# Asteroseismology on supernova gravitational waves

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collaborate with

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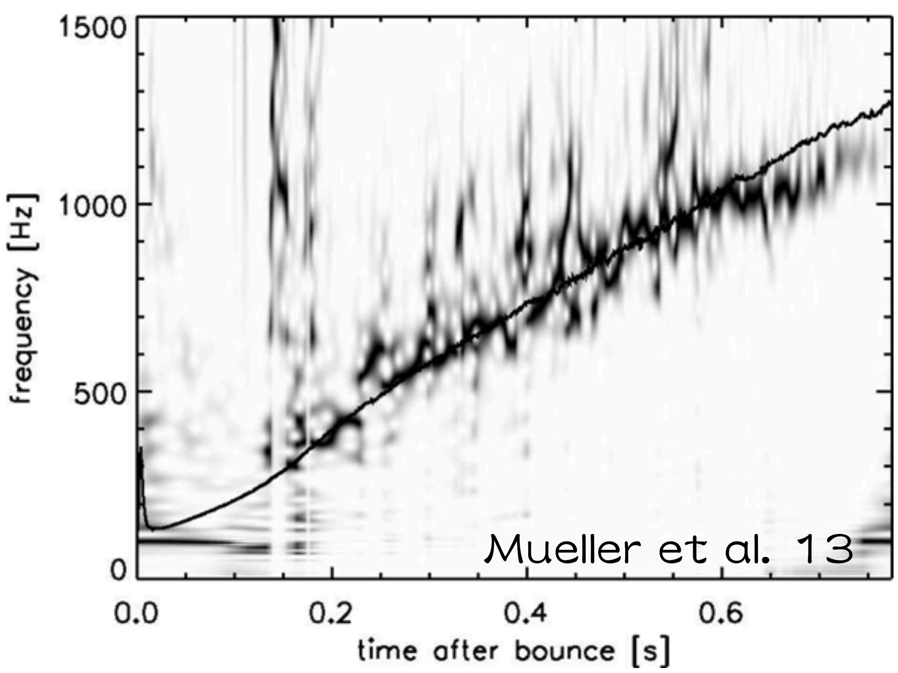
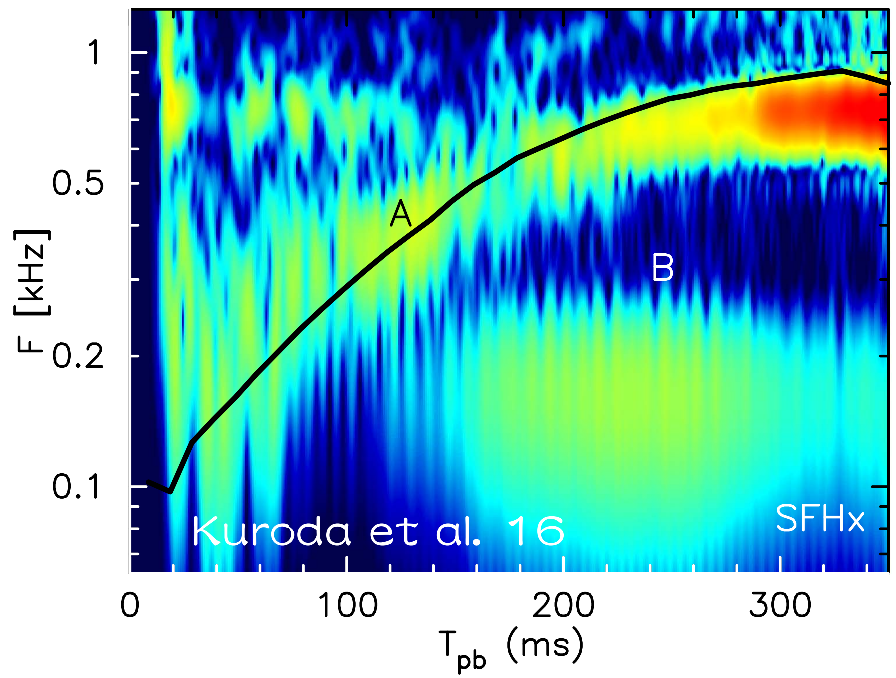
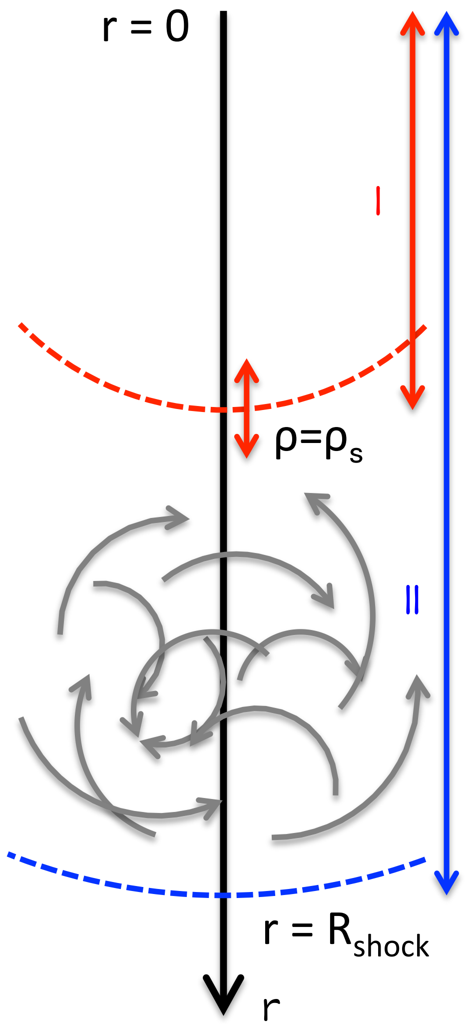
# Next candidate of GW sources

