**【 中性子星の観測と理論 – 研究活性化ワークショップ 2021– 】**
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# Effects of three-baryon forces on kaon condensation in hyperon-mixed matter

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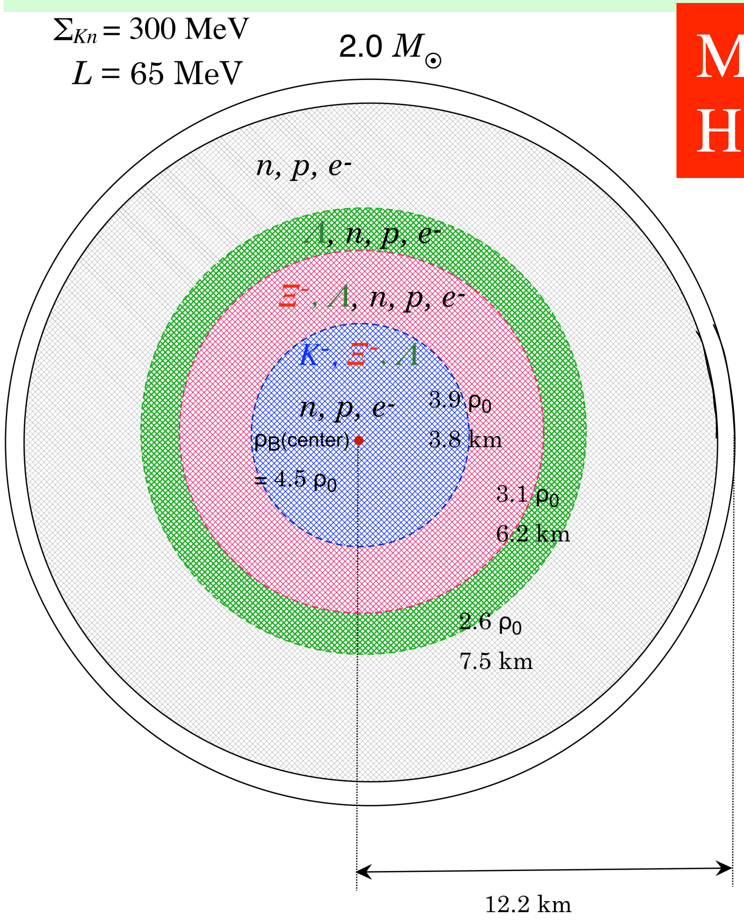
arXiv: 2106.03449 [nucl-th]

# 1. Introduction

## 1-1 Various phases in dense matter and Equation of state (EOS)

Possibility of kaon condensed phase in hyperon-mixed matter



Multi-strangeness Hadron phase

Lattice QCD, many-body theory

Heavy-ion collisions
Hypernuclear experiments
Kaonic nuclei (J-PARC, …)

- K-N interactions
- Y-N, Y-Y interactions
- properties of baryons, mesons in medium

Neutron−Star Observations

Xray (NICER), $\gamma$ ray, neutrino,
Gravitational wave
(LIGO,VIRGO … )

- Mass, Radius
- surface temperature
- magnetar
- Supernovae
- Glitches

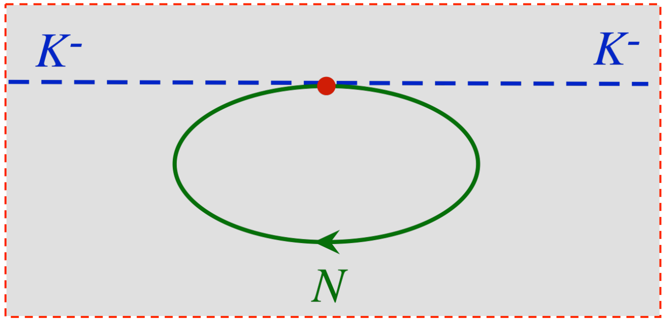
# 1-2 Kaon condensation (Bose-Einstein condensation of antikaon $K^-$)

[D.B. Kaplan and A. E. Nelson, Phys. Lett. B175 (1986), 57.]

[T. Tatsumi, Prog. Theor. Phys. 80, 22(1988).]

[T. Muto and T. Tatsumi, Phys. Lett. B283(1992), 165.]

[H. Fujii, T. Maruyama, T. Muto, and T. Tatsumi,
Nucl. Phys. A597 (1996), 645.]



## ・Softening of EOS

Delayed collapse of protoneutron stars ←1st order phase transition

⇒ Mini-black hole scenario

[G. E. Brown and H. A. Bethe, Astrophy. J. 423(1994), 659.]

Mini-collapse to a kaon-condensed $N_\bigstar$ ⇒ anomalous γ-ray bursts

[H. Fujii, T. Maruyama, T. Muto, and T. Tatsumi, Nucl. Phys. A597 (1996), 645.]

## ・Rapid cooling of neutron stars

[H. Fujii, T. Muto, T. Tatsumi, R. Tamagaki,
Nucl. Phys. A578 (1994), 758; Phys. Rev. C 50 (1994), 3140.]

# 1-3 Coexistence of kaon condensation and hyperons

Problem: [(Y+K) phase] necessarily leads to very soft EOS

↕ [T. Muto, Phys. Rev. C77, 015810 (2008).] Self-bound stars

Observation of massive neutron stars

M(PSR J1614-2230) = (1.97 ± 0.04) $M_{\odot}$

[ P. Demorest, T.Pennucci, S. Ransom,M. Roberts and J.W.T.Hessels, Nature 467 (2010) 1081.]

M(PSR J0348+0432) = (2.01 ± 0.04) $M_{\odot}$

[J. Antoniadis et al., Science 340, 6131 (2013).]

M(PSR J2215+5135) = (2.27 + 0.17-0.15) $M_{\odot}$

[M.Linares, T. Shahbaz, J.Casares, Astrophys. J. 859, 54 (2018).]

Millisecond pulsars in compact binaries

M(PSR J0740+6620) = (2.14 + 0.10-0.09) $M_{\odot}$

[H.T.Cromartie et al., Nat.Astron.4, 72(2020.)]

M(PSR J1810+1744) = (2.13 ± 0.04) $M_{\odot}$

[R. W. Romani et al., Astrophys. J. L. 908, L46 (2021).]

# Many-Body Repulsive Forces are necessary at high densities in order to stiffen the EOS at high densities.

## Possible Solutions to the " Hyperon Puzzle"

- Universal YNN, YYN, YYY repulsions

[ S. Nishizaki, Y. Yamamoto and T. Takatsuka, Prog. Theor. Phys. 108 (2002) 703. ]

[R. Tamagaki, Prog. Theor. Phys. 119 (2008), 965. ] : String-Junction model

- Multi-pomeron exchange potential

[Y. Yamamoto, T. Furumoto, N. Yasutake, and Th.A. Rijken,
Phys. Rev. C 90, 045805 (2014). ]

- RMF extended to BMM, MMM type diagrams

[K. Tsubakihara and A. Ohnishi, Nucl. Phys. A 914 (2013), 438; arXiv:1211.7208.]

# 1-4 Purpose of this study

Introducing the Universal Three-Baryon Repulsive Force (UTBR)
and Three-Nucleon Attraction (TNA),

- Energy per particle in symmetric nuclear matter (SNM) as a function of baryon density $\rho_{B}$. ← reproduce saturation properties of SNM within the Relativistic mean-field theory (RMF)

- Onset density of the (Y+K) phase and properties of neutron stars (M-R )

↑ Effects of TNA ( choice of the slope *L*) on stiffness of the EOS for the (Y+K) phase in high densities.

