原子核構造で探る中性子星

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August 10, 2021
〜中性子星の観測と理論〜研究活性化ワークショップ 2021 (Online)
理化学研究所

SCHOOL OF SCIENCE
THE UNIVERSITY OF TOKYO

RIKEN
NiSHiNA
CENTER

原子核構造で探る~~中性子星~~ 核物質

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Self Introduction

My Research Topics—**First-Principles Quantum Many-Body Problems**

- Research Tools: **Density Functional Theory (密度汎関数理論)**
  —A tool to calculate ground-state wave functions
- Target Systems: **Atomic Nuclei, Atoms, Molecules, Solids**
  —Any quantum many-body systems
- Recent Research Topics
  - Chemical properties of super-heavy elements
  - Isospin sym. breaking of nuclear int. and Coulomb int. in atomic nuclei
  - Charge distribution of atomic nuclei
  - Interplay between electronic systems, muons, and atomic nuclei ($\beta^{\pm}$-decay, electron/muon capture, etc)
  - Fundamental studies of density functional theory
  - ...

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- Today, I will talk my research topics not far from neutron star research

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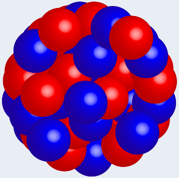
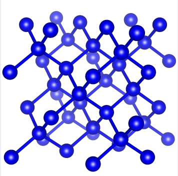
What Are Quantum Many-body Problems (量子多体問題)?



Atomic Nuclei

Particles	Protons and neutrons
Interaction	Nuclear interaction Coulomb interaction
External field	None
# of particles	$\lesssim 300$



Atoms, Moleculs, and Solids

Particles	Electrons
Interaction	Coulomb interaction
External field	Coulomb fields (formed by atomic nuclei)
# of particles	$O(10)$–$O(10000)$

- Many-particle systems which obey Schrödinger or Dirac equation
- It is impossible to solve Schrödinger or Dirac equation directly
  → Efficient method to solve the equation is required

Right figure is drawn by using VESTA

Introduction

How to Solve Quantum Many-Body Problems?

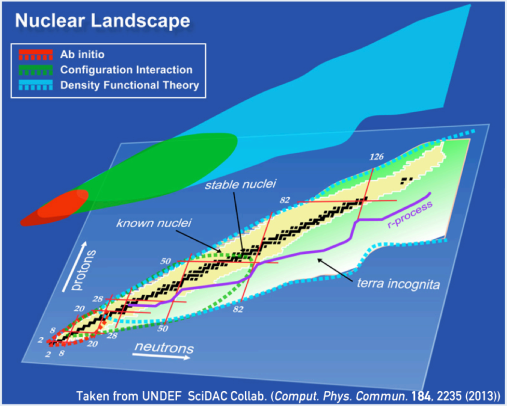
- Our motivation is to calculate w.f. (or density), energy spectra, . . . without introducing any model (at a minimum)
- First, let me focus on ground-state properties
- Such methods are called “microscopic methods”, which can be divided into two classes

Wave function methods	Derive Ψ_{gs} and E_{gs} directly High accuracy but huge numerical costs (e.g. post Hartree-Fock, quantum Monte Carlo)
Density functional theory	Variation of energy density functional Problems are truncated into non-int. systems Low numerical costs but moderate accuracy

Difficulty in Nuclear Structure Calculation

- Nuclear force in vacuum is well known (scat. exp., lat. QCD, χEFT)
- Nuclear force in medium is different from bare one, and its detail form is still under discussion

Applicable Range of Many-Body Methods in Nuclear Chart

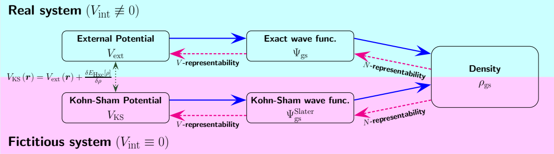


Taken from UNDEF SciDAC Collab. (*Comput. Phys. Commun.* **184**, 2235 (2013))

At this moment,
only DFT can be applied to
(almost) the whole nuclear chart

What Are Density Functional Theory (密度汎関数理論)?

