Stiffening of matter in **quark-hadron continuity**

--- *peak in sound velocity* ---

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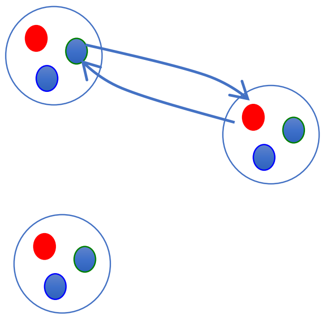
Ref) TK, [2106.06687](https://arxiv.org/abs/2106.06687) [nucl-th]

A picture being developed

[Masuda-Hatsuda-Takatsuka '12; TK-Powell-Song-Baym '14]

- few meson exchange
- nucleons only

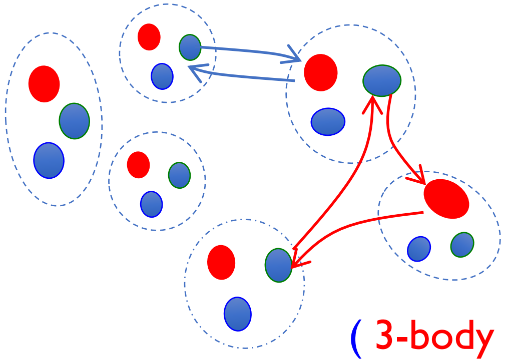


ab-initio nuclear cal.

e.g.) ChEFT, variational

steady progress

- many-quark exchange
- structural change,...

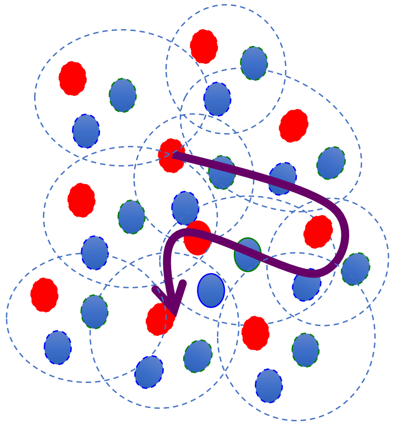


(3-body)

most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

(pQCD)

[Freedman-McLerran, Kurkela+,...]

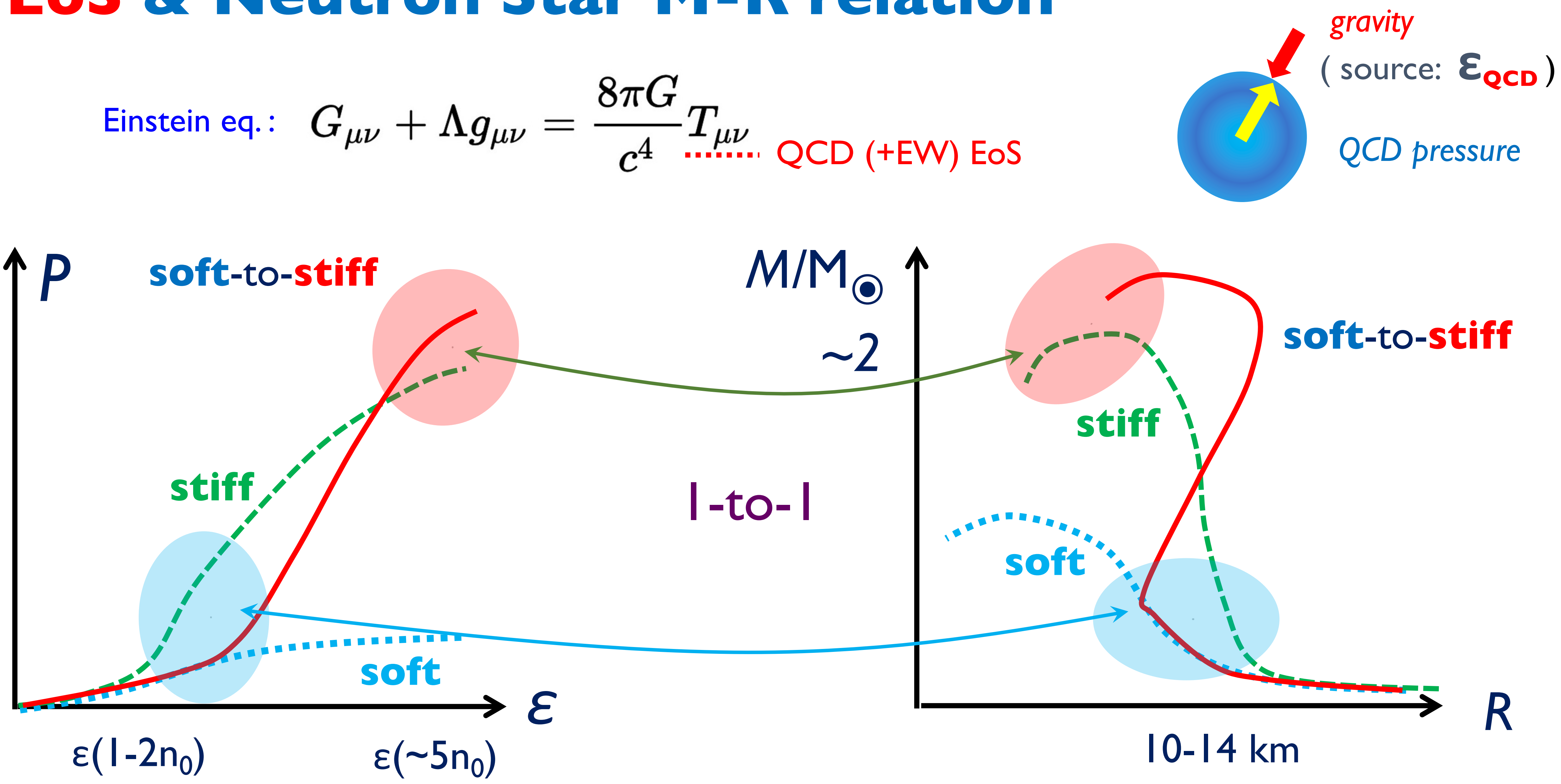
~ 1.4 $M_\odot$ ~ 2 $M_\odot$ n_B

Hints from NS

~ $2n_0$ ~ $5n_0$ ~ $40n_0$

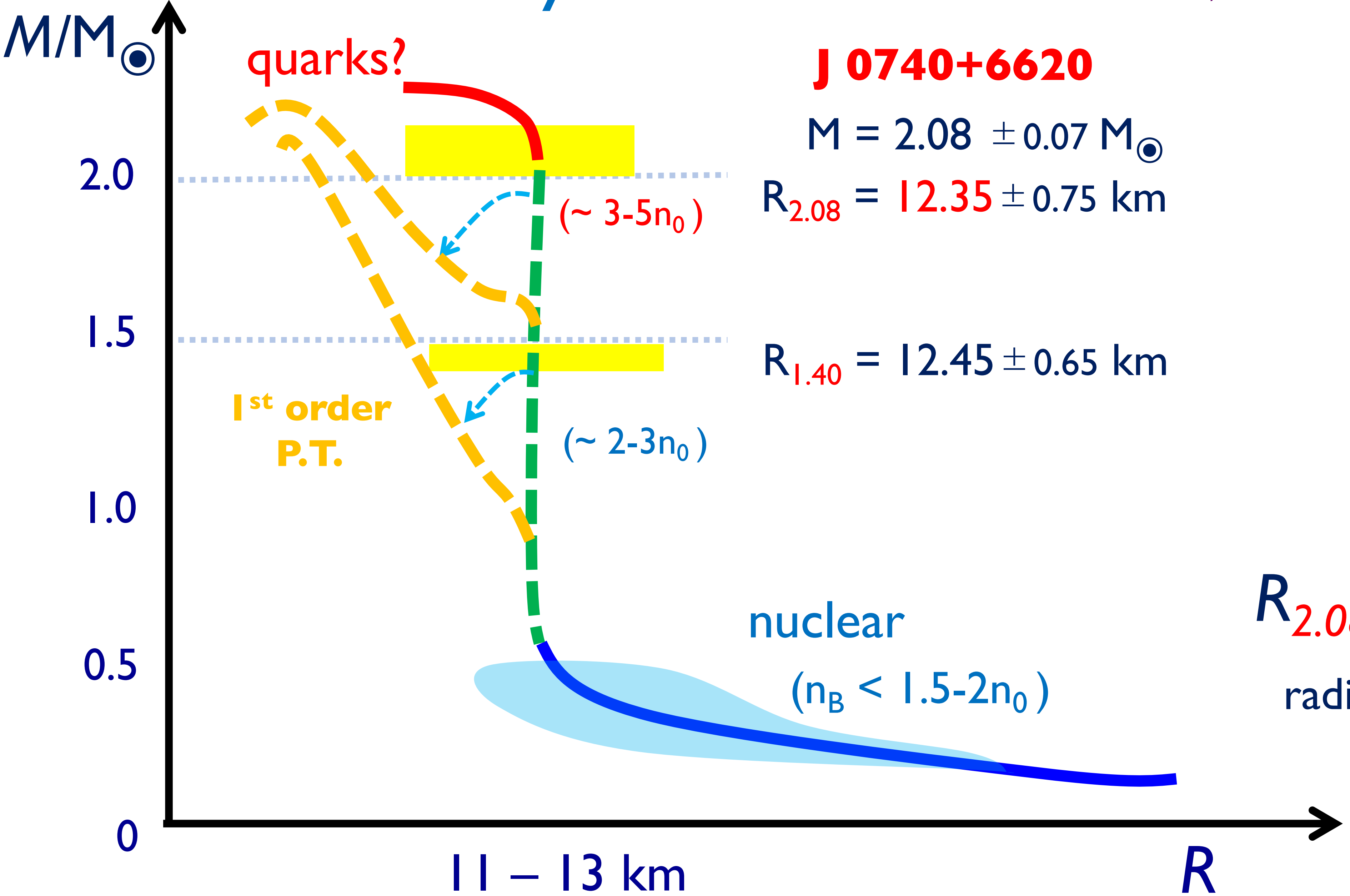
EoS & Neutron Star M-R relation

Einstein eq. : $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ ...... QCD (+EW) EoS



Density vs *M-R* curves

Ref) Lattimer & Prakash (2001)



J 0740+6620

$M = 2.08 \pm 0.07\ M_\odot$

$R_{2.08} = 12.35 \pm 0.75$ km

$R_{1.40} = 12.45 \pm 0.65$ km

based on

NICER

J0740+6620 data

+ J0030+0451 data

+ GW170817

+ *nuclear constraints*

[Miller+ '21]

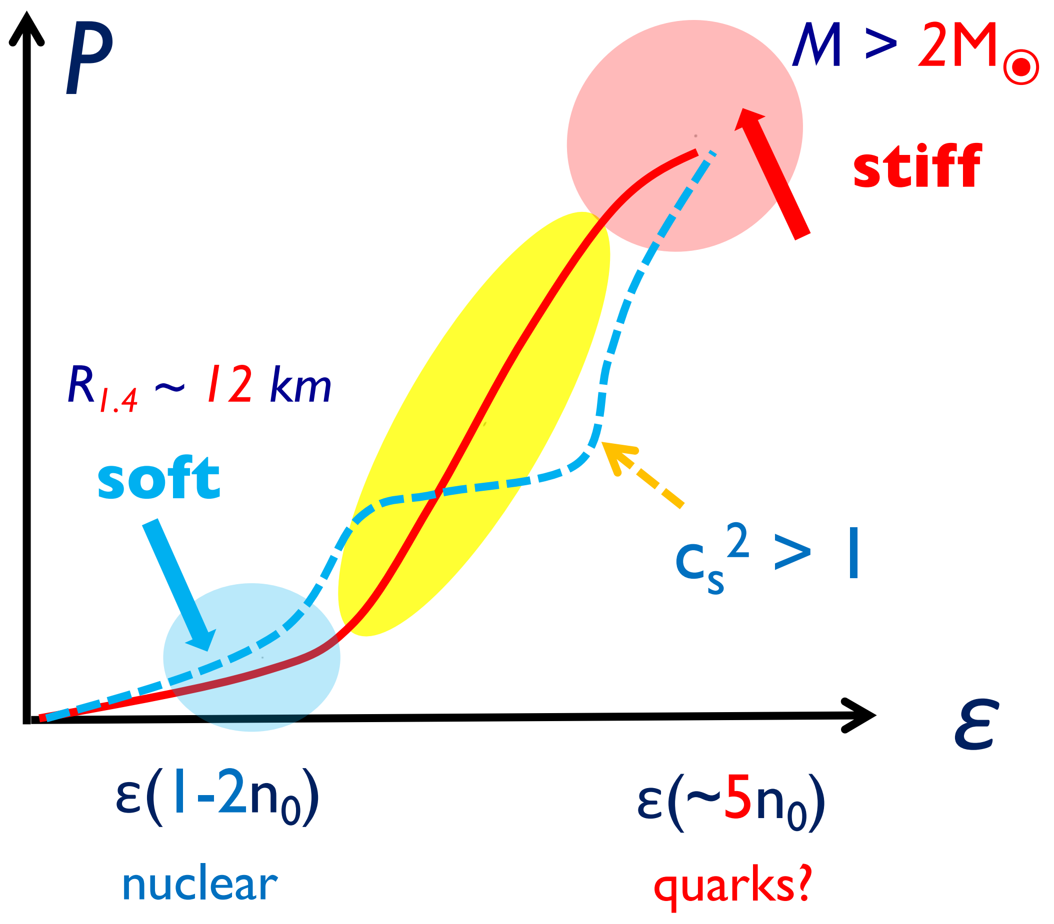
$R_{2.08} \sim R_{1.40}$ (!)

