Productions of Tetra-Quark Mesons in B Decays and Related

K. Terasaki
YITP, Kyoto U.,
Kanazawa U.

§1. Introduction – Tetra-Quark Mesons and Their Candidates

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Appendix: 1, 2, 3, 4, 5, 6

§1. Introduction

Tetra-quark mesons :

Jaffe, PRD 15, 267 and 281 (1977),
(K. T., PRD 68, 011501(R) (2003))

- **Flavor wavefunctions :**

$$\{qq\bar{q}\bar{q}\} = [qq][\bar{q}\bar{q}] \oplus \underset{\substack{\Uparrow \\ \text{Ignored}}}{(qq)(\bar{q}\bar{q})} \oplus \{[qq](\bar{q}\bar{q}) \oplus (qq)[\bar{q}\bar{q}]\}, \qquad q = u, d, s, (\text{and } c)$$

- **Two different color configurations :** $\bar{3}_c \times 3_c$ **and** $6_c \times \bar{6}_c$

$$\text{Forces between two quarks:} \begin{cases} * \text{ Attractive when they are of } \bar{3}_c \\ * \text{ Repulsive when they are of } 6_c \end{cases}$$

Han and Nambu, PR 139, B(1006 (1965); S. Hori, PTP 36, 131 (1966)

$\Rightarrow m_{\bar{3}_c \times 3_c} < m_{6_c \times \bar{6}_c}$

- **When** $\bar{3}_c \times 3_c$ **is taken as the lower lying state (heavy mesons)**

	$[qq][\bar{q}\bar{q}]$	$(qq)(\bar{q}\bar{q})$	$\{[qq](\bar{q}\bar{q}) \oplus (qq)[\bar{q}\bar{q}]\}$	
J^P:	0^+	$0^+, 1^+, 2^+$	1^+	$\Leftarrow$ Dominant ($\bar{3}_c \times 3_c$)
	Scalar	(Ignored)	Axial-vector	
	$0^+, 1^+, 2^+$	0^+	1^+	$\Leftarrow$ Minor ($6_c \times \bar{6}_c$)

Typical candidates of tetra-quark mesons

$\underline{D_{s0}^{+}(2317)}$: $J^P = 0^+$

- Observed in $D_s^+\pi^0$ but no signal in $D_s^{*+}\gamma$ in inclusive e^+e^- annihilation

Babar, PRL 90, 242001 (2003); CLEO, PRD 68, 032002 (2003)

– $R(D_{s0}^{+}(2317))|_{\rm CLEO} = \left.\dfrac{\Gamma(D_{s0}^{+}(2317) \to D_s^{*+}\gamma)}{\Gamma(D_{s0}^{+}(2317) \to D_s^{+}\pi^0)}\right|_{\rm CLEO} < 0.059$

– $\Gamma_{D_{s0}(2317)} < 3.8$ MeV PDG08

◇ Hierarchy of hadron interactions:

Isospin conserving ≫ electromagnetic ≫ isospin non-conserving
$(\sim O(1))$ $(\sim O(\sqrt{\alpha}))$ $(\sim O(\alpha))$

⇒ $D_{s0}^{+}(2317)$ should be an iso-triplet charm-strange scalar state.

⇓ K.T., PRD 68, 011501(R) (2003)

$$\hat{F}_I^+ \sim [cn][\bar{s}\bar{n}]_{I=1}$$

◇ Narrow width ⇐ Small overlap of color and spin wave functions
(Discussed in Appendix 2)

◇ Iso-singlet partner: $\Gamma(\hat{F}_0^+ \to D_s^{*+}\gamma) \gg \Gamma(\hat{F}_0^+ \to D_s^+\pi^0)$

◇ Conventional $\{c\bar{s}\}$: $\Gamma(D_{s0}^{*+} \to D_s^{*+}\gamma) \gg \Gamma(D_{s0}^{*+} \to D_s^+\pi^0)$

c.f. $\Gamma(D_s^{*+} \to D_s^+\gamma)_{\rm exp} \gg \Gamma(D_s^{*+} \to D_s^+\pi^0)_{\rm exp}$

(Discussed in Appendix 3)

Hayashigaki and K. T., PTP 114, 1191 (2005); hep-ph/0410393

- **In B decays :** $D_{s0}^{+}(2317) = \tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^{0}] \neq \tilde{D}_{s0}^{+}(2317)[D_s^{*+}\gamma]$
  - **Belle** PRL <u>91</u>, 262002(2003)

$$\begin{cases} Br(B \to \bar{D}\tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^{0}]) = (8.5^{+2.1}_{-1.9} \pm 2.6) \times 10^{-4} \\ \boxed{Br(B \to \bar{D}\tilde{D}_{s0}^{+}(2317)[D_s^{*+}\gamma]) \ = (2.5^{+2.0}_{-1.8}(< 7.5)) \times 10^{-4}} \end{cases}$$

  - **Babar** Acta Phys. Polon. B36, 2315(2005)

$$\begin{cases} Br(B_u^{+} \to \bar{D}^{0}\tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^{0}])_{\rm Babar} = (1.0 \pm 0.3 \pm 0.1^{+0.4}_{-0.2}) \times 10^{-3} \\ Br(B_d^{0} \to D^{-}\tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^{0}])_{\rm Babar} = (1.8 \pm 0.4 \pm 0.3^{+0.6}_{-0.4}) \ \times 10^{-3} \end{cases}$$

$$\Updownarrow$$

$$\heartsuit \begin{cases} D_{s0}^{+}(2317) = \tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^{0}] & \Rightarrow \quad [cn][\bar{s}\bar{n}]_{I=1} \sim \hat{F}_I^{+} \\ \tilde{D}_{s0}^{+}(2317)[D_s^{*+}\gamma] & \Rightarrow \quad [cn][\bar{s}\bar{n}]_{I=0} \sim \hat{F}_0^{+} \end{cases}$$

Note on $D_{s0}^{+}(2317)$:

∗ **Rates for $\tilde{D}_{s0}^{+}(2317)[D_s^{+}\pi^{0}]$ and $\tilde{D}_{s0}^{+}(2317)[D_s^{*+}\gamma]$ productions in B decays are not very much different from each other.**

∗ **$\tilde{D}_{s0}^{+}(2317)[D_s^{*+}\gamma] = \hat{F}_0^{+}$ production is suppressed in $e^{+}e^{-}$ annihilation.**

(Discussed in §4.)

$\underline{X(3872)}$: $J^{PC} = 1^{++}$

- $X(3872) \to \pi^+\pi^- J/\psi$
  - Mass of $X(3872)$: $m_{X(3872)} = 3871.56 \pm 0.22$ MeV PDG10
  - Width of $X(3872)$: $\Gamma_{X(3872)} < 2.3$ MeV PDG10
  - Large isospin non-conservation:
    $X(3872) \to \rho^0 J/\psi \to \pi^+\pi^- J/\psi$

    Belle, hep-ex/0505037;
    CDF, PRL <u>96</u>, 102002 (2006)

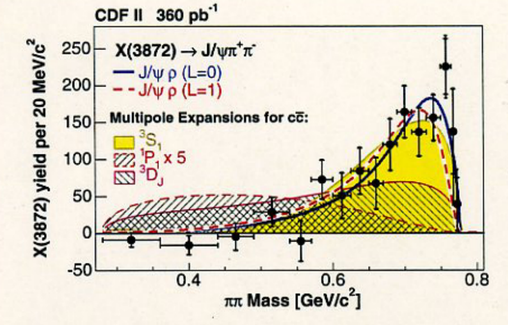


FIG. 3 (color online). The dipion mass spectrum for the $X(3872)$ and fits to various hypotheses (see text). The fitted curve for the $^1P_1$ model is scaled up by a factor of 5 for better visibility.

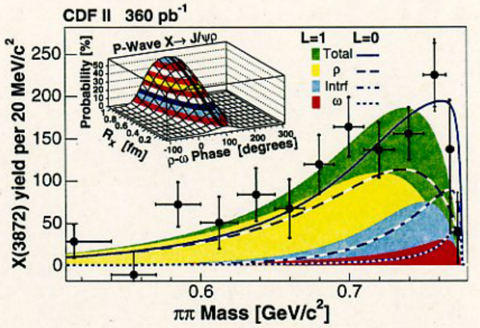


FIG. 4 (color online). A blowup of the $X(3872)$ spectrum with $J/\psi\rho$ fits which include $\rho - \omega$ interference (95° phase) with relative amplitudes set by $\mathcal{R}_{3/2} = 1.0$. Fits for both $L = 0$ (lines) and 1 (shaded regions) are shown, along with their decomposition into ρ, ω, and interference terms. The inset shows $L = 1$ fit probabilities as a function of ϕ and R_X in 5% contours.

  - No signal of $X(3872)^{\pm} \Rightarrow I = 0$ Babar, PRD <u>71</u>, 031501 (2005)
  - $X(3872) \to \omega J/\psi \to \pi^+\pi^-\pi^0 J/\psi$

    Belle, hep-ex/0505037
    Babar, hep-ex/1005.5190
  - $$\frac{\Gamma(X(3872) \to \pi^+\pi^- J/\psi)}{\Gamma(X(3872) \to \pi^+\pi^-\pi^0 J/\psi)} = 1.0 \pm 0.4 \pm 0.3$$ Belle, hep-ex/0505037

- Isospin symmetry in production of $X(3872)$: Belle, hep-ex/0809.1224
  - $\left.\dfrac{Br(B^0 \to X(3872)K^0)}{Br(B^+ \to X(3872)K^+)}\right|_{\text{Belle}} = 0.82 \pm 0.22 \pm 0.05$

    × Contradict to the $D^0\bar{D}^{*0}$ molecule Törnqvist, hep-ph/0308277
  - Small mass difference between $X_d \sim [cd][\bar{c}\bar{d}]$ and $X_u \sim [cu][\bar{c}\bar{u}]$

    $(\Delta m_X)_{\text{Belle}} = (m_{X_d} - m_{X_u})_{\text{Belle}} = 0.18 \pm 0.89 \pm 0.26$ MeV

    × Contradict to diquark-antidiquark model

    $(\Delta m_X) = 7 \pm 2$ MeV Maiani et al., PRD <u>71</u>, 014028 (2005)
- Angular analysis $\Rightarrow J^P(X(3872)) = 1^+$ Belle, hep-ex/0505038
- Observation of radiative decay(s) $\Rightarrow C = +$
  - Ratio of decay rates

    $$R(\gamma J/\psi) \equiv \frac{Br(X(3872) \to \gamma J/\psi)}{Br(X(3872) \to \pi^+\pi^- J/\psi)} = \begin{cases} 0.14 \pm 0.05, & \text{Belle} \\ 0.33 \pm 0.12, & \text{Babar} \end{cases}$$

    Belle, hep-ex/0505037; Babar, hep-ex/0809.0042
  - Sizable mixing of $\chi'_{c1} = 2\,{}^3P_1\{c\bar{c}\}$ is questionable.

    $$\frac{Br(X(3872) \to \gamma\psi')}{Br(X(3872) \to \gamma J/\psi)} = \begin{cases} \text{No signal of } \gamma\psi' & \text{Belle, hep-ex/1010.2331} \\ 3.4 \pm 1.4 & \text{Babar, hep-ex/0809.0042} \end{cases}$$

- The meausred production cross section of prompt $X(3872)$

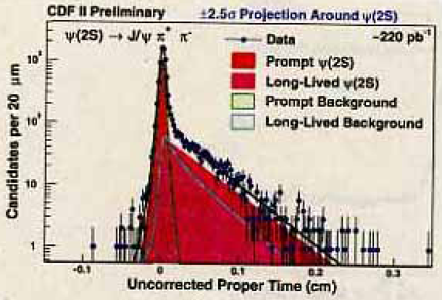


FIG. 3: The uncorrected proper-time projection of the full $\psi(2S)$ likelihood probability density function over the restricted mass range ±2.5 standard deviations around the $\psi(2S)$ resonance.

