

Efforts on search for gravitational waves from neutron stars

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第4回中性子星の観測と理論: 研究活性化ワークショップ
2021/8/10-12

GW sources

Compact Binary Coalescence (CBC)

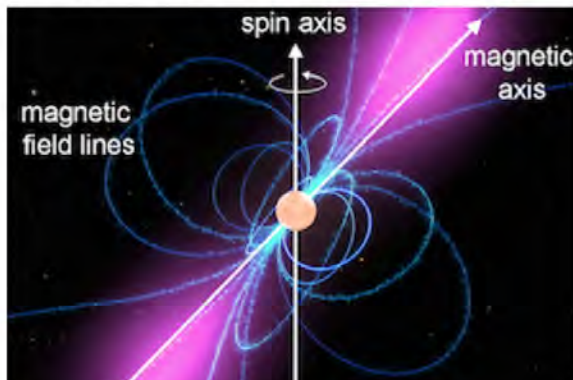


Burst



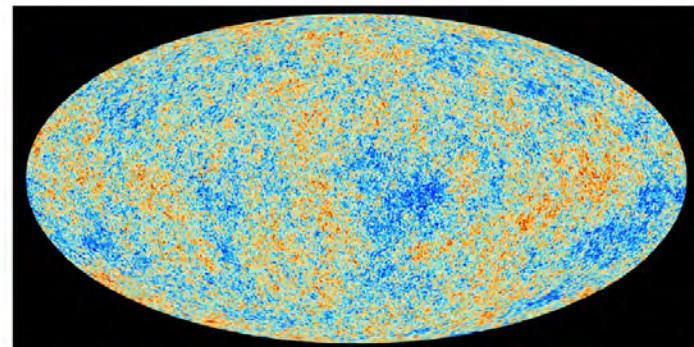
SN, GRB, FRB,
Cosmic string, ...

Continuous Waves (CW)



Rotating NSs, boson cloud, ...

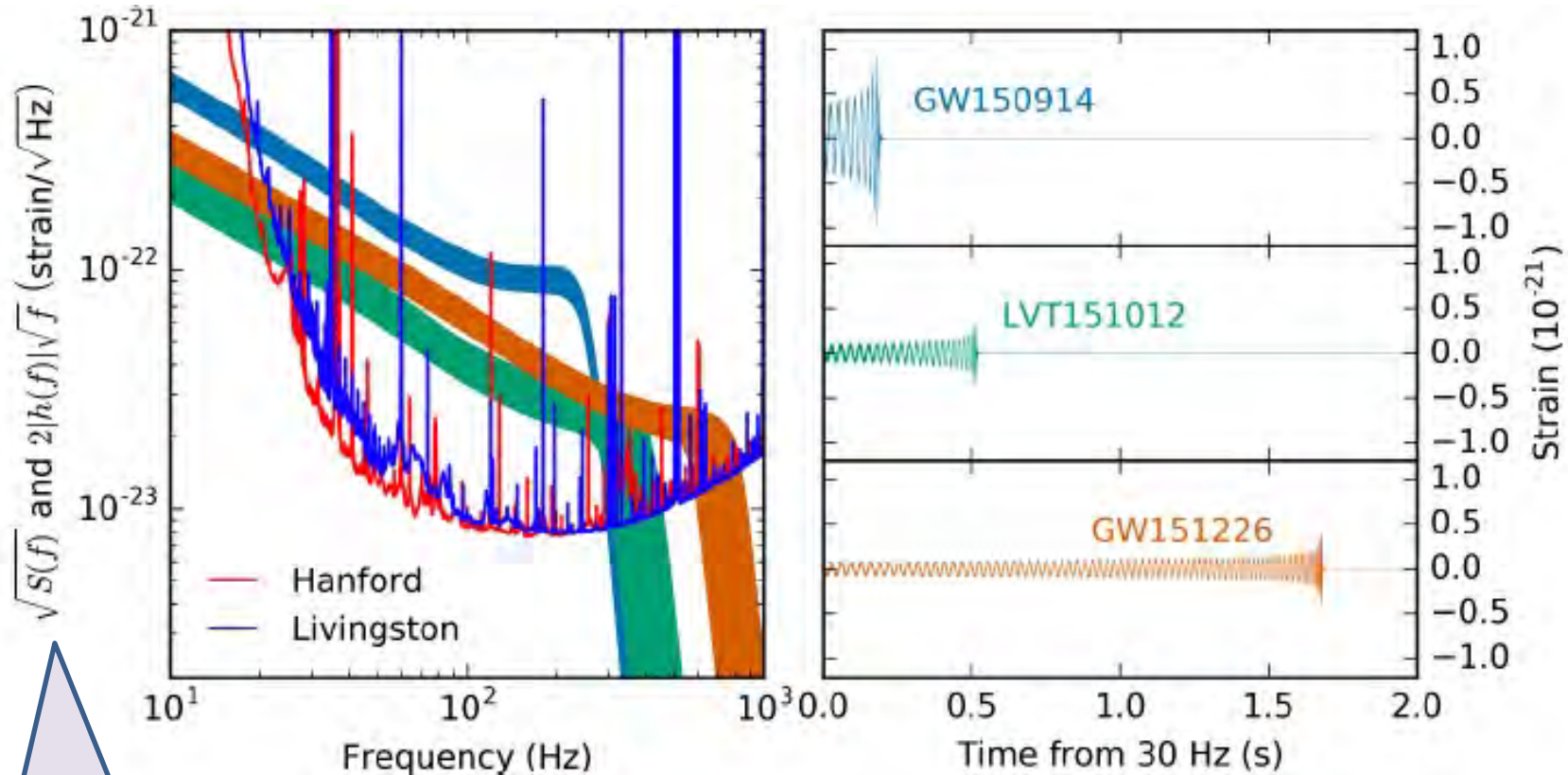
Stochastic Background



Astrophysical foreground, Cosmic string,
cosmological background,...

GW detector sensitivity & CBC signal

LVC, PRX 6, 041015 (2016)

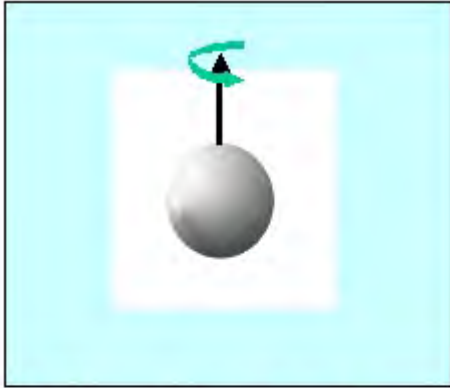


$$\sqrt{S_n} [1/\sqrt{\text{Hz}}]$$

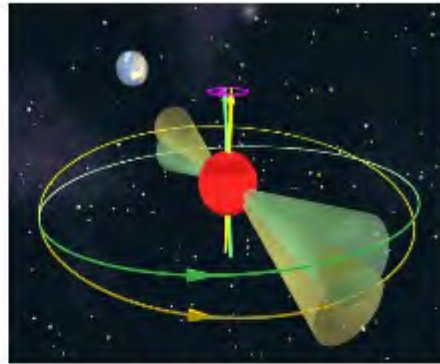
- Signals *larger* than “the sensitivity curve” can be detected.
- CBC lasts only a fraction of second (BBH) ~ a minute (BNS).

Continuous GW sources

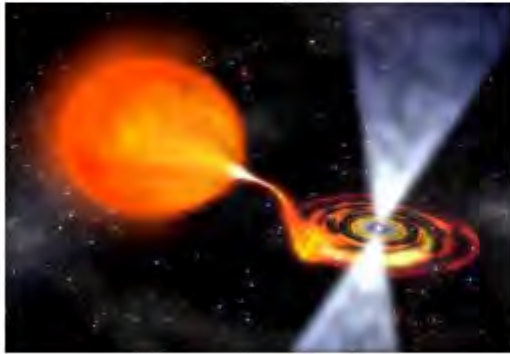
This talk concerns with GW from spinning motion of NS.



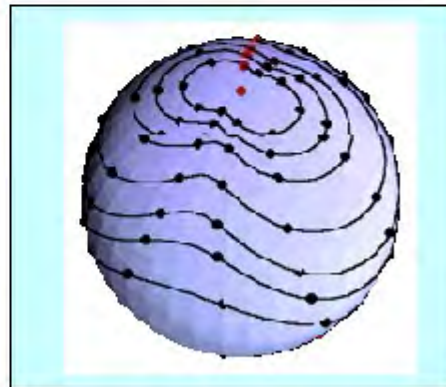
Mountain on a star



Wobbling star



Accreting star



Oscillating star

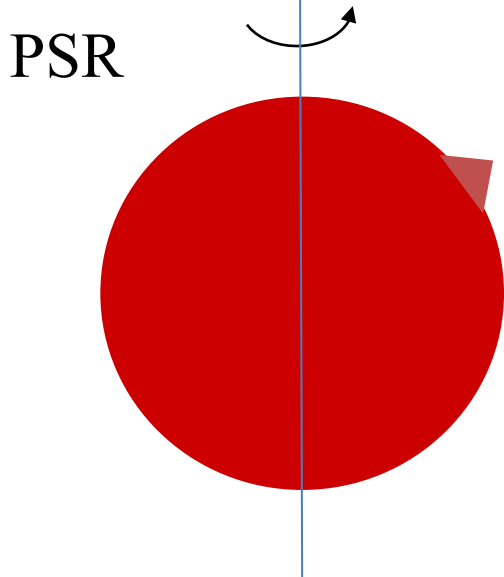
Stability of stellar rotation means simple waveform
(with some exceptions...)

$$h_{ij}^{TT} = \frac{2\ddot{I}_{ij}^{TT}}{r} \propto \sin[\Phi(t)]$$
$$\Phi(t) = \omega_{GW}t + \phi_0$$

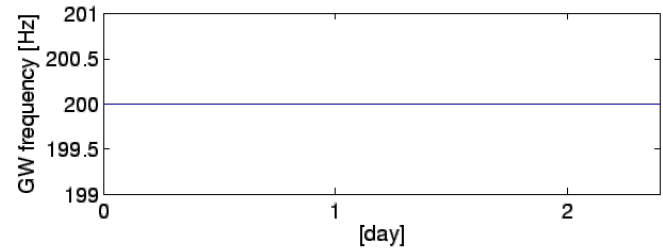
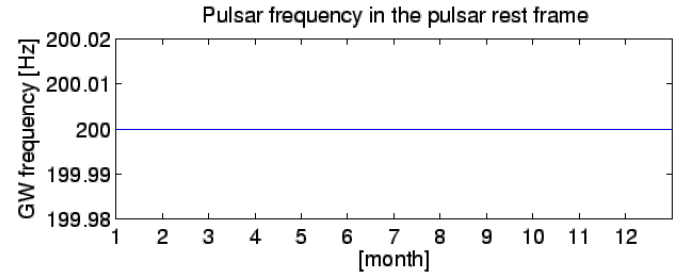
It's so weak that $T_{\text{obs}} \sim \text{years}$ is necessary.

Angular resolution:
 \sim a few arcseconds

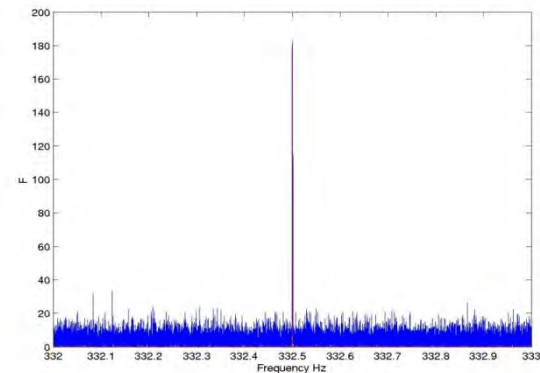
Detection of GWs from a pulsar using a detector at rest with respect to the pulsar



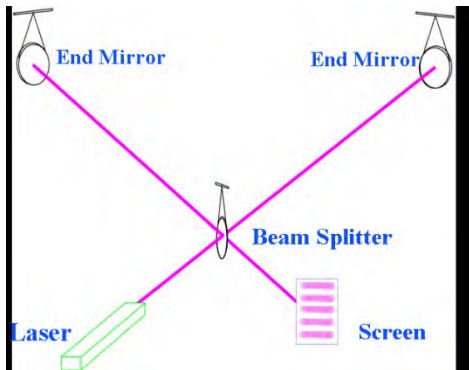
GW frequency at the detector



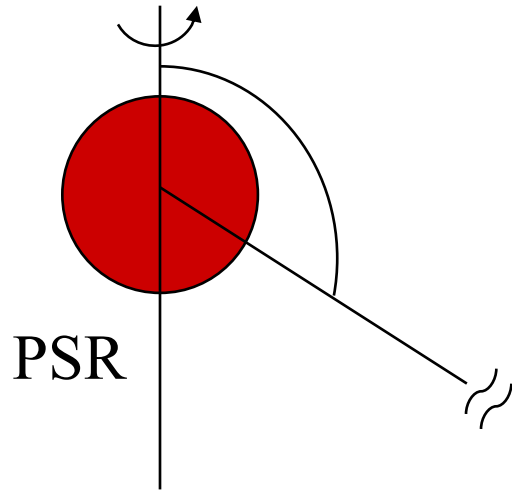
(Fast) Fourier Transform



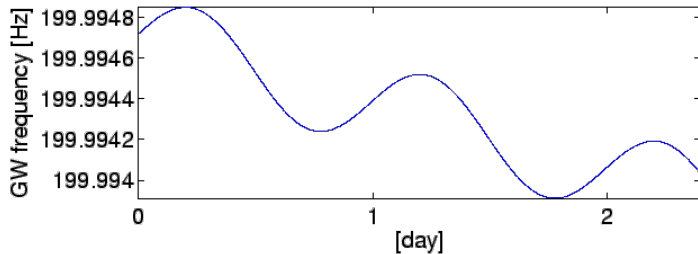
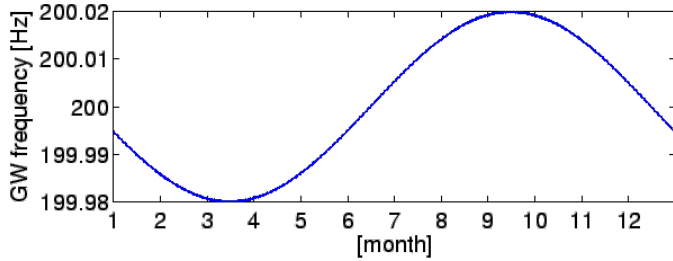
Putative
detector at rest
w.r.t. PSR



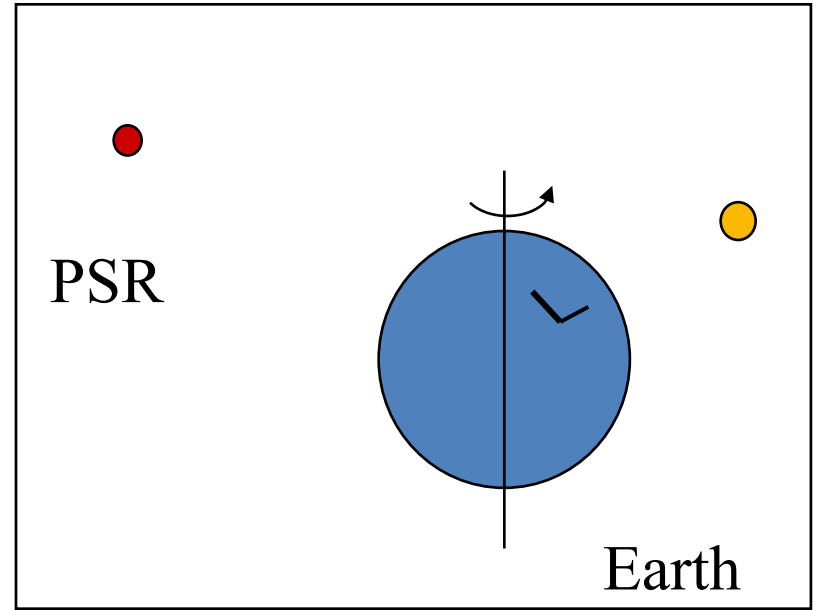
Detection of GWs from a pulsar using a ground-based detector



