Charmed meson scatterings from lattice QCD

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1. Introduction

Motivation

- Experimental progress
  - Exotic hadrons Ex) X(3872), Y(3940), etc
  - $Z^+(4430)$
    - Belle (2007)
      - Decay : $Z^+(4430) \rightarrow \pi^+ \psi'$

likely to be $(c\bar{c})+(u\bar{d})$

      - Mass : 4433 ±5 [MeV/c²]
      - Width : 45^{+35}_{-18} [MeV/c²]
      - Quantum number : $I^G = 1^+$

Strong candidate for 4-quark state

  - Babar found no peak (2009)

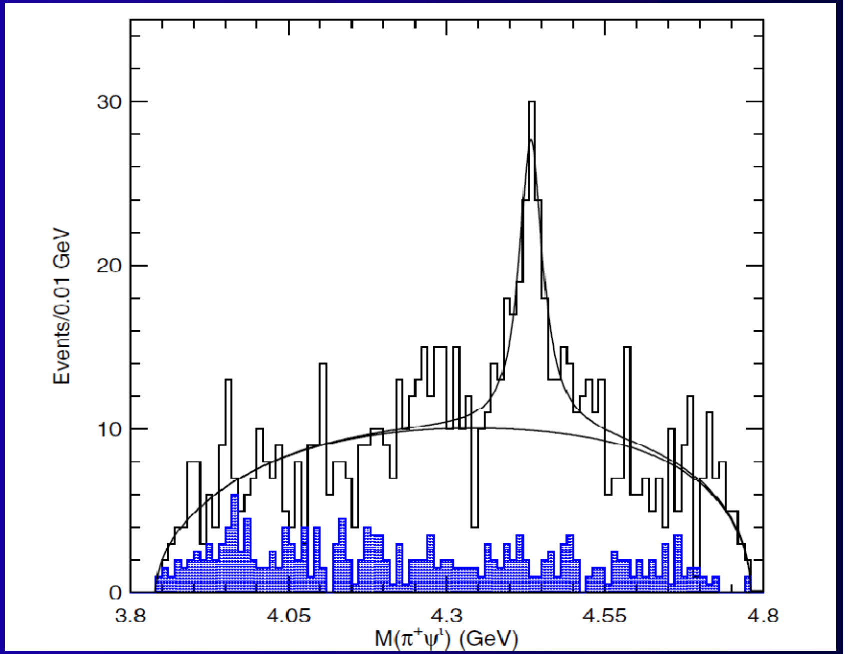


Fig : $\pi^+\psi'$ invariant mass distribution (Belle 2007)

Theoretical works

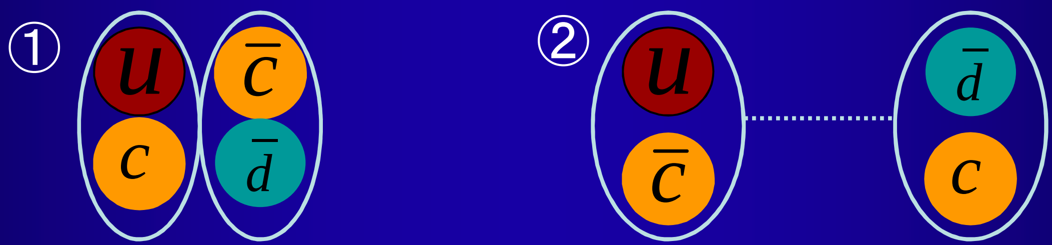
- Physical pictures for $Z^+(4430)$

① Tetraquark state

- Tightly bound state of diquark - diantiquark pairs

② Molecular state

- Loosely bound state of D^* (2010) and D_1(2420) in the S-wave



As a first step to clarify the nature of $Z^+(4430)$,
we study the channels coupled to this hadoron from lattice QCD

$$D_1 D^*, D D_0^*, \chi_{c1}(1P)\rho, J/\psi a_1$$

Recent work : the scattering lengths of D*D1 (CLQCD, 2009) not eigenstate of I^G

2. Details of calculation

Setup

Gauge Configuration

- Quenched Approximation
- Lattice constant : $a \approx 0.07$ [fm], $a^{-1} \approx 2810$ [MeV]
- Lattice Size : L^3 x T = 24^3x48, (16^3x48)
- Statistics : N = 2000 (for 24^3x48), 3000 (for 16^3x48)