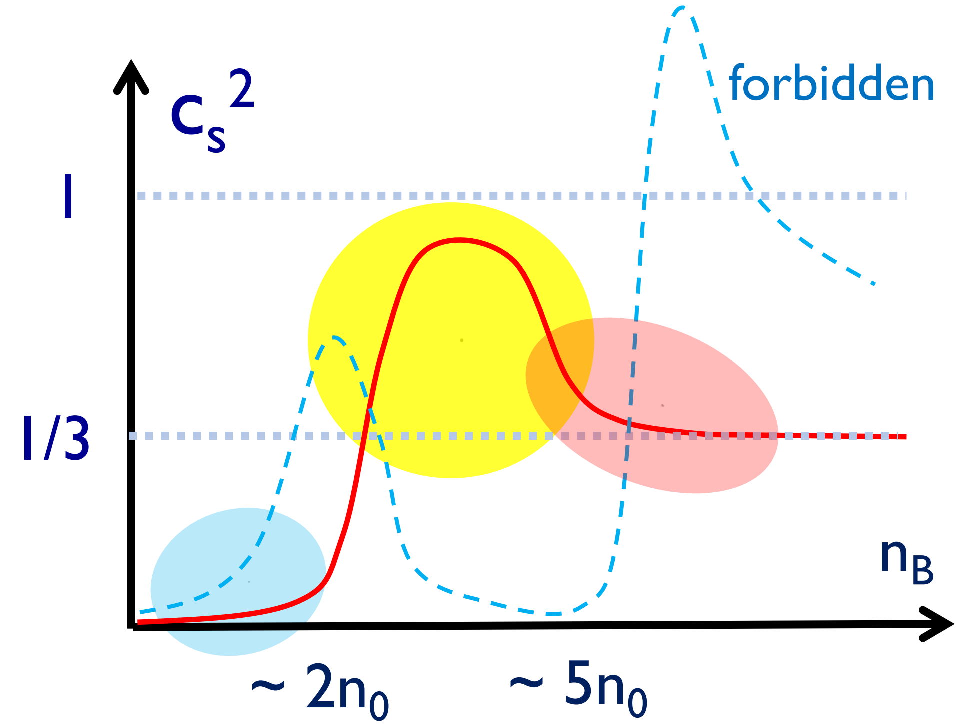
radical softening unlikely

for 2-5 n_0

Soft to stiff is challenging

speed of sound: $c_s^2 = dP/d\varepsilon < 1$ (*causality*)





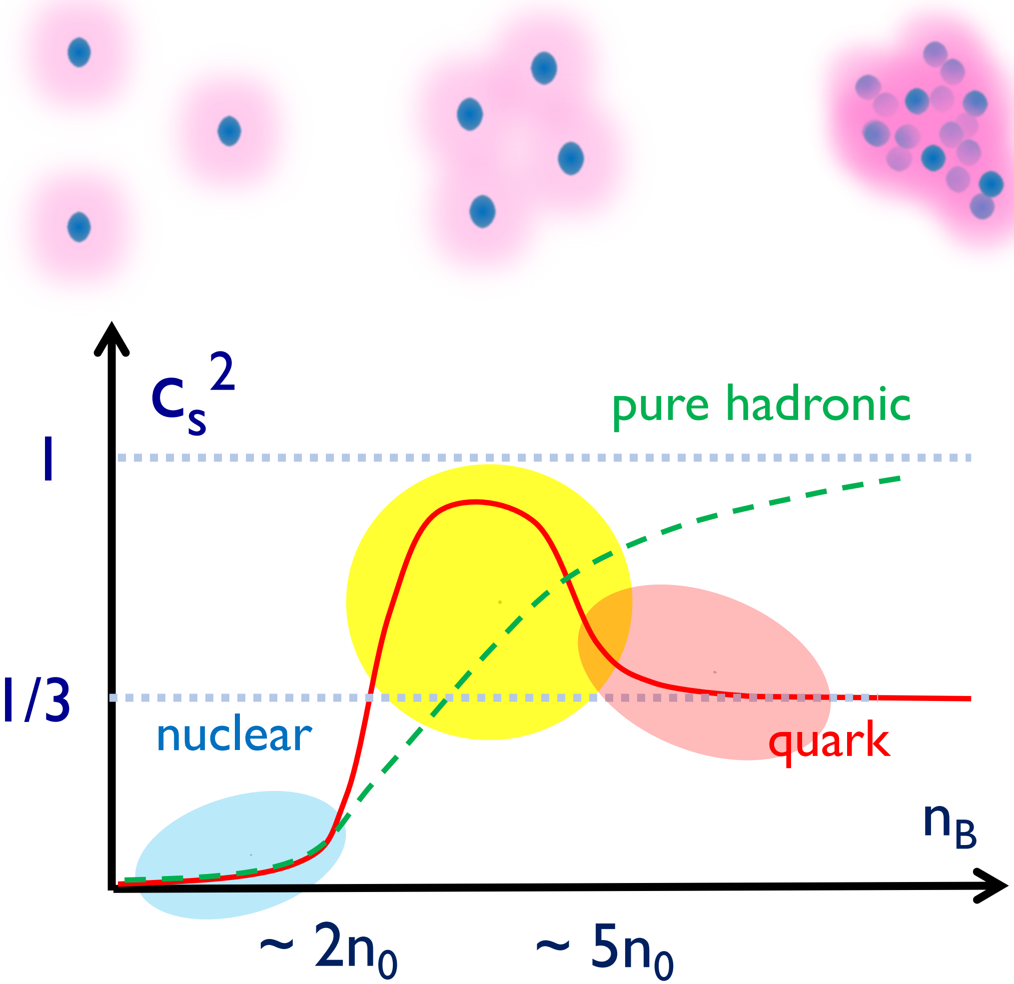
the simplest: *quark-hadron continuity*

→ we take it as our **baseline**

Goals in this work

Direct descriptions of the "c_s-peak"

[cf) McLerran-Reddy (MR), PRL '19,...]



the statements include:

1) *robust in a quark-hadron continuity model*

2) *appears before baryon cores overlap*

3) *nuclear repulsive forces are NOT major driving forces*

[nuclear int. will be ignored at LO]

Problem

Switching from baryonic to quark bases

$\longrightarrow$ *source of confusions in hybrid modeling*

(e.g. normalization of energy)

Strategy

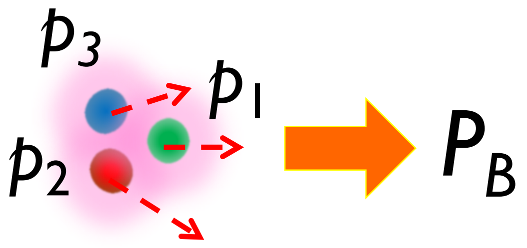
Follow quark states from nuclear to quark matter

(within a *single* model)

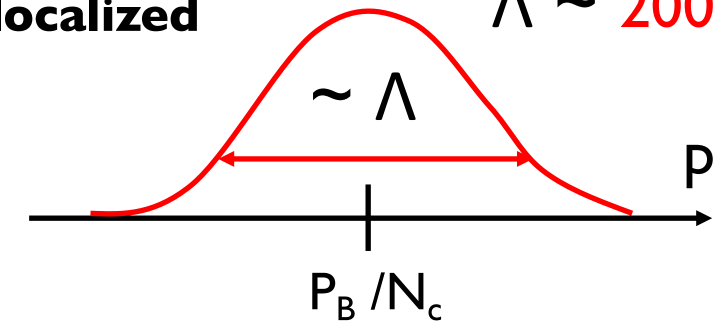
Quarks in a baryon

N_c (=3): number of colors

probability density: $$Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B) = \mathcal{N} e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2}$$



localized



$\Lambda \sim$ 200 MeV

mean: $$\langle \boldsymbol{P}_B \rangle = N_c \int_{\boldsymbol{p}} \boldsymbol{p} Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B)$$

variance: $$\left\langle \left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2 \right\rangle \sim \Lambda^2$$

energetic !

$$\langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B} = \mathcal{N} \int_{\boldsymbol{p}} E_q(\boldsymbol{p}) e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2} \simeq \langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B=0} + \frac{1}{6}\left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i} \right\rangle_{\boldsymbol{P}_B=0} \left(\frac{\boldsymbol{P}_B}{N_c}\right)^2 + \cdots$$

average energy (quark)

x N_c ↓ $\sim N_c (M_q + \Lambda)$ — baryon mass

$\gg$

x N_c ↓ $\sim P_B^2 / (N_c E_q)$ — baryon kin. energy

Occupation probability of quark states

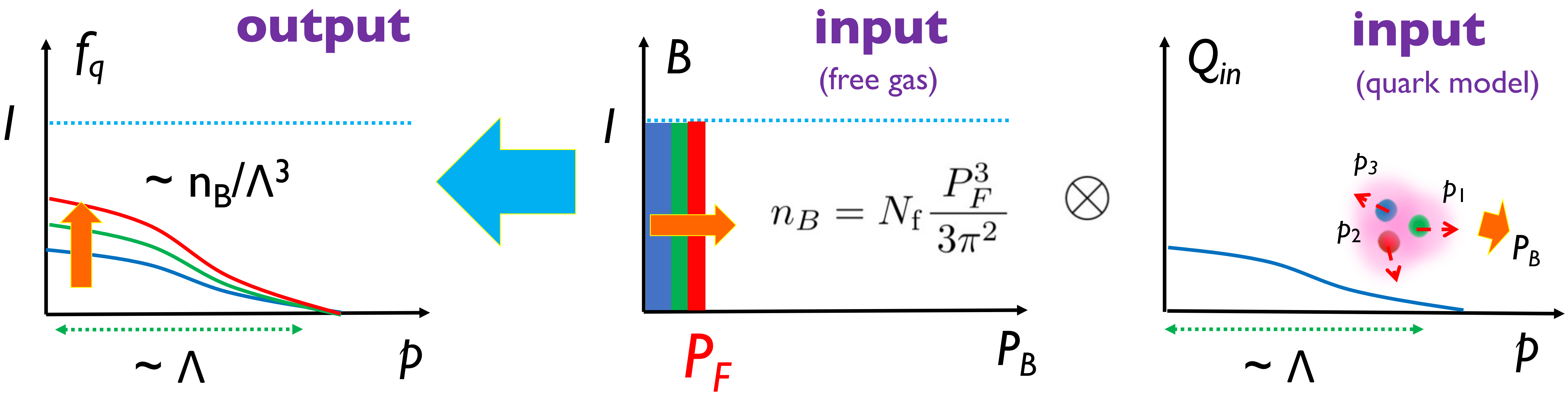
occupation **probability** of **quark** state with p

occupation **probability** of **baryon** state with P_B

quark mom. **pro. density** in a baryon

$$f_q(p; n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(P_B; n_B) Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B)$$

e.g.) in **dilute** baryonic matter

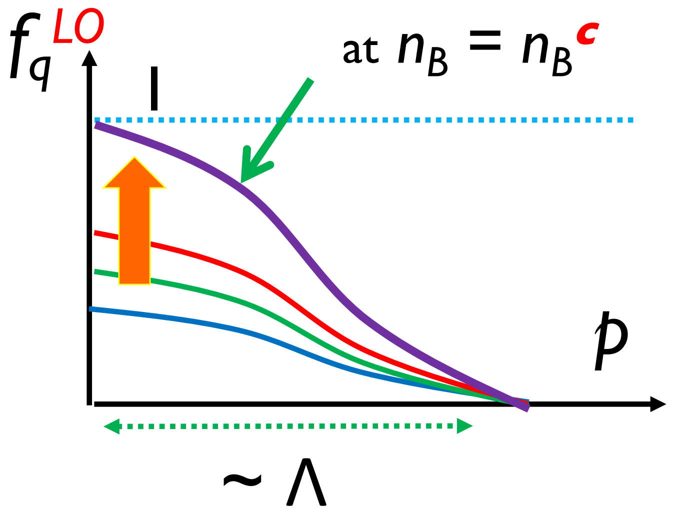


Quarks in ideal baryon gas

$$f_q(p;n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(P_B;n_B) Q_{\rm in}(\boldsymbol{p},\boldsymbol{P}_B)$$

for ideal baryon gas:

$$f_q(p;n_B) = \frac{n_B}{n_B^c} \mathrm{e}^{-p^2/\Lambda^2} + O(1/N_c^2)$$



LO:

- the **shape** does not change
- the **height** grows linearly in n_B

energy density

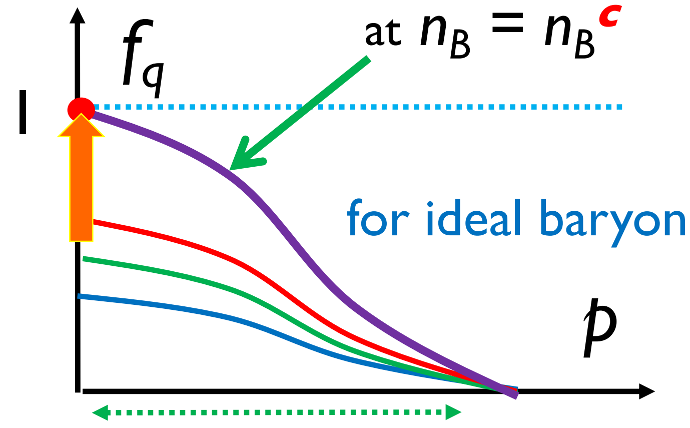
$$\varepsilon(n_B) = N_c \int_{\mathbf{p}} E_q(p) \left(f_q^{\rm LO}(p;n_B) + O(1/N_c^2) \right) = n_B M_B + O(1/N_c)$$

kin. energy

pressure: $\mathcal{P} = n_B^2 \partial(\varepsilon/n_B)/\partial n_B$ ~ 0 + O(1/Nc)

~ const. very soft

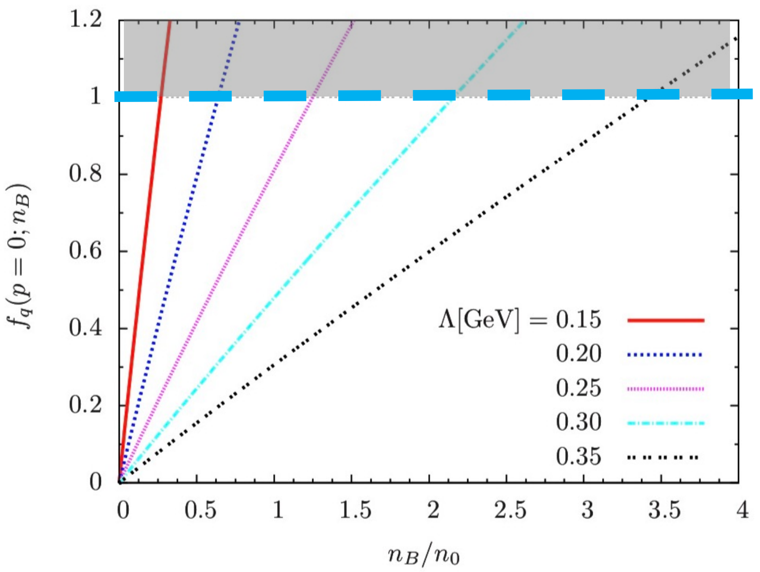
Saturation of quark states

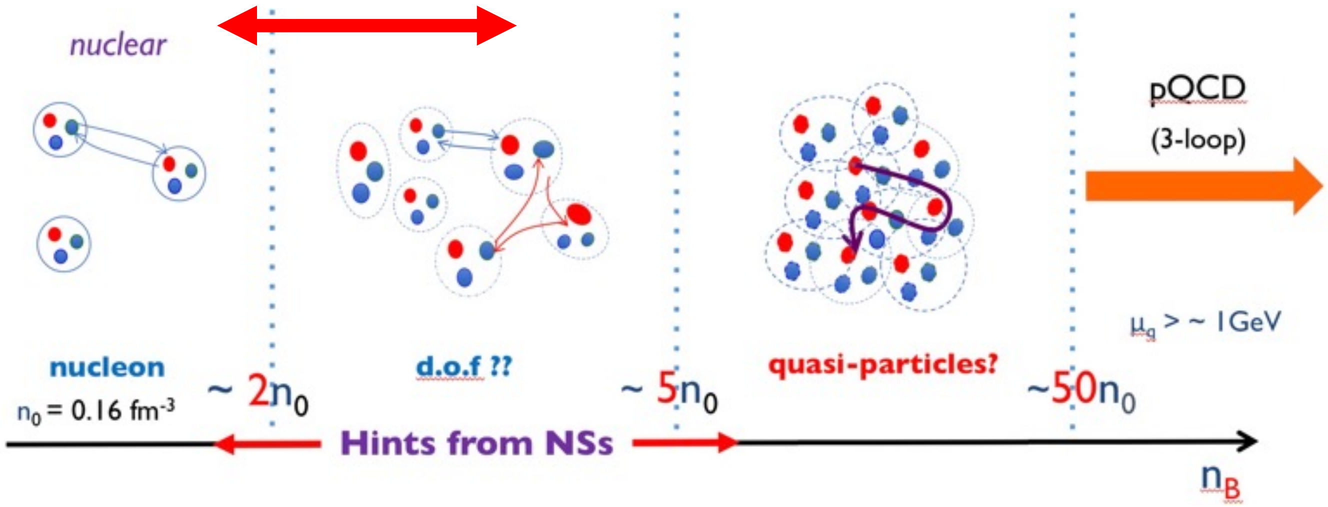


$\Lambda \sim$ 0.15-0.35 GeV $\rightarrow n_B^c \sim$ 0.2-3.5 n_0

should happen **before** baryon cores overlap

cf) *Soft Deconfinement* [Fukushima-TK-Weise, '20]



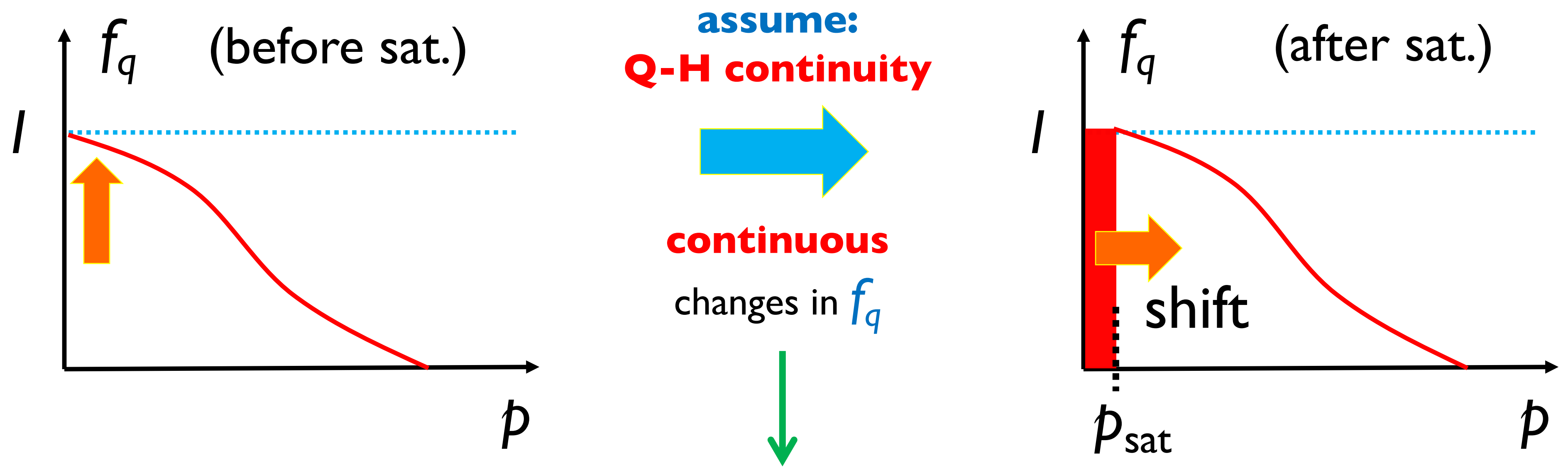


Quark matter formation: $f_q\,(p\,;n_B)$

a **model** after the saturation:

same shape

$$f_q^{\text{after}} = \theta(p_{\text{sat}} - p) + \theta(p - p_{\text{sat}})\, \underline{f_q(p - p_{\text{sat}}; n_B^c)}$$

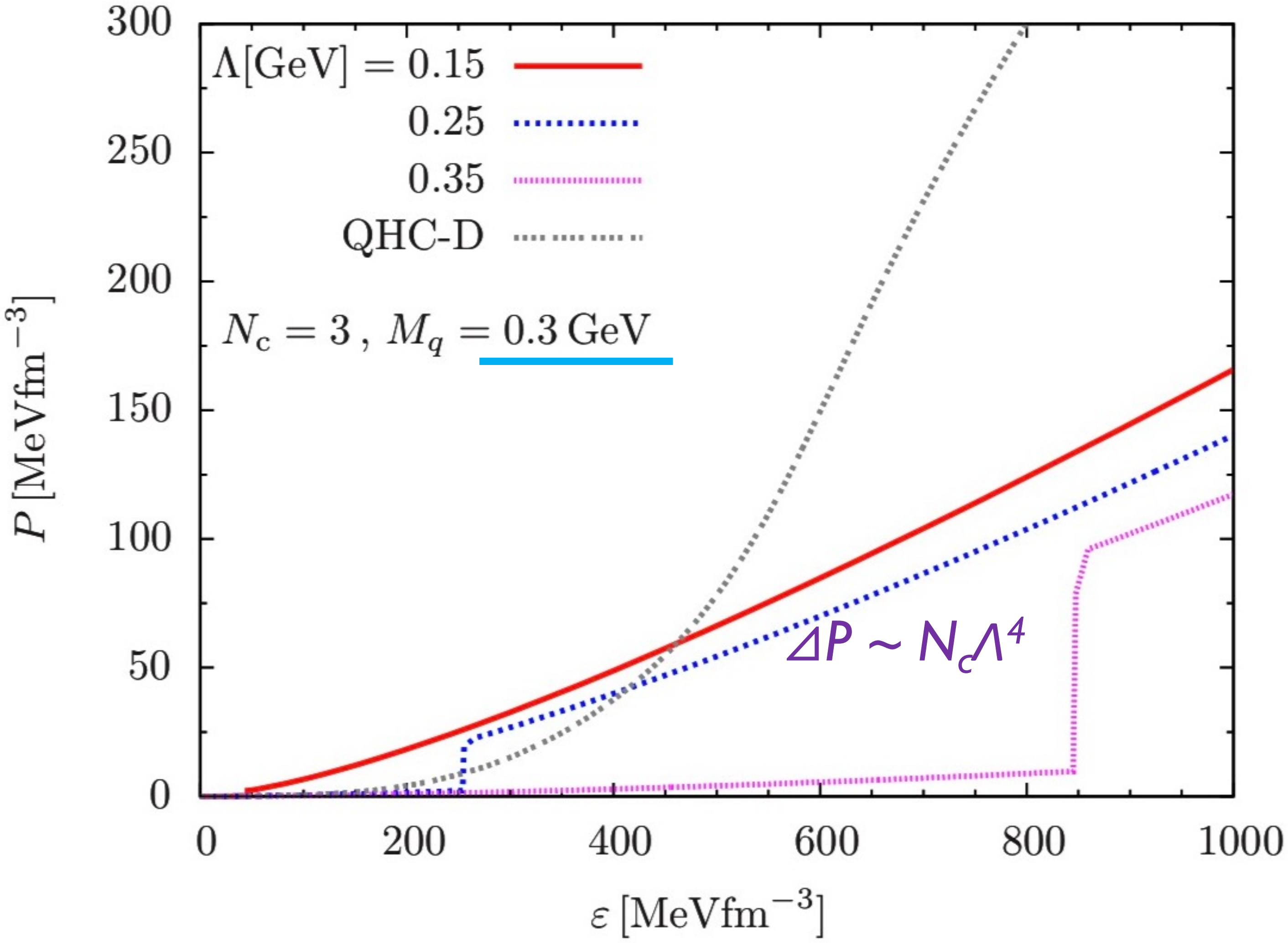


ε, n_B are **continuous** before and after the saturation.