- core-collapse supernovae
  - compered to binary merger, system is more spherically symmetric
    - less energy of gravitational waves
  - many numerical simulations show the existence of GW signals
  - to understand the physics behind GW signals, we adopt perturbative approach, i.e., asteroseismology



# difference in two approaches

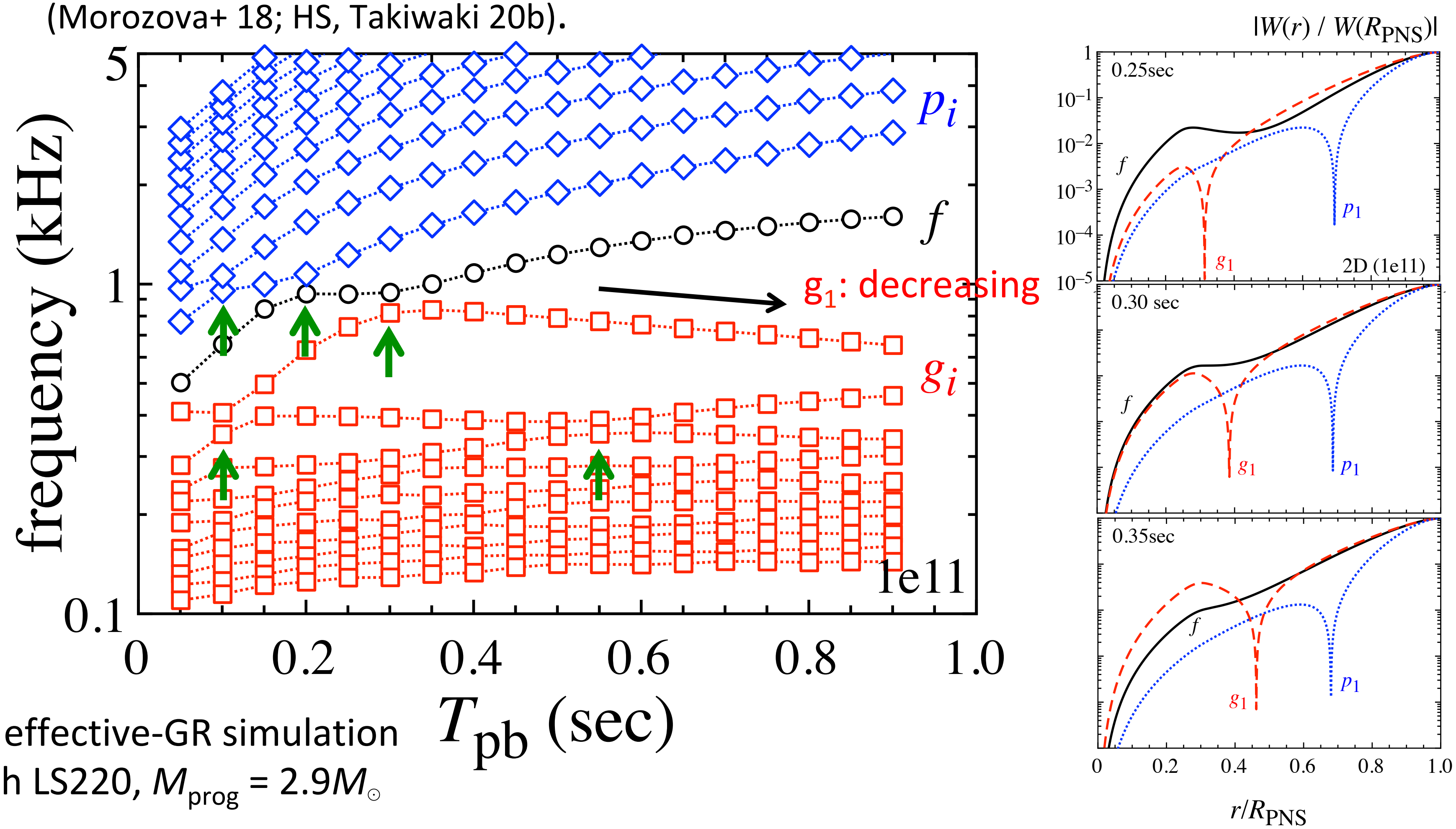



- computational domain
  - Model I : only inside $R_{PNS}$ defined by $\rho_s$
  - Model II : up to $R_{shock}$
- Boundary condition for solving the eigenvalue problem
  - Model I : $\Delta p = 0$ @$r = R_{PNS}$
  - Model II : $\delta\xi^r = 0$ @$r = R_{shock}$
  - mathematically, problem to solve is complete different
  - for the both models, the BC is a kind of assumption (not exact one)
- advantage
  - Model I : matter motion is relatively small
    mode classification is as usual
  - Model II : boundary is uniquely determined
- disadvantage
  - Model I : uncertainty in choice of $\rho_s$
  - Model II : matter motion may not be negligible outside $R_{PNS}$
    mode classifications is different from the standard one.

