Magnetic field confined by elasticity in neutron star crust

- YK & Suzuki(2020) arXiv:2004.08006 MNRAS(2020)
- YK, Kisaka & Fujisawa(2021) arXiv:2011.03239 MNRAS(2021)
- YK, Kisaka & Fujisawa(2021) arXiv:2106.14337 MNRAS(2021) in press
- +a

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～中性子星の観測と理論～研究活性化ワークショップ 2021

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Magnetic field confined in crust

Working hypothesis
-> CCO(Magnetar)
-> Formation ?　Matsumoto-san's talk

Outer part ($\Delta r/R$ <0.1) of NS

- Core $\rho = 1.4 \times 10^{14}\ g/cm^3$ nuclear density
  Pressure p=$3 \times 10^{33}\ erg/cm^3$
  Max. B $B < 2 \times 10^{17} G$ $\sim Mev/fm^3$
- Outer BC $\rho = 4 \times 10^{11}\ g/cm^3$ neutron drip / Solid crust
  Pressure p=$5 \times 10^{29}\ erg/cm^3$
  Max. B $B < 3 \times 10^{15} G$ (enough) $\sim kev/fm^3$
  Shear $\mu = 3 \times 10^{27} \rightarrow 10^{30}\ erg/cm^3$

-> Deformation of NS "Mountain"

Core vs crustal field

Core physics may appear via crust   Crust first!   Negative -> Core next!

- Complicated treatment for core physics

  Many species(e,n,p,…) superfluid/superconductor +….

- Simple and possible observational consequence for crust case

  Ion lattice + e -> B-evolution by Hall drift & Joule loss

  Deformation of crust,   "mountain on NS "

  -> GW?

  Beyond limit   Fracture or not?   -> Observational events?

  -> Quake? Models of FRB?

  -> Plastic flow? Effect on magnetic field evolution   (YK+20,21)

  +…

Outline of my talk

1. (Introduction) Braking strain of solid
2. Astrophysical consequences

   -> A mountain on NS by elastic or magnetic forces

   -> GW? (Brief review) ----> Ito-san's talk

   Deformation induced by magnetic evolution (YK+,21a)

3. Equilibrium of magnetized crust (NEW) (YK+,21b)

- Observation consequences beyond threshold

  -> Quake? Models of FRB?

  -> Plastic flow?

  Some discussion in literature by speculation, but is omitted here

Breaking strain of solid crystals

- Material property in high density regime

  Crust of NS($\rho \approx 10^{11} - 10^{14} g/cm^3$) outer part < 1km thick

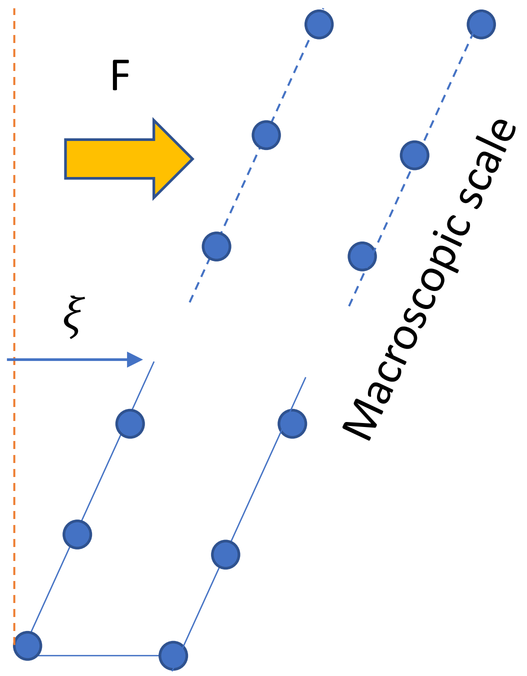
  pressure(p$\approx 10^{29} - 10^{33} erg/cm^3$)

$$\sigma_{ij} = \nabla_i \xi_j + \nabla_j \xi_i - \frac{2}{3} g_{ij} (\nabla \cdot \xi),\ h_i = \nabla_j \left(\mu \sigma_i^j \right)$$

  shear modulus $\mu \sim \rho$ Limit of strain σ_c

- Experimental approach is impossible (at present)
- Theory (Analytic $\sigma_c \sim 0.01$ /Numerical $\sigma_c \sim 0.1$)