↑ Suppression mechanisms of Kaon condensates due to UTBR and TNA
(Boson degree of freedom)

- Rapid cooling mechanisms in the (Y+K) phase

# 2. Formulation

## 2-1 Our interaction model for the (Y+K) phase

### K-Baryon and K-K interactions

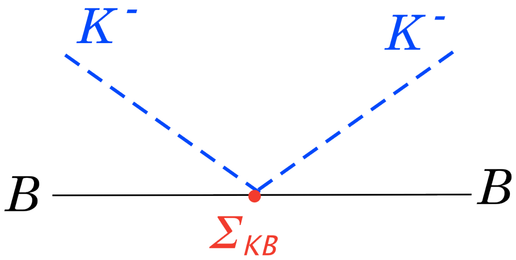
specified by chiral symmetry

[classical K$^-$ field]

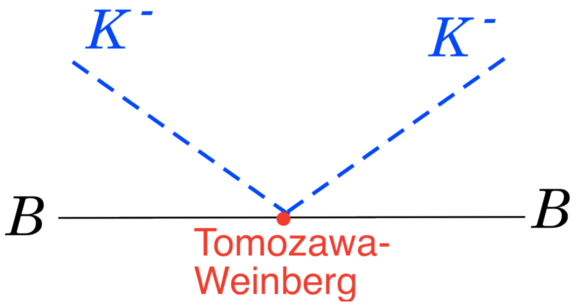
$$K^-(r) = \frac{f}{\sqrt{2}}\theta(r)$$

### Effective chiral Lagrangian

$$\mathcal{L}_K = f^2\left[\frac{1}{2}(\mu_K \sin\theta)^2 - m_K^2(1-\cos\theta) + 2\mu_K X_0(1-\cos\theta)\right]$$



S wave KB interaction



$$m_K^{*2} \equiv m_K^2 - \frac{1}{f^2}\sum_i \rho_i^s \Sigma_{Ki}$$

$$X_0 \equiv \left(\rho_p + \frac{1}{2}\rho_n - \frac{1}{2}\rho_{\Sigma^-} - \rho_{\Xi^-}\right)/(2f^2)$$

# 2-2 Baryon-Baryon interaction

## Minimal Relativistic Mean-Field theory (RMF)

Baryons: $(p, n, \Lambda, \Sigma^-, \Xi^-)$ Mesons: $\sigma, \omega, \rho, \sigma^*, \phi$

## 2-Body Baryon Interaction Lagrangian

(Meson-exchange)

$$
\begin{aligned}
\mathcal{L}_{B,M} &= \sum_B \overline{B}(i\gamma^\mu D_\mu - M_B^*)B + \frac{1}{2}(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2) \boxed{-U(\sigma)} + \frac{1}{2}(\partial^\mu\sigma^*\partial_\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}) \\
&- \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}R^{\mu\nu}R_{\mu\nu} + \frac{1}{2}m_\rho^2 R^\mu R_\mu - \frac{1}{4}\phi^{\mu\nu}\phi_{\mu\nu} + \frac{1}{2}m_\phi^2\phi^\mu\phi_\mu
\end{aligned}
$$

$$\partial_\mu \to \mathcal{D}_\mu^B \equiv \partial_\mu + ig_{\omega B}\omega_\mu + i\tilde{g}_{\rho B}\vec{I}^{(B)}\cdot\vec{R}_\mu + ig_{\phi B}\phi_\mu$$

$$M_B^* = M_B - g_{\sigma B}\sigma - g_{\sigma^* B}\sigma^* - \Sigma_{Kb}(1 - \cos\theta)$$

without introduction of the nonlinear self-interacting σ potential U(σ):

$$\cancel{U(\sigma) = bM_N(g_{\sigma N}\sigma)^3/3 + c(g_{\sigma N}\sigma)^4/4}$$ : omitted

# 3-Baryon interactions

**UTBR** String-Junction Model 2

[R. Tamagaki,
Prog. Theor. Phys. 119 (2008), 965. ]

$$W(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = W_0 g(\mathbf{r}_1 - \mathbf{r}_3) g(\mathbf{r}_2 - \mathbf{r}_3)$$

$W_0$ ~ 2 GeV

$$g(\mathbf{r}_i - \mathbf{r}_j) = \exp(-\lambda(\mathbf{r}_i - \mathbf{r}_j)^2)$$

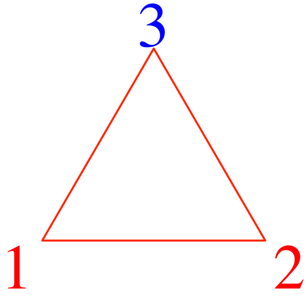
$\lambda = 1/\eta_C{}^2$ $\eta_C = 0.5$ fm for SJM2

(range of repulsive core)

Effective 2-body potential

short-range correlation function

$$U_{\mathrm{SJM}}(1,2;\rho_{\mathrm{B}}) = \rho_{\mathrm{B}} \int d^3\mathbf{r}_3 W(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_3) f^2_{\mathrm{src}}(\mathbf{r}_1 - \mathbf{r}_3) f^2_{\mathrm{src}}(\mathbf{r}_2 - \mathbf{r}_3)$$



$$U_{\mathrm{SJM2}}(r;\rho_{\mathrm{B}}) = V_r \rho_{\mathrm{B}} (1 + c_r \rho_{\mathrm{B}}/\rho_0) \exp[-(r/\lambda_r)^2)]$$

$V_r = 95$ MeV・fm$^3$ $c_r = 0.024$ $\lambda r = 0.86$ fm

for SJM2

$$\widetilde{U}_{\mathrm{SJM}}(r;\rho_{\mathrm{B}}) = f_{\mathrm{SRC}}(r) U_{\mathrm{SJM}}(r;\rho_{\mathrm{B}})$$

**TNA** [B. Friedman and V. R. Pandharipande, Nucl. Phys. A361 (1981) 502;
I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. A 359 (1981) 349 ]

$$\mathcal{E}(\mathrm{TNA})/\rho_\mathrm{B} = \gamma_2 \rho_\mathrm{B}^2 \exp[-\eta_a \rho_\mathrm{B}]\left[3 - 2\left(\frac{\rho_n-\rho_p}{\rho_\mathrm{B}}\right)^2\right]$$

Isospin-dependence ← $(\tau_1 \cdot \tau_2)^2$

## TNA is related with isospin-dependence of the EOS around $\rho_0$

$$L \equiv 3\rho_0 \left(\frac{\partial S}{\partial \rho_B}\right)_{\rho_B=\rho_0, x=1/2} = L^{(\mathrm{kin})} + L^{(\rho)} + L^{(\mathrm{TNA})}.$$

$$L^{(\mathrm{kin})} = \frac{p_F^2}{6\widetilde{E}(p_F)}\left[2 - \left(\frac{p_F}{\widetilde{E}(p_F)}\right)^2 + \frac{2}{\pi^2}\frac{(g_{\sigma N}M_N^*)^2(p_F/\widetilde{E}(p_F))^3}{m_\sigma^2 + g_{\sigma N}^2 I(p_F)}\right]$$

$$L^{(\rho)} = \frac{3}{2}\left(\frac{g_{\rho N}}{m_\rho}\right)^2 \rho_0$$

$$\boxed{L^{(\mathrm{TNA})} = 6\gamma_2\rho_0^2(\eta_a\rho_0 - 2)e^{-\eta_a\rho_0}} \quad (<0)$$

*L* is given
→ attraction of TNA
is tuned.

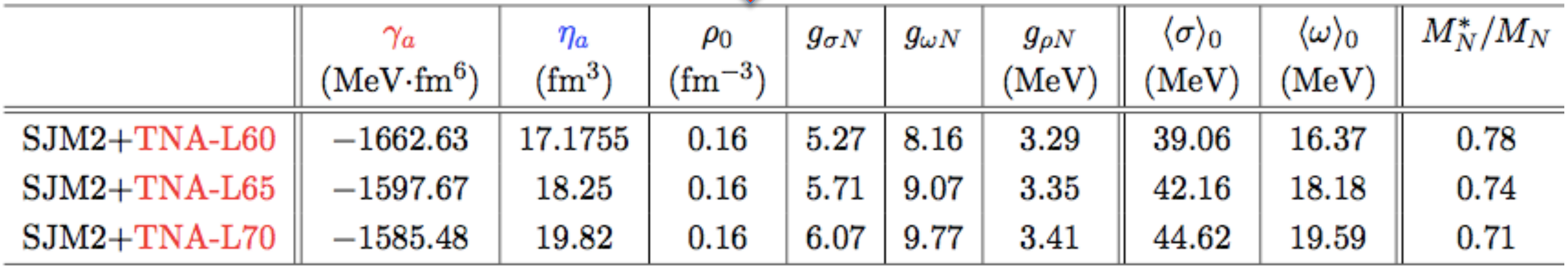
# 2-3 Saturation properties in our model

## Parameters in TNA

【Constraints 】

- Meson-N Coupling constants are determined so as to satisfy the saturation properties of symmetric nuclear matter (SNM)