any nontrivial consequences ?

jumps in μ_B, P

(Nc = 3, M_q = 0.3 GeV)



a model of f_q looks innocent,

but its consequence is unphysical:

ε, n_B are **continuous,**

jumps in μ_B & P

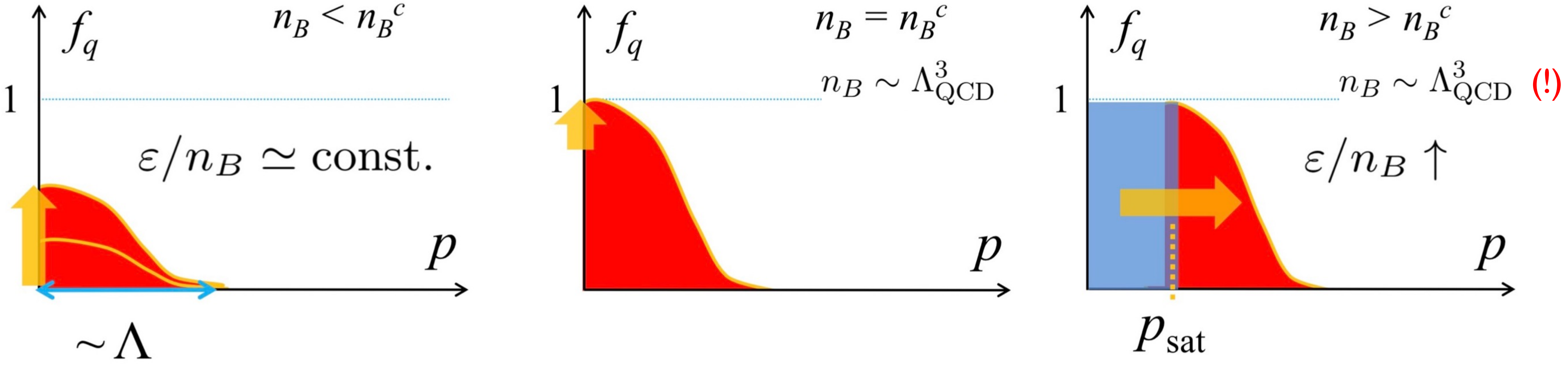
($\Delta\mu_B \sim N_c\Lambda$)

opposite to usual 1st order P.T. (!)

Quark matter formation: *EoS*

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$ energy per particle

[cf) McLerran-Reddy (MR), PRL '19,...]



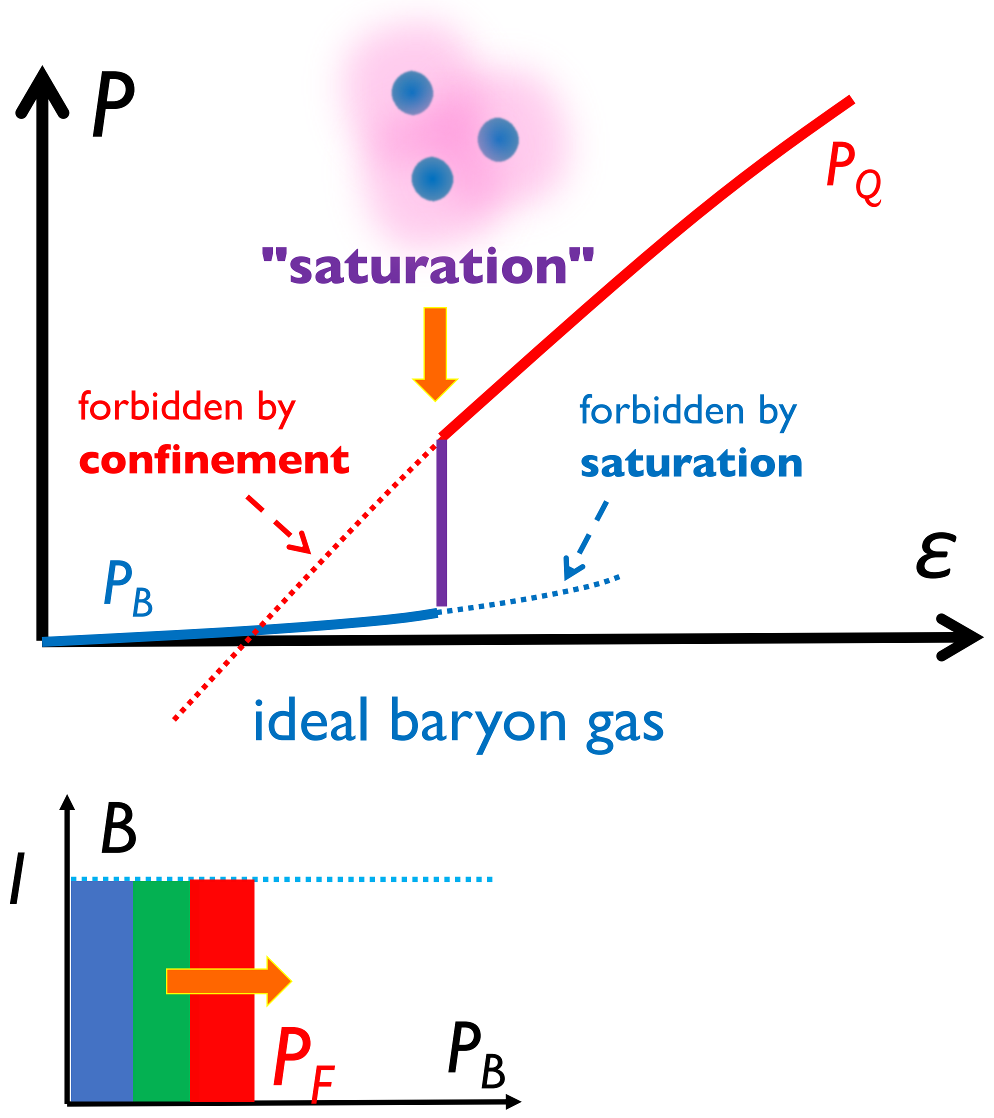
$\varepsilon \sim n_B \times N_c \Lambda_{\rm QCD}$

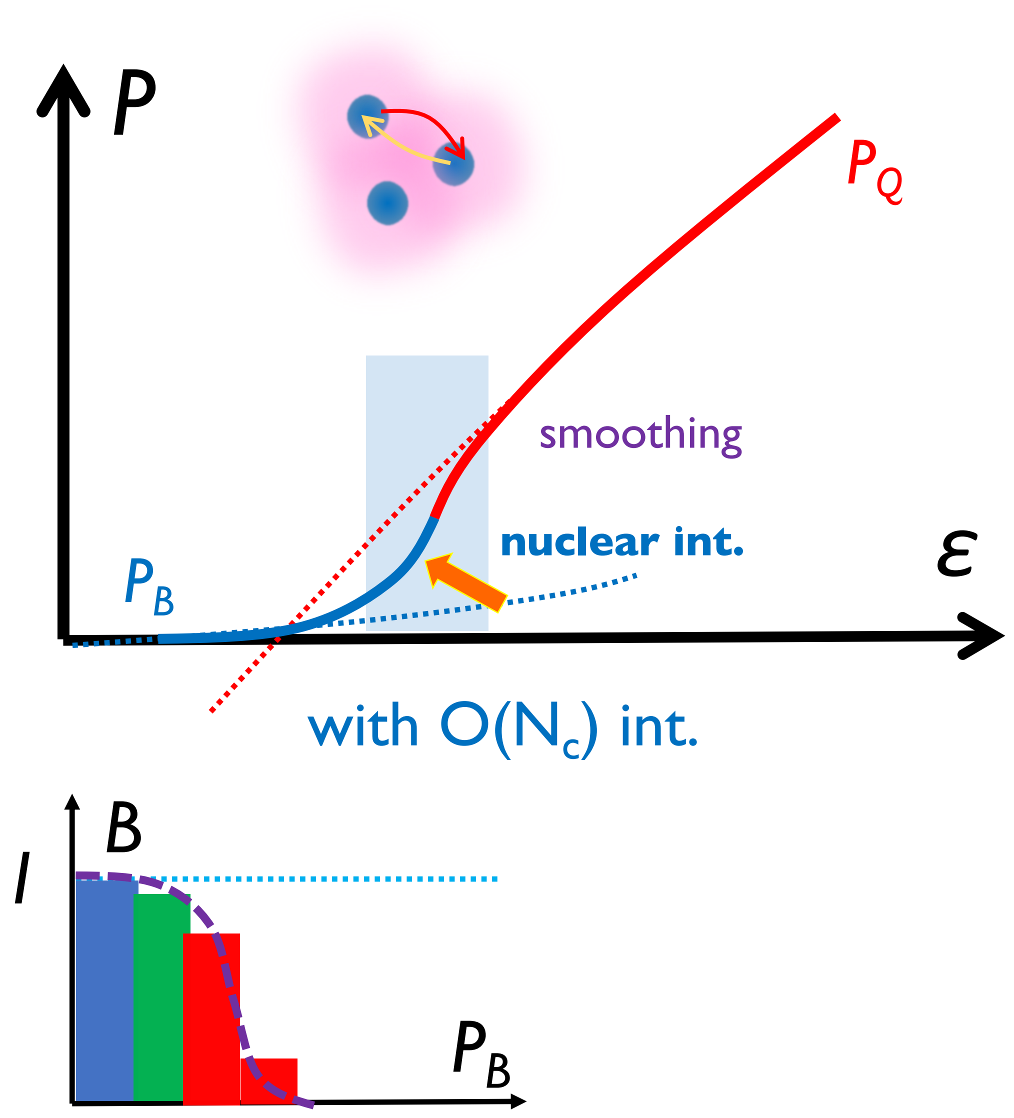
$\varepsilon \sim N_c \Lambda_{\rm QCD}^4$

$\mathcal{P} \sim n_B^{5/3}/N_c\Lambda_{\rm QCD}$ $\qquad$ $\mathcal{P} \sim \Lambda_{\rm QCD}^4/N_c$ $\qquad$ $\mathcal{P} \sim N_c \Lambda_{\rm QCD}^4$ (!)

