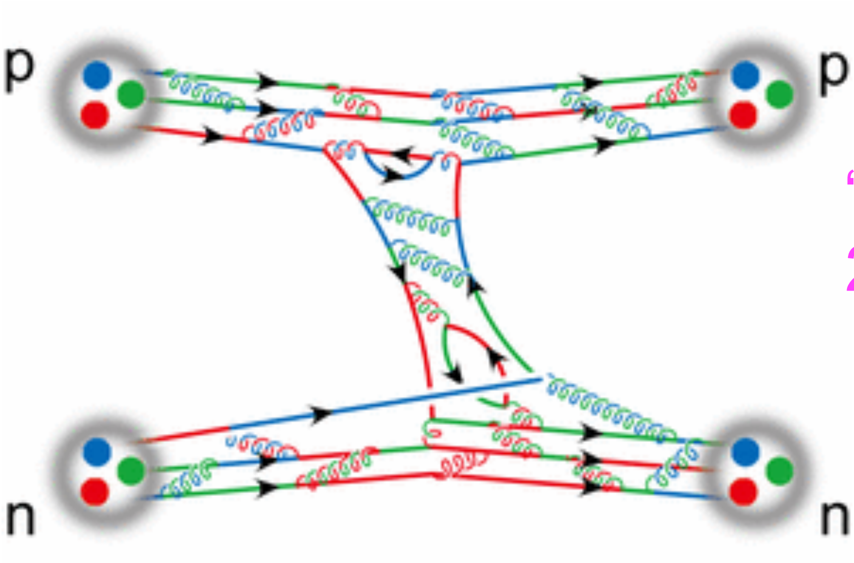
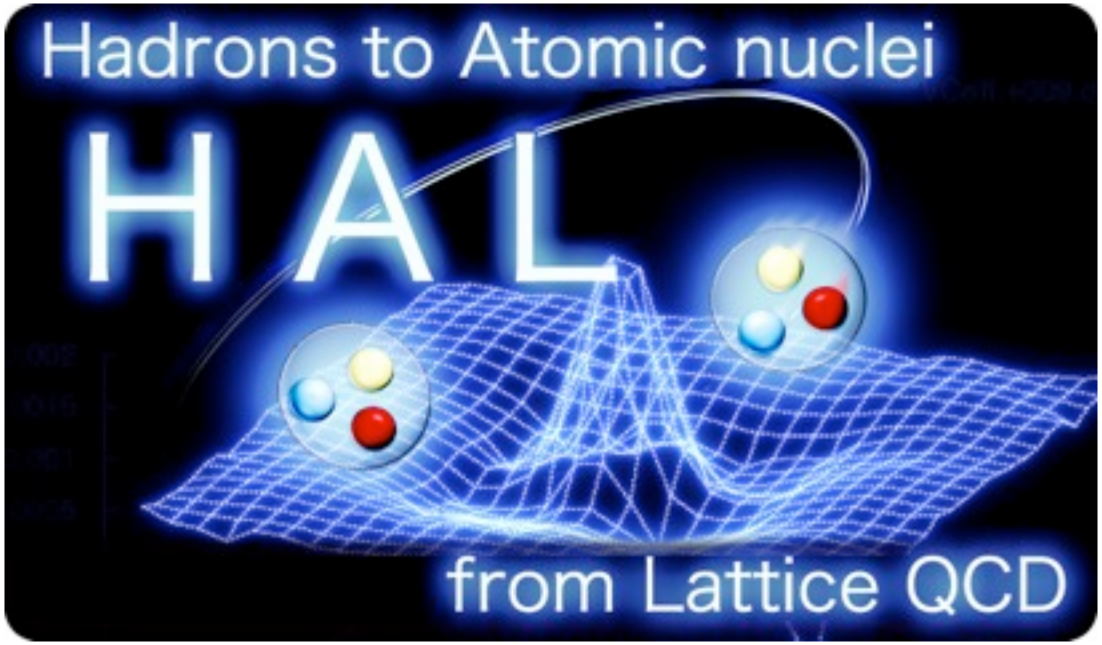
Lattice QCD and Hadron Physics
-Toward Search for New Hadrons-

Sinya AOKI
University of Tsukuba



"New Hadron" Workshop 2010,
2/28-3/1, 2011 @ Nishina Hall, RIKEN

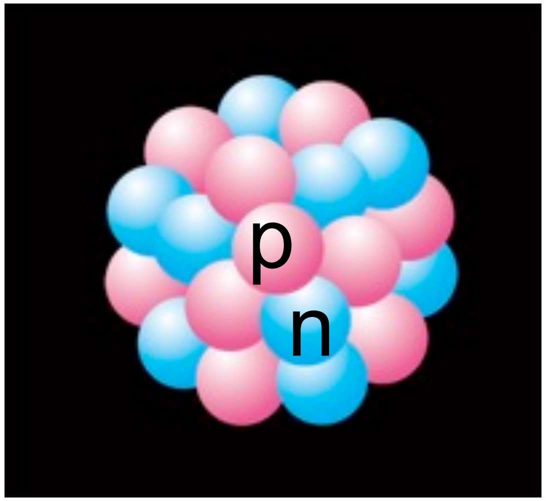
HAL QCD Collaboration



S. Aoki (Tsukuba)
T. Doi (Tsukuba)
T. Hatsuda (Tokyo)
Y. Ikeda (Riken)
T. Inoue (Nihon)
N. Ishii(Tsukuba)
K. Murano (KEK)
H. Nemura (Tohoku)
K. Sasaki (Tsukuba)

1. Introduction

What binds protons and neutrons inside a nuclei ?



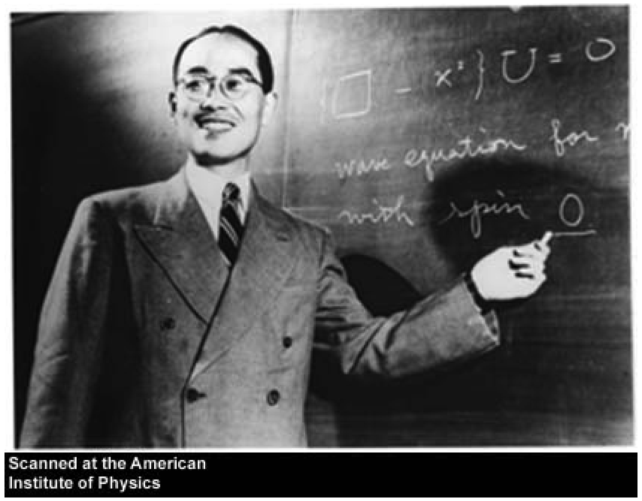
gravity: too weak

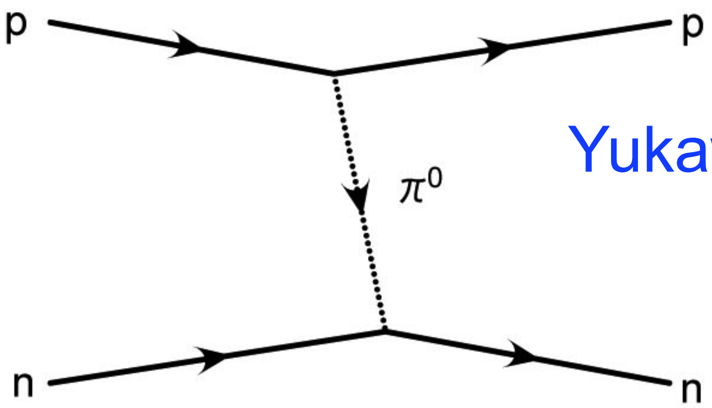
Coulomb: repulsive between pp
no force between nn, np

New force (nuclear force) ?

1935 H. Yukawa

introduced virtual particles (mesons) to explain the nuclear force





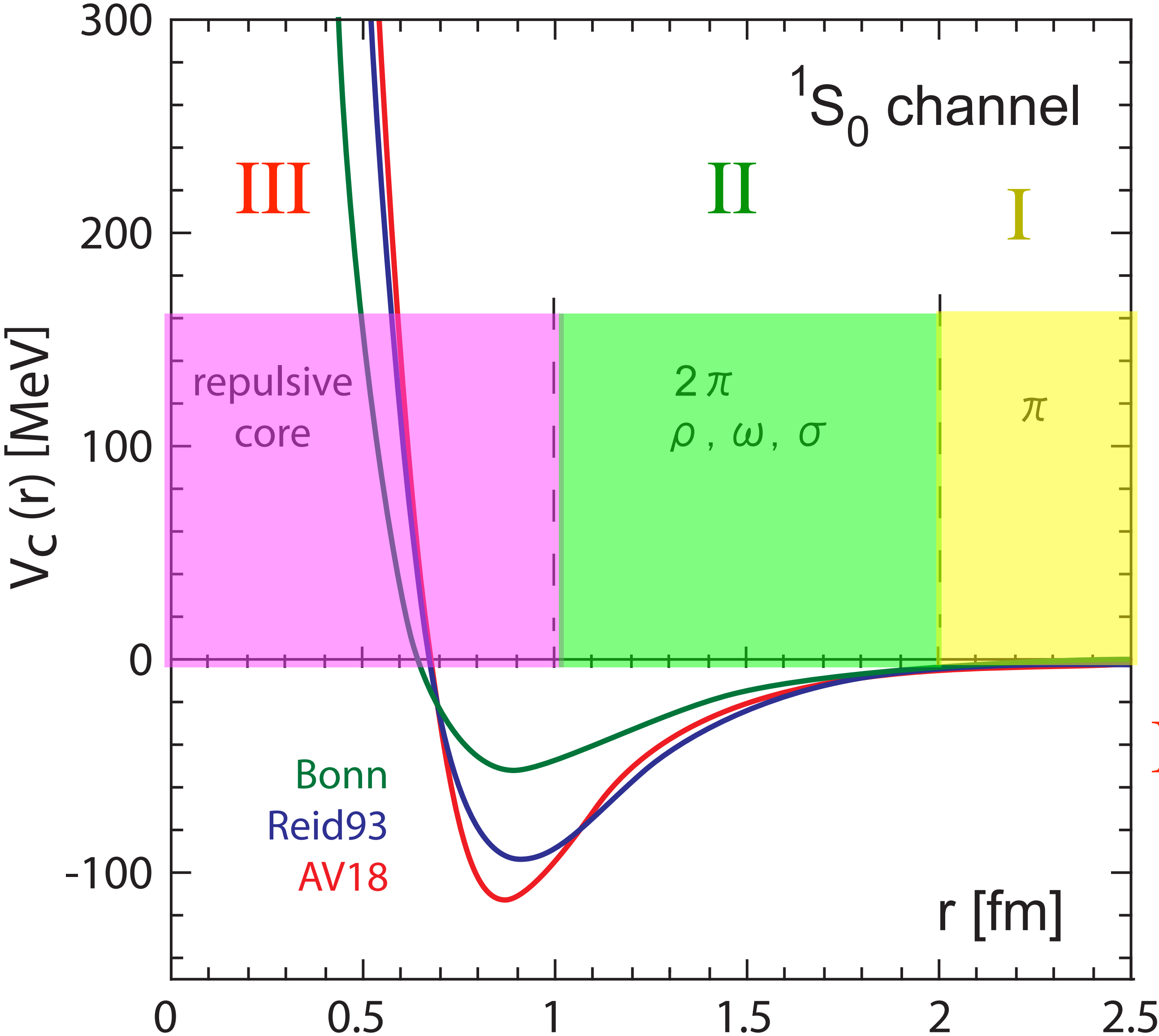
Yukawa potential

$$V(r) = \frac{g^2}{4\pi}\frac{e^{-m_\pi r}}{r}$$

1949 Nobel prize

Phenomenological NN potential

(~40 parameters to fit 5000 phase shift data)



I One-pion exchange

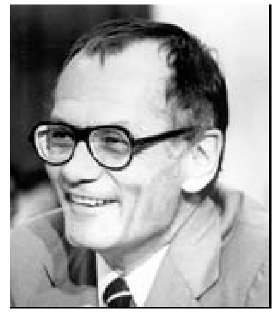
Yiukawa(1935)

II Multi-pions

Taketani et al.(1951)

III Repulsive core

Jastrow(1951)

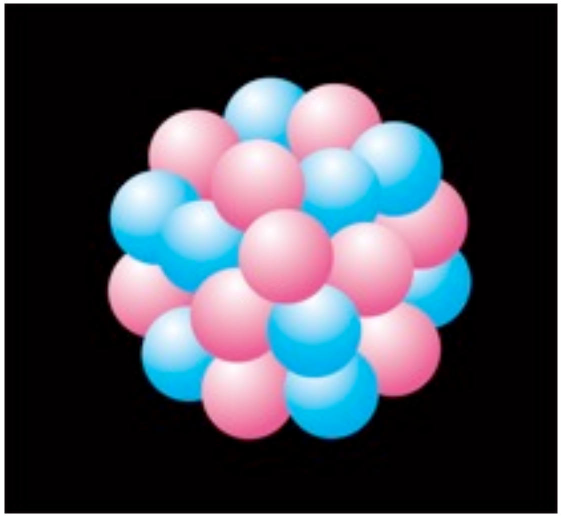


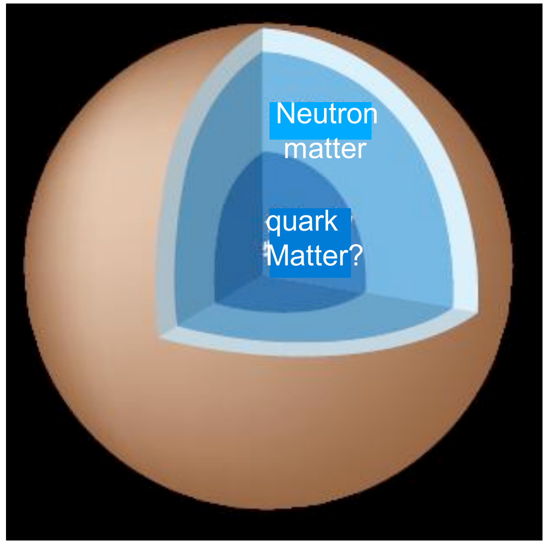
Repulsive core is important

stability of nuclei

maximum mass of neutron star

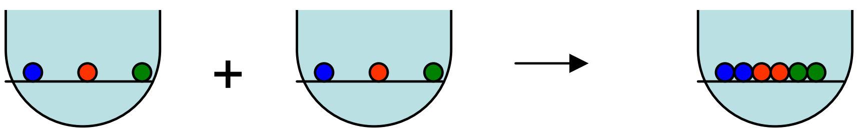
explosion of type II supernova

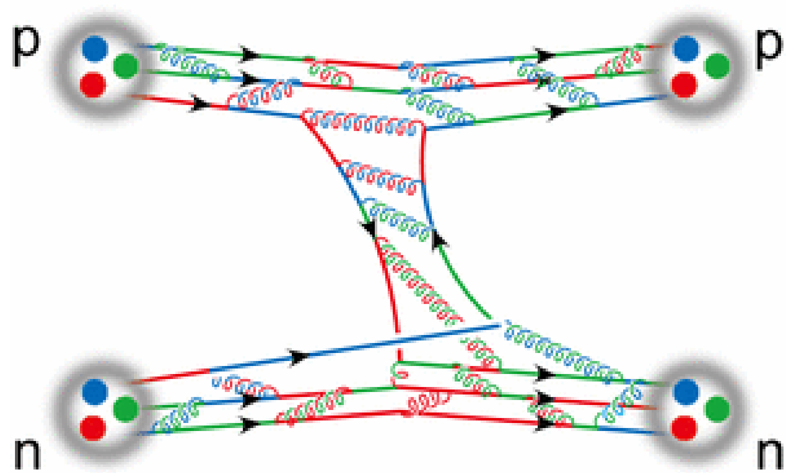




Origin of RC: “The most fundamental problem in Nuclear physics.”

