Toward QCD-based construction of the equation of state

藤本 悠輝

(東京大学)

Reference:

Y. Fujimoto, K. Fukushima, "Equation of state of cold and dense QCD matter in resummed perturbation theory," arXiv:2011.10891.

12 August 2021, 4th neutron star workshop

Content of the talk

1. Refinement of the perturbative QCD calculation
2. Observational constraint and quark cores of neutron stars

Content of the talk

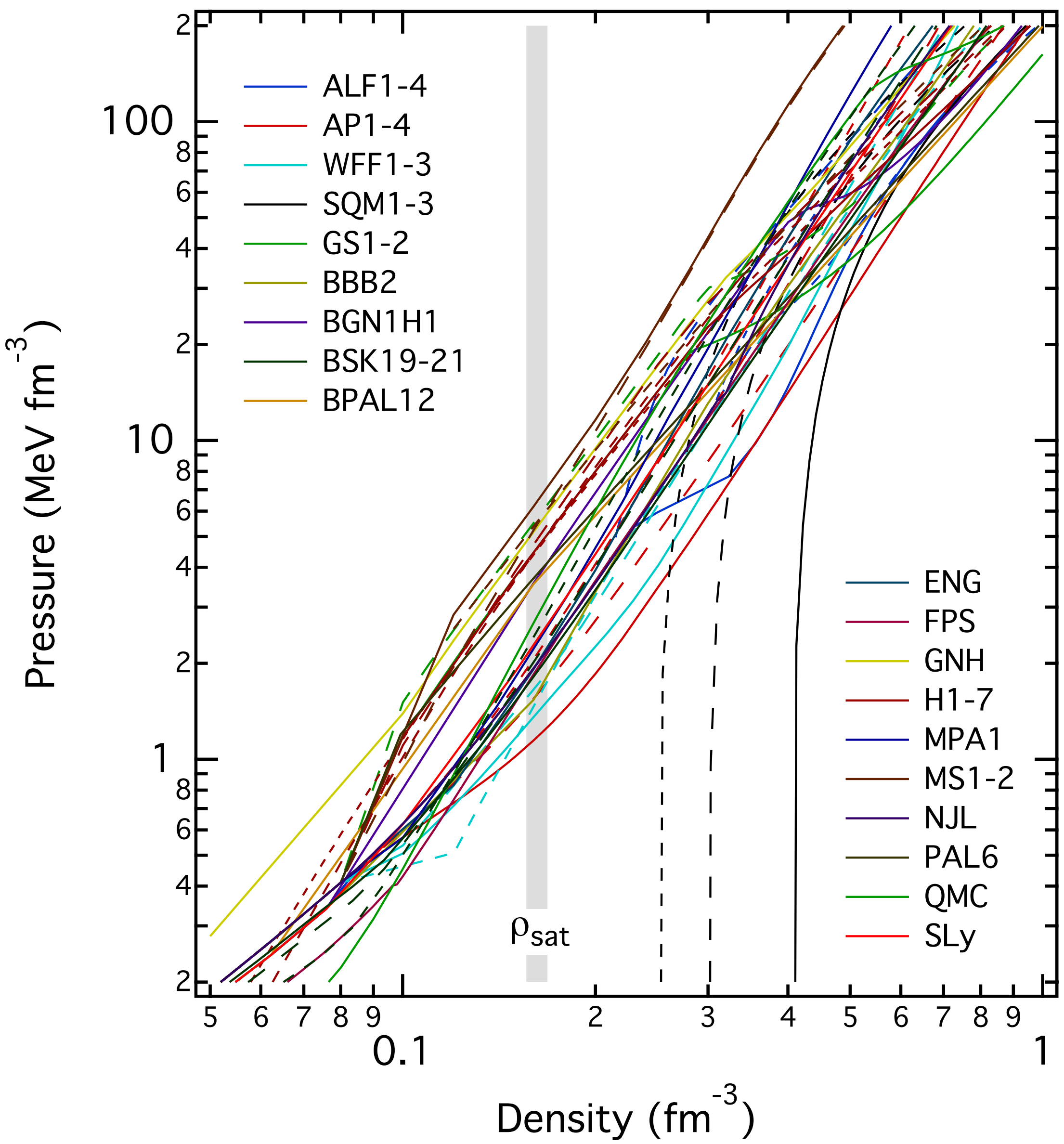
1. **Refinement of the perturbative QCD calculation**

2. Observational constraint and quark cores of neutron stars

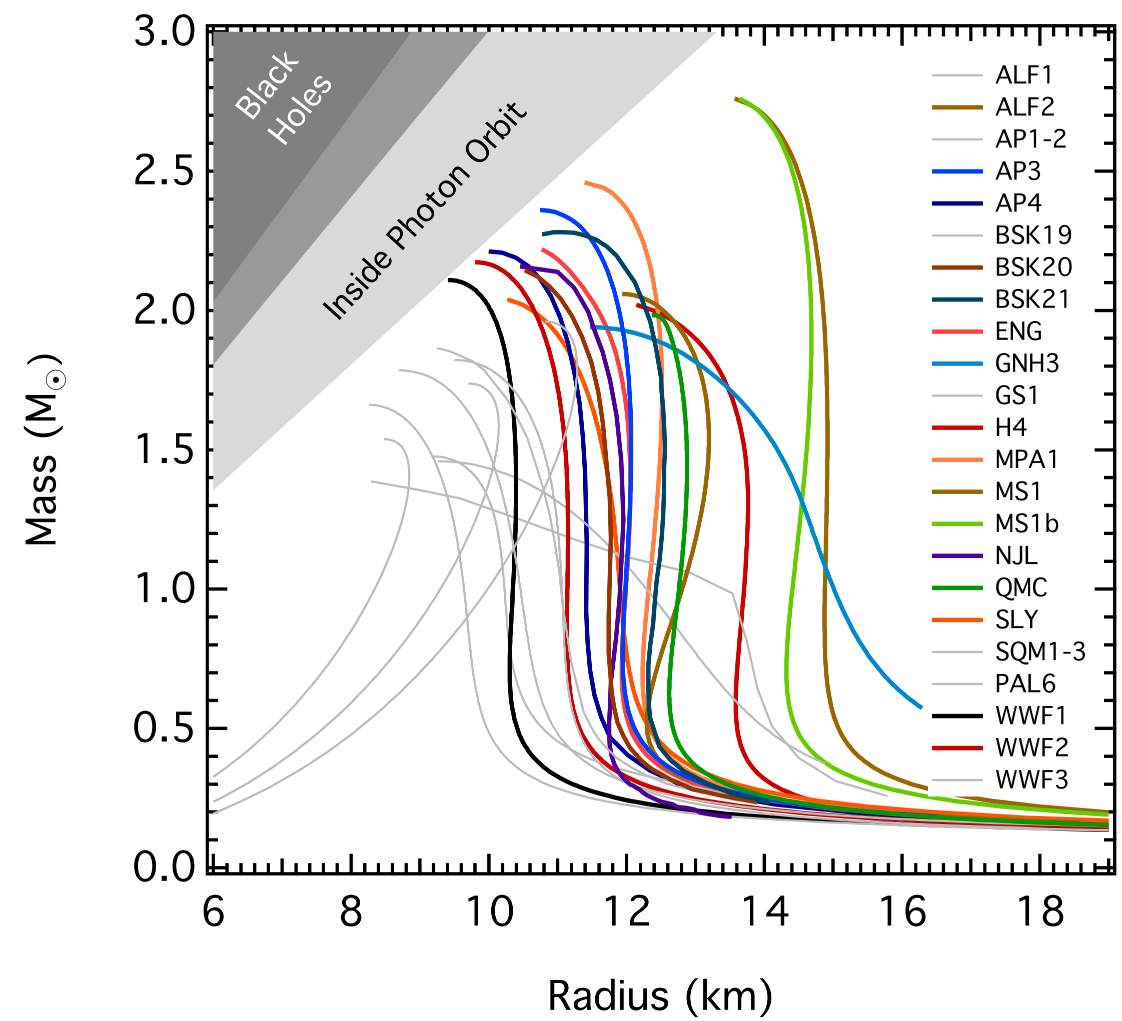
Equation of state (EoS)

EoS: pressure function $p(n_\mathrm{B})$, $p(\varepsilon)$, or $p(\mu_\mathrm{B})$

(n_B: baryon density, ε: energy density, μ_B: chemical potential)