More realistic picture

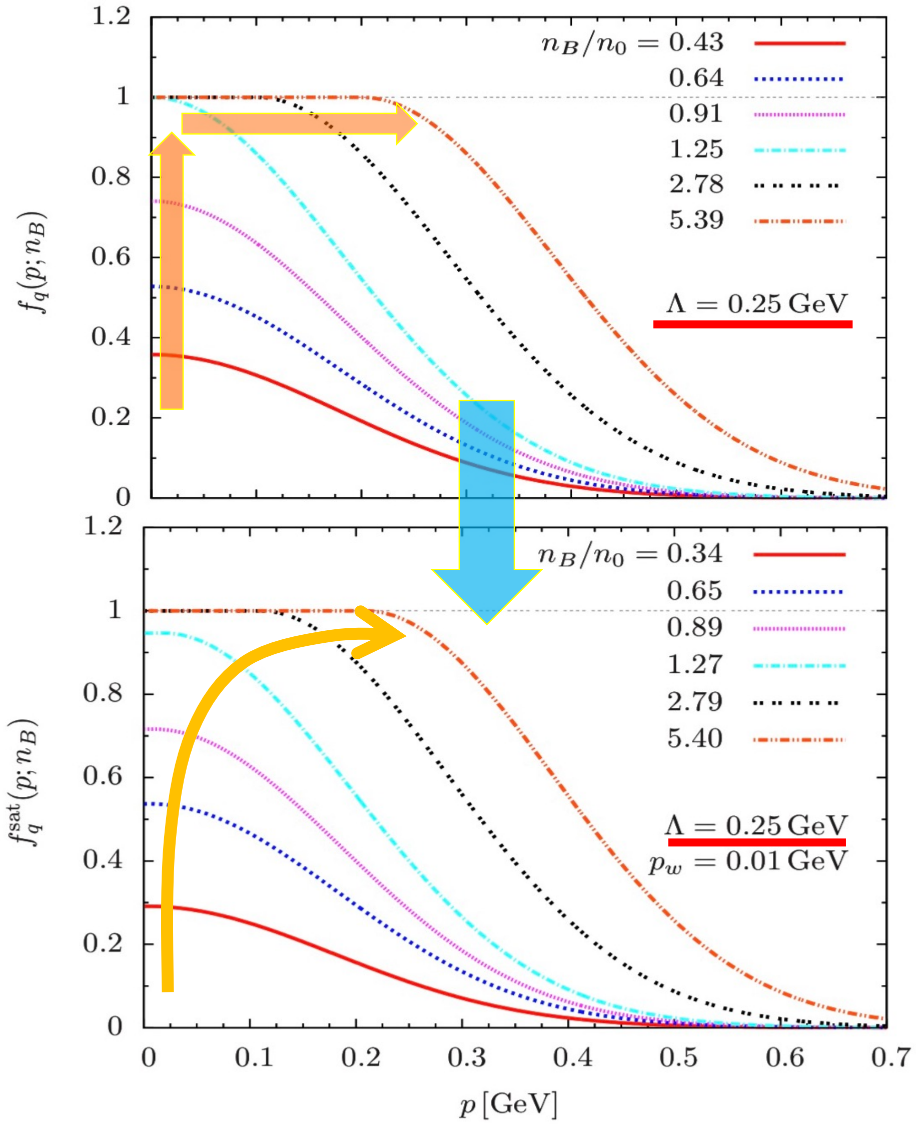
[P_B & P_Q from a **single** model; f_q continuous]

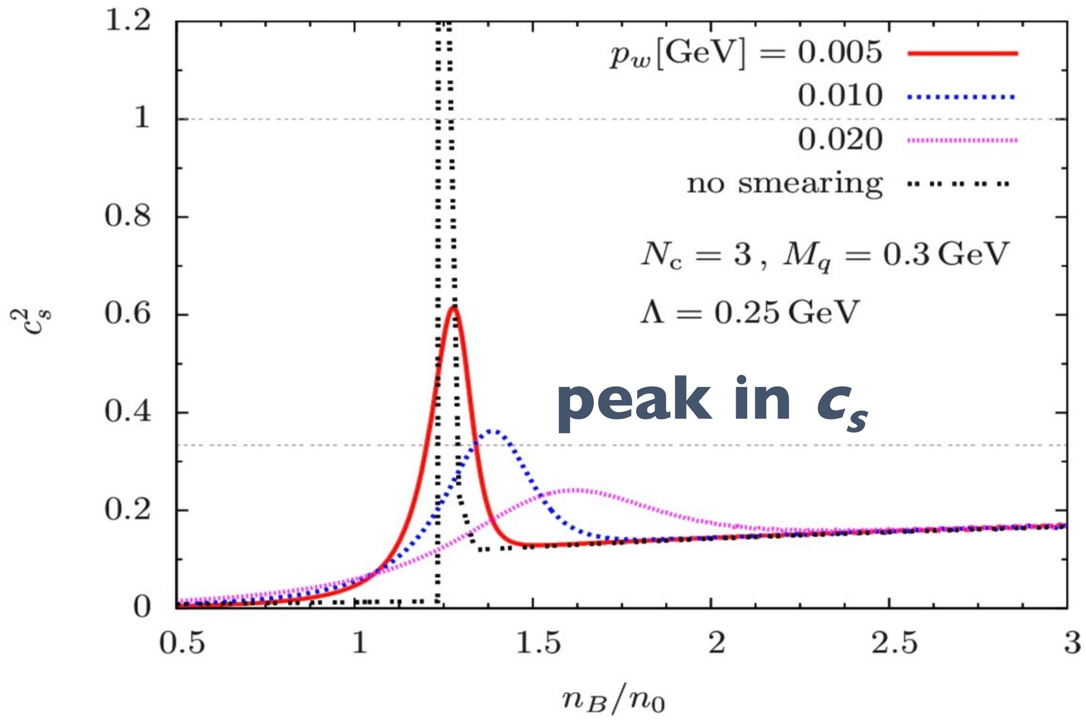




Smooth version

$$f_q^{\rm sat}(p;p_{\rm sat}) = \underline{\tanh(p_{\rm sat}/p_w)} \times \left[\,\theta(p_{\rm sat}-p) + \theta(p-p_{\rm sat})\,{\rm e}^{-(\tilde{p}-\tilde{p}_{\rm sat})^2}\right]$$





- *location* primarily determined by Λ (or baryon size)
- *width* primarily determined by p_w (or interactions)
- interactions: *NOT driving forces*, just *temper* the peak

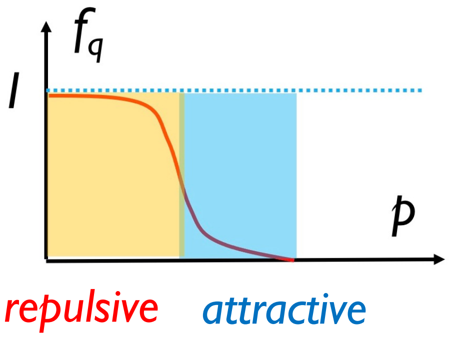
quark energy; *parameterization of MF*

$$\mathcal{V}_{\mathrm{CE}}[f_q] = -C_E^{\mathcal{A}} \times \left(1 - \underline{f_q^{\beta}}\right) + C_E^{\mathcal{S}} \underline{f_q^{\beta}}$$

for f_q (p) << 1

$$\mathcal{V}_{\mathrm{CE}}[f_q] \simeq -C_E^{\mathcal{A}}$$

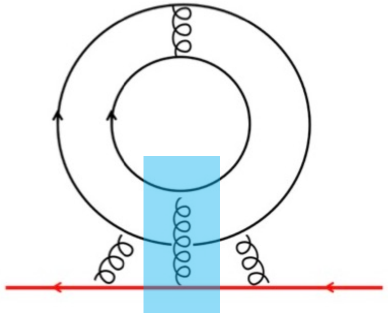
dilute in momentum space



for f_q (p) ~ 1

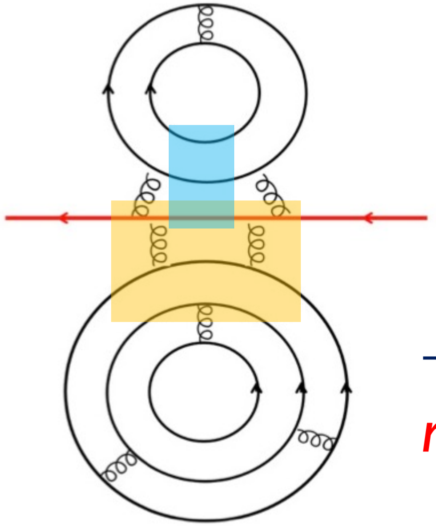
$$\mathcal{V}_{\mathrm{CE}}[f_q] \simeq C_E^{\mathcal{S}}$$

for *saturated levels*



color-*antisym.* channels dominate

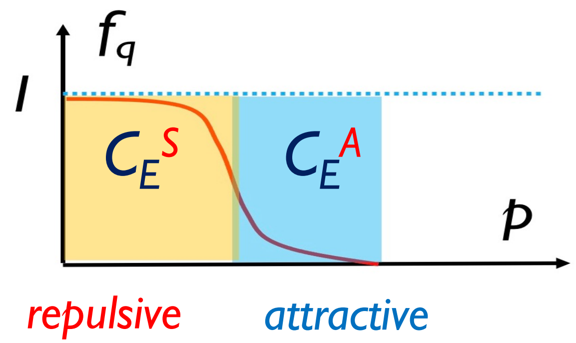
→ the quark feels *attractive* correlations

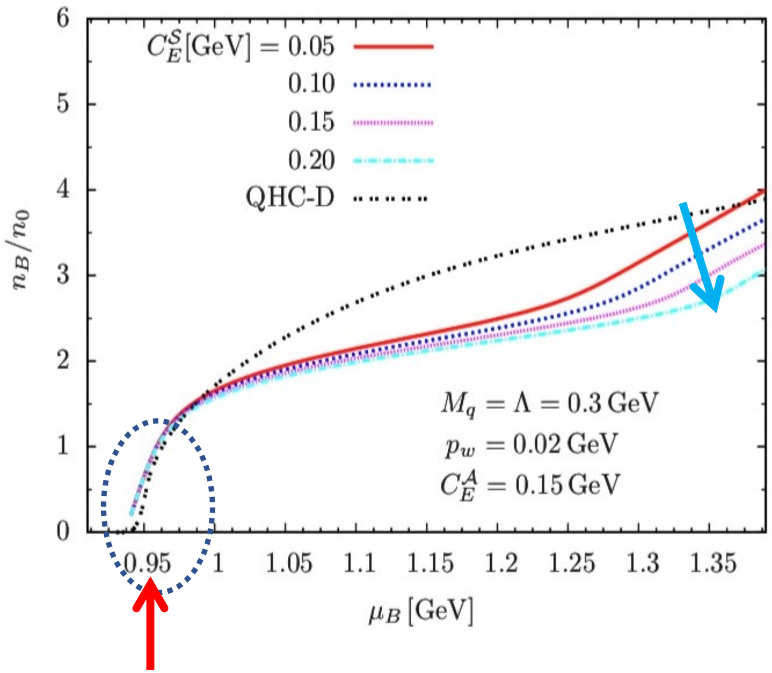


color-*sym.* channels also enter

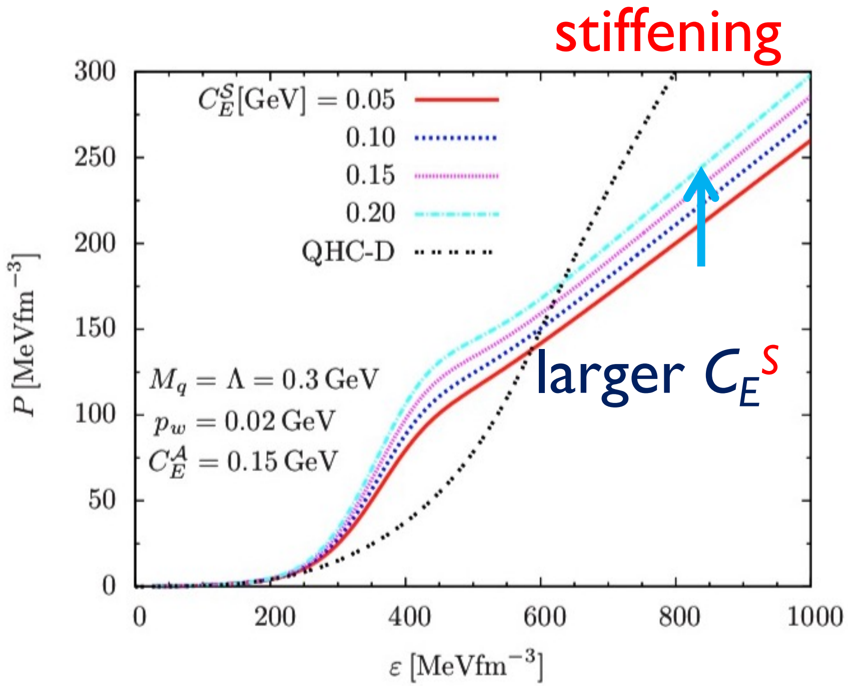
→ the quark feels *repulsive* correlations also