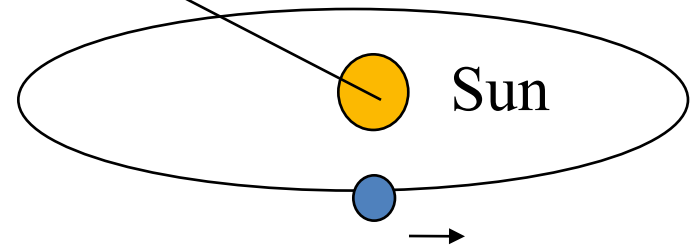
Pulsar frequency in the Earth detector frame



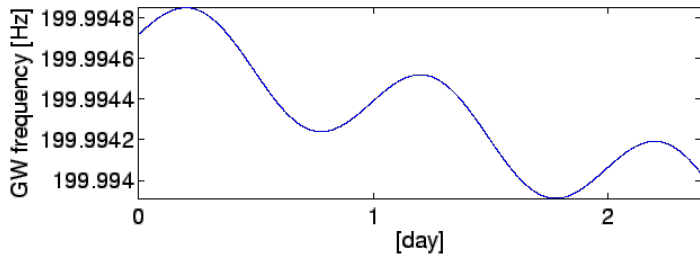
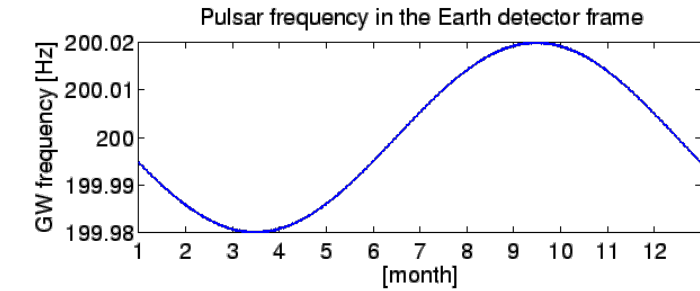
地球上の検出器での重力波周波数



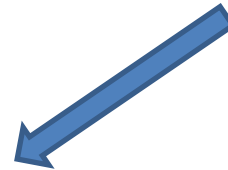
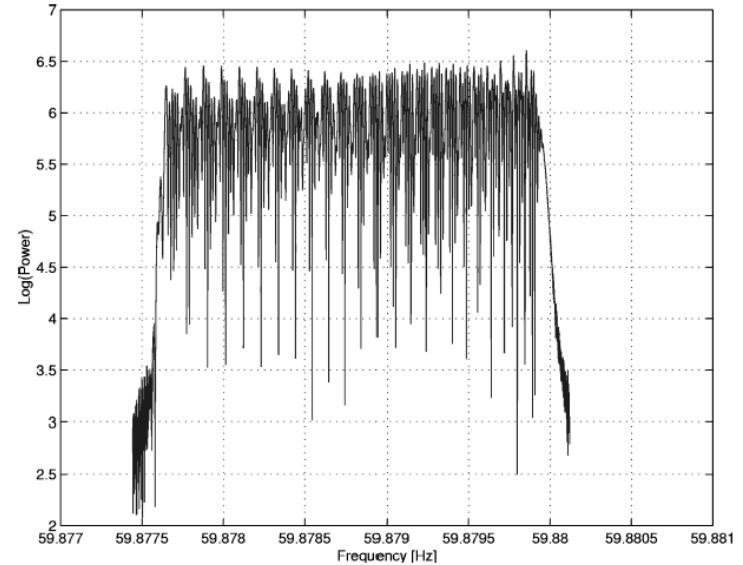
Doppler effect



Detection of GWs from a pulsar using a ground-based detector

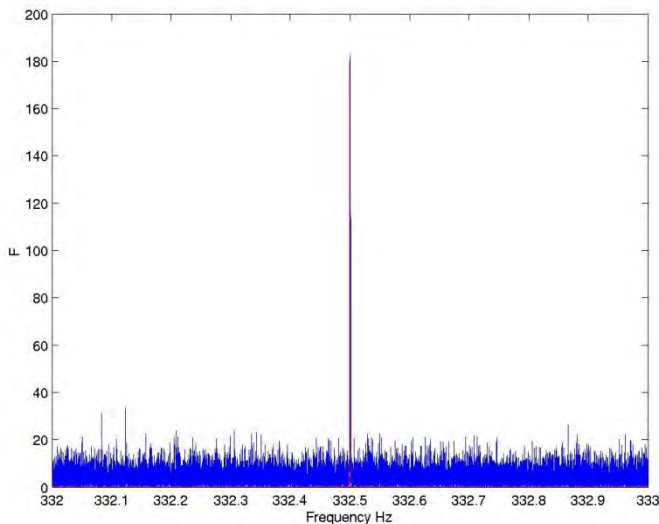


Fourier Transform



Demodulation:

Increase the signal to noise ratio by collecting the powers spread over frequencies due to the Doppler effects.



$$h_0 \propto S_n^{\frac{1}{2}} T_{\text{obs}}^{-\frac{1}{2}}$$

どれくらい細かく天域を見ていく必要があるのか = 波源方向の推定精度

- 1つの周波数ビンに全てのパワーを集めたい。
- 周波数ビンは1/(積分時間)で細くなる。
- 検出器出力を単純にフーリエ変換すると、空の \vec{n} 方向にある周波数 f_0 の波源は時間に依存するDopplerドリフトで

$$f = f_0 \left(1 + \frac{\vec{v}(t) \cdot \vec{n}}{c} \right)$$

の帯域で信号を生成する。

- $\Delta\theta = |\Delta n|$ だけ波源からずれた方向を探索すると、

$$\Delta f \sim \frac{|\Delta \vec{v}(t)| \cdot \Delta\theta}{c} f_0$$

程度周波数領域で広がった信号を得る。 $|\Delta \vec{v}(t)|$ は観測時間中の地球の公転・自転速度変化の大きさ。

- $f_0(\Delta v/c)(\Delta\theta) < 1/T$ の精度で天域を探索する必要がある。
- あるいは、方向決定精度は、

$$\Delta\theta < \frac{1}{f_0 \left(\frac{v_{orb}}{c} \right) w_{orb} T^2} \sim 10 \text{ arcsec} \left(\frac{10^6 \text{ s}}{T} \right)^2 \left(\frac{100 \text{ Hz}}{f_0} \right)$$

注: 回折限界 ~ 2秒角@100Hz

- あるいは探索すべき (= de-modulationすべき) 空の方向は $4\pi/\Delta\theta \sim T^2$ で増えていく。

Computational cost

- Template search:
 - spin down parameters & source sky position.

$$\lambda \equiv [\{f^{(k)}\}_{k=1}^{s_{\max}}, \theta, \phi].$$

- Number of templates (T: integration time):

$$\mathcal{N} \propto \int d^n \lambda \sqrt{g} \sim O((fT)^2) \prod_{k=1}^{s_{\max}} f^{(k)} T^{k+1}$$

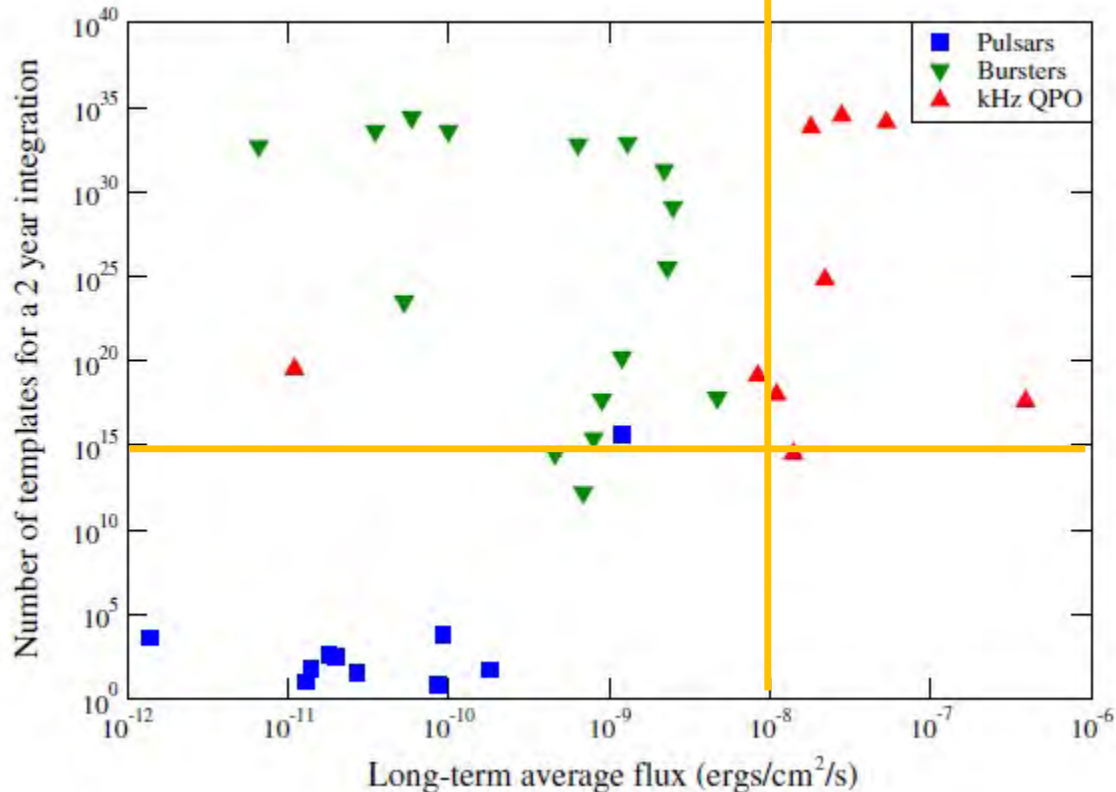
- Conduct incoherent search instead:

$$h_0 \propto S_n^{\frac{1}{2}} (T_{\text{coh}} T_{\text{obs}})^{-\frac{1}{4}}$$

instead of

$$h_0 \propto S_n^{\frac{1}{2}} T_{\text{obs}}^{-\frac{1}{2}}$$

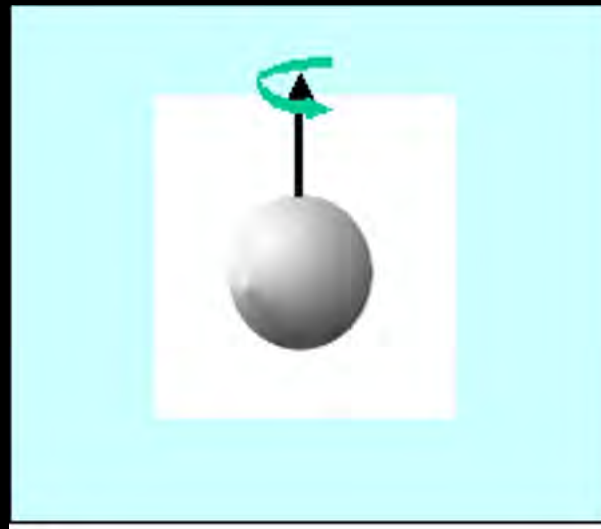
Known accreting pulsar search: Number of templates

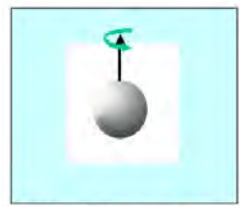


- Detectable if long-term average flux is larger than 10^{-8} .
- 10^{15} templates data can be analyzed by LIGO at the time of iLIGO

Watt and Krishnan (2009)

“Isolated pulsar search”





Mountain on a star

What we can learn

Amplitude of GW or “degree of non-axisymmetry ϵ ” of NS

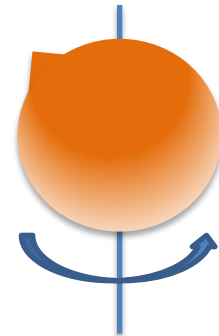
$$h_0 = \frac{4\pi^2 G}{c^4} \sqrt{\frac{8\pi}{15}} \frac{Q_{22} f_{GW}^2}{r} = \frac{4\pi^2 G}{c^4} \frac{\epsilon I_{zz} f_{GW}^2}{r}$$

$$= 1.1 \times 10^{-27} \left(\frac{\epsilon}{10^{-7}} \right) \left(\frac{I_{zz}}{10^{45} \text{g} \cdot \text{cm}^2} \right) \left(\frac{r}{1 \text{kpc}} \right)^{-1} \left(\frac{f_{GW}}{200 \text{Hz}} \right)^2$$

- Degree of non-axisymmetry

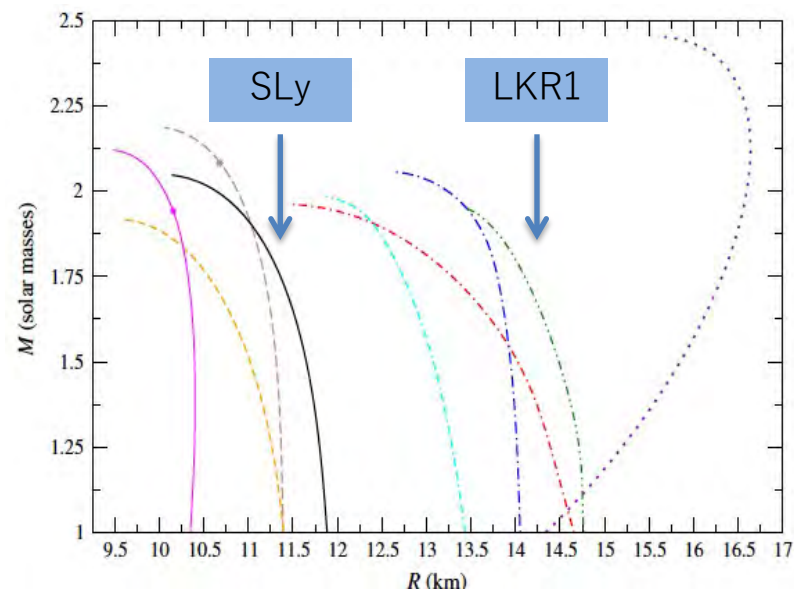
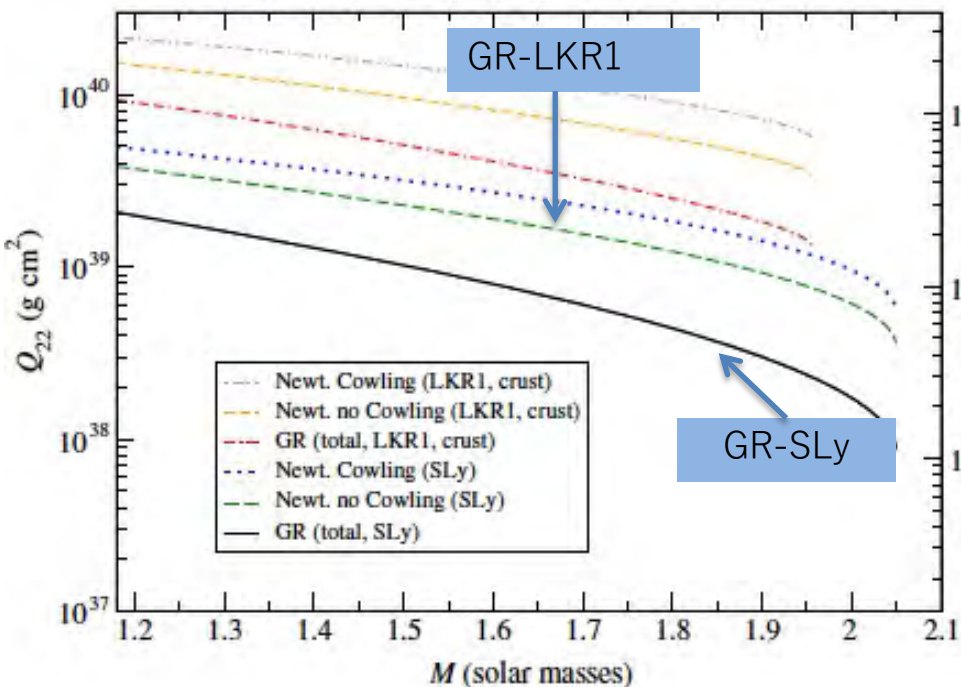
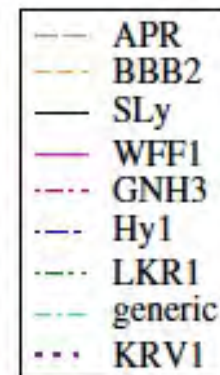
$$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}} \quad \epsilon I_{zz} = \sqrt{\frac{8\pi}{15}} Q_{22}$$

