中性子星内部における
中性子P波超流動のスピン分極相

arXiv:2108.01256 [nucl-th]

S. Yasui (Keio U.)

with M. Nitta, D. Inotani (Keio U.), T. Mizushima (Osaka U.)

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中性子P波超流動のスピン分極相

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nuclear physics

S. Yasui (Keio U.)

particle physics &
condensed matter physics

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condensed matter physics

Contents

1. Neutron $^3P_2$ superfluid: introduction

2. Various phases of Neutron $^3P_2$ superfluid

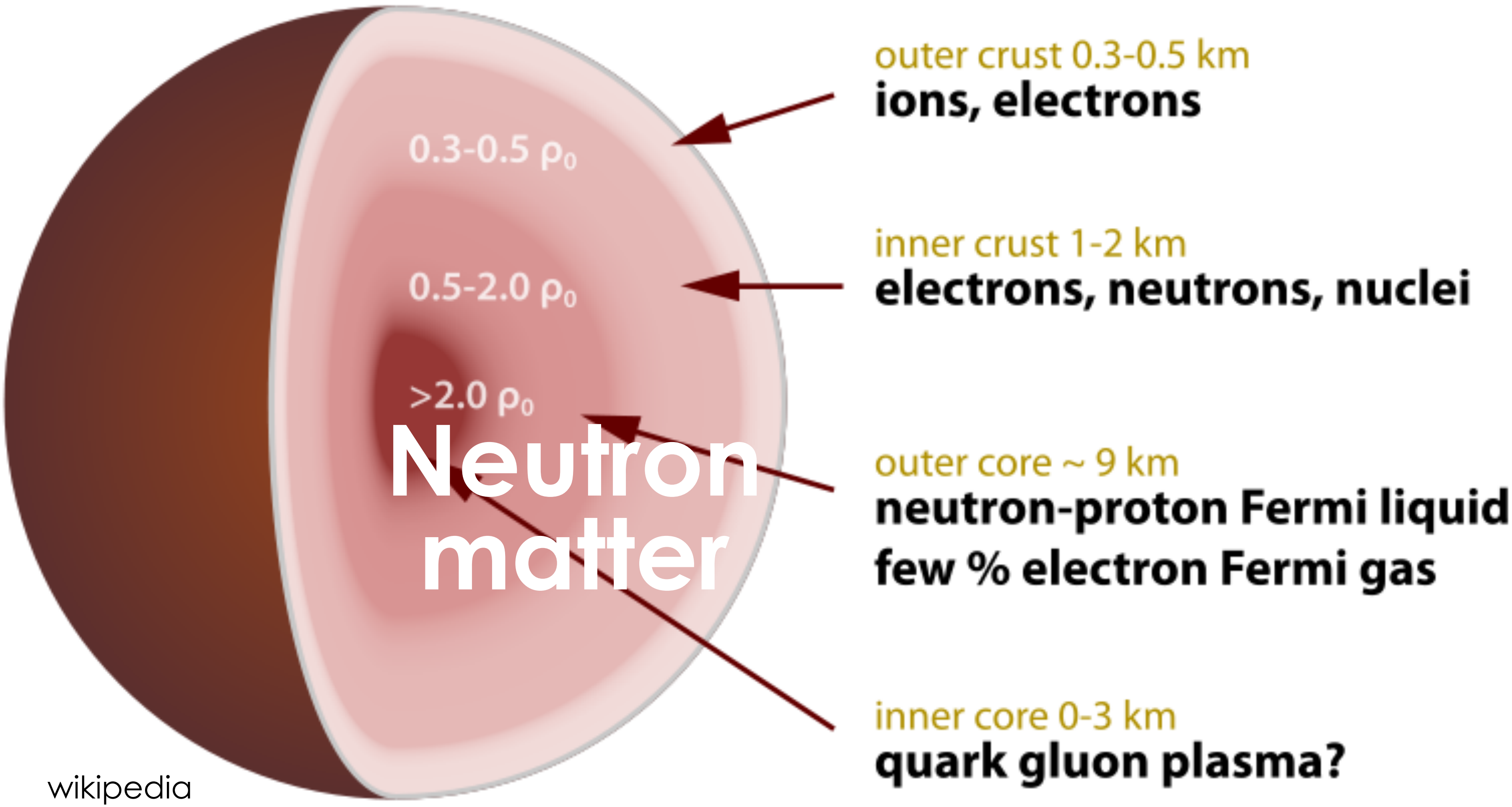
3. Spin polarized phase (today's topic)

4. Conclusion & perspectives

Contents

1. Neutron $^3P_2$ superfluid

Neutron $^3P_2$ superfluid



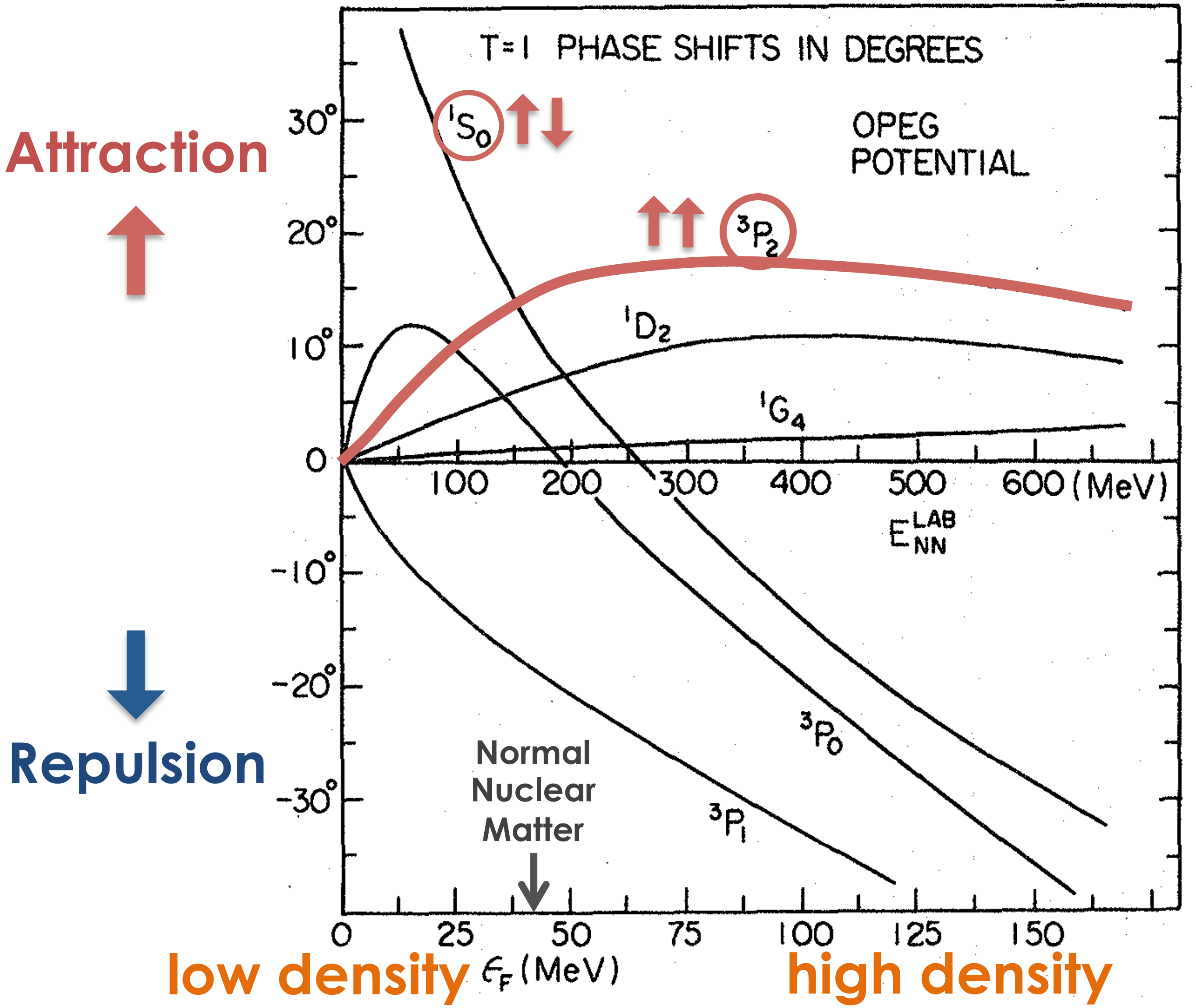
1. Neutron $^3P_2$ superfluid

Neutron-neutron (NN) interaction

Attraction? or Repulsion?

$^{2S+1}L_J$
S: total spin
L: angular momentum
J: total (L+S)

Takatsuka, Tamagaki, PTP Suppl. 112, 27 (1993)



1. Neutron $^3P_2$ superfluid

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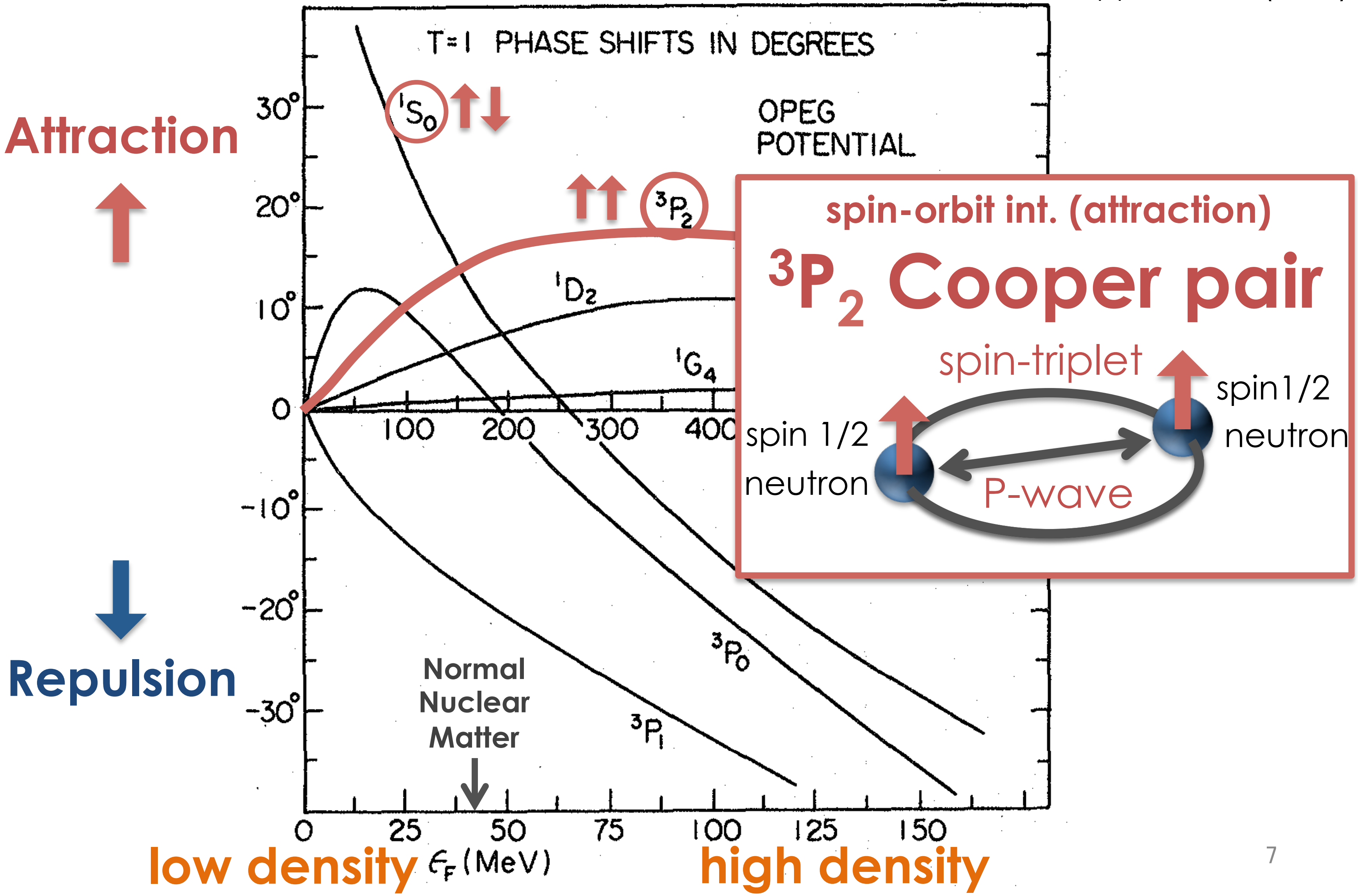
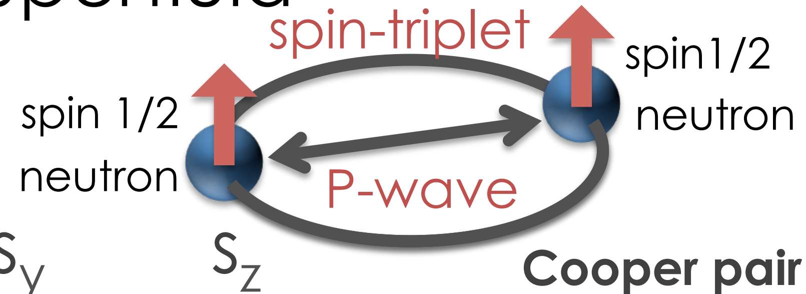