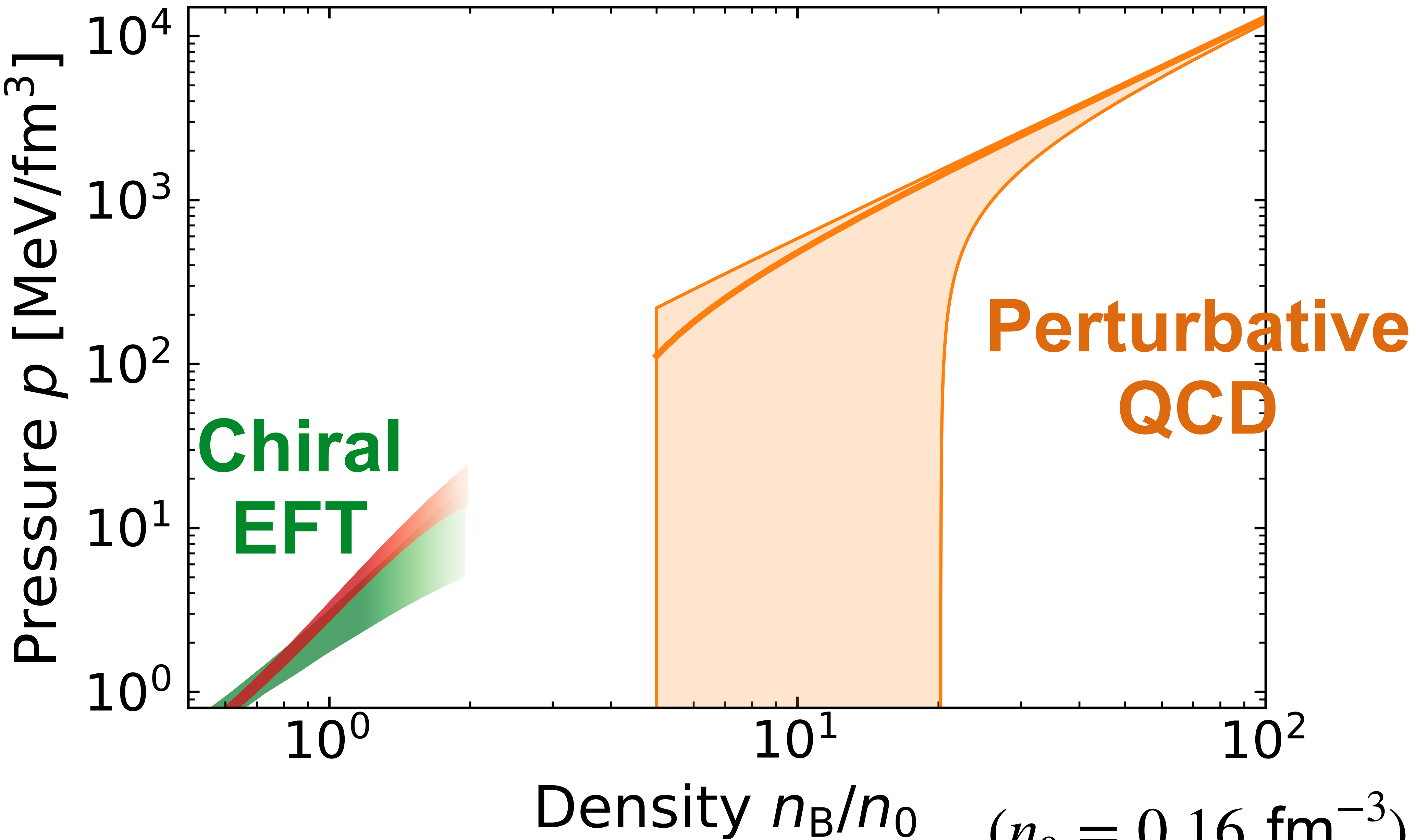
Özel,Freire (2016)



There are many different calculations…

A QCD point of view on the NS EoS

Ab initio and model-independent constraints from QCD point of view:



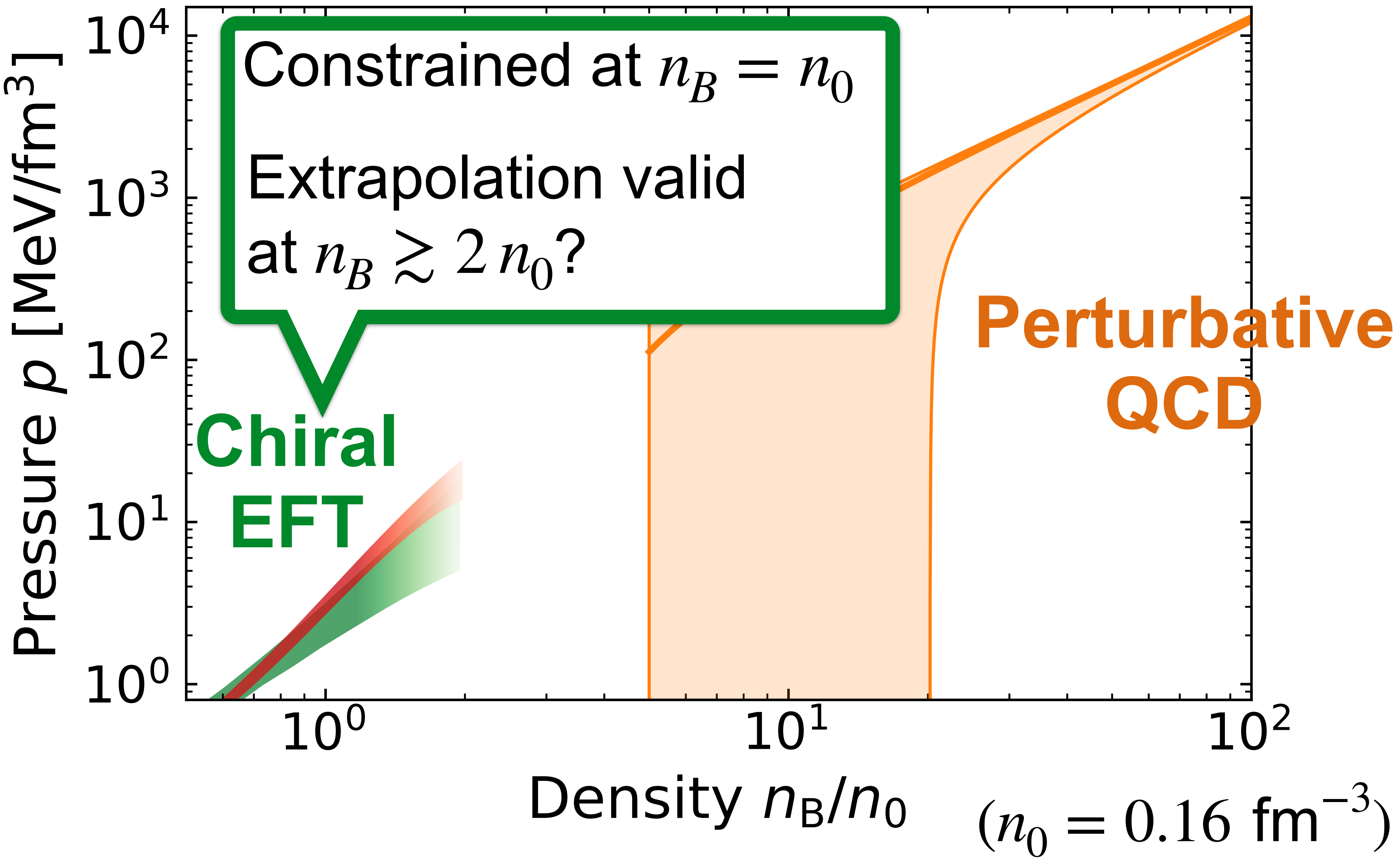
$(n_0 = 0.16\ \mathrm{fm}^{-3})$

ChEFT: Tews,Carlson,Gandolfi,Reddy (2018); Drischler,Furnstahl,Melendez,Phillips (2020)

pQCD: Freedman,McLerran (1978); Baluni (1979); Kurkela,Romatschke,Vuorinen,Gorda,Sappi (2009-)

A QCD point of view on the NS EoS

Ab initio and model-independent constraints from QCD point of view:



ChEFT: Tews,Carlson,Gandolfi,Reddy (2018); Drischler,Furnstahl,Melendez,Phillips (2020)

pQCD: Freedman,McLerran (1978); Baluni (1979); Kurkela,Romatschke,Vuorinen,Gorda,Sappi (2009-)