Fermion

- Fermion action : Wilson action
- Hopping parameter
  - κ= 0.136 (for c), 0.152,0.1525,0.1528,0.1531 (for u, d)

$\eta_c(1S) \approx 2980$ [MeV]

$m_\pi \approx 500 \sim 800$ [MeV]

Lüscher's formula (1) : 1+1 D

- Spatial Size : L , Temporal Size: ∞ , Interaction range : R

b.c. : $\phi(x+nL)=\phi(x)$ n : integer

A) Interaction OFF :

⇒ $kL = 2\pi\ n$

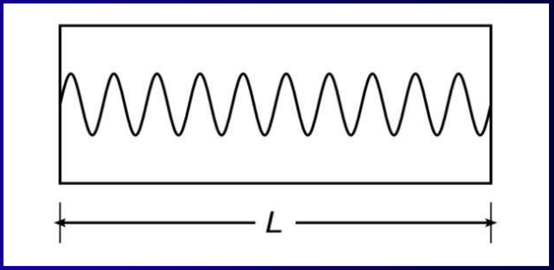


Fig: Wave function in the finite box (Interaction : OFF)

B) Interaction ON : if $|x| >> R/2$

⇒ $kL+2\delta(k)=2\pi\ n$

$L >> R \Rightarrow \delta(k)\ (L: \text{finite}) \rightarrow \delta(k)\ (L:\infty)$

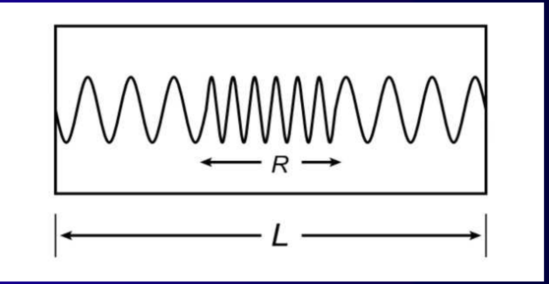


Fig: Wave function in the finite box (Interaction : ON)

Method to calculate $\delta(k)$

① calculation of mass "m" from 2pt correlator

② calculation of 2 particle energy "W" from 4pt correlator

$$W \equiv 2\sqrt{m^2+k^2}$$

③ subtract "k" from "m" and "W" ⇒ $\delta(k)$

Lüscher's formula (2) : 3+1 D

- For 3+1 dimension   Lüscher(1991)

$$\begin{cases} (\Delta + k^2)\ \phi(\vec{x}) = 0 & \text{: Helmholtz equation} \\ \text{b.c.}\ :\ \phi(\vec{x}) = \phi(\vec{x} + \vec{n}L) & R < |\vec{x}| < L/2 \end{cases}$$

$\vec{n}$: 3D integer vectors

$$k \cot \delta_0(k) = \frac{1}{\pi L} Z(1, \eta) \quad ①$$

$$Z(s,x) \equiv \sum_{\vec{m} \in Z^3} (m^2 - x)^{-s}$$

$$\eta \equiv \frac{k^2 L^2}{4\pi^2} \approx \frac{mL^2}{4\pi^2}(W - 2m)$$

η > 0 : repulsive
η < 0 : attractive

- Expansion by "$a_0 / \pi L$" Lüscher(1986)

$$W - 2m = -\frac{4\pi a_0}{m L^3}\left(1 + c_1 \frac{a_0}{\pi L} + c_2 \left(\frac{a_0}{\pi L}\right)^2 + \cdots \right) \quad ②$$