Note: Pauli principle is not essential for the “RC”.





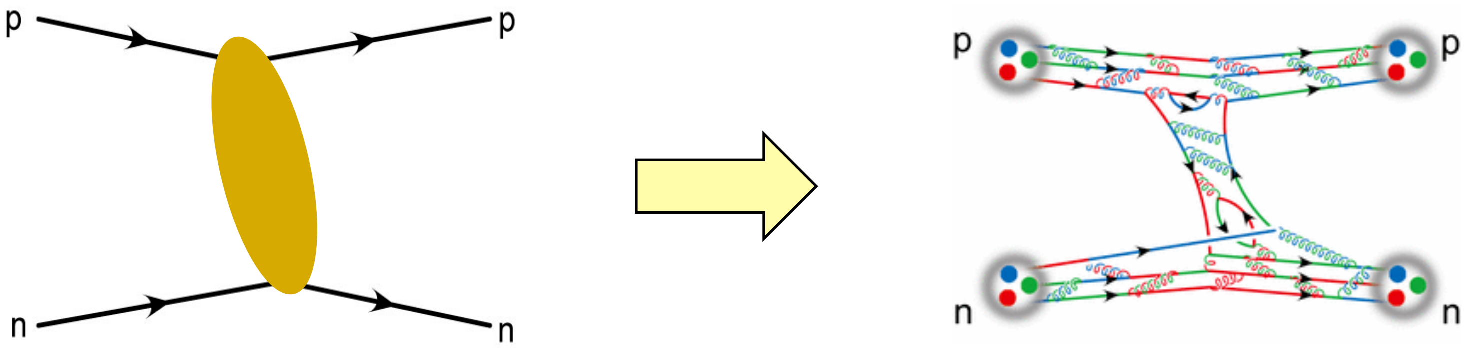
QCD based explanation is needed

Lattice QCD can explain ?

Plan of my talk

1. Introduction
2. Strategy in (lattice) QCD to extract "potential"
3. Octet baryon interactions
   (1) Potentials in the flavor SU(3) symmetric limit
   (2) H-dibaryon in the flavor SU(3) limit
4. Search for new hadron in lattice QCD
   (1) Proposal for S=-2 inelastic scattering
   (2) H-dibaryon in Nature: resonance or bound state ?
5. Summary and Discussion

2. Strategy in (lattice) QCD to extract "potential"



Challenge to Nambu's statement

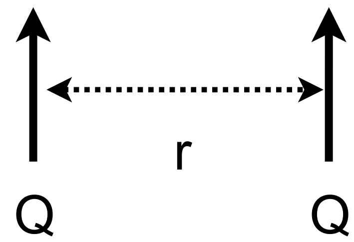
Y. Nambu, "Quarks : Frontiers in Elementary Particle Physics", World Scientific (1985)

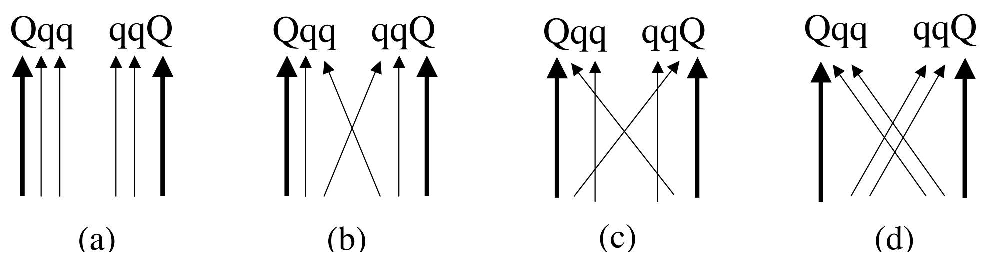
> **"Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task."**

Definition of “Potential” in (lattice) QCD ?

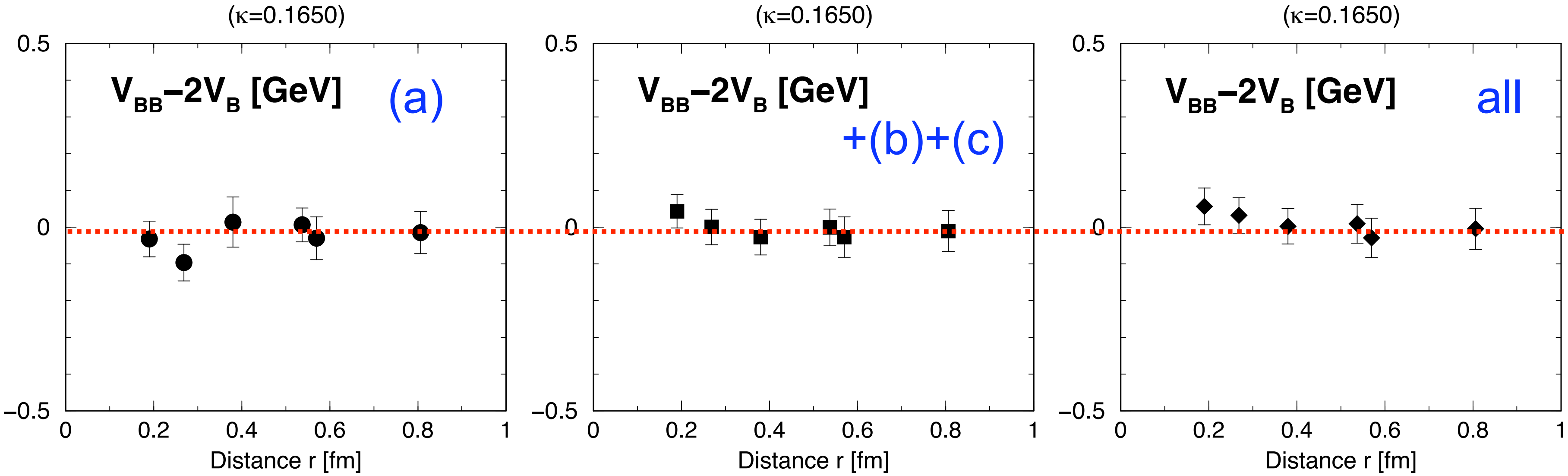
Previous attempt Takahashi-Doi-Suganuma, AIP Conf.Proc. 842,249(2006)

calculate energy of Qqq +Qqq as a function of r between 2Q.
Q:static quark, q: light quark





Quenched result



Almost no dependence on r !

cf. Recent successful result in the strong coupling limit
(deForcrand-Fromm, PRL104(2010)112005)

Quantum Field Theoretical consideration

S.Aoki, LesHouches 2009

- S-matrix below inelastic threshold. Unitarity gives

$$E < E_{th}$$

$$S = e^{2i\delta}$$

- Nambu-Bethe-Salpeter (NBS) Wave function

$$E = 2\sqrt{\mathbf{k}^2 + m_N^2}$$

$$\varphi_E(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | 6q, E\rangle$$

QCD eigen-state with energy E and #quark =6

$$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x): \text{ local operator}$$

off-shell T-matrix

$$\varphi_E(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} + \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{E_k + E_p}{8E_p^2} \frac{T(\mathbf{p}, -\mathbf{p} \leftarrow \mathbf{k}, -\mathbf{k})}{\mathbf{p}^2 - \mathbf{k}^2 - i\epsilon}$$

$$+ \ \mathcal{I}(\mathbf{r})$$

inelastic contribution $\propto O(e^{-\sqrt{E_{th}^2 - E^2}|\mathbf{r}|})$

C.-J.D.Lin et al., NPB69(2001) 467
CP-PACS Coll., PRD71 (2005) 094504

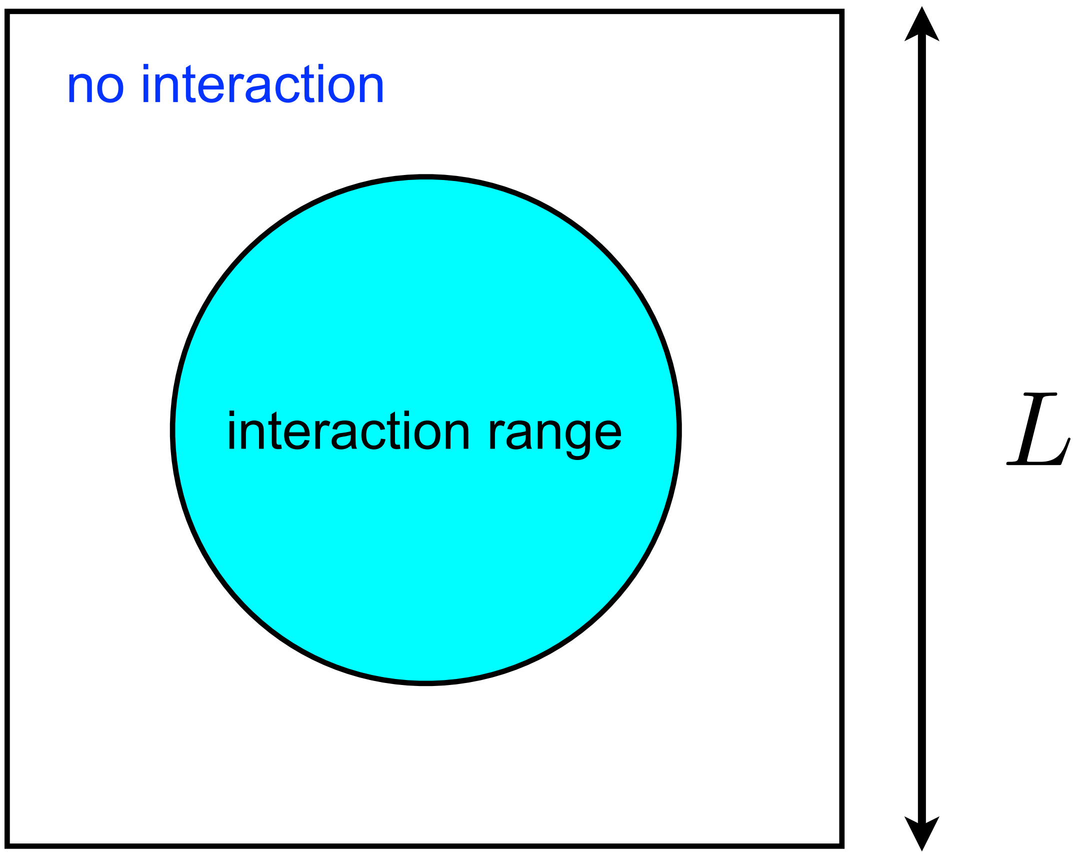
Asymptotic behavior

$$r = |\mathbf{r}| \to \infty$$

$$\varphi_E^l(r) \longrightarrow A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} \qquad l = 0, 1, 2, \cdots$$

partial wave

$\delta_l(k)$ is the scattering phase shift



Finite volume

allowed value: k_n^2

Lueshcer's formula

$\delta_l(k_n)$

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89.

We define the potential as

$$[\epsilon_k - H_0]\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y}) \qquad \epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

Velocity expansion $\quad U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$

Okubo-Marshak (1958)

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_\mathbf{1}\cdot\sigma_\mathbf{2}) + V_T(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

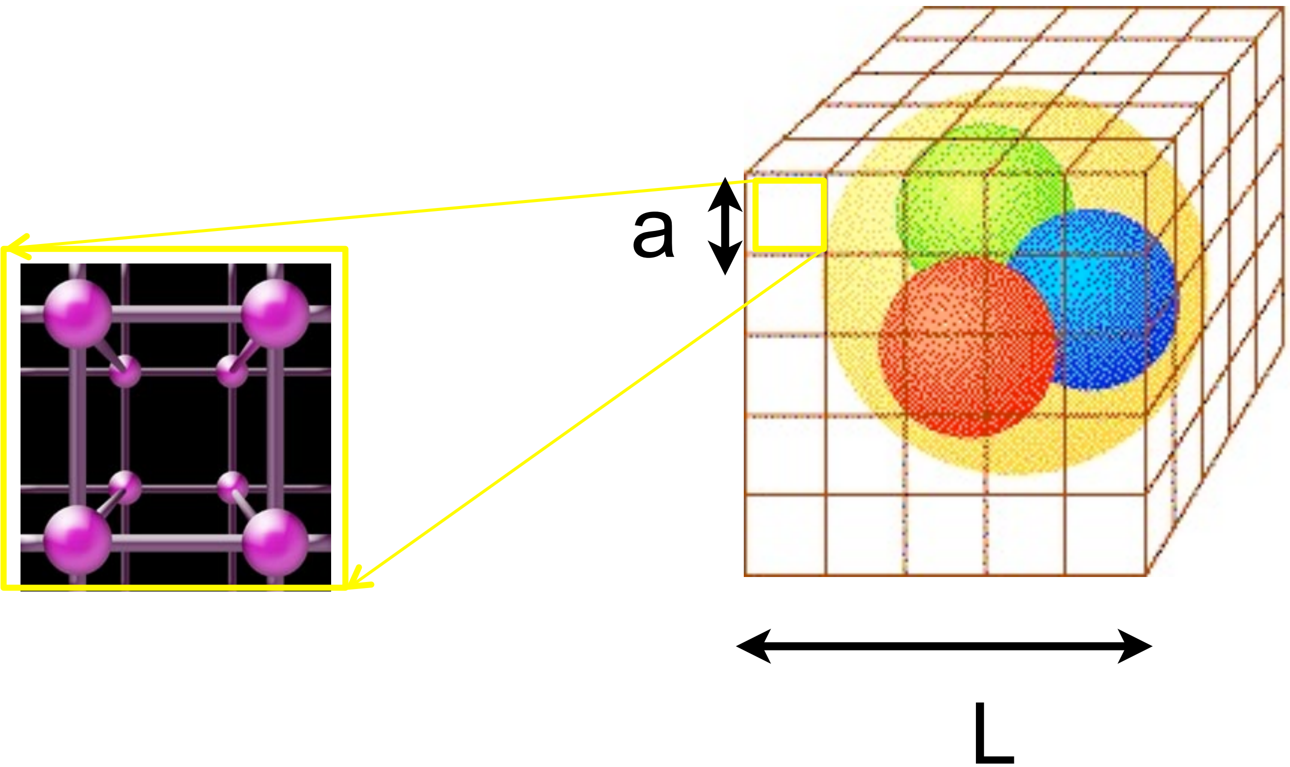
LO LO LO NLO NNLO

tensor operator $\quad S_{12} = \frac{3}{r^2}(\sigma_\mathbf{1}\cdot\mathbf{x})(\sigma_\mathbf{2}\cdot\mathbf{x}) - (\sigma_\mathbf{1}\cdot\sigma_\mathbf{2})$

spins

We calculate observables: phase shift, binding energy etc. using this approximated potential.