EoS with interactions

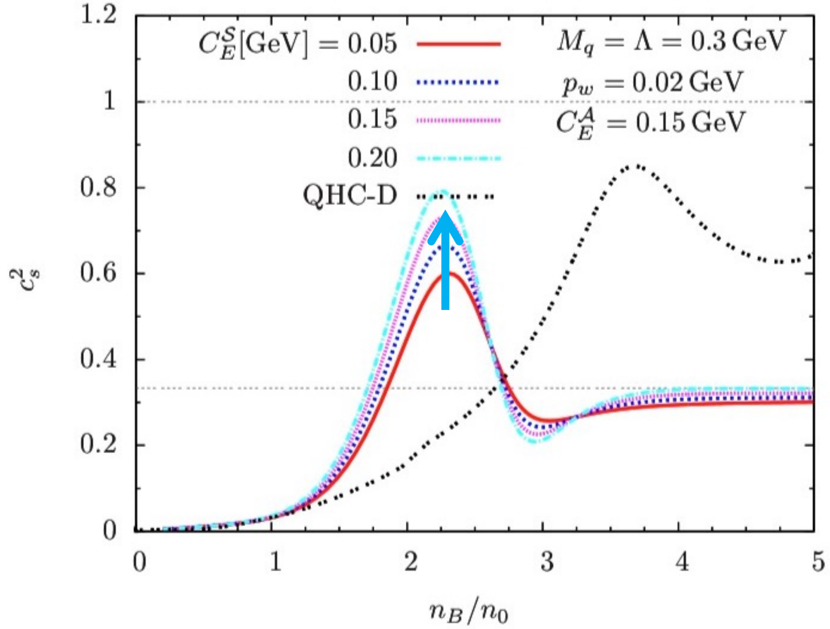




adjust C_E^A (fit M_B = 939 MeV)

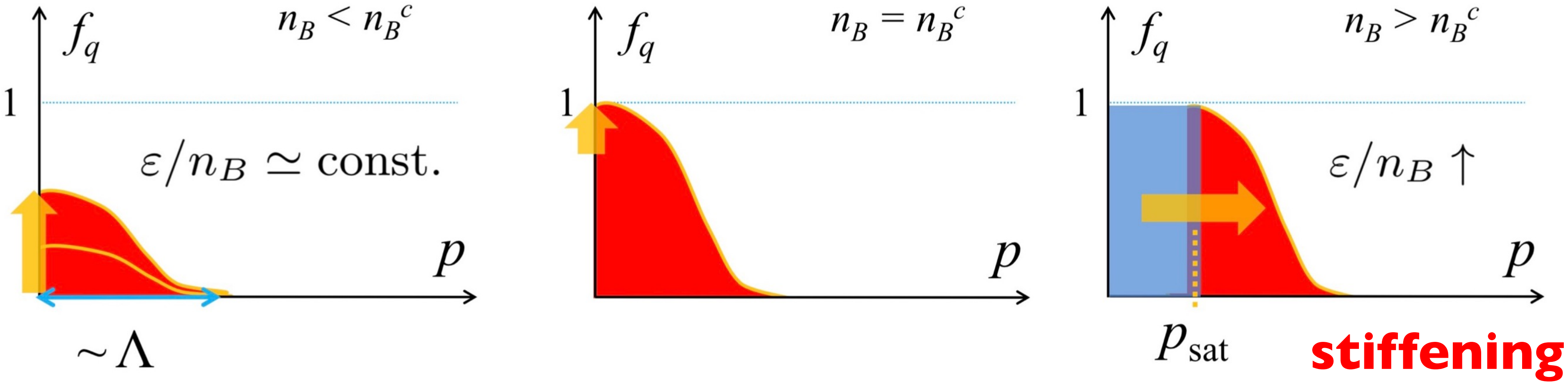


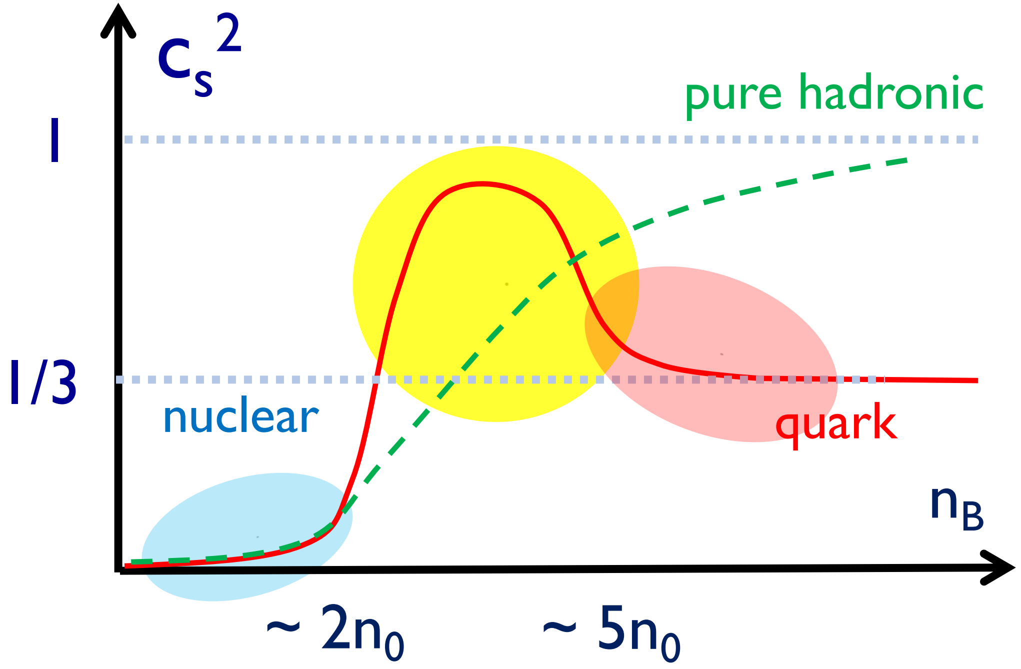
high density stiffening



peak in c_s

Summary





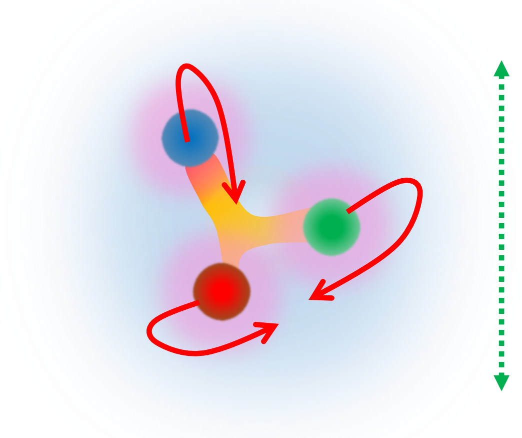
- *robust in a quark-hadron continuity model*
- *appears before baryon cores overlap*
- *nuclear forces are NOT driving forces for the peak*

Back up

Stiff quark matter EoS ? : *intuitive questions*

quarks in a baryon are energetic

baryonic matter

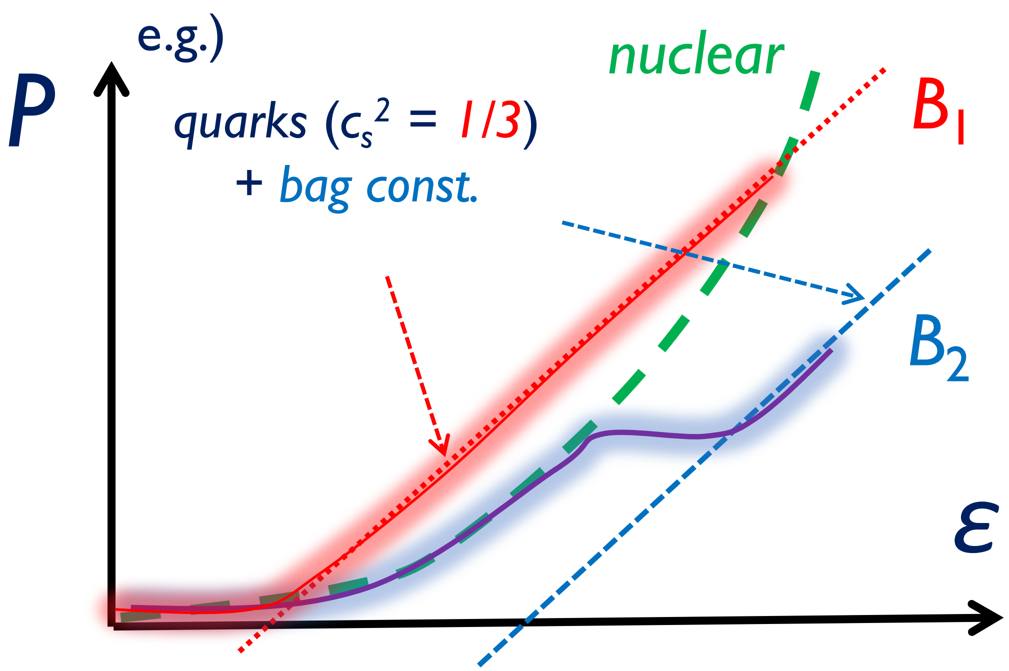


but they do not directly contribute to the pressure

$\varepsilon >> P$

Q1) deconfinement → stiff EoS ?

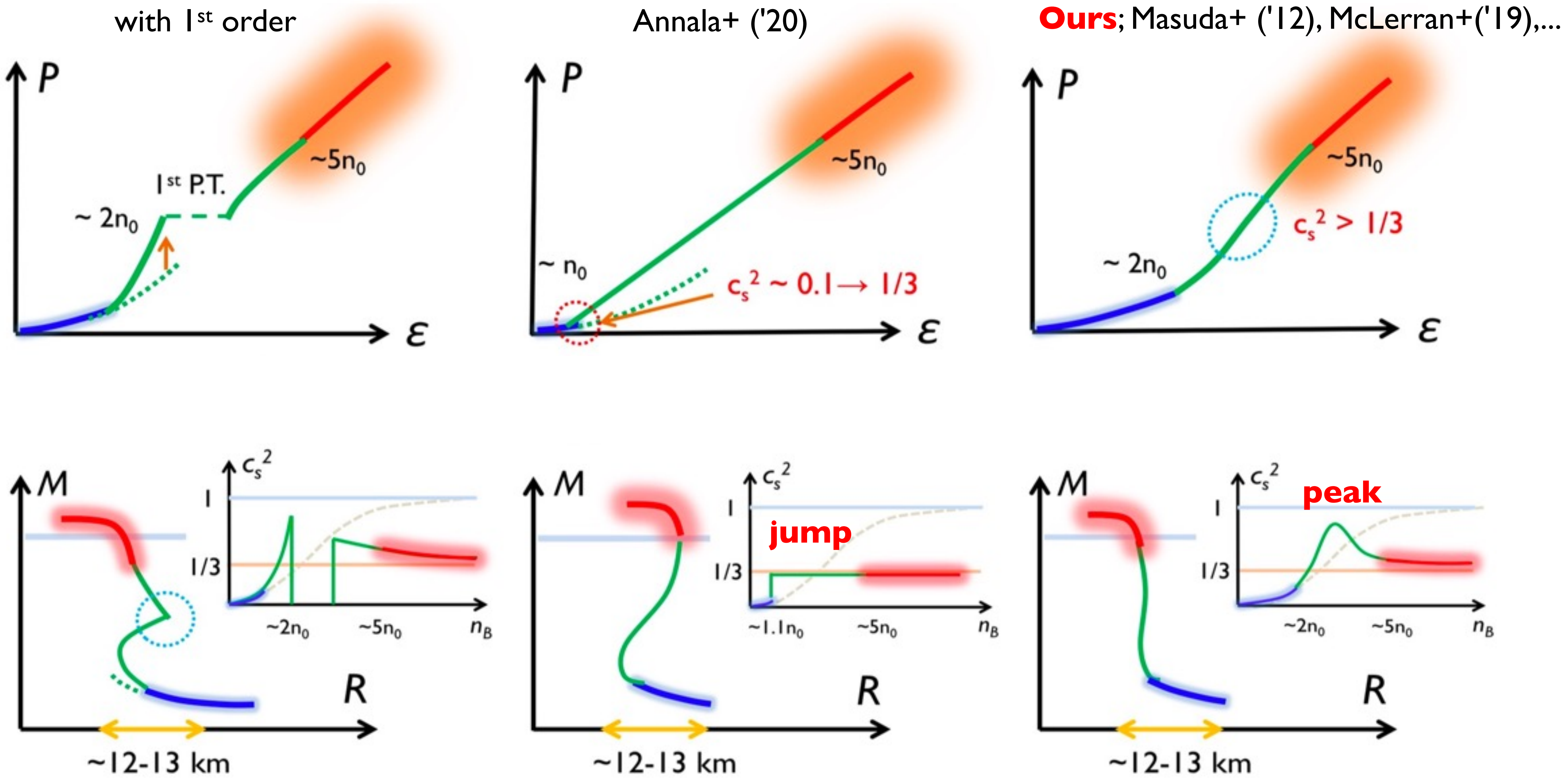
can be both soft and stiff



Q2) *normalization* ?