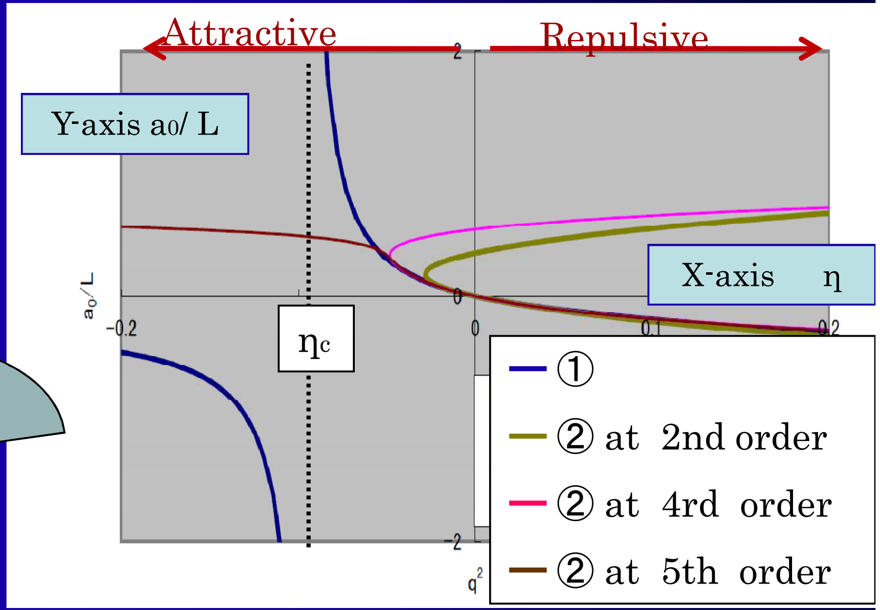


Fig: scattering lengths for η

② fails in the attractive region

Channels coupled to Z(4430)

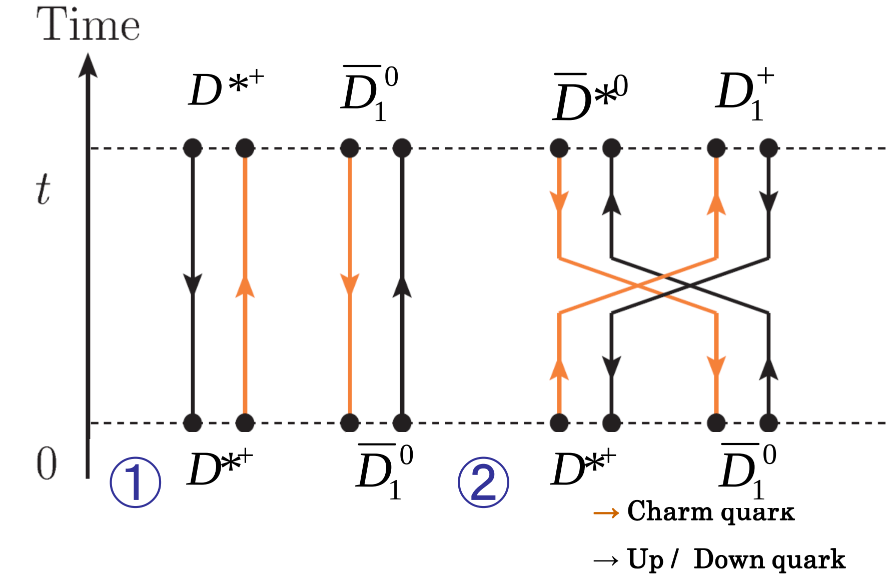
- Meson pairs ($I^G(J^P)=1^+(0^-)$) in s-wave