- “Height of a mountain” in some sense (Johnson-McDaniel PRD 2013)



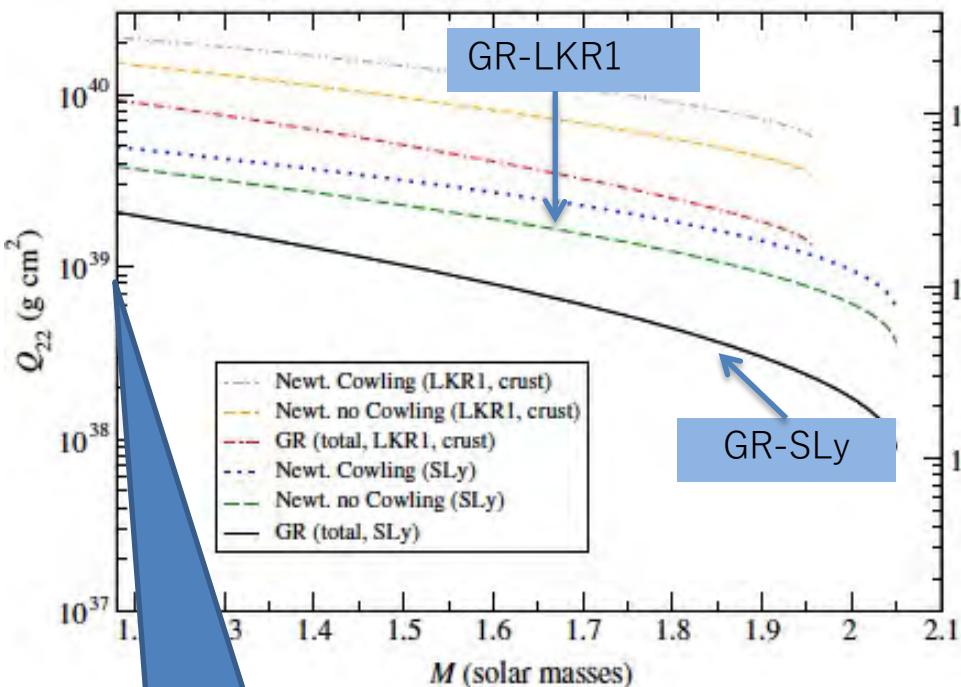
Calculations on theoretical maximum

(Johnson-McDaniel PRD 2013 & Johnson-McDaniel & Owen, PRD 2013)

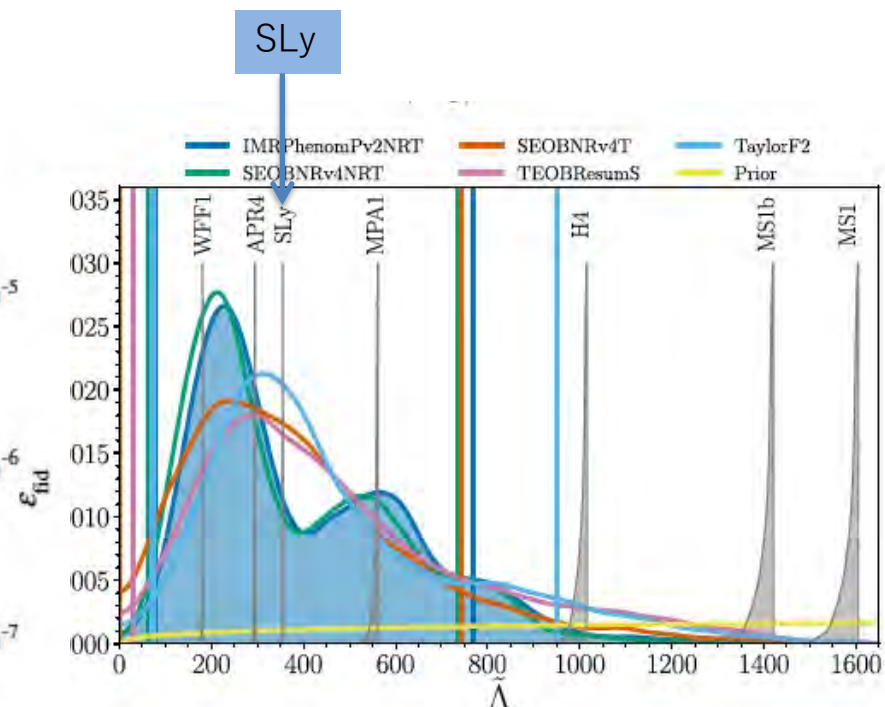


Calculations on theoretical maximum

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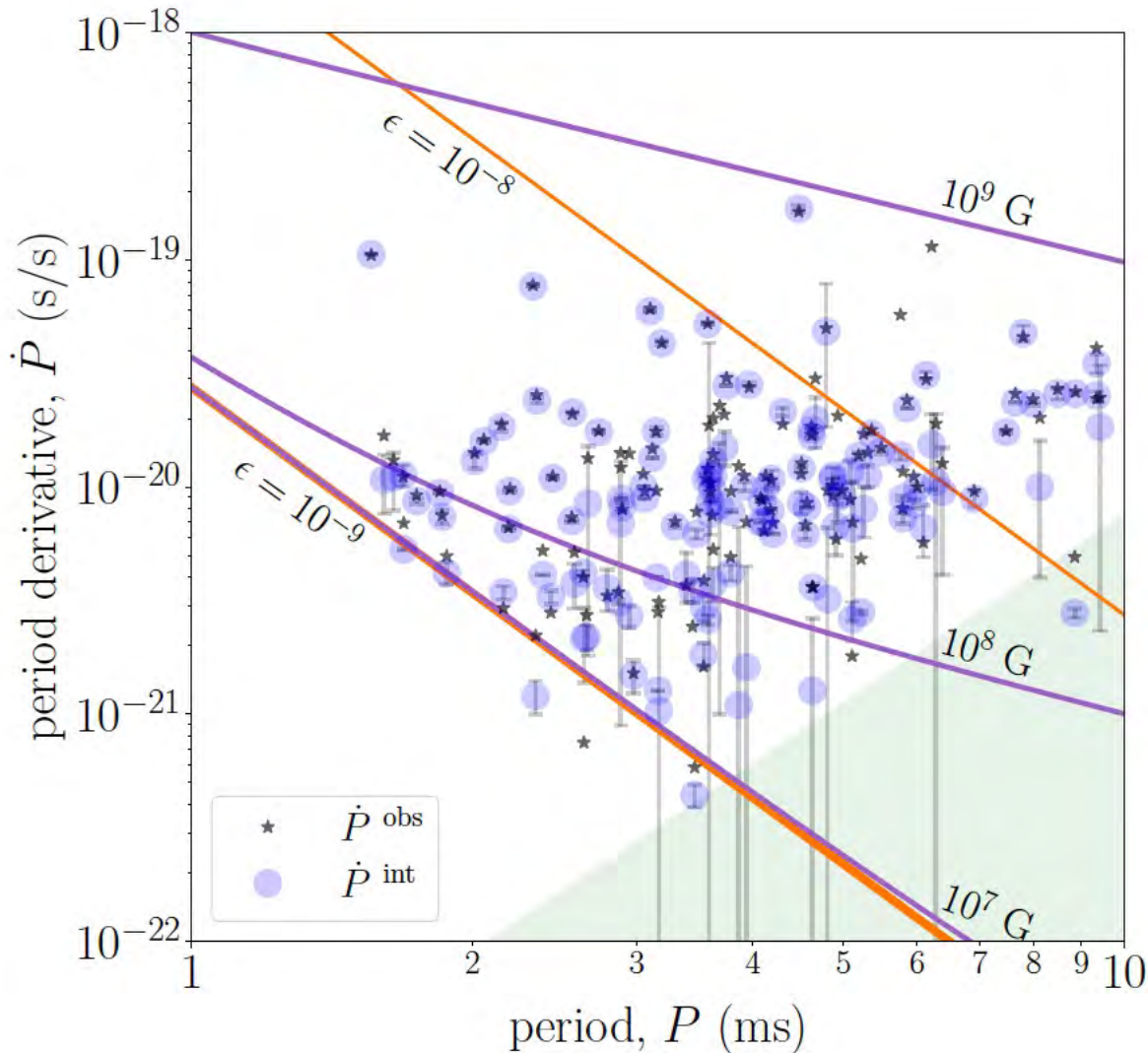
$Q_{22} \sim 10^{32} \text{ kg m}^2$



LVC (2019) GWTC-1 “catalogue paper”, low spin case

GW170817 → actual EOS is softish.

Lower limit for ϵ ?



$$\left(\frac{\dot{P}}{10^{-20} \text{ s/s}}\right) = 0.98 \left(\frac{1 \text{ ms}}{P}\right) \left(\frac{B_s}{10^8 \text{ Gauss}}\right)^2 + 2.7 \left(\frac{1 \text{ ms}}{P}\right)^3 \left(\frac{\epsilon}{10^{-9}}\right)^2.$$

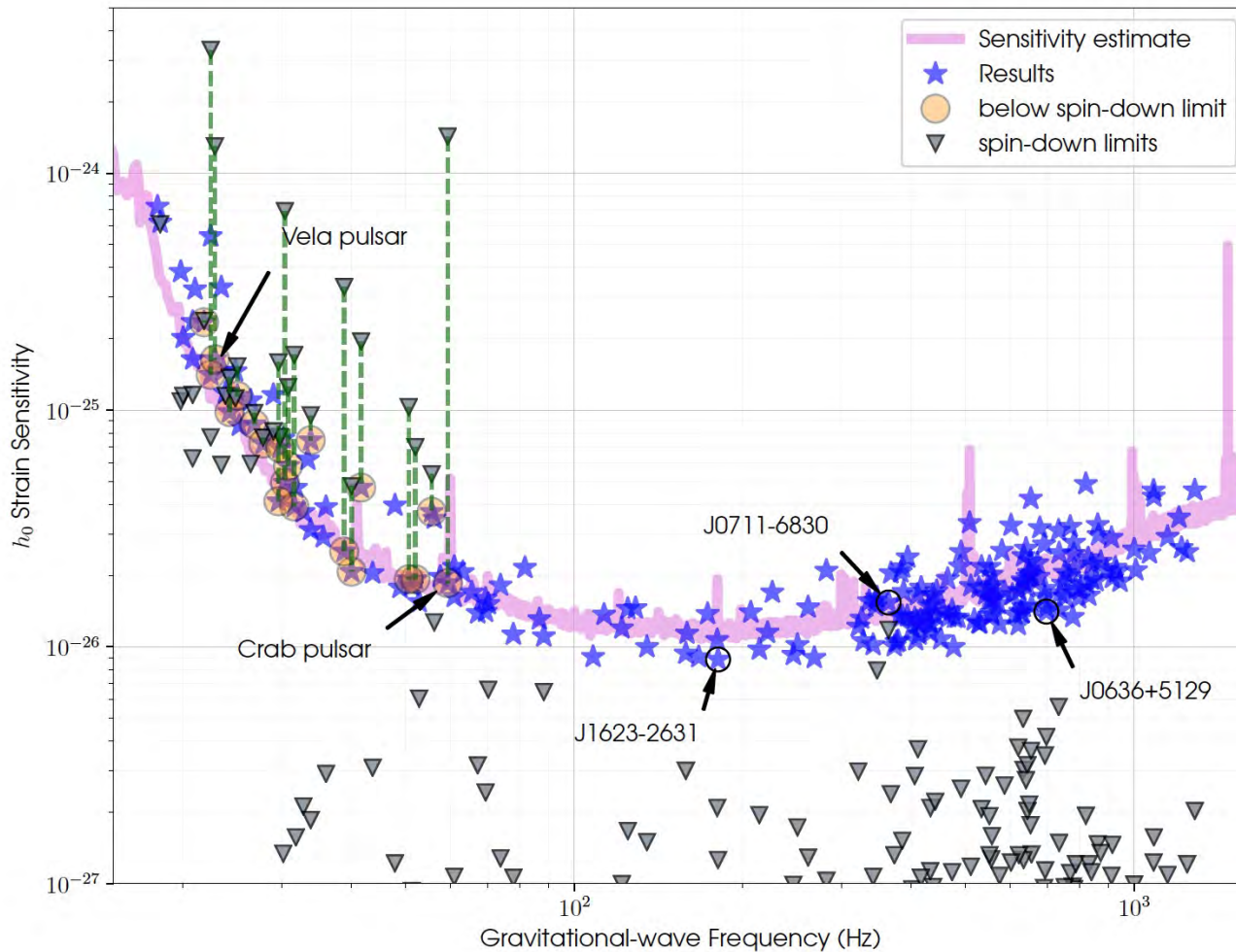
It looks like there is a lower limit on ϵ .

Woan et al. 2018, ApJL, 863, L40.

See also
Gittins & Andersson
MNRAS 2019.