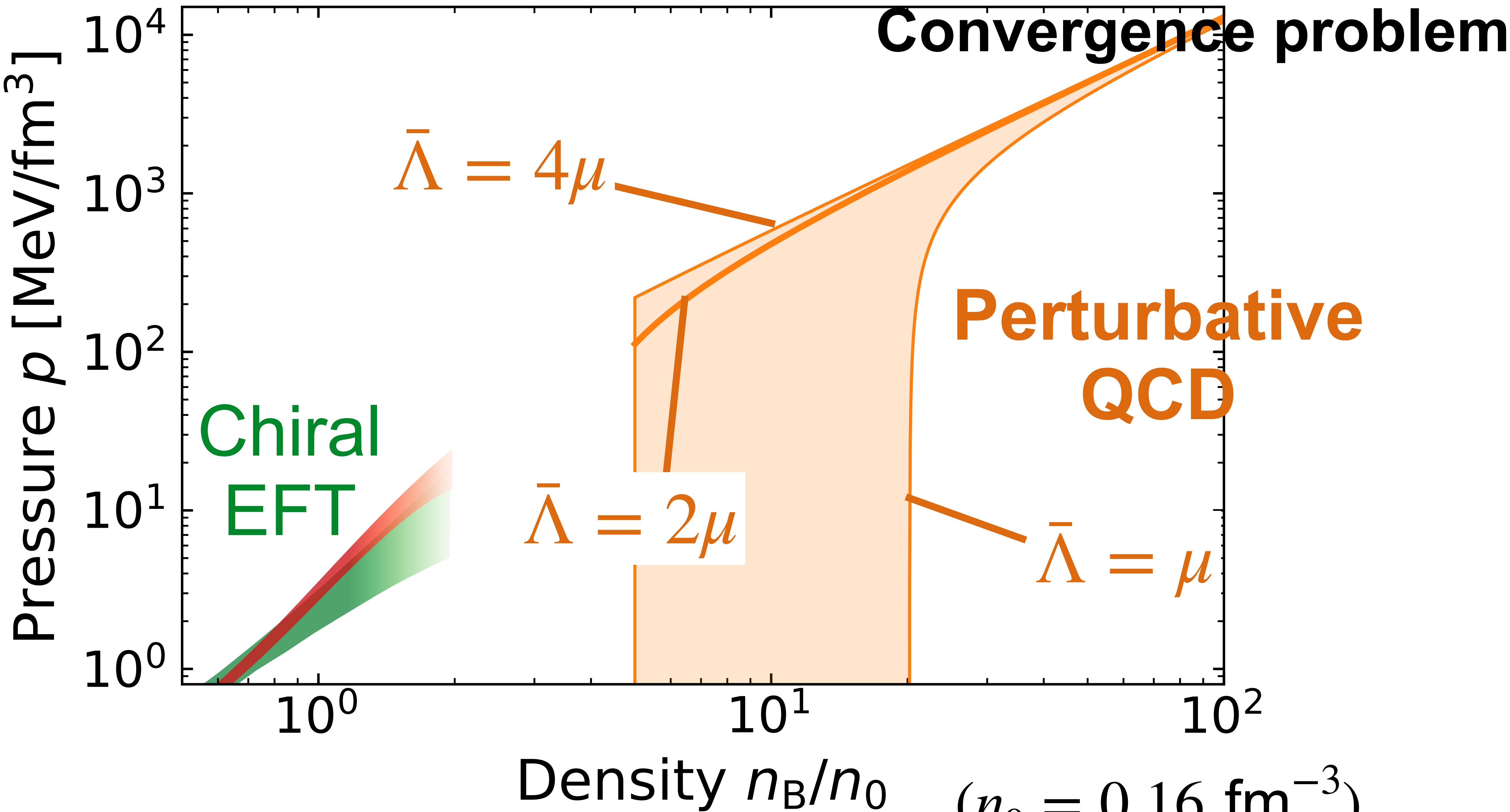
Problem of the pQCD calculation

$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 \log(\bar{\Lambda}^2/\Lambda^2_{\overline{\rm MS}})}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\log^2(\bar{\Lambda}^2/\Lambda^2_{\overline{\rm MS}})}{\log(\bar{\Lambda}^2/\Lambda^2_{\overline{\rm MS}})}\right]$$

$\bar{\Lambda}$: undetermined const.

uncertainty: originate from scale variation

$\bar{\Lambda} = \mu,\ 2\mu,\ 4\mu$



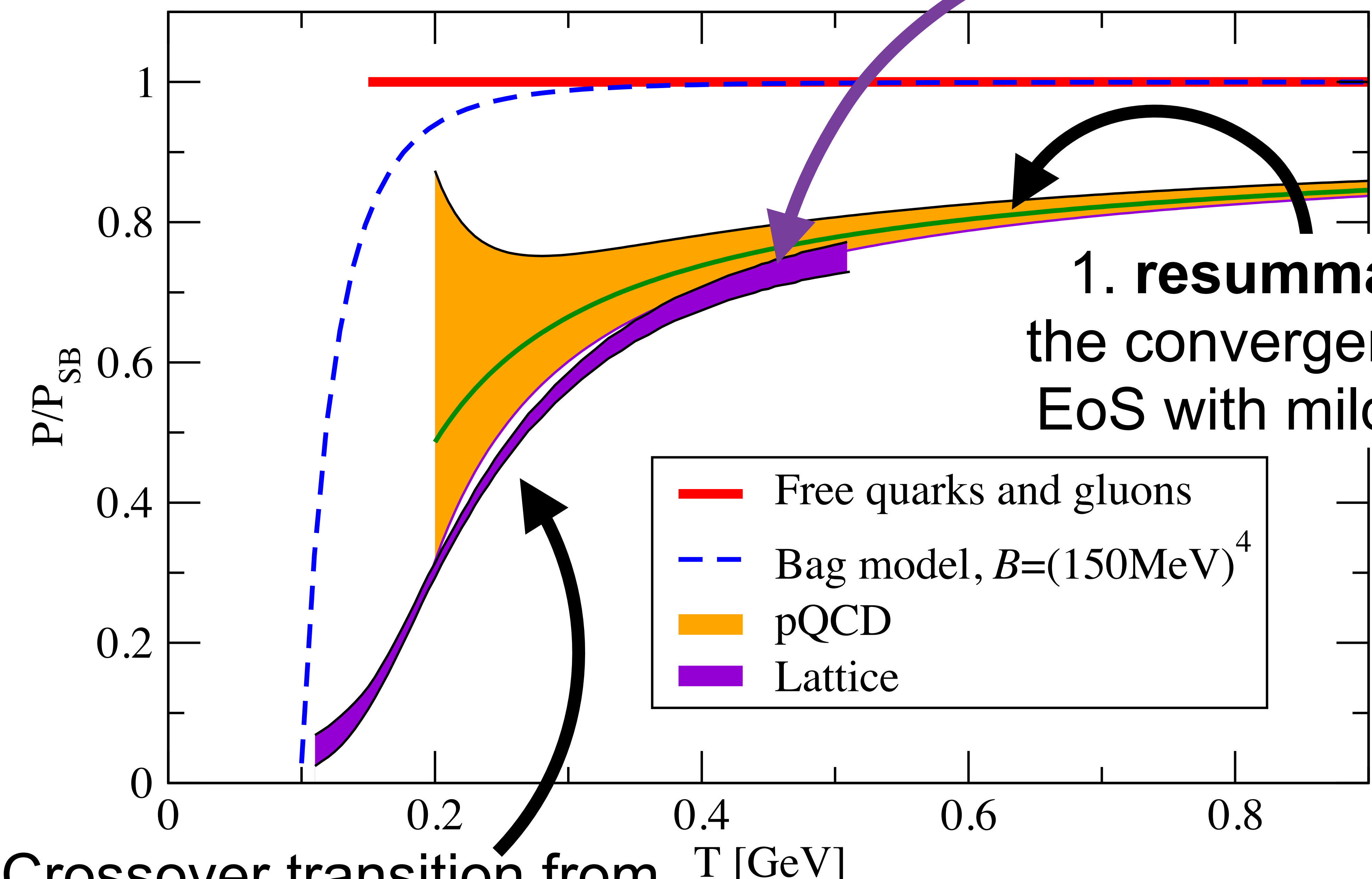
$(n_0 = 0.16\ \mathrm{fm}^{-3})$

ChEFT: Tews,Carlson,Gandolfi,Reddy (2018); Drischler,Furnstahl,Melendez,Phillips (2020)

pQCD: Freedman,McLerran (1978); Baluni (1979); Kurkela,Romatschke,Vuorinen,Gorda,Sappi (2009-)

Lessons from high temperature QCD

Nice thing about hot QCD: there is **Lattice QCD** calculation



1. **resummation** cures the convergence problem; EoS with mild uncertainty

2. Crossover transition from hadron to quark. **NO** 1st order phase transition

Figure taken from Fraga,Kurkela,Vuorinen (2013)