- Proposed by Hohenberg, Kohn, and Sham in 1964–1965
- Idea—Combination of Thomas-Fermi approx. and Hartree-Fock calc.
- Basic Theory—**Hohenberg-Kohn Theorem**
  ① There is one-to-one correspondence between ρ_{gs} and V_{ext}
  Key point is V_{ext} can be uniqly specified once ρ_{gs} is known
  ② G.S. energy can be expressed as $E_{\mathrm{gs}} = \int V_{\mathrm{ext}}\left(\boldsymbol{r}\right) \rho_{\mathrm{gs}}\left(\boldsymbol{r}\right) \, d\boldsymbol{r} + F\left[\rho_{\mathrm{gs}}\right]$,
  where F is universal functional w.r.t. V_{int}
- Practical Method—**Kohn-Sham Scheme**



Real system $(V_{\mathrm{int}} \not\equiv 0)$

External Potential V_{ext}

Exact wave func. Ψ_{gs}

Density ρ_{gs}

V-**representability**

N-**representability**

$V_{\mathrm{KS}}\left(\boldsymbol{r}\right) = V_{\mathrm{ext}}\left(\boldsymbol{r}\right) + \frac{\delta E_{\mathrm{Hxc}}[\rho]}{\delta \rho}$

Kohn-Sham Potential V_{KS}

Kohn-Sham wave func. $\Psi_{\mathrm{gs}}^{\mathrm{Slater}}$

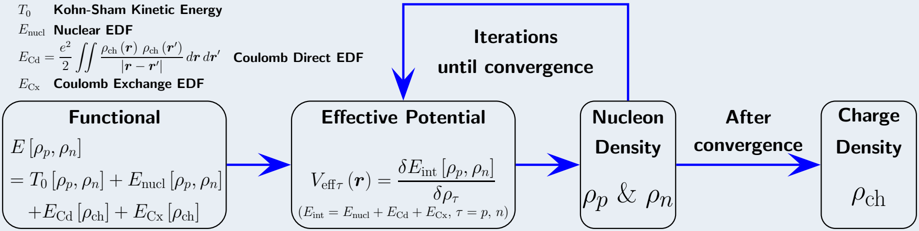
V-**representability**

N-**representability**

Fictitious system $(V_{\mathrm{int}} \equiv 0)$

- Since problems are truncated into "non-interacting fictitious system" using E_{Hxc}, which includes information of V_{int},
  numerical cost is drastically reduced
- E_{Hxc} governs accuracy of DFT calculation

How to Solve Ψ_{gs} in Nuclear DFT?



T_0 **Kohn-Sham Kinetic Energy**

E_{nucl} **Nuclear EDF**

$E_{\mathrm{Cd}} = \frac{e^2}{2} \iint \frac{\rho_{\mathrm{ch}}(\boldsymbol{r})\ \rho_{\mathrm{ch}}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}\, d\boldsymbol{r}\, d\boldsymbol{r}'$ **Coulomb Direct EDF**

E_{Cx} **Coulomb Exchange EDF**

Functional

$$E[\rho_p, \rho_n] = T_0[\rho_p, \rho_n] + E_{\mathrm{nucl}}[\rho_p, \rho_n] + E_{\mathrm{Cd}}[\rho_{\mathrm{ch}}] + E_{\mathrm{Cx}}[\rho_{\mathrm{ch}}]$$

Effective Potential

$$V_{\mathrm{eff}\,\tau}(\boldsymbol{r}) = \frac{\delta E_{\mathrm{int}}[\rho_p, \rho_n]}{\delta \rho_\tau}$$

$(E_{\mathrm{int}} = E_{\mathrm{nucl}} + E_{\mathrm{Cd}} + E_{\mathrm{Cx}},\ \tau = p,\ n)$

Iterations until convergence

Nucleon Density

ρ_p & ρ_n

After convergence

Charge Density

ρ_{ch}

- In DFT for electronic systems, EDF can be derived theoretically
- Nuclear DFT includes to HF calc. with δ-type effective int. (Skyrme int.)
- Other types of density-dep. int. (Gogny, relativistic, etc)
  are also referred to as “DFT” for simplicity
- Although bare nuclear int. is known experimentally or theoretically, nuclear int. in medium is not known well
- Thus, in DFT for nuclear systems, E_{nucl} is fitted to experimental data

Skyrme Hartree-Fock / Skyrme DFT

- Ground-state energy density can be written as ($E_{\mathrm{int}} = \int \mathcal{E}_{\mathrm{int}}\left(\boldsymbol{r}\right)\, d\boldsymbol{r}$)