LIGO-Virgo O2 result for known pulsar search

[Astrophysical Journal 879, 10 \(2019\)](#)



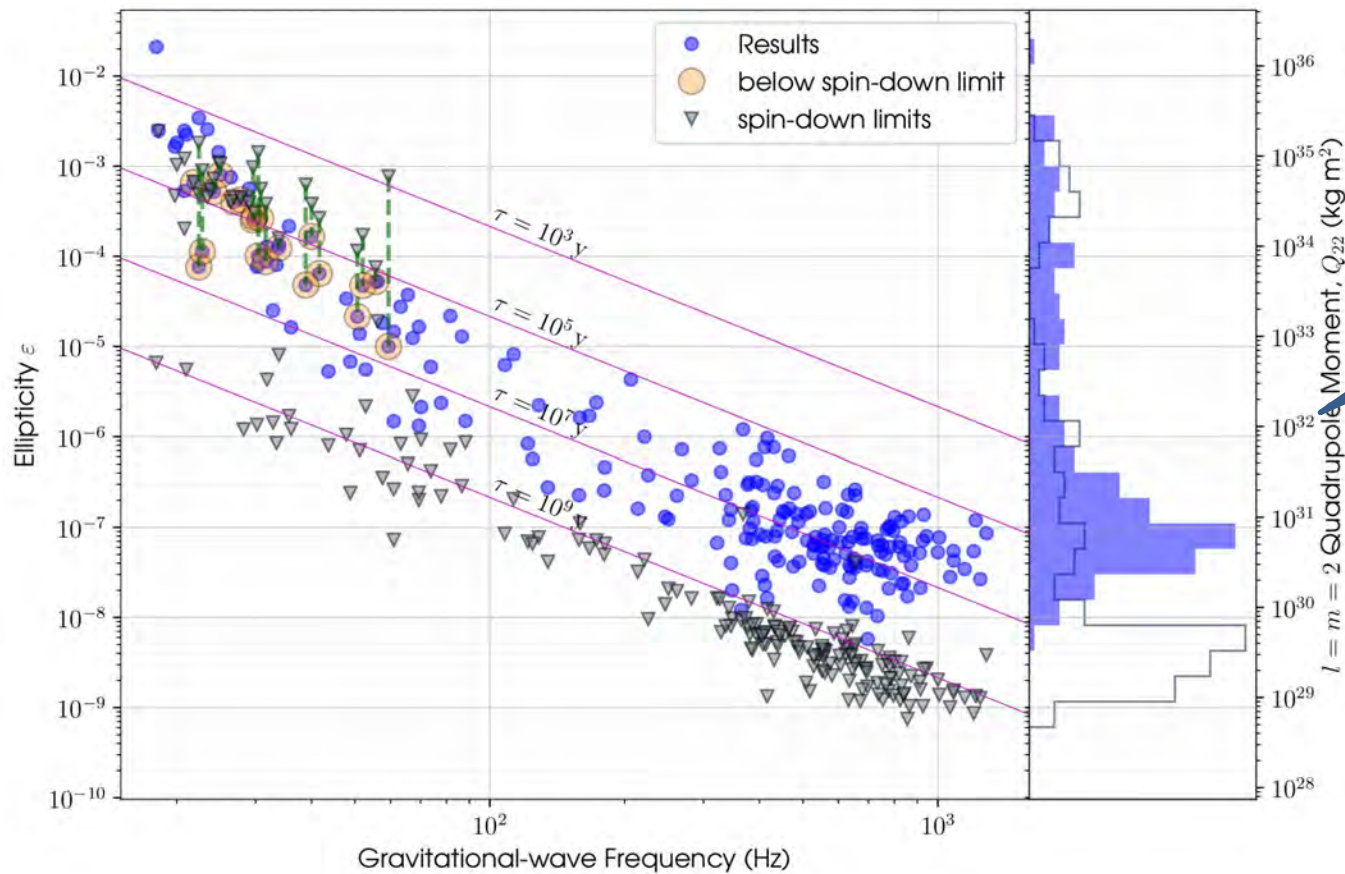
Upper limits on GW
below spin down limits
for several PSRs

NEW! O2+O3
PSRJ0537-6910:
Spin down limits beaten
($f_{\text{rot}} = 62\text{Hz}$)
LVK+ ApJL2021

Spin-down limit:
GW amplitude that
explains the
observed spin down
rate

LIGO-Virgo O2 result for known pulsar search

[Astrophysical Journal 879, 10 \(2019\)](#)



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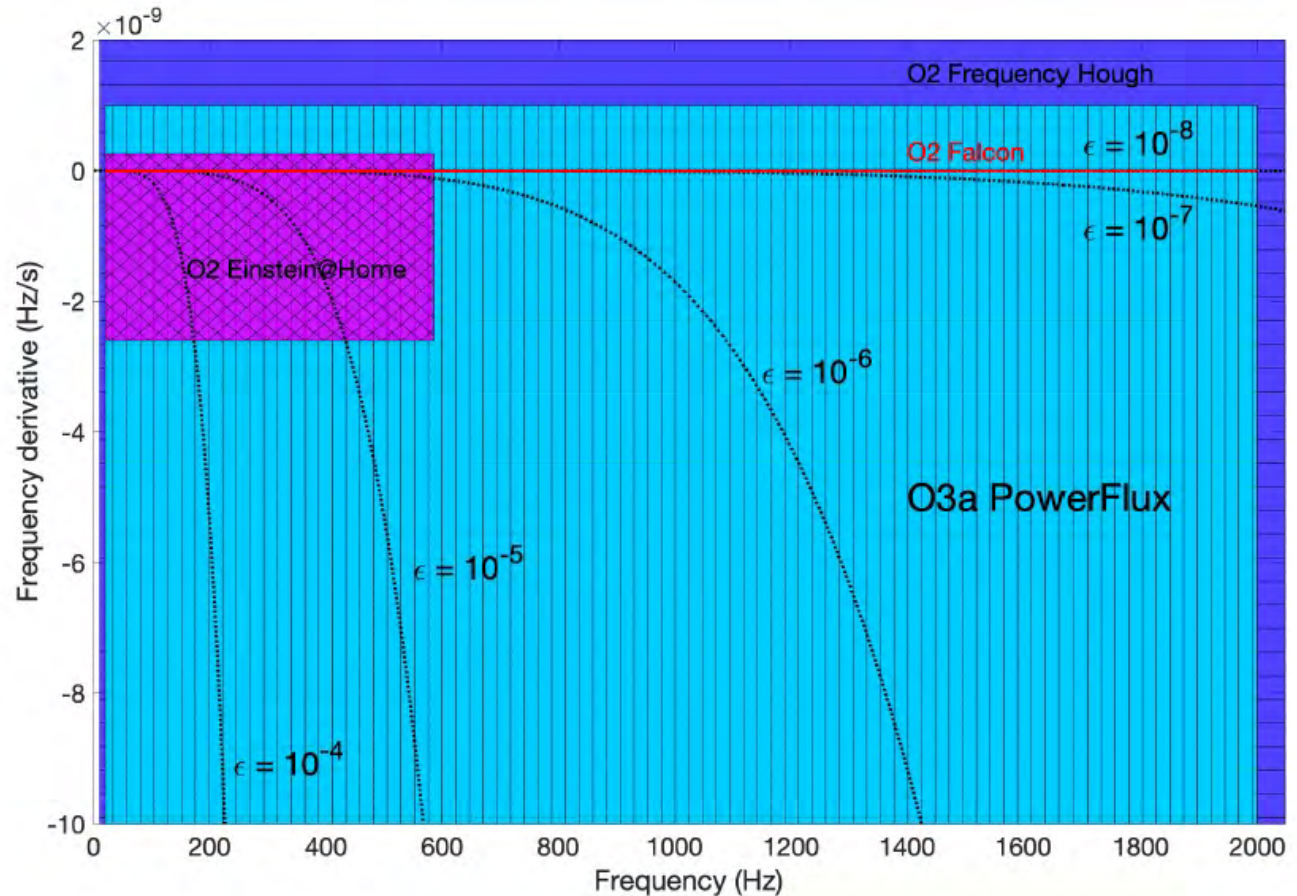
LVK O3a result for isolated unknown NS search

LVK arXiv:2107.00600v1

O3:
2019 1 Apr. – 27 Mar.
with 1 month
commissioning break
in Oct.

aLIGO duty factor:
71.2% (H1)
75.8% (L1)

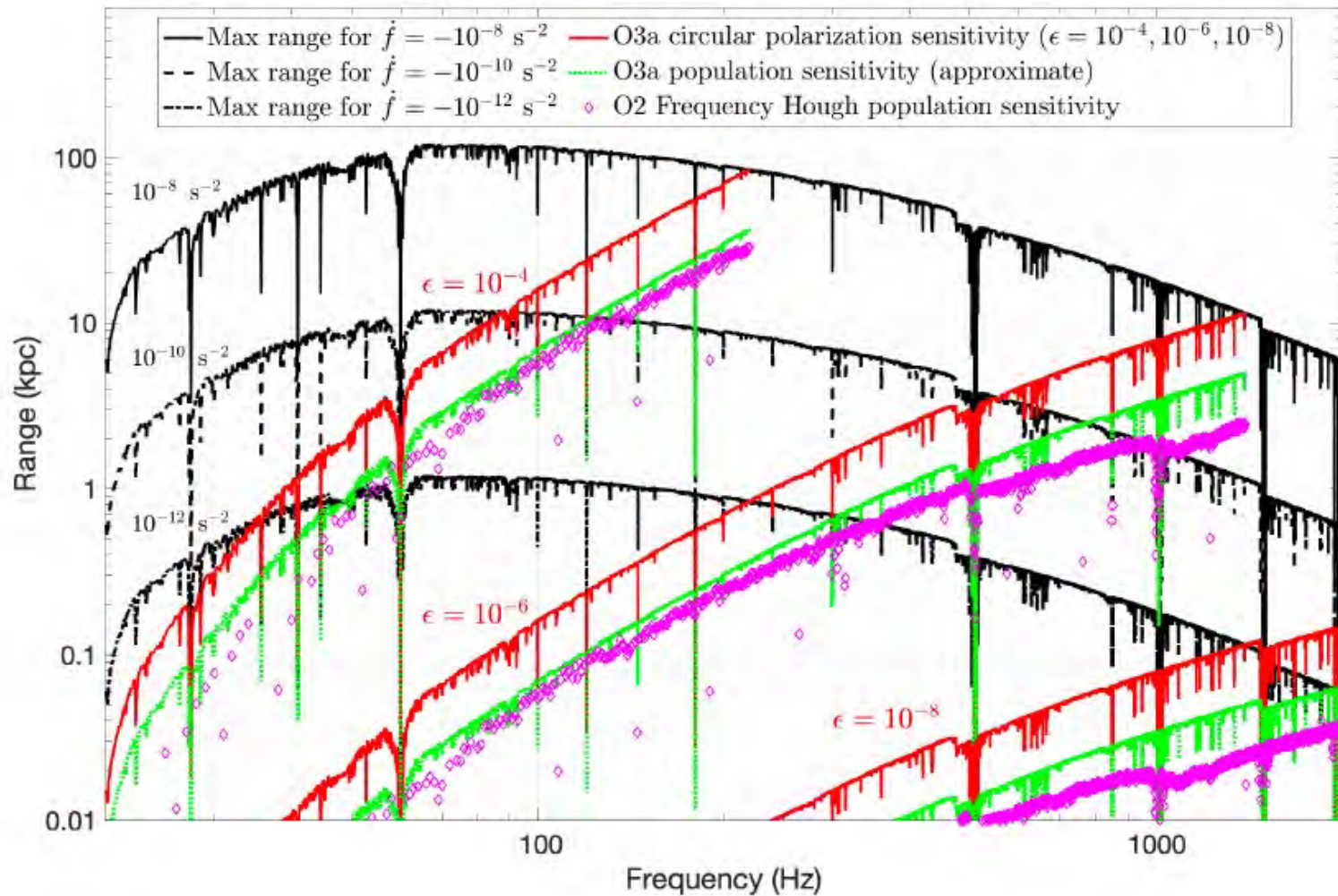
Calibration error:
< 7%, < 4deg.



The parameter region searched is much larger than that
previously performed

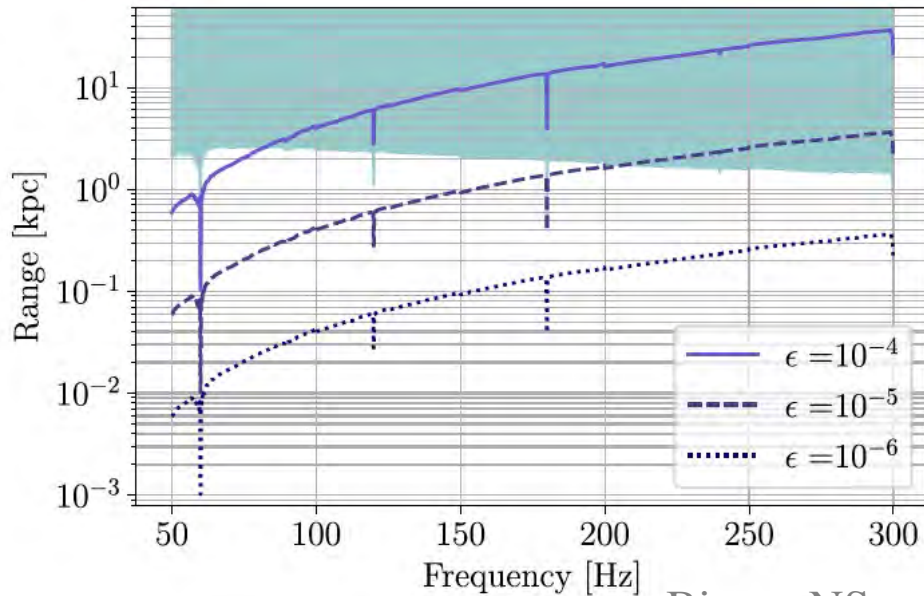
LVK O3a result for isolated unknown NS search

LVK arXiv:2107.00600v1

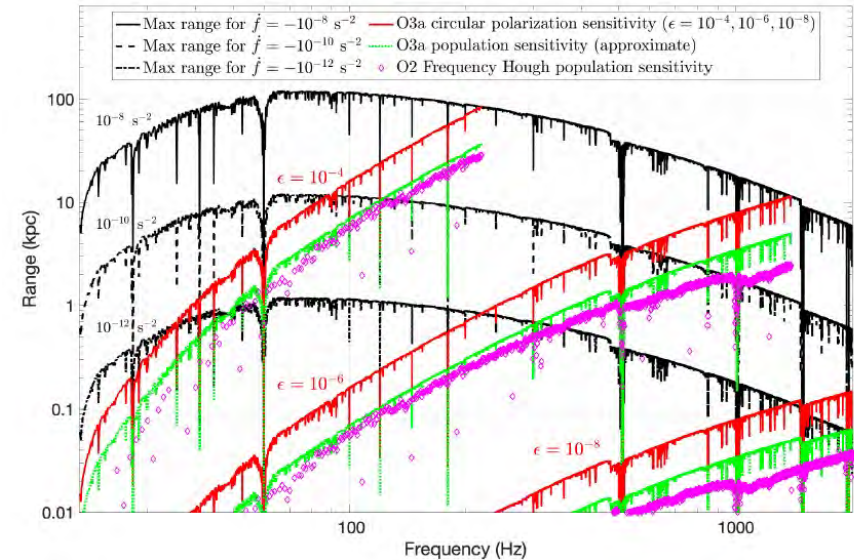


LIGO-Virgo O3a result for unknown NSs in binary systems

LVC PRD103,064017 (2021)



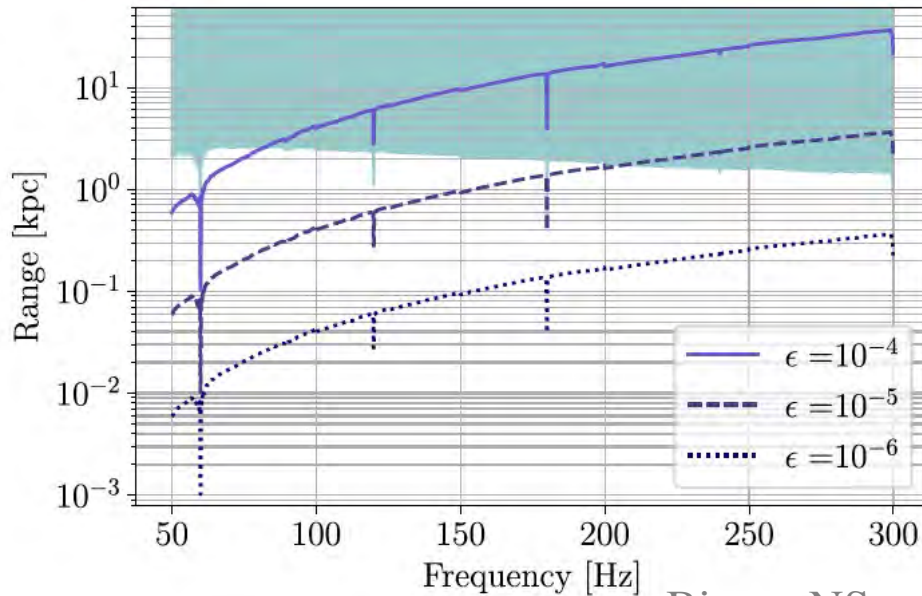
Binary NS



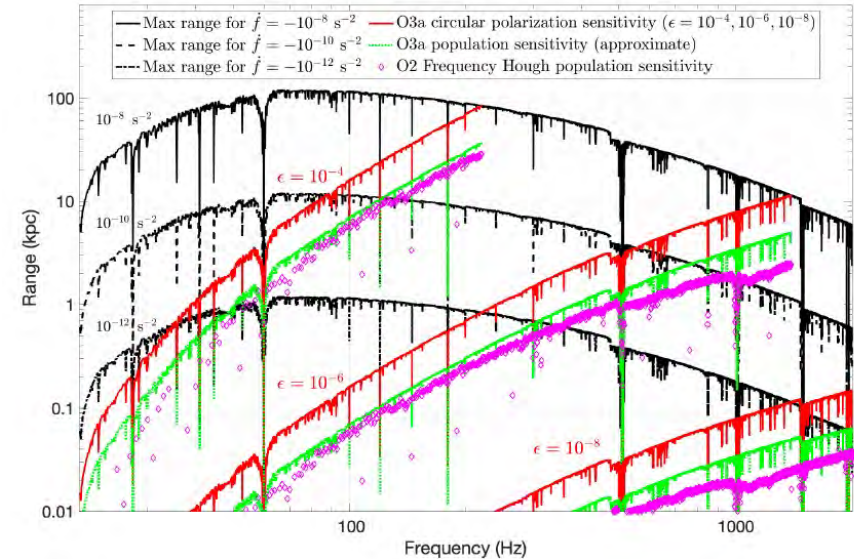
Isolated NS

LIGO-Virgo O3a result for unknown NSs in binary systems

LVC PRD103,064017 (2021)



