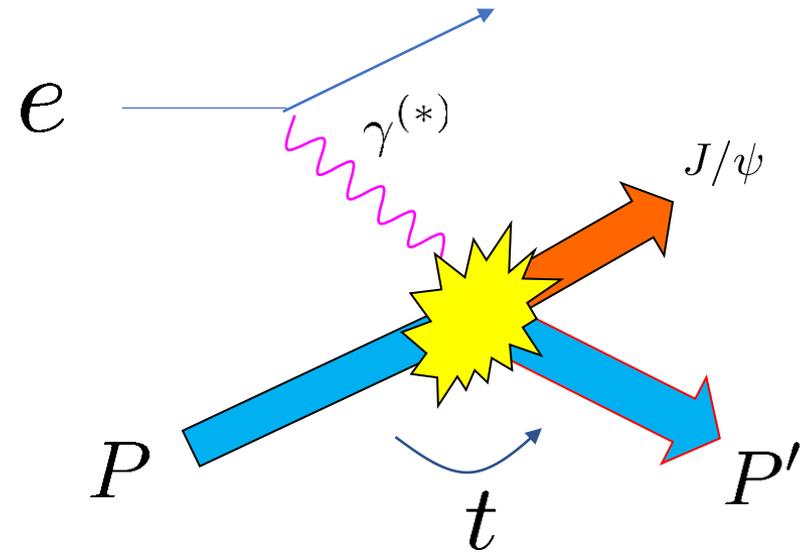
Gluon D-term from near-threshold J/ψ -production

YH, Yang (2018); Boussarie, YH (2020)

Low-energy, near-threshold quarkonium production sensitive to gluon gravitational form factors

$$W_{th} = M_p + M_{J/\psi} \approx 4.04 \text{ GeV}$$

Ongoing experiment at JLab, future measurement at EIC, RHIC



Sensitive also to the gluon condensate
 \rightarrow Proton mass problem

Conclusion

- In 10-15 years from now, DIS experiments will be running in the US, China and maybe also in Europe.
- Spin/TMD/GPD main physics topics. Theory rapidly evolving.