- Beyond it , plastic flow or quake
- Possibly it is checked in astrophysical object



Breaking strain

“Introduction to Solid State Physics”

Charles Kitell

$\sigma_c = 10^{-2}$ to 10^{-5}
1/6 for an ideal case

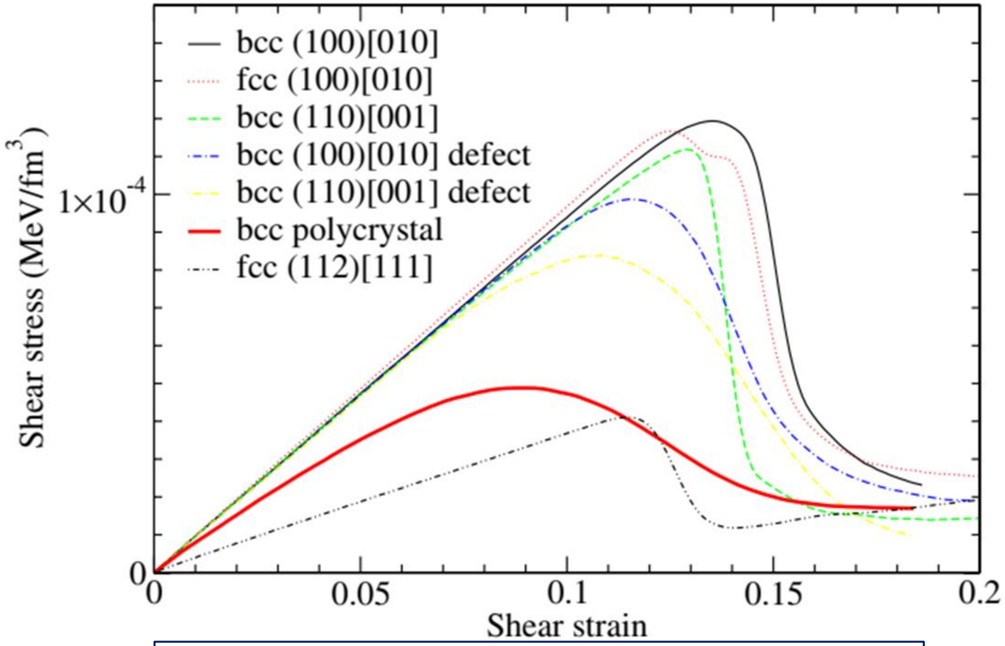
表 18・1 ずれ弾性率と弾性限度の比較（Mott による）

	ずれ弾性率 G [dyn/cm²]	弾性限度 σ_c [dyn/cm²]	$\frac{G}{\sigma_c}$
Sn（単結晶）	1.9×10^{11}	1.3×10^{7}	15 000
Ag（単結晶）	2.8×10^{11}	6×10^{6}	45 000
Al（単結晶）	2.5×10^{11}	4×10^{6}	60 000
Al（純粋，多結晶）	2.5×10^{11}	2.6×10^{8}	900
Al（市販圧延品）	$\sim2.5\times10^{11}$	9.9×10^{8}	250
ジュラルミン	$\sim2.5\times10^{11}$	3.6×10^{9}	70
Fe（軟，多結晶）	7.7×10^{11}	1.5×10^{9}	500
炭素鋼（熱処理）	$\sim8\times10^{11}$	6.5×10^{9}	120
ニッケル・クロム鋼	$\sim8\times10^{11}$	1.2×10^{10}	65

Molecular dynamics simulation

Horowitz + (09)PRL102,191102

$\sigma_c = 0.1$



$10^{29} erg/cm^3 \sim 10^{-4} p$

Mountain on NS

e.g. Haskell +(06) Johnson-McDaniel +(13)