# avoided crossing in GW frequency

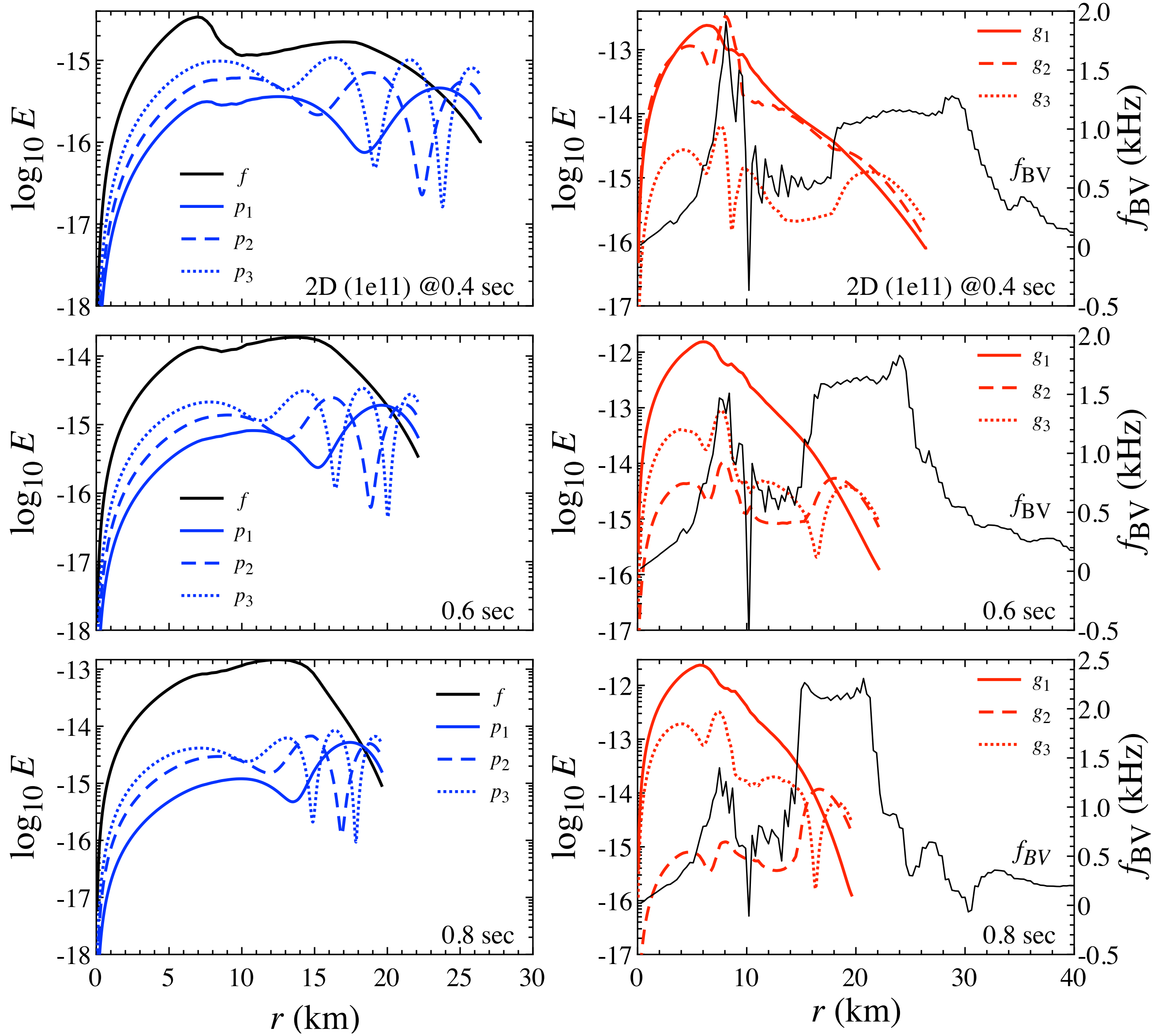
(Sotani&Takiwaki 20b)

- one can observe the phenomena of avoided crossing between the eigenmodes.
- the f- & $g_1$-modes frequencies are almost independent from the selection of $\rho_c$ (Morozova+ 18; HS, Takiwaki 20b).




2D effective-GR simulation with LS220, $M_{prog} = 2.9M_\odot$

# pulsation energy density




$$E(r) \sim \frac{\omega^2 \varepsilon}{r^4} \left[ W^2 + \ell(\ell+1) r^2 V^2 \right]$$