Binary NS

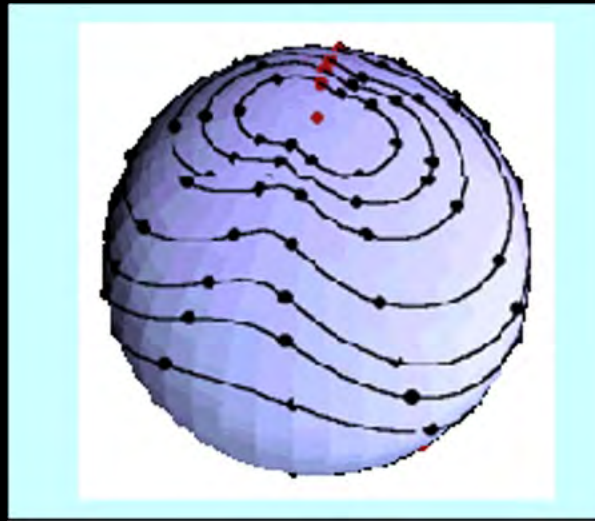


Isolated NS

$$\delta f_g = \frac{1}{T_c}, \delta \theta \propto \frac{c}{v_E T_c f_g}, \delta a \propto \frac{1}{T_c f_g \Omega},$$

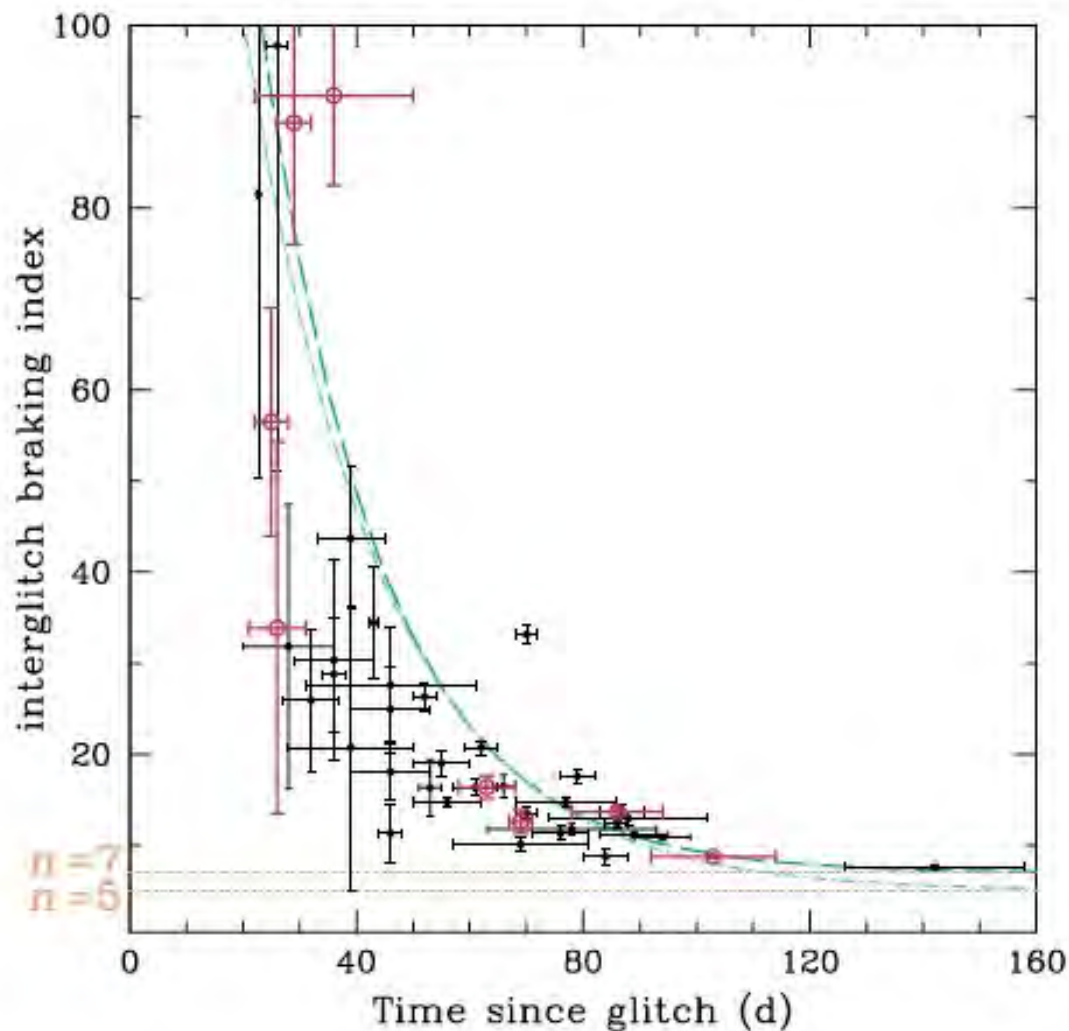
$$\delta \Omega \propto \frac{1}{T_c f_g a \Omega T_o}, \delta t_a \propto \frac{1}{T_c f_g a \Omega^2}$$

“R-mode search”



PSRJ0537-6910

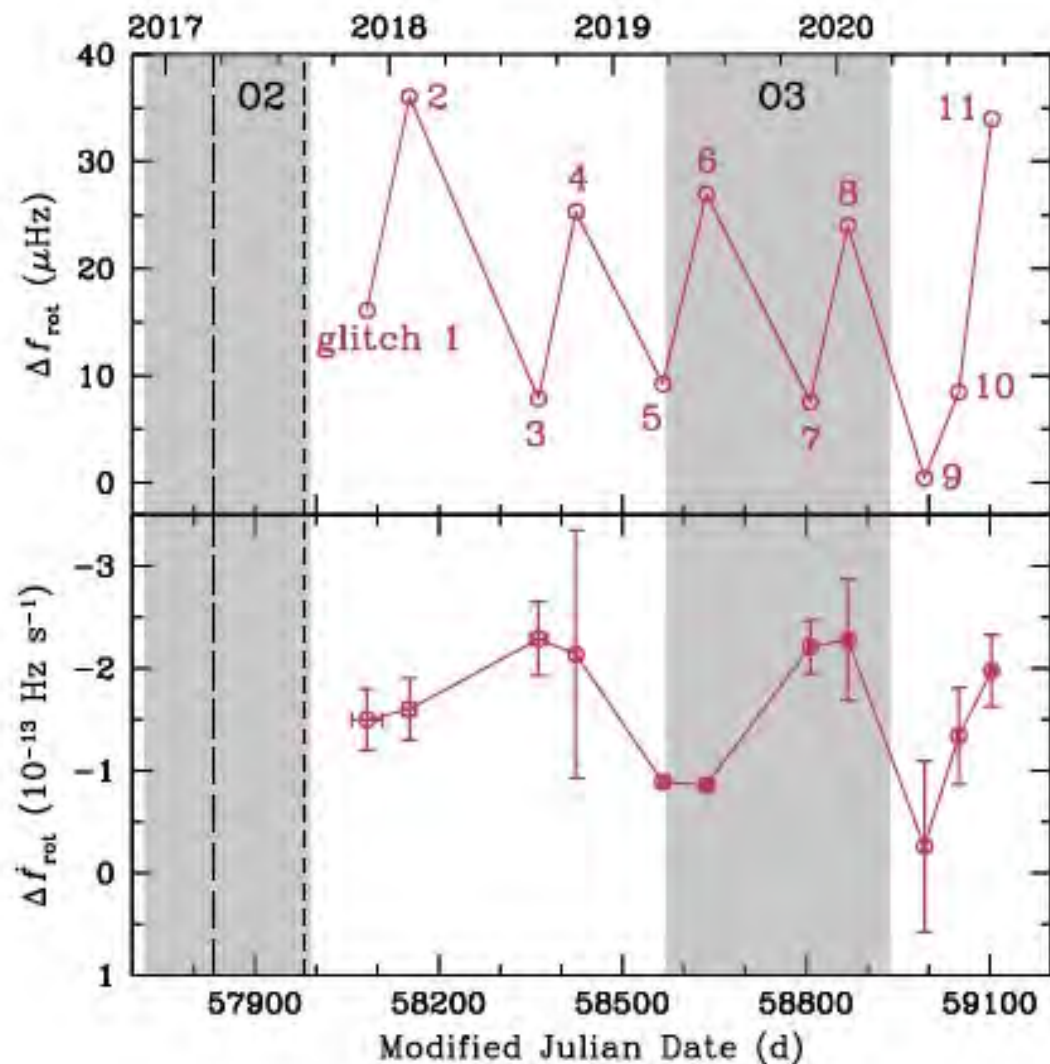
LVK+, [Astrophys. J. Lett. 913, L27 \(2021\)](#)



- Young (1-5kyr) PSR
- 49.6kpc (LMC)
- Found by RXTE (1998)
- NICER obs. 2017-2020
- $f_{\text{rot}} = 62\text{Hz}$
- $\text{Edot} \sim 5 \times 10^{38} \text{ erg s}^{-1}$
(highest among known ATNF PSRs)
- Breaking idx: $n = -1.25$
(20yrs obs)

PSRJ0537-6910

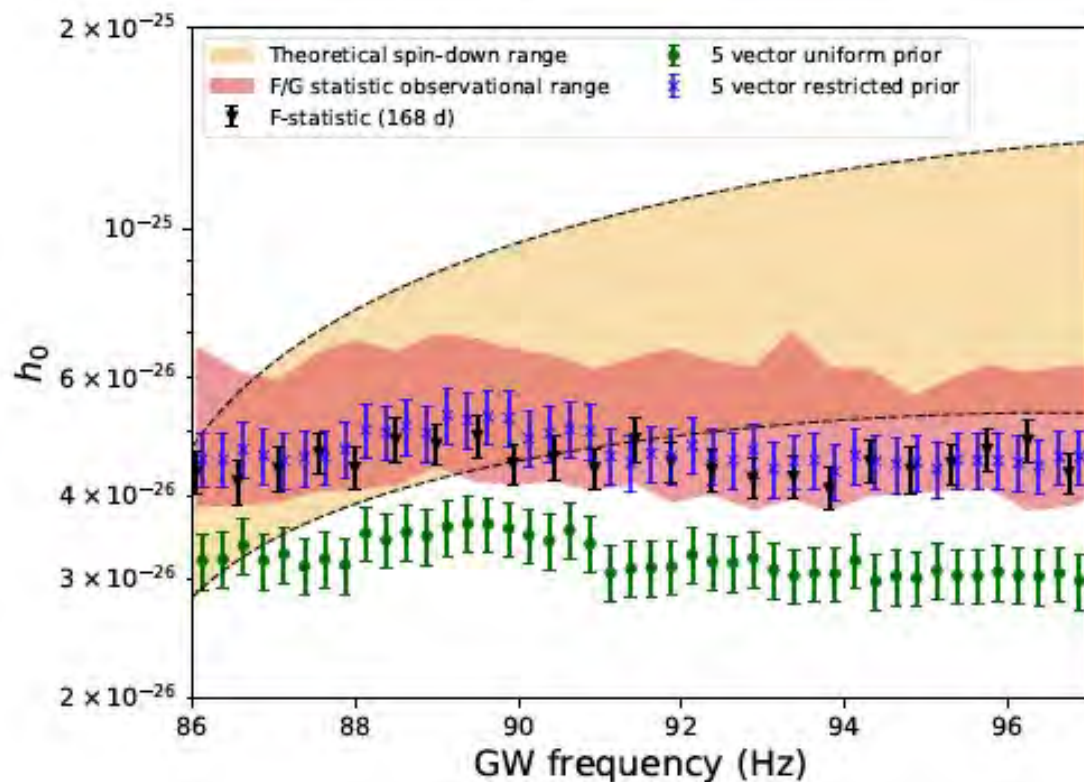
LVK+, [Astrophys. J. Lett. 913, L27 \(2021\)](#)



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(20yrs obs)

PSRJ0537-6910

LVK+, arXiv:2104.14417



R-mode freq. depends on
EOS & NS mass

$$f = A\nu - B \left(\frac{\nu^2}{\nu_K^2} \right) \nu,$$

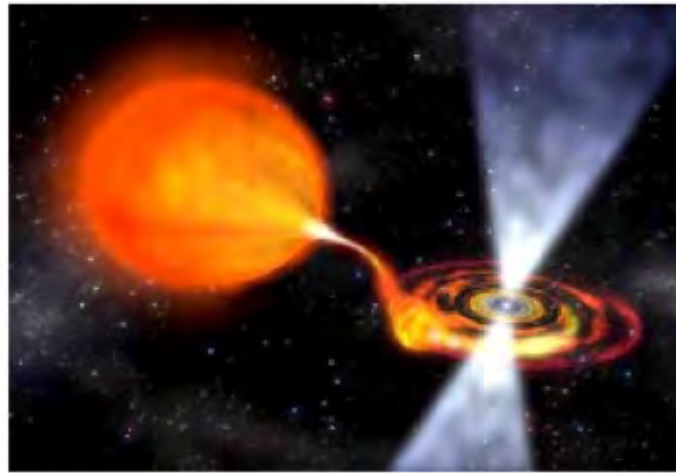
$$\dot{f} = A\dot{\nu} - 3B \left(\frac{\nu^2}{\nu_K^2} \right) \dot{\nu},$$

$$\ddot{f} = A\ddot{\nu} - \left(3 + \frac{6}{n} \right) B \frac{\nu^2}{\nu_K^2} \ddot{\nu}.$$

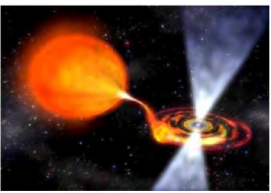
$$1.39 < A < 1.57,$$

$$0 < B < 0.195.$$

“Binary pulsar search”