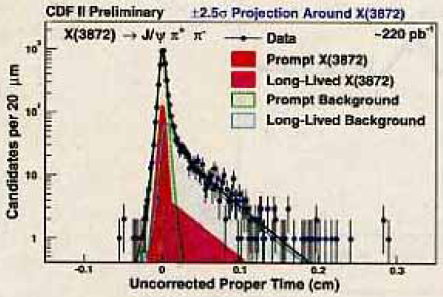


FIG. 6: The uncorrected proper-time projection of the full $X(3872)$ likelihood PDF over the restricted mass range ±2.5 standard deviations around the $X(3872)$.

CDF II Collaboration,
CDF note 7159 (2004);
URL http://www-cdf.fnal.gov

Notes:

- The observed isospin non-conservation in the $X(3872) \to \pi^+\pi^- J/\psi$ decay can be understood by the $\omega\rho^0$ mixing (with $|g_{\omega\rho}| \simeq 3 \times 10^{-3}$ GeV^2)
  - ∗ well-known as the origin of isospin non-conservation in nuclear forces
    Miller et al., Ann. Rev. Nucl. Part. Sci. 56, 253 (2006)
  - ∗ enhanced in the decay, because $\left|m_\omega^2 - m_\rho^2\right| \ll \left|m_\omega^2\right|$
- Measured values of $R(\gamma J/\psi)$ favor a tetra-quark over a charmonium.
  (Appendix 6) K. T., PTP 122, 1285 (2009); hep-ph/0904.3368
- The measured cross section for prompt $X(3872)$ favors
  a compact object (like a tetra-quark meson) over
  an extended object (like a loosely bound molecule).
  Bignamini et al., hep-ph/0906.0882

$X(3875)$

- $B \to \{D^0 \bar{D}^0 \pi^0\} K$

  Mass: $m^{\text{Belle}}_{D^0 \bar{D}^0 \pi^0} = 3875.4 \pm 0.7^{+1.2}_{-2.0}$ MeV

  Width: $\Gamma^{\text{Belle}}_{D^0 \bar{D}^0 \pi^0} = 3.0^{+2.1}_{-1.7}$ MeV $(\gtrsim \Gamma_{X(3872)})$

  Belle, hep-ex/0606055

- $B \to \{D^0 \bar{D}^{*0} (+\bar{D}^0 D^{*0})\} K$

  Mass: $m^{\text{Belle}}_{D^0 \bar{D}^{*0}} = 3872.9^{+0.6+0.4}_{-0.4-0.5}$ MeV

  Width: $\Gamma^{\text{Belle}}_{D^0 \bar{D}^{*0}} = 3.9^{+2.5+0.8}_{-1.3-0.3}$ MeV $(\gtrsim \Gamma_{X(3872)})$

  Belle, hep-ex/0810.0358v3

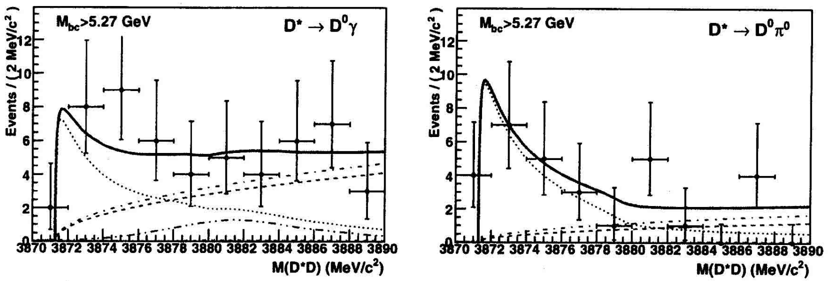


FIG. 4: Distribution of M_{D^*D} for $M_{\text{bc}} > 5.27\,\text{GeV}$, for $D^{*0} \to D^0\gamma$ (left) and $D^{*0} \to D^0\pi^0$ (right). The points with error bars are data, the dotted curve is the Flatté distribution, the dashed curve is the background, the dash-dotted curve is the sum of the background and the $B \to D^*DK$ component, the dot-dot-dashed curve is the contribution from D^0–$\bar{D}^0$ reflections, and the solid curve is the total fitting function.

- $B \to \{D^0 \bar{D}^{*0} (+\bar{D}^0 D^{*0})\} K$

  Mass: $m^{\text{Babar}}_{D^0 \bar{D}^{*0}} = 3875.1^{+0.7}_{-0.5} \pm 0.5$ MeV

  Width: $\Gamma^{\text{Babar}}_{D^0 \bar{D}^{*0}} = 3.0^{+1.9}_{-1.4} \pm 0.9$ MeV $(\gtrsim \Gamma_{X(3872)})$

  Babar, PRD 77, 011102(R) (2008)

$$\Rightarrow \boxed{\begin{array}{l} X(3875) = X_- \sim \{[cn](\bar{c}\bar{n}) - (cn)[\bar{c}\bar{n}]\} \\ X(3872) = X_+ \sim \{[cn](\bar{c}\bar{n}) + (cn)[\bar{c}\bar{n}]\} \end{array}}$$

- **Why no peak around 3875 MeV in the $\pi\pi J/\psi$ mass distribution ?**
  - **Contribution of non-resonant $(\pi\pi)_{\rm non-res}$ is much smaller than resonant contribution $(\pi\pi)_{\rm res}$ as seen in $X(3872) \to \pi^+\pi^- J/\psi$**

    CDF, PRL <u>96</u>, 102002 (2006)
  - **$f_0(600) = \sigma$ is extremely broad ($600 - 1000$ MeV)** **PDG10**

    $\Rightarrow$ Not very easy to find $X(3875) \to (\pi\pi)_\sigma J/\psi$
  - **$X(3875) \to f_0(1370)$(or $f_0(1500),\ f_0(1710))J/\psi$ is suppressed because their thresholds are far beyond $m_{X(3875)}$**
- **Search for $X(3875) \to \eta J/\psi$ to confirm $X(3875) = X_-$**

§2. Tetra-Quark Mesons with Exotic Quantum Numbers

- **Tetra-quark interpretation of $D_{s0}^{+}(2317)$ and $X(3872)$ has been favored by experiments, although their quantum numbers are not exotic.**
  ⇒ Observation of their partners with exotic quantum numbers is needed to establish existence of tetra-quark mesons.
- **Tetra-quark mesons with exotic quantum numbers hep-ph/1008.2992v2**
  - **Scalar meson with $C = -S = 1$:**
    - $I = 0$: $\hat{E}^0 \sim [cs][\bar{u}\bar{d}]$
  - **Axial-vector mesons with $C = 2$:**
    - $I = 0$: $H_A^+ \sim (cc)[\bar{u}\bar{d}]$
    - $I = 1/2$: $\left\{\begin{array}{l} K_{ccA}^{++} \sim (cc)[\bar{d}\bar{s}] \\ K_{ccA}^{+} \sim (cc)[\bar{u}\bar{s}] \end{array}\right.$
  - **Axial-vector mesons with $C = -S = 1$:**
    - $I = 0$: $E_{(cs)A}^0 \sim (cs)[\bar{u}\bar{d}]$
    - $I = 1$: $\left\{\begin{array}{l} E_{[cs]A}^{+} \sim [cs](\bar{d}\bar{d}) \\ E_{[cs]A}^{0} \sim [cs](\bar{u}\bar{d}) \\ E_{[cs]A}^{-} \sim [cs](\bar{u}\bar{u}) \end{array}\right.$

$$\{qq\bar{q}\bar{q}\} = \underbrace{[qq][\bar{q}\bar{q}]}_{\text{Scalar}} \oplus (qq)(\bar{q}\bar{q}) \oplus \underbrace{\{[qq](\bar{q}\bar{q}) \oplus (qq)[\bar{q}\bar{q}]\}}_{\text{axial-vector}}, \qquad (q = u, d, s, \{\text{and } c\})$$

Scalar and axial-vector tetra-quark mesons with exotic quantum numbers, their crudely estimated masses, and possible decay modes. Mass values are estimated by using a quark counting with $\Delta_{cs} = m_c - m_s \simeq 1.0$ GeV and $\Delta_{sn} = m_s - m_n \simeq 0.1$ GeV.

Tetra-quark meson	Mass$^{(*)}$ (GeV)	Possible decay mode			
		2-body	Threshold	3-body	Radiative (or Weak)
$\hat{E}^0$	2.32	$\langle D\bar{K}\rangle$	2.36	—	$(\bar{K}\pi)\bar{K}$ $(\bar{K}\pi\pi)\bar{K}$ $\cdots$
H^+_{Acc}	3.87	$\langle DD^*\rangle$?	3.88	$\langle DD\pi\rangle$?	$DD\gamma$
K_{Acc}	3.97	$\langle DD_s^{*+}\rangle$? $\langle D^*D_s^+\rangle$?	3.98	— $\langle DD_s\pi\rangle$?	$DD_s\gamma$ —
$E_{A(sc)}$ $E_{A[sc]}$	2.97	$\bar{K}D^*$ $\bar{K}^*D$	2.61 2.86	$\bar{K}D\pi$	$\bar{K}D\gamma$

(*) Input data:

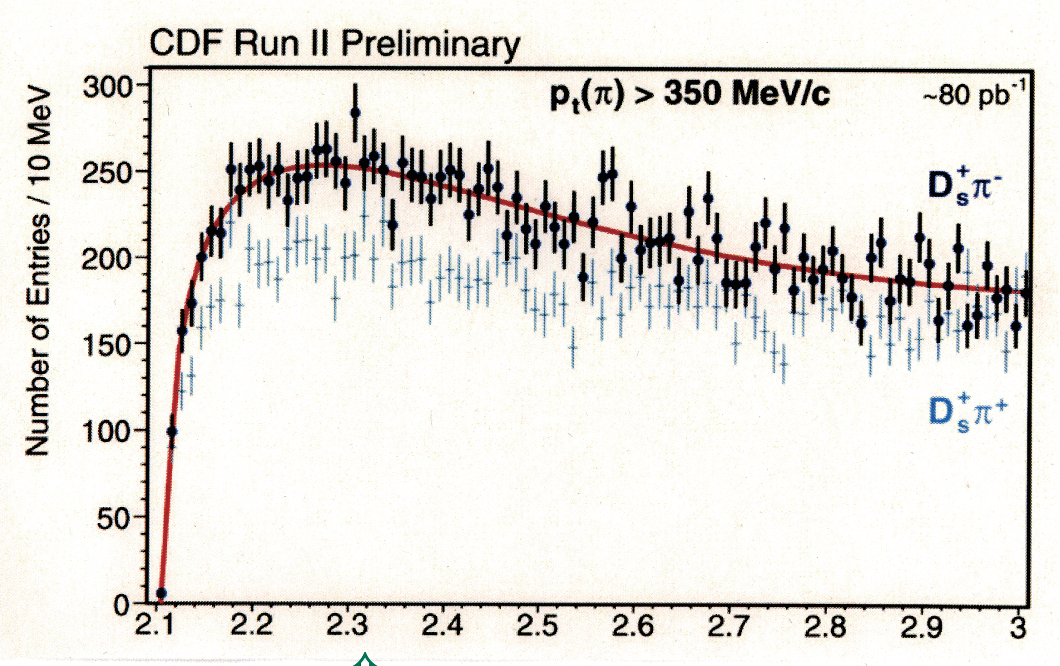
$m_{D_{s0}(2317)} = 2317.8 \pm 0.6$ MeV and $m_{X(3872)} = 3872.2 \pm 0.8$ MeV

§3. Existing Searches for Tetra-Quark Mesons

Assigning $D_{s0}^{+}(2317)$ to the iso-triplet $\hat{F}_I^{+} \sim [cn][\bar{s}\bar{n}]_{I=1}$ is favored.