1. Neutron $^3P_2$ superfluid

Tabakin (1968), Hoffenberg, Glassgold, Richardson, Ruderman (1970), Tamagaki (1970), Takatsuka, Tamagaki (1971), Takatsuka (1972), ...

$$\boxed{{}^3P_2}$$



Order parameter
(neutron-neutron condensate)

$$A(t,\boldsymbol{x}) = A_0 \begin{array}{c} \begin{array}{ccc} s_x & s_y & s_z \end{array} \\ \begin{pmatrix} r & 0 & 0 \\ 0 & -(1+r) & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} \begin{array}{l} \\ q_x \\ q_y \\ q_z \end{array}$$

symmetric
traceless-tensor

spin × momentum

1. Neutron $^3P_2$ superfluid

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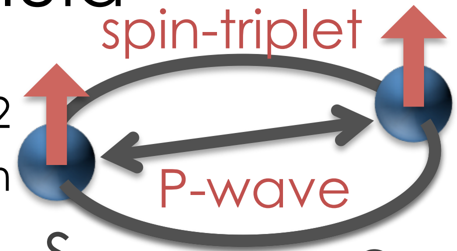

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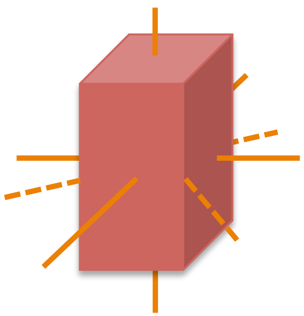
symmetric traceless-tensor

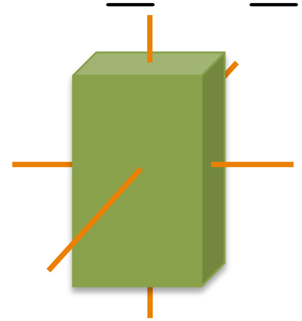
spin × momentum

Internal parameter: $-1 \leq r \leq -1/2$

D_4 

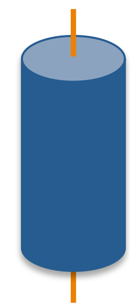
D_4-BN: D_4 biaxial nematic
(r=-1)

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

D_2 

D_2-BN: D_2 biaxial nematic
(-1<r<-1/2)

$$\begin{pmatrix} r & 0 & 0 \\ 0 & -1-r & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

O(2) 

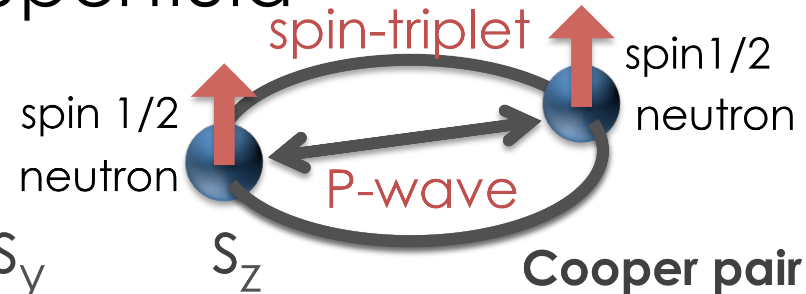
UN: uniaxial nematic
(r=-1/2)

$$\begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1. Neutron $^3P_2$ superfluid

Tabakin (1968), Hoffenberg, Glassgold, Richardson, Ruderman (1970), Tamagaki (1970), Takatsuka, Tamagaki (1971), Takatsuka (1972), ...

$$\boxed{{}^3P_2}$$



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(neutron-neutron condensate)

$$A(t,\boldsymbol{x}) = A_0 \begin{array}{c} \begin{array}{ccc} S_x & S_y & S_z \end{array} \\ \begin{pmatrix} r & 0 & 0 \\ 0 & -(1+r) & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} \begin{array}{l} \\ q_x \\ q_y \\ q_z \end{array}$$

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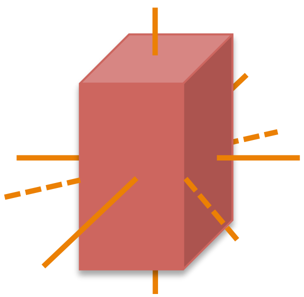
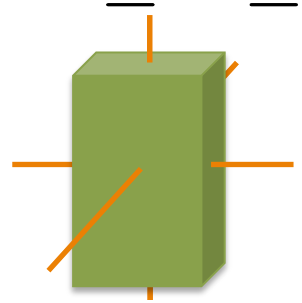
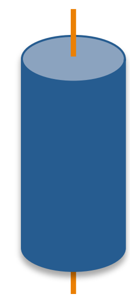
D_4 

D_2 

$O(2)$ 

Phase	H	G/H	π_0	π_1	π_2	π_3	π_4
UN	O(2)	$U(1)\times[SO(3)/O(2)]$	0	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}_2$
D_2 BN	D_2	$U(1)\times[SO(3)/D_2]$	0	$\mathbb{Z}\oplus\mathbb{Q}$	0	$\mathbb{Z}$	$\mathbb{Z}_2$
D_4 BN	D_4	$[U(1)\times SO(3)]/D_4$	0	$\mathbb{Z}\times_h D_4^*$	0	$\mathbb{Z}$	$\mathbb{Z}_2$

1. Neutron $^3P_2$ superfluid

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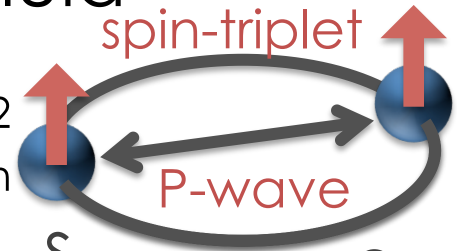
$$\boxed{^3P_2}$$



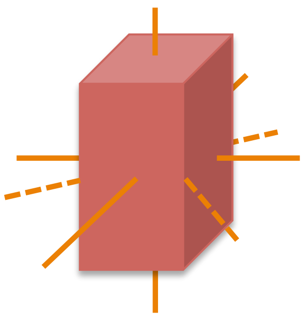
Order parameter
(neutron-neutron condensate)

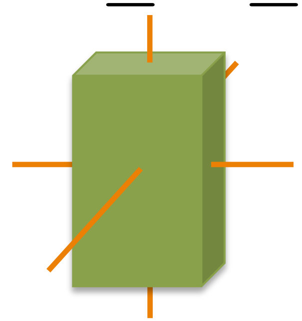
$$A(t,\boldsymbol{x}) = A_0 \begin{array}{c} \begin{array}{ccc} S_x & S_y & S_z \end{array} \\ \left(\begin{array}{ccc} r & 0 & 0 \\ 0 & -(1+r) & 0 \\ 0 & 0 & 1 \end{array}\right) \end{array} \begin{array}{c} \\ q_x \\ q_y \\ q_z \end{array}$$

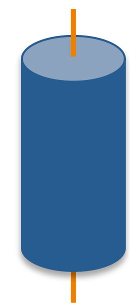
symmetric
traceless-tensor

spin × momentum

Internal parameter: $-1 \leq r \leq -1/2$

D_4 

D_2 

O(2) 

D_4-BN: D_4 biaxial nematic
(r=-1)

D_2-BN: D_2 biaxial nematic
(-1<r<-1/2)

UN: uniaxial nematic
(r=-1/2)

Topology

Quantized vortex: K. Masuda M. Nitta, PRC93, 035804 (2016)
Gapless Majorana fermions: T. Mizushima, K. Masuda, M. Nitta, PRB95, 140503 (2017)
Soliton on vortex: C. Chatterjee, M. Haberichter, M. Nitta, PRC96, 055807 (2017)
Half-quantized non-Abelian vortex: K. Masuda, M. Nitta, PTEP2020, 013D01 (2020)
Y. Masaki, T. Mizushima, M. Nitta, arXiv:2107.02448 [cond-mat.supr-con]
and more ...

5 min.

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1. Neutron ${}^3P_2$ superfluid: introduction

2. Various phases of Neutron ${}^3P_2$ superfluid

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2. Various phases of Neutron $^3P_2$ superfluid

Hamiltonian
Fermion theory

$$\mathcal{H} = \int d\boldsymbol{r}\psi_a^\dagger(\boldsymbol{r})\xi_{ab}(-i\boldsymbol{\nabla})\psi_b(\boldsymbol{r})$$
$$+\frac{1}{2}\int d\boldsymbol{r}_1\int d\boldsymbol{r}_2 \mathcal{V}_{a,b}^{c,d}(\boldsymbol{r}_{12})\psi_a^\dagger(\boldsymbol{r}_1)\psi_b^\dagger(\boldsymbol{r}_2)\psi_c(\boldsymbol{r}_2)\psi_d(\boldsymbol{r}_1)$$

attraction in $^3P_2$ channel

$$\xi(\boldsymbol{k}) = \xi_0(\boldsymbol{k}) - \frac{1}{2}\gamma_n\boldsymbol{\sigma}\cdot\boldsymbol{B}$$

spin-magnetic field int.

Bogoliubov-de Gennes (BdG) theory

Fermion

F. Tabakin, Single Phys. Rev. 174, 1208 (1968)
M. Hoffberg, A. E. Glassgold, R. W. Richardson, M. Ruderman, Phys. Rev. Lett. 24, 775 (1970)
R. Tamagaki, Progress of Theoretical Physics 44, 905 (1970)
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M. Baldo, O. Elgaroey, L. Engvik, M. Hjorth- Jensen, H. J. Schulze, Phys. Rev. C58, 1921 (1998)
V. V. Khodel, V. A. Khodel, J. W. Clark, Nucl. Phys. A679, 827 (2001)
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S. Maurizio, J. W. Holt, P. Finelli, Phys. Rev. C90, 044003 (2014)
S. K. Bogner, R. J. Furnstahl, A. Schwenk, Prog. Part. Nucl. Phys. 65, 94 (2010)
S. Srinivas and S. Ramanan, Phys. Rev. C94, 064303 (2016)
T. Mizushima, K. Masuda, M. Nitta, Phys. Rev. C93, 035804 (2016)
T. Mizushima, K. Masuda, M. Nitta, Phys. Rev. B95, 140503 (R) ()2017
T. Mizushima, S. Yasui, M. Nitta, Phys. Rev. Research2, 013194 (2020)
T. Mizushima, S. Yasui, D. Inotani, M. Nitta, arXiv:2108,01256 [nucl-th]
...