- Pressure in crust p=$10^{29} - 10^{33} erg/cm^3$

Ellipticity estimated by pressure or energy ratio

- Mountain supported by elastic force

----> Ito-san's talk

$\varepsilon \approx \mu\sigma_c(4\pi R^2 \Delta R)/(GM^2/R) \approx 10^{-6}$ for $\mu = 10^{30} erg/cm^3$

Detailed calculations in literature, e.g, elastic mountain 10^{-8} (Gittens+ 21)

- Mountain supported by magnetic force

$\varepsilon \approx B^2 R^3/(GM^2/R) \approx 10^{-6}$ for $B = 10^{14} G$

Ellipticity(deformation) of NS will be observed by GW

Magnetic deformation of NS revisited

Suvorov+(16) proposed large deformation ~10^{-6} induced by Hall evolution in normal radio pulsars B~ 10^{12} G

Current upper limit of ellipticity <3x10^{-5}   (-> next silde)

Table 5. Comparison of our models with observational limits of seven selected pulsars, for which $h_0 \lesssim 4h_0^{sd}$ (Aasi et al. 2014). $|\boldsymbol{B}_s|$ and τ_c were computed from data given in Aasi et al. (2014). The fourth column shows the observational upper limits on ϵ from LIGO and Virgo (Aasi et al. 2014), the fifth column shows ϵ as predicted by the AL model, and the sixth column shows ϵ as predicted by the BL model.

Suvorov+ MNRAS(16)

Two models

Pulsar	$\lvert\boldsymbol{B}_s\rvert$ (10^{12} G)	τ_c (yr)	Obs. limit $\lvert\epsilon\rvert$	AL-predicted $\lvert\epsilon\rvert$	BL-predicted $\lvert\epsilon\rvert$
J0534+2200 (Crab)	1.84	1.3×10^4	8.6×10^{-5}	9.6×10^{-10}	2.9×10^{-7}
J0537−6910	1.42	4.9×10^3	1.2×10^{-4}	5.8×10^{-8}	3.2×10^{-8}
J1813−1246	1.43	4.3×10^4	3.5×10^{-4}	9.6×10^{-10}	2.9×10^{-7}
J1833−1034	5.52	4.8×10^3	5.7×10^{-3}	5.8×10^{-8}	3.2×10^{-8}
J1913+1011	0.54	1.7×10^5	2.2×10^{-4}	1.3×10^{-8}	2.6×10^{-6}
J0835−4510 (Vela)	5.26	1.1×10^4	6.0×10^{-4}	9.6×10^{-10}	2.9×10^{-7}
J1952+3252	0.74	1.1×10^5	3.0×10^{-4}	1.3×10^{-8}	2.6×10^{-6}

References　**Upper limit from GW**

- Crab　J0534+2200

Ellipticity <1x10^{-5}　LIGO/Virgo O2

- B1951+32/J1952+3252

Ellipticity <2x10^{-5}　LIGO/Virgo O2

B. P. Abbott et al 2019 ApJ 879 10

- PSR0537－6910

Ellipticity <3x10^{-5}　LIGO/Virgo O3(2020 Oct)