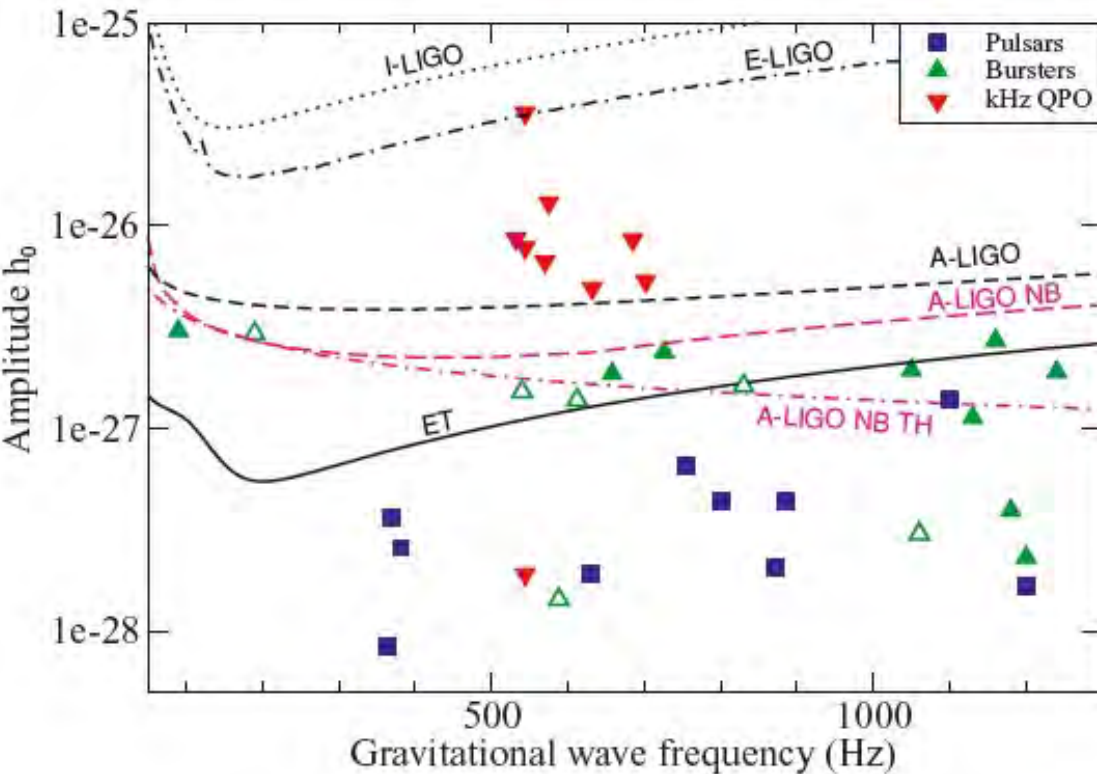


Accreting star



Accreting star

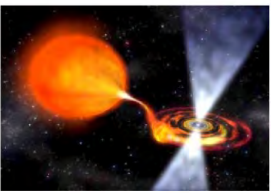
Known accreting pulsar search: best case scenario



Watts et al. (2008)

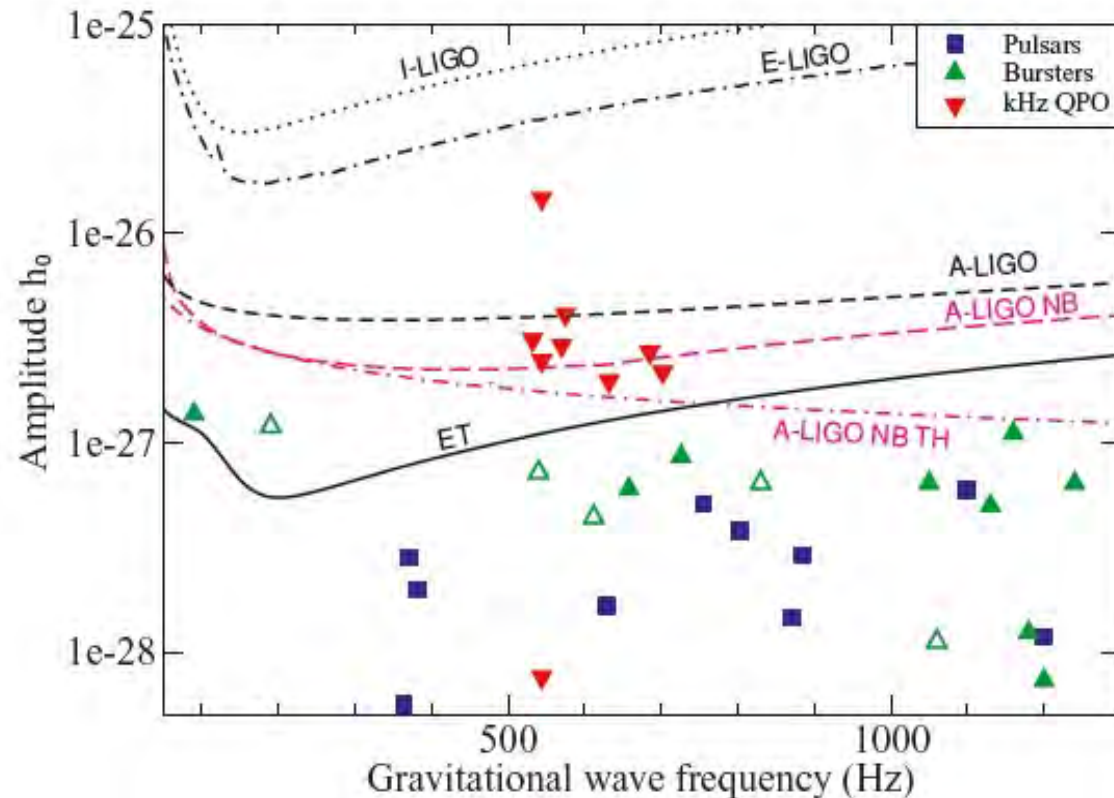
- Single template search
- 2 years integration
- quadrupole mode
- long-term average flux
- Perfect balance
- Open: frequency not known for sure.
- No limitation on computing power
- kHz QPOs are marginally detectable.

$$h_0^{\text{torg.bal.}} = 3.4 \times 10^{-26} \left(\frac{1.4 M_{\odot}}{M} \right)^{\frac{1}{4}} \left(\frac{F_X / X}{3.9 \times 10^{-7} \text{ erg cm}^{-2} \text{ s}^{-1}} \right)^{\frac{1}{2}} \left(\frac{r_m}{10 \text{ km}} \right)^{\frac{1}{4}} \left(\frac{R_m}{10 \text{ km}} \right)^{\frac{1}{2}} \left(\frac{600 \text{ Hz}}{f_{\text{GW}}^{27/38}} \right)^{\frac{1}{2}}$$



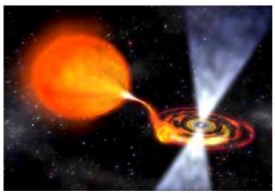
Accreting star

Known accreting pulsar search: better case scenario



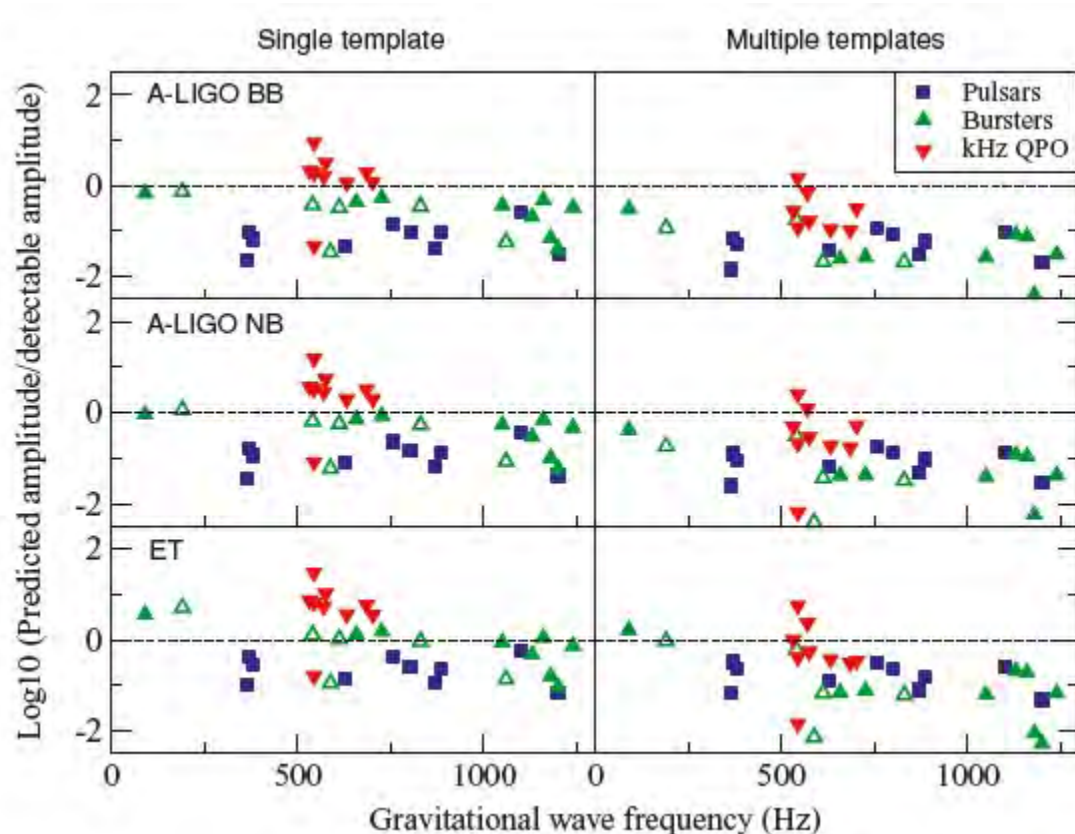
Watts et al. (2008)

- Multi templates search only “look-elsewhere” effect is taken into account.
- 2 years integration
- quadrupole mode
- long-term average flux
- Perfect balance
- Open: frequency not known for sure.
- No limitation on computing power
- SCO-X1 is marginally detectable.



Accreting star

Known accreting pulsar search: realistic scenario



Watt et al. (2008)

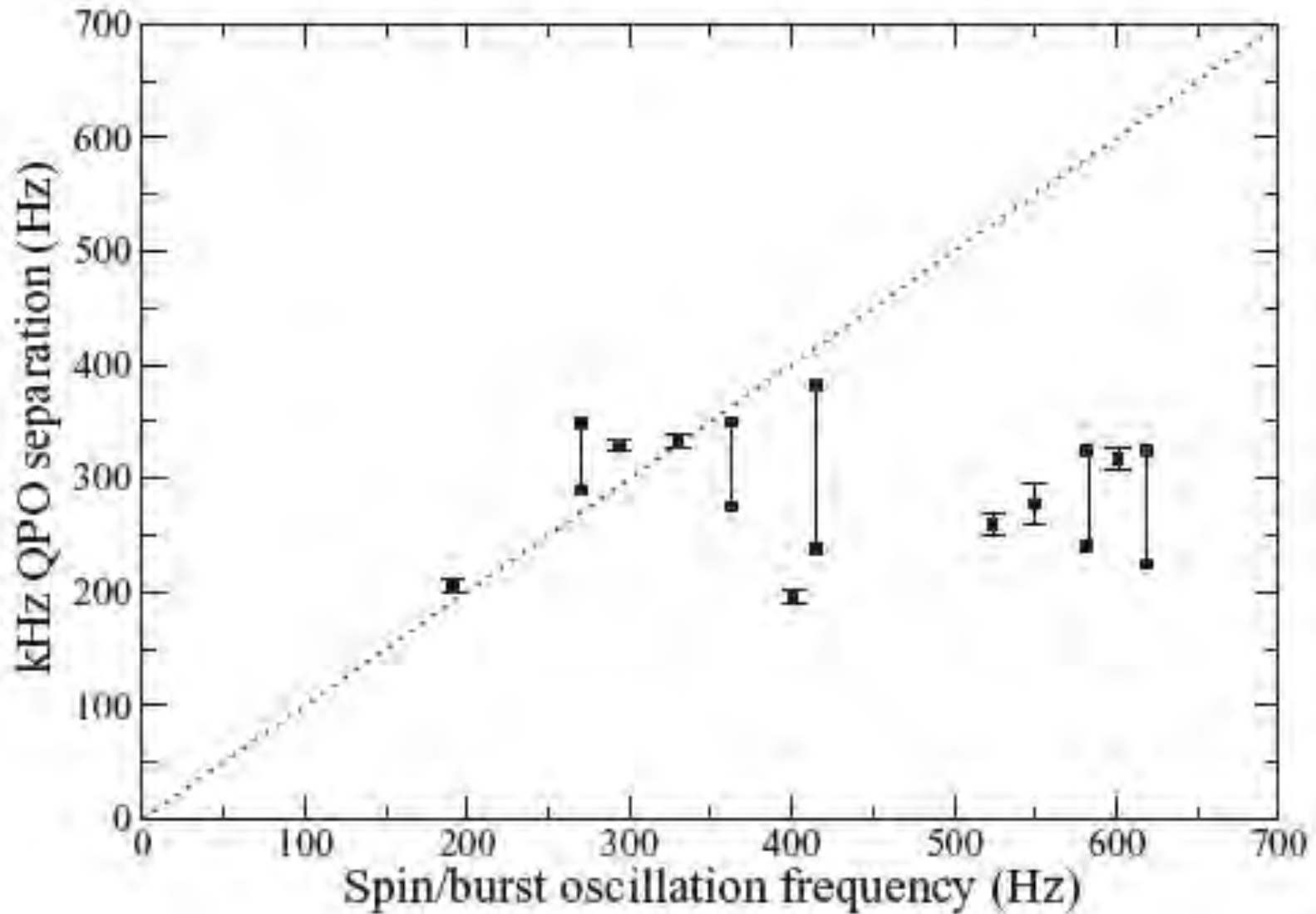
- Multi templates search: “look-elsewhere” effect and computational limitation are taken into account.
- 2 years integration
- quadrupole mode
- long-term average flux
- Perfect balance
- Open: frequency not known for sure.
- 50 times faster computers for AdLIGO, 100 times faster for ET.
- SCO-X1 is marginally detectable.



Accreting star

Can we infer spin frequency from kHz QPO?

Watts et al. (2008)



Sco-X1 result: O2, Zhang et al. 2021

- Cross correlation method $\text{SNR} \propto (T_{\text{coh}} T_{\text{obs}})^{1/4}$
- aLIGO O2: \sim 8-month, \sim 100 days worth
- Ephemeris by Wang et al. (2018)

Parameter	Range	Grid Spacing
f_{GW} (Hz)	[40, 180]	$\sim 2 \times 10^{-6}$ [Hz]
$a \sin i$ (lt-s)	[1.45, 3.25]	$\sim \frac{0.17 \text{ [lt - s Hz]}}{f_{\text{GW}}}$
T_{asc} (GPS s) ^a	$1178556229 \pm 3 \times 139$	$\sim \frac{1576 \text{ [lt - s]}}{f_{\text{GW}} a \sin i}$
P_{orb} (s)	$68023.86 \pm 3 \times 0.04$	$\sim \frac{18 \text{ [lt - s]}}{f_{\text{GW}} a \sin i}$

Position is known.