$$f_{\rm BV} = {\rm sgn}(\mathcal{N}^2) \sqrt{|\mathcal{N}^2|/2\pi}$$

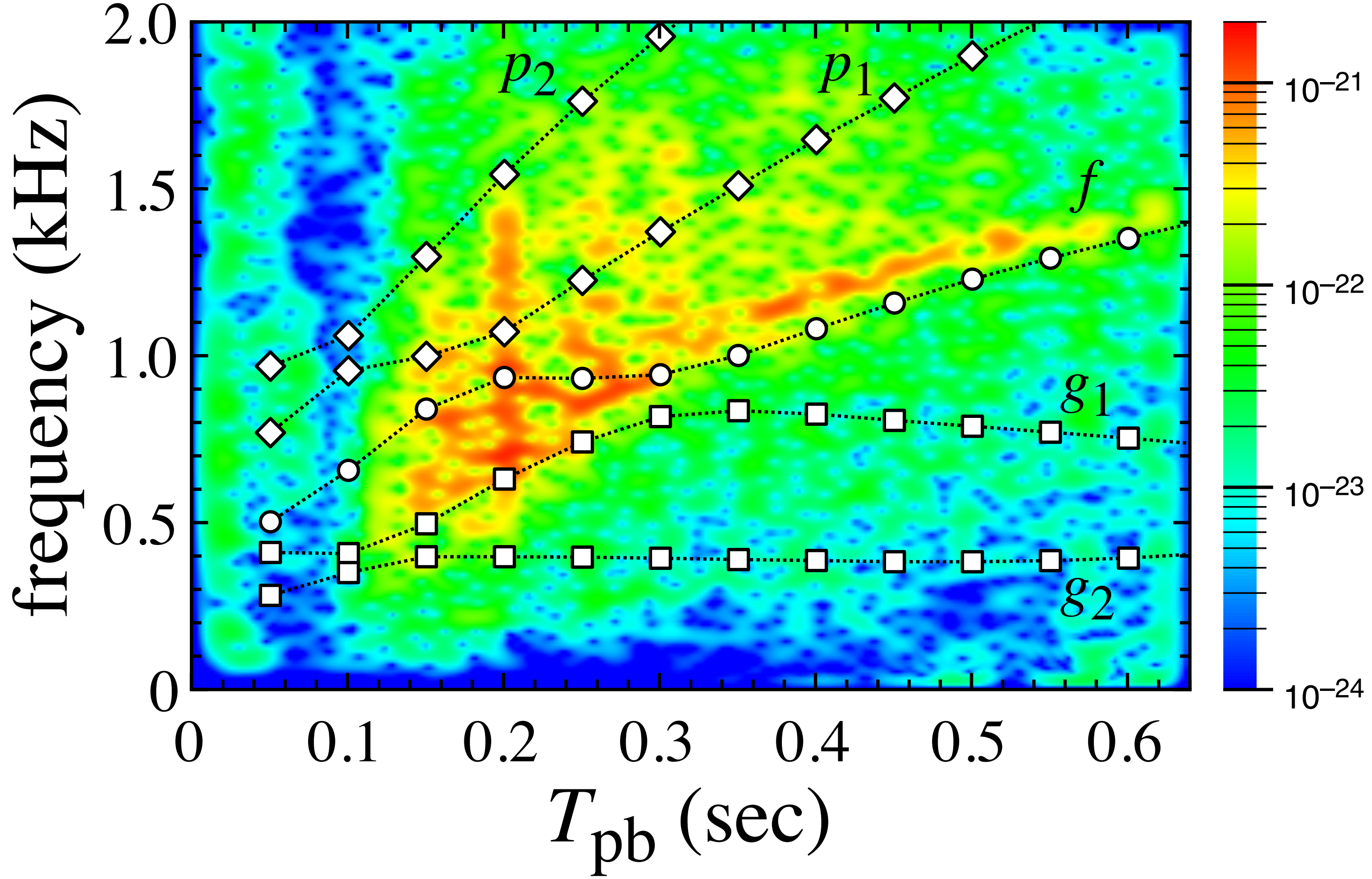
$$\mathcal{N}^2 = -e^{2\Phi - 2\Lambda} \frac{\Phi'}{\varepsilon + p} \left( \varepsilon' - \frac{p'}{c_s^2} \right)$$

- f- & $g_1$-modes are not dominant @PNS surface
  → f- & $g_1$-modes weakly depend on $\rho_s$

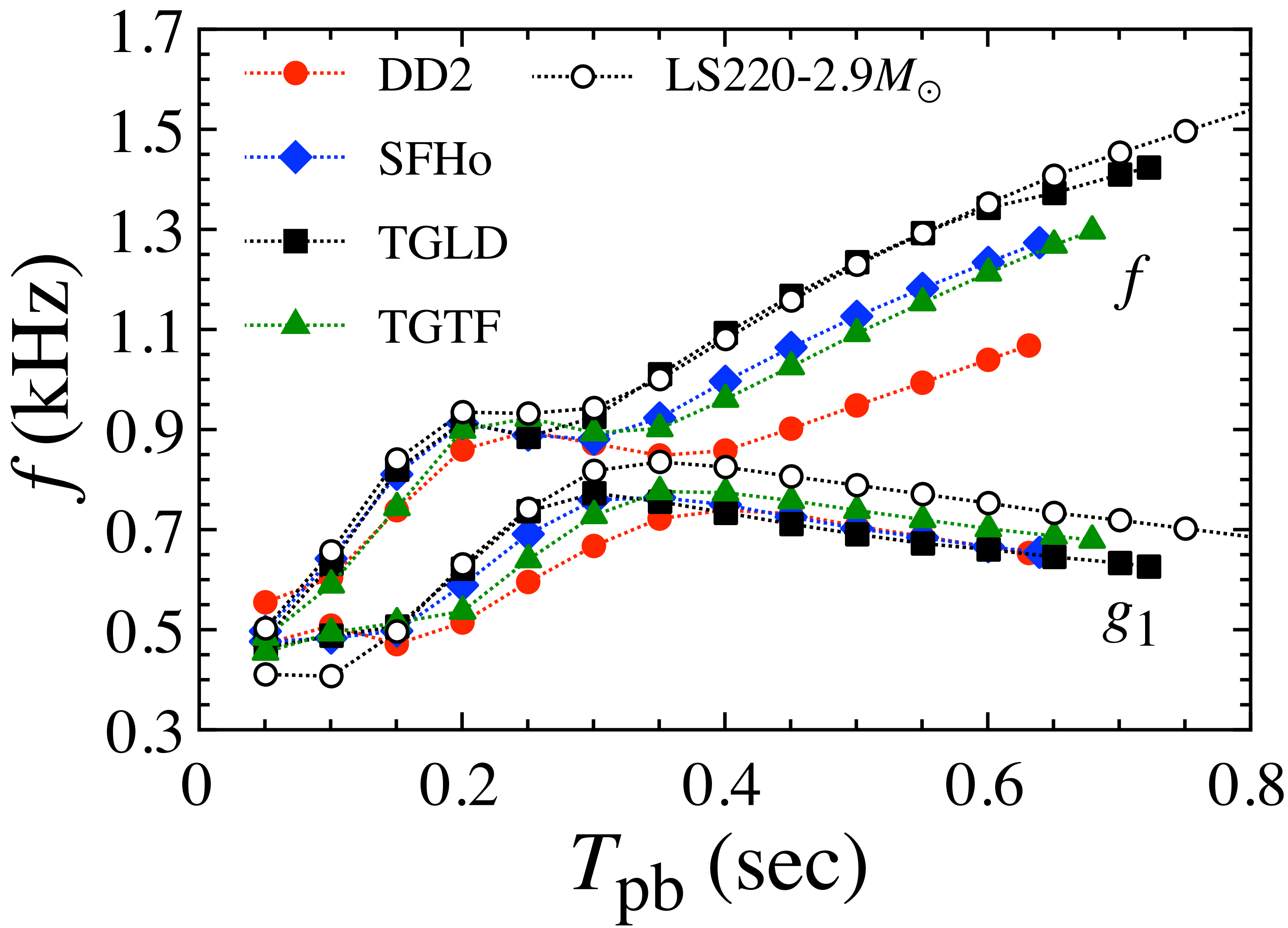
- $g_i$-modes related to $f_{BV}$
- $g_1$-mode is strongly associated with BV freq. @r=8km, which decreases with time
  → decrease of $g_1$-mode