Resummation saves the pQCD

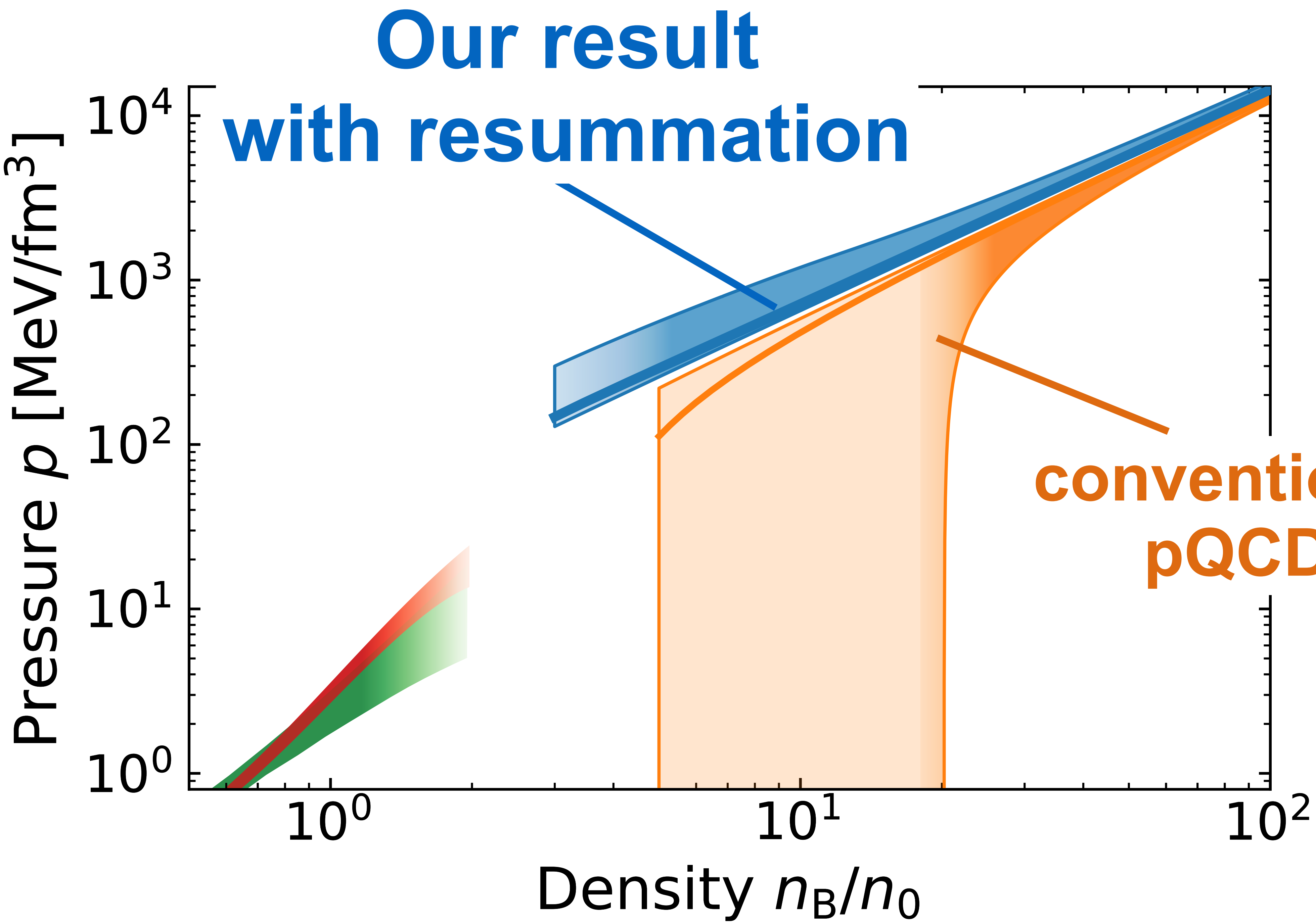
Fujimoto, Fukushima: 2011.10891 (2020)

$$\alpha_s(\bar\Lambda) = \frac{4\pi}{\beta_0 \log(\bar\Lambda^2/\Lambda^2_{\overline{\rm MS}})}\left[1-\frac{2\beta_1}{\beta_0^2}\frac{\log^2(\bar\Lambda^2/\Lambda^2_{\overline{\rm MS}})}{\log(\bar\Lambda^2/\Lambda^2_{\overline{\rm MS}})}\right]$$

$\bar\Lambda$: undetermined const.

uncertainty: originate from scale variation

$$\bar\Lambda = \mu,\ 2\mu,\ 4\mu$$



What we calculate here

Quark contribution to the pressure p:

$$p(\mu) = \mathrm{Tr}\log \underline{S^{-1}}$$

HDL resummed full propagator

$$= \sum\!\!\!\!\!\!\!\!\int_{\{K\}} \ln\det\left[\not{k} - M_f - \underline{\Sigma(i\tilde{\omega}_n + \mu_f, k)}\right]$$

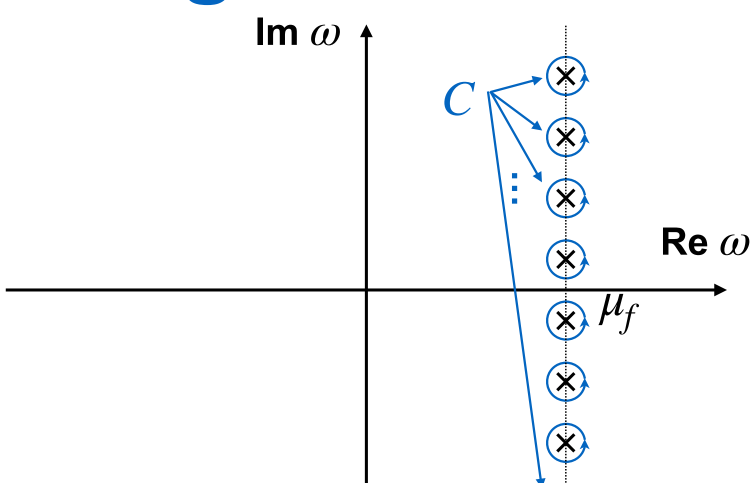
Self-energy of quark matter
→ leads to quasi-particles
& screening

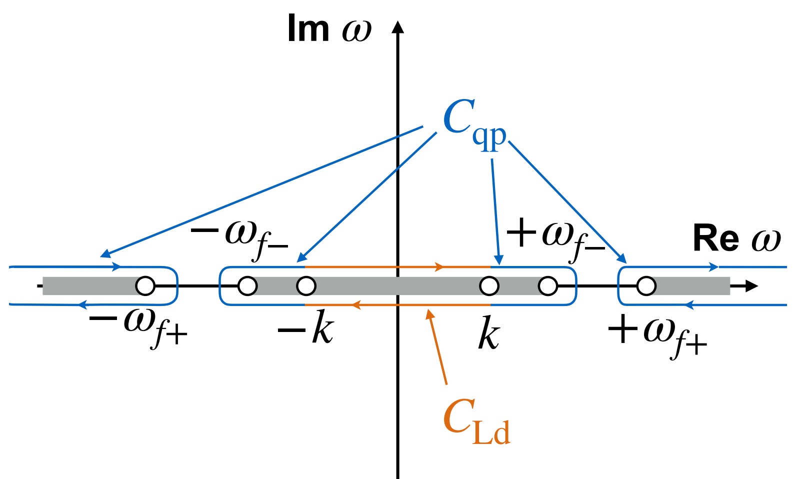
Self-energy Σ in HDL approximation:

$$\Sigma \equiv \frac{m_q^2}{k}\gamma^0 Q_0\left(\frac{k_0}{k}\right) + \frac{m_q^2}{k}\boldsymbol{\gamma}\cdot\hat{\boldsymbol{k}}\left[1 - \frac{k_0}{k}Q_0\left(\frac{k_0}{k}\right)\right]$$

(Legendre function: $Q_0(x) = \frac{1}{2}\log\frac{x+1}{x-1}$)

Integration contour deformation:





tedious calculation…

Result from the HDL resummed QCD

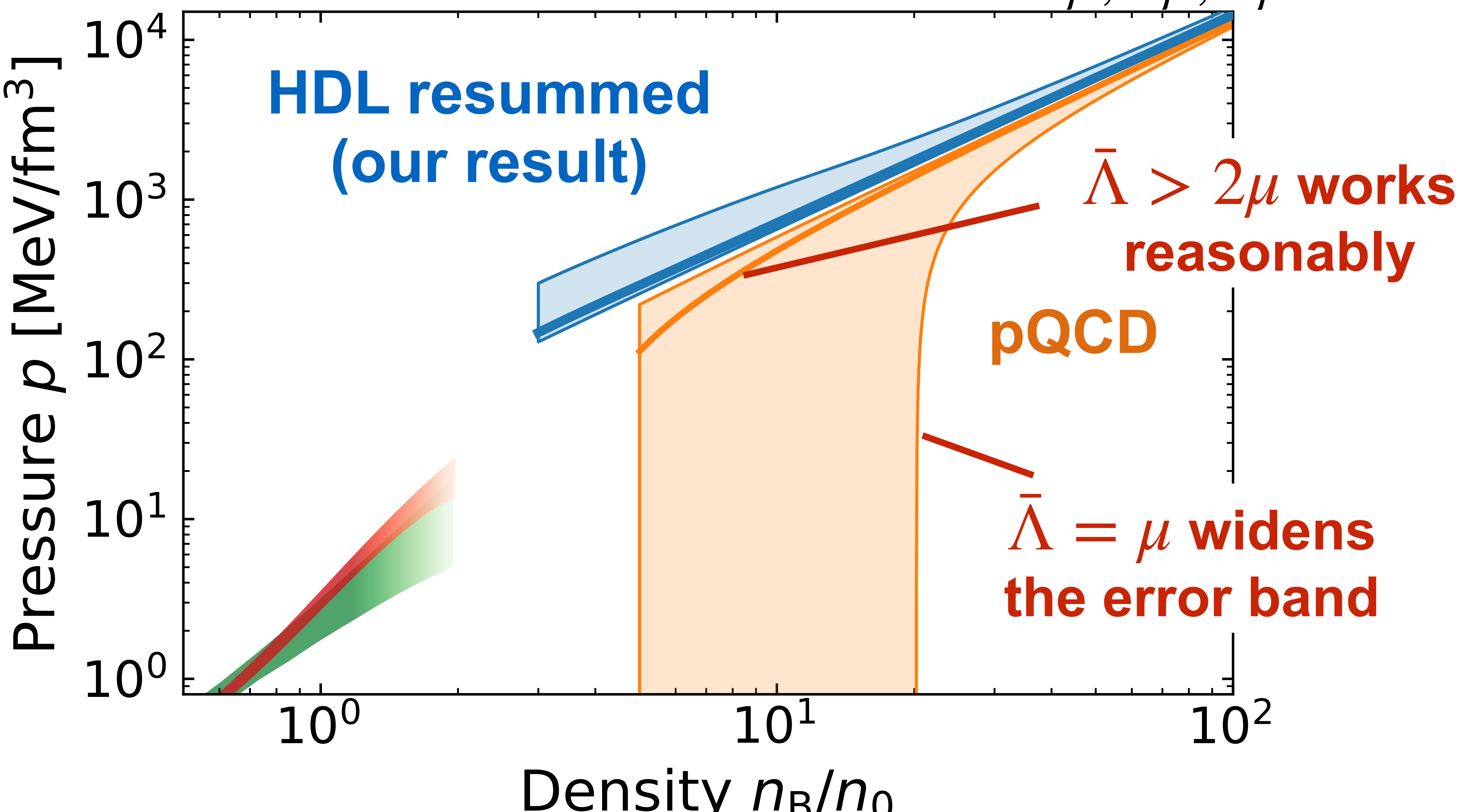
Fujimoto, Fukushima: 2011.10891 (2020)

$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 \log(\bar{\Lambda}^2/\Lambda_{\overline{\mathrm{MS}}}^2)}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\log^2(\bar{\Lambda}^2/\Lambda_{\overline{\mathrm{MS}}}^2)}{\log(\bar{\Lambda}^2/\Lambda_{\overline{\mathrm{MS}}}^2)}\right]$$

$\bar{\Lambda}$: undetermined const.

uncertainty: originate from scale variation

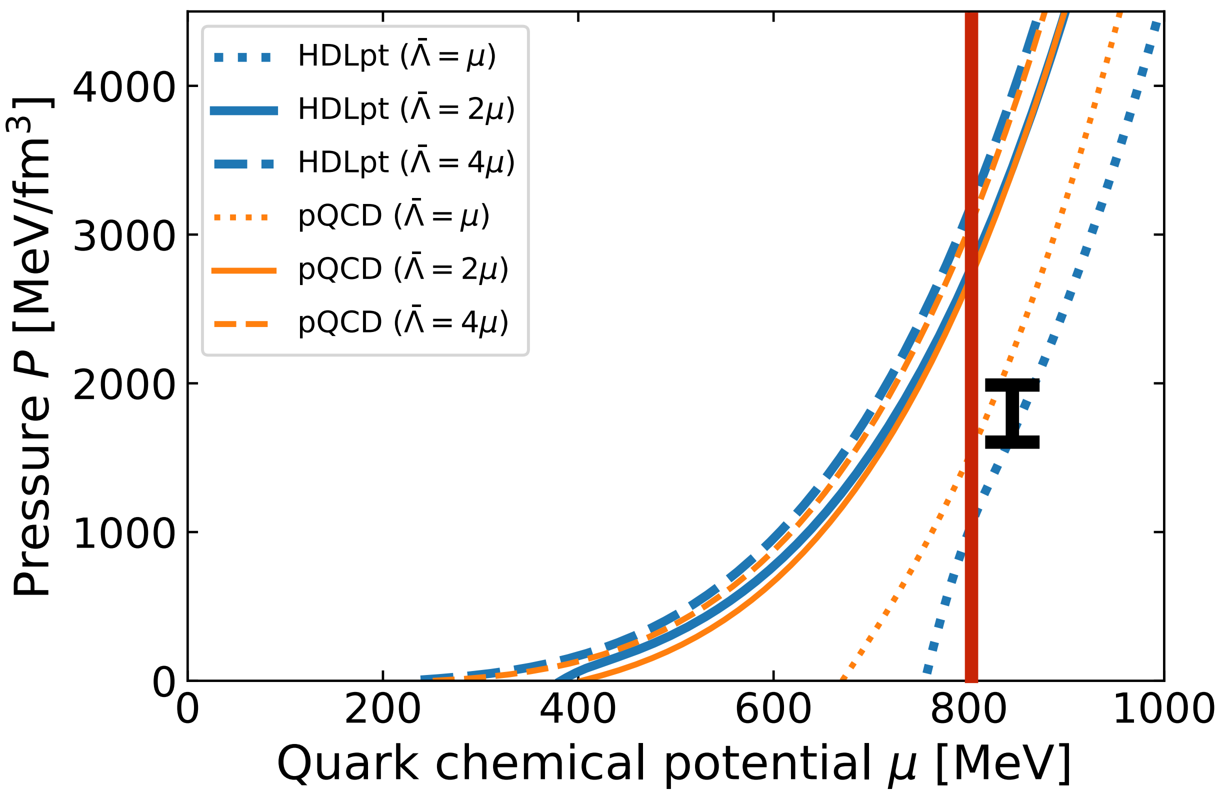
$\bar{\Lambda} = \mu,\ 2\mu,\ 4\mu$



Heuristic argument

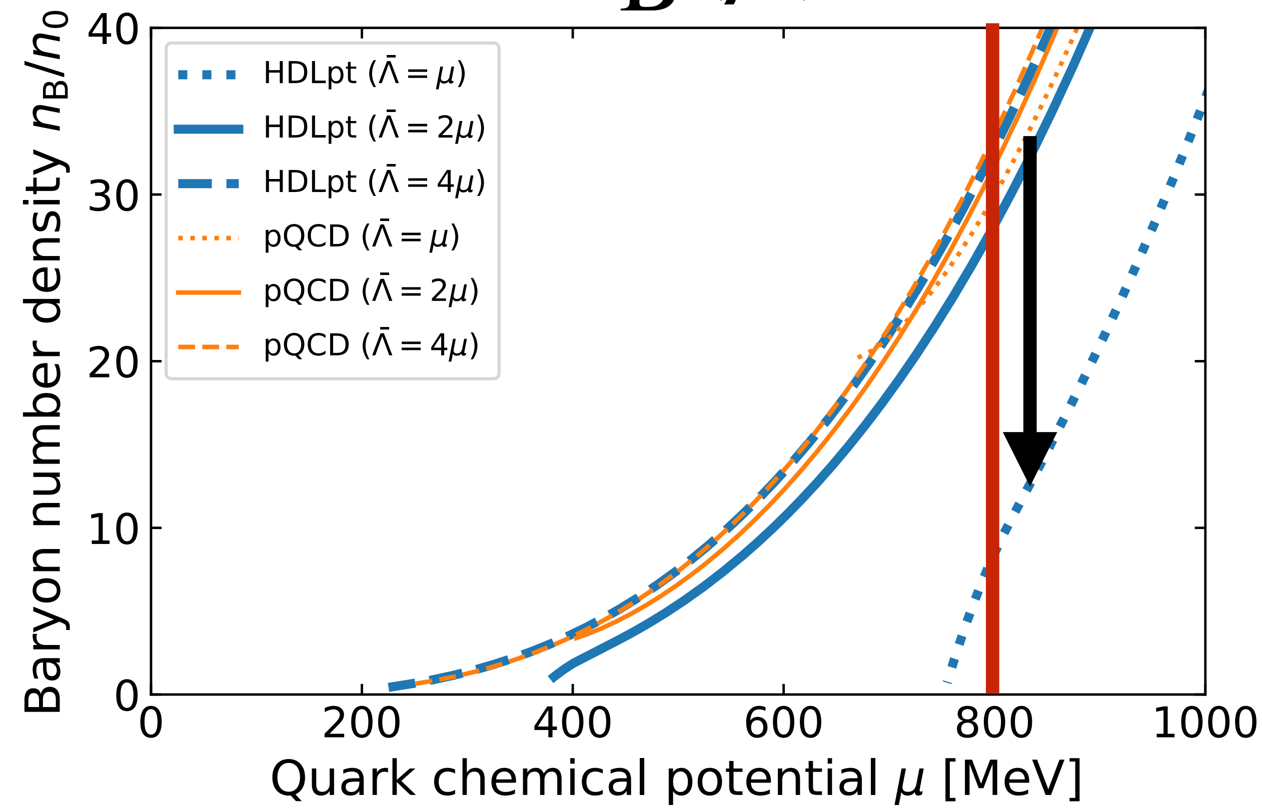
Fujimoto, Fukushima: 2011.10891 (2020)