($\rho_0 = 0.16\ \mathrm{fm}^{-3}$) ($E_\mathrm{B}$=16.3 MeV) (K=240 MeV) ($S_0$=31.5 MeV)

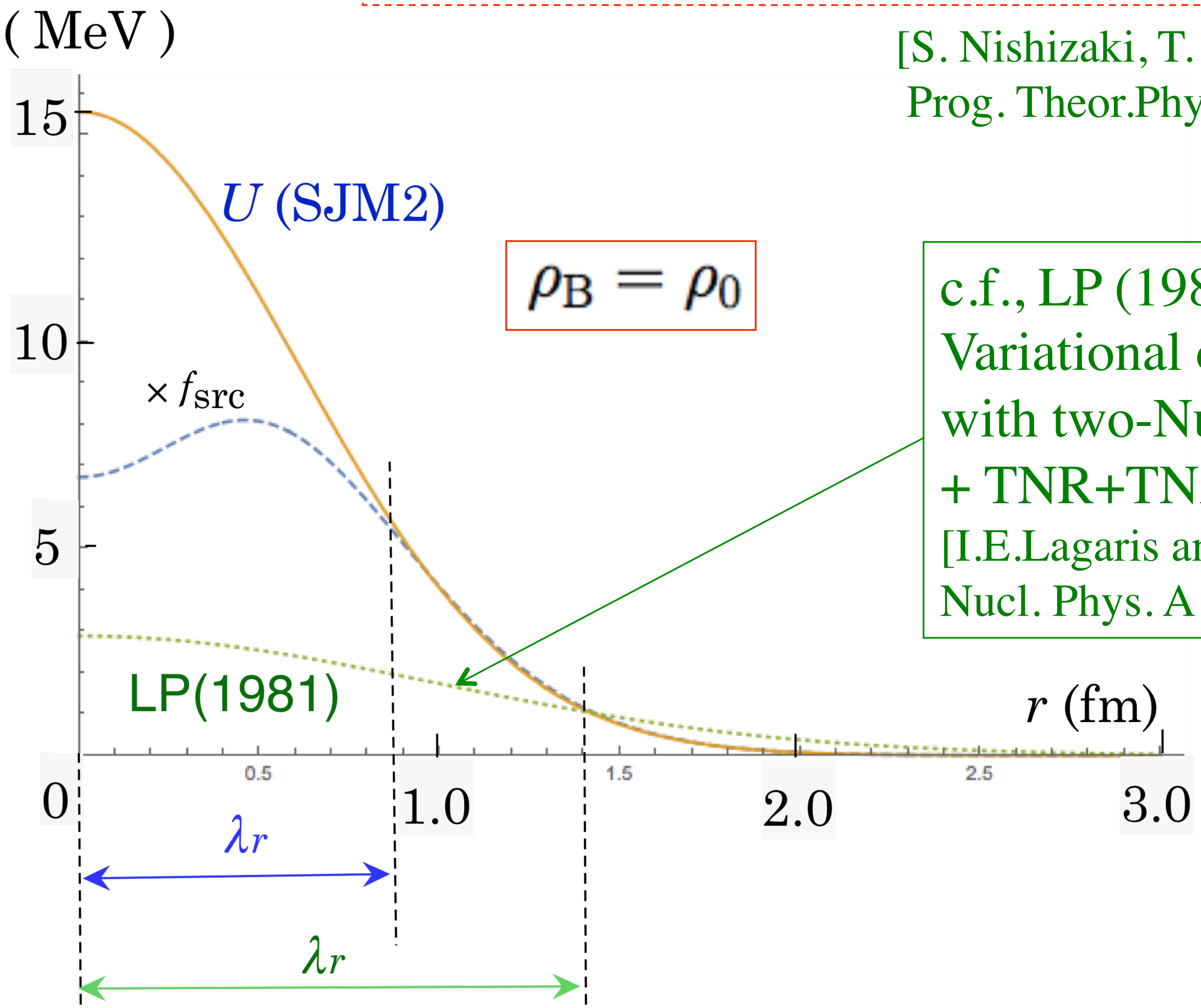


| | $\gamma_a$ (MeV·fm$^6$) | $\eta_a$ (fm$^3$) | $\rho_0$ (fm$^{-3}$) | $g_{\sigma N}$ | $g_{\omega N}$ | $g_{\rho N}$ (MeV) | $\langle\sigma\rangle_0$ (MeV) | $\langle\omega\rangle_0$ (MeV) | $M_N^*/M_N$ |
|---|---|---|---|---|---|---|---|---|---|
| SJM2+TNA-L60 | −1662.63 | 17.1755 | 0.16 | 5.27 | 8.16 | 3.29 | 39.06 | 16.37 | 0.78 |
| SJM2+TNA-L65 | −1597.67 | 18.25 | 0.16 | 5.71 | 9.07 | 3.35 | 42.16 | 18.18 | 0.74 |
| SJM2+TNA-L70 | −1585.48 | 19.82 | 0.16 | 6.07 | 9.77 | 3.41 | 44.62 | 19.59 | 0.71 |

# Effective 2-body potential from the UTBF

$$U_{\mathrm{TNR}}(r;\rho_{\mathrm{B}}) = V_r\,(1-\exp(-\eta_r\rho_{\mathrm{B}}))\,\exp[-(r/\lambda_r)^2]$$

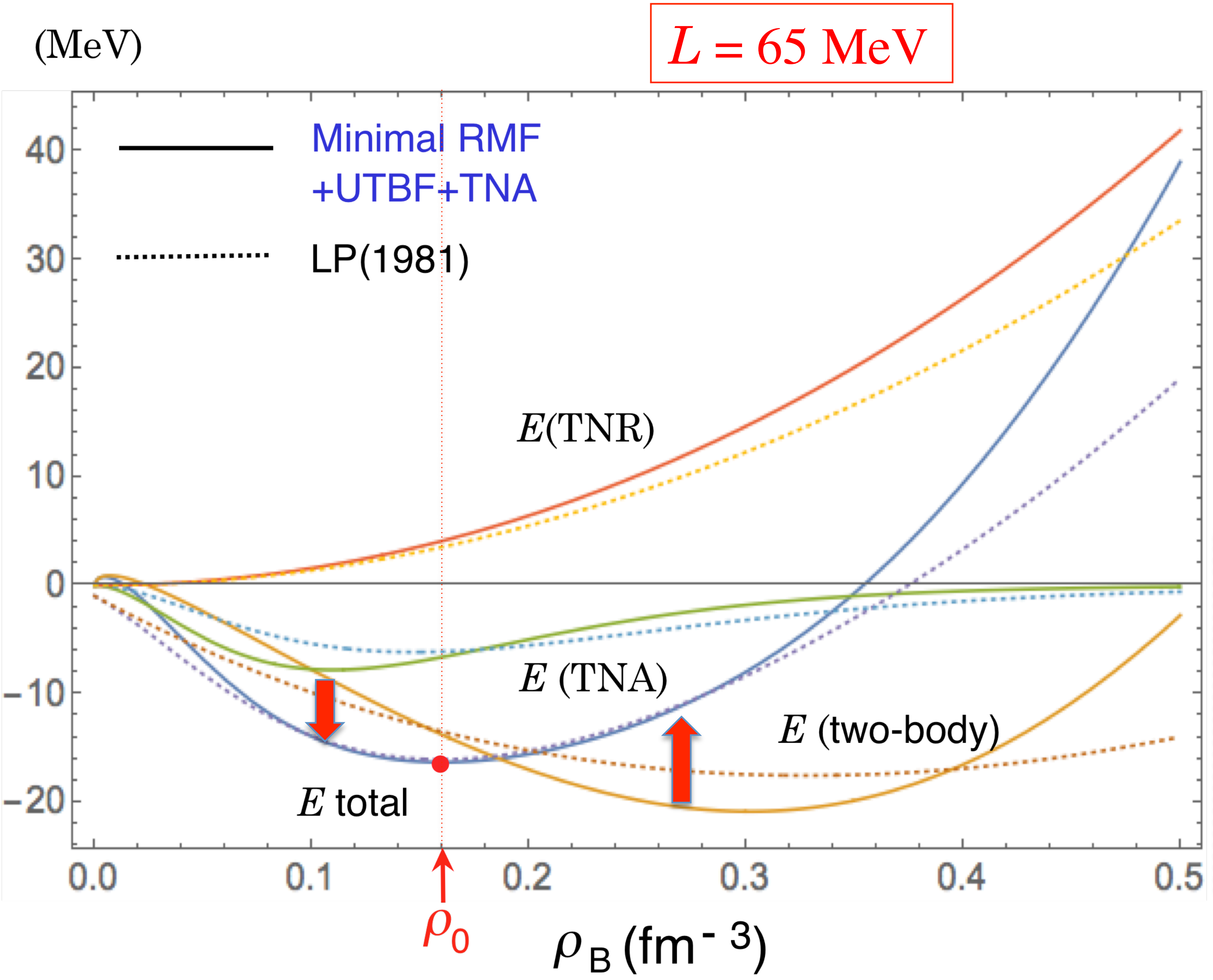
[S. Nishizaki, T. Takatsuka and J. Hiura,
Prog. Theor.Phys. 92 (1994), 93]



c.f., LP (1981):
Variational calculation
with two-Nucleon int. V14
+ TNR+TNA
[I.E.Lagaris and V. R. Pandharipande,
Nucl. Phys. A 359 (1981),349]