$\Rightarrow$ Existence of $\hat{F}_I^{++}$, $\hat{F}_I^{0}$, $\hat{F}_0^{+}$, $\hat{D}_0$, $\cdots$ is expected.

- Existing searches for $D_{s0}^{++}(2317) = \hat{F}_I^{++}$ and $D_{s0}^{0}(2317) = \hat{F}_I^{0}$
  - Search for $\hat{F}_I^{0}$ and $\hat{F}_I^{++}$
    - $D_s^{+}\pi^{\pm}$ spectra at CDF ⇐ No clear signal



$\Uparrow$

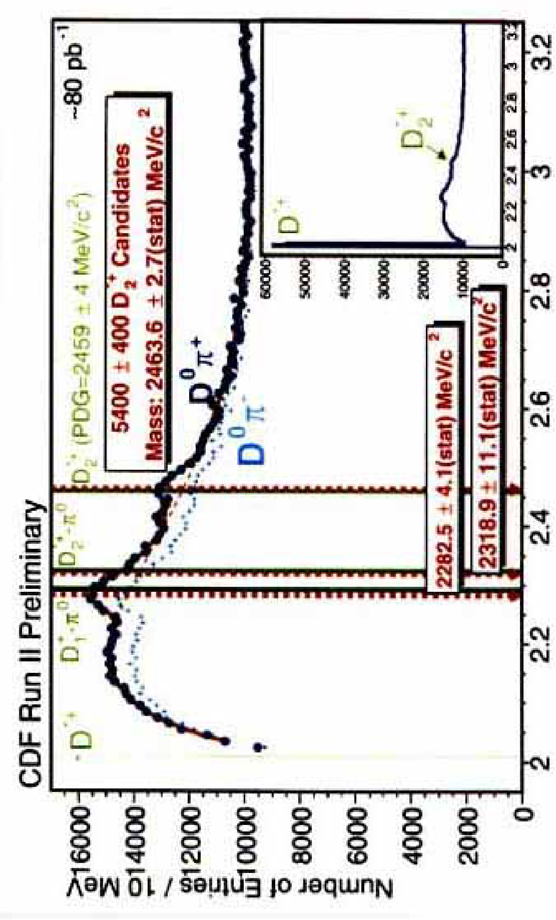
$\hat{F}_I^{+}$

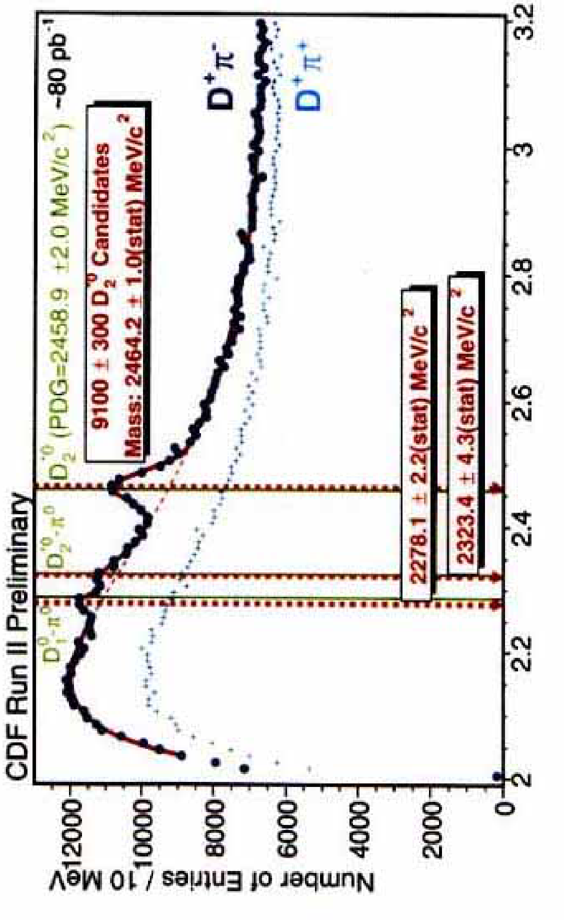
M. Shapiro,
FPCP conf.,
Paris, 2003

$$\Rightarrow \begin{cases} \text{Production mechanism ?} \\ \text{Re-analyze more precisely the } D_s^{+}\pi^{\pm} \text{ spectra around } 2.3 \text{ GeV} \end{cases}$$

– **Search for $\hat{D}_0 \sim [cn][\bar{u}\bar{d}]$ (and the conventional $D_0^* \sim \{c\bar{n}\}$)**

  - **$D\pi$ mass spectra at CDF M. Shapiro, FPCP conf., Paris, 2003**
    (False peaks between 2.2 GeV and 2.4 GeV)





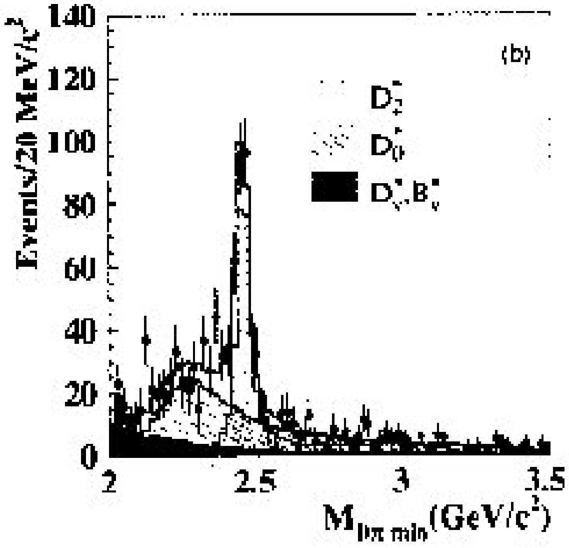
⇐ D_0^*

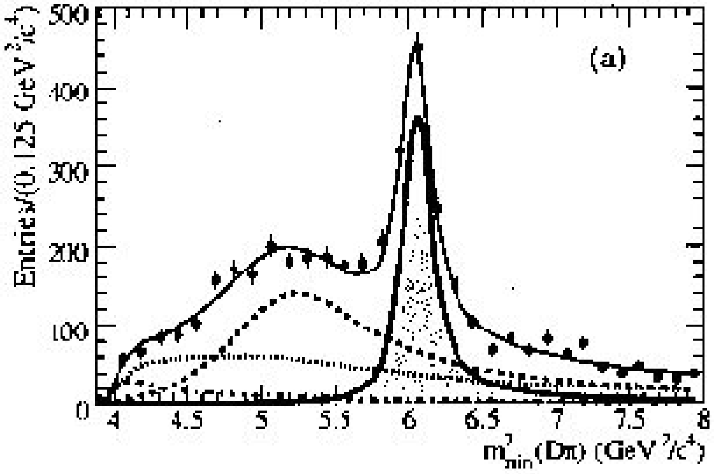
⇐ $\hat{D}_0$

⇒ **Re-analyze the spectra between 2.0 – 2.4 GeV,**
and find $\hat{D}_0$ and D_0^* behind the false peaks.

∗ $D\pi$ mass spectra at Belle and Babar

Belle, PRD 69, 112002 (2004)   Babar, hep-ex/0901.1291v2





⇑ ⇑
$\hat{D}$ D_0^*

⇑ ⇑
$\hat{D}$ D_0^*

- ∗ Re-analyze the above $D\pi$ enhancement, and
- ∗ Find a structure of the broad enhancement containing a narrow peak ($\hat{D}$) and a broad (D_0^*)   (Discussed in Appendix 4)

$*$ DK mass spectra at CLEO

D_{s0}^{*+} around 2.4 GeV

$\Downarrow$

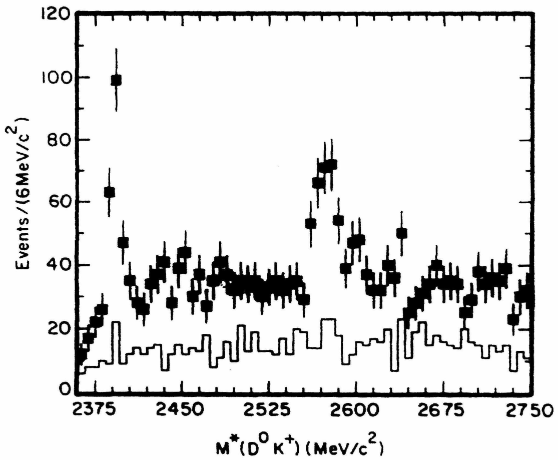


FIG. 1. M^*, "corrected" invariant mass, of $(K^-\pi^+[\pi^0])K^+$ combinations. Data points are for $K^-\pi^+[\pi^0]$ combinations in the D^0 signal region; the histogram shows M^* for $(K^-\pi^+[\pi^0])K^+$ combinations where the $K^-\pi^+[\pi^0]$ combinations were chosen in D^0 sidebands.

CLEO,
Phys. Rev. Lett.
<u>72</u>, 1972 (1994)

Re-analyze more precisely the spectrum, and find D_{s0}^{*+} behind the false peak at 2.39 GeV.

– **$D_s^+\pi^\pm$ spectra in <u>inclusive</u> e^+e^- annihilation:**

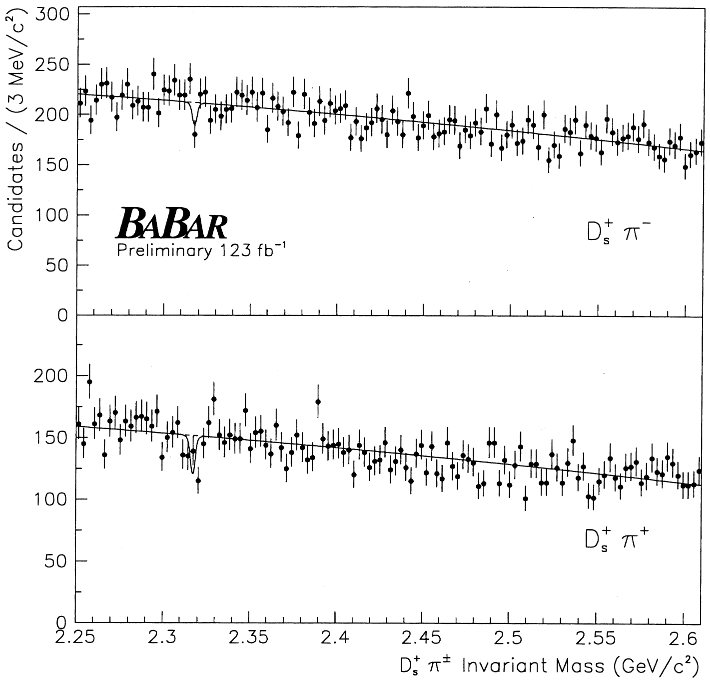


Figure 10: The $D_s^+\pi^-$ (top) and $D_s^+\pi^+$ (bottom) invariant mass distributions for candidates that satisfy the requirements discussed in the text. The solid curve is the result of a unbinned likelihood fit.