$p(\mu)$



Pressure does not differ at constant μ

$n_{\rm B}(\mu)$



Density is screened in HDL resummation at constant μ

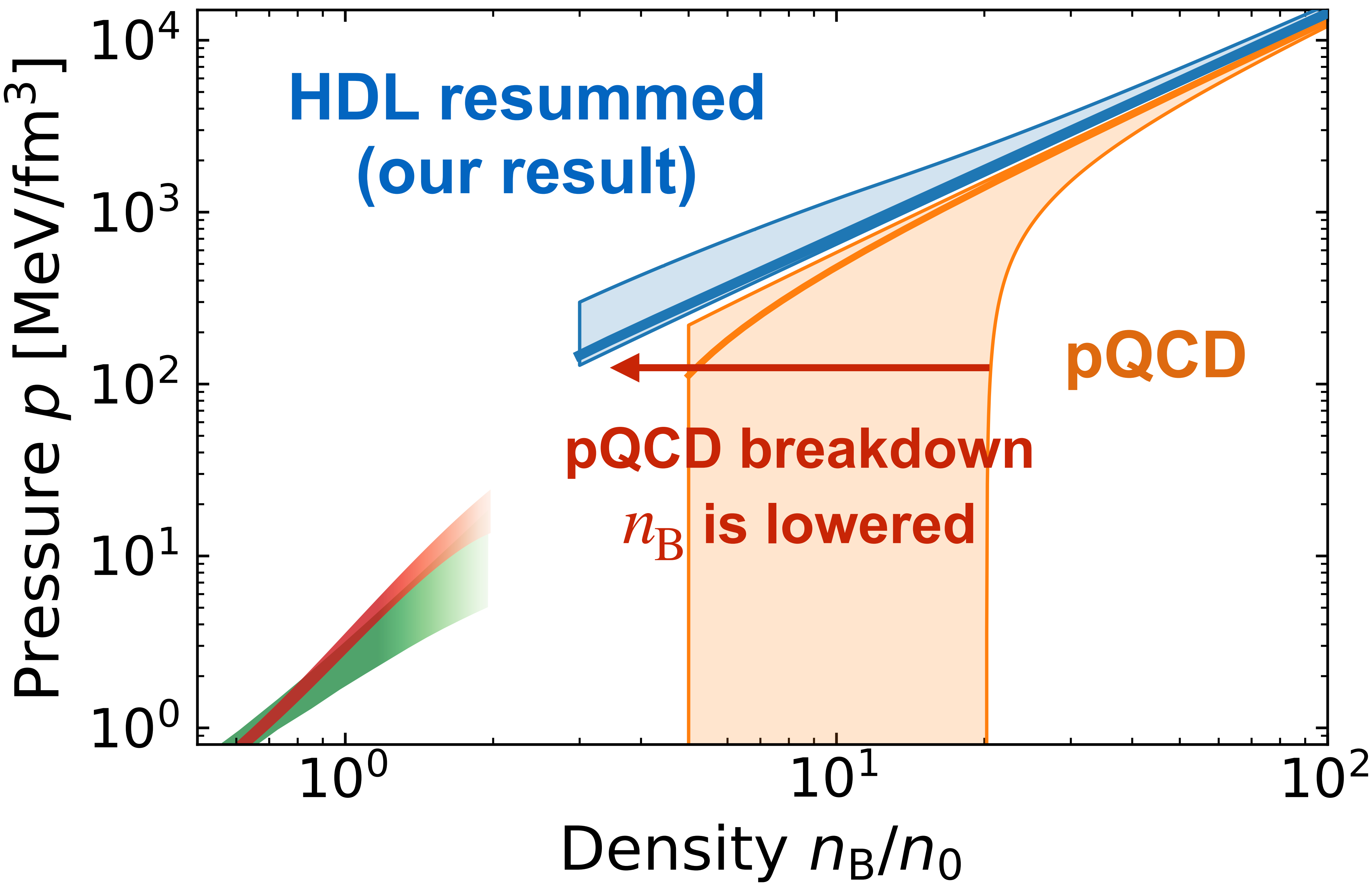
→ in HDL resummation,
the same value of p realizes at lower $n_{\rm B}$
especially for $\bar{\Lambda} = \mu$

Result from the HDL resummed QCD

Fujimoto, Fukushima: 2011.10891 (2020)

$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 \log(\bar{\Lambda}^2/\Lambda^2_{\overline{\mathrm{MS}}})}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\log^2(\bar{\Lambda}^2/\Lambda^2_{\overline{\mathrm{MS}}})}{\log(\bar{\Lambda}^2/\Lambda^2_{\overline{\mathrm{MS}}})}\right]$$

uncertainty: originate from scale variation $\bar{\Lambda} = \mu,\ 2\mu,\ 4\mu$



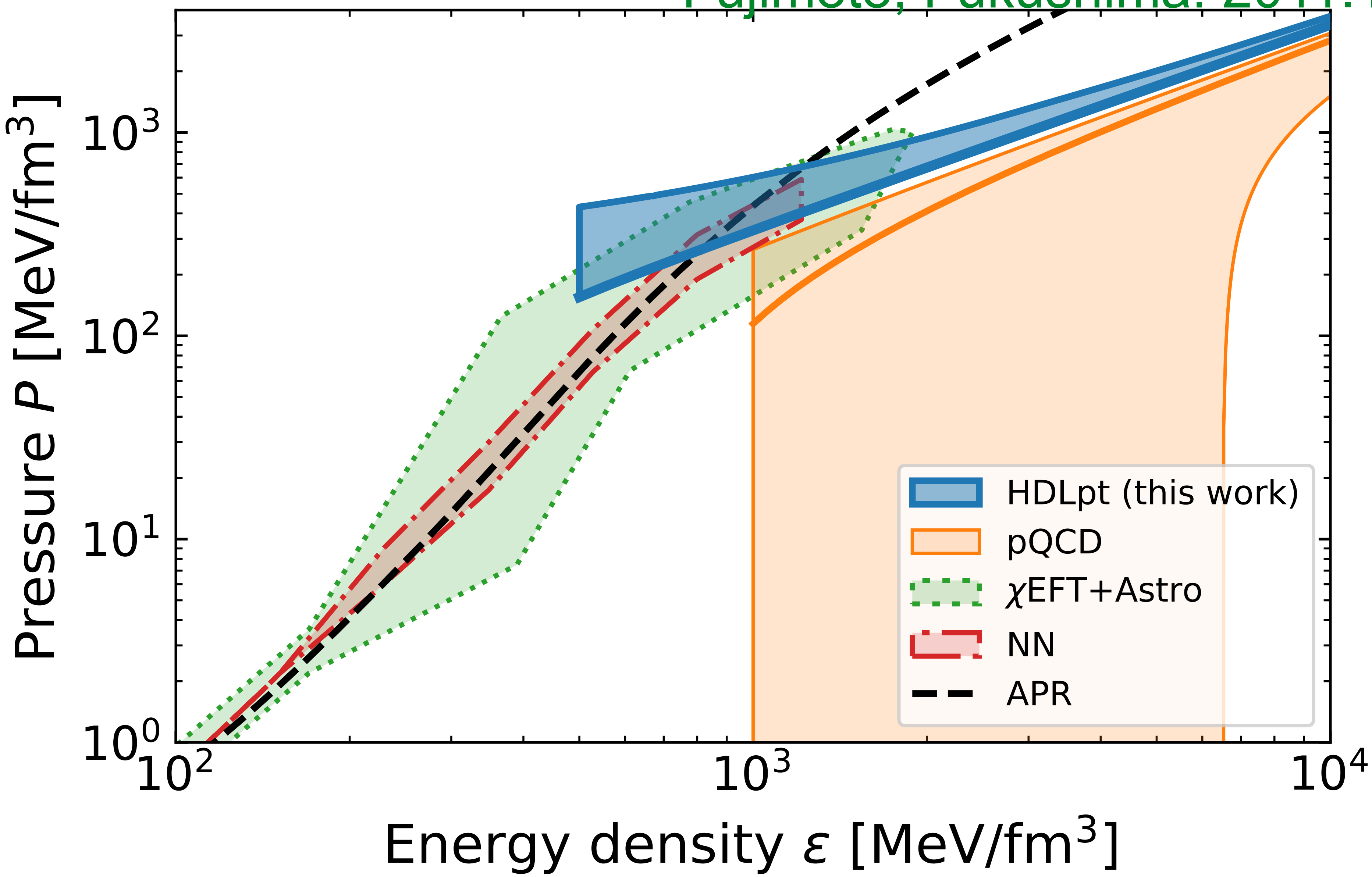
Content of the talk

1. Refinement of the perturbative QCD calculation

2. **Observational constraint and quark cores of neutron stars**

Smooth matching to the nuclear EoS

Fujimoto, Fukushima: 2011.10891 (2020)