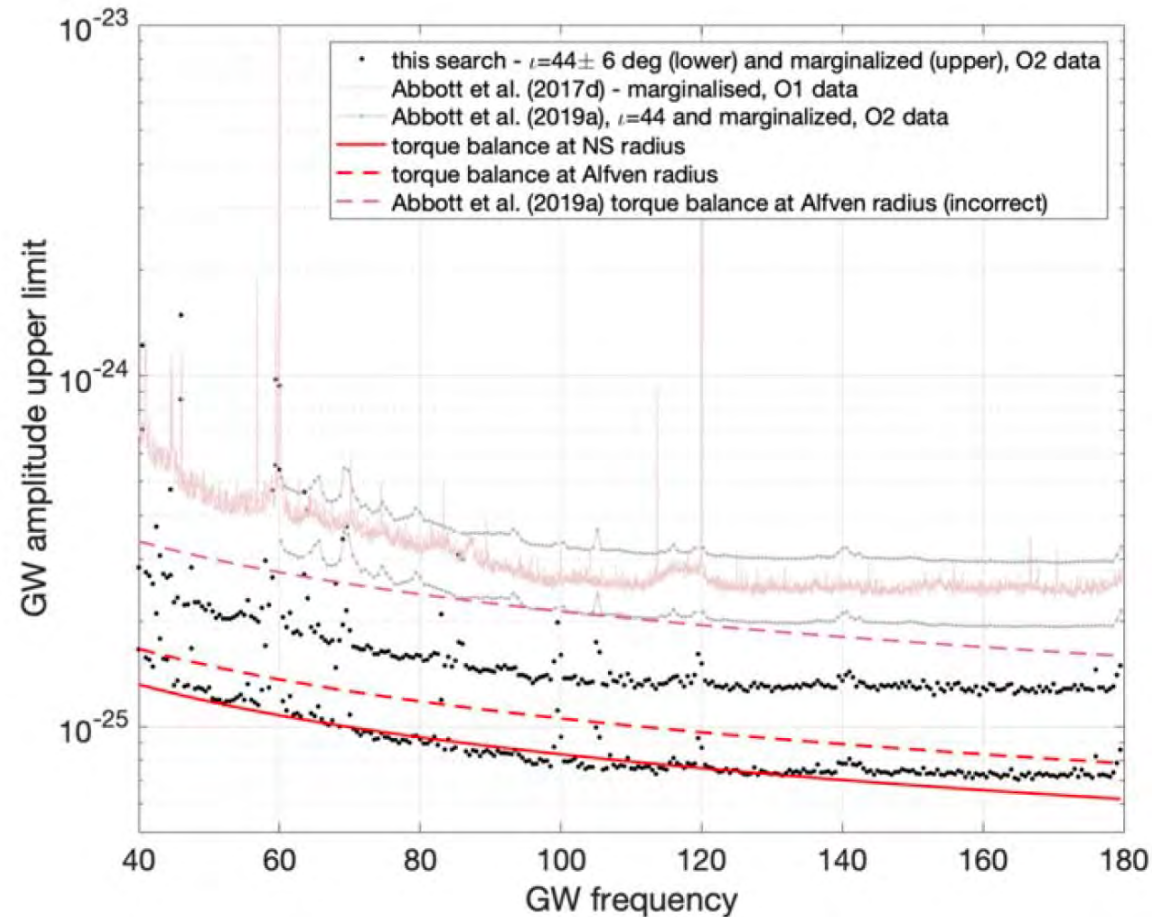
Can assume the inclination to be $44^\circ \pm 6^\circ$

No constraints on the spin frequency

Several binary parameters needs to be searched over.

Grid spacings determined by loss of signal to noise ratio (SNR)

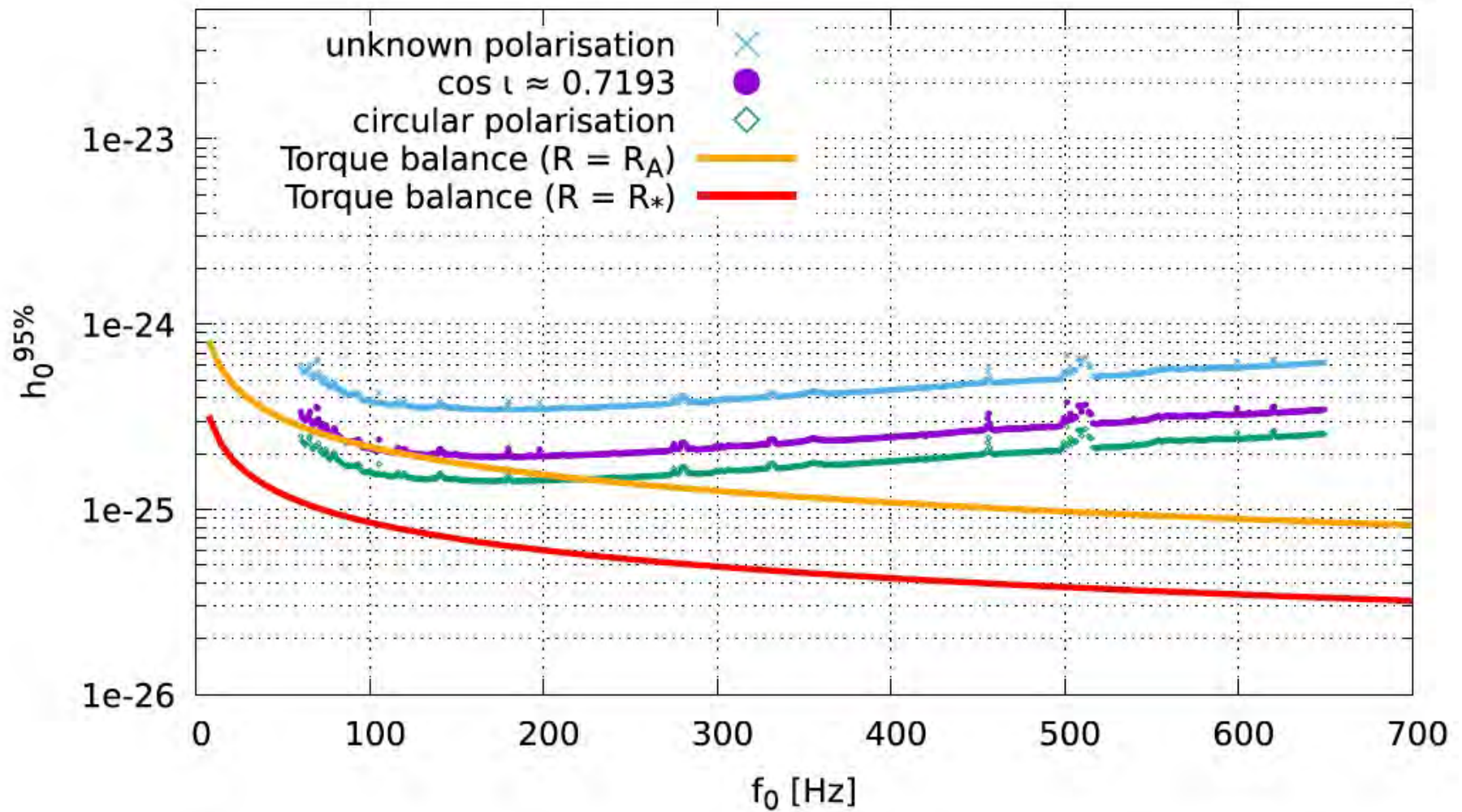
Sco-X1 result: O2, Zhang et al. 2021



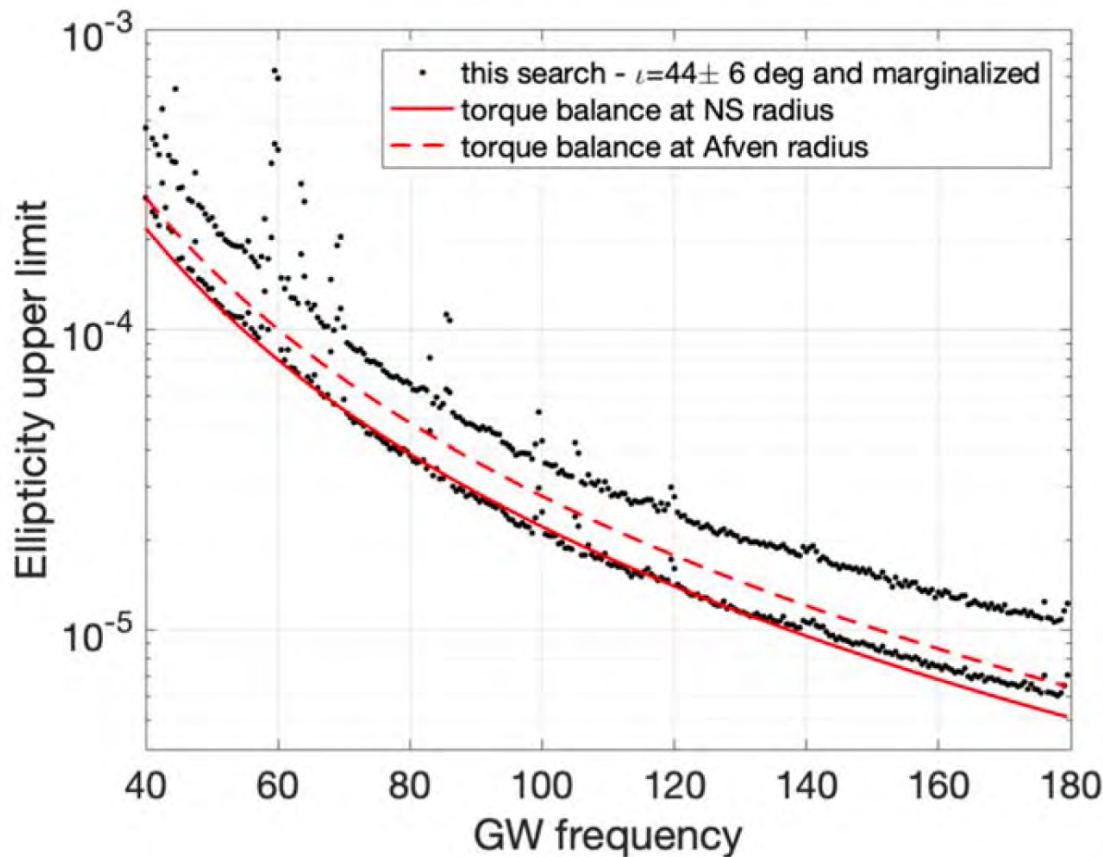
$$\begin{aligned}
 h_0^{\text{torq.bal.}} &= \sqrt{\frac{10}{Xc^3} \frac{F_X R}{f_{\text{GW}}}} \sqrt{\frac{Gr_m}{M}} \\
 &= 3.4 \times 10^{-26} \left(\frac{1.4 M_\odot}{M} \right)^{\frac{1}{4}} \left(\frac{r_m}{10 \text{ km}} \right)^{\frac{1}{4}} \\
 &\quad \times \left(\frac{F_X/X}{3.9 \times 10^{-7} \text{ erg cm}^{-2} \text{ s}^{-1}} \right)^{\frac{1}{2}} \\
 &\quad \times \left(\frac{R}{10 \text{ km}} \right)^{\frac{1}{2}} \left(\frac{600 \text{ Hz}}{f_{\text{GW}}} \right)^{\frac{1}{2}}.
 \end{aligned}$$

Reached the torque balance limit for the first time

Cf. Sco-X1 result: O2, LVC 2019



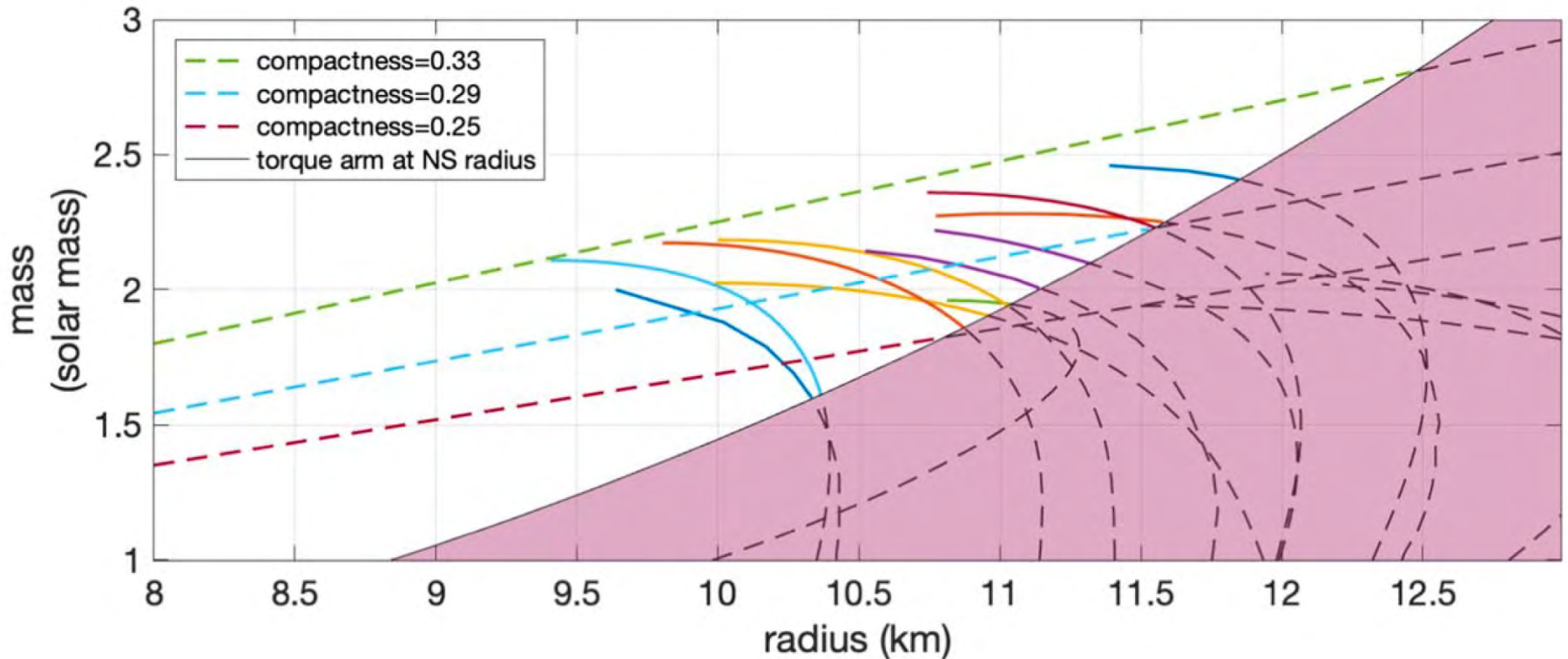
Sco-X1 result: O2, Zhang et al. 2021



$$\left\{ \begin{array}{l} h_0 = \frac{4\pi^2 G}{c^4} \frac{I \varepsilon f_{\text{GW}}^2}{d} \\ \text{with } \varepsilon = \frac{I_{xx} - I_{yy}}{I} \end{array} \right.$$