# 3. Energy per particle in symmetric nuclear matter (SNM)

## ---- comparison with Lagaris-Pandharipande (1981) ----



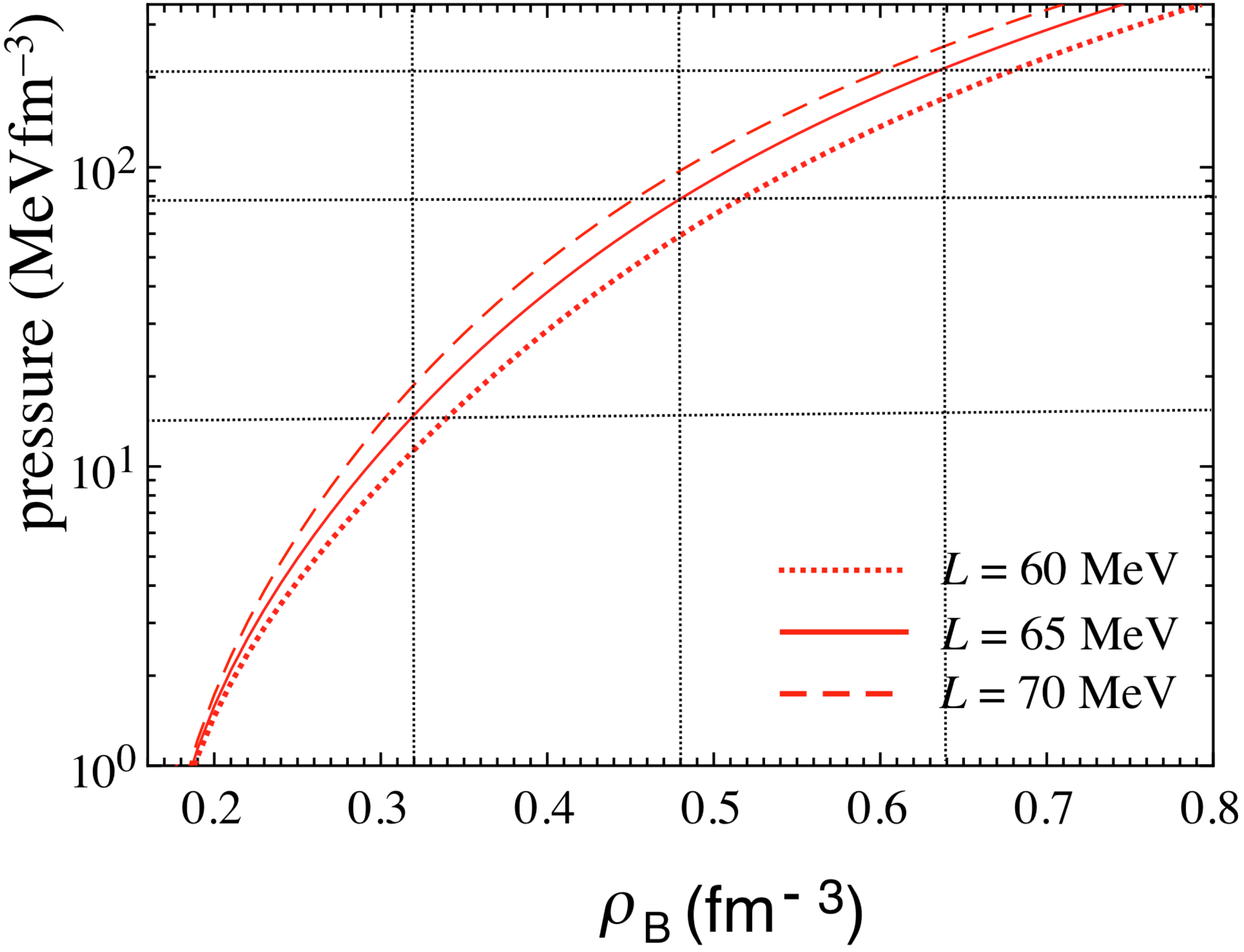
E(TNR) is similar to LP.

↑

Volume-integral for TNR potential is similar.

TNA is more attractive for $\rho_B < \rho_0$, and less attractive for $\rho_B > \rho_0$, than LP.