Ginzburg-Landau (GL) theory

Boson

R. W. Richardson, Phys. Rev. D5, 1883 (1972)
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P. Muzikar, J. A. Sauls, . W. Serene, Phys. Rev. D21, 1494 (1980)
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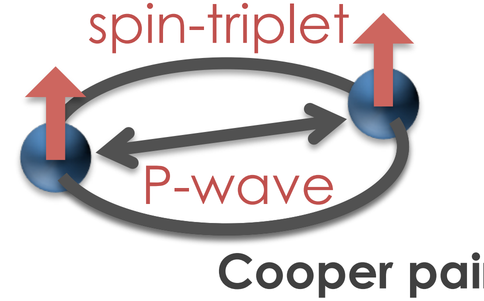
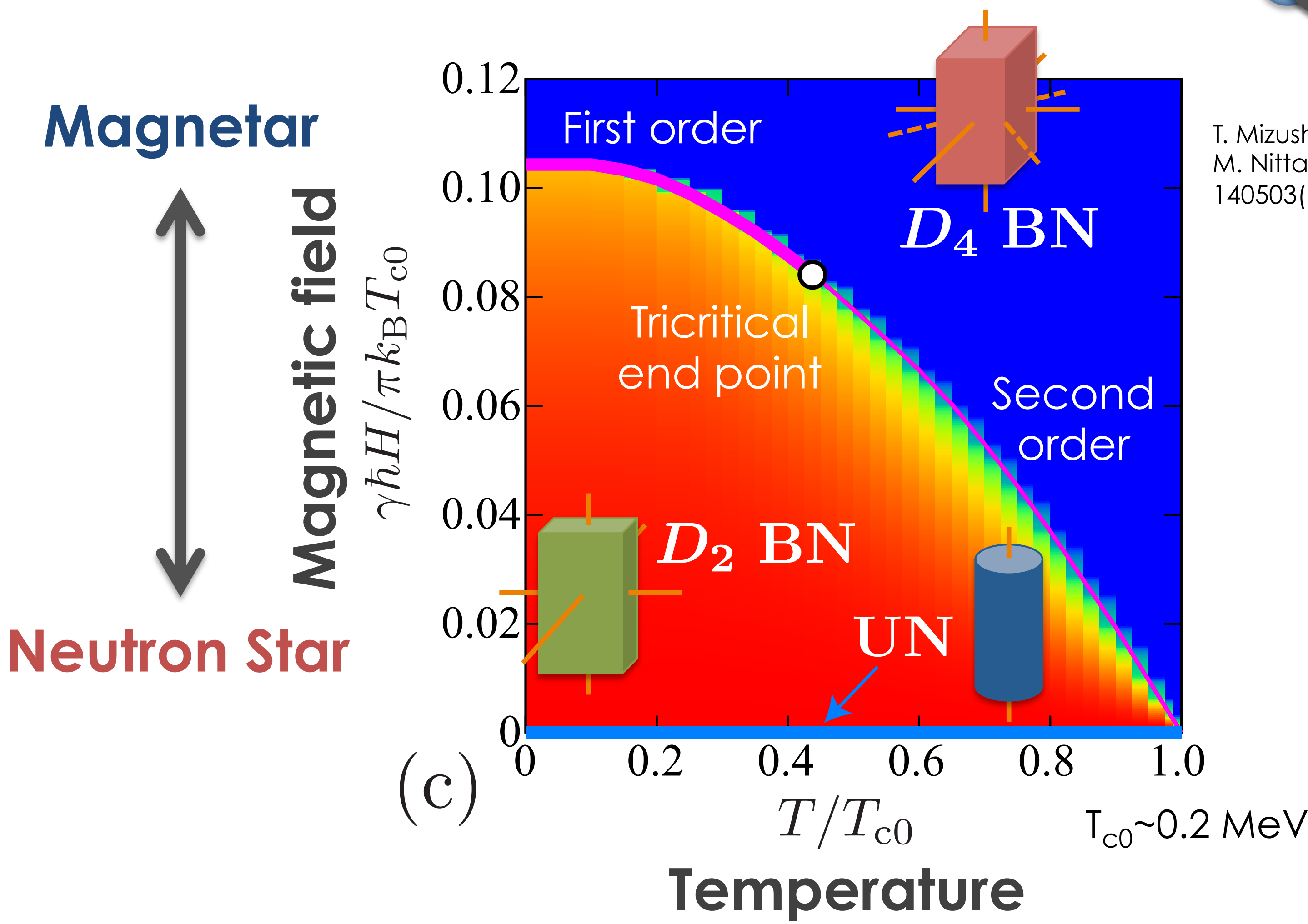
K. Masuda, M. Nitta, Phys. Rev. C93, 035804 (2016)
K. Masuda and M. Nitta, PTEP2020, 013D01 (2020)
S. Yasui, C. Chatterjee, and M. Nitta, Phys. Rev. C101, 025204 (2020)
S. Yasui, M. Nitta, Phys. Rev. C101, 015207 (2020)
S. Yasui, C. Chatterjee, M. Nitta, Phys. Rev. C99, 035213 (2019)
S. Yasui, C. Chatterjee, M. Kobayashi, M. Nitta, Phys. Rev. C100, 025204 (2019)
T. Mizushima, S. Yasui, M. Nitta, Phys. Rev. Research2, 013194 (2020)
S. Yasui, D. Inotani, M. Nitta, Phys. Rev. C101, 055806 (2020)
T. Mizushima, S. Yasui, D. Inotani, M. Nitta, arXiv:2108,01256 [nucl-th]
...

2. Various phases of Neutron $^3P_2$ superfluid

Bogoliubov-de Gennes (BdG) theory

$$\mathscr{H}(\boldsymbol{k}) = \begin{pmatrix} \varepsilon(\boldsymbol{k}) & i\boldsymbol{\sigma}\cdot\boldsymbol{d}(\boldsymbol{k})\sigma_2 \\ i\sigma_2\boldsymbol{\sigma}\cdot\boldsymbol{d}^*(-\boldsymbol{k}) & -\varepsilon^{\mathrm{T}}(-\boldsymbol{k}) \end{pmatrix}$$





T. Mizushima, K. Masuda, M. Nitta, Phys. Rev. B95, 140503(R) (2017)

2. Various phases of Neutron $^3P_2$ superfluid

Ginzburg-Landau (GL) theory

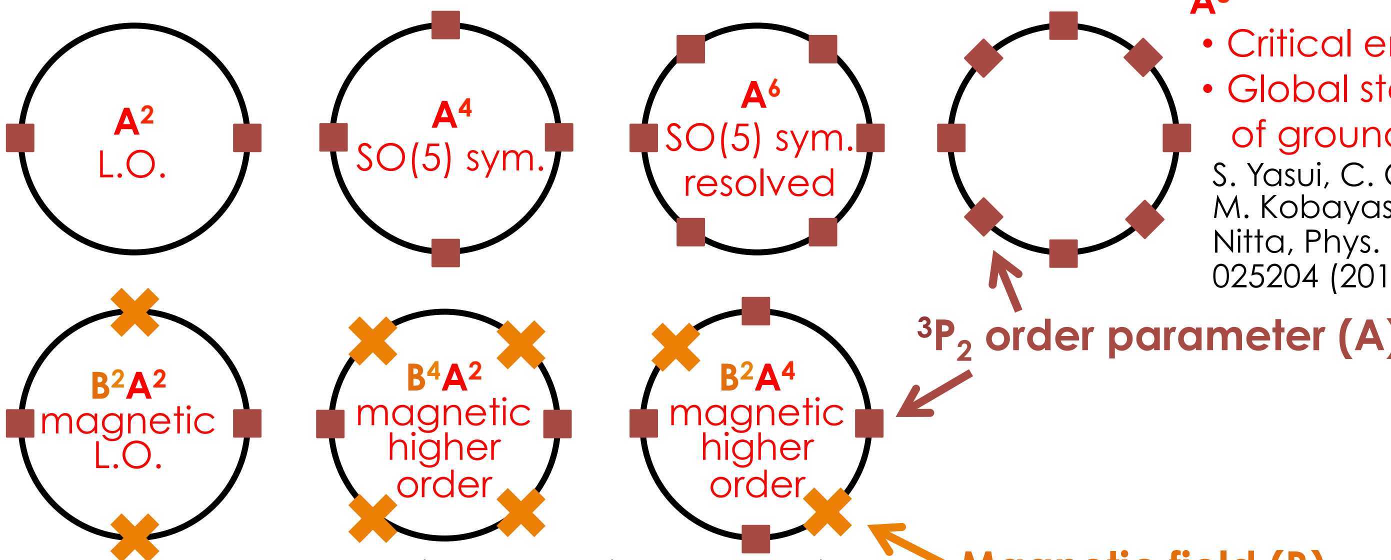
(A: condensate, B: magnetic field)

Tabakin (1968), Hoffenberg, Glassgold, Richardson, Ruderman (1970), Tamagaki (1970), Takatsuka, Tamagaki (1971), Takatsuka (1972), ...

$$\boxed{A \sim \psi S^a \nabla^b \psi}$$

$$f = A^2 + A^4 + A^6 + A^8 + B^2A^2 + B^4A^2 + B^2A^4 + \ldots$$

✔ Loop expansion for neutrons



A^8
- Critical endpoint
- Global stability of ground state

S. Yasui, C. Chatterjee, M. Kobayashi, and M. Nitta, Phys. Rev. C100, 025204 (2019).

S. Yasui, C. Chatterjee, and M. Nitta, Phys. Rev. C99, 035213 (2019).

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$$f = A^2 + A^4 + A^6 + A^8 + B^2A^2 + B^4A^2 + B^2A^4 + \ldots$$

$$
\begin{aligned}
f[A] = {} & K^{(0)}\left(\nabla_{xi}A^{ba*}\nabla_{xi}A^{ab} + \nabla_{xi}A^{ia*}\nabla_{xj}A^{aj} + \nabla_{xi}A^{ja*}\nabla_{xj}A^{ai}\right) \\
& +\alpha^{(0)}(\mathrm{tr}A^*A) + \beta^{(0)}\left((\mathrm{tr}\,A^*A)^2 - (\mathrm{tr}\,A^{*2}A^2)\right) \\
& +\gamma^{(0)}\Big(-3(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^2)(\mathrm{tr}\,A^{*2}) + 4(\mathrm{tr}\,A^*A)^3 + 6(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^{*2}A^2) + 12(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^*AA^*A) \\
& \quad -6(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*A^3) - 6(\mathrm{tr}\,A^2)(\mathrm{tr}\,A^{*3}A) - 12(\mathrm{tr}\,A^{*3}A^3) + 12(\mathrm{tr}\,A^{*2}A^2A^*A) + 8(\mathrm{tr}\,A^*AA^*AA^*A)\Big) \\
& +\delta^{(0)}\Big((\mathrm{tr}\,A^{*2})^2(\mathrm{tr}\,A^2)^2 + 2(\mathrm{tr}\,A^{*2})^2(\mathrm{tr}\,A^4) - 8(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*AA^*A)(\mathrm{tr}\,A^2) - 8(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*A)^2(\mathrm{tr}\,A^2) \\
& \quad -32(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^*A^3) - 32(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*AA^*A^3) - 16(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*A^2A^*A^2) \\
& \quad +2(\mathrm{tr}\,A^{*4})(\mathrm{tr}\,A^2)^2 + 4(\mathrm{tr}\,A^{*4})(\mathrm{tr}\,A^4) - 32(\mathrm{tr}\,A^{*3}A)(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^2) \\
& \quad -64(\mathrm{tr}\,A^{*3}A)(\mathrm{tr}\,A^*A^3) - 32(\mathrm{tr}\,A^{*3}AA^*A)(\mathrm{tr}\,A^2) - 64(\mathrm{tr}\,A^{*3}A^2A^*A^2) - 64(\mathrm{tr}\,A^{*3}A^3)(\mathrm{tr}\,A^*A) \\
& \quad -64(\mathrm{tr}\,A^{*2}AA^{*2}A^3) - 64(\mathrm{tr}\,A^{*2}AA^*A^2)(\mathrm{tr}\,A^*A) + 16(\mathrm{tr}\,A^{*2}A^2)^2 + 32(\mathrm{tr}\,A^{*2}A^2)(\mathrm{tr}\,A^*A)^2 \\
& \quad +32(\mathrm{tr}\,A^{*2}A^2)(\mathrm{tr}\,A^*AA^*A) + 64(\mathrm{tr}\,A^{*2}A^2A^{*2}A^2) - 16(\mathrm{tr}\,A^{*2}AA^{*2}A)(\mathrm{tr}\,A^2) + 8(\mathrm{tr}\,A^*A)^4 \\
& \quad +48(\mathrm{tr}\,A^*A)^2(\mathrm{tr}\,A^*AA^*A) + 192(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^*AA^{*2}A^2) + 64(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^*AA^*AA^*A) \\
& \quad -128(\mathrm{tr}\,A^*AA^{*3}A^3) + 64(\mathrm{tr}\,A^*AA^{*2}AA^*A^2) + 24(\mathrm{tr}\,A^*AA^*A)^2 + 128(\mathrm{tr}\,A^*AA^*AA^{*2}A^2) \\
& \quad +48(\mathrm{tr}\,A^*AA^*AA^*AA^*A)\Big) \\
& +\beta^{(2)}\boldsymbol{B}^tA^*A\boldsymbol{B} + \beta^{(4)}|\boldsymbol{B}|^2\boldsymbol{B}^tA^*A\boldsymbol{B} \\
& +\gamma^{(2)}\Big(-2|\boldsymbol{B}|^2(\mathrm{tr}\,A^2)(\mathrm{tr}\,A^{*2}) - 4|\boldsymbol{B}|^2(\mathrm{tr}\,A^*A)^2 + 4|\boldsymbol{B}|^2(\mathrm{tr}\,A^*AA^*A) + 8|\boldsymbol{B}|^2(\mathrm{tr}\,A^{*2}A^2) \\
& \quad +\boldsymbol{B}^tA^2\boldsymbol{B}(\mathrm{tr}\,A^{*2}) - 8\boldsymbol{B}^tA^*A\boldsymbol{B}(\mathrm{tr}\,A^*A) + \boldsymbol{B}^tA^{*2}\boldsymbol{B}(\mathrm{tr}\,A^2) + 2\boldsymbol{B}^tAA^{*2}A\boldsymbol{B} \\
& \quad +2\boldsymbol{B}^tA^*A^2A^*\boldsymbol{B} - 8\boldsymbol{B}^tA^*AA^*A\boldsymbol{B} - 8\boldsymbol{B}^tA^{*2}A^2\boldsymbol{B}\Big)
\end{aligned}
$$