Lattice QCD



- well-defined statistical system (finite a and L)
- gauge invarinat
- fully non-perturbative

→ Monte-Calro simulations

Quenched QCD : neglects creation-anihilation of quark-anitiquak pair
Full QCD : includes creation-anihilation of quark-anitiquak pair

NBS wave function from lattice QCD

$$\langle 0|n_\beta(\mathbf{y},t)p_\alpha(\mathbf{x},t)\overline{\mathcal{J}}_{pn}(t_0)|0\rangle = \langle 0|n_\beta(\mathbf{y},t)p_\alpha(\mathbf{x},t)\sum_n |E_n\rangle\langle E_n|\overline{\mathcal{J}}_{pn}(t_0)|0\rangle$$

$$= \sum_n A_n \langle 0|n_\beta(\mathbf{y},t)p_\alpha(\mathbf{x},t)|E_n\rangle e^{-E_n(t-t_0)} \xrightarrow[t\to\infty]{} A_0 \varphi^{E_0}_{\alpha\beta}(\mathbf{x}-\mathbf{y}) e^{-E_0(t-t_0)}$$

$$A_n = \langle E_n|\overline{\mathcal{J}}_{pn}(t_0)|0\rangle$$

Wall source $\overline{\mathcal{J}}_{pn}(t_0) = p^{\mathrm{wall}}(t_0)n^{\mathrm{wall}}(t_0)$ $\quad q(\mathbf{x},t_0) \to q^{\mathrm{wall}}(t_0) = \sum_{\mathbf{x}} q(\mathbf{x},t_0)$

with Coulomb gauge fixing

$L = 0 \qquad P = +$

spin $\quad \frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$

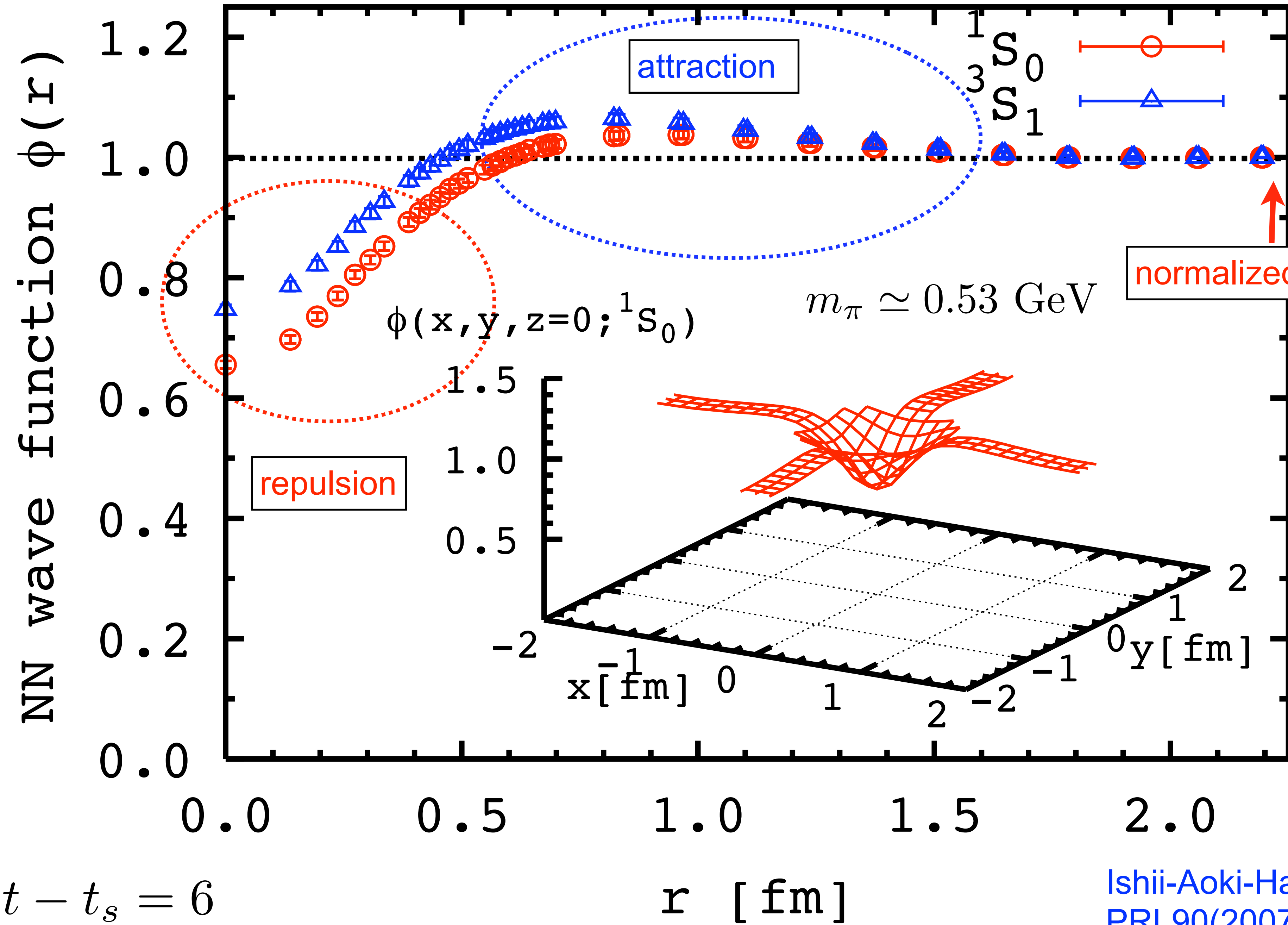
$^{2S+1}L_J$ ⇨ $^3S_1 \qquad {}^1S_0$

2011年2月28日月曜日

NN wave function

Quenched QCD

a=0.137fm



$t - t_s = 6$

Ishii-Aoki-Hatsuda, PRL90(2007)0022001

2011年2月28日月曜日

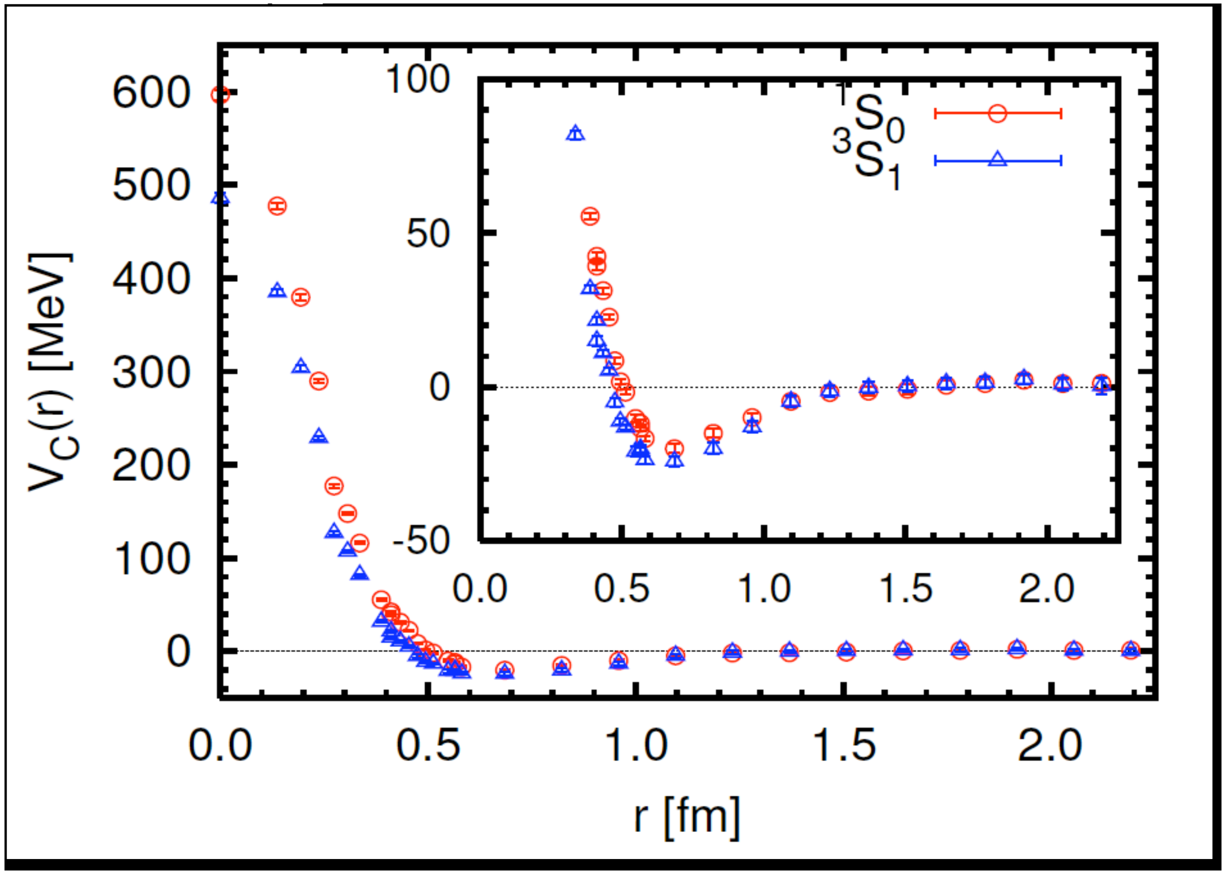
(quenched) potentials

LO (effective) central Potential

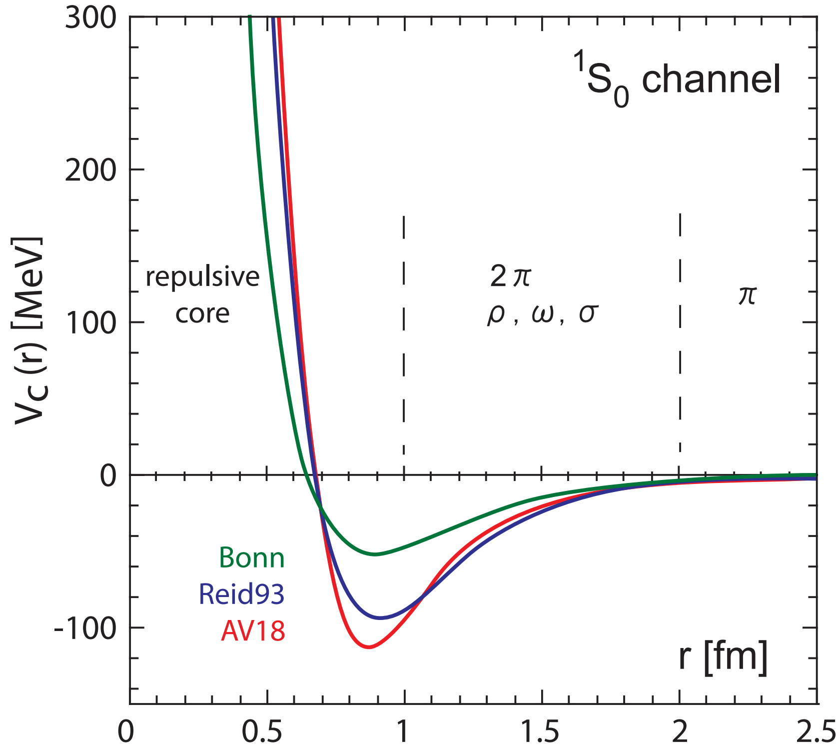
$$V(r;{}^1S_0) = V_0^{(I=1)}(r) + V_\sigma^{(I=1)}(r)$$

$$V(r;{}^3S_1) = V_0^{(I=0)}(r) - 3V_\sigma^{(I=0)}(r)$$

$E \simeq 0$ $\quad m_\pi \simeq 0.53$ GeV



a=0.137 fm L=4.4fm



Qualitative features of NN potential are reproduced !

Ishii-Aoki-Hatsuda, PRL90(2007)0022001

This paper has been selected as one of 21 papers in Nature Research Highlights 2007

Frequently Asked Questions

[Q1] Scheme/Operator dependence of the potential

- the potential depends on the definition of the wave function, in particular, on the choice of the nucleon operator N(x). (Scheme-dependence)
  - local operator = convenient choice for reduction formula
- Moreover, the potential itself is NOT a physical observable. Therefore it is NOT unique and is naturally scheme-dependent.
  - Observables: scattering phase shift of NN, binding energy of deuteron

QM: (wave function,potential) → observables
QFT: (asymptotic field,vertex) → observables
EFT: (choice of field, vertex) → observables

- Is the scheme-dependent potential useful ? Yes !
  - useful to understand/describe physics
  - a similar example: running coupling
    - Although the running coupling is scheme-dependent, it is useful to understand the deep inelastic scattering data (asymptotic freedom).
- “good” scheme ?
  - good convergence of the perturbative expansion for the running coupling.
  - good convergence of the derivative expansion for the potential ?
    - completely local and energy-independent one is the best and must be unique if exists. (Inverse scattering method)

[Q2] Energy dependence of the potential

Non-local, E-independent ⟺ Local, E-dependent

$$\left(E + \frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y})$$

$$V_E(\mathbf{x})\varphi_E(\mathbf{x}) = \left(E + \frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x})$$

non-locality can be determined order by order in velocity expansion (cf. ChPT)

$$V(\mathbf{x},\nabla) = V_C(r) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + \{V_D(r),\nabla^2\} + \cdots$$

Numerical check in quenched QCD

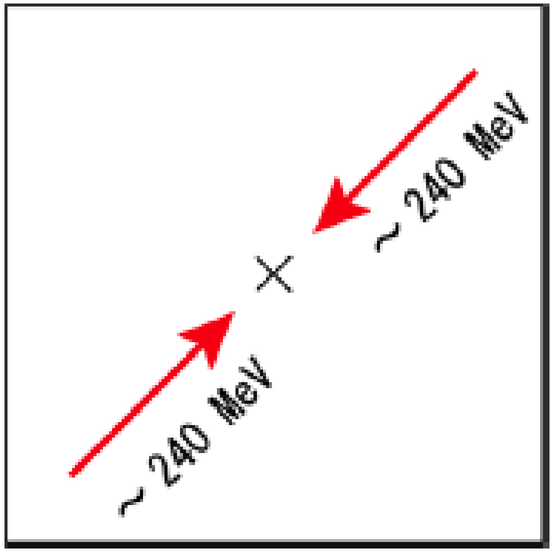
$m_\pi \simeq 0.53$ GeV

a=0.137fm

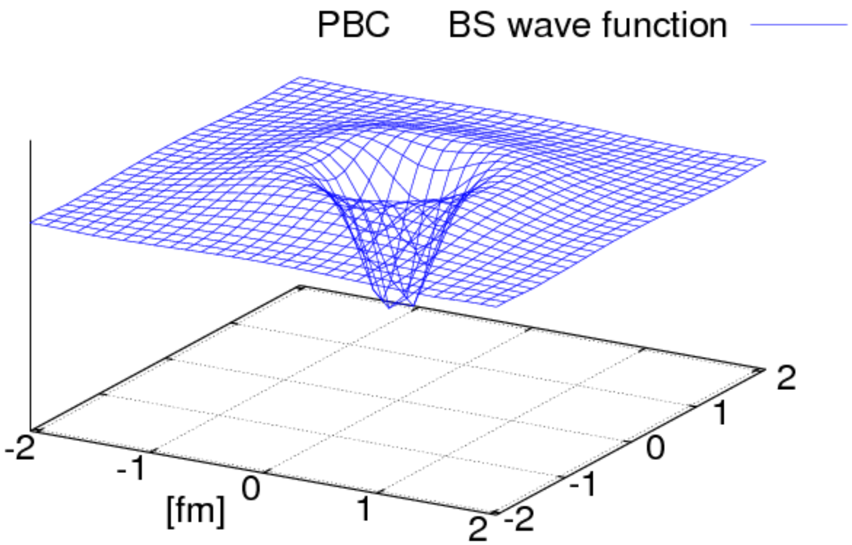
K. Murano, N. Ishii, S. Aoki, T. Hatsuda

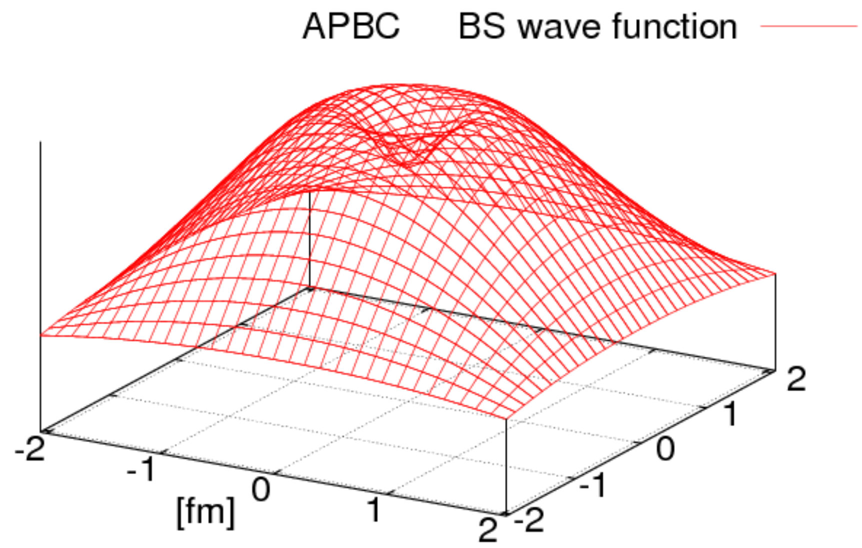
PoS Lattice2009 (2009)126.