Still far from the plausible upper limit on the ellipticity

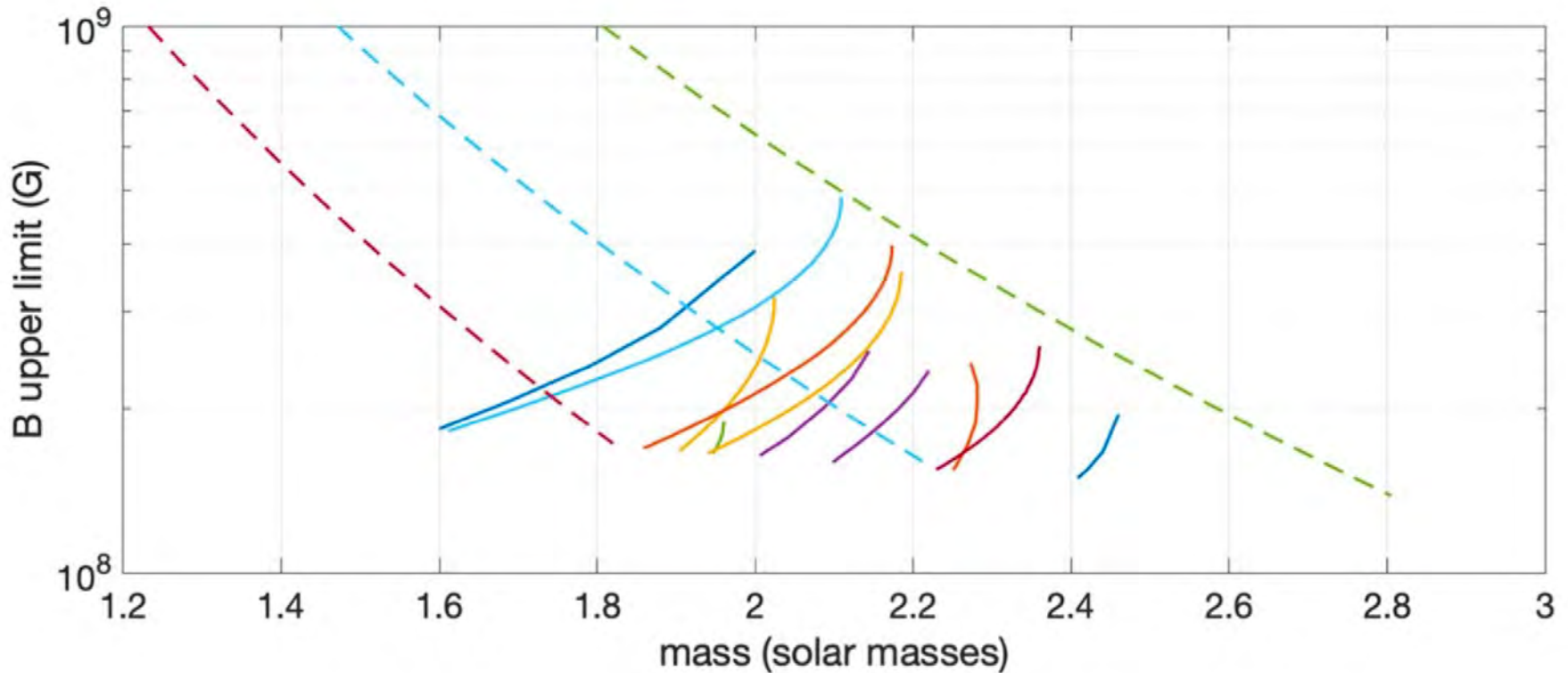
Sco-X1 result: O2, Zhang et al. 2021



From the torque balance limit equation and the derived upper limit on the GW strain, one can put the constraint on the mass and radius of NS, assuming the **Alfven radius = the NS radius**, $f_{\text{GW}} = 117.5\text{Hz}$, $i = 44^\circ$, $X=1$

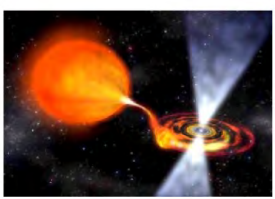
$$[M_1^{-\frac{1}{4}} R_1^{\frac{3}{4}}]^{\text{excl}} \geq X \frac{1}{2} \frac{h_0^{95\%}}{3.4 \times 10^{-26}} f_{\text{GW}_1}^{\frac{1}{2}} F_1^{-\frac{1}{2}}.$$

Sco-X1 result: O2, Zhang et al. 2021



From the torque balance limit equation and the derived upper limit on the GW strain, one can put the constraint on the mass and radius of NS, assuming the [Alfven radius > the NS radius](#), $f_{\text{GW}} = 117.5\text{Hz}$, $i = 44^\circ$, $X=1$

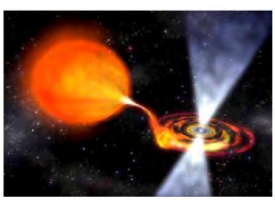
$$\left[M_1^{\frac{3}{14}} R_1^{\frac{6}{7}} B_1^{\frac{1}{7}} \right]^{\text{excl}} \geq \frac{X^{\frac{3}{7}} h_0^{95\%}}{\xi^{\frac{1}{4}} 4.3 \times 10^{-26}} f_{\text{GW}_1}^{\frac{1}{2}} F_1^{-\frac{3}{7}} d_1^{\frac{1}{7}}.$$



Accreting star

Summary

- kHz QPOs would be marginally detectable by LVK. Sco-X1 is most promising.
- Obstacles against possible detection are:
 - Computing power
 - Poor knowledge on the pulsar properties, especially spin frequency (and binary motion information).
 - Need to find a relationship between kHz QPO freq. and spin freq.
 - Need to have more precise and accurate measurements of pulsar and binary properties.
- Zhang et al. (2021) has reached the torque balance upper limit but only 40-180Hz
- LVC (2019) has searched over 60-650Hz incl. plausible freq. range.



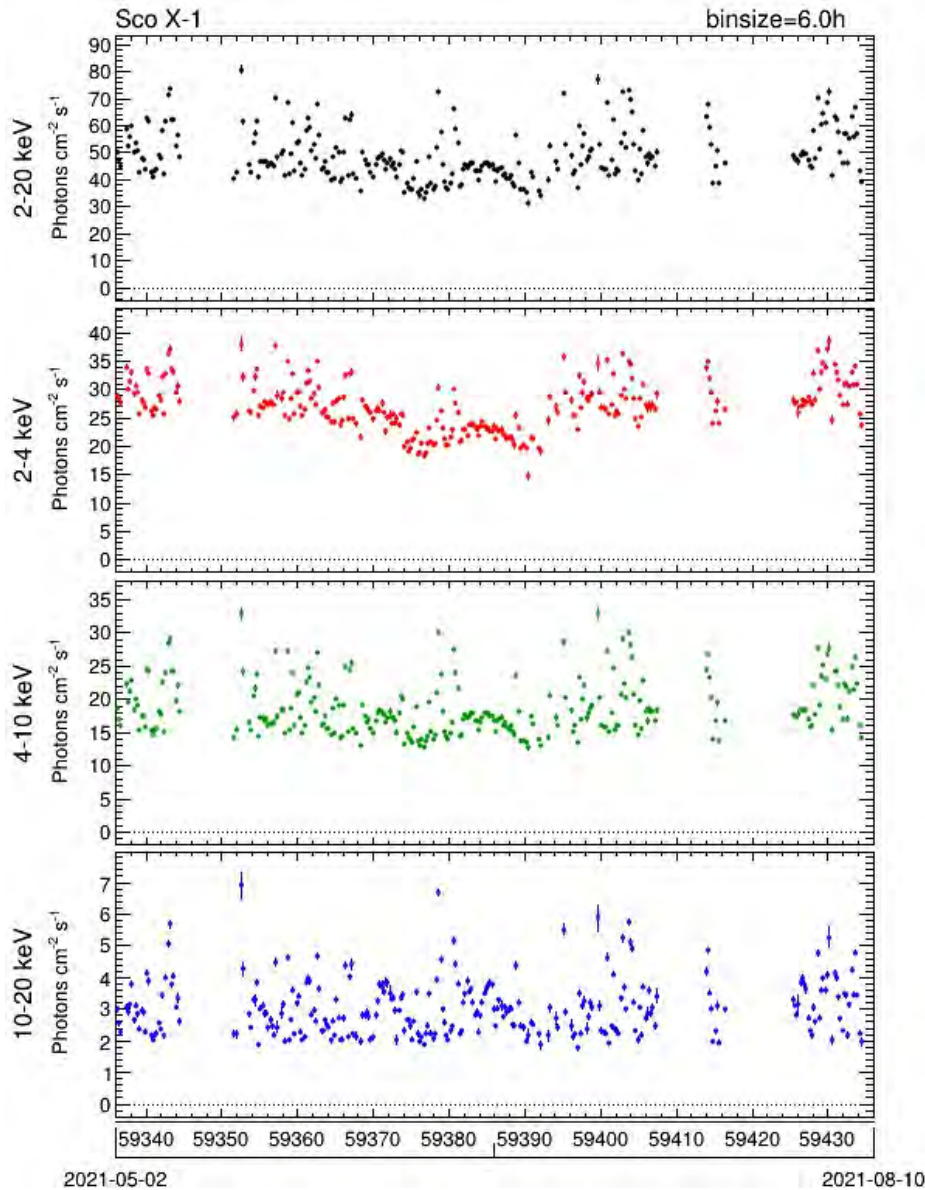
Accreting star

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NinjaSat(EM obs.)

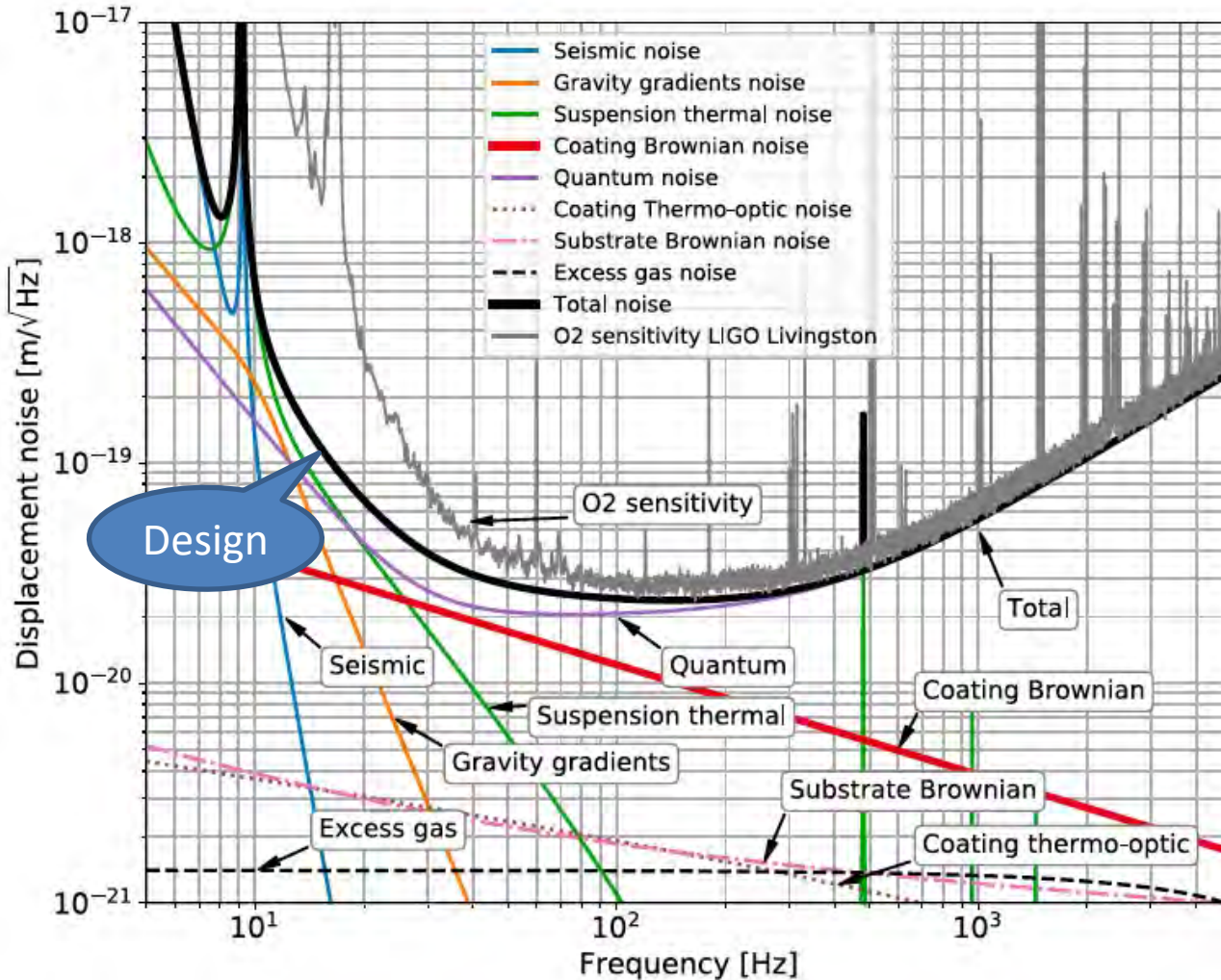
Appendix



- MAXI observation of Sco-X1

<http://maxi.riken.jp/top/index.html>

Main noise sources of the GW detector sensitivity (aLIGO)



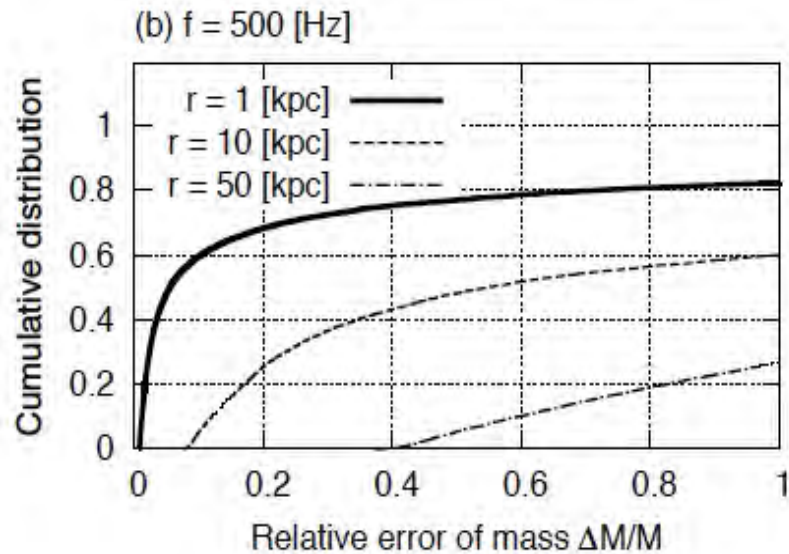
- GW detector sensitivity curve determined by various noise sources
- High frequency: Laser shot noise colored by reduction during photon round-trip

Something different: Mass of isolated NS

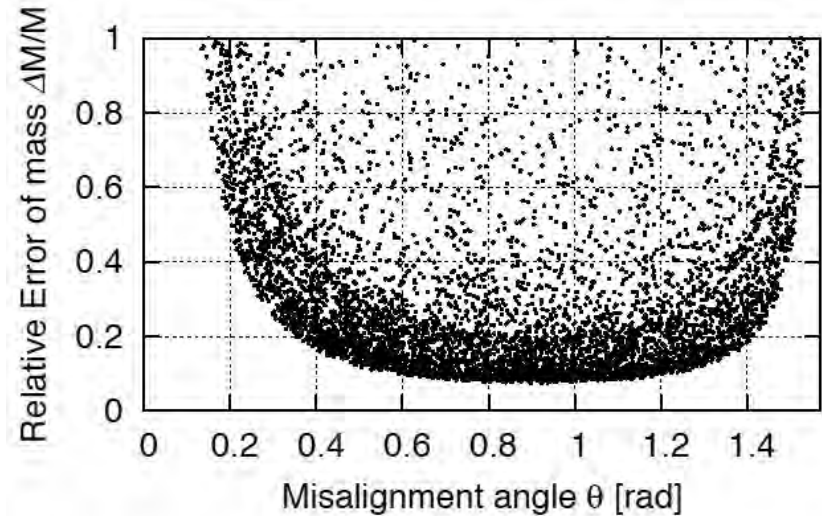
Ono, Eda and Itoh, PRD (2015).

- If we detect GW from NS spinning motion and find $Q_{22} < 10^{31} \text{ kg m}^2$, no constraint on EOS will be available since almost any EOS can explain it.
- Needs to measure mass of that NS:
 - For a NS in binary, we can use orbital motion.
 - For an isolated NS, observation of two GW modes from that NS helps (Ono, Eda, & Itoh, PRD 2015), but perhaps in the 3rd generation GW Telescope era.

Result of the Fisher analysis



Einstein Telescope, 3 years integration.



Ono, Eda and Itoh, PRD (2015).

Maximum possible wobble angle (Jones & Andersson, 2002, cf. Horowitz & Kadau, 2009):

$$\theta_{\max} \approx 0.45 \left(\frac{100 \text{ Hz}}{f} \right)^2 \left(\frac{u_{\text{break}}}{10^{-3}} \right) \text{ rad.}$$

PSR B1828-11: ~ 3 degrees (Stairs, Lyne & Shemar, Nature 2000, Link & Epstein, ApJ 2001)