NN EoS extracted from X-ray NS data using neural network

Fujimoto,Fukushima,Murase (2017-)

χEFT+Astro $n_B \le n_0$: ChEFT EoS

$n_B > n_0$: extrapolation w/ the $2M_\odot$ pulsar constraint

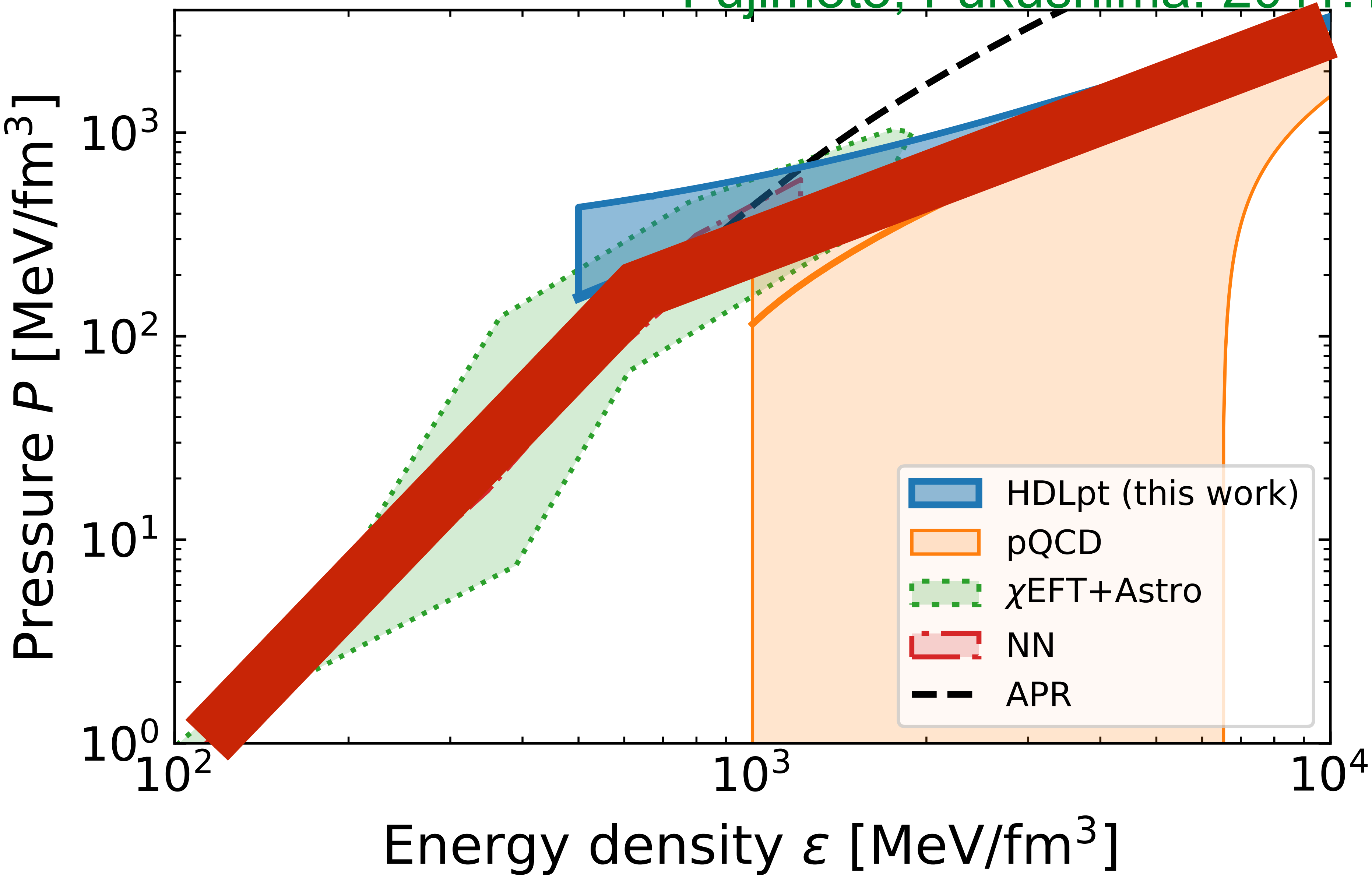
Hebeler,Lattimer,Pethick,Schwenk (2013)

APR conventional nuclear EoS

Akmal,Pandharipande,Ravenhall (1998)

Smooth matching to the nuclear EoS

Fujimoto, Fukushima: 2011.10891 (2020)



NN EoS extracted from X-ray NS data using neural network

Fujimoto,Fukushima,Murase (2017-)

χEFT+Astro $n_{\mathrm{B}} \leq n_0$: ChEFT EoS

$n_{\mathrm{B}} > n_0$: extrapolation w/ the $2M_\odot$ pulsar constraint

Hebeler,Lattimer,Pethick,Schwenk (2013)

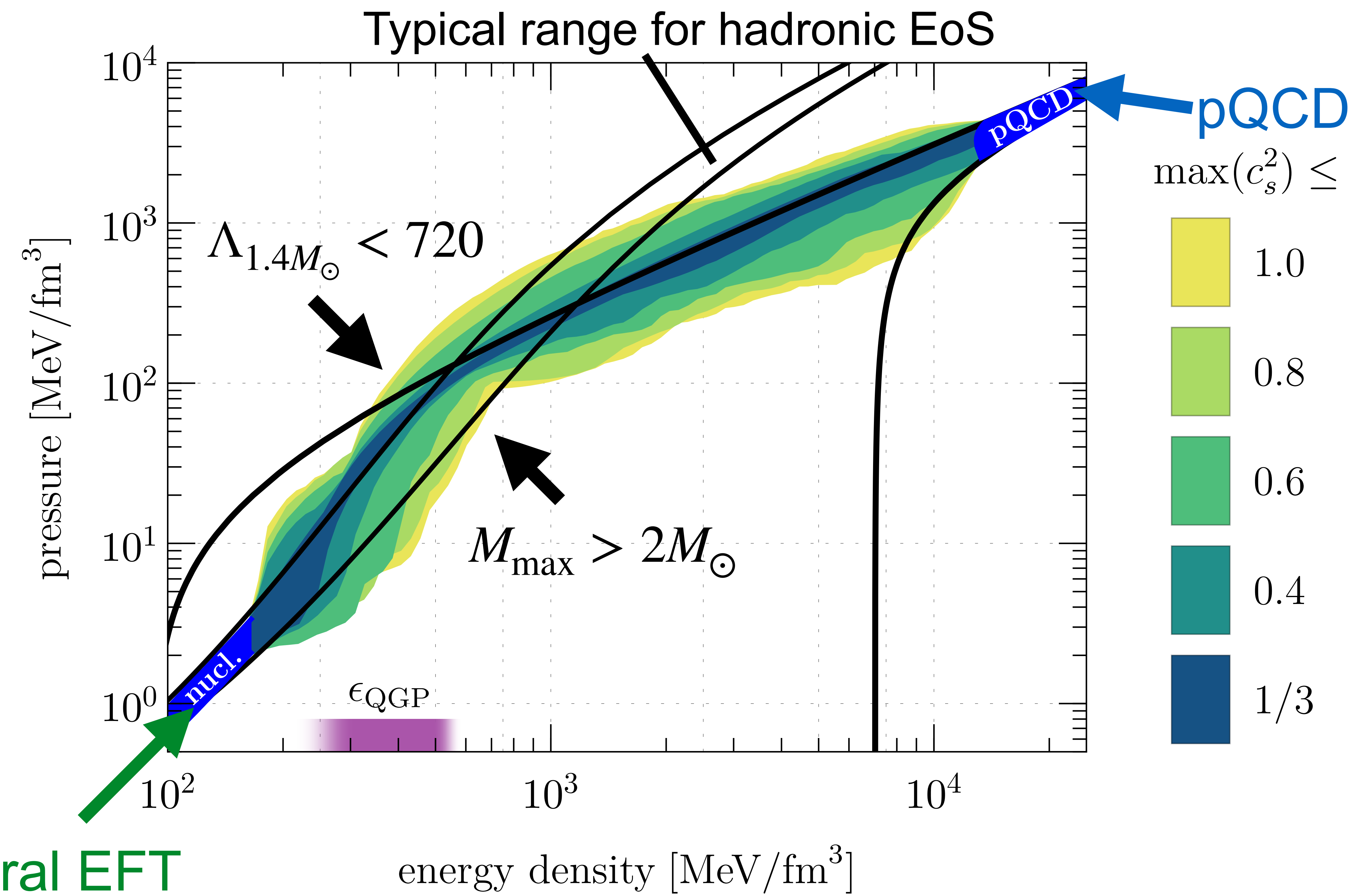
APR conventional nuclear EoS

Akmal,Pandharipande,Ravenhall (1998)

QCD + observational constraint

Annala,Gorda,Kurkela,Nättilä,Vuorinen, Nat.Phys.(2020)