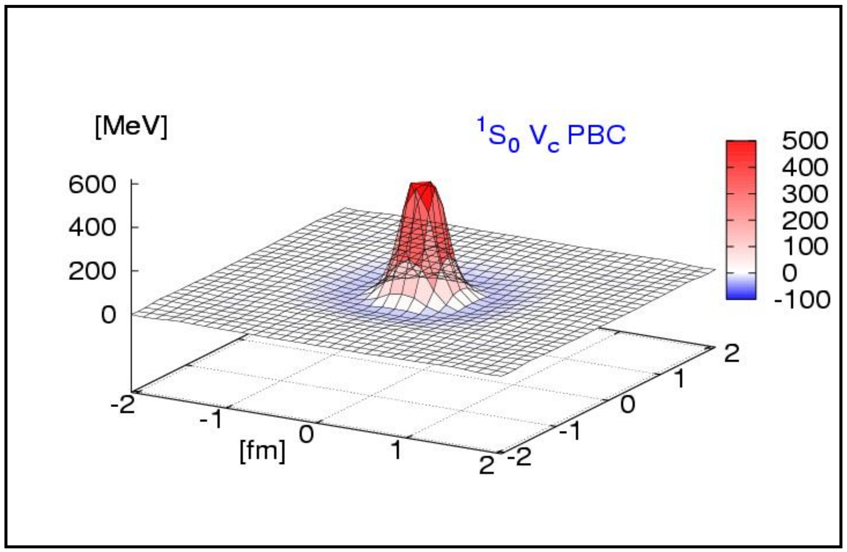
Anti-Periodic B.C.

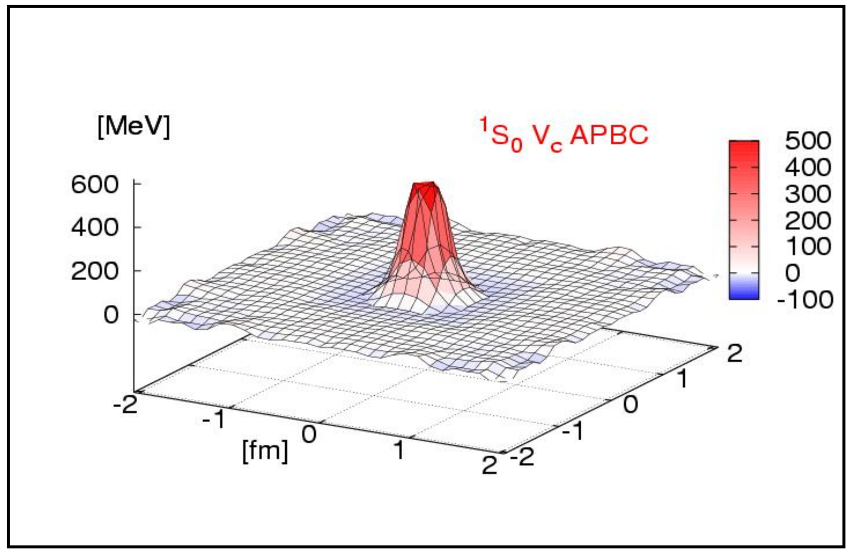


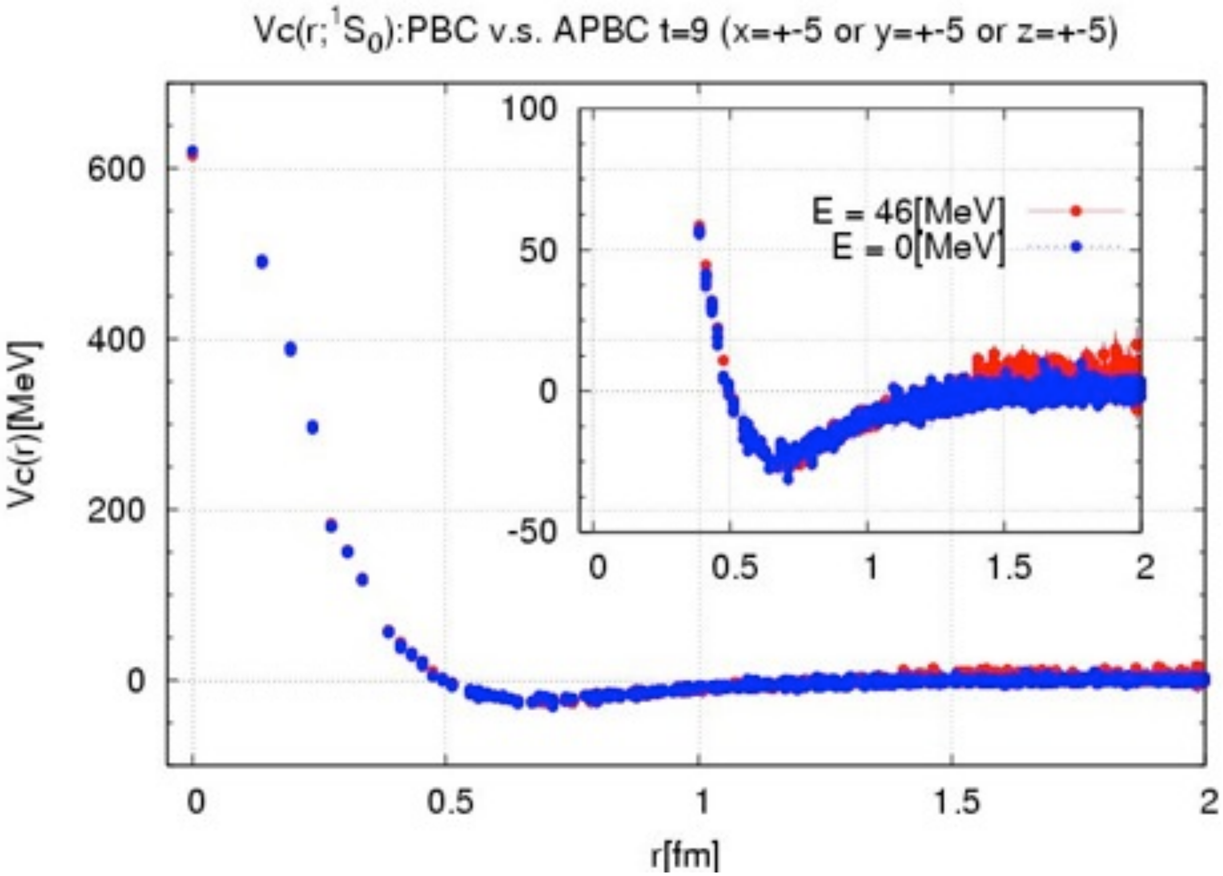
- PBC (E～0 MeV)
- APBC (E～46 MeV)







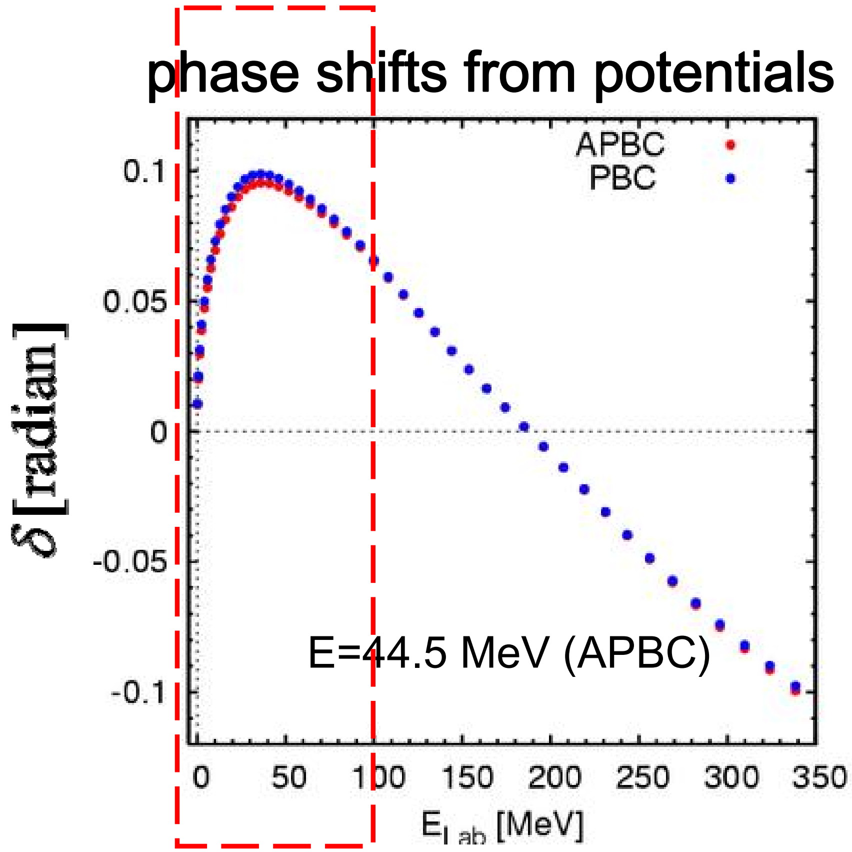




Quenched QCD

$m_\pi \simeq 0.53$ GeV

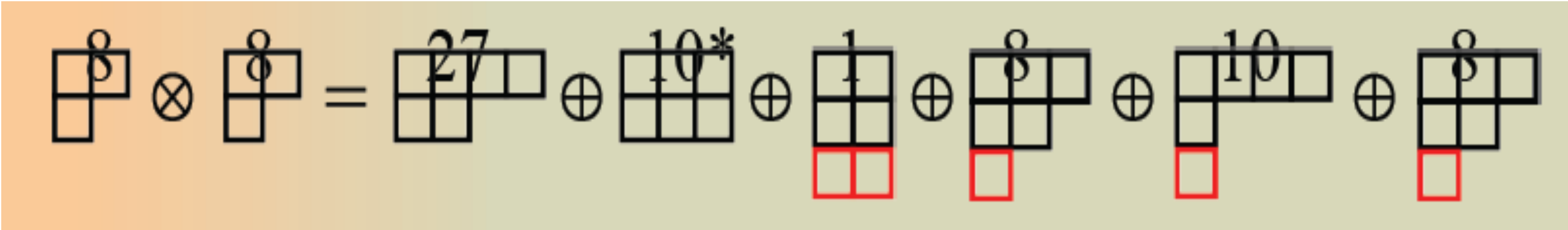
a=0.137fm

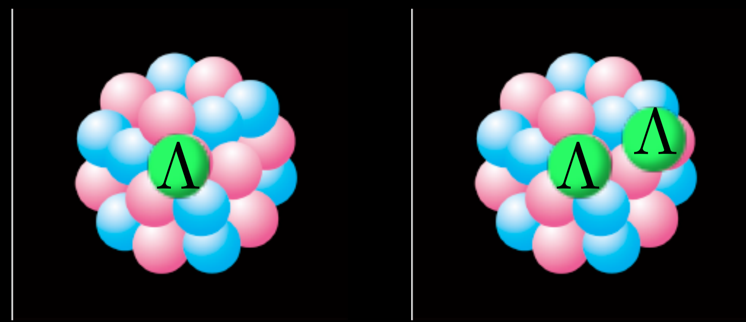


E-dependence of the local potential turns out to be very small at low energy in our choice of wave function.

3. Octet baryon interactions

Octet Baryon interactions



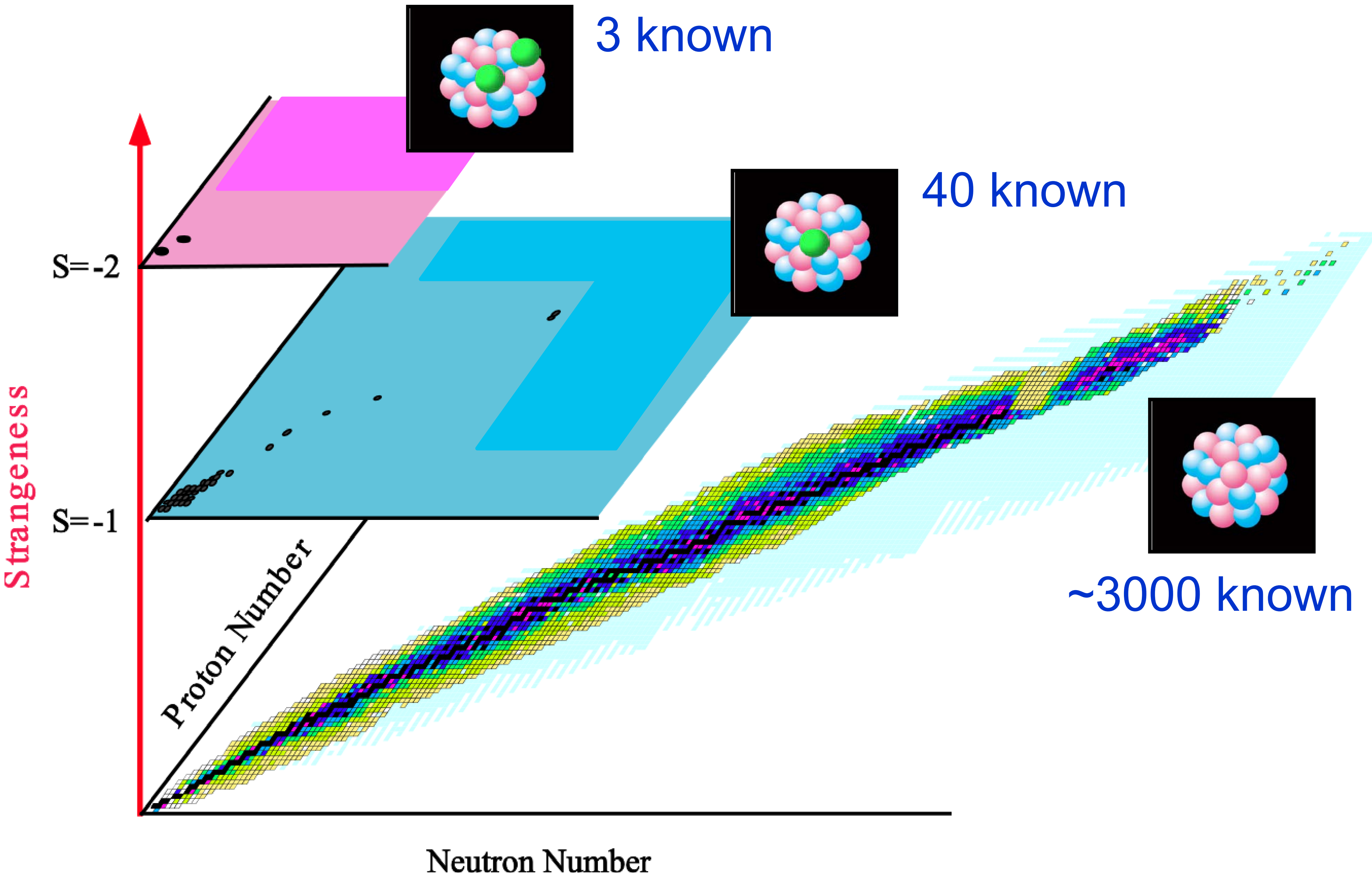


- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

also in GSI

- prediction from lattice QCD
- difference between NN and YN ?