No signal of $\hat{F}_I^0$ and $\hat{F}_I^{++}$ **Babar, PRD <u>74</u>, 032007 (2006)**

$\Downarrow$

Suppression of $\hat{F}_I^0$ and $\hat{F}_I^{++}$ production in e^+e^- annihilation

§4. Productions of Tetra-Quark Mesons

- Productions of tetra-quark mesons within the minimal $q\bar{q}$ pair creation
  - e^+e^- annihilation:
    - ∗ Hierarchy of charm-strange meson production:

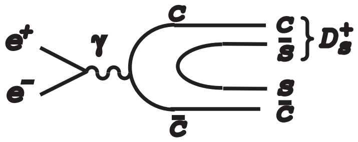
Babar, PRD 74, 032007 (2006) hep-ex/0604031

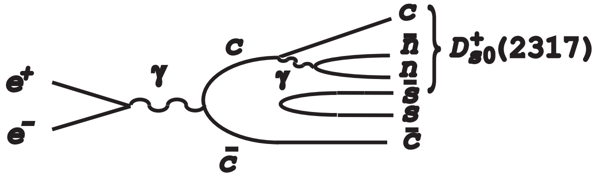
$N_{c\bar{c}}(D_s^+)$ > $N_{c\bar{c}}(D_{s0}^+(2317))$ > $N_{\Upsilon(4S)}(D_{s0}^+(2317))$

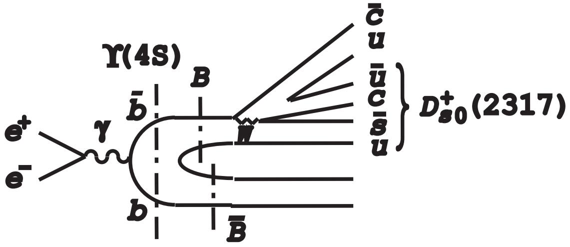
$\sim 10^7$ for $232 fb^{-1}$ near 10.6 GeV

$\sim 10^6$ for $232 fb^{-1}$ near 10.6 GeV

$\sim 10^5$ for $211 fb^{-1}$ on $\Upsilon(4S)$







⇓

Productions of $\hat{F}_I^0$ and $\hat{F}_I^{++}$ are suppressed compared with $D_{s0}^+(2317) = \hat{F}_I^+$.

Notes:

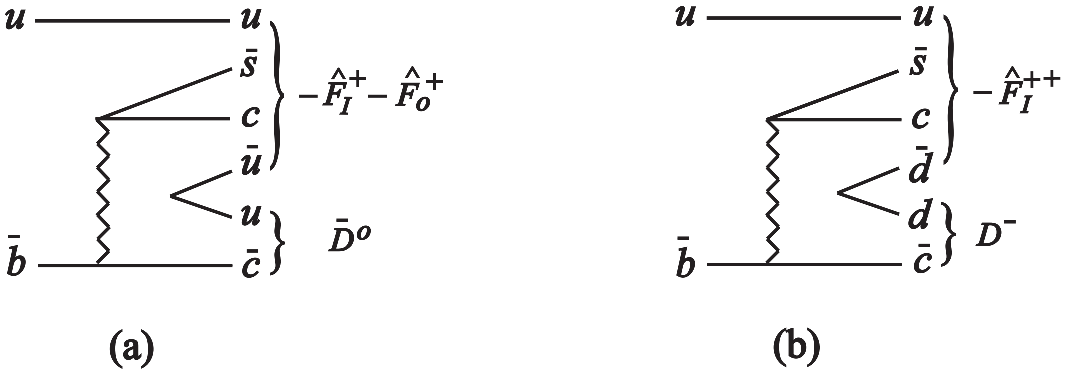
- ∗ Suppression of $\hat{F}_0^+$ production $\left(\frac{X_\omega}{X_\rho} \sim \frac{1}{10}\right.$ of $\hat{F}_I^+$ production $\left.\right)$

  K.T., AIP Conf. Proc. 1030, 190 (2008); hep-ph/0804.2295
- ∗ Production of $\hat{F}_I^{++}$ on $\Upsilon(4S)$ might be of the order of $N_{\Upsilon(4S)}(D_{s0}^+(2317))$ ⇐ Large enough to observe ?

- **Productions of $\hat{F}_I^{++}$, $\hat{F}_I^{+}$, $\hat{F}_I^{0}$ and $\hat{F}_0^{+}$ in B decays**

K. T., PTP 116, 435 (2006); EPJA 31, 676 (2007)

  - In $B_u^+ \to \bar{D}\hat{F}_I$ and $\bar{D}^0\hat{F}_0^+$ decays :



(a) (b)

$\hat{F}_I^{++} \sim [cu][\bar{s}\bar{d}]$

$\hat{F}_I^{+} \sim [cn][\bar{s}\bar{n}]_{I=1}$

$\hat{F}_0^{+} \sim [cn][\bar{s}\bar{n}]_{I=0}$

Fig.1: Production of $\hat{F}_0^+$, $\hat{F}_I^+$ and $\hat{F}_I^{++}$ in B_u^+ decays

(a) Production of $\hat{F}_I^+$ and $\hat{F}_0^+$ with $\bar{D}^0$

$$* \begin{cases} Br(B_u^+ \to \bar{D}^0\tilde{D}_{s0}^+(2317)[D_s^+\pi^0])_{\text{Belle}} = (8.1^{+3.0}_{-2.7} \pm 2.4) \times 10^{-4} \\ Br(B_u^+ \to \bar{D}^0\tilde{D}_{s0}^+(2317)[D_s^{*+}\gamma])_{\text{Belle}} = (2.5^{+2.1}_{-1.6}(< 7.6)) \times 10^{-4} \end{cases}$$

$$* \quad Br(B_u^+ \to \bar{D}^0\tilde{D}_{s0}^+(2317)[D_s^+\pi^0])_{\text{Babar}} = (1.0 \pm 0.3 \pm 0.1^{+0.4}_{-0.2}) \times 10^{-3}$$

(b) Production of $\hat{F}_I^{++}$ with D^-

$$* \boxed{\begin{array}{l} Br(B_u^+ \to D^-\hat{F}^{++}) \sim Br(B_u^+ \to \bar{D}^0\hat{F}_0^+) \\ \sim Br(B_u^+ \to \bar{D}^0\hat{F}_I^+) \sim 10^{-(4-3)} \end{array}} \text{ (same type of diagrams)}$$

where $\hat{F}_I^+ = \tilde{D}_{s0}^+(2317)[D_s^{*+}\gamma]) = D_{s0}^+(2317)$

– **In $B_d^0 \to \bar{D}\hat{F}_I$ and $\bar{D}\hat{F}_0^+$ decays :**

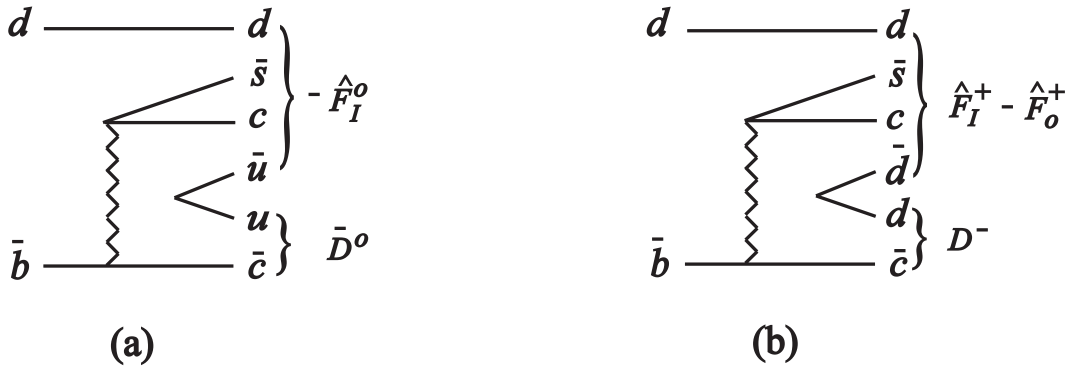


Fig. 2: Production of $\hat{F}_I^0$, $\hat{F}_I^+$ and $\hat{F}_0^+$ in B_d^0 decay

(a) Production of $\hat{F}_I^0$ with $\bar{D}^0$

$$* \quad \boxed{Br(B_d^0 \to \bar{D}^0\hat{F}_I^0) \sim Br(B_d^0 \to D^-\hat{F}_I^+)_{\text{exp}} \sim 10^{-(4-3)}}$$

(Same type of diagrams)

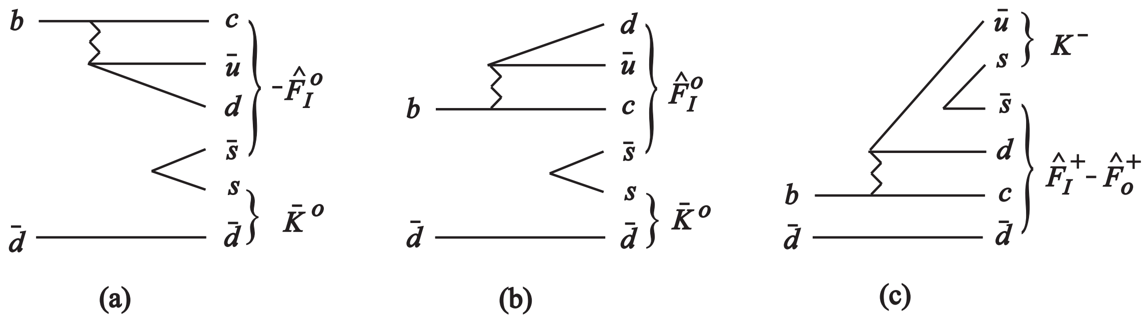
(b) Production of $\hat{F}_I^+$ and $\hat{F}_0^+$ with D^-

$$* \begin{cases} Br(B_d^0 \to D^-\tilde{D}_{s0}^+(2317)[D_s^+\pi^0])_{\text{Babar}} = (1.8 \pm 0.4 \pm 0.3^{+0.6}_{-0.4}) \times 10^{-3} \\ Br(B_d^0 \to D^-\tilde{D}_{s0}^+(2317)[D_s^+\pi^0])_{\text{Belle}} = (8.6^{+3.3}_{-2.6} \pm 2.6) \times 10^{-4} \end{cases}$$

$$* \quad Br(B_d^0 \to D^-\tilde{D}_{s0}^+(2317)[D_s^{*+}\gamma])_{\text{Belle}} = (2.7^{+2.9}_{-2.2}(< 9.5)) \times 10^{-4}$$

Note: $Br(B_d^0 \to \bar{D}^0\hat{F}_I^0) \sim Br(B_u^+ \to D^-\hat{F}^{++}) \sim 10^{-(4-3)}$ **is large enough to observe $D_{s0}^{++} = \hat{F}_I^{++}$ and $D_{s0}^0 = \hat{F}_I^0$ in B decays.**

- **In $\bar{B}_d^0 \to \bar{K}\hat{F}_I$ and $K^-\hat{F}_0^+$ decays :**



(a) (b) (c)

Fig. 3: Production of $\hat{F}_I$ and $\hat{F}_0^+$ with $\bar{K}$ in $\bar{B}_d^0$ decay

$\bar{B}_d^0 \to K^-\hat{F}_I^+$ decay is given by (c) and the measured rate:

$$Br(\bar{B}_d^0 \to K^- D_{s0}^+(2317)) \cdot Br(D_{s0}^+(2317) \to D_s^+\pi^0) = (5.3^{+1.5}_{-1.3} \pm 0.7 \pm 1.4) \times 10^{-5}$$

Belle

Because $Br(D_{s0}^+(2317) \to D_s^+\pi^0) \simeq 1$ from $R(D_{s0}^+(2317))_{\mathrm{CLEO}} < 0.059$

$\Rightarrow \boxed{Br(\bar{B}_d^0 \to K^-\hat{F}_0^+) \sim Br(\bar{B}_d^0 \to K^-\hat{F}_I^+) \sim 10^{-(5-4)}}$ with $D_{s0}^+(2317) = \hat{F}_I^+$

Suppressed because of an $s\bar{s}$ pair creation

Note: $s\bar{s}$ component is much smaller than $n\bar{n}$ in the sea of nucleon.