$$
\begin{aligned}
\mathcal{E}_{\mathrm{nucl}}\left[\rho_p, \rho_n\right] = {} & \frac{t_0}{2}\left[\left(\frac{x_0}{2}+1\right)\rho^2 - \left(x_0 + \frac{1}{2}\right)\sum_{\tau}\rho_{\tau}^2\right] + \frac{t_1}{4}\left[\left(\frac{x_1}{2}+1\right)\rho t - \left(x_1 + \frac{1}{2}\right)\sum_{\tau}\rho_{\tau} t_{\tau}\right] \\
& + \frac{t_2}{4}\left[\left(\frac{x_2}{2}+1\right)\rho t - \left(x_2 + \frac{1}{2}\right)\sum_{\tau}\rho_{\tau} t_{\tau}\right] - \frac{3t_1}{16}\left[\left(\frac{x_1}{2}+1\right)\rho\,\Delta\rho + \left(x_1 + \frac{1}{2}\right)\sum_{\tau}\rho_{\tau}\,\Delta\rho_{\tau}\right] \\
& + \frac{t_2}{16}\left[\left(\frac{x_2}{2}+1\right)\rho\,\Delta\rho + \left(x_2 + \frac{1}{2}\right)\sum_{\tau}\rho_{\tau}\,\Delta\rho_{\tau}\right] + \frac{t_3}{12}\left[\left(\frac{x_3}{2}+1\right)\rho^2 - \left(x_3 + \frac{1}{2}\right)\sum_{\tau}\rho_{\tau}^2\right]\rho^{\alpha} \\
& - \frac{\theta_{\mathrm{SO}}}{8}\left(t_1 x_1 + t_2 x_2\right)\sum_{\tau}\boldsymbol{J}_{\tau}^2 - \frac{\theta_{\mathrm{SO}}}{16} t_1\left(x_1 - 1\right) + t_2\left(x_2 + 1\right)\boldsymbol{J}_n \cdot \boldsymbol{J}_p \\
& - \frac{W_0}{2}\rho\nabla\cdot\boldsymbol{J} - \frac{W_0'}{2}\sum_{\tau}\rho_{\tau}\cdot\boldsymbol{J}_{\tau}
\end{aligned}
$$

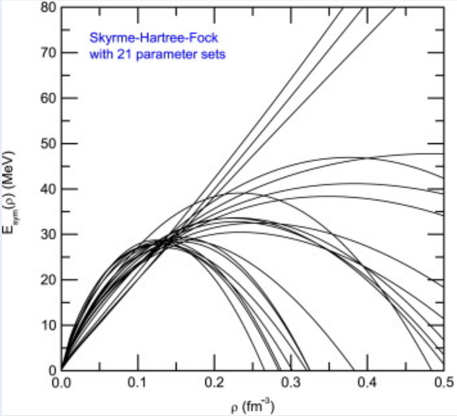
$\tau = p, n$, $\rho\left(\boldsymbol{r}\right) = \sum\left|\varphi_j\left(\boldsymbol{r}\right)\right|^2$, $t\left(\boldsymbol{r}\right) = \sum\left|\nabla\varphi_j\left(\boldsymbol{r}\right)\right|^2$, $\boldsymbol{J}\left(\boldsymbol{r}\right) = \sum\varphi^{\dagger}\left(\boldsymbol{r}\right)\boldsymbol{\sigma}\times\nabla\varphi_j\left(\boldsymbol{r}\right)$

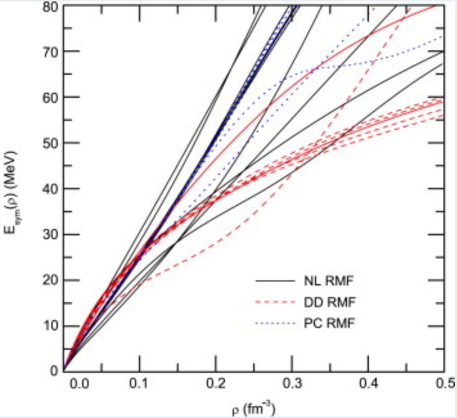
- Parameters are determined to satisfy nuclear EoS (e.g. APR) and/or experimental binding energies, charge radii, etc

Skyrme. *Nucl. Phys.* **9**, 615 (1958)
Vautherin and Brink. *Phys. Rev. C* **5**, 626 (1972)
Reinhard, Dean, Nazarewicz *et al.* *Phys. Rev. C* **60**, 014316 (1999)

Skyrme Zoo?