$A^2 \to$ kinetic term

$A^2 \to$ L.O.

$A^4 \to$ SO(5) symmetry (pseudo NG boson)

$A^6 \to$ SO(5) sym. resolved

A^8
- Critical endpoint
- Global stability of ground state

$B^2A^2 \to$ L.O.

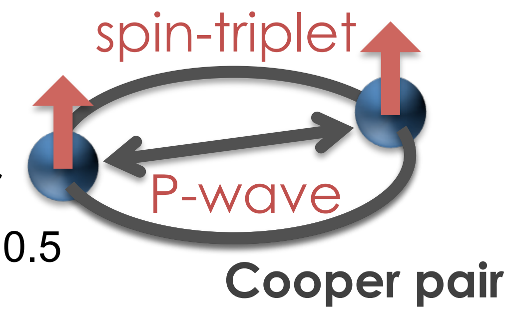
$B^4A^2 \to$ magnetic higher order

$B^2A^4 \to$ magnetic higher order

2. Various phases of Neutron $^3P_2$ superfluid

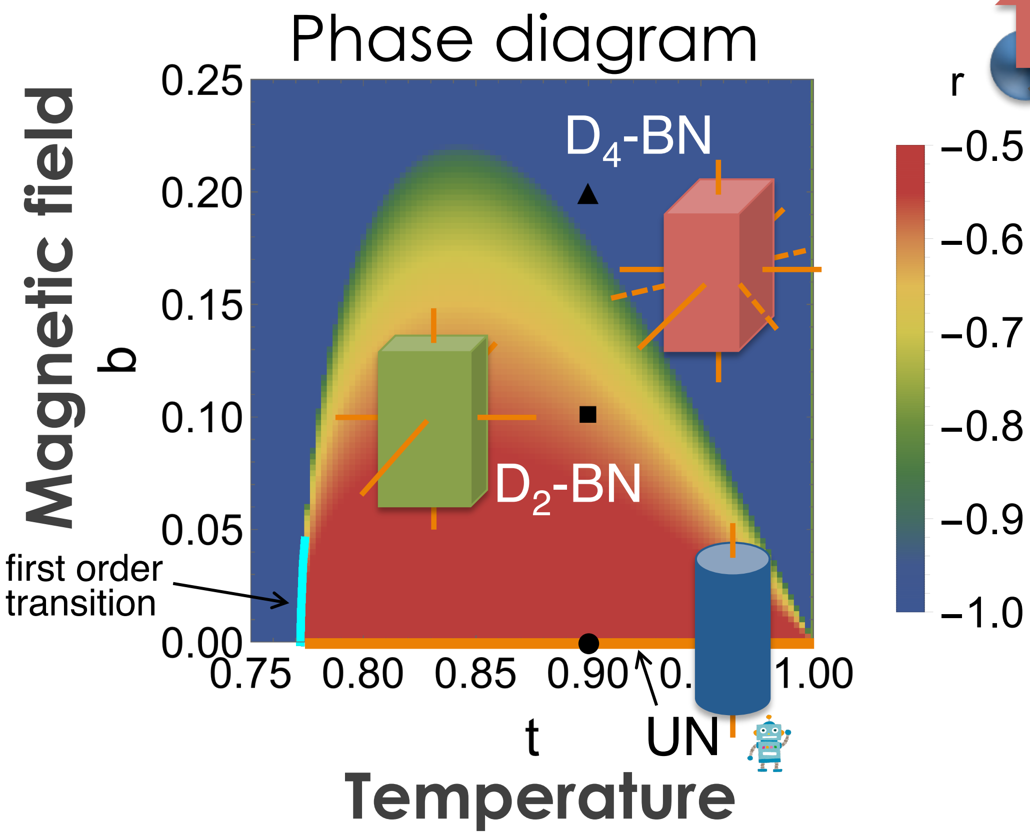
Ginzburg-Landau (GL) theory

(A: condensate, B: magnetic field)



Magnetar

Neutron star



magnetic field	zero	weak	strong
bulk phase	UN	D_2-BN	D_4-BN

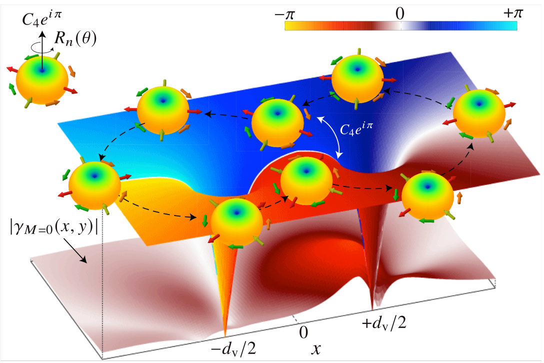
S. Yasui, C. Chatterjee, M. Kobayashi, and M. Nitta, Phys. Rev. C100, 025204 (2019)
T. Mizushima, S. Yasui and M. Nitta, Phys. Rev. Research 2, 013194 (2020)

2. Various phases of Neutron $^3P_2$ superfluid

Possible phenomena in neutron stars/magnetars ...

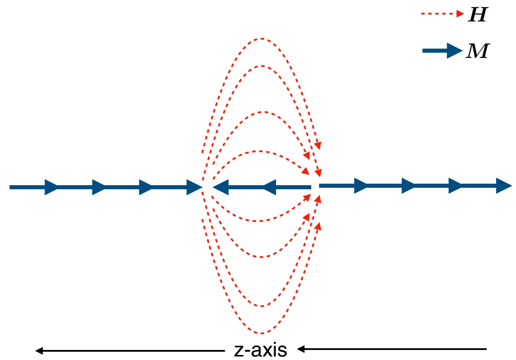
Half-quantized vortices

Y. Masaki, T. Mizushima, M. Nitta, arXiv:2107.02448 [cond-mat.supr-con]



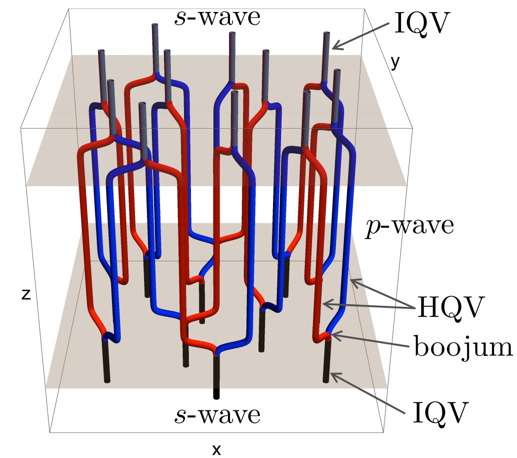
Soliton excitations

C. Chatterjee, M. Haberichter, M. Nitta, PRC96, 055807 (2017)



Vortex networks

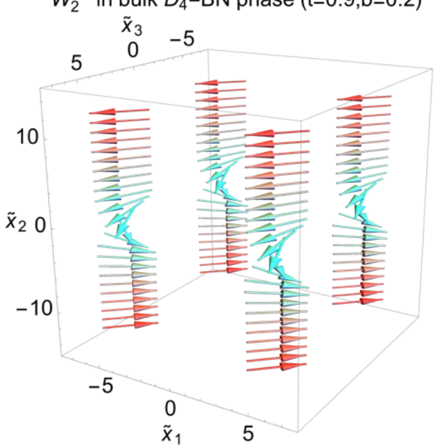
G. Marmoni, S. Yasui, M. Nitta, arXiv:2010/09032 [astro-ph.HE]



Domain wall

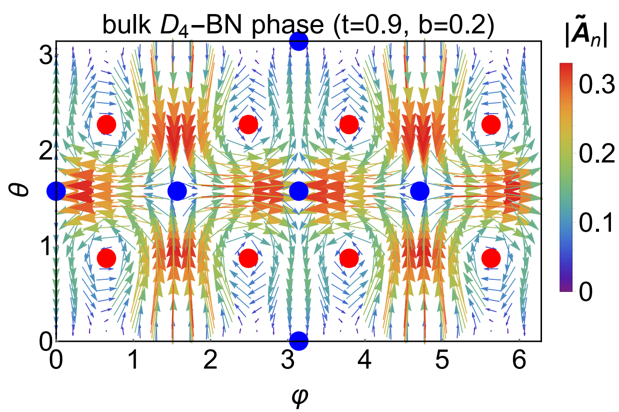
S. Yasui, M. Nitta, PRC101, 015207 (2020)

W_2^{13} in bulk D_4–BN phase (t=0.9,b=0.2)



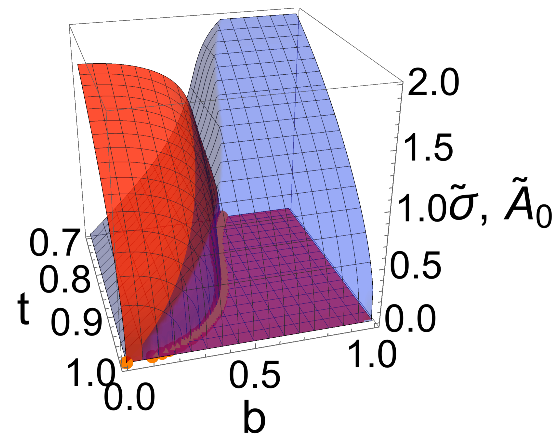
Surface defects

S. Yasui, C. Chatterjee, M. Nitta, PRC101, 025204 (2020)



$^1S_0$-$^3P_2$ coexistence

S. Yasui, D. Inotani, M. Nitta, PRC101, 055806 (2020)



... and more!

10 min.