Arxiv2012.12926

Our theoretical results

Right figure shows evolution of ellipticity for three models

Unfortunately for GW community, our quadrupole deformation is ~10^{-8}

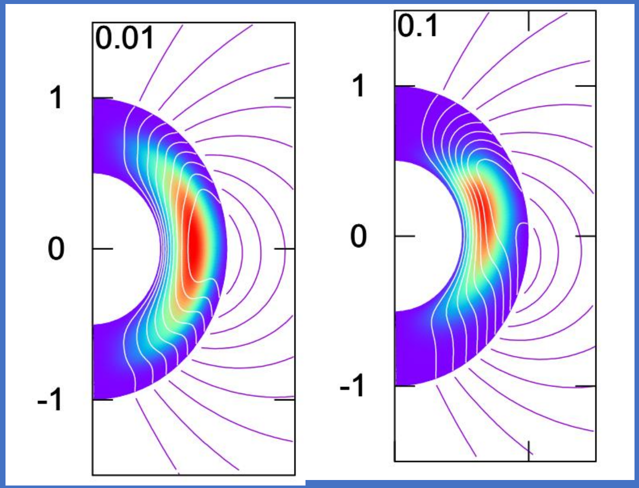
mass moment

- Deformation in shape and that in quadrupole moment are different

  Shape deformation in crust is ~ 3x10^{-5}

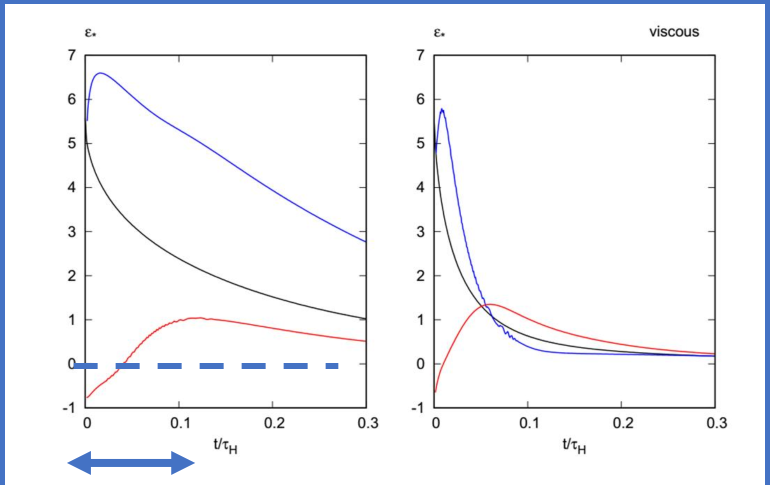
- Different model/treatment

=> Large deformation by core field



Evolution of magnetic field

Evolution of deformation



10^5 yr

Evolution of magnetic deformation in neutron star crust
Kojima, Kisaka & Fujisawa, MNRAS(21)

Is elastic force effective for mag. confinement?

New

In crust $\rho = 4 \times 10^{11} - 1.4 \times 10^{14} g/cm^3$

p= $5 \times 10^{29} - 3 \times 10^{33}$ $erg/cm^3 \sim B^2$

Shear $\mu = 3 \times 10^{27} - 10^{30}$ erg/cm^3 $\mu/p \sim 10^{-2} - 10^{-3}$

Elastic force is too small

✓Magnetic energy is

$E_B \sim 10^{46} (B_{14})^2 erg$ (which depends on geometry)

enough to activities

✓Elastic energy is

$E_{els} \sim 10^{44} \bar{\mu} (\sigma_c/0.1)^2 erg$

➢Solution for stability of magnetized neutron star

Mixed poloidal-toroidal fields are stable in conjecture, but toroidal- dominated models are not yet calculated.

See also Bera + (20)

Unimportant!

No !

A pitfall in argument

Magneto-elastic equilibrium of a neutron-star crust

- Static equilibrium determined by
- $\nabla\left(T_{grav} + T_{fluid} + T_{mag} + T_{ela}\right) = 0$
- $|T_{grav}| \sim |T_{fluid}| \gg |T_{mag}|, \ |T_{ela}|$

- Vector decomposition of the Lorentz force

  Irrotational + Solenoidal vector

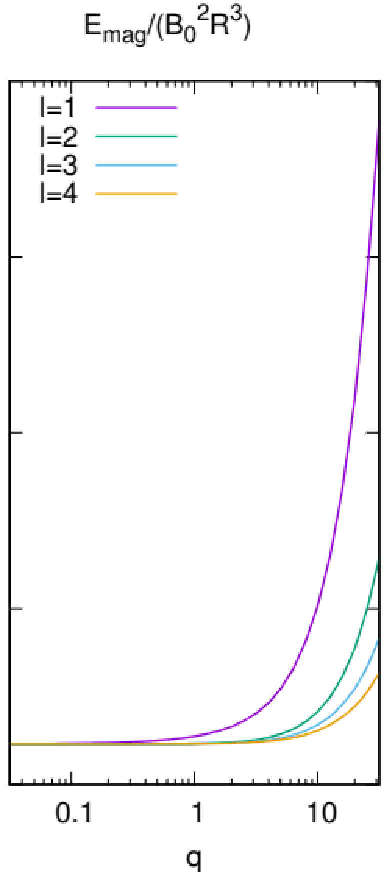
  $(f = -\nabla F + \nabla \times A)$

- Solenoidal part is balanced with elastic force
- Irrotational part is balanced with dominant forces