- Although “form” of Skyrme EDF is unique, there are more than 200 of Skyrme EDF in the market
- Different Skyrme EDF predict different E_{sym} even around ρ_{sat}
- There are many attempts to derive EDF theoretically





Li, Chen, and Ko. *Phys. Rep.* **464**, 113 (2008)

Introduction

EDF as A Way to Connect Theories and Experiments

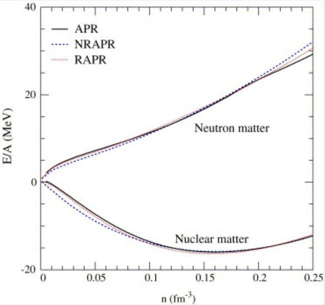
- Since EDFs are determined to reproduce properties of finite nuclei, EDF includes “information” of finite nuclei
  eg. EoS properties around saturation density
- DFT calculation using such empirical EDFs can be used to connect theoretical calculations and experiments

Nuclear Structure and Neutron Star

- Nuclear Equation of State
  - Properties (ρ vs E) of infinite nuclear matter
  - One of inputs of calculation of NS properties
- Although nuclear EoS is an important input of NS calculation, infinite nuclear matter is fictitious system
- Study of atomic nuclei helps to understand nuclear matter
  We can “check” whether EoS prediction is correct

Introduction

Nuclear EoS and Symmetry Energies



$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,0) + E_{\mathrm{sym}}(\rho)\beta^2 + \dots$$

$$E_{\mathrm{sym}}(\rho) = J + L\frac{\rho-\rho_{\mathrm{sat}}}{3\rho_{\mathrm{sat}}} + \frac{K_{\mathrm{sym}}}{2}\left(\frac{\rho-\rho_{\mathrm{sat}}}{3\rho_{\mathrm{sat}}}\right)^2 + \dots$$

Total density $\rho = \rho_p + \rho_n$

Isospin asymmetry $\beta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$

Symmetry energy E_{sym}

Saturation density $\rho_{\mathrm{sym}} \simeq 0.16\,\mathrm{fm}^{-3}$

- L corresponds to slope of nuclear matter EoS around ρ_{sat}
- K_{sym} corresponds to curvature of nuclear matter EoS around ρ_{sat}

Steiner, Prakash, Lattimer, and Ellis. *Phys. Rep.* **411**, 325 (2005)

Introduction

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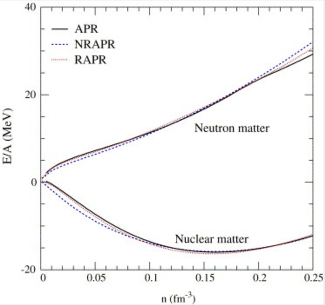

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- L corresponds to slope of nuclear matter EoS around ρ_{sat}
- K_{sym} corresponds to curvature of nuclear matter EoS around ρ_{sat}
- Question: Can we “measure” such parameters??

Steiner, Prakash, Lattimer, and Ellis. *Phys. Rep.* **411**, 325 (2005)

Recent Experiments toward Nuclear EoS

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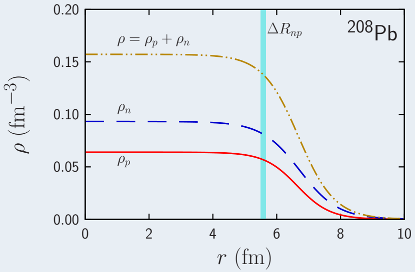
Recent Experiments toward Nuclear EoS

Basic Ideas of Nuclear Interaction

- $V_{pp} = V_{nn} = V_{pn}^{T=1}$ (Isospin symmetry)
- Due to isospin sym., $N = Z$ is the most stable if no Coulomb int.

Basic Ideas of Nuclear Properties



- $\rho \simeq \rho_{\text{sat}} \approx 0.16\,\text{fm}^{-3}$ at core region
- If no Coulomb interactoin
  $\rightarrow N = Z$ are stable $\rightarrow R_p = R_n$
- Due to Coulomb interaction
  $\rightarrow N > Z$ are stable at heavy nucl.
  $\rightarrow R_p < R_n$
- $\Delta R_{np} = R_n - R_p$: Neutron skin

- Is ΔR_{np} related to nuclear EoS?

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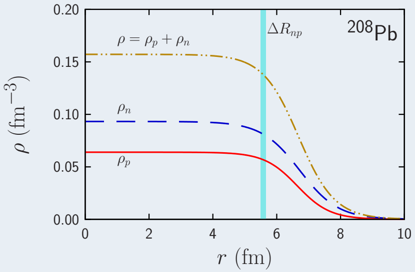

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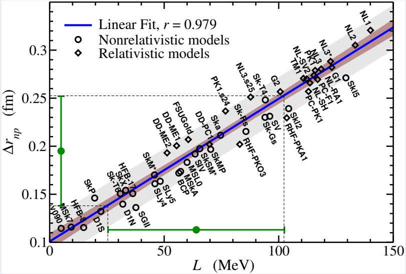
- $\Delta R_{np} = R_n - R_p$: Neutron skin

- Is ΔR_{np} related to nuclear EoS? **Yes!**

L parameter vs ΔR_{np}



- Different EDF assumes different L
- Different EDF predicts different ΔR_{np}
- L and ΔR_{np} are highly correlated