	combination	threshold[MeV]	quantum number	eigen state
S-wave	$D \otimes D_0^*$	4270	$\frac{1}{2} \otimes \frac{1}{2}(0^- \otimes 0^+)$	$\frac{1}{\sqrt{2}}\left(\lvert D^+ \bar{D}_0^{*0}\rangle - \lvert D_0^{*+} \bar{D}^0\rangle\right)$
$(J^P = 0^-)$	$D_1 \otimes D^*$	4430	$\frac{1}{2} \otimes \frac{1}{2}(1^+ \otimes 1^-)$	$\frac{1}{\sqrt{2}}\left(\lvert \bar{D}_1^0 D^{*+}\rangle + \lvert \bar{D}^{*0} D_1^+\rangle\right)$
	$\chi_{c1}(1P) \otimes \rho$	4280	$0^+ \otimes 1^+(1^{++} \otimes 1^{--})$	$\lvert \chi_{c1}(1P)\, \rho\rangle$
	$J/\psi \otimes a_1$	4360	$0^- \otimes 1^-(1^{--} \otimes 1^{++})$	$\lvert J/\psi\, a_1\rangle$

Candidate for Z(4430)

Term to product eigenstate of G-parity

- Correlation functions for 2-particle state (ex. D1D*)



① Exchange of the gluon

② State mixing

Ex) D_1D^*

$$\bar{D}_1^0 D^{*+} \to \bar{D}^{*0} D_1^+$$
$$\bar{D}^{*0} D_1^+ \to \bar{D}_1^0 D^{*+}$$

- As a approximation, we removed the charm and anti-charm quark's annihilation diagrams.

3. Results

Classfication of each channel

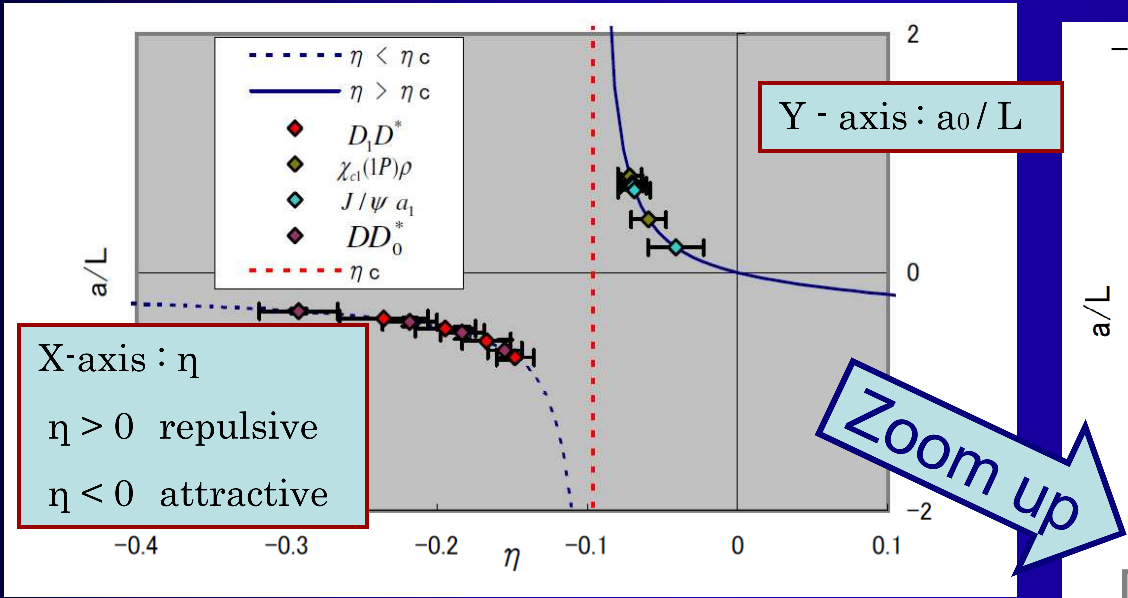
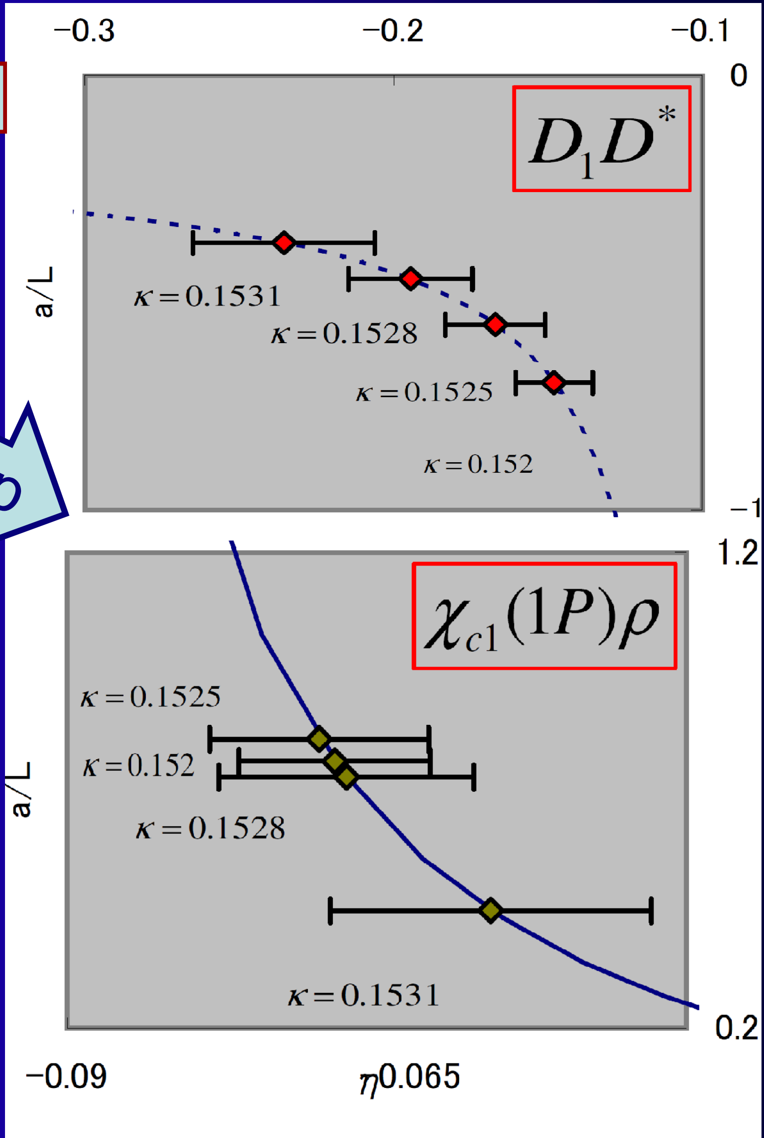