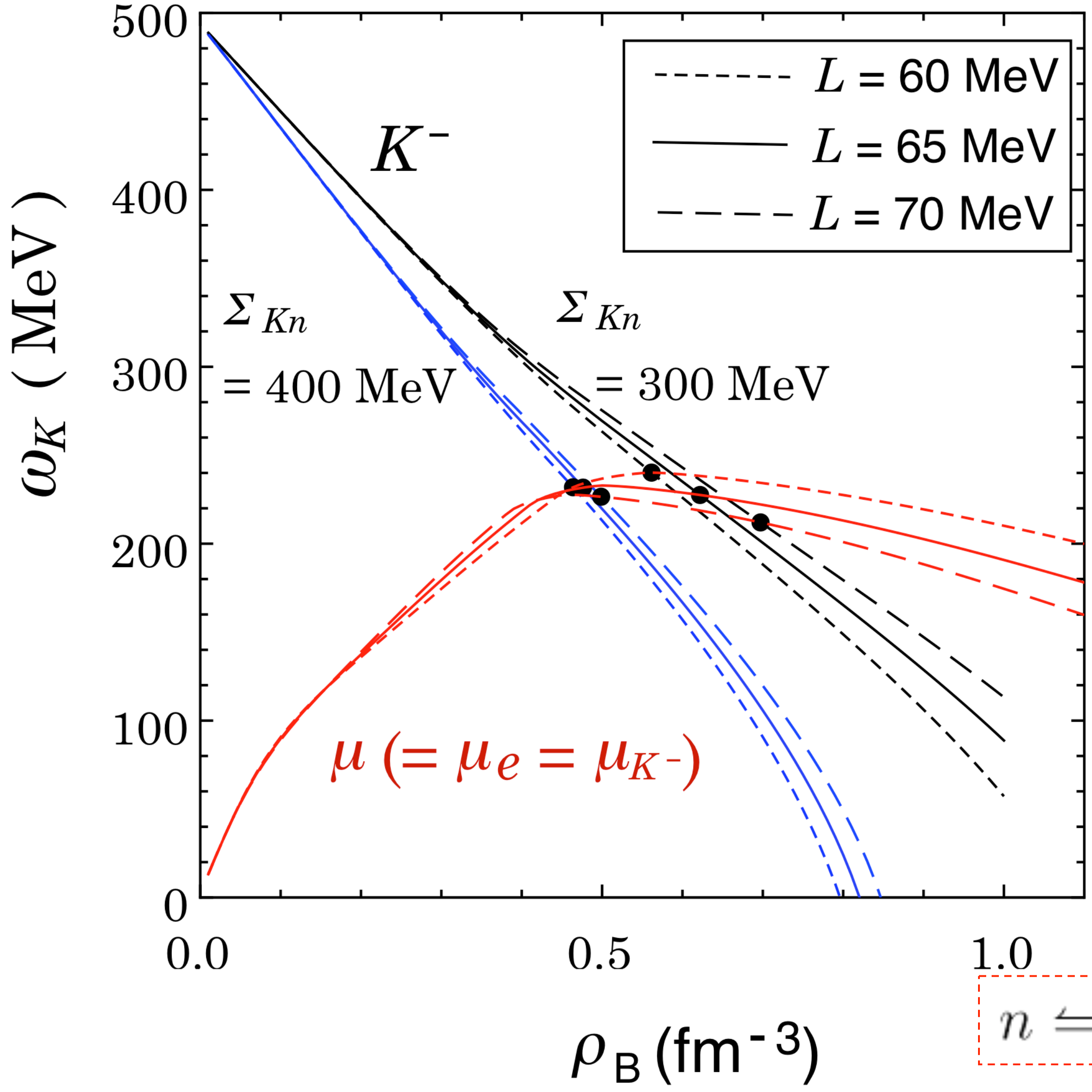
# Pressure-density curves in SNM



# 4. Results

## 4-1 Onset density of $K^-$ condensation

### Lowest K- energy as a function of $\rho_B$



$$\Sigma_{Kn} = 300\ \mathrm{MeV}$$

$$\rho_B^c(K^-) = (3.5 - 4.4)\rho_0$$

**for $L = (60 - 70)$ MeV**

$$\Sigma_{Kn} = 400\ \mathrm{MeV}$$

$$\rho_B^c(K^-) = (2.9 - 3.1)\rho_0$$

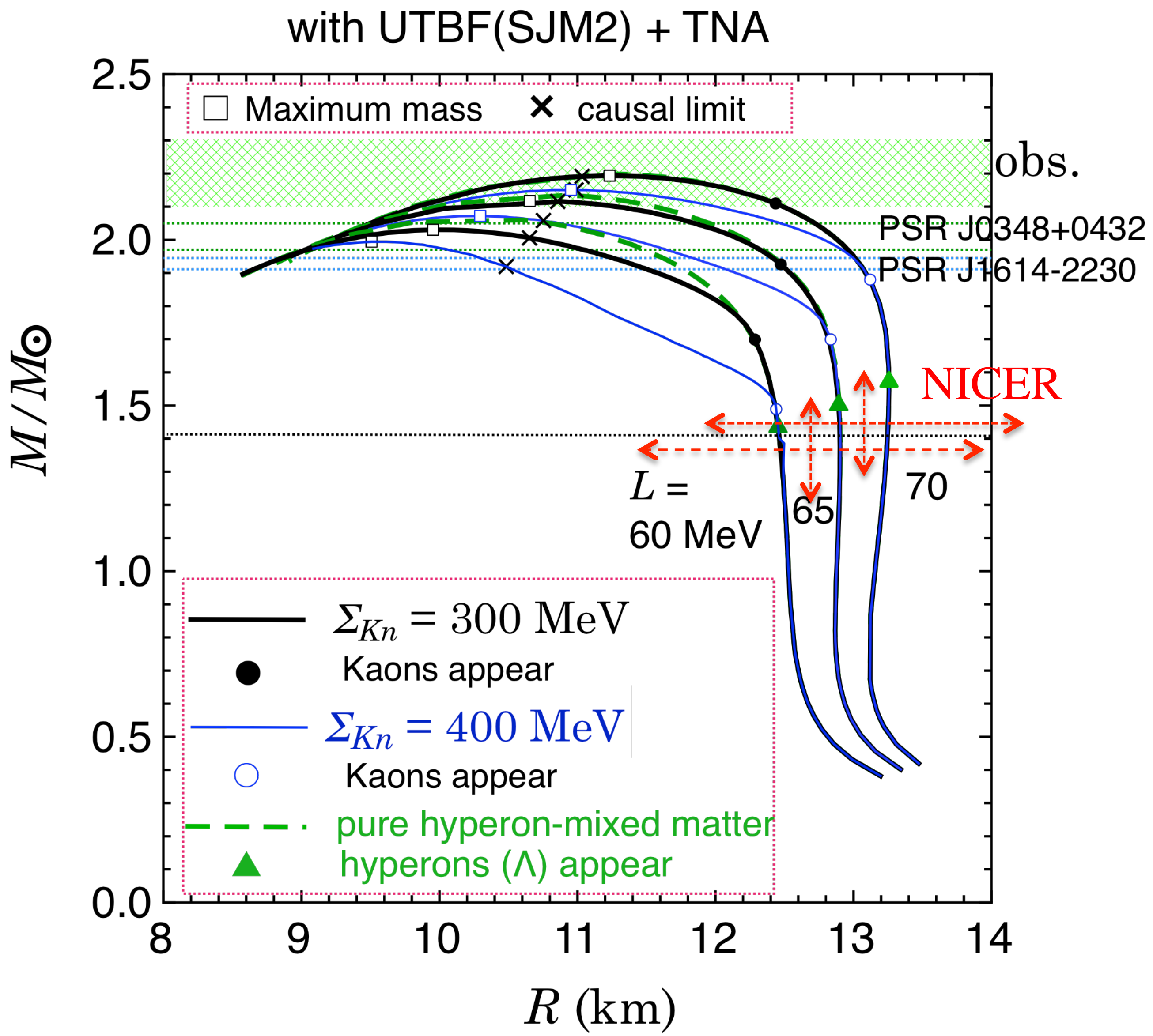
**for $L = (60 - 70)$ MeV**

Chemical equilibrium
for weak processes

$n \leftrightharpoons p\, K^-$ $\quad n \leftrightharpoons p\, e^-\ (\bar{\nu}_e)$

# 4-3 Effects of $L$ on the structure of $N_\star$

## Gravitational Mass – radius R relations



# 【Consistency with N$_{☆}$ observations 】

$$M_{\max} = (2.1 - 2.3) M_{\odot}$$

Mass *and* Radius of PSR J0030+0451 [from NICER]

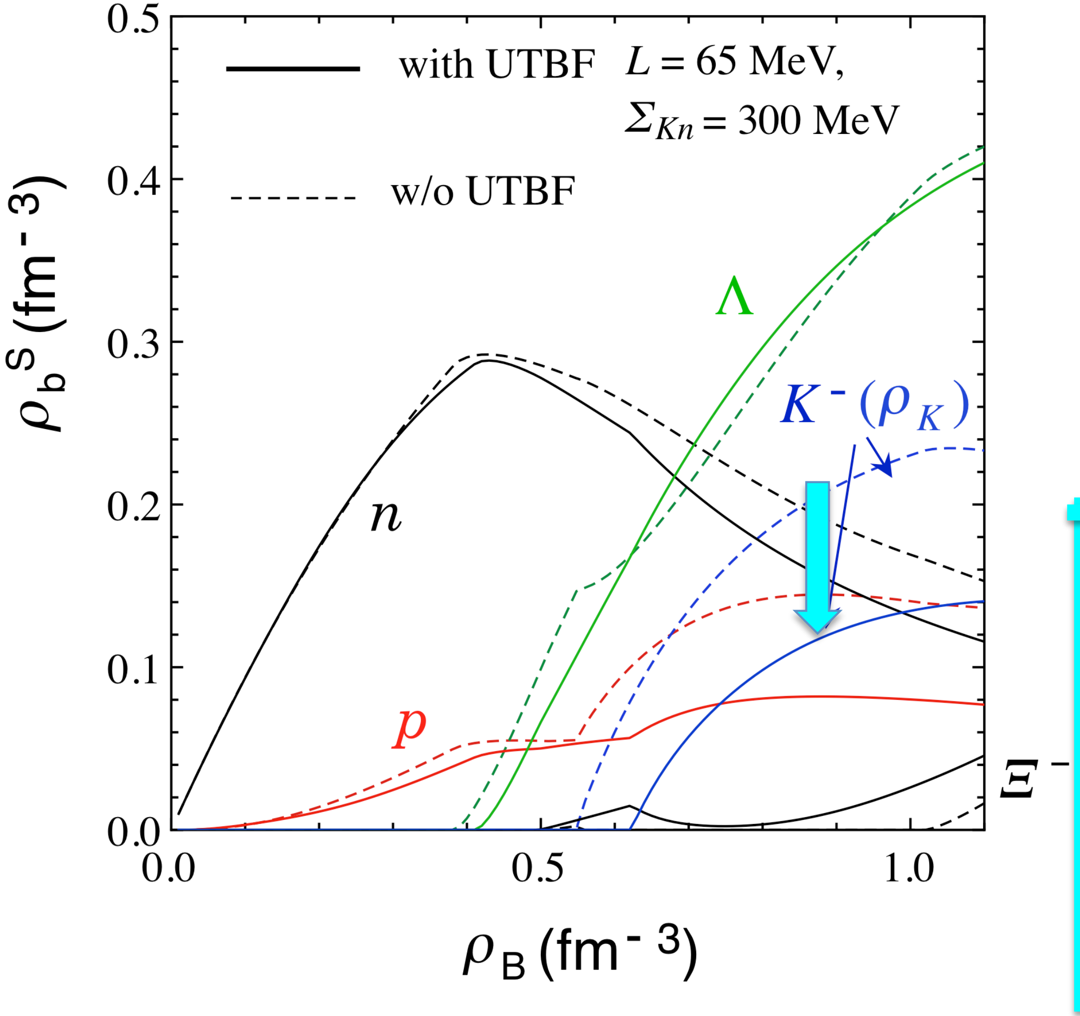
[ T.E.Riley et al.Astrophys. J. 887, L21 (2019); M.C.Miller et al., *ibid* L24 ]