Phenomenology: Bernard, PPNP 60, 82 (2008)
Lattice QCD: JLQCD, PRD 78, 05402 (2008)

$$y = \frac{\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} = \begin{cases} 0.07 \lesssim y \lesssim 0.22 & \text{Phenomenology} \\ \lesssim 0.030(16)_{\mathrm{stat}}(^{+6}_{-8})_{\mathrm{extrap}}(^{+1}_{-2})_{m_s} & \text{Semi-quench} \end{cases}$$

- **In $\bar{B}_u^- \to K^- \hat{F}_I^0$ decay :**

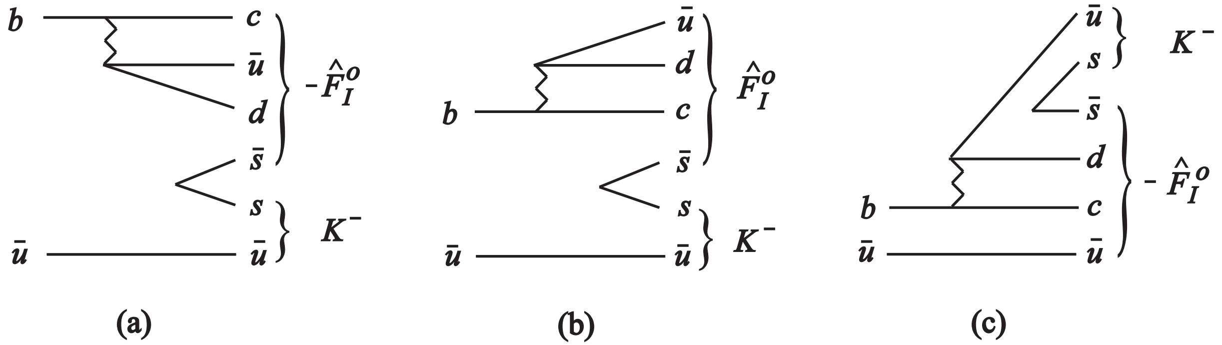


Fig. 4: Production of $\hat{F}_I^0$ with K^- in B_u^- decay

Destructive contribution betweeb (a) and (b)

$\Rightarrow$ Dominant contribution to the $\bar{B}_u^- \to K^- \hat{F}_I^0$ decay is given by (c) which is of the same type as Fig. 3(c).

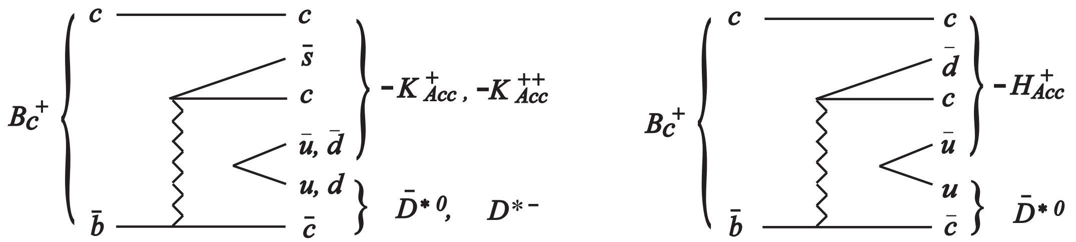
$\Rightarrow$ $Br(\bar{B}_u^- \to K^- \hat{F}_I^0) \sim Br(\bar{B}_d^0 \to K^- \hat{F}_I^+) \sim 10^{-(5-4)}$

(Suppressed because of an $s\bar{s}$ creation)

Productions of tetra-quark mesons with exotic quantum numbers

K. T., PTP 125, 199 (2011); hep-ph/1008.2992v3

- Productions of tetra-quark mesons with $C = 2$



(a) (b)

$K_{Acc} \sim (cc)[\bar{n}\bar{s}]$

$H_{Acc}^{+} \sim (cc)[\bar{u}\bar{d}]$

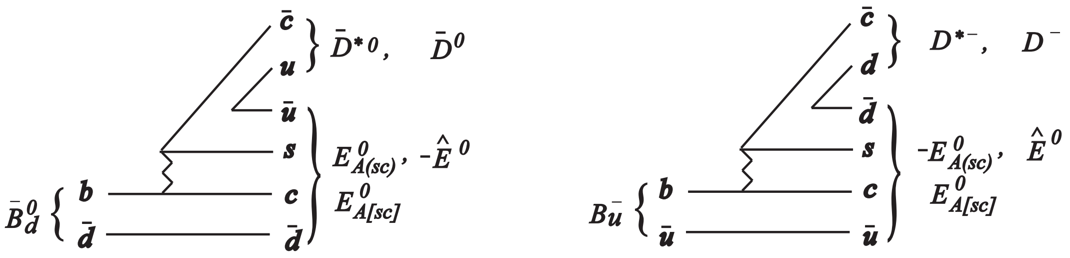
Fig. 5: Production of axial-vector K_{Acc} and H_{Acc}^{+}

Figs. 5(a) and (b) are of the same type as Fig. 1(a) and Fig. 2(b)

$$\Rightarrow \boxed{\begin{aligned} & Br(B_c^+ \to \bar{D}^0 K_{Acc}^{+}) \sim Br(B_c^+ \to D^- K_{Acc}^{++}) \\ & \sim |V_{cs}/V_{cd}|^2 Br(B_c \to \bar{D}^0 H_{Acc}^{+}) \\ & \sim Br(B_u^+ \to \bar{D}^0 \hat{F}_I^{+}) \sim Br(B_d^0 \to \bar{D}^- \hat{F}_I^{+}) \sim 10^{-(4-3)} \end{aligned}}$$

The H_{Acc}^{+} production is the CKM suppressed.

- **Production of tetra-quark mesons with $C = -S = 1$**



(a) (b)

Fig. 6: Production of scalar $\hat{E}^0$, and axial-vector $E^0_{A(cs)}$ with $I = 0$ and $E^0_{A[cs]}$ with $I = 1$

Figs. 6(a) and (b) are of the same type as Fig. 3(c).

$$\Rightarrow \left\{ \begin{array}{l} Br(B_u^- \to D^- E^0_{A(cs)}) \sim Br(B_u^- \to D^- E^0_{A[cs]}) \sim Br(B_u^- \to D^- \hat{E}^0) \\ \sim Br(\bar{B}^0_d \to \bar{D}^0 E^0_{A(cs)}) \sim Br(\bar{B}^0_d \to \bar{D}^0 E^0_{A[cs]}) \sim Br(\bar{B}^0_d \to \bar{D}^0 \hat{E}^0) \\ \sim \underset{\Uparrow}{\underline{10}} \times Br(\bar{B}^0_d \to K^- \hat{F}_I^+) \sim 10^{-(4-3)} \text{ with } \hat{F}_I^+ = D^+_{s0}(2317) \end{array} \right.$$

No suppression because of no $s\bar{s}$ pair creation

§5. Summary

- **Evidences for existence of tetra-quark mesons :**
  - Ratio of decay rates

$$\left.\frac{\Gamma(D_{s0}^{+}(2317) \to D_s^{*+}\gamma)}{\Gamma(D_{s0}^{+}(2317) \to D_s^{+}\pi^0)}\right|_{\mathrm{CLEO}} < 0.059$$

$\Rightarrow$ $D_{s0}^{+}(2317)$ should be assigned to $\hat{F}_I^{+} \sim [cn][\bar{s}\bar{n}]_{I=1}$.

$\Rightarrow$ Existence of $D_{s0}^{++}(2317) = \hat{F}_I^{++}$ and $D_{s0}^{++}(2317) = \hat{F}_I^{0}$

◇ $\left\{\begin{array}{l} * \text{ Suppression of } \hat{F}_I^{++} \text{ and } \hat{F}_I^{0} \text{ production in } e^+e^- \text{ annihilation} \\ * \ \hat{F}_I^{++} \text{ and } \hat{F}_I^{0} \text{ can be observed in } B \text{ decays} \end{array}\right.$

  - $X(3872) \to \pi^+\pi^- J/\psi$ can be understood by the $\omega\rho^0$ mixing.
  - $\dfrac{\Gamma(X(3872) \to \gamma J/\psi)}{\Gamma(X(3872) \to \pi^+\pi^- J/\psi)}$ favors a tetra-quark over a charmonium.
  - Sizable mixing of χ'_{c1} is questionable, because no signal of $X(3872) \to \gamma\psi'$ in the measurement by Belle in contrast with Babar.
  - Production of prompt $X(3872)$ favors a tetra-quark over a molecule.
  - Approximate degeneracy between $X(3872)$ and $X(3875)$:

$$\left\{\begin{array}{l} X(3872) = X_+ \sim \{[cn](\bar{c}\bar{n}) + (cn)[\bar{c}\bar{n}]\}_{I=0} \\ X(3875) = X_- \sim \{[cn](\bar{c}\bar{n}) - (cn)[\bar{c}\bar{n}]\}_{I=0} \end{array}\right.$$

◇ Search for $X(3875) \to \eta J/\psi$ to confirm $X(3875) = X_-$

- Narrow widths of (lower lying) open- and hidden-charm tetra-quark mesons (with $\bar{3}_c \times 3_c$) is understood by a small overlap of color and spin wavefunctions

- Hidden-charm scalar tetra-quark mesons :
  - A peak around 3.2 GeV in the $\eta\pi^0$ distribution observed by Belle

$\Downarrow$

A candidate of $\hat{\delta}^c \sim [cn][\bar{c}\bar{n}]_{I=1}$ around 3.3 GeV ($\Leftarrow m_c$ at 2 GeV scale ?)
(much lower than predictions by the other models)

- Open-charm scalar mesons:
  - Re-analyze the broad $D\pi$ enhancement just below the D_2^* peak

$$\Rightarrow \left\{ \begin{array}{l} \text{Find } D_0^* \sim \{c\bar{n}\} \text{ with } \Gamma(D_0^*) \sim 50 \text{ MeV} \\ \qquad\qquad \Rightarrow \text{Reject an artificially large } \Gamma(D_{s0}^{*+} \to D_s^+\pi^0) \\ \text{Find } \hat{D}_0 \sim [cn][\bar{u}\bar{d}] \text{ with } \Gamma(\hat{D}) \lesssim 10 \text{ MeV} \end{array} \right.$$

  - Search for $D_{s0}^{*+} \sim \{c\bar{s}\}$ in the DK channel around 2.4 GeV region:
    - Find D_{s0}^{*+} as a peak with a $\sim$ 40 MeV width behind the false peak

- **Production rates for tetra-quark mesons with exotic quantum numbers**

  - $*$ **Doubly charged and neutral partners of $\hat{F}_I^+ = D_{s0}^+(2317)$:**

    $$Br(B_u^+ \to D^- \hat{F}^{++}) \sim Br(B_d^0 \to \bar{D}^0 \hat{F}_I^0) \sim Br(B_d^0 \to D^- \hat{F}_I^+) \sim 10^{-(4-3)}$$

  - $*$ **$\hat{F}_I^+ = D_{s0}^+(2317)$ production with K^-:**

    $$Br(\bar{B}_d^0 \underset{\Uparrow}{\to} K^- \hat{F}_I^+) \sim 10^{-(5-4)} \sim \frac{1}{10} Br(\bar{B}_d^0 \underset{\Uparrow}{\to} D^- \hat{F}_I^+)$$

    (an $s\bar{s}$ pair creation) **(no $s\bar{s}$ pair creation)**

  - $*$ **Production of $C = 2$ mesons:**

    $$Br(B_c^+ \to \bar{D}^0 K_{Acc}^+) \sim Br(B_c^+ \to D^- K_{Acc}^{++}) \; (\Leftarrow \textbf{Large enough ?})$$

    $$\sim |V_{cs}/V_{cd}|^2 Br(B_c \to \bar{D}^0 H_{Acc}^+) \sim Br(B_u^+ \to \bar{D}^0 \hat{F}_I^+) \sim 10^{-(4-3)}$$

  - $*$ **Production of $C = -S = 1$ mesons:**

    $$Br(B_u^- \to D^- E_{A(cs)}^0) \sim Br(B_u^- \to D^- E_{A[cs]}^0) \sim Br(B_u^- \to D^- \hat{E}^0)$$

    $$\sim Br(\bar{B}_d^0 \to \bar{D}^0 E_{A(cs)}^0) \sim Br(\bar{B}_d^0 \to \bar{D}^0 E_{A[cs]}^0) \sim Br(\bar{B}_d^0 \to \bar{D}^0 \hat{E}^0)$$

    $$\sim Br(\bar{B}_d^0 \to D^- \hat{F}_I^+) \sim 10^{-(4-3)}$$

Conclusion:

Tetra-quark mesons with exotic quantum numbers can be observed in B decays, as $D_{s0}^+(2317)$ and $X(3872)$.