# comparison with GW signals in numerical simulation

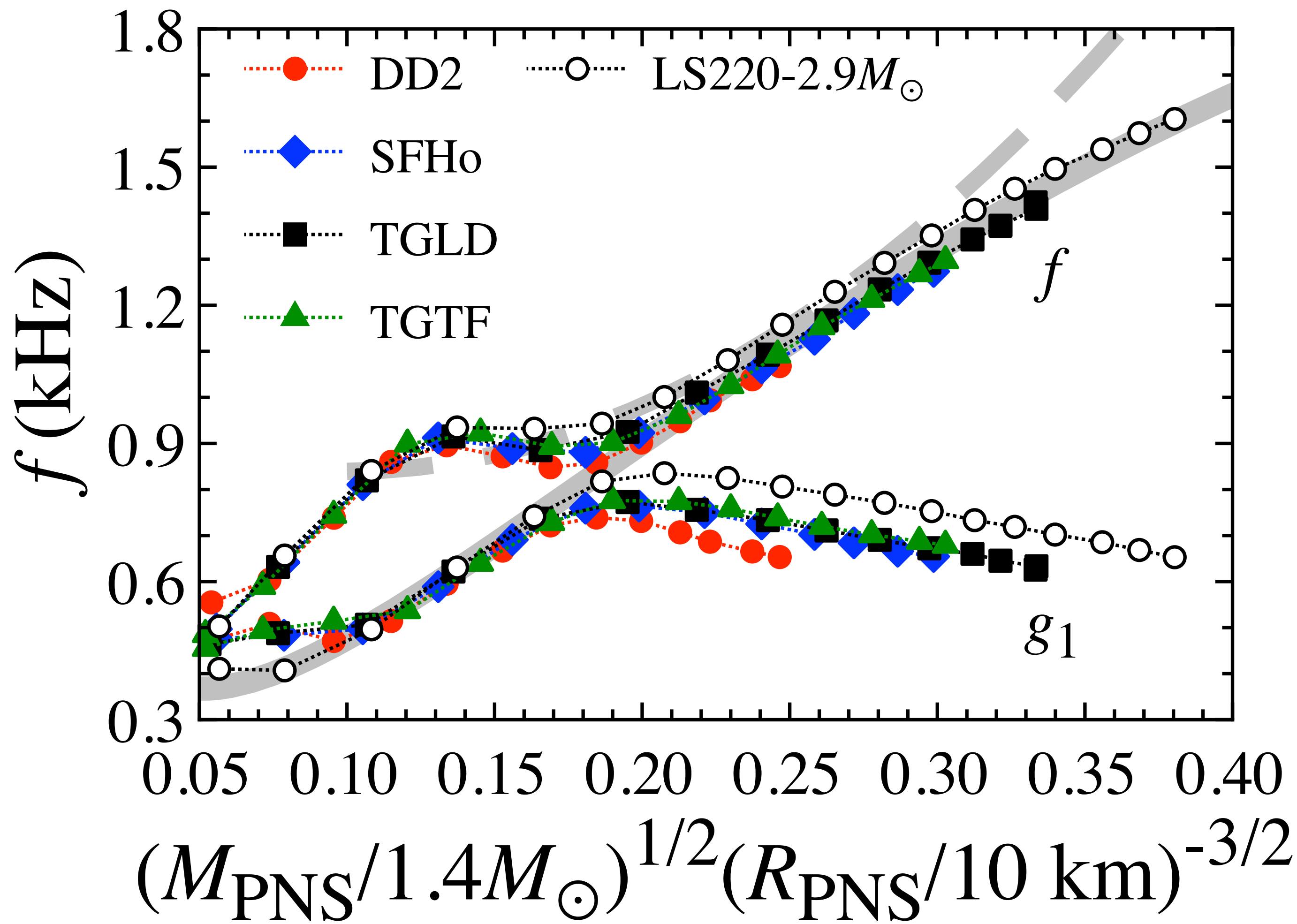
- GW signals correspond to $g_1$-mode in early phase and f-mode after avoided crossing.