Contents

1. Neutron $^3P_2$ superfluid: introduction

2. Various phases of Neutron $^3P_2$ superfluid

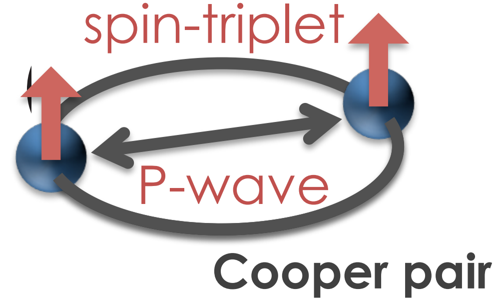
3. **Spin polarized phase (today's topic)**

4. Conclusion & perspectives

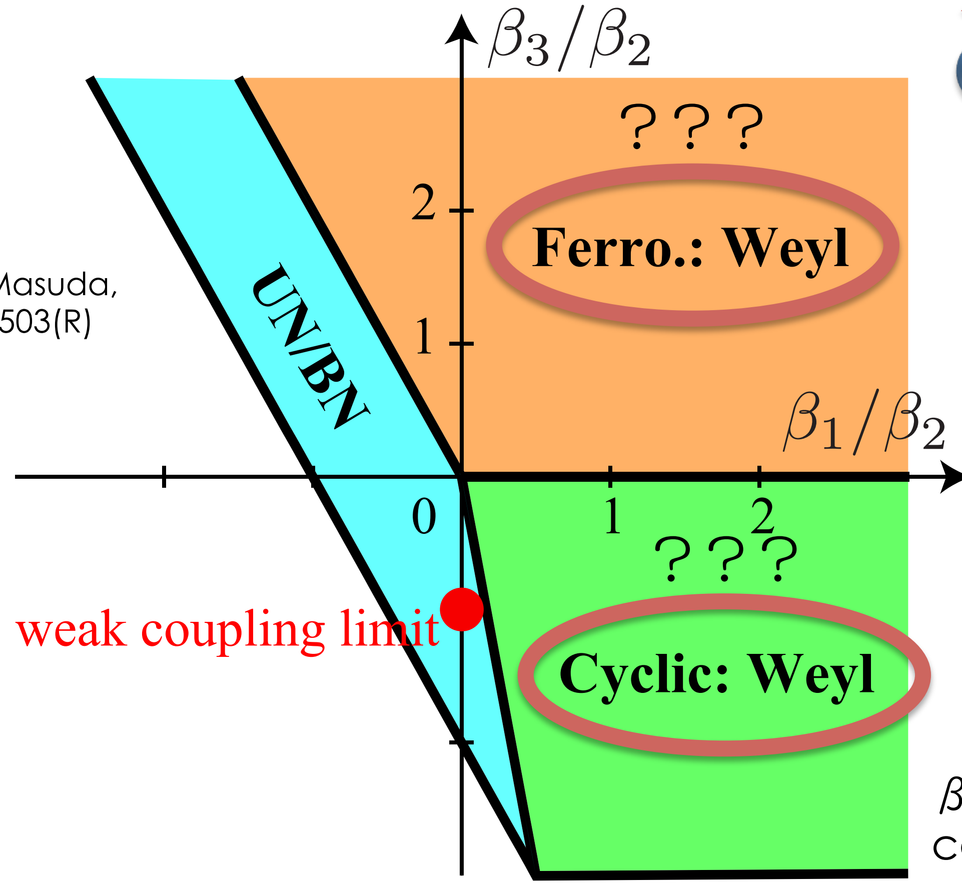
3. Spin polarized phase

"Phase diagram" in Ginzburg-Landau theory

Cf. V. Z. Vulovic, J. A. Sauls, PRD29, 2705 (1984)



Taken from . Mizushima, K. Masuda, M. Nitta, Phys. Rev. B95, 140503(R) (2017)



β_1, β_2, β_3: coefficients in GL theory

So far we have discussed UN/BN phase.

What's about the other phases？

3. Spin polarized phase

$^3P_2$ order parameter spin × momentum

Phase	O.P. [see Eq. (28)]	H	$R = G/H$	$\pi_1(R)$	$\#_{\mathrm{NG}}$	$\#_{\mathrm{qNG}}$ [66]
Uniaxial nematic	$r = -1/2, \kappa = 0$	$D_\infty \simeq \mathrm{O}(2)$	$\mathrm{U}(1) \times \mathbb{R}P^2$	$\mathbb{Z} \oplus \mathbb{Z}_2$ [43, 67]	3	2
Biaxial nematic	$r \in (-1, -1/2), \kappa = 0$	D_2	$\mathrm{U}(1) \times \mathrm{SO}(3)/D_2$	$\mathbb{Z} \oplus \mathbb{Q}$ [43, 67]	4	1
	$r = -1, \kappa = 0$	D_4	$[\mathrm{U}(1) \times \mathrm{SO}(3)]/D_4$	$\mathbb{Z} \times_h D_4^*$ [43, 44, 65]	4	1
Cyclic	$r = e^{i2\pi/3}, \kappa = 0$	T	$[\mathrm{U}(1) \times \mathrm{SO}(3)]/T$	$\mathbb{Z} \times_h T^*$ [65, 68–70]	3	—
Magnetized biaxial nematic	$r \in (-1, -1/2), \kappa \in (0, 1)$	0	$\mathrm{SO}(3) \times \mathrm{U}(1)$	$\mathbb{Z}_2 \oplus \mathbb{Z}$	4	—
	$r = -1, \kappa \in (0, 1)$	C_4	$[\mathrm{U}(1) \times \mathrm{SO}(3)]/\mathbb{Z}_4$	$\mathbb{Z} \times_h C_4^*$	4	—
Ferromagnetic	$r = -1, \kappa = 1$	$\mathrm{U}(1)_{J_z+2\Phi}$	$\mathrm{SO}(3)_{J_z-2\Phi}/\mathbb{Z}_2$	$\mathbb{Z}_4$ [69, 71]	3	—
	Eq. (26)	$\mathrm{U}(1)_{J_z+\Phi}$	$\mathrm{SO}(3)_{J_z-\Phi}/\mathbb{Z}_2$	$\mathbb{Z}_4$ [69, 71]	3	—

3. Spin polarized phase

$^3P_2$ order parameter   spin × momentum

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Magnetized biaxial nematic	$r\in(-1,-1/2),\ \kappa\in(0,1)$	0	$SO(3)\times U(1)$	$\mathbb{Z}_2\oplus\mathbb{Z}$	4	—
	$r=-1,\ \kappa\in(0,1)$	C_4	$[U(1)\times SO(3)]/\mathbb{Z}_4$	$\mathbb{Z}\times_h C_4^*$	4	—
Ferromagnetic	$r=-1,\ \kappa=1$	$U(1)_{J_z+2\Phi}$	$SO(3)_{J_z-2\Phi}/\mathbb{Z}_2$	$\mathbb{Z}_4$ [69, 71]	3	—
	Eq. (26)	$U(1)_{J_z+\Phi}$	$SO(3)_{J_z-\Phi}/\mathbb{Z}_2$	$\mathbb{Z}_4$ [69, 71]	3	—

Magnetized biaxial nematic $\xrightarrow{\kappa\to 1}$ Ferromagnetic

$$\mathcal{A}_{\mu i} = \Delta \begin{pmatrix} 1 & i\kappa & 0 \\ i\kappa & r & 0 \\ 0 & 0 & -1-r \end{pmatrix}_{\mu i}$$

$$\kappa \in (0,1)$$

$$\mathcal{A}^{\rm FM}_{\mu i} = \Delta \begin{pmatrix} 1 & \pm i & 0 \\ \pm i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{\mu i}$$

$$\kappa = 1$$

3. Spin polarized phase

$^3P_2$ order parameter spin × momentum

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$\kappa \in (0, 1)$ κ : new parameter $\kappa = 1$

Net average spin of $^3P_2$ Cooper pair

$$\langle S^z_{\rm pair} \rangle = 2\kappa(1-r)\Delta^2/3$$

$\kappa \neq 0 \rightarrow$ spin polarization

3. Spin polarized phase

What does cause the spin polarization？

① Strong coupling effect

J. A. Sauls, J. W. Serene, PRD17, 1524 (1978)
V. Z. Vulovic, J. A. Sauls, PRD29, 2705 (1984)
D. N. Voskresensky, PRD101, 056011 (2020)

② Violation of particle-hole symmetry

T. Mizushima, D. Inotani, S. Yasui, M. Nitta , arXiv:2108.01256 [nucl-th]

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T. Mizushima, D. Inotani, S. Yasui, M. Nitta , arXiv:2108.01256 [nucl-th]

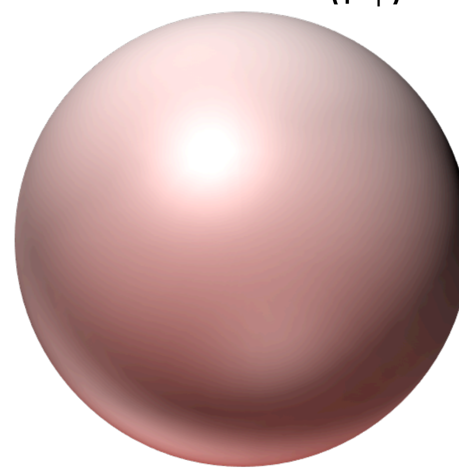
Fermi momentum (p_F) → ∞

Fermi momentum (p_F) : finite

Momentum space

Plane

→



Sphere

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J. A. Sauls, J. W. Serene, PRD17, 1524 (1978)
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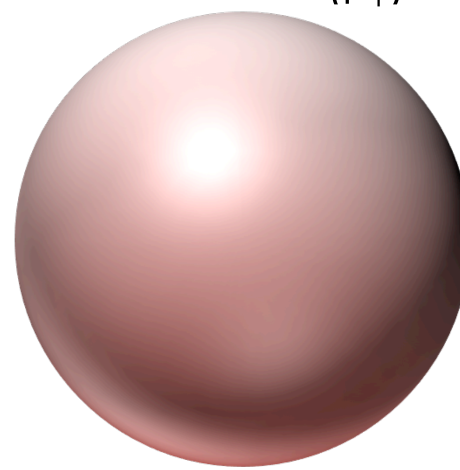
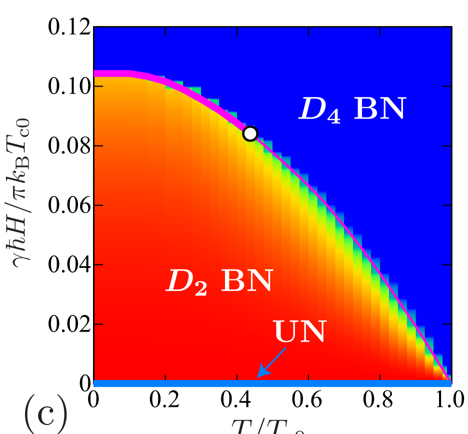


Sphere

Phase diagram



T. Mizushima, K. Masuda, M. Nitta, Phys. Rev. B95, 140503(R) (2017)

?