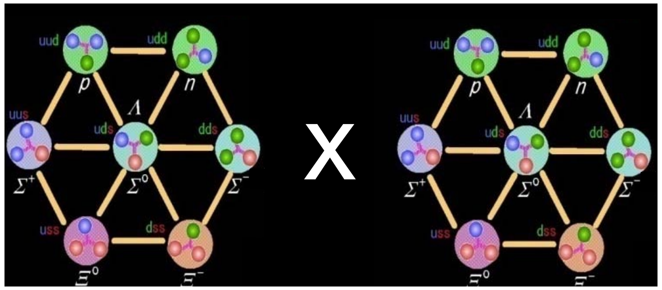
3D Nuclear chart



(1) Potentials in the flavor SU(3) symmetric limit

$$m_u = m_d = m_s$$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Symmetric   Anti-symmetric

6 independent potential in flavor-basis

$$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r) \longleftarrow {}^1S_0$$

$$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r) \longleftarrow {}^3S_1$$

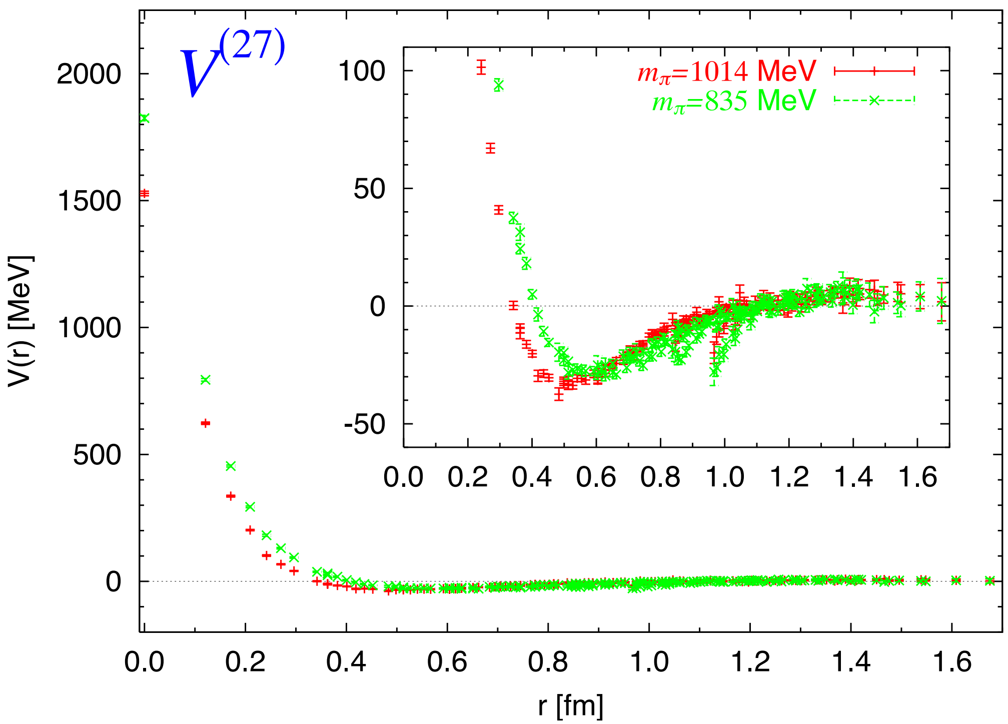
Potential(full QCD)

Inoue *et al.* (HAL QCD Coll.), PTP124(2010)591

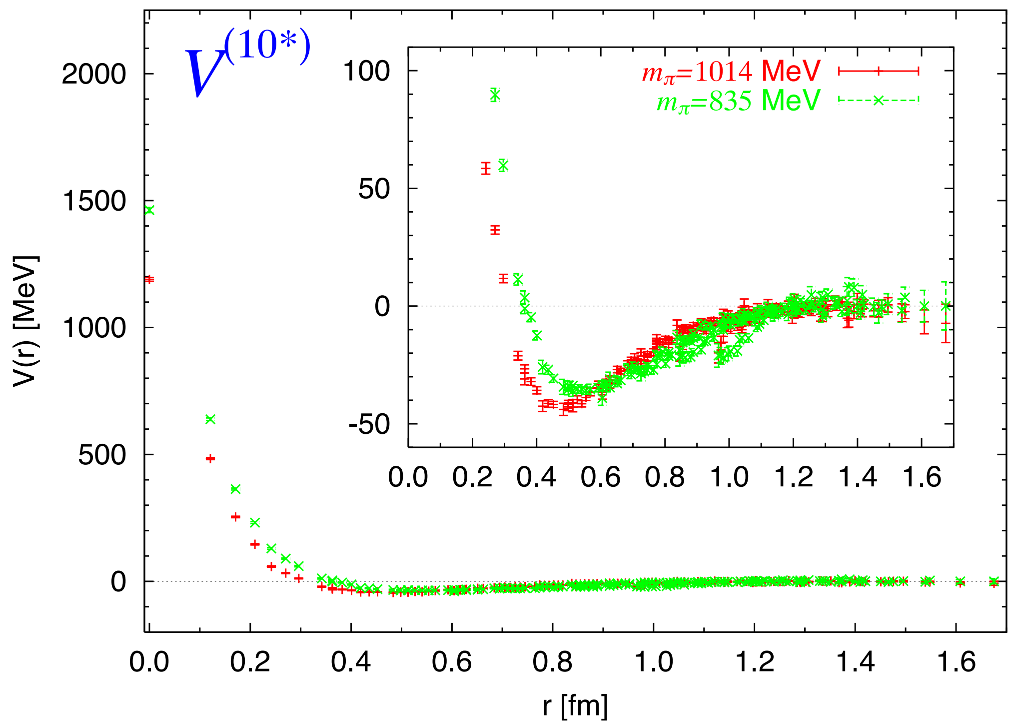
a=0.12 fm, L=2 fm

BG/L@KEK

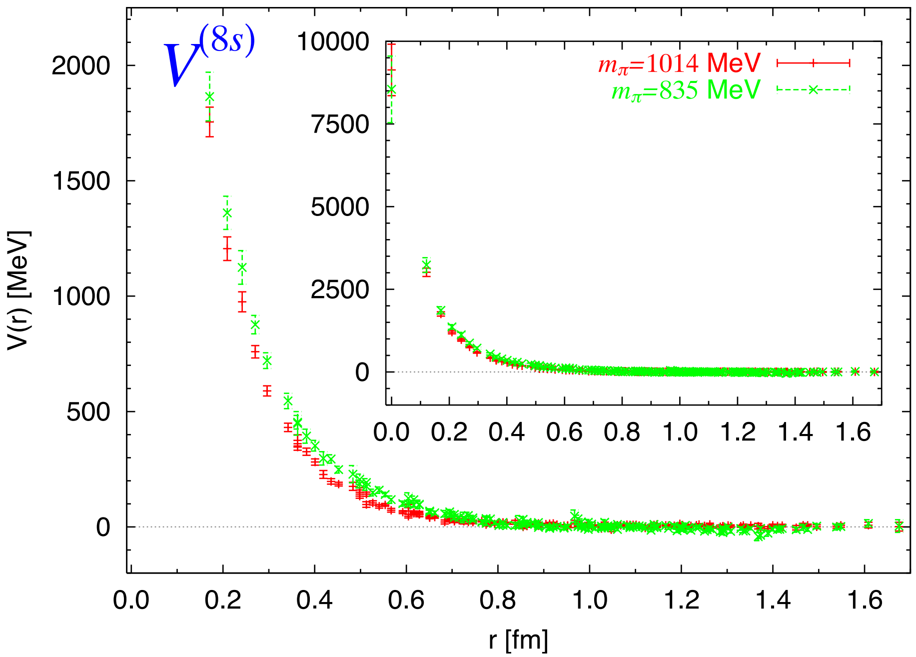
S(spin)=0



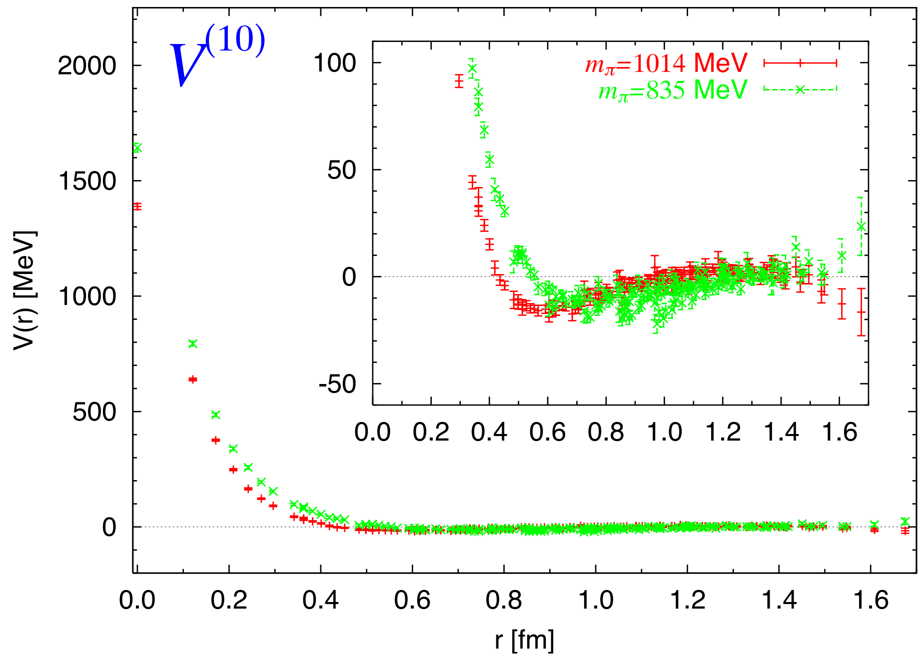
S(spin)=1



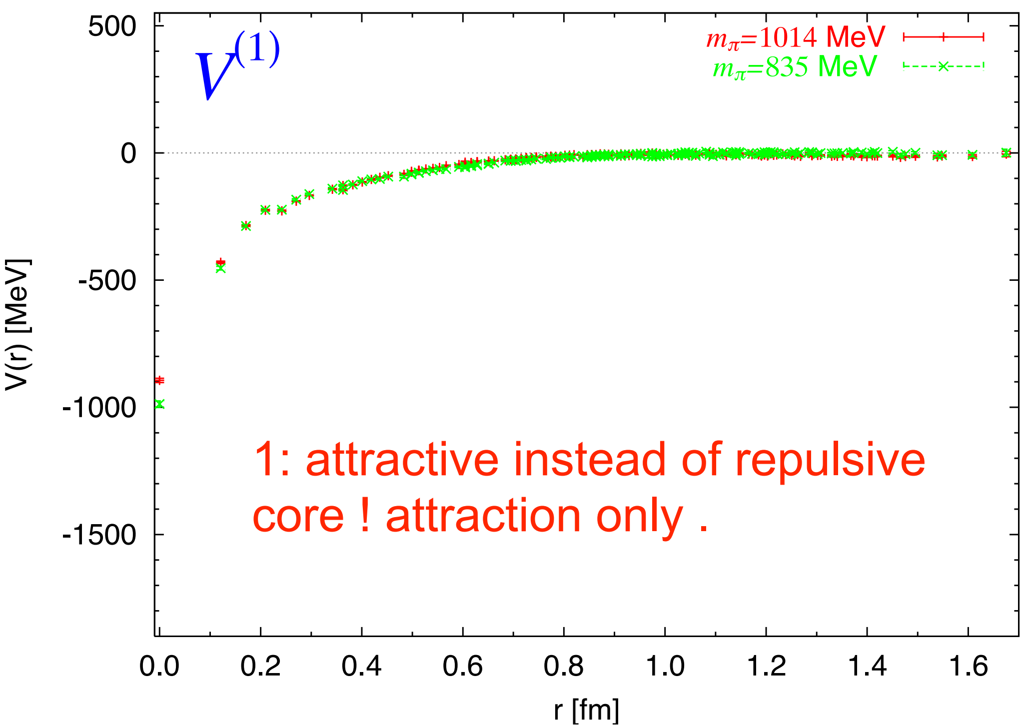
27, 10*: channels NN belongs
same behaviors as NN potentials



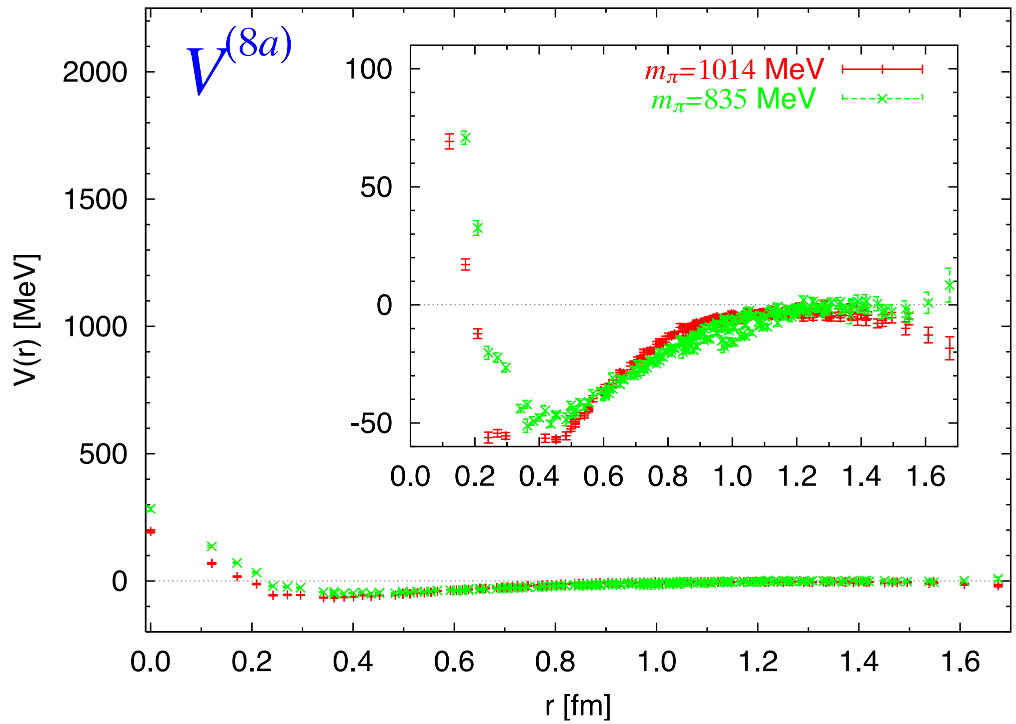
8s: strong repulsive core. repulsion only.