# dep. of GW signals on PNS models



# universal relation



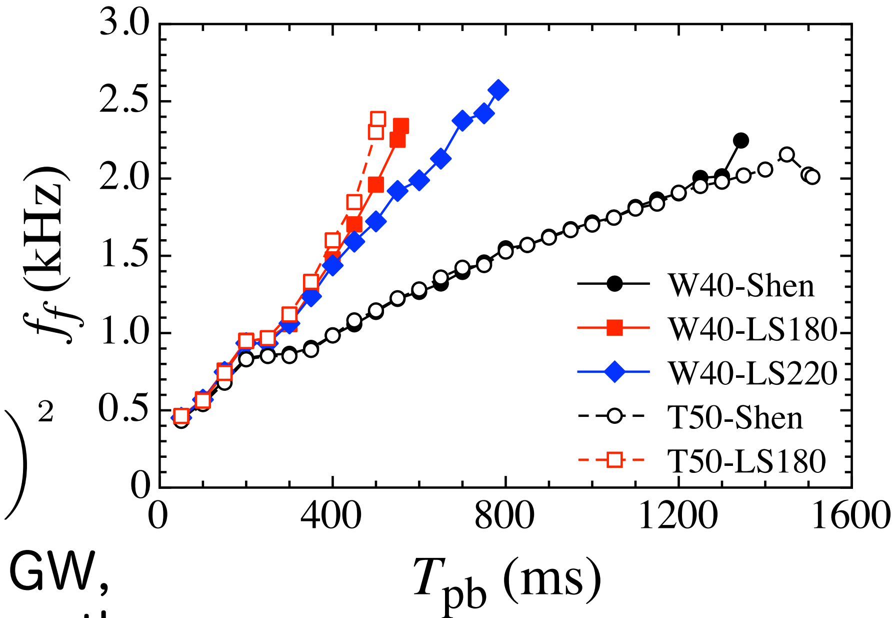

$$f\ (\mathrm{kHz}) = -1.410 - 0.443\ln(x) + 9.337x - 6.714x^2$$

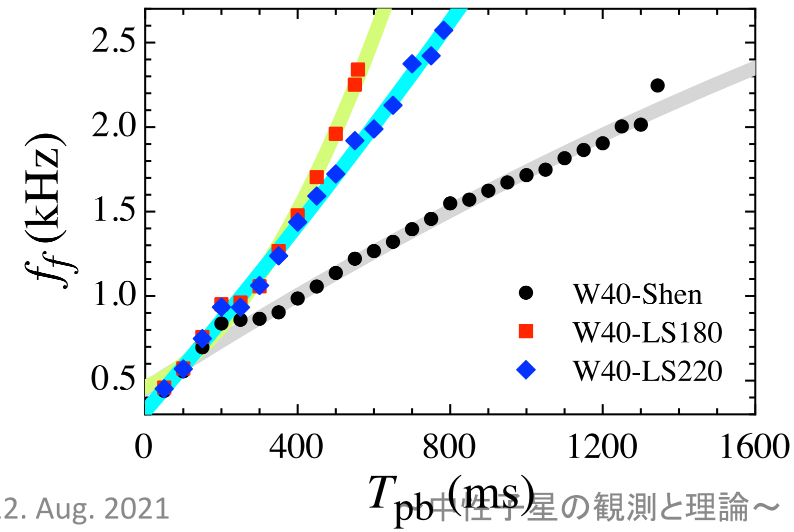
# dep. on PNS models for BH formation

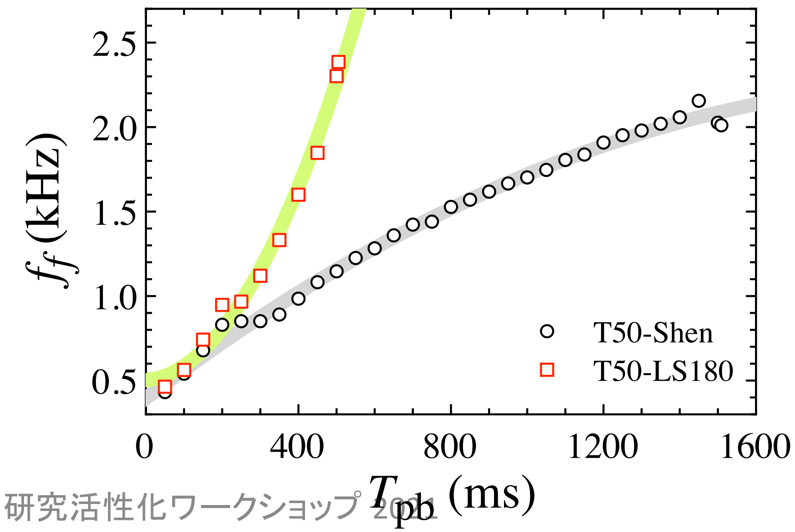
- Time evolution of f-mode GW strongly depends on the progenitor models.
- In any case, it can be well fitted as a function of $T_{pb}$, such as

$$f_f(\mathrm{kHz}) = c_0 + c_1 \left(\frac{T_{\rm pb}}{1000\,\mathrm{ms}}\right) + c_2 \left(\frac{T_{\rm pb}}{1000\,\mathrm{ms}}\right)^2$$

  - one can expect high fre. f-mode GW, even though it is not detected directly.



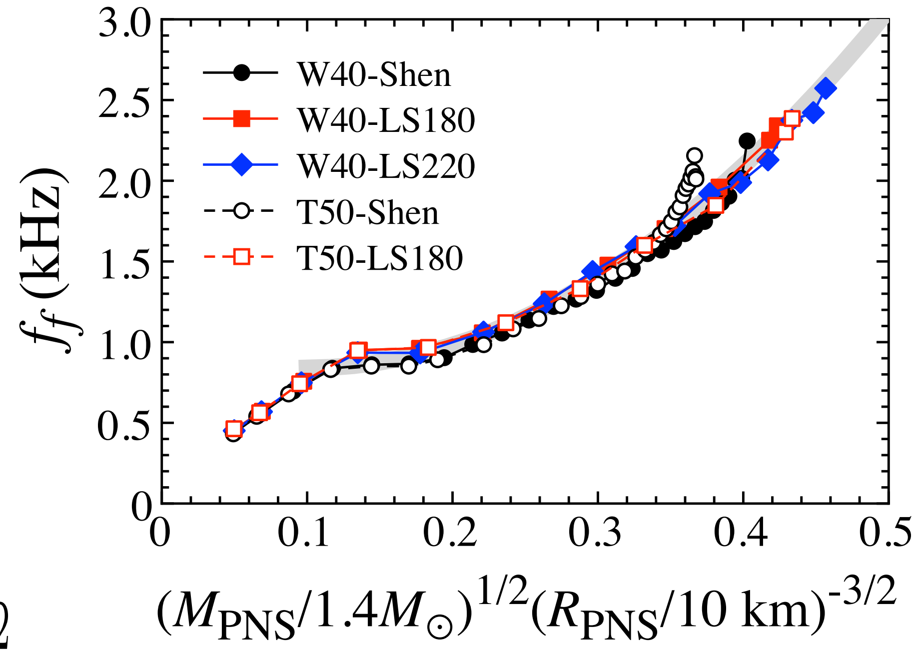



# Universality in f-mode GWs

- The f-mode frequencies are well-expressed as a function of stellar average density, <u>independently of progenitor models.</u>

$$f_f(\mathrm{kH}) = 0.9733 - 2.7171X + 13.7809X^2$$

$$X \equiv \left(M_{\mathrm{PNS}}/1.4M_\odot\right)^{1/2}\left(R_{\mathrm{PNS}}/10\,\mathrm{km}\right)^{-3/2}$$




- Through the f-mode GW obs., one can extract the PNS average density, which leads to the time evolution of PNS average density.