3. Spin polarized phase

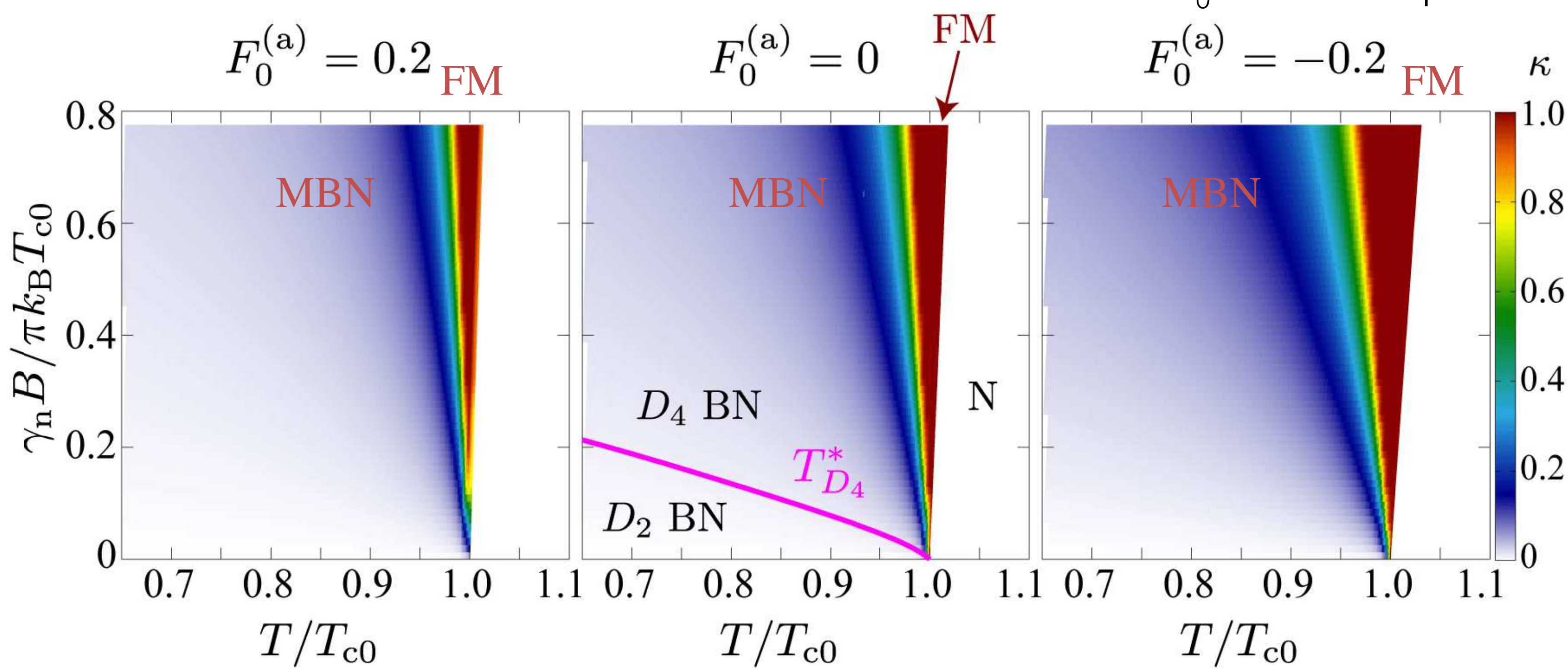
Bogoliubov-de Gennes (BdG) theory

Δ parameter: $\frac{1}{v} = \frac{1}{2}\left[\frac{(1+\kappa)^2}{1+\kappa^2}\mathcal{F}_+ + \frac{(1-\kappa)^2}{1+\kappa^2}\mathcal{F}_-\right],$

κ parameter: $\frac{\kappa}{v} = \frac{1}{2}\left[(1+\kappa)\mathcal{F}_+ - (1-\kappa)\mathcal{F}_-\right],$

$$\mathcal{F}_\alpha = \sum_{\boldsymbol{k}} \frac{\hat{k}_x^2 + \hat{k}_y^2}{2E_\alpha(\boldsymbol{k})} \tanh\left(\frac{E_\alpha(\boldsymbol{k})}{2T}\right)$$

$F_0^{(a)}$: Landau parameter

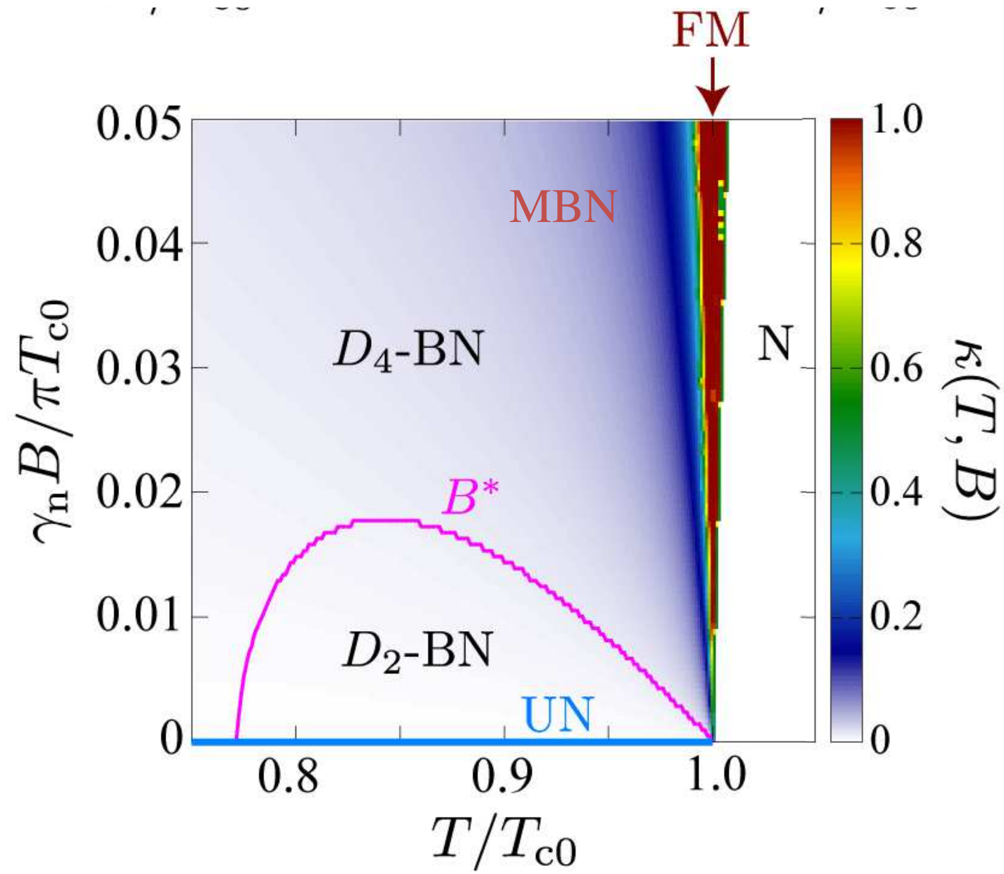


Magnetized biaxial-nematic (MBN) and Ferromagnetic (FM) phase appears!

3. Spin polarized phase

Ginzburg-Landau (GL) theory

$$f(\tau) = f_8^{(0)}(\tau) + f_2^{(\leq 4)}(\tau) + f_4^{(\leq 2)}(\tau) \quad \tau_{\mu i} = \tau_0 \begin{pmatrix} r & i\kappa & 0 \\ i\kappa & 1 & 0 \\ 0 & 0 & -1-r \end{pmatrix}_{\mu i}$$



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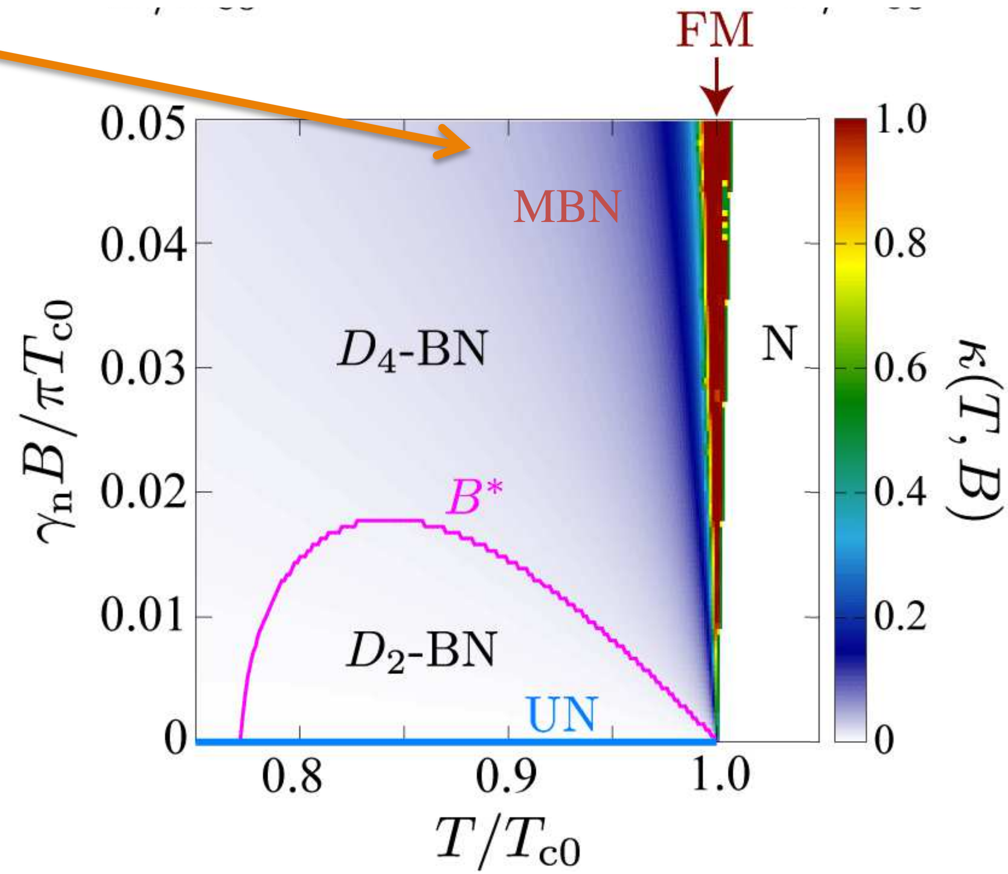
$$f(\tau) = f_8^{(0)}(\tau) + f_2^{(\leq 4)}(\tau) + f_4^{(\leq 2)}(\tau) \quad \tau_{\mu i} = \tau_0 \begin{pmatrix} r & i\kappa & 0 \\ i\kappa & 1 & 0 \\ 0 & 0 & -1-r \end{pmatrix}_{\mu i}$$

More chance to be observed in magnetars.

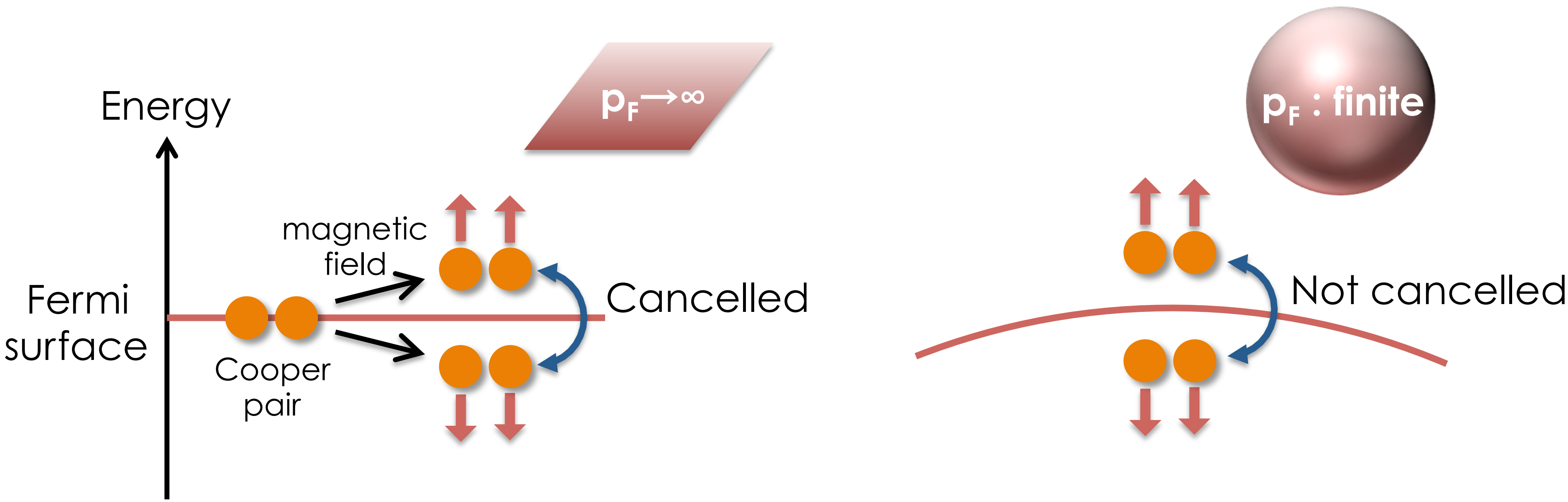


Magnetized biaxial-nematic (MBN) and Ferromagnetic (FM) phase appears!