Myers and Swiatecki. *Ann. Phys.* **55**, 395 (1969)

Roca-Maza, Centelles, Viñas, and Warda. *Phys. Rev. Lett.* **106**, 252501 (2011)

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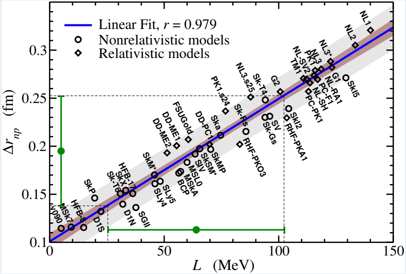

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- Once ΔR_{np} is measured,
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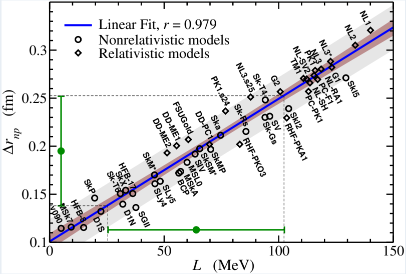

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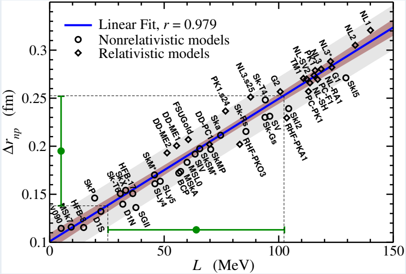

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- Why L and ΔR_{np} are correlated?
- Let us assume all EDFs give the same R_p (due to fitting)
- As L increases, pressure of outermost neutrons increases

  (Pressure is propotional to L, if ρ is fixed)

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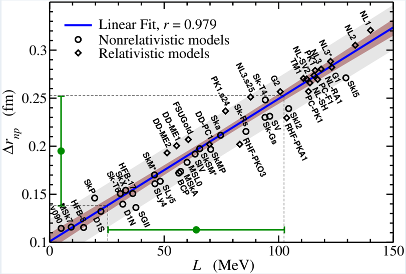

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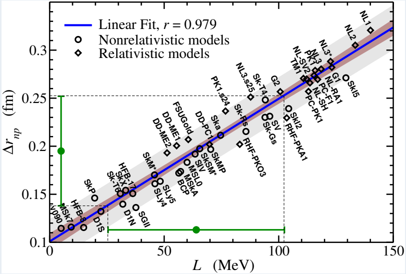

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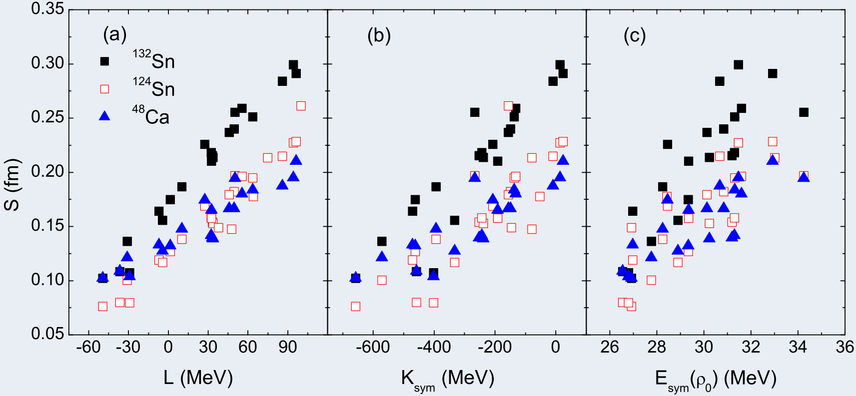
  (Pressure is propotional to L, if ρ is fixed)
- Neutrons prefer to extend as L increases → R_n (i.e., ΔR_{np}) increases

Myers and Swiatecki. *Ann. Phys.* **55**, 395 (1969)

Roca-Maza, Centelles, Viñas, and Warda. *Phys. Rev. Lett.* **106**, 252501 (2011)

Recent Experiments toward Nuclear EoS

How about Other Parameters?



- Correlations between J vs ΔR_{np} and K_{sym} vs ΔR_{np} are weak?