Appendix

1. Low Lying Tetra-Quark Mesons
2. Narrow Width of Heavy Mesons with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$
3. Decay Properties of Open-Charm Tetra-Quark Mesons
4. Conventional Charmed Scalar Mesons
5. Various Approaches to the I-spin Non-Conservation in $X(3872) \rightarrow \pi^+\pi^- J/\psi$ Decay
6. Isospin Non-Conservation through $\omega\rho^0$ Mixing

Appendix 1. Low Lying Tetra-Quark Mesons

- **Light scalar $[qq][\bar{q}\bar{q}]$ mesons:** Jaffe, PRD 15, 267 and 281 (1977)

	$S=1$	$S=0$	
$I=1$		$\hat{\delta}^s \sim [ns][\bar{n}\bar{s}]_{I=1}$	
$I=\frac{1}{2}$	$\hat{\kappa} \sim [ud][\bar{n}\bar{s}]$		
$I=0$		$\hat{\sigma}^s \sim [ns][\bar{n}\bar{s}]_{I=0}$	$\hat{\sigma} \sim [ud][\bar{u}\bar{d}]$
Mass (‡) (GeV)	0.90	1.10	0.65
Candidate	$\kappa(800)$	$a_0(980)$, $f_0(980)$	$f_0(600)$

(‡) MIT bag model with a large mixing between the $\bar{3}_c \times 3_c$ and $6_c \times \bar{6}_c$

⇒
- ♡ Mass hierarchy of the observed low-lying scalar mesons
- ♡ Approximate degeneracy between $a_0(980)$ and $f_0(980)$
- ♡ Broad widths of light scalar tetra-quark mesons because of a large mixing between the $\bar{3}_c \times 3_c$ and $6_c \times \bar{6}_c$ states (Non-perturbative property of QCD at a scale $\lesssim 1$ GeV)

- **Open-charm scalar $[cq][\bar{q}\bar{q}]$ mesons (with $\bar{3}_c \times 3_c$):**

K. T., PRD 68, 011501(R) (2003)

S	$I = 1$	$I = \frac{1}{2}$	$I = 0$	Mass (‡) (GeV)	Candidate
1	$\hat{F}_I \sim [cn][\bar{s}\bar{n}]_{I=1}$			2.32	$D_{s0}^+(2317) = \tilde{D}_{s0}^+(2317)[D_s^+\pi^0]$;
			$\hat{F}_0^+ \sim [cn][\bar{s}\bar{n}]_{I=0}$	2.32	$\tilde{D}_{s0}^+(2317)[D_s^{*+}\gamma]$
0	$\hat{D} \sim [cn][\bar{u}\bar{d}]$ $\hat{D}^s \sim [cs][\bar{n}\bar{s}]$		2.22 2.42		
-1			$\hat{E}^0 \sim [cs][\bar{u}\bar{d}]$	2.32	

(‡) Quark counting with $\begin{cases} \Delta_{sn} = m_s - m_n \simeq m_{D_s} - m_D \simeq 0.1 \text{ GeV} \\ m_{\hat{F}_I^+} = m_{D_{s0}} \simeq 2.32 \text{ GeV (input data)} \end{cases}$

Note: $\hat{E}^0$ is only one charmed scalar with an exotic set of C and S.

- **Hidden-charm scalar $[cq][\bar{c}\bar{q}]$ mesons (with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$):**

Strangeness (S)	1	0		
$I = 1$		$\hat{\delta}^c \sim [nc][\bar{n}\bar{c}]_{I=1}$		
$I = \frac{1}{2}$	$\hat{\kappa}^c \sim [nc][\bar{s}\bar{c}]$			
$I = 0$			$\hat{\sigma}^c \sim [nc][\bar{c}\bar{n}]_{I=0}$	$\hat{\sigma}^{sc} \sim [sc][\bar{c}\bar{s}]$
Mass (GeV) (‡)	$\sim$3.4	$\sim$3.3	$\sim$3.3	$\sim$3.5
OZI-allowed Decay	$\eta_c K$	$\eta_c \pi$	$\eta_c \eta$	$\eta_c \eta$
Threshold (GeV)	3.48	3.12	3.53	3.53

(‡) **Quark counting with $\Delta_{cs} \simeq 1$ GeV, $\Delta_{sn} \simeq 0.1$ GeV,**
$m_{\hat{F}_I^+} = m_{D_{s0}} = 2317.8 \pm 0.6$ GeV (Input data)

Notes:
- $*$ **A peak around 3.2 GeV in the $\eta\pi^0$ channel**
  Belle, PRD 80, 032001 (2009)
  ⇒ A candidate of $\hat{\delta}^c$
- $*$ **No state with exotic quantum numbers**

- Hidden-charm axial-vector $[cq](\bar{c}\bar{q}) \oplus (cq)[\bar{c}\bar{q}]$ mesons with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$

S	1	0			-1
$I = 1$		$C^c_{a_1(b_1)}(\pm)$			
$I = 1/2$	$C^c_{K_1}(\overline{60}_f)$ $C^c_{K_1}(60_f)$				$C^c_{\bar{K}_1}(\overline{60}_f)$ $C^c_{\bar{K}_1}(60_f)$
$I = 0$			$C^c_1(\pm)$	$C^{cs}_1(\pm)$	
Mass (*) (GeV)	~ 4.0	~ 3.9(‡)		~ 4.1	~ 4.0
Candidate	?	?	$C^c_1(\pm) = X_\pm = \left\{\begin{array}{l} X(3872) \\ X(3875) \end{array}\right.$	$C^{cs}_1(\pm) = \left\{\begin{array}{l} J/\psi\phi(4143) \\ ? \end{array}\right.$	?

(*) Quark counting, (‡) Input data

– Neutral non-strange states form always pairs with opposite G-parities:

$$\left.\begin{array}{l} C^c_1(\overline{60}_f) \sim \{[cn](\bar{c}\bar{n})\}_{I=0} \\ C^c_1(60_f) \sim \{(cn)[\bar{c}\bar{n}]\}_{I=0} \end{array}\right\rangle \text{ are not eigenstates of } G\text{-parity.}$$

$\Downarrow$ $C^c_1(60_f)$-$C^c_1(\overline{60}_f)$ mixing to form G-parity eigenstates

Approximately degenerate pair of $I = 0$ states with opposite G-parities

$$\left\{\begin{array}{l} C^c_1(+) \to J/\psi\pi\pi\pi,\ J/\psi\pi\pi \Rightarrow X(3872) \\ C^c_1(-) \to D^0\bar{D}^{*0},\ D^0\bar{D}^0\pi^0 \Rightarrow X(3875) \end{array}\right.$$

Appendix 2. Narrow Width of Heavy Mesons with $\bar{\mathbf{3}}_c \times \mathbf{3}_c$

- **Variety of color and spin configurations** in heavy tetra-quark mesons
  - Open-charm scalar:

    For example,

$$
\begin{aligned}
\overset{\hat{F}_I}{\overset{\Downarrow}{|[cn]^{1_s}_{\bar{3}_c}[\bar{s}\bar{n}]^{1_s}_{3_c}\rangle^{1_s}_{1_c}}}
&= \overbrace{-\sqrt{\frac{1}{4}}\times\sqrt{\frac{1}{3}}}^{\beta_0(\hat{F}_I)}\,\overset{D_s^+}{\overset{\Downarrow}{|\{c\bar{s}\}^{1_s}_{1_c}}}\overset{\pi^0}{\overset{\Downarrow}{\{n\bar{n}\}^{1_s}_{1_c}}}\rangle^{1_s}_{1_c}
+\overbrace{\sqrt{\frac{3}{4}}\times\sqrt{\frac{1}{3}}}^{\beta_1(\hat{F}_I)}\,\overset{D_s^{*+}}{\overset{\Downarrow}{|\{c\bar{s}\}^{3_s}_{1_c}}}\overset{\rho^0}{\overset{\Downarrow}{\{n\bar{n}\}^{3_s}_{1_c}}}\rangle^{1_s}_{1_c} \\
&\quad -\underset{\underset{\text{spin}}{\Uparrow}}{\sqrt{\frac{1}{4}}}\times\underset{\underset{\text{color}}{\Uparrow}}{\sqrt{\frac{2}{3}}}\,|\{c\bar{s}\}^{1_s}_{8_c}\{n\bar{n}\}^{1_s}_{8_c}\rangle^{1_s}_{1_c}
+\underset{\underset{\text{spin}}{\Uparrow}}{\sqrt{\frac{3}{4}}}\times\underset{\underset{\text{color}}{\Uparrow}}{\sqrt{\frac{2}{3}}}\,|\{c\bar{s}\}^{3_s}_{8_c}\{n\bar{n}\}^{3_s}_{8_c}\rangle^{1_s}_{1_c}
\end{aligned}
$$

∗ The above decomposition is reshuffled by a gluon exchange:

- Rare in heavy mesons (at the heavy meson mass scale)

  ⇒ Small overlap of color and spin wfs. between $\hat{F}_I^+$ and $D_s^+\pi^0$

  ⇒ Narrow widths of heavy tetra-quark mesons with $\mathbf{3}_c \times \bar{\mathbf{3}}_c$
- Often in light mesons (at the light meson mass scale)

  ⇒ Nearly "fall-apart" ⇒ Broad !