3. Spin polarized phase

Why does the spin polarization appear?

Intuitive understanding...



Phase space (↑ ↑) = Phase space (↓ ↓)

Phase space (↑ ↑) > Phase space (↓ ↓)

Fermi surface curvature induces the spin polarization!

18 min.

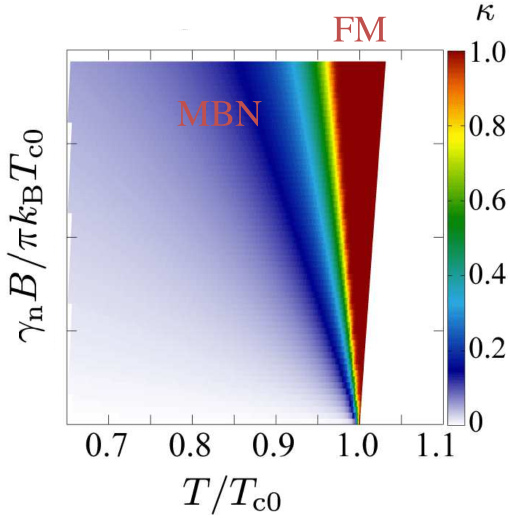
Contents

4. Conclusion and perspectives

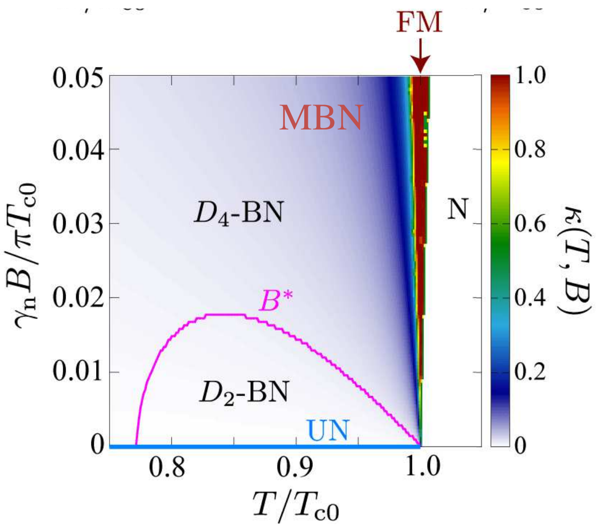
① We study $^3P_2$ superfluid in neutron stars.

② We find magnetized biaxial-nematic (MBN) and ferromagnetic (FM) phase as a spin polarized phase.

Bogoliubov-de Gennes (BdG) theory



Ginzburg-Landau (GL) theory



$$\langle S^z_{\mathrm{pair}} \rangle = 2\kappa(1-r)\Delta^2/3$$

$\kappa \neq 0$ → spin polarization

③ MBN/FM phase will exist in neutron stars and magnetars.

Open Challenge

Phase diagram？

- Thermodynamic properties
- Transport coefficients (cooling process)
- Other New phases
- Hyperon matter
- Non-uniform phase (FFLO) D. Inotani, S. Yasui, T. Mizushima, M. Nitta, Phys. Rev. A103, 053308 (2021)

Topological objects？

- Fractionally quantized vortices K. Masuda, M. Nitta, PRC93, 035804 (2016), PTEP202 (2020) 013
- Solitons in vortices C. Chatterjee, M. Haberichter, M. Nitta, PRC96, 055807 (2017)
- Gapless fermions T. Mizushima, K. Masuda, M. Nitta, PRB95, 140503 (2017)
- Boojum
  M. Cipriani, W. Vinci and M. Nitta, Phys. Rev. D 86, 121704 (2012)
  G. Alford, G. Baym, F. Fukushima, T. Hatsuda, M. Tachibana, Phys. Rev. D99, 036004 (2019)
  C. Chatterjee, M. Nitta, S. Yasui, Phys. Rev. D99, 034001 (2019)
  A. Cherman, S. Sen, L. G. Yaffe, Phys, Rev, D100, 034015 (2019)
  G. Maromorini, S. Yasui, M. Nitta, arXiv:2010.09032 [astro-ph.HE]

20 min.

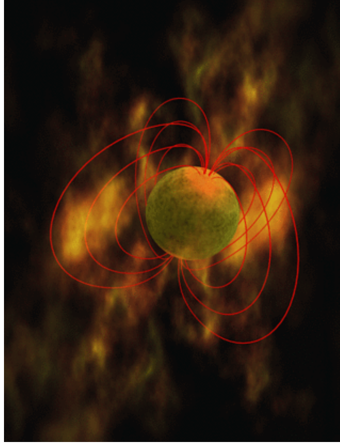
End

Appendix

V. M. Kaspi and A. M. Beloborodov, Annual Review of Astronomy and Astrophysics 55, 261 (2017).

Magnetars

Strong magnetic field



Name[b]	P (s)	B^{c} (10^{14} G)	Age[d] (kyr)	$\dot{E}^{e}$ 10^{33} erg s^{-1}	D^{f} (kpc)	$L_X{}^{g}$ 10^{33} erg s^{-1}	Band[h]
CXOU J010043.1–721134	8.02	3.9	6.8	1.4	62.4	65	–
4U 0142+61	8.69	1.3	68	0.12	3.6	105	OIR/H
SGR 0418+5729	9.08	0.06	36,000	0.00021	∼2	0.00096	–
SGR 0501+4516	5.76	1.9	15	1.2	∼2	0.81	OIR/H
SGR 0526–66	8.05	5.6	3.4	2.9	53.6	[illegible]	–
1E 1048.1–5937	[illegible]	3.9	[illegible]	3.3	9.0	[illegible]	OIR
(PSR J1119–6127)	[illegible]	4.1	[illegible]	[illegible]	8.4	[illegible]	R/H
1E 1547.0–5408	[illegible]	3.2	0.69	[illegible]	4.5	[illegible]	O?/R/[illegible]
[illegible]	[illegible]	2.7	4.0	[illegible]	∼9	[illegible]	R
SGR 1627–41	[illegible]	2.2	2.2	[illegible]	11	[illegible]	–
CXOU J164710.2–455216	[illegible]	<0.66	>420	[illegible]	3.9	[illegible]	–
[illegible]	[illegible]	4.7	9.0	[illegible]	3.8	42	O?/[illegible]
CXOU J171405.7–381031	[illegible]	5.0	0.95	[illegible]	∼13	56	–
SGR J1745–2900	[illegible]	2.3	[illegible]	[illegible]	8.3	<0.11	R/H
SGR 1806–20	[illegible]	20	[illegible]	45	8.7	163	OIR/H
XTE J1810–197	5.54	2.1	11	1.8	3.5	0.043	OIR/R
Swift J1822.3–1606	8.44	0.14	6,300	0.0014	1.6	>0.0004	–
SGR 1833–0832	7.56	1.6	34	0.32	–	–	–
Swift J1834.9–0846	[illegible]	1.4	4.9	[illegible]	4.2	<0.0084	–
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	–
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	–
3XMM J185246.6+003317	11.56	<0.41	>1,300	<0.0036	∼7	<0.006	–
SGR 1900+14	5.20	7.0	0.9	[illegible]	12.5	90	H
SGR 1935+2154	[illegible]	2.2	3.6	17	–	–	–
1E 2259+586	[illegible]	0.59	[illegible]	[illegible]	[illegible]	17	OIR/H
SGR 0755–2933	–	–	–	–	–	–	–
SGR 1801–23	–	–	–	–	–	–	–
SGR 1808–20	–	–	–	–	–	–	–
AX J1818.8–1559	–	–	–	–	–	–	–
AX J1845.0–0258	6.97	–	–	–	–	2.9	–
SGR 2013+34	–	–	–	–	–	–	–

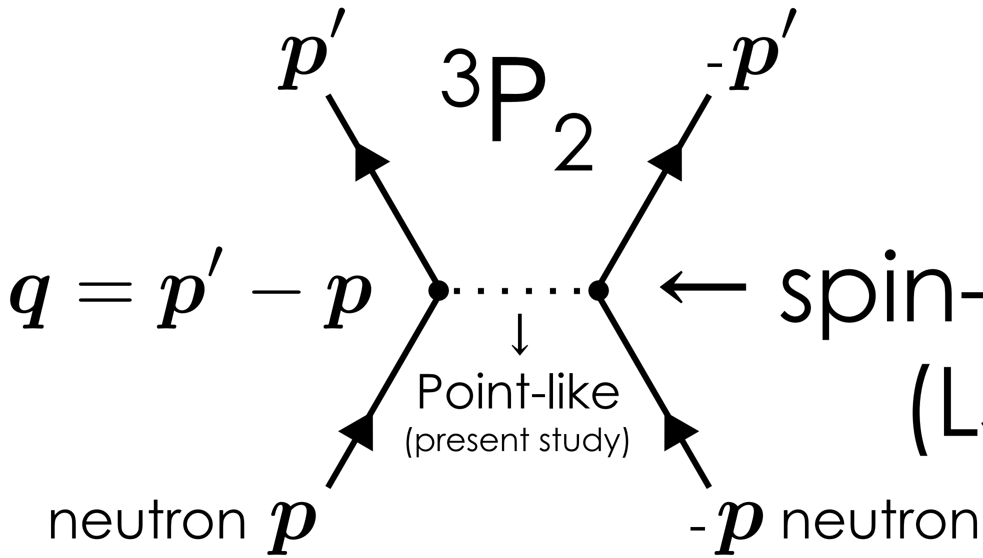
$B = 10^{15}$ G

Normal neutron stars:

$B = 10^{13}$ G

1. Neutron $^3P_2$ superfluid

$^3P_2$: most attractive interaction between two neutrons



← spin-orbit int. (LS pot.)

Tensor-type condensate

symmetric & traceless

$$\boxed{\frac{1}{2}\left(s^a q^b + s^b q^a\right) - \frac{\delta^{ab}}{3}\boldsymbol{s}\cdot\boldsymbol{q}}$$

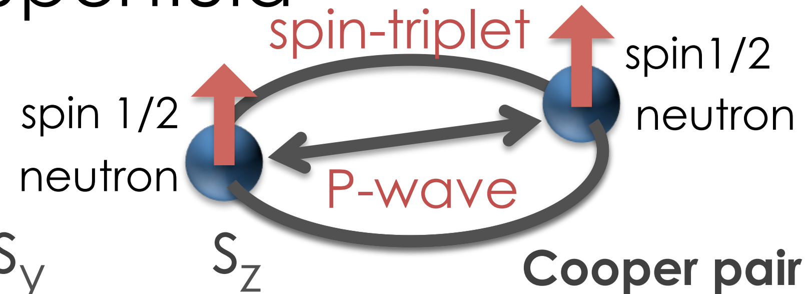
spin-momentum space (a,b=1,2,3)

Neutron $^3P_2$ superfluid →
- **Rapid neutrino cooling ?**
- **Tolerance to strong magnetic field ?**
- **Property of topological matter ?**

1. Neutron $^3P_2$ superfluid

Tabakin (1968), Hoffenberg, Glassgold, Richardson, Ruderman (1970), Tamagaki (1970), Takatsuka, Tamagaki (1971), Takatsuka (1972), ...