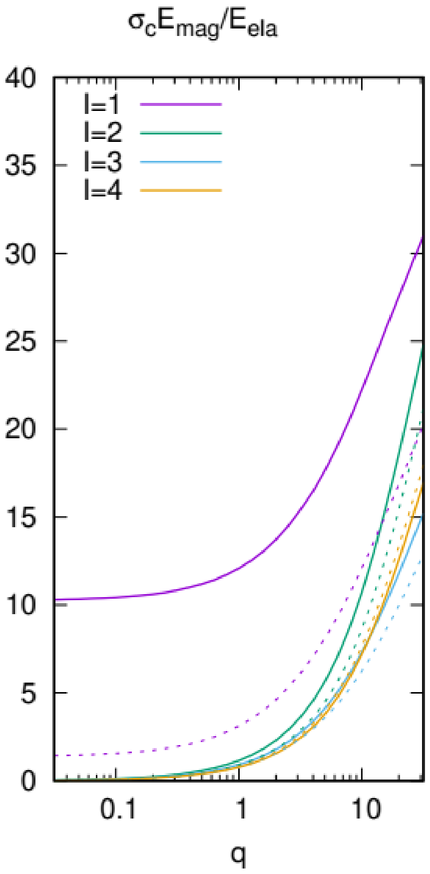
Results

Magnetic energy

ΔE_B



Energy ratio

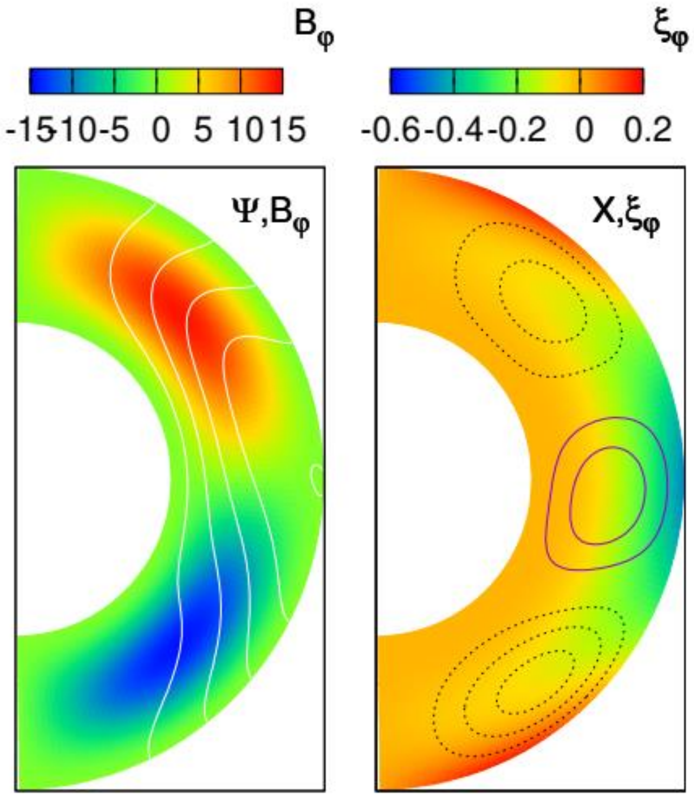


Strong magnetic force

Strong elastic force

A large amount of mag. E stored

YK+ MN(21b)



Magnetic field and elastic displacement

Crust depth enlarged by a factor 5

Summary

- Implication of magneto-elastic equilibrium
  -> Effective /stable confinement of magnetic field in crust
  Large toroidal magnetic field is also allowed
  Mixed toroidal-poloidal field may be realized in NS case.
  => Core field for magnetars?
  (-> never justify magnetar burst+ evolution models)
- Braking strain $\sigma_c \sim 0.1$? by theory How do we check?

We may discuss it in astrophysical objects/events, after lots of theoretical studies, e.g, effect of magnetic field.