Chen, Ko, and Li. *Phys. Rev. C* **72**, 064309 (2005)

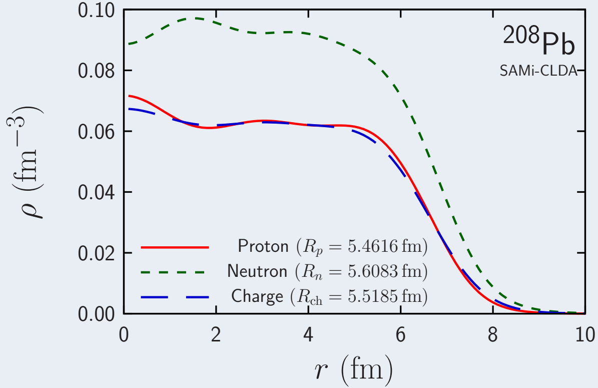
How Can ΔR_{np} Measured?

- **Hadron Scat.** Matter density distribution $\rho = \rho_p + \rho_n$, ρ_n etc
  - **Pros** Direct measurement of ρ_τ
  - **Cons** Model dependent (interaction)
- **Electron Scat.** Charge form factor $F_{\text{ch}} = \tilde{\rho}_{\text{ch}}$
  - **Pros** Direct measurement of ρ_{ch}, no theo. ambiguity
  - **Cons?** $\rho_{\text{ch}} \to \rho_p$ and ρ_n (see next slide)
- **Laser spec.** Charge radius R_{ch}
  - **Pros** High precision, no theo. ambiguity
  - **Cons?** Only relative values are measured
- **Magnetic Scat.** Magnetic form factor F_{M}
  - **Pros** No theo. ambiguity
  - **Cons** Only sensitive to valence nucleons
- **PV Scat.** Weak form factor F_{wk} or weak radius R_{wk}?
  - **Pros** Accesible to ρ_n, no theo. ambiguity?
  - **Cons** Difficult measurement (long beam time)

Proton Density and Charge Density

- Nucleons have “finite” charge/weak densities (radii)
- ρ_τ is distribution of “nucleon CoM”,
  i.e., nucleons are assumed to be point particles
- ρ_{ch} and ρ_{wk} include finite-size effects of nucleons

$$\tilde{\rho}_{\mathrm{ch}}(q) = \tilde{G}_{\mathrm{E}p}(q)\,\tilde{\rho}_p(q) + \tilde{G}_{\mathrm{E}n}(q)\,\tilde{\rho}_n(q)$$



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  i.e., nucleons are assumed to be point particles
- ρ_{ch} and ρ_{wk} include finite-size effects of nucleons
  $$\tilde{\rho}_{\mathrm{ch}}(q) = \tilde{G}_{\mathrm{E}p}(q)\,\tilde{\rho}_p(q) + \tilde{G}_{\mathrm{E}n}(q)\,\tilde{\rho}_n(q)$$
- Correspondingly, R_{ch} and R_{wk} are different from R_p and R_n
  $$R_{\mathrm{ch}}^2 = R_p^2 + r_{\mathrm{E}p}^2 + \frac{N}{Z} r_{\mathrm{E}n}^2 + \frac{1}{M^2 Z} \sum_{a\tau} \kappa_\tau \mathcal{N}_{a\tau} \langle \boldsymbol{l} \cdot \boldsymbol{\sigma} \rangle$$
  $$R_{\mathrm{wk}}^2 = \frac{Z Q_{\mathrm{wk}p}}{Z Q_{\mathrm{wk}p} + N Q_{\mathrm{wk}n}} \left(R_p^2 + r_{\mathrm{wk}p}^2 \right) + \frac{N Q_{\mathrm{wk}n}}{Z Q_{\mathrm{wk}p} + N Q_{\mathrm{wk}n}} \left(R_n^2 + r_{\mathrm{wk}n}^2 \right) + R_{\mathrm{wkSO}}^2$$
  $r_{\mathrm{E}\tau}$: Nucleon charge radius, $r_{\mathrm{wk}\tau}$: Nucleon weak radius, $Q_{\mathrm{wk}p} \approx 0.0710$, $Q_{\mathrm{wk}n} \approx -0.9902$
- $\rho_{\mathrm{ch}} \to \rho_p$ and ρ_n is not easy
- $r_{\mathrm{E}p}$ is well known (despite proton radius puzzle), $r_{\mathrm{E}n}^2$ is known indirectly
  → R_p can be extract from R_{ch}

Horowitz and Piekarewicz. *Phys. Rev. C* **86**, 045503 (2012)

Proton Density and Charge Density

- Nucleons have “finite” charge/weak densities (radii)
- ρ_τ is distribution of “nucleon CoM”,
  i.e., nucleons are assumed to be point particles
- ρ_{ch} and ρ_{wk} include finite-size effects of nucleons

$$\tilde{\rho}_{\mathrm{ch}}(q) = \tilde{G}_{\mathrm{E}p}(q)\,\tilde{\rho}_p(q) + \tilde{G}_{\mathrm{E}n}(q)\,\tilde{\rho}_n(q)$$