Gravitational waves from neutron-star mergers

[B.P.Abbott et al., Phys. Rev. Lett.121, 161101 (2018).] $R_1, R_2 = (11.9 \pm 1.4)$ km

[F.J.Fattoyev, J.Piekarewicz, and C.J.Horowitz, Phys. Rev. Lett.120, 172702 (2018).]

# Density-dependence of baryon scalar density $\rho_b^s$ ($p$, $n$, $\Lambda$, $\Xi^-$) and K- density $\rho_K$



Kaon condensation develops

⇒ Self-suppression of

K-baryon scalar attraction

$$\sum_b \overline{\psi}_b \left( i\gamma^\mu D_\mu^{(b)} - \widetilde{M}_b^* \right) \psi_b$$

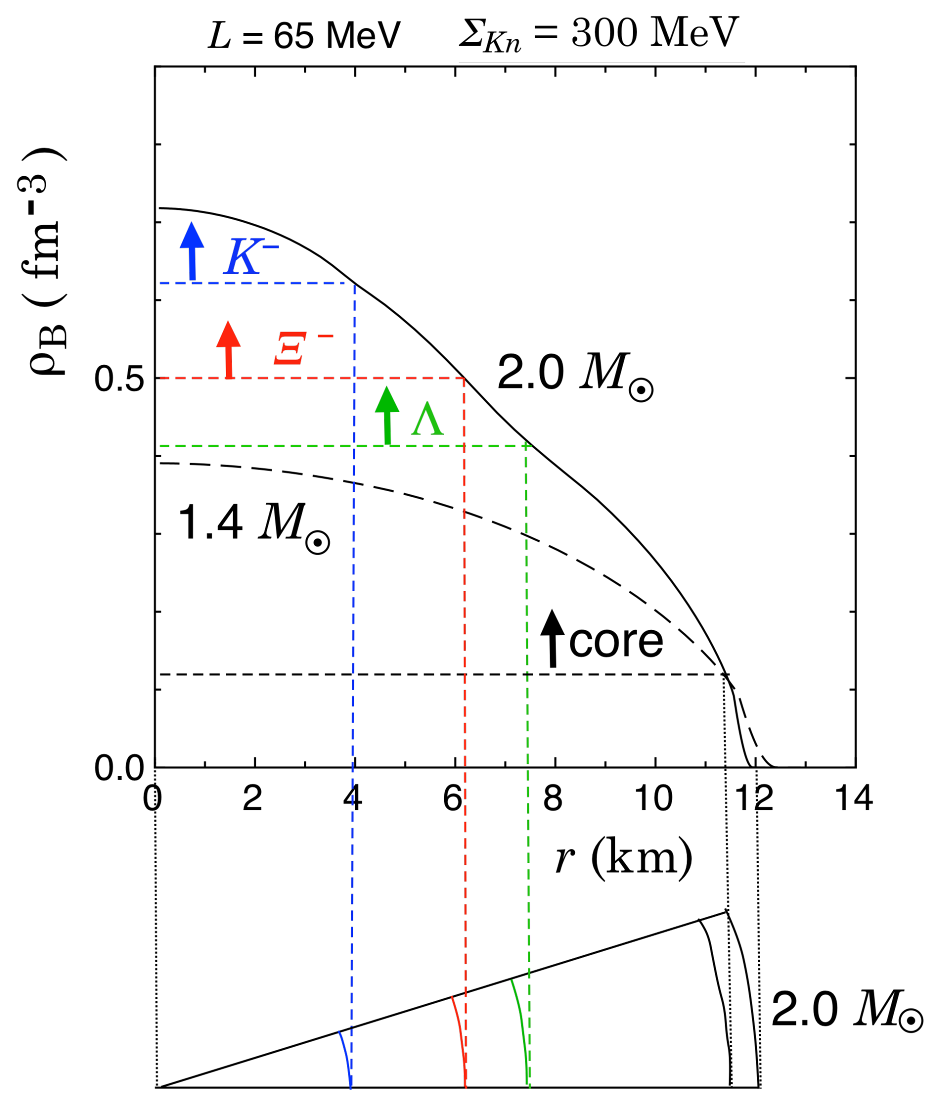
$$\to \sum_b \widetilde{M}_b^* \rho_b^s$$

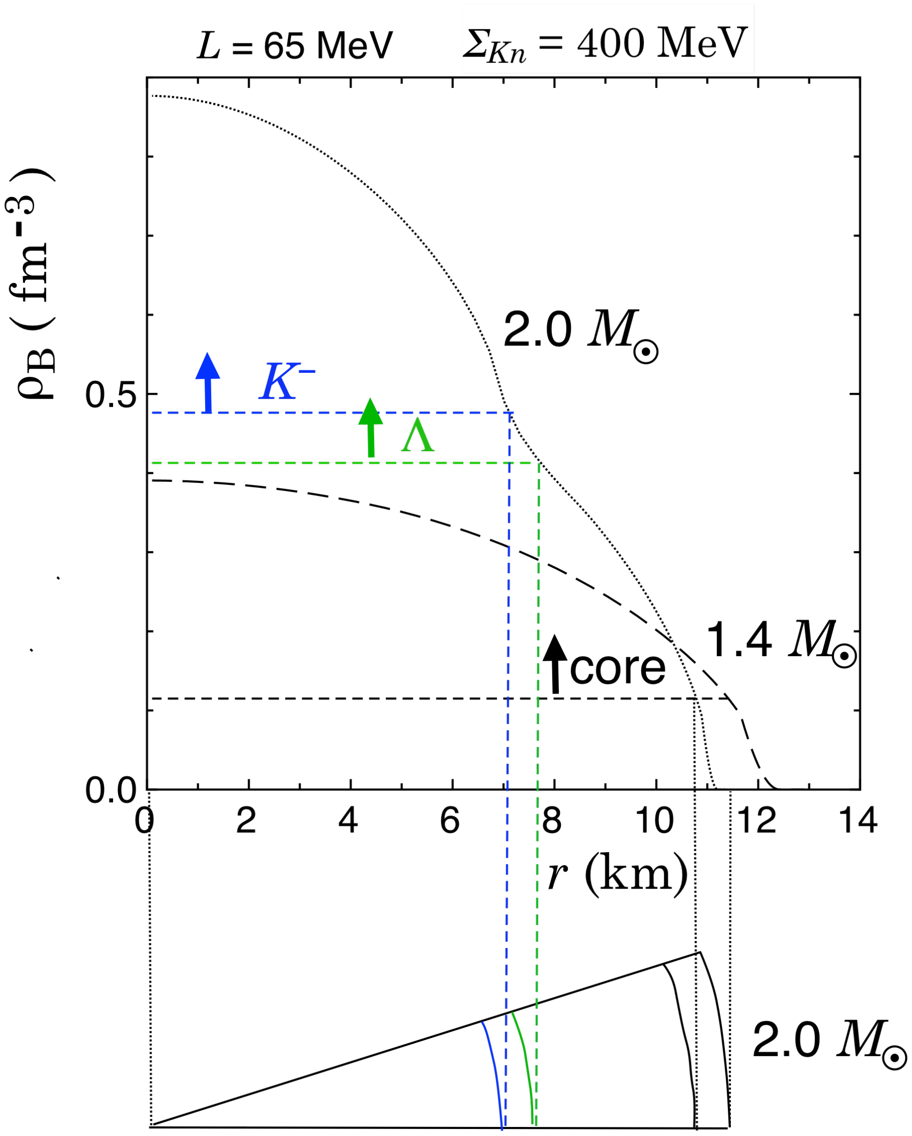
$$\widetilde{M}_b^* = M_b - g_{\sigma b}\sigma - g_{\sigma^* b}\sigma^* - \Sigma_{Kb}(1-\cos\theta)$$