Switching bases from *baryons* to *quarks* → **blurs** the picture

Comparisons with other scenarios



Interactions for stiff EoS : *a guide*

cf) [TK-Powell-Song-Baym, '14]

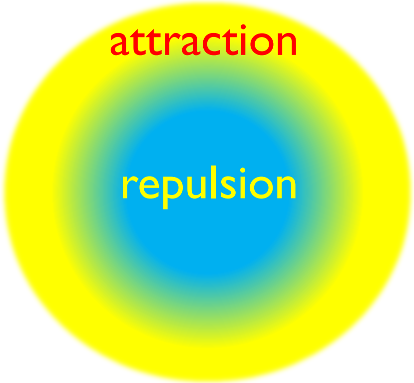
kin. energy interactions

$$\varepsilon(n) = \underline{a}n^{4/3} + \underline{b}n^{\alpha} \quad \xrightarrow{\mu = \frac{\partial \varepsilon}{\partial n}} \quad \mu = \frac{4}{3}\underline{a}n^{1/3} + \alpha \underline{b}n^{\alpha-1}$$

conformal interactions

$$\xrightarrow{P = \mu n - \varepsilon} \quad P = \frac{\varepsilon}{3} + \underline{b}\left(\alpha - \frac{4}{3}\right)n^{\alpha}$$

ideal combination



both the *sign* & *density dep.* are important

For α > **4/3**: b > 0 (repulsion) $\rightarrow$ stiff EoS (e.g. bulk repulsion, ~ + $n_B^{\mathbf{2}}/\Lambda^2$)

For α < **4/3**: b < 0 (attraction) $\rightarrow$ stiff EoS (e.g. surface pairings, ~ $-\Lambda^2 n_B^{\mathbf{2/3}}$)

Quantum numbers ?

quark quantum numbers; N_c, N_f, 2-spins (for a given spatial w.f.)

how many baryon species are needed to saturate quark states?

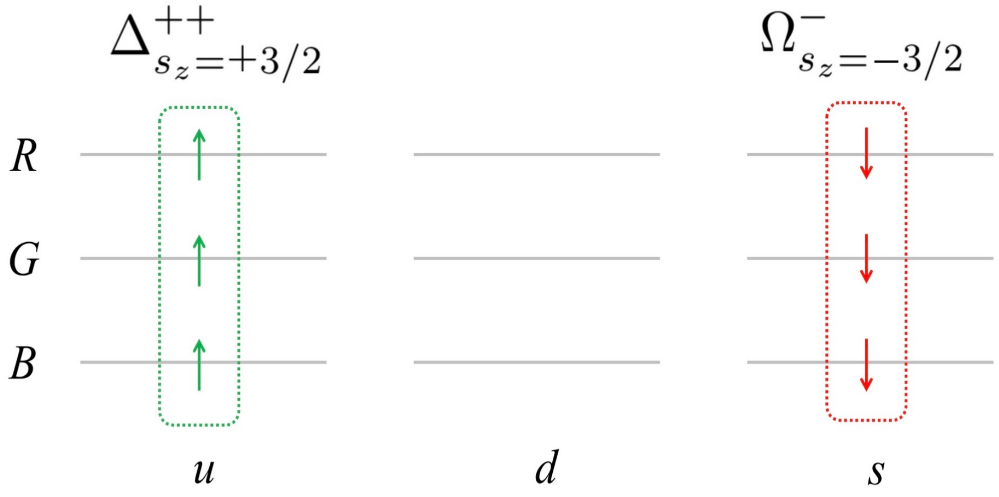
$\rightarrow$ we need only **$2N_f$ = 6** species for $N_f = 3$

(full members of singlet, octet, decuplet are NOT necessary)

convenient **color-flavor-spin** bases

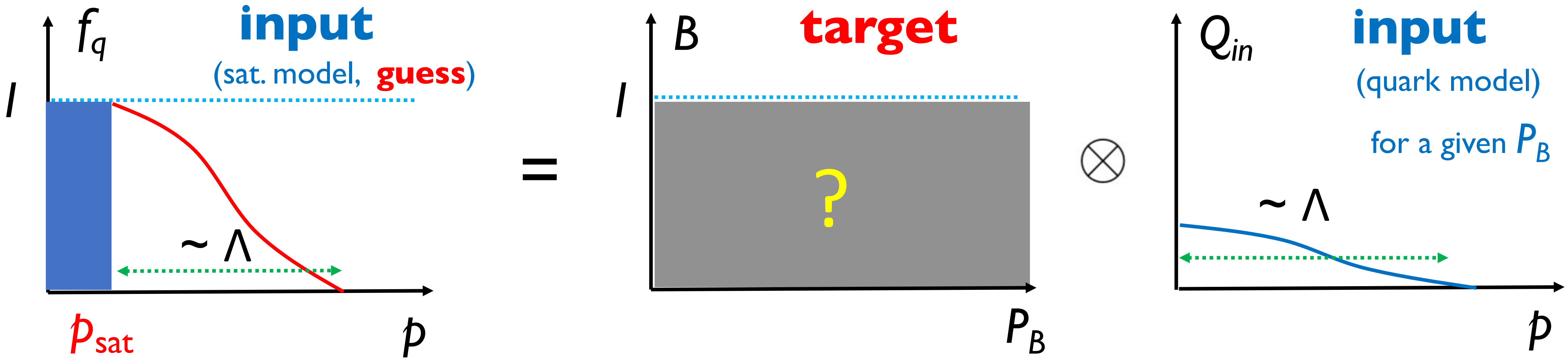
[neglect N-Δ splitting etc. for simplicity]

$$\Delta^{++}_{s_z=\pm 3/2} = [u_R\!\uparrow u_G\!\uparrow u_B\!\uparrow],\quad [u_R\!\downarrow u_G\!\downarrow u_B\!\downarrow],$$
$$\Delta^{-}_{s_z=\pm 3/2} = [d_R\!\uparrow d_G\!\uparrow d_B\!\uparrow],\quad [d_R\!\downarrow d_G\!\downarrow d_B\!\downarrow],$$
$$\Omega^{-}_{s_z=\pm 3/2} = [s_R\!\uparrow s_G\!\uparrow s_B\!\uparrow],\quad [s_R\!\downarrow s_G\!\downarrow s_B\!\downarrow],$$



Inversion problem: from f_q to B

$$f_q(p; n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(P_B; n_B) Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B)$$



How does *baryon occ. probability* look *after* the saturation ?

Inversion problem: motivations to study B

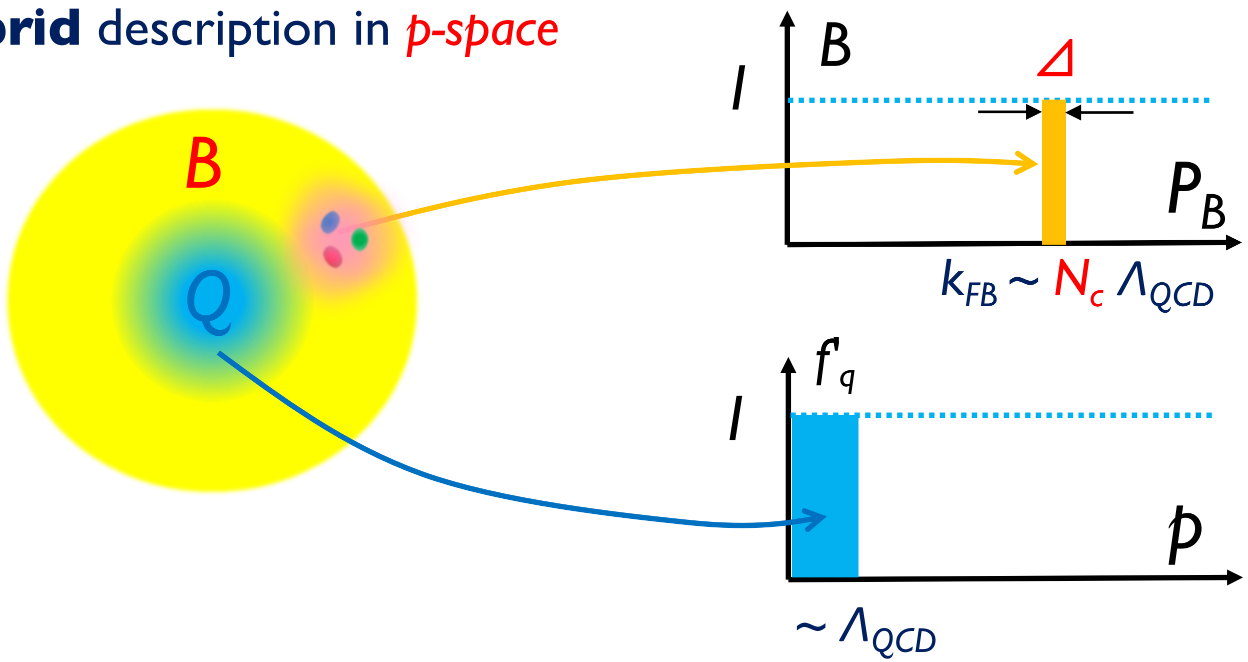
- perhaps convenient to *use the baryonic bases* for *low E* physics

$P(\mu_B)|_{\beta\text{-eq}} \Rightarrow P(\mu_B, \mu_Q, T, \ldots)$

extensions of the *quark-hadron continuity*

- relations to the *McLerran-Reddy* (*MR*) model

hybrid description in *p-space*



important parameter

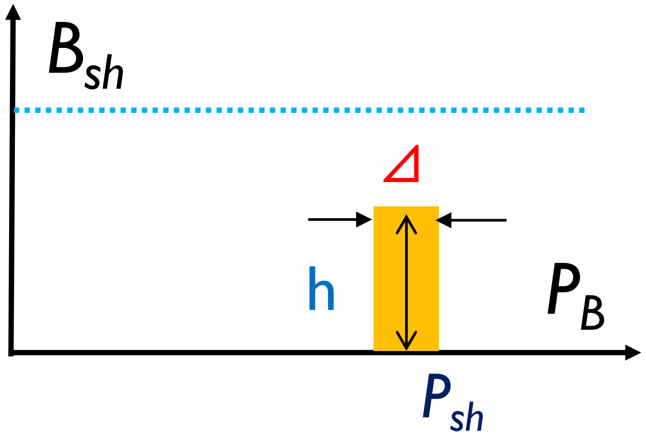
$$\Delta = \frac{\Lambda^3}{k_{\rm FB}^2} + \kappa \frac{\Lambda}{N_c^2}$$

why this form?