Fig : scattering lengths for each channel, and each quark mass (L=24)



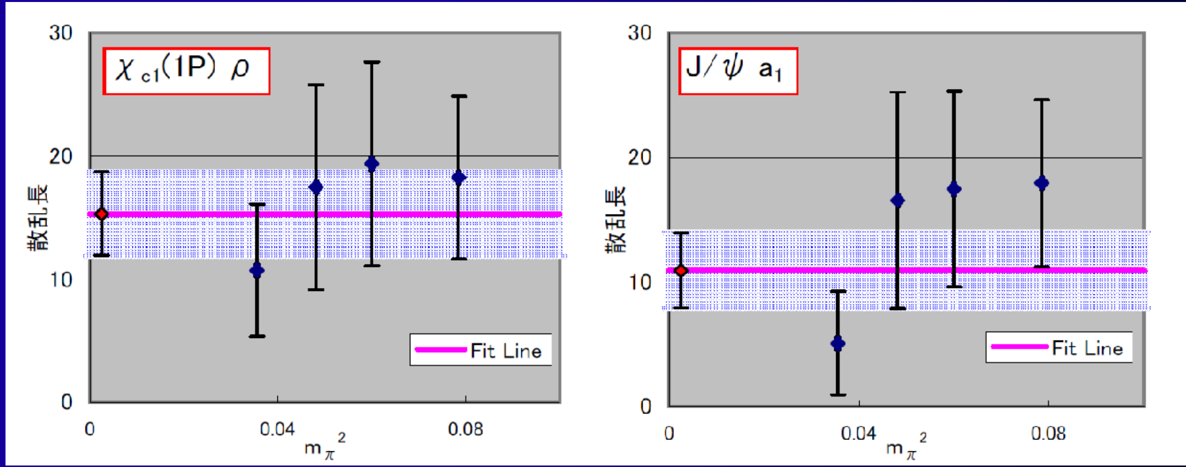
- Charmonium – light meson channel
  - $\eta < 0,\ a_0 > 0$
  - ⇨ Attractive scattering
- Charmed meson – Charmed meson channel
  - $\eta < \eta_c < 0,\ a_0 < 0$
  - ⇨ Bound state ?