# For PNS with maximum mass

- PNS <u>at the moment when it collapses to BH</u>, corresponds to the PNS model with maximum mass.↑

  one can know via neutrino observation

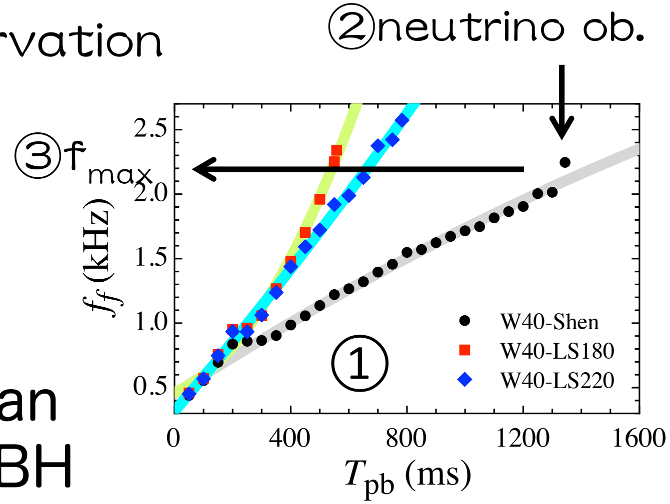
- How to determine the PNS property

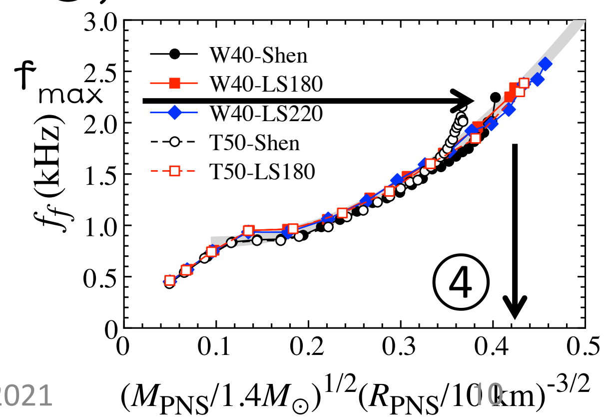
① With the data of the f-mode GW, one can fit the time evolution of the f-mode GW

② Owning to the neutrino observation, one can know the moment when PNS collapses to BH

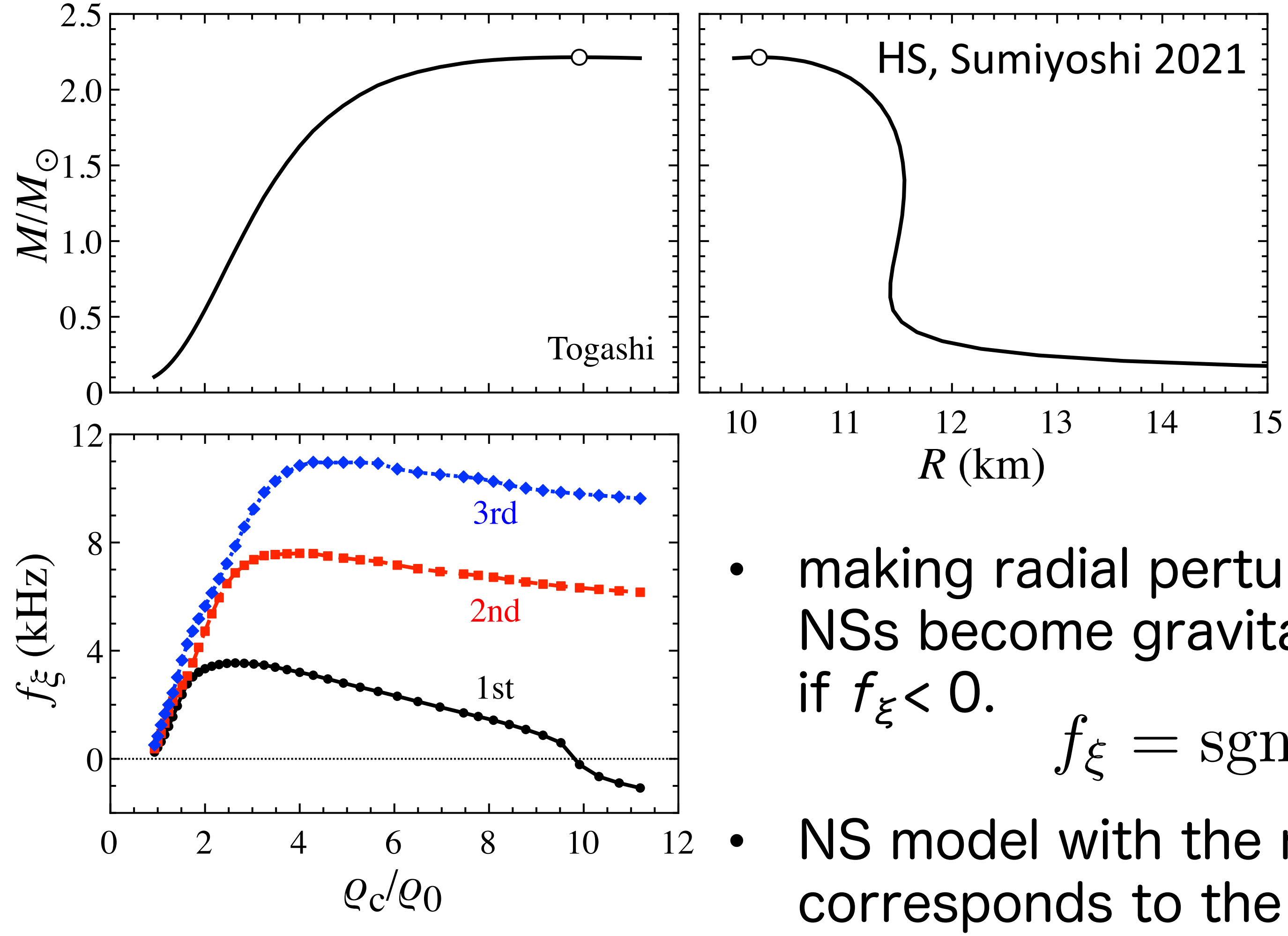
③ The f-mode frequency is expected via ① and ②, <u>even if the f-mode freq. at the final phase would not be detected</u>.