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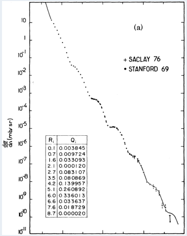
Measurements of ρ_{ch}, R_{ch}, R_{wk}

ρ_{ch} Many stable nuclei have been measured by 1980s
Unstable nuclei will be (planned to be) measured at RIKEN and GSI
$d\sigma/d\Omega \sim F_{\mathrm{ch}}$ for wide range of q is measured

R_{ch} Many nuclei includes unstable nuclei are available



R_i	Q_i
0.1	0.003845
0.7	0.009724
1.6	0.033093
2.1	0.000120
2.7	0.083107
3.5	0.080869
4.2	0.139957
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Frois *et al. Phys. Rev. Lett.* **38**, 152 (1977)

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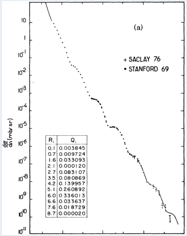
R_{wk} $A_{\mathrm{PV}} = \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}}$ is measured for only one $q = 0.00616\,\mathrm{GeV}$
→ Using theoretical A_{PV}-R_{wk}, ΔR_{np} relation, R_{wk} & ΔR_{np} are extracted



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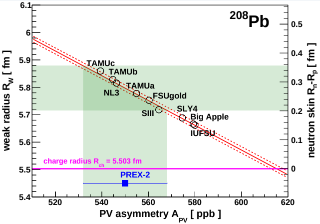

Frois *et al.* *Phys. Rev. Lett.* **38**, 152 (1977)

Adhikari *et al.* (PREX Collab.) *Phys. Rev. Lett.* **126**, 172502 (2021)

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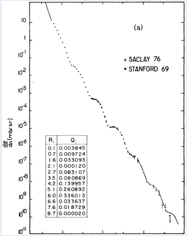

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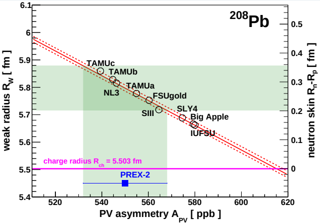
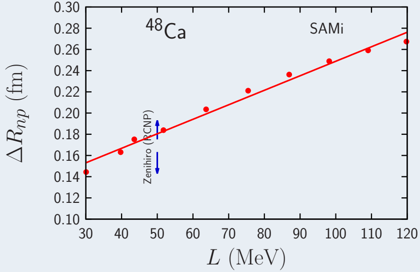


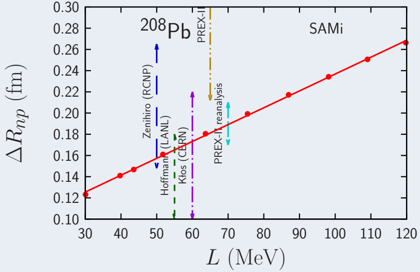
- They did NOT measure R_{wk} or ρ_{wk}

Frois *et al. Phys. Rev. Lett.* **38**, 152 (1977)

Adhikari *et al.* (PREX Collab.) *Phys. Rev. Lett.* **126**, 172502 (2021)

Estimate L Parameter





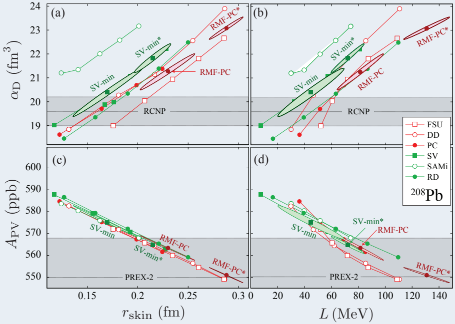
Ca 0.168 ± 0.025 fm (41 ± 18 MeV RCNP)

Pb 0.140 ± 0.040 fm (40 ± 25 MeV LANL), 0.160 ± 0.060 fm (52 ± 38 MeV CERN)
$0.211^{+0.054}_{-0.063}$ fm (84 ± 40 MeV RCNP), 0.283 ± 0.071 fm (129 ± 45 MeV PREX-II)

- PREX-II ΔR_{np} (and L) is too large compared to hadron scat. one, although some data are overlapped a little ($0.21 < \Delta R_{np} < 0.22$ fm)
- Effect of Coulomb to L-ΔR_{np}
- Effect of ISB to L-ΔR_{np}