10: strong repulsive core. weak attraction.



1: attractive instead of repulsive core ! attraction only .



8a: weak repulsive core. strong attraction.

(2) H-dibaryon in the flavor SU(3) symmetric limit

Attractive potential in the flavor singlet channel

→ possibility of a bound state (H-dibaryon)

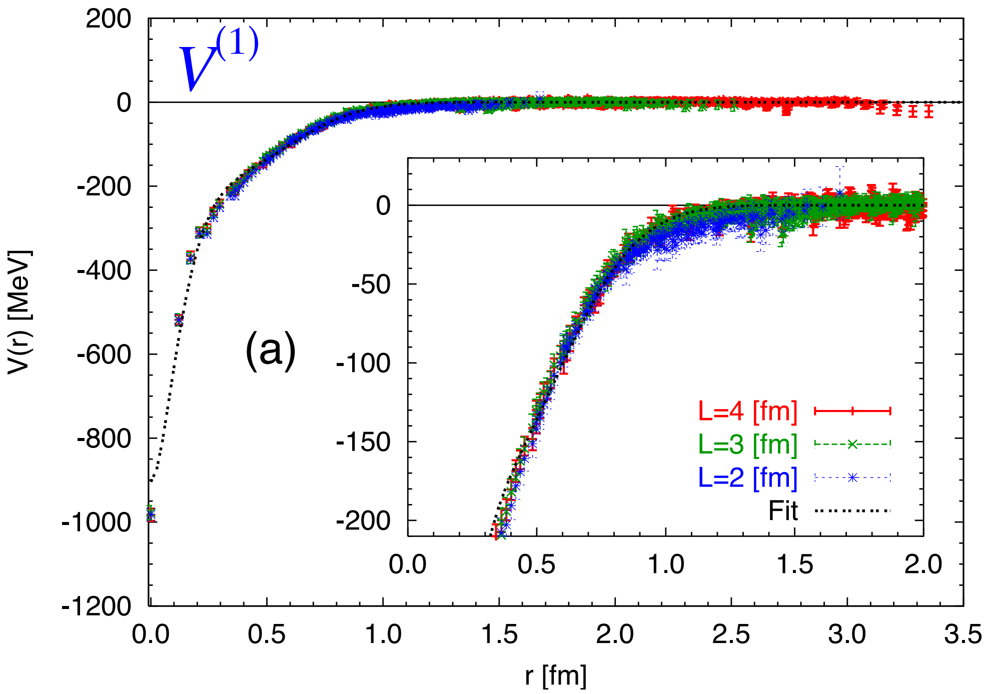
$$\Lambda\Lambda - N\Xi - \Sigma\Sigma$$

However, it is difficult to distinguish a bound state from scattering states in the finite volume.
Additional calculations in different volumes are needed.

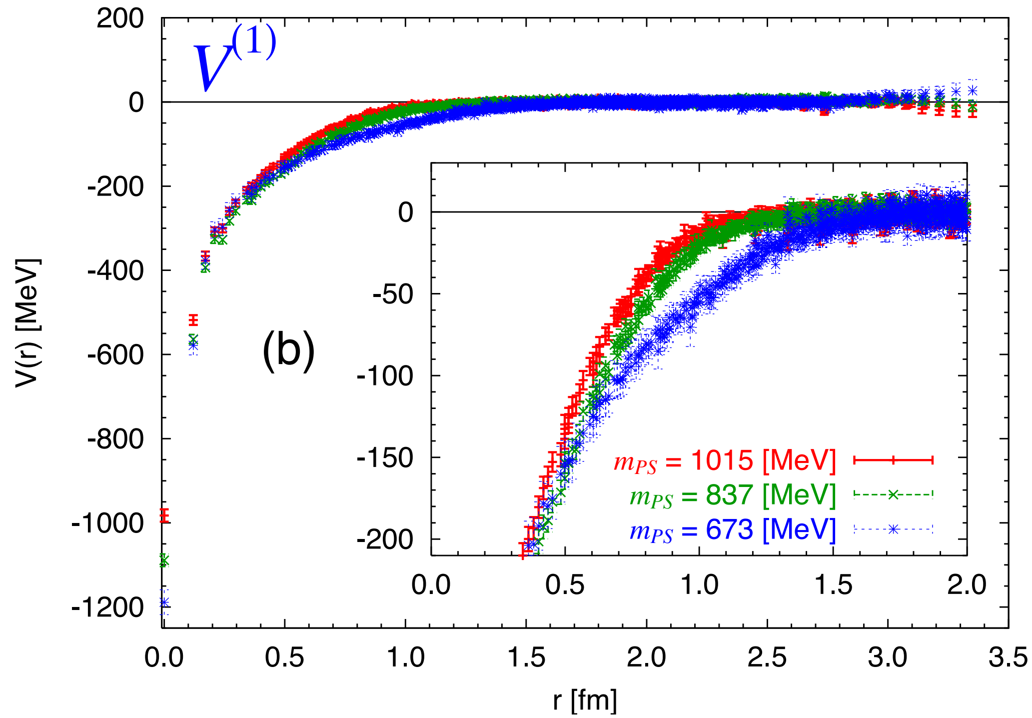
Inoue *et al.* (HAL QCD Coll.), “Bound H-dibaryon in Flavor SU(3) Limit of Lattice QCD”, arXiv:1012.5928[hep-lat].

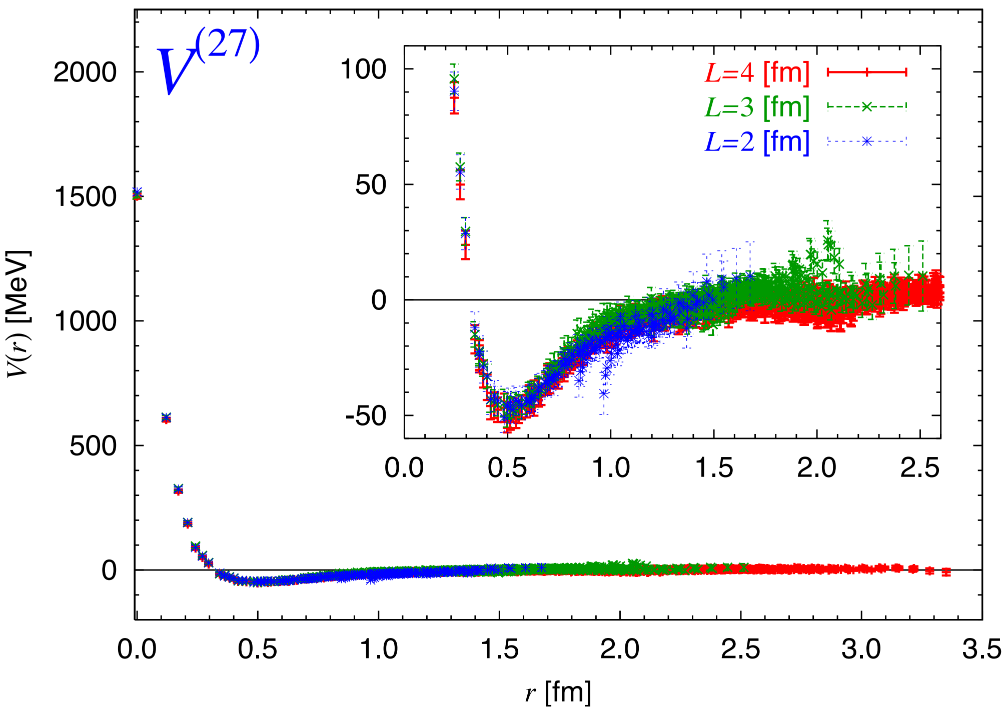
T2K-Tsukuba BG/L@KEK

volume dependence



pion mass dependence





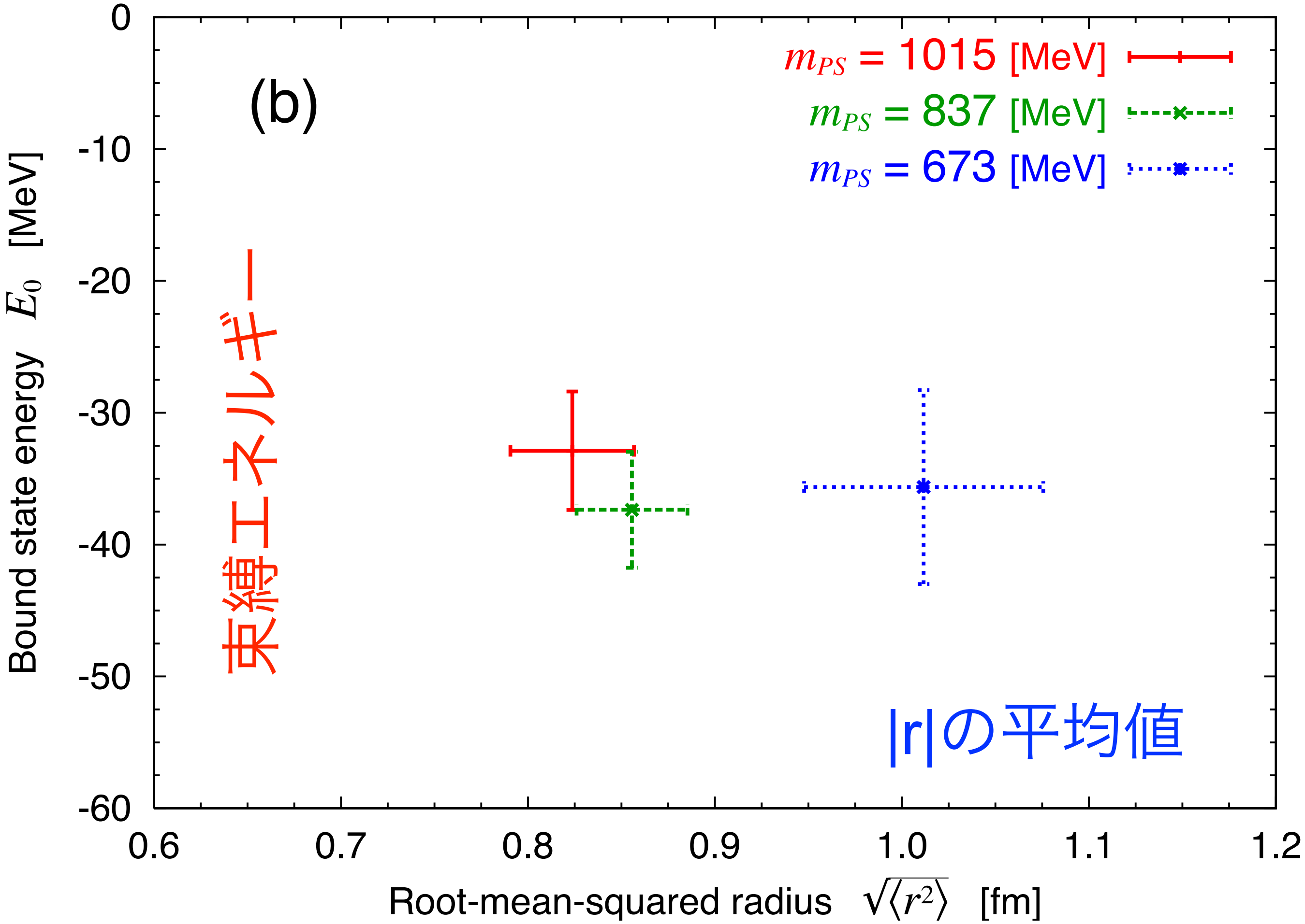
L=3 fm is enough for the potential.

lighter the pion mass,
stronger the attraction

fit the potential at L=4 fm by

$$V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$$

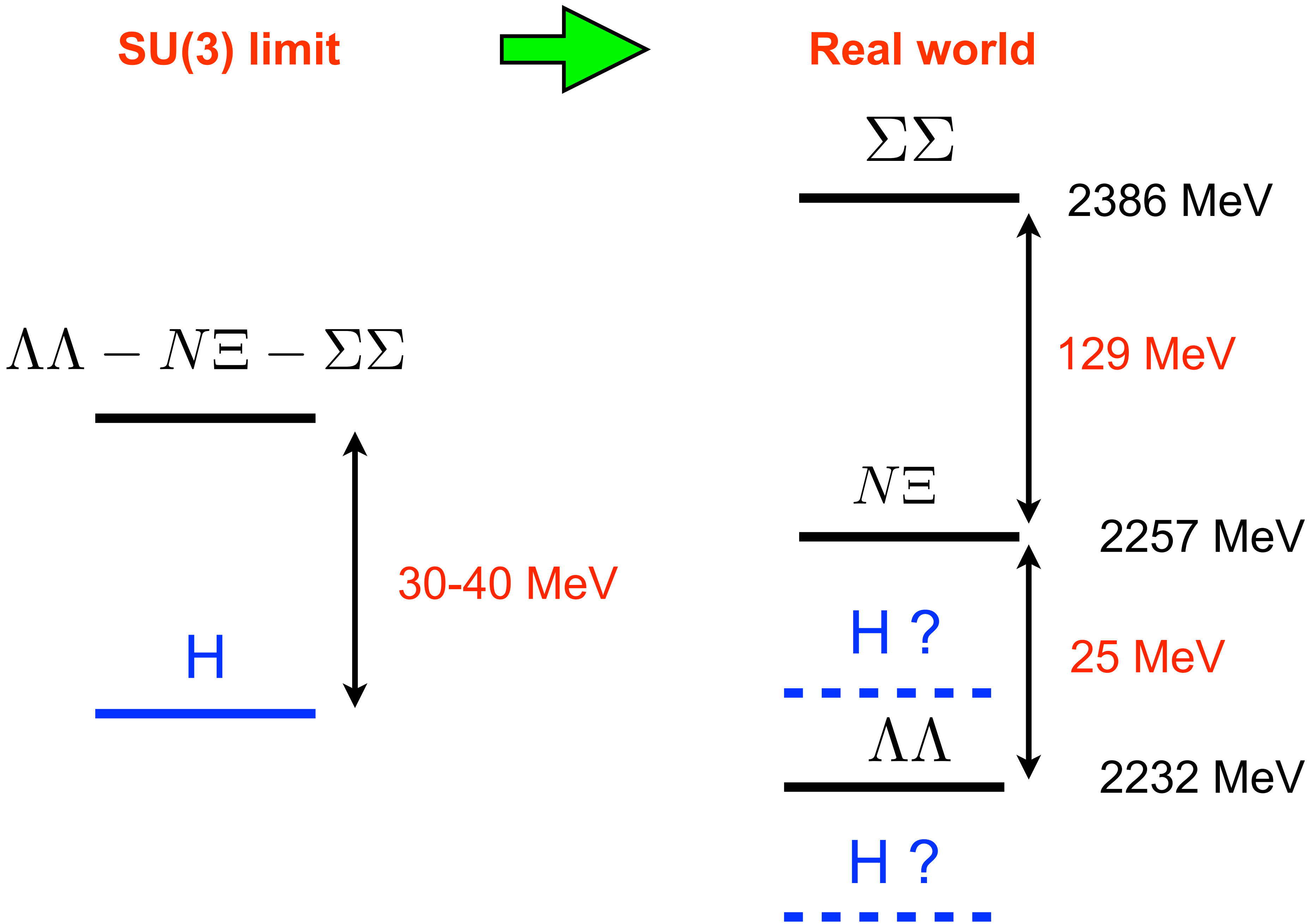
Solve Schroedinger equation in the infinite volume with the fitted potential(@4fm) ⇒ A bound state(H-dibaryon) exists !