$\boxed{^3P_2}$



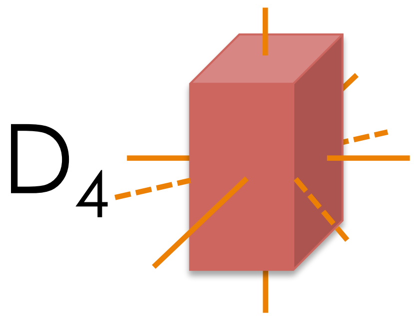
Order parameter
(neutron-neutron condensate)

$$A(t,\boldsymbol{x}) = A_0 \begin{array}{cc} & \begin{array}{ccc} S_x & S_y & S_z \end{array} \\ & \begin{pmatrix} r & 0 & 0 \\ 0 & -(1+r) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{array}{c} q_x \\ q_y \\ q_z \end{array} \end{array}$$

symmetric traceless-tensor

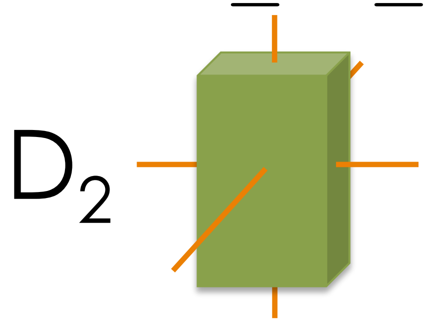
spin × momentum

Internal parameter: $-1 \leq r \leq -1/2$



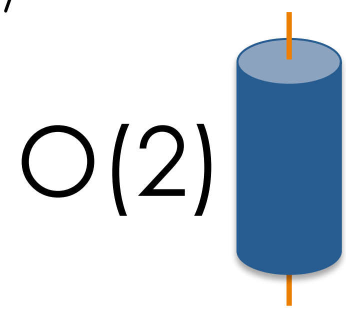
D_4-BN: D_4 biaxial nematic
(r=-1)

$\mathrm{U(1)} \times \mathrm{SO(3)}_{\mathrm{L+S}} \rightarrow \mathrm{D}_4$



D_2-BN: D_2 biaxial nematic
(-1<r<-1/2)

$\mathrm{U(1)} \times \mathrm{SO(3)}_{\mathrm{L+S}} \rightarrow \mathrm{D}_2$



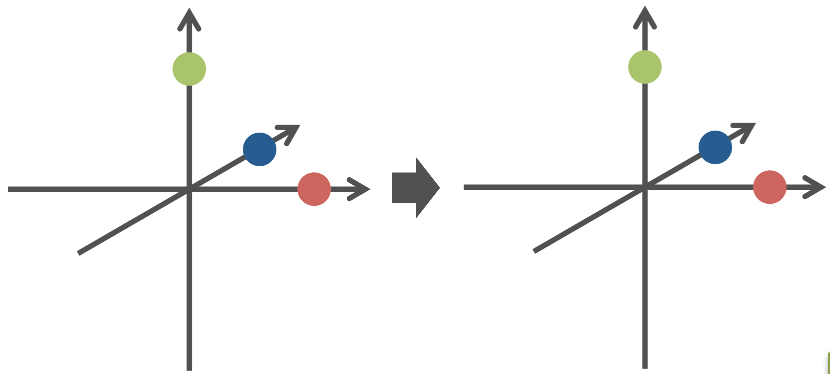
UN: uniaxial nematic
(r=-1/2)

$\mathrm{U(1)} \times \mathrm{SO(3)}_{\mathrm{L+S}} \rightarrow \mathrm{O(2)}$

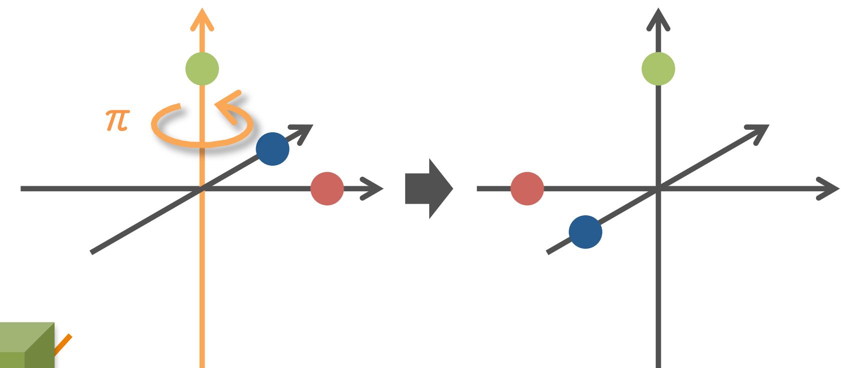
D_n symmetry: invariance both (i) under n-times rotation around one rotation axis and (ii) under two-times rotation around the n axes that are perpendicular to the rotation axis in (i).

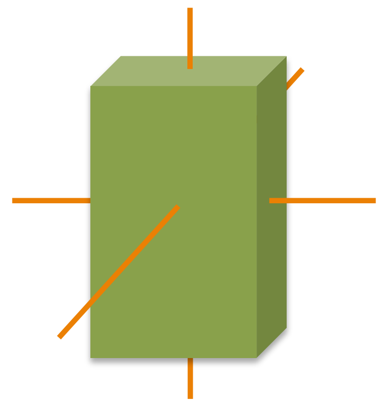
D_2 symmetry

$$1_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

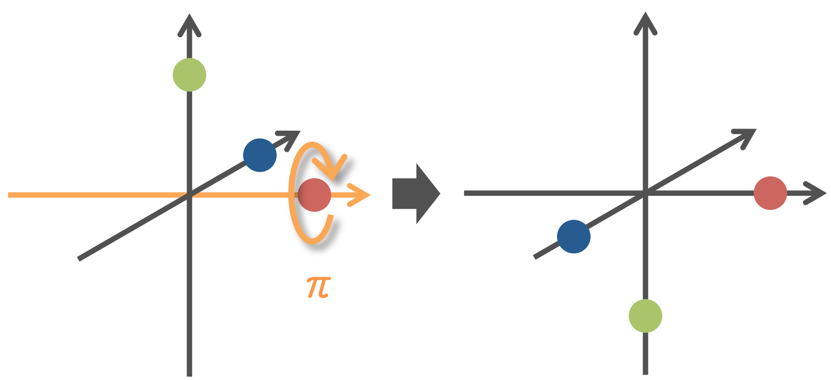


$$I_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

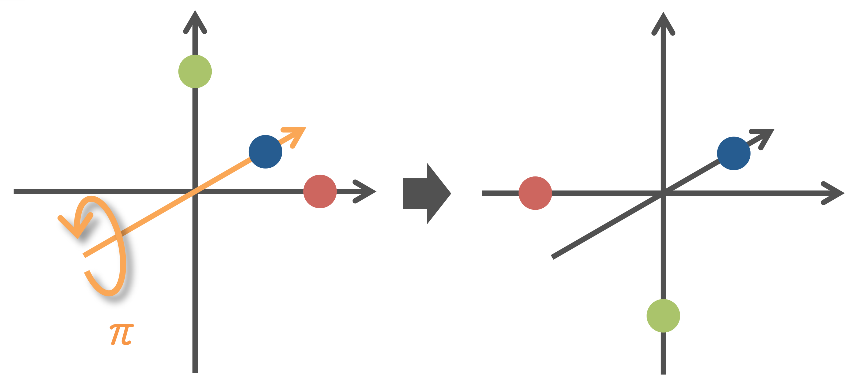




$$I_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$



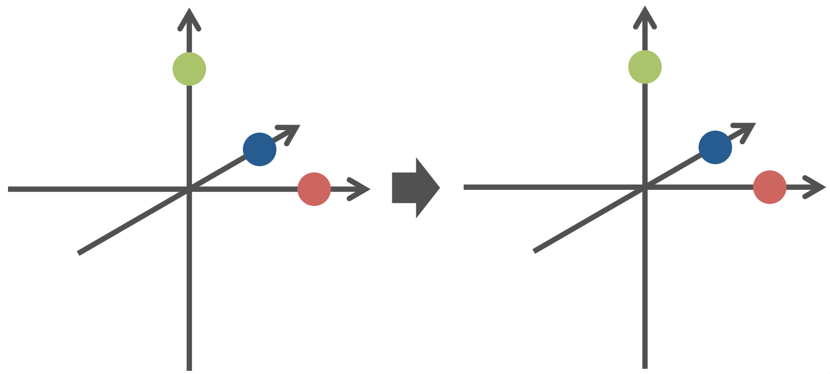
$$I_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$



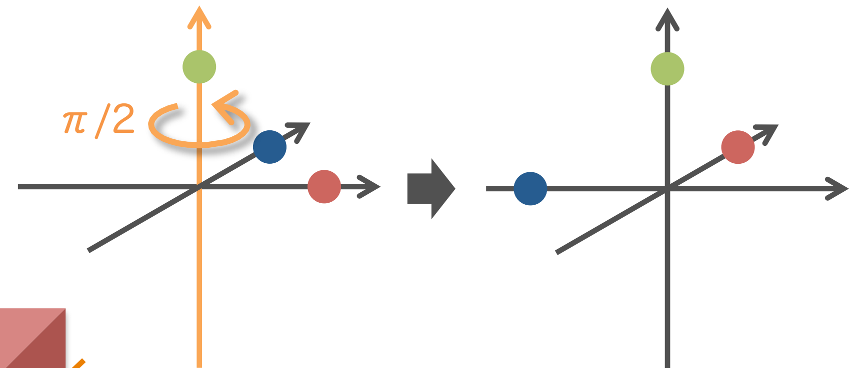
D_n symmetry: invariance both (i) under n-times rotation around one rotation axis and (ii) under two-times rotation around the n axes that are perpendicular to the rotation axis in (i).

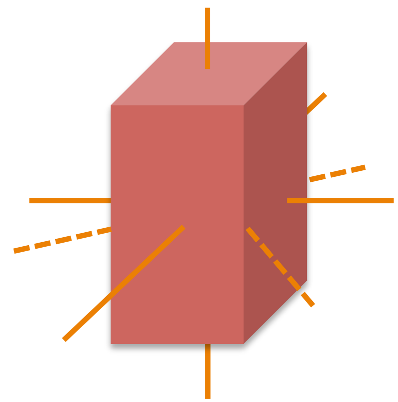
D_4 symmetry

$$1_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

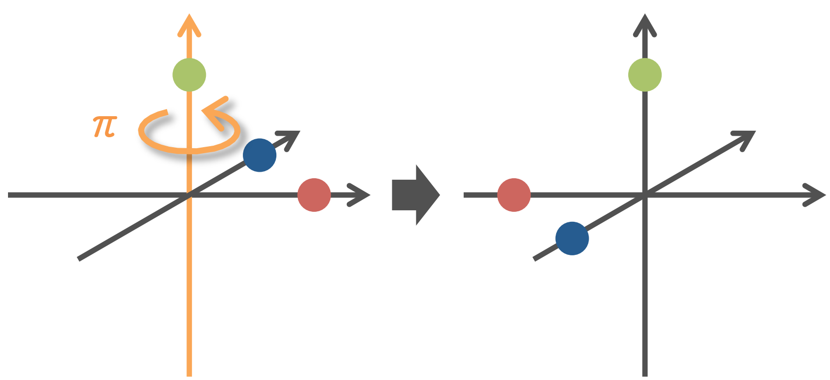


$$R = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

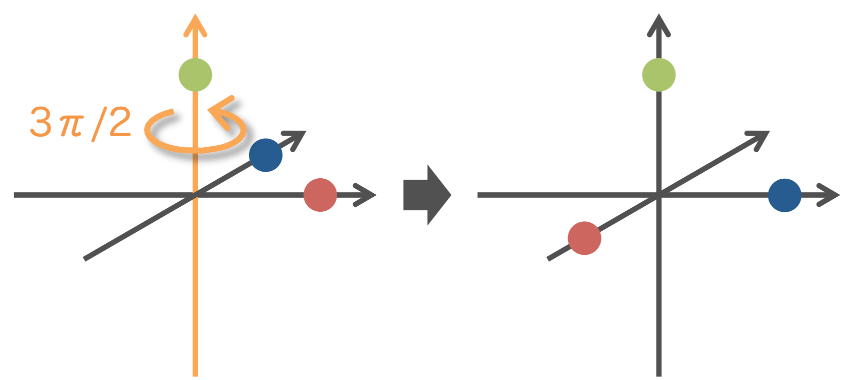




$$I_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$I_3R = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



D_n symmetry: invariance both (i) under n-times rotation around one rotation axis and (ii) under two-times rotation around the n axes that are perpendicular to the rotation axis in (i).

D_4 symmetry

$$I_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad I_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$I_1 R = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad I_2 R = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$