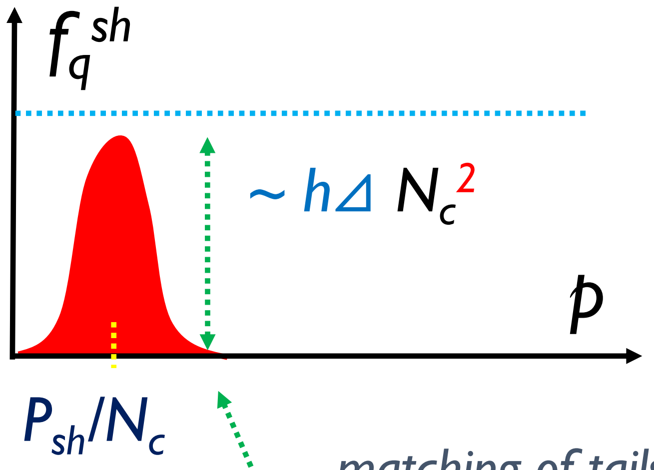
- phenomenological [McLerran-Reddy, PRL '19]
- derivation in excluded vol. model [Jeong-McLerran-Sen, '19]

A trial: *shell form*

$$\mathcal{B}^{\rm sh}(P_B;P_{\rm sh}) = h\theta(P_{\rm sh}-P_B)\theta(P_B-P_{\rm sh}-\Delta)$$

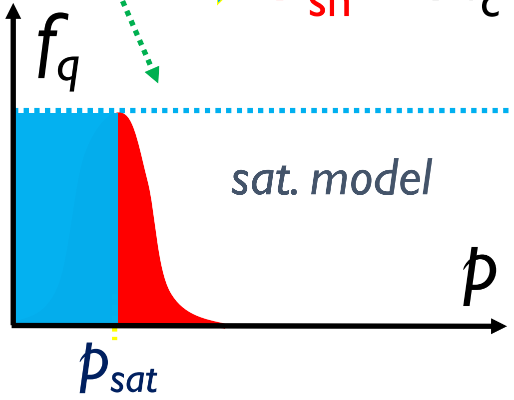


Q_{in}



matching of tails

$P_{sh} \sim N_c\, p_{sat}$



$$f_q^{\rm sh}(p) \simeq h\Delta \frac{N_c^3}{\sqrt{\pi}}\frac{\tilde{P}_{\rm sh}}{\tilde{p}}\, e^{-\tilde{p}^2-\tilde{P}_{\rm sh}^2}\left(e^{2\tilde{p}\tilde{P}_{\rm sh}} - e^{-2\tilde{p}\tilde{P}_{\rm sh}}\right)$$

$$f_q^{\rm sh}(p) \sim h\Delta N_c^2\, e^{-(\tilde{p}-\tilde{P}_{\rm sh})^2}$$

$P_{sh} \sim N_c \Lambda$

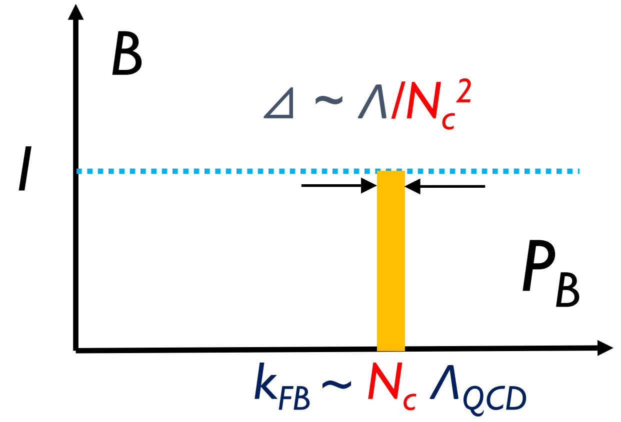
Constraints from f_q (*for* $P_{sh} \sim N_c \Lambda$)

$$f_q^{\rm sh}(p) \sim h\Delta N_c^2\, e^{-(\tilde p - \tilde P_{\rm sh})^2}$$

constraint : $f_q^{sh} < 1$ ➡ $h\Delta < \Lambda/N_c^2$

a possible scaling form:

$$[h\Delta](P_{\rm sh}) \sim c_0 \Lambda \left(\frac{\Lambda^2}{P_{\rm sh}^2} + \frac{c_1}{N_{\rm c}} \frac{\Lambda}{P_{\rm sh}} + \frac{c_2}{N_{\rm c}^2} \right)$$



MR-model (thin shell model)

$h = 1$ & $\Delta = \dfrac{\Lambda^3}{k_{\rm FB}^2} + \kappa \dfrac{\Lambda}{N_c^2}$ ($c_1 = 0$)

MR-model: *EoS*

$P_{sh} \sim N_c \Lambda$ baryon relativistic

but $$n_B^{(\text{shell})} \simeq \frac{h}{\pi^2}\left(P_{\rm sh}^3 - (P_{\rm sh}-\Delta)^3\right) \sim h\Delta P_{\rm sh}^2$$ $n_B^{(bulk)} \sim \Lambda^3$

$$\simeq c_0\Lambda^3 + c_1\Lambda^2\frac{P_{\rm sh}}{N_{\rm c}} + c_2\Lambda\left(\frac{P_{\rm sh}}{N_{\rm c}}\right)^2$$

$n_B \sim \Lambda^3$ (!) << $(N_c\Lambda)^3$

(kin.) energy density:

$$\varepsilon - m_B n_B \sim h\Delta \times [\, E(P_{sh}) - m_B \,] \times 4\pi P_{sh}^2$$

consistent with quark's

$$\sim \Lambda/N_c^2 \times (N_c\Lambda)^2/m_B \times (N_c\Lambda)^2 \sim N_c\Lambda^4$$

relativistic pressure ~ $N_c\Lambda^4$ within $n_B \sim \Lambda^3$ → *stiff* EoS

Inversion problem: from f_q to B

vector — vector — matrix

$$f_q(p; n_B) = \int_{\boldsymbol{P}_B} \underline{\mathcal{B}(P_B; n_B)} Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B)$$

formally: $\vec{f}_q = Q_{\rm in}\vec{\mathcal{B}}$ for a given n_B

[**constraint** : $0 < B < 1$]

a *practical* problem : e.g., an optimization for $\{\alpha\}$ parameterized form $B[\alpha]$:

$$\mathcal{H}(\vec{\alpha}) = (\vec{f}_q - Q_{\rm in}\vec{\mathcal{B}}_\alpha)^T(\vec{f}_q - Q_{\rm in}\vec{\mathcal{B}}_\alpha) + \underline{\mathcal{I}_{\rm cost}(\mathcal{B}_\alpha)}$$

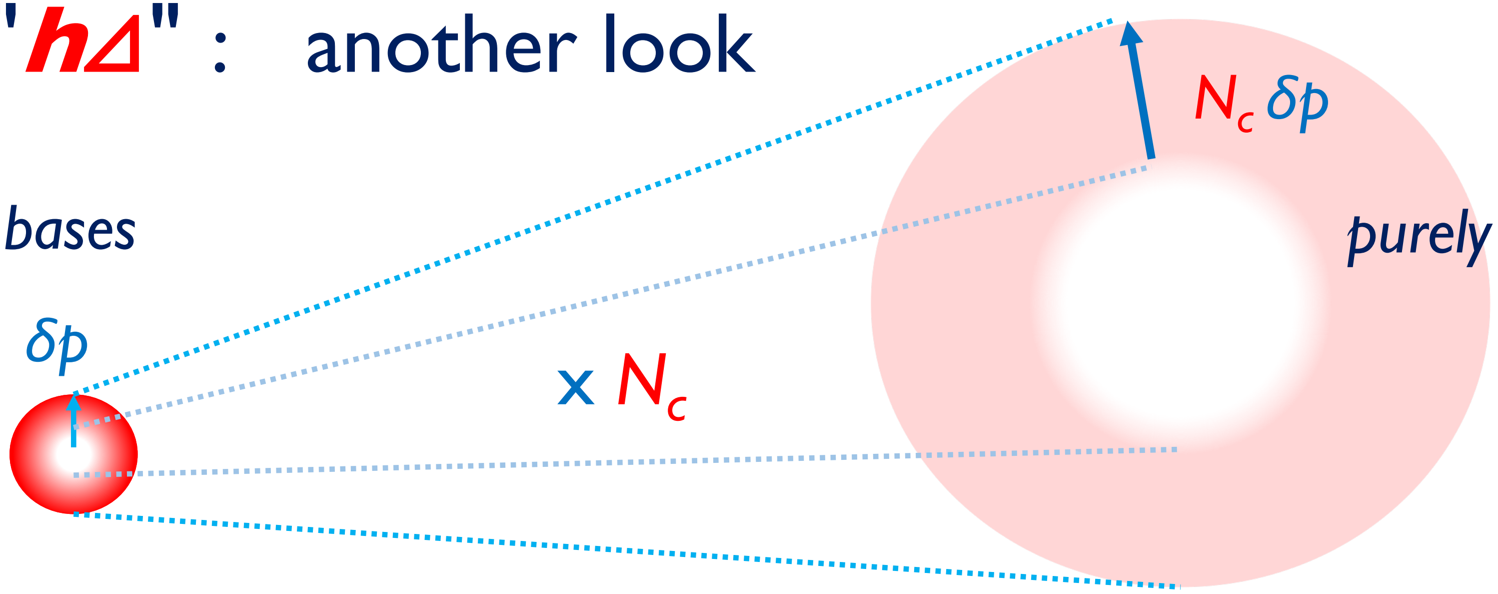
some penalty when violating the constraint.

what is the reasonable parameterization for B ?

More on "$h\Delta$" : another look

purely quark bases *purely baryonic bases*

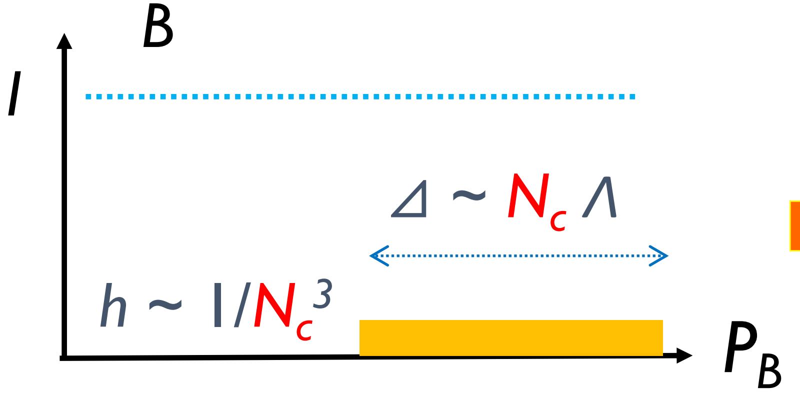
δn_B



$$\delta n_B \;\; (= \delta n_{qR} = \delta n_{qG} = \delta n_{qB} \;)$$

(need for consistency)

$$p^2\,\delta p \;\Longleftrightarrow\; \underbrace{(N_c p)^2 \times N_c\,\delta p}_{\text{naive}} \;\; \underbrace{\times\, 1/N_c^{\,3}}_{\text{occ. probability}}$$



$\Rightarrow$ $$h\Delta \sim \Lambda/N_c^{\,2}$$

phase space density is **more dilute** for **composite** particles

Introduction (7 slides): 6.5 min

1, title	[0.5 min]
2, 3-winodw	[1.5 min]
3, M-R relations	[1min]
4, NS; NICER	[1min]
5, Soft-to-Stiff	[1.5min]
6, Goals	[1.5 min]

main (10 slides): 14 min

[6min]

8, strategy	[1 min]
9, quarks in a baryon	[2 min]
10, fq = B Qin	[1.5 min]
11, ideal baryon gas	[1.5 min]

[7min]

12, saturation of quarks	[1 min]
13, quark matter formation	[1 min]
14, jumps	[1 min]
15, P = d(e/n)/dn ...	[1.5min]
16, more realistic	[1 min]
17, smear	[1.5 min]

18, int
19, EoS with int

20, summary