Extrapolation to the physical limit ($m_\pi = 140$[MeV])

① For scattering stetes,

$a_0 = c_0$ (c_0:constant)

$$a_{0\,\chi_{c1}(1P)\rho} = 1.07 \pm 0.24\,[\mathrm{fm}]$$
$$a_{0\,J/\psi a_1} = 0.77 \pm 0.21\,[\mathrm{fm}]$$



Y-axis : a_0 (scattering lengths) X-axis : m^2_π

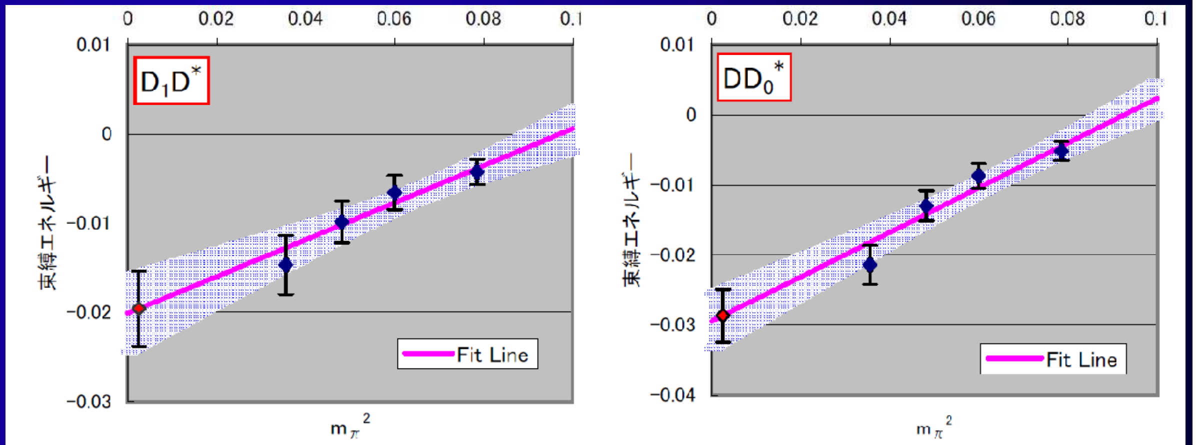
② For bound state(?),

$$\gamma + k\cot\delta_0(k)\,|_{k^2=-\gamma^2} \approx 0$$

$$\Rightarrow\quad E_b \equiv -\frac{\gamma^2}{2\mu}$$

$E_b = c_0 + c_1 m_\pi^2$ (c_0, c_1:constant)

$$E_{b\,D_1D^*} = -55 \pm 12\,[\mathrm{MeV}]$$
$$E_{b\,DD_0^*} = -81 \pm 11\,[\mathrm{MeV}]$$



Y-axis : E_b (binding energy) X-axis : m^2_π

4. Summary & Future Plan

Summary

- We have studied charmed meson scatterings coupled to Z(4430). $I^G(J^P)=1^+(0^-)$
- The scattering lengths obtained show that

  the interactions in these channels are all attractive .

① Charmonium-light meson pair $\bar{c}c-\bar{d}u$: weak

→ little dependence on quark mass

② Charmed- meson pair $\bar{c}u-\bar{d}c$: strong

→ dependence on quark mass by the state mixing from G-parity

→ suggests that Z(4430) is a molecular of D_1 and D*

Future Plan

- Improvement : larger lattice box, full QCD simulation, …
- Other hadrons : $Z_1(4050)$, $Z_2(4250)$, Tcc ($cc-\bar{d}\bar{u}$), …