Baryon scalar density

$$\rho_b^s = \frac{2}{(2\pi)^3}\int_{|\mathbf{p}|\le p_F(b)} d^3\mathbf{p}\, \frac{\widetilde{M}_b^*}{(|\mathbf{p}|^2+\widetilde{M}_b^{*2})^{1/2}}$$

# Density distributions --- $L = 65$ MeV ---



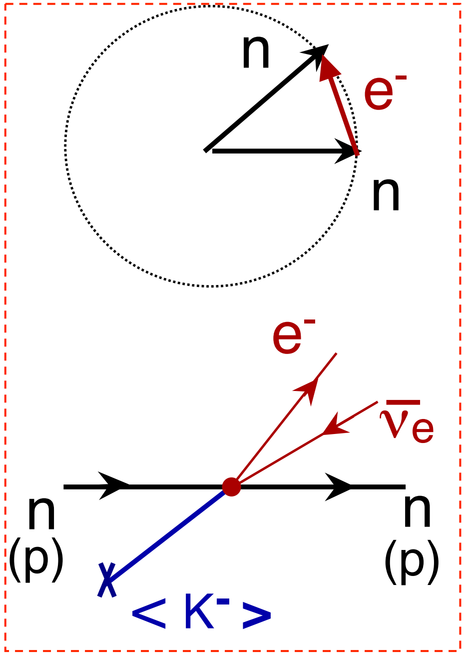


# 5 Unified description of EOS with (Y+K) phase and neutron-star cooling

## 5-1 Rapid cooling mechanisms via neutrino emission

[ T. Tatsumi, Prog. Theor. Phys. 80 (1988), 22. ]

[H. Fujii, T. Muto, T. Tatsumi, R. Tamagaki,
Nucl. Phys. A578 (1994), 758; Phys. Rev. C 50 (1994), 3140.]



### Kaon-induced Urca process

$$\begin{aligned} n\ (p) + \langle K^- \rangle &\rightarrow n\ (p) + e^- + \bar{\nu}_e\ , \\ n\ (p) + e^- &\rightarrow n\ (p) + \langle K^- \rangle + \nu_e \end{aligned}$$

Weak Hamiltonian $H_W = \frac{G_F}{\sqrt{2}} J_h^{\mu\dagger} l_\mu + \mathrm{h.c.}$

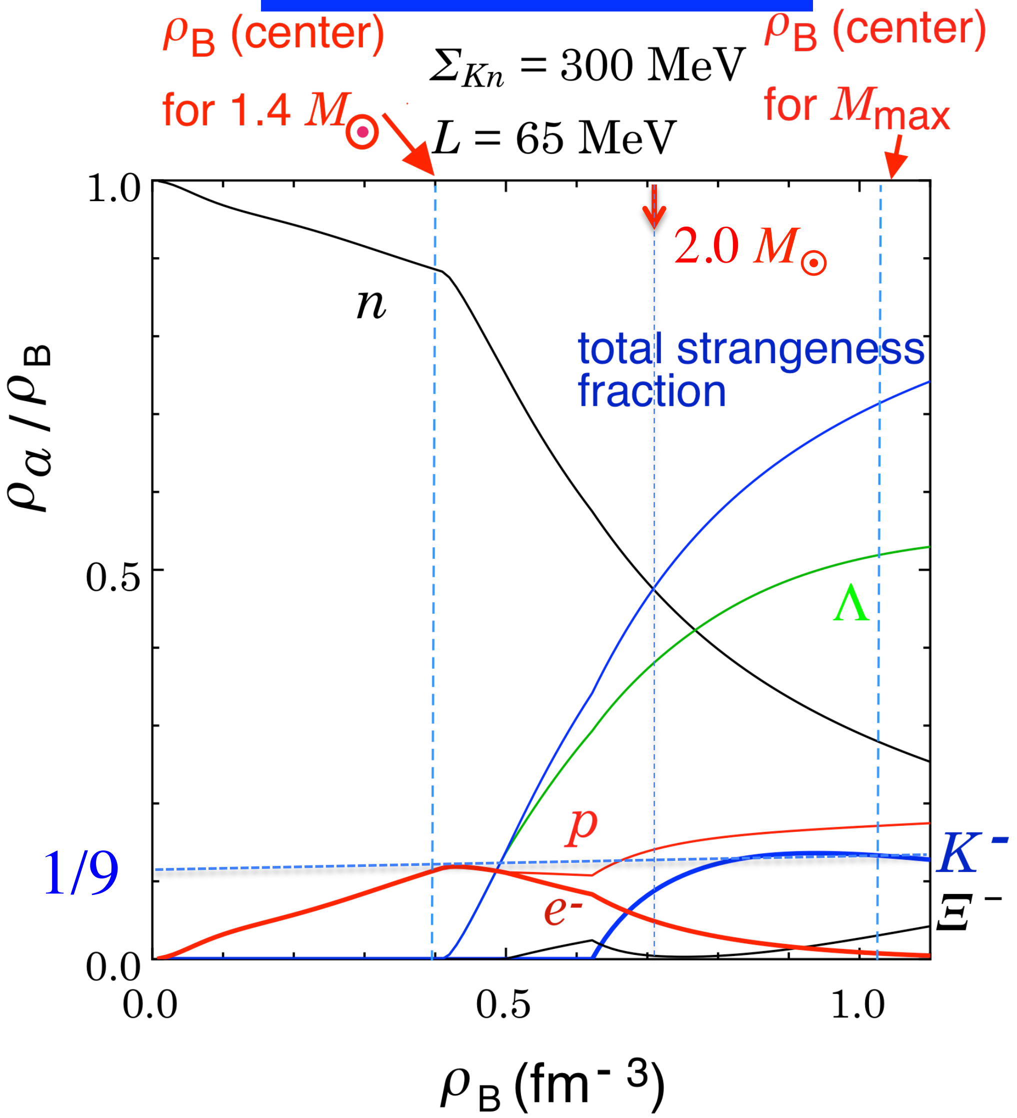
Hadron current $J_{\mathrm{h}}^\mu = \cos\theta_c (V^\mu_{1+i2} - A^\mu_{1+i2}) + \sin\theta_c (V^\mu_{4+i5} - A^\mu_{4+i5})$

$$\begin{aligned} \widetilde{J}_{\mathrm{h}}^\mu &= \hat{U}_K^{-1} J_{\mathrm{h}}^\mu \hat{U}_K \\ &= e^{-i\mu_K t}\Big[\cos\theta_c\Big\{(V^\mu_{1+i2} - A^\mu_{1+i2})\cos(\theta/2) + i(V^\mu_{6-i7} - A^\mu_{6-i7})\sin(\theta/2)\Big\} \\ &+ \sin\theta_c\Big\{(V^\mu_4 - A^\mu_4) + i\cos\theta(V^\mu_5 - A^\mu_5) - \frac{i}{2}\sin\theta\Big(V^\mu_3 - A^\mu_3 + \sqrt{3}(V^\mu_8 - A^\mu_8)\Big)\Big\}\Big] \end{aligned}$$

# 5-2 Rapid cooling processes in the (Y+K) phase

e.g. $\boxed{\Sigma_{Kn} = 300\ \mathrm{MeV}}$ $\boxed{L = 65\ \mathrm{MeV}}$ case

## Particle fractions



**For $M \lesssim 1.4 M_\odot$,**

[ Modified Urca process ]

$$n + n \rightarrow n + p + e^- + \bar{\nu}_e\ ,$$
$$n + p + e^- \rightarrow n + n + \nu_e$$

**For $M \gtrsim 1.4 M_\odot$,**

[hyperon ($\Lambda$) Urca process ]