Real world ?

An H-dibaryon exists in the flavor SU(3) limit !
binding energy = 30-40 MeV, weak quark mass dependence.

4. Search for new hadrons in lattice QCD


SU(3) limit
Real world
$\Lambda\Lambda - N\Xi - \Sigma\Sigma$
30-40 MeV
H
$\Sigma\Sigma$
2386 MeV
129 MeV
$N\Xi$
2257 MeV
H ?
25 MeV
$\Lambda\Lambda$
2232 MeV
H ?

(1) Proposal for S=-2 In-elastic scattering

$$m_N = 939 \text{ MeV},\ m_\Lambda = 1116 \text{ MeV},\ m_\Sigma = 1193 \text{ MeV},\ m_\Xi = 1318 \text{ MeV}$$

S=-2 System(I=0)

$$M_{\Lambda\Lambda} = 2232 \text{ MeV} < M_{N\Xi} = 2257 \text{ MeV} < M_{\Sigma\Sigma} = 2386 \text{ MeV}$$

The eigen-state of QCD in the finite box is a mixture of them:

$$|S=-2, I=0, E\rangle_L = c_1(L)|\Lambda\Lambda, E\rangle + c_2(L)|\Xi N, E\rangle + c_3(L)|\Sigma\Sigma, E\rangle$$

$$E = 2\sqrt{m_\Lambda^2 + \mathbf{p}_1^2} = \sqrt{m_\Xi^2 + \mathbf{p}_2^2} + \sqrt{m_N^2 + \mathbf{p}_2^2} = 2\sqrt{m_\Sigma^2 + \mathbf{p}_3^2}$$

In this situation, we can not directly extract the scattering phase shift in lattice QCD.

HAL's proposal

Let us consider 2-channel problem for simplicity.

NBS wave functions for 2 channels at 2 values of energy:

$$\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = \langle 0|\Lambda(\mathbf{x})\Lambda(\mathbf{0})|E_\alpha\rangle$$
$$\Psi_\alpha^{\Xi N}(\mathbf{x}) = \langle 0|\Xi(\mathbf{x})N(\mathbf{0})|E_\alpha\rangle$$

$$\alpha = 1, 2$$

They satisfy

$$(\nabla^2 + \mathbf{p}_\alpha^2)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = 0$$
$$(\nabla^2 + \mathbf{q}_\alpha^2)\Psi_\alpha^{\Xi N}(\mathbf{x}) = 0$$

$$|\mathbf{x}| \to \infty$$

We define the “potential” from the coupled channel Schroedinger equation:

$$\left(\frac{\nabla^2}{2\mu_{\Lambda\Lambda}} + \frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}}\right)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

diagonal — off-diagonal

$$\left(\frac{\nabla^2}{2\mu_{\Xi N}} + \frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}\right)\Psi_\alpha^{\Xi N}(\mathbf{x}) = V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Xi N\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

off-diagonal — diagonal

μ: reduced mass

$$\begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix} \begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix}$$

$X \neq Y$

$$E_\alpha = \frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}},\ \frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}$$

$X, Y = \Lambda\Lambda$ or ΞN

$$\begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix}^{-1} \begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix}$$

Using the potentials: $\begin{pmatrix} V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x}) & V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x}) \\ V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x}) & V^{\Xi N\leftarrow\Xi N}(\mathbf{x}) \end{pmatrix}$

we solve the coupled channel Schroedinger equation in the infinite volume with an appropriate boundary condition.

For example, we take the incomming $\Lambda\Lambda$ state by hand.

In this way, we can avoid the mixture of several “in”-states.

$$|S=-2, I=0, E\rangle_L = c_1(L)|\Lambda\Lambda, E\rangle + c_2(L)|\Xi N, E\rangle + c_3(L)|\Sigma\Sigma, E\rangle$$

Lattice is a tool to extract the interaction kernel (“T-matrix” or “potential”).

Preliminary results from HAL QCD Collaboration

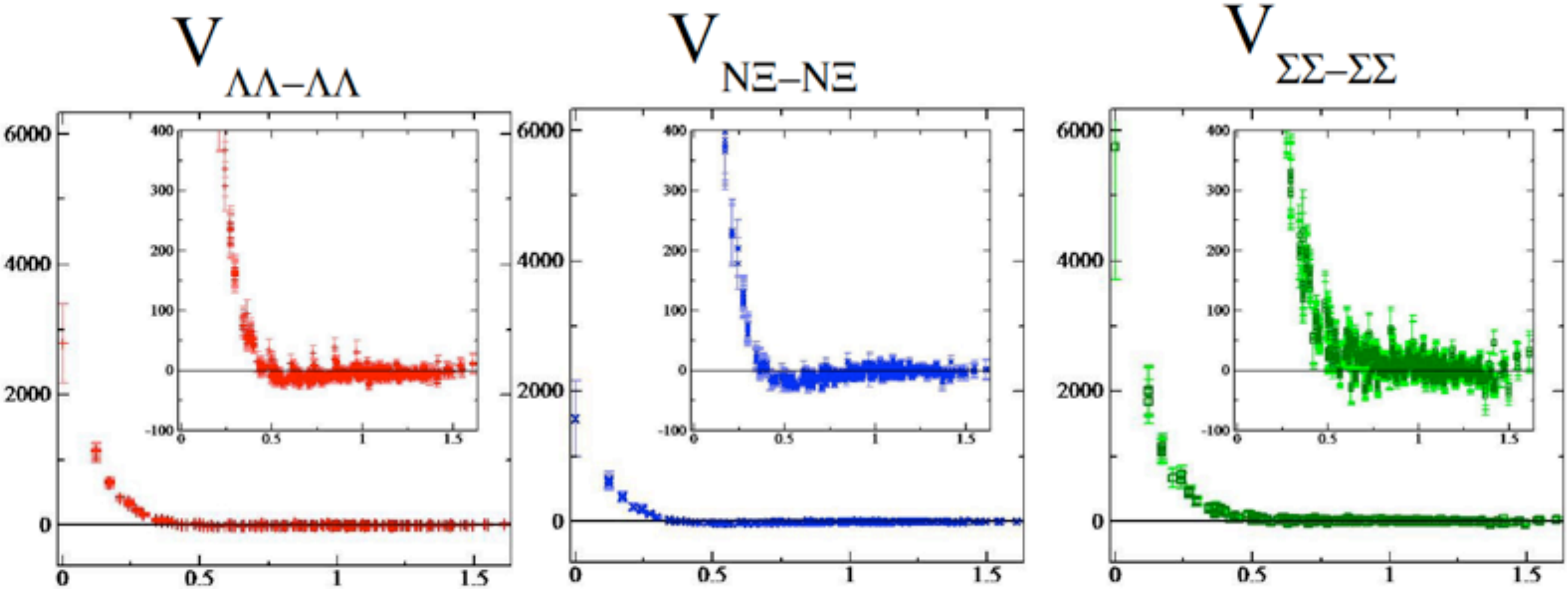
Sasaki for HAL QCD Collaboration

2+1 flavor full QCD

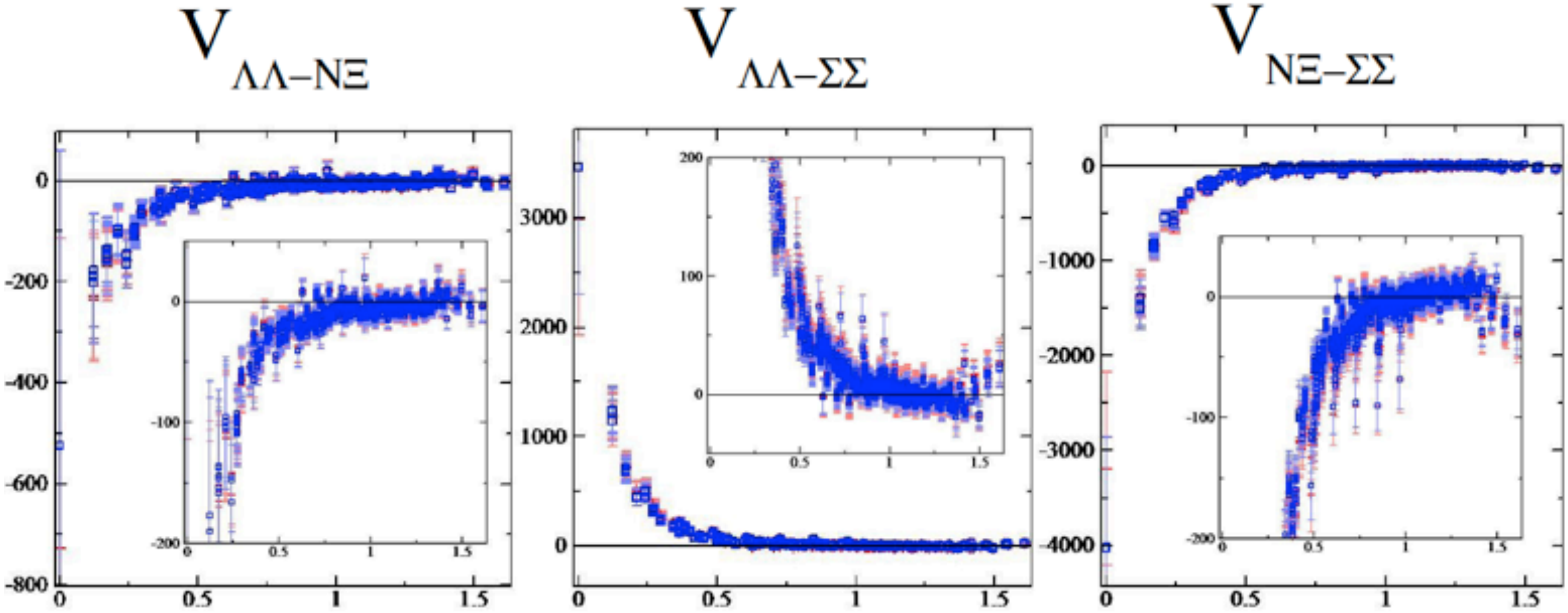
a=0.1 fm, L=2.9 fm

$m_\pi \simeq 870$ MeV

Diagonal part of potential matrix



Non-diagonal part of potential matrix



$$V_{A-B} \simeq V_{B-A}$$

Hermiticity ! (non-trivial check)

(2) H-dibaryon in Nature: resonance or bound state ?

I. S=-2 singlet state become the bound state in flavor SU(3) limit.

II. In the real world (s is heavier than u,d), a resonance state or a bound state appears.

III. We can determine which possibility is realized by solving the coupled channel Schroedinger equation with the 3 x 3 potential matrix.

IV. Trial demonstration for III.

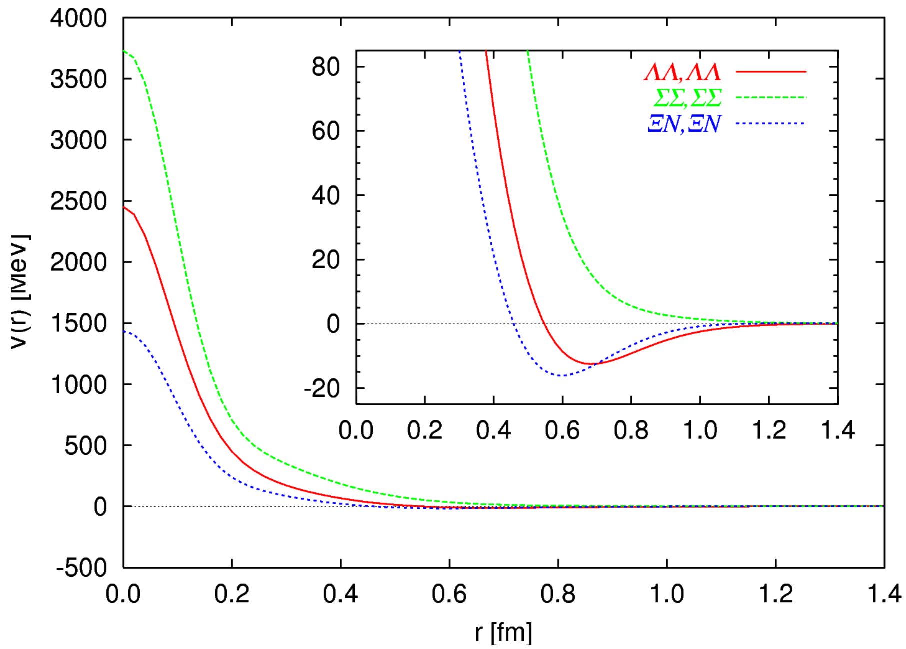
IV.1. Use potential in SU(3) limit

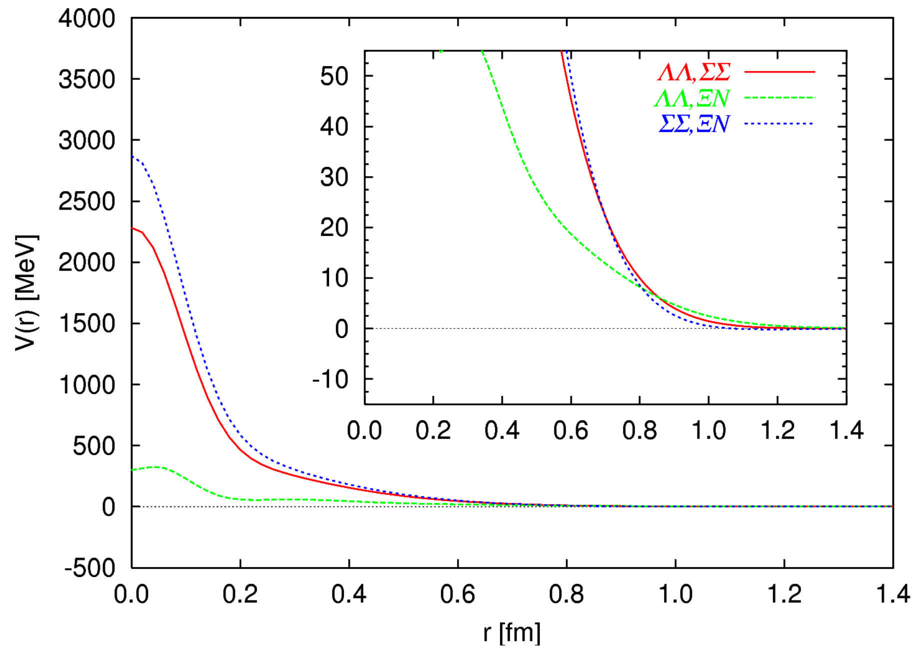
Inoue for HAL QCD Collaboration

IV.2. Introduce mass difference only from 2+1 simulation

Potentials in particle basis in SU(3) limit

$$\begin{pmatrix} \Lambda\Lambda \\ \Sigma\Sigma \\ \Xi N \end{pmatrix} = U \begin{pmatrix} |27\rangle \\ |8\rangle \\ |1\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{(27)} & & \\ & V^{(8)} & \\ & & V^{(1)} \end{pmatrix} U^t \longrightarrow \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{\Sigma\Sigma} & V^{\Lambda\Lambda}_{\Xi N} \\ & V^{\Sigma\Sigma} & V^{\Sigma\Sigma}_{\Xi N} \\ & & V^{\Xi N} \end{pmatrix}$$