④ Via the universal relation of the f-mode, one can extract the average density of PNS with maximum mass



# Stability analysis for cold NSs



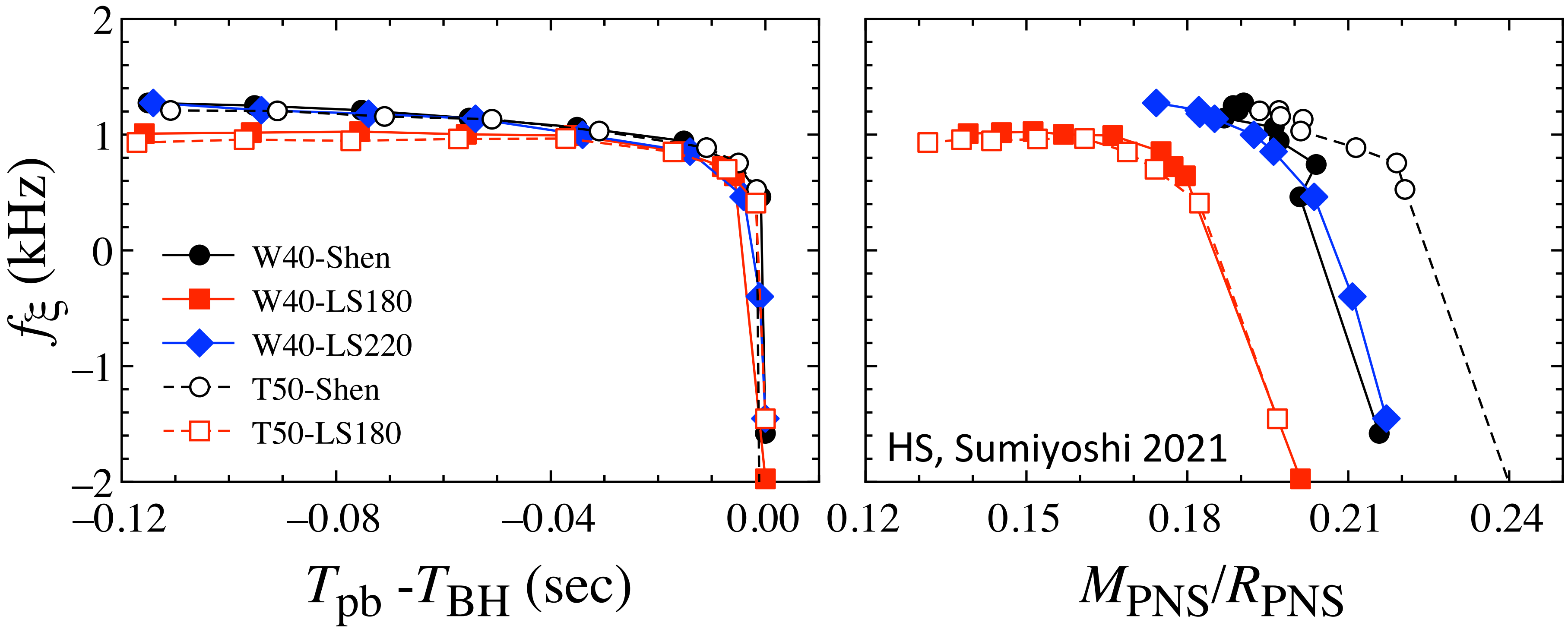

- making radial perturbation analysis, NSs become gravitationally unstable, if $f_\xi < 0$.

$$f_\xi = \mathrm{sgn}(\omega^2)\sqrt{|\omega^2|}/2\pi$$

- NS model with the maximum mass corresponds to the onset of instability.

# Stability of PNS @final phase

- before the apparent horizon appears inside the PNS, the PNS seems to become gravitationally unstable



# summary

- we examine the GW freq. from PNSs
- f- & $g_1$-modes in later phase are almost independent of $\rho_s$
- $g_1$-mode frequency decreases with time, which is related to the decrease of $f_{BV}$ inside the PNS
- GW signals in numerical simulations correspond to $g_1$- & f-modes
  - we find the empirical formula for GW signals
  - via the GW observations, one could extract the PNS average density
  - we should check the universality
- Owning to the neutrino observation, one would determine the average density of PNS with maximum mass by detecting the f-mode GW.
- PNS becomes gravitationally unstable before the apparent horizon appears inside the PNS.
- We will taken into account the effect of the radial velocity as background properties.