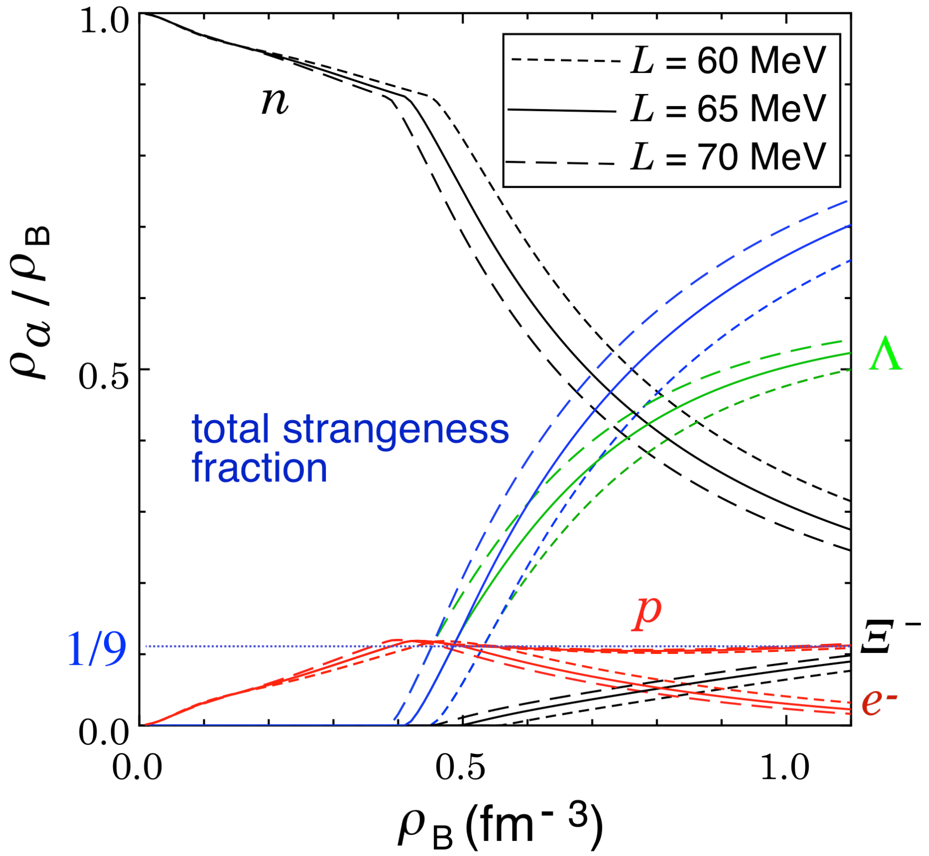
$$\Lambda \rightarrow p + e^- + \bar{\nu}_e$$
$$p + e^- \rightarrow \Lambda + \nu_e$$

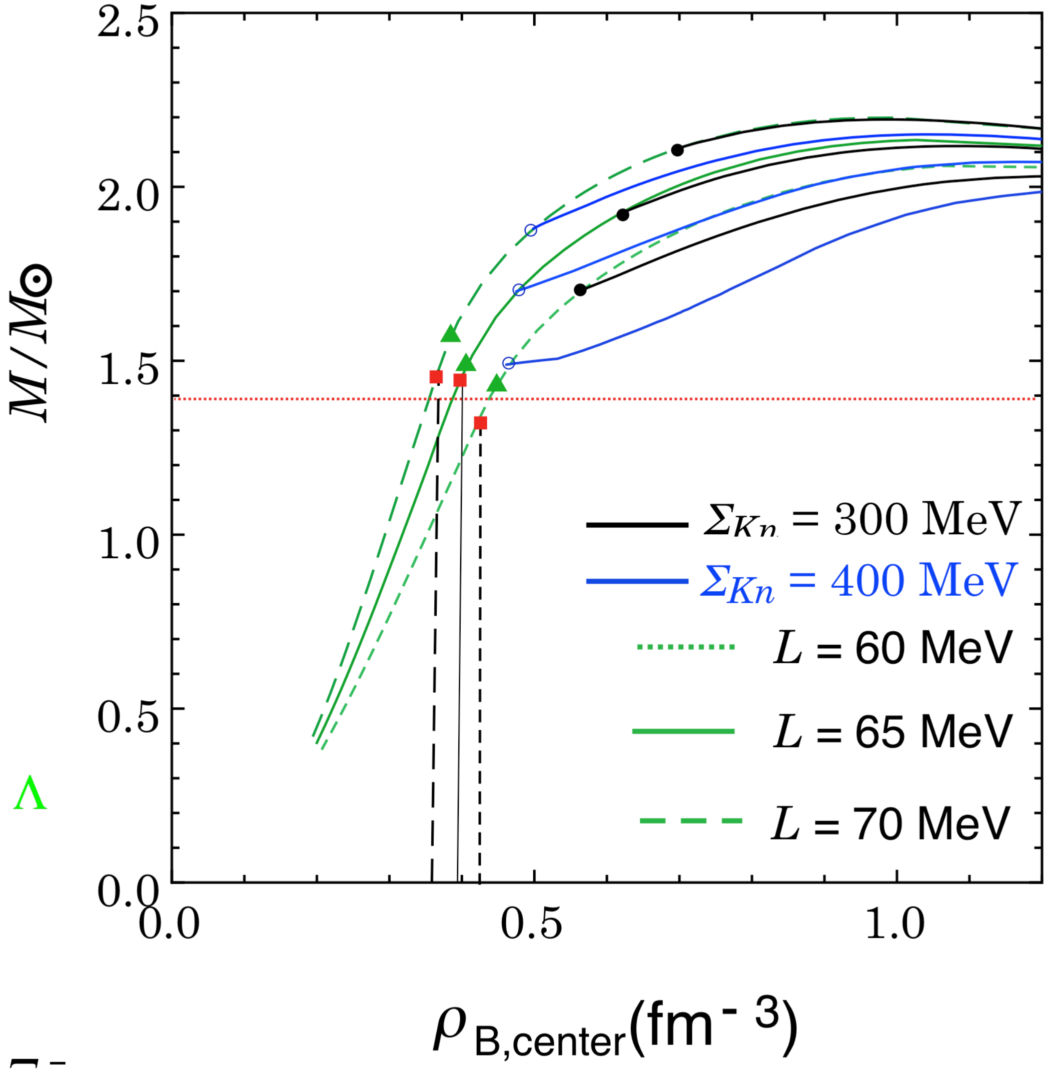
[Kaon-induced Urca process ]

$$n\ (p) + \langle K^- \rangle \rightarrow n\ (p) + e^- + \bar{\nu}_e\ ,$$
$$n\ (p) + e^- \rightarrow n\ (p) + \langle K^- \rangle + \nu_e$$

# Gravitational Mass – central density relations

## Particle fractions in pure hyperon-mixed matter





● Threshold density at center for Direct Urca process

▲ Onset of Λ in the center

# 6. Summary and concluding remarks

By the use of the Minimal RMF [omitting the nonlinear self-interacting scalar potential U(σ)] for B-B interactions and taking into account the UTBR and TNA for 3-body interactions,

we have considered effects of the TNA, by choosing the allowable values of the slope $L$ [= (60, 65, 70) MeV] , on the whole EOS for the (Y+K) phase in high densities.

## EOS and neutron star structure

- Kaon condensates do appear in the center of the core, for heavy $N_{☆}$ with $M$= (1.7 ~ 2.1) $M_{\odot}$, the detailed values depending on $L$ and $\Sigma_{Kn}$

- For the canonical mass (~ $1.4M_{\odot}$ ) stars, the ground state in the core consists of only n, p, e- , and even hyperons ($\Lambda$ . . . ) do not appear.

- The $L$ is correlated with the TNA, which is resposible to the EOS of SNM and quantities at $\rho_0$ ( $g_{\sigma N}, g_{\omega N}, g_{\rho N}, \langle\sigma\rangle_0, \langle\omega\rangle_0$ ).
Hence $L$ also affects the EOS at high densities.

# Effects of the (Y+K) phase on cooling scenario of N☆

**For** $M \lesssim 1.4M_\odot$**,**

Main cooling processes are given by the modified Urca process.

**For** $M \gtrsim 1.4M_\odot$**,**

Hyperon (Λ) Urca process starts and becomes a dominant cooling process. For further heavy neutron stars, the kaon-induced Urca becomes a main cooling process.

## Future issues

・Systematic study by modifying the repulsive interaction volume, $V_r \times (\lambda_r)^3$ , and with allowable $L$ from TNA, how it affects the stiffness of the EOS.

・Effects of the (Y+K) EOS on thermal evolution of neutron stars : rapid cooling mechanisms → cooling curves and consistency with observation of surface temperatures.