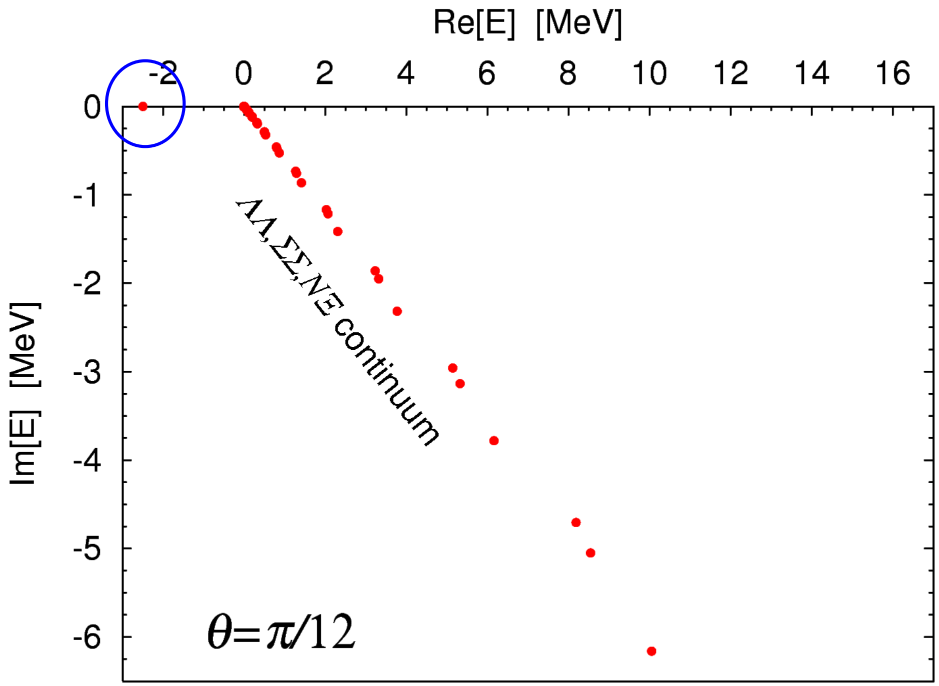


where $T_0^{(1)}=-25,\ T_0^{(8)}=25,\ T_0^{(27)}=-5$ [MeV] are used

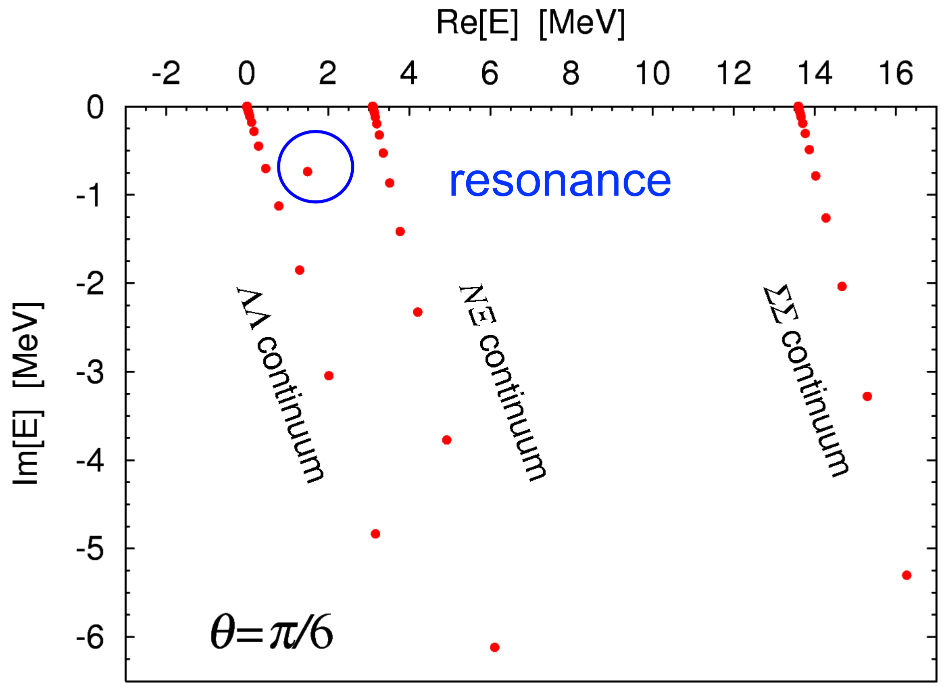
$S=-2, I=0, {}^1S_0$ scattering

$$E^{(1)} = -40 \text{ MeV}$$

bound state



SU(3) limit

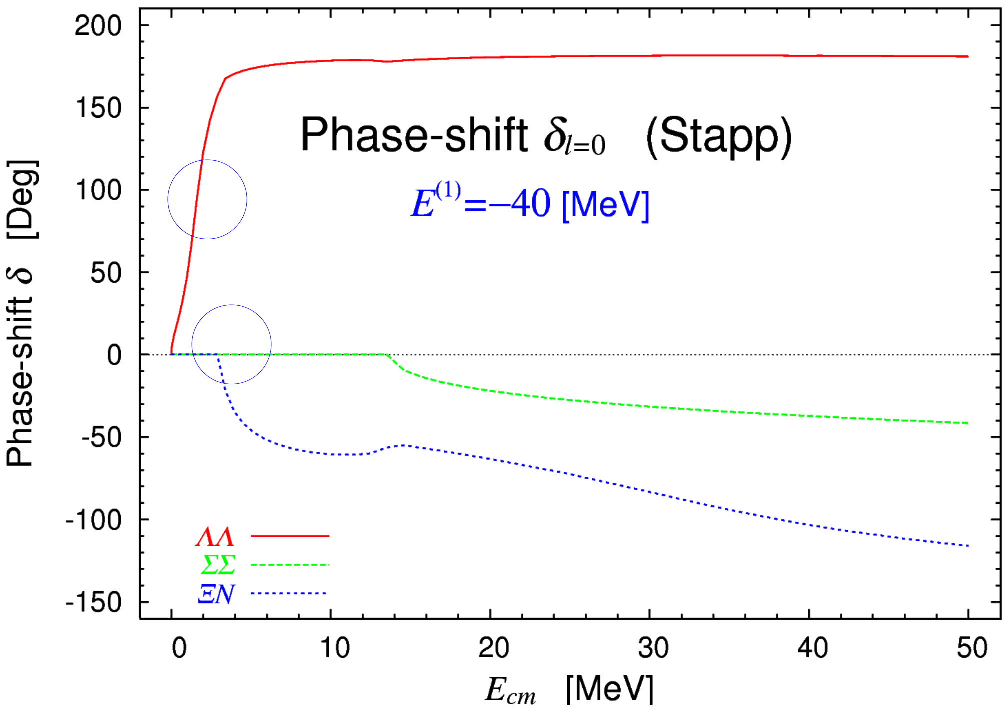


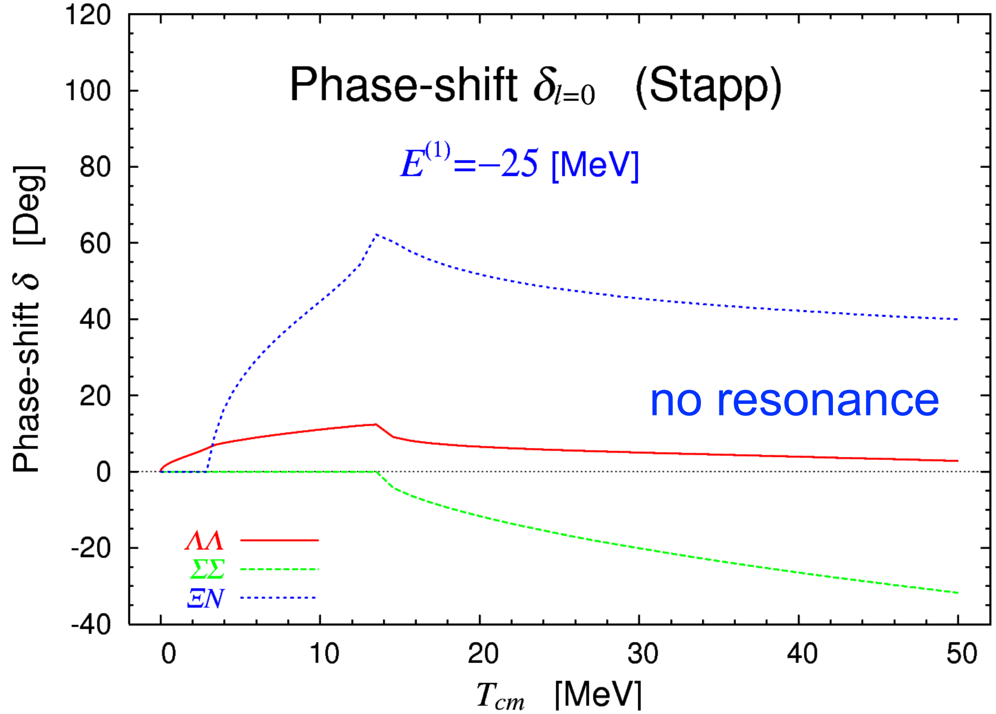
"2+1 flavor"

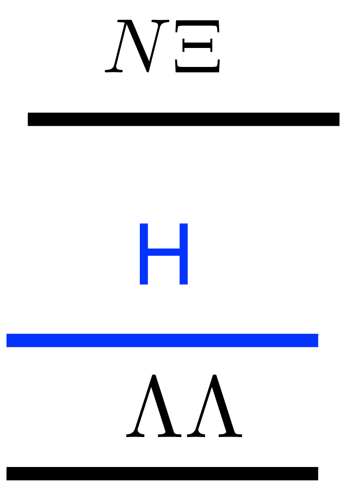
Phase shift in "2+1 flavors"

$$E^{(1)} = -40 \text{ MeV}$$

$$E^{(1)} = -25 \text{ MeV}$$







bound state from NΞ

resonance from ΛΛ

5. Summary and Discussion

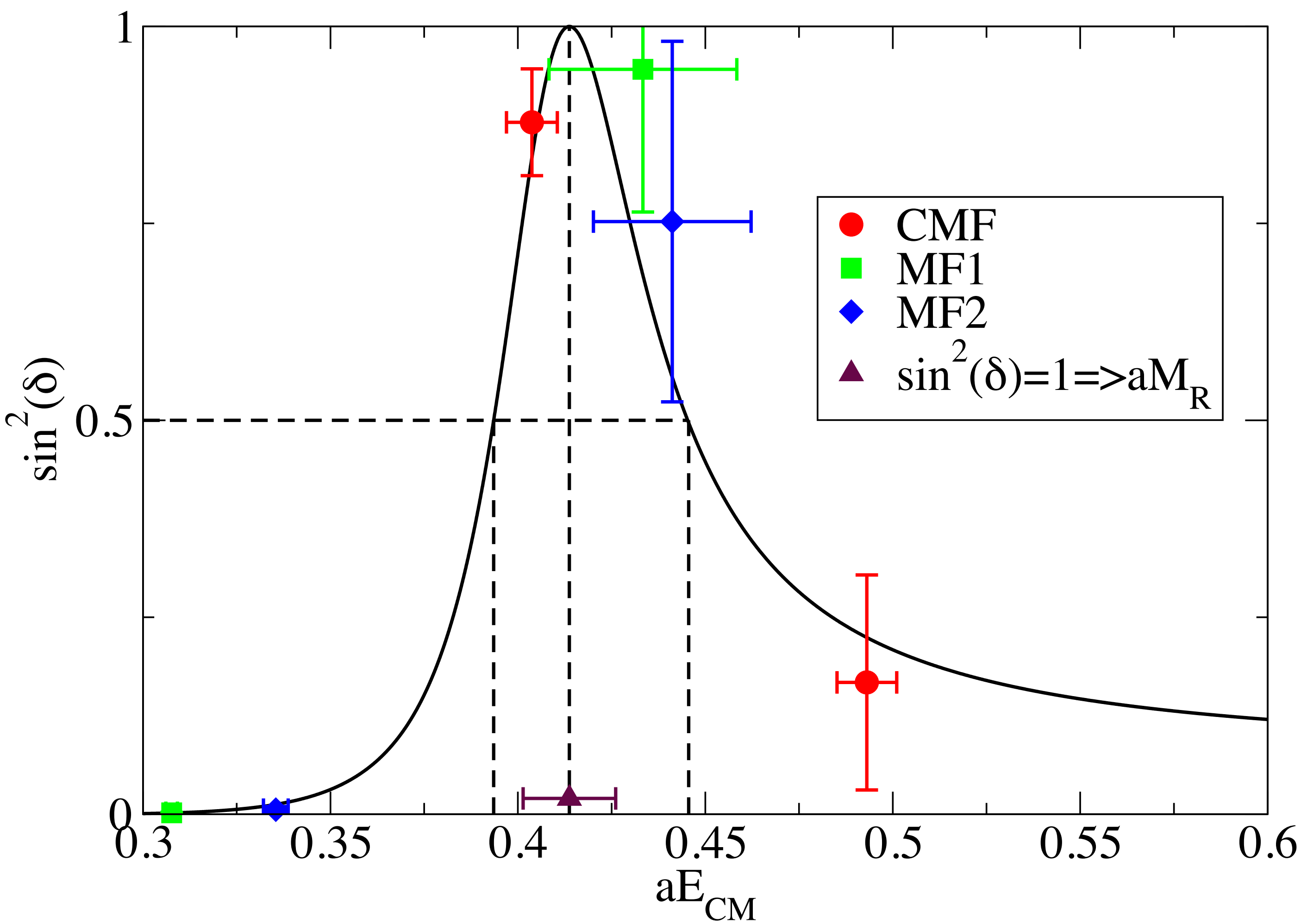
- We can investigate an existence of new hadrons in QCD using the potential (HAL QCD's method).
    - Calculate potential (matrix) in lattice QCD on a finite box.
    - Calculate phase shift by solving (coupled channel) Shroedinger equation in infinite volume.
    - bound/resonance/scattering
- H-dibaryon
    - bound state exists in the SU(3) limit
    - resonance or bound state in Nature ?
- Exotic particle search by potentials between stable particles
    - penta-quark, X, Y etc.
    - potentials contain all informations -> bound state(energy,size)/ resonance state(energy,width)
    - extension to "potentials" among 3 or more particles

unstable particle as a resonance

$\pi^+\pi^-$ scattering (ρ meson width)

Finite volume method

ETMC: Feng-Jansen-Renner, PLB684(2010)



$$\varphi_E(\mathbf{x}) = \langle 0|\pi(\mathbf{x},0)\pi(\mathbf{0},0)|\rho, E\rangle \Rightarrow V(\mathbf{x}) \Rightarrow \sin^2\delta(s)?$$