Naito, Colò, Liang, Roca-Maza, and Sagawa. *To Be Submitted*

Recent Analysis on PREX-II

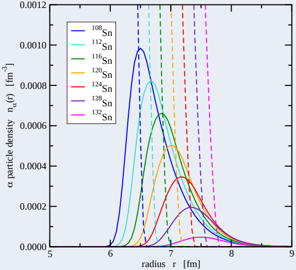


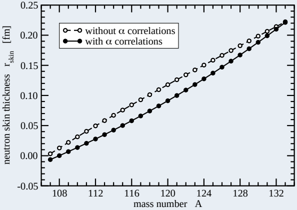
- Refitted functionals with different J (DD, PC, RD, SV), A_{PV} and α_{D} are re-analized
- Obtained results: $\Delta R_{np} = 0.19 \pm 0.02\,\mathrm{fm}$, $L = 54 \pm 8\,\mathrm{MeV}$
  → compatible to previous results

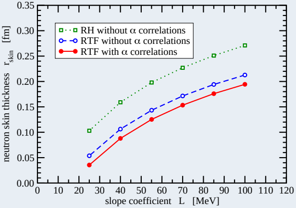
Reinhard, Roca-Maza, and Nazarewicz. arXiv:2105.15050 [nucl-th]

Recent Experiments toward Nuclear EoS

Another Ambiguity?—α-Particle Formation at Surface?







- α particle forms at surface → ΔR_{np} becomes smaller
- Experimentally, it was confirmed in Sn isotopes (RCNP)

Typel. *Phys. Rev. C* **89**, 064321 (2014)

Tanaka *et al. Science* **371**, 260 (2021)

Recent Experiments toward Nuclear EoS

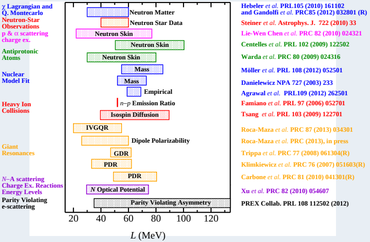
Another Proxy to Measure R_n?

- Kurasawa and Suzuki proposed that $Q_{\mathrm{ch}}^4 = \int \rho_{\mathrm{ch}}(r)\, r^4 \, d\boldsymbol{r}$ includes R_n
$$Q_{\mathrm{ch}}^4 = Q_p^4 + \frac{10}{3}\left(r_{\mathrm{E}p}^2 R_2^p + \frac{N}{Z} r_{\mathrm{E}n}^2 R_2^n\right) + q_{\mathrm{E}p}^4 + \frac{N}{Z} q_{\mathrm{E}n}^4 + Q_{\mathrm{chSO}}^4$$
Problem is that Q_p^4 cannot be determined experimentally
- Kurasawa, Suda, and Suzuki proposed that
Q_{ch}^4 and R_n^2 has strong correlation
and Q_{ch}^4 can be measured via electron scattering
→ R_n can be extracted from electron scattering
- We found that such correlation collapses once pairing is considered
- Extracting R_n from electron scattering is still under discussion

Kurasawa and Suzuki. *Prog. Theor. Exp. Phys.* **2019**, 113D01 (2019)
Kurawasa, Suda, and Suzuki. *Prog. Theor. Exp. Phys.* **2021**, 013D02 (2021)

Naito, Colò, Liang, and Roca-Maza. *Phys. Rev. C* **104**, 024316 (2021)

Other Experimental Progress



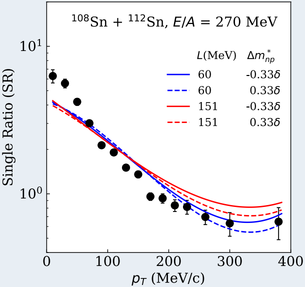
- Properties of excited states

  (isovector giant quadrupole resonance)

  and response to electric field

  (dipole polarizability)

  are also used to constrain L



- $\pi^{\pm}$ production on $\mathrm{Sn} + \mathrm{Sn}$ collision is measured
- Ratio of momentum spectra of π^+ and π^- depends on L and $\Delta m^*_{np}/\beta$

  (Calc: Quantum Molecular Dynamics)
- Estimated value: $42 < L < 117\,\mathrm{MeV}$

Viñas, Centelles, Roca-Maza, and Warda. *Eur. Phys. J. A* **50**, 27 (2014)

Estee *et al.* (SπRIT Collab.) *Phys. Rev. Lett.* **126**, 162701 (2021)

Summary

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- Estimate EoS parameters using nuclear properties is ongoing
- In my impression although PREX-II result is impressive, the values of R_{wk}, ΔR_{np}, and L are still under discussion
    - PREX-II did NOT measure R_{wk} but instead A_{PV}
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Thank you for attention!!