- **Hidden-charm axial-vector:**
  - **Small overlap of color and spin wavefunctions :**

$$\overset{\substack{X(3872),[X(3875)]\\ \Downarrow}}{|[cu]^{1_s}_{\bar{3}_c}(\bar{c}\bar{u})^{3_s}_{3_c}\rangle^{3_s}_{1_c}} = \overbrace{\frac{1}{2}\sqrt{\frac{1}{3}}}^{\beta_{01}}|\overset{\substack{D^0\\ \Downarrow}}{\{c\bar{u}\}^{1_s}_{1_c}}\overset{\substack{\bar{D}^{*0}\\ \Downarrow}}{\{u\bar{c}\}^{3_s}_{1_c}}\rangle^{3_s}_{1_c} + \overbrace{\sqrt{\frac{1}{2}}\sqrt{\frac{1}{3}}}^{\beta_{11}}|\overset{\substack{D^{*0}\\ \Downarrow}}{\{c\bar{u}\}^{3_s}_{1_c}}\overset{\substack{\bar{D}^{*0}\\ \Downarrow}}{\{u\bar{c}\}^{3_s}_{1_c}}\rangle^{3_s}_{1_c}$$

$$+\frac{1}{2}\sqrt{\frac{1}{3}}|\overset{\substack{D^{*0}\\ \Downarrow}}{\{c\bar{u}\}^{3_s}_{1_c}}\overset{\substack{\bar{D}^{0}\\ \Downarrow}}{\{u\bar{c}\}^{1_s}_{1_c}}\rangle^{3_s}_{1_c} + \cdots$$

$$= \frac{1}{2}\sqrt{\frac{1}{3}}|\overset{\substack{[\eta_c\\ \Downarrow}}{\{c\bar{c}\}^{1_s}_{1_c}}\overset{\substack{\omega]\\ \Downarrow}}{\{u\bar{u}\}^{3_s}_{1_c}}\rangle^{3_s}_{1_c} + \sqrt{\frac{1}{2}}\sqrt{\frac{1}{3}}|\overset{\substack{J/\psi\\ \Downarrow}}{\{c\bar{c}\}^{3_s}_{1_c}}\overset{\substack{\omega\\ \Downarrow}}{\{u\bar{u}\}^{3_s}_{1_c}}\rangle^{3_s}_{1_c}$$

$$+\frac{1}{2}\sqrt{\frac{1}{3}}|\overset{\substack{[J/\psi\\ \Downarrow}}{\{c\bar{c}\}^{3_s}_{1_c}}\overset{\substack{\eta]\\ \Downarrow}}{\{u\bar{u}\}^{1_s}_{1_c}}\rangle^{3_s}_{1_c} + \cdots$$

$\Rightarrow$ **Narrow width of $X(3872), [X(3875)]$ in the tetra-quark model**

Note: Large difference of color and spin wf. overlap between heavy and light mesons $\left(|\beta_0|^2 \sim \frac{1}{12},\ |\beta_{01}|^2 \sim \frac{1}{12},\ |\beta_{11}|^2 \sim \frac{1}{6}\right)$

Appendix 3. Decay Properties of Tetra-Quark Mesons

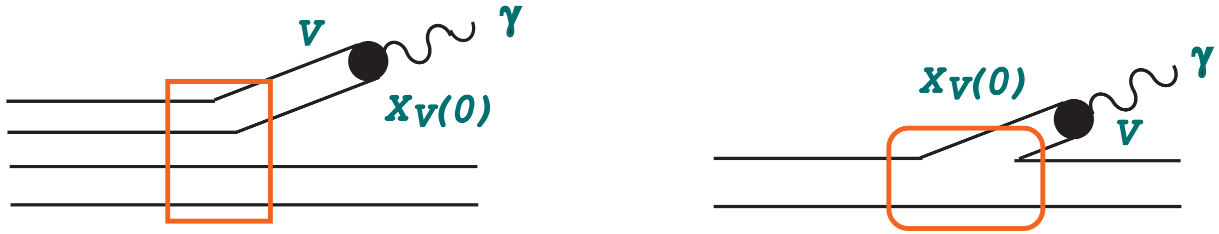
- To estimate decay rates:
  - $SU_f(4)$ and its breaking for strong vertices
    - In the case of tetra-quark mesons,
      $SU_f(4)$ relates vertices at different energy scales
      $\Rightarrow$ Different overlap of color and spin wfs.
      (Light flavors $\Leftrightarrow$ Heavy flavors)
    - Deviation of spatial wf. overlap from the flavor symmetry limit
      $\Rightarrow$ Deviation of $f_+(0)$ from unity
    - $SU_f(3)$: $f_+^{(\pi^- K^0)}(0) = 0.961 \pm 0.008$ Leutwyler & Roos
    - $SU_f(4)$: $f_+^{(\bar{K} D)}(0) = 0.74 \pm 0.03$ PDG96

$= 0.747 \pm 0.019$ HPQCD, hep-lat/1008.4562

$$\left| \frac{f_+^{(\pi D)}(0)}{f_+^{(\bar{K} D)}(0)} \right| = 1.00 \pm 0.13 \quad \text{E687}$$

$= 0.90 \pm 0.08.$ CLEO

- The vector meson dominance (VMD) in radiative decays



$(V = \rho^0, \omega, \phi, \psi)$

  - Photon-vector meson coupling strengths

Photon-vector meson coupling strengths in GeV^2.
The sign of X_V is determined by using the quark model.

V	$X_V(k^2 = 0)$ in GeV^2	$X_V(k^2 = m_V^2)$ in GeV^2
ρ^0	0.033 ± 0.003	0.0357 ± 0.0008
ω	0.011 ± 0.001	0.0109 ± 0.0002
ϕ	-0.018 ± 0.004	-0.0238 ± 0.0003
ψ	~ 0.054	0.380 ± 0.013

K. T., NC Lett. 31, 457 (1981); NC 66A, 475 (1981)

- **Rate for isospin conserving $D_{s0}^{+}(2317) = \hat{F}_I^{+} \to D_s^{+}\pi^0$ decay**
  - **Flavor $SU_f(4)$ relations with corrections to its symmetry breaking:**

    $$\sqrt{2}\langle D_s^{+}|A_{\pi^0}|\hat{F}_I^{+}\rangle = \cdots = \langle \eta^s|A_{\pi^-}|\hat{\delta}^{s+}\rangle (SB)\beta_0$$

    - **$(SB) \simeq 0.7 - 0.8$: deviation of spatial wavefunction overlap from the $SU_f(4)$ symmetry limit**
    - **Color and spin wf. overlap: $|\beta_0|^2 \sim \dfrac{1}{12}$**
  - **Input data : $\Gamma(a_0(980)) \simeq \Gamma(a_0(980) \to \eta\pi) \simeq 50 - 100$ MeV PDG04**

$$\Rightarrow \boxed{\Gamma(\hat{F}_I^{+}) \simeq \Gamma(\hat{F}_I^{+} \to D_s^{+}\pi^0) \sim (3-6)\ \text{MeV}}$$ **(Sufficiently small)**

A. Hayashigaki and K. T., PTP <u>114</u>, 1191 (2005); hep-ph/0410393

- **Radiative and isospin non-conserving decays of D_s^{*+} and charm-strange scalars:**
  - **Radiative decays under the VMD**

$$\left(\begin{array}{l} \text{Deviation from the } SU_f(4) \text{ symmetry limit of} \\ \text{the spatial w.f. overlap} \sim 0.7-0.8 \text{ is corrected} \end{array} \right)$$

Decay	Overlap ($\lvert\beta_1\rvert^2$)	Decay rate (keV)	Input Data
$D_s^{*+} \to D_s^+ \gamma$	1	$0.4-0.5$	$\Gamma(\omega \to \pi^0\gamma)_{\rm exp}$ $= 0.734 \pm 0.035$ MeV
$D_{s0}^{*+} \to D_s^{*+}\gamma$	1	$15-20$	$\Gamma(\chi_{c0} \to \psi\gamma)_{\rm exp}$ $= 119 \pm 15$ keV
$\hat{F}_0^+ \to D_s^{*+}\gamma$	$1/4$	$2-3$	$\Gamma(\phi \to a_0(980)\gamma)_{\rm exp}$ $= 0.32 \pm 0.03$ keV
$\hat{F}_I^+ \to D_s^{*+}\gamma$	$1/4$	$20-25$	

A. Hayashigaki and K. T., PTP 114, 1191 (2005); hep-ph/0410393

Remarks:

* $\Gamma(D_s^{*+} \to D_s^+\gamma)_{\rm VMD} \sim \Gamma(D_s^{*+} \to D_s^+\gamma)_{\rm CQM} = 0.4$ keV
* $\Gamma(D_{s0}^{*+} \to D_s^+\gamma)_{\rm VMD} \sim 10 \times \Gamma(D_{s0}^{*+} \to D_s^+\gamma)_{\rm CQM}$

CQM: Bardeen et al., PRD 68, 054024 (2003)

- Isospin non-conserving $D_s^+\pi^0$ decays of charm-strange mesons:
  - Isospin non-conservation through the $\eta\pi^0$ mixing:

    $\epsilon = 0.0105 \pm 0.0013$ Dalitz and Von Hippel, PL 10, 153 (1964)
  - $\left(\begin{array}{l}\text{Deviation from the } SU_f(4) \text{ symmetry limit of} \\ \text{the spatial wf. overlap } \sim 0.7 - 0.8 \text{ is corrected}\end{array}\right)$

Decay	Overlap (β_0)	Decay rate (keV)	Input Data
$D_s^{*+} \to D_s^+\pi^0$	1	$0.02 - 0.03$	$\Gamma(\rho \to \pi\pi)_{\rm exp}$ $\simeq 150$ MeV
$D_{s0}^{*+} \to D_s^+\pi^0$	1	$0.3 - 0.4$	$\Gamma(K_0^* \to K^+\pi^-)_{\rm exp}$ $= 182 \pm 24$ MeV
$\hat{F}_0^+ \to D_s^+\pi^0$	1/12	$0.2 - 0.5$	$\Gamma(a_0(980) \to \eta\pi)_{\rm exp}$ $= 50 - 100$ MeV

A. Hayashigaki and K. T., PTP 114, 1191 (2005); hep-ph/0410393

Remarks:

- $\Gamma(D_s^{*+} \to D_s^+\pi^0)_{\rm our} \simeq 20 - 30$ eV $\simeq 3 \times \Gamma(D_s^{*+} \to D_s^+\pi^0)_{\rm BEH}$
- $\Gamma(D_{s0}^{*+} \to D_s^+\pi^0)_{\rm our} \simeq 0.3 - 0.4$ keV $\ll \Gamma(D_{s0}^{*+} \to D_s^+\pi^0)_{\rm BEH} \simeq 20$ keV

BEH (HHχPT): Bardeen et al., PRD 68, 054024 (2003)

Ratios of decay rates:

$$R(D_s^{*+})^{-1}_{\rm exp} = 0.062 \pm 0.003\ , \quad R(D_{s0}^{+}(2317))_{\rm CLEO} < 0.059$$

	VMD	Input Data
$R(D_s^{*+})^{-1}$	0.06	$\Gamma(\rho \to \pi\pi)_{\rm exp} \simeq 150$ MeV, $\Gamma(\omega \to \pi^0\gamma)_{\rm exp} = 0.734 \pm 0.035$ keV
$R(\hat{F}_I^+)$	$(4.5 - 9) \times 10^{-3}$	$\Gamma(a_0(980) \to \eta\pi)_{\rm exp} = 50 - 100$ MeV, $\Gamma(\phi \to a_0(980)\gamma)_{\rm exp} = 0.32 \pm 0.03$ keV
$R(\hat{F}_0^+)$	$5 - 10$	
$R(D_{s0}^{*+})$	50	$\Gamma(K_0^* \to K^+\pi^-)_{\rm exp} = 182 \pm 24$ MeV, $\Gamma(\chi_{c0} \to \psi\gamma)_{\rm exp} = 119 \pm 15$ keV

Remarks:

- Our approach is well calibrated by $R(D_s^{*+})^{-1}_{\rm exp}$.
- $R(\hat{F}_I^+) \simeq (4.5 - 9) \times 10^{-3}$ satisfies well $R(D_{s0}^{+}(2317))_{\rm CLEO} < 0.059$.
  $\Rightarrow$ $D_{s0}^{+}(2317)$ should be assigned to $\hat{F}_I^+$ $\Rightarrow$ $D_{s0}^{0}(2317)$, $D_{s0}^{++}(2317)$?
- $R(D_{s0}^{*+}) \simeq 50 > R(\hat{F}_0^+) \simeq 5 - 10 \gg 0.059 > R(D_{s0}^{+}(2317))_{\rm CLEO}$.
  $\Rightarrow$ Assignment of $D_{s0}^{+}(2317)$ to an $I = 0$ state ($\hat{F}_0^+$ or D_{s0}^{*}) is rejected.

$$R(D_s^{*+})^{-1} = \frac{\Gamma(D_s^{*+} \to D_s^+\pi^0)}{\Gamma(D_s^{*+} \to D_s^+\gamma)}, \; R(S) = \frac{\Gamma(S \to D_s^{*+}\gamma)}{\Gamma(S \to D_s^+\pi^0)}, \quad (S = D_{s0}^{*+},\ \hat{F}_0^+,\ \hat{F}_I^+\ )$$

Comments:

- **If $D_{s0}^{+}(2317)$ were an iso-singlet state like**

  a $^{3}P_{0}\ \{c\bar{s}\}$, a chiral partner of D_{s}^{+}, a DK molecule, $\cdots$,

  then, the rate for the isospin non-conserving decay should satisfy

  $$\Gamma(\{D_{s0}^{+}\}_{I=0} \to D_{s}^{+}\pi^{0}) \gg \Gamma(\{D_{s0}^{+}\}_{I=0} \to D_{s}^{*+}\gamma)$$

  $$\begin{cases} \sim 2\ \text{keV}, & \text{CQM } (m_s \simeq 500\ \text{MeV ???}): \\ & \text{Bardeen et al., PRD } \underline{68}\text{, 054024 (2003)} \\ (\sim 20\ \text{keV}, & \text{VMD} \qquad \text{A.H. \& K.T.)} \end{cases}$$

  - $\Gamma(D_{s0}^{*+} \to D_{s}^{+}\pi^{0}) \gtrsim 20\ \text{keV}$ **(HHχPT) to satisfy** $R(D_{s0}^{+})|_{\text{CLEO}} < 0.059$

    $$\Rightarrow \begin{cases} SU_f(3) \text{ implies that } \Gamma(D_0^{*} \to D\pi) \gtrsim 500\ \text{MeV} > \Gamma(D_0^{*})_{\text{Babar}} \\ \text{HH}\chi\text{PT cannot reproduce } R(D_s^{*+})^{-1}_{\text{exp}}. \end{cases}$$

    $$\begin{cases} \text{Cho and Wise, PRD } \underline{49}\text{, 6228 (1994),} \\ \text{Bardeen et al., PRD } \underline{68}\text{, 054024 (2003)} \end{cases}$$

  - **Dynamically generated molecules with**

    $$\Gamma(D_{s0}^{+}(2317) \to D_{s}^{+}\pi^{0}) \gtrsim \text{several tens keV}$$

    - **Where the conventional D_{s0}^{*+} and D_{0}^{*}, and how broad ?**
    - **Basic (chiral) Lagrangian should be calibrated by $R(D_s^{*+})^{-1}_{\text{exp}}$.**

Appendix 4. Conventional Charmed Scalar Mesons

- Current algebra $\oplus$ $SU_f(4)$ with $\underline{\text{corrections from its symmetry breaking}}$

$\Uparrow$

20 -30 % deviation of spatial wf. overlap
from unity (i.e., $SU_f(4)$ symmetry limit)

- Input data: $\left\{ \begin{array}{l} m_{K_0^*} = 1412 \pm 6 \text{ MeV}, \\ \Gamma_{K_0^*} = 294 \pm 23 \text{ MeV}, \quad Br(K_0^* \to K\pi) = 93 \pm 10\ \% \end{array} \right.$

$\underline{D_0^* \sim \{c\bar{n}\}}$

– Mass: $m_{D_0^*} \simeq 2.3$ GeV (tentative)

$\Rightarrow$ Width: $\Gamma(D_0^*) \simeq \Gamma(D_0^* \to D\pi) \sim 50 \text{ MeV} \ll \Gamma(D_0)_{\text{Babar}}$

$$\diamondsuit \left\{ \begin{array}{l} \text{Re-analyze the } D\pi \text{ mass distribution just below the } D_2^* \text{ peak} \\ \text{Find a structure containing } \left\{ \begin{array}{l} D_0^* \text{ with } \sim 50 \text{ MeV width} \\ \hat{D} \text{ with a width } \lesssim 10 \text{ MeV} \end{array} \right. \end{array} \right.$$

$\underline{D_{s0}^{*+} \sim \{c\bar{s}\}}$

– Mass: $m_{D_{s0}^{*+}} \simeq m_{D_0^*} + \Delta_{sn} \simeq 2.4$ GeV

$\Rightarrow$ Width: $\Gamma(D_{s0}^*) \simeq \Gamma(D_{s0}^* \to DK) \sim 40$ MeV

◇ Search for a DK peak (with a width ~ 40 MeV) around 2.4 GeV

K.T. and B.H. McKellar, PTP $\underline{114}$, 205 (2005)

Appendix 5. Various Approaches to Large I-Spin Non-Conservation in $X(3872)$ Decays

- **Explicit violation of Isospin conservation:**
  - Molecular model:

    $X(3872) \sim D^0\bar{D}^{*0} + c.c.$ Törnquist, hep-ph/0308277

    Inconsistent with the isospin symmetry in $X(3872)$ productions

    Braaten and Kusunoki, PRD 71, 074005 (2005)
  - Diquark-antidiquark model with a large mass difference

    $\Delta m = (m_{X_d} - m_{X_u}) \geq 7 \pm 2$ MeV

    Maiani et al., PRD 71, 014028 (2005)

    Inconsistent with $(\Delta m)_{\text{Belle}} = 0.22 \pm 0.90 \pm 0.27$ MeV
- **Dynamical violation of Isospin conservation (through $D\bar{D}^*$-loop):**
  - Unitarized chiral model:

    Gamermann and Oset, PRD 76, 074016 (2007)
  - $D\bar{D}^*$ molecule mixed with $\{c\bar{c}\}$: Takizawa and Takeuchi, EPJ web of Conference 3 (2010), 03026.

Problem:
- $*$ Phenomenologically known ω-ρ^0 mixing is not considered.
- $*$ Disfavored by production of prompt $X(3872)$?

Appendix 6. Isospin Non-Conservation through $\omega\rho^0$ Mixing:

K.T., PTP <u>122</u>, 1285 (2009); hep-ph/0904.3368v2

– Basic assumptions

- Isospin conservation in the ordinary strong interactions
  $\Rightarrow$ No problem in productions of $X(3872)$
- Isospin non-conservation only through $\omega\rho^0$ mixing which is well-known as the origin of <u>iso-spin non-conservation in nuclear forces</u>
  Miller et al., Ann. Rev. Nucl. Part. Sci. <u>56</u>, 253 (2006)
  and reproduces the measured rate for the $\omega \to \pi\pi$ decay

$$\Rightarrow \left\{ \begin{array}{l} \text{Three steps of } X(3872) \to \pi^+\pi^- J/\psi \text{ decay:} \\ \qquad \left(\begin{array}{ll} \text{(i)} & \text{Isospin conserving } X(3872) \to \omega J/\psi, \\ \text{(ii)} & \omega\text{-}\rho^0 \text{ mixing } X(3872) \to \omega J/\psi \to \rho^0 J/\psi, \\ \text{(iii)} & \rho^0 \to \pi^+\pi^- \text{ decay} \end{array} \right. \\ X(3872) \to \gamma J/\psi \text{ under the VMD:} \\ \qquad \left(\begin{array}{ll} \text{(i)} & X(3872) \to \omega J/\psi \to \gamma J/\psi, \\ \text{(ii)} & X(3872) \to \omega J/\psi \to \rho^0 J/\psi \to \gamma J/\psi, \\ \text{(iii)} & X(3872) \to J/\psi J/\psi \to \gamma J/\psi, \\ & \qquad \text{only if } X(3872) \text{ is a charmonium)} \end{array} \right. \end{array} \right.$$

- Decay rates for the $\Gamma(X(3872)\to\pi^+\pi^- J/\psi)$ and $\Gamma(X(3872)\to\gamma J/\psi)$
  - (Broad) widths of ρ and ω $\Leftarrow$ Breit-Wigner form of propagaters
  - Values of parameters involved in $\Gamma(X(3872)\to\pi^+\pi^- J/\psi)$:
    - $|g_{\rho^0\pi^+\pi^-}| \simeq 5.98$ from $\Gamma(\rho\to\pi\pi)\simeq\Gamma_\rho\simeq 149\ \text{MeV}$
    - $|g_{\omega\rho^0}| = (3.4\pm 0.5)\times 10^{-3}\ \text{GeV}^2$
      from $\Gamma_\omega = 8.49\pm 0.08\ \text{MeV}$, $Br(\omega\to\pi^+\pi^-) = 1.53^{+0.11}_{-0.13}\ \%$
      $\Rightarrow \Gamma(X(3872)\to\omega J/\psi\to\rho^0 J/\psi\to\pi^+\pi^- J/\psi)$ is enhanced,
      because $\left|\dfrac{g_{\omega\rho}}{m_\omega^2-m_\rho^2}\right| \gg \left|\dfrac{g_{\omega\rho}}{m_\omega^2}\right|$.
    - Remaining unknown parameter : $g_{X\omega J/\psi}$
  - Values of parameters involved in $\Gamma(X(3872)\to\gamma J/\psi)$:
    - γV coupling strengths on the photon mass-shell:
      $$\begin{cases} X_\rho(0) = 0.033\pm 0.003\ \text{GeV}^2, \\ X_\omega(0) = 0.011\pm 0.001\ \text{GeV}^2, \\ X_{J/\psi}(0) = 0.050\pm 0.013\ \text{GeV}^2, \end{cases}$$
    - Ratio of the ψ- to ω-pole contribution
      $$K = 2\frac{m_\omega^2}{m_\psi^2}\frac{X_\psi(0)}{X_\omega(0)}\frac{g_{XJ/\psi J/\psi}}{g_{X\omega J/\psi}} \Rightarrow \begin{cases} |K|\ll 1,\ \text{if } X\sim\{cn\bar{c}\bar{n}\}, \\ |K|\gg 1,\ \text{if } X\sim\{c\bar{c}\}. \end{cases}$$
    - Remaining unknown parameter(s) : $g_{X\omega J/\psi}$ (and K)

- Ratio of decay rates, $R = \dfrac{\Gamma(X(3872) \to \gamma J/\psi)}{\Gamma(X(3872) \to \pi^+\pi^- J/\psi)}$
  - $R_{\text{tetra}} \simeq 0.33$, (no free parameter, because of $|K| \ll 1$)
  - $R_{c\bar{c}} \gg 1$, (because of $|K| \gg 1$),

    where $K = 2\dfrac{m_\omega^2}{m_\psi^2}\dfrac{X_\psi(0)}{X_\omega(0)}\dfrac{g_{XJ/\psi J/\psi}}{g_{X\omega J/\psi}} \Rightarrow \begin{cases} |K| \ll 1, \text{ if } X \sim \{cn\bar{c}\bar{n}\}, \\ |K| \gg 1, \text{ if } X \sim \{c\bar{c}\} \end{cases}$
  - Experimental data on the ratio:
    - * $R_{\text{Belle}} = 0.14 \pm 0.05$
    - * $R_{\text{Babar}} = 0.33 \pm 0.12$

  $\Rightarrow R_{c\bar{c}} \gg R_{\text{tetra}} \simeq R_{\text{Babar}} \sim R_{\text{Belle}}$

The measured ratios of R seems to favor the tetra-quark interpretation of $X(3872)$ over the charmonium.

- Question to a sizable mixing of χ'_{c1}

$$\frac{Br(X(3872) \to \gamma\psi')}{Br(X(3872) \to \gamma J/\psi)} = \begin{cases} \text{No signal : } < 2.1 \text{ (90 \% CL)} & \text{Belle} \\ 3.4 \pm 1.4 & \text{Babar} \end{cases}$$