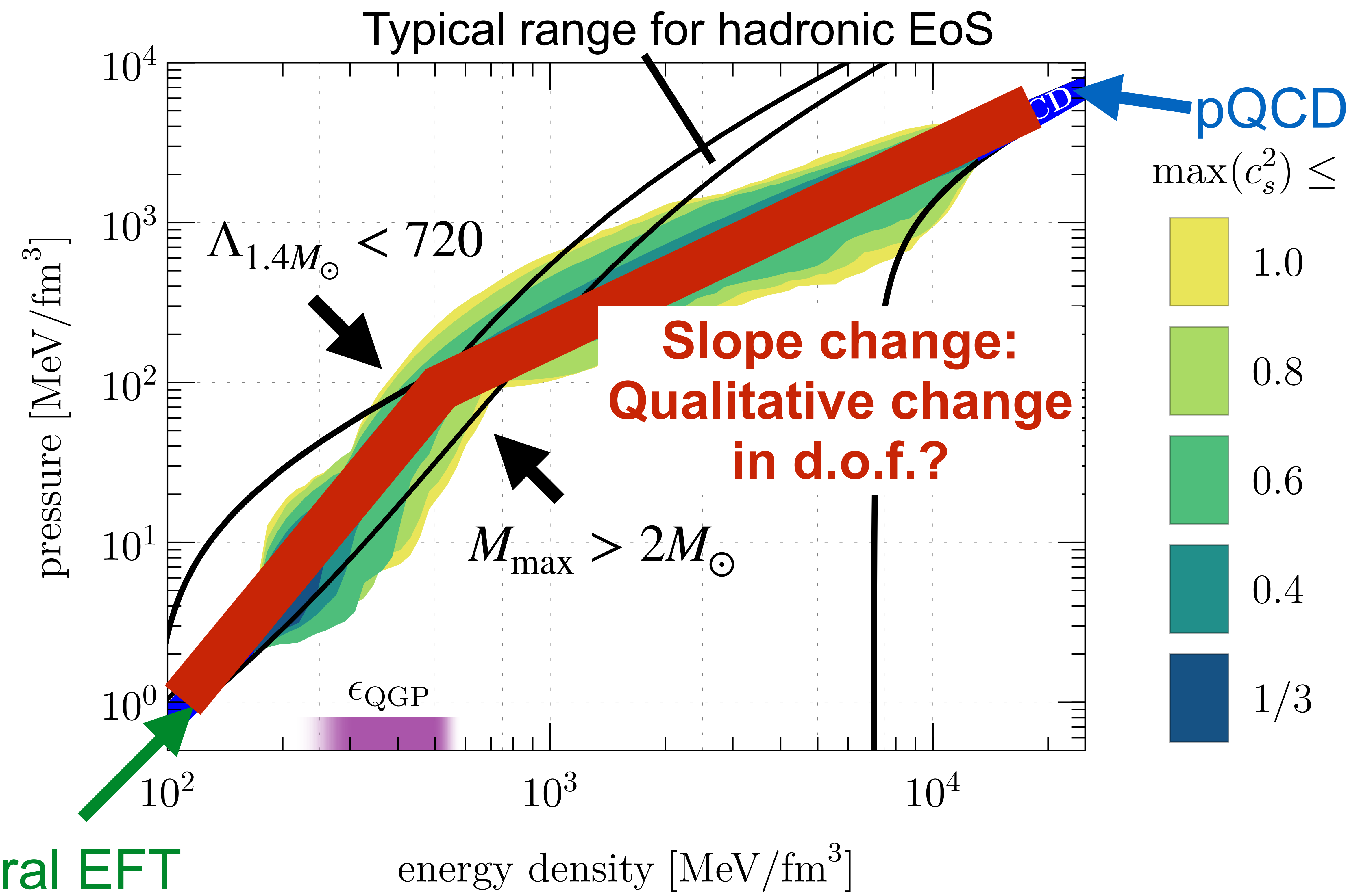
~ 570,000 interpolated EoSs are plotted:



QCD + observational constraint

Annala,Gorda,Kurkela,Nättilä,Vuorinen, Nat.Phys.(2020)

~ 570,000 interpolated EoSs are plotted:



QCD + observational constraint

Annala,Gorda,Kurkela,Nättilä,Vuorinen, Nat.Phys.(2020)

- EoS is characterized by several parameters:

- **Speed of sound**: $c_s^2 = \dfrac{dP(\varepsilon)}{d\varepsilon}$

- **Polytropic index**: $\gamma = \dfrac{d\log P}{d\log\varepsilon}$ (cf. polytrope: $P = K\varepsilon^\gamma$)

- **Ratio to free Fermi-Dirac pressure**: $P_{\mathrm{FD}}(\mu_{\mathrm{B}}) = \dfrac{\mu_{\mathrm{B}}^4}{12\pi^2}$

  … effective measure for number of d.o.f

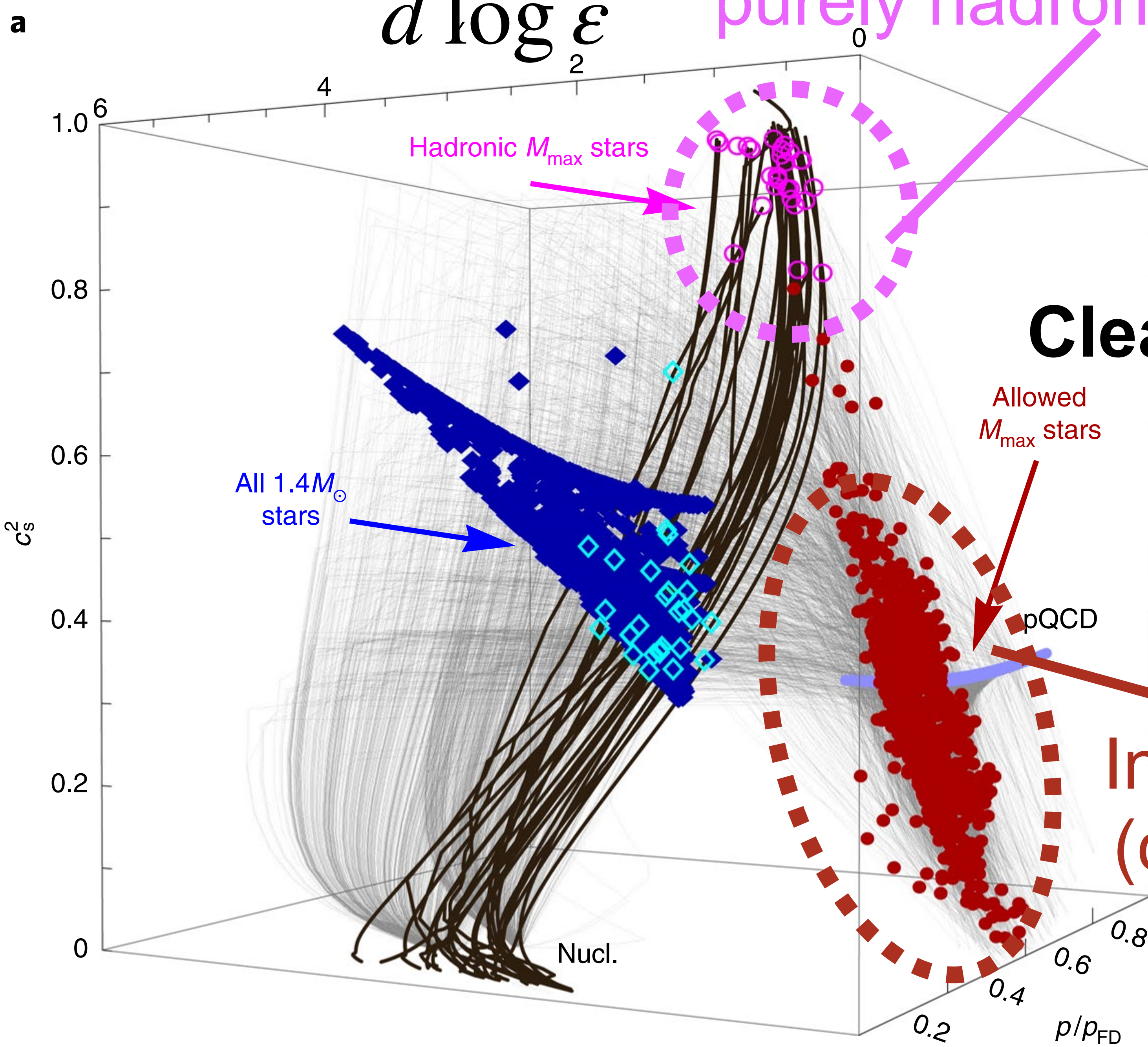
3d-plot of many interpolated EoSs

Annala,Gorda,Kurkela,Nättilä,Vuorinen, Nat.Phys.(2020)

$$\gamma = \frac{d\log P}{d\log\varepsilon}$$

Heavy neutron stars from purely hadronic model EoS

$$c_s^2 = \frac{dP(\varepsilon)}{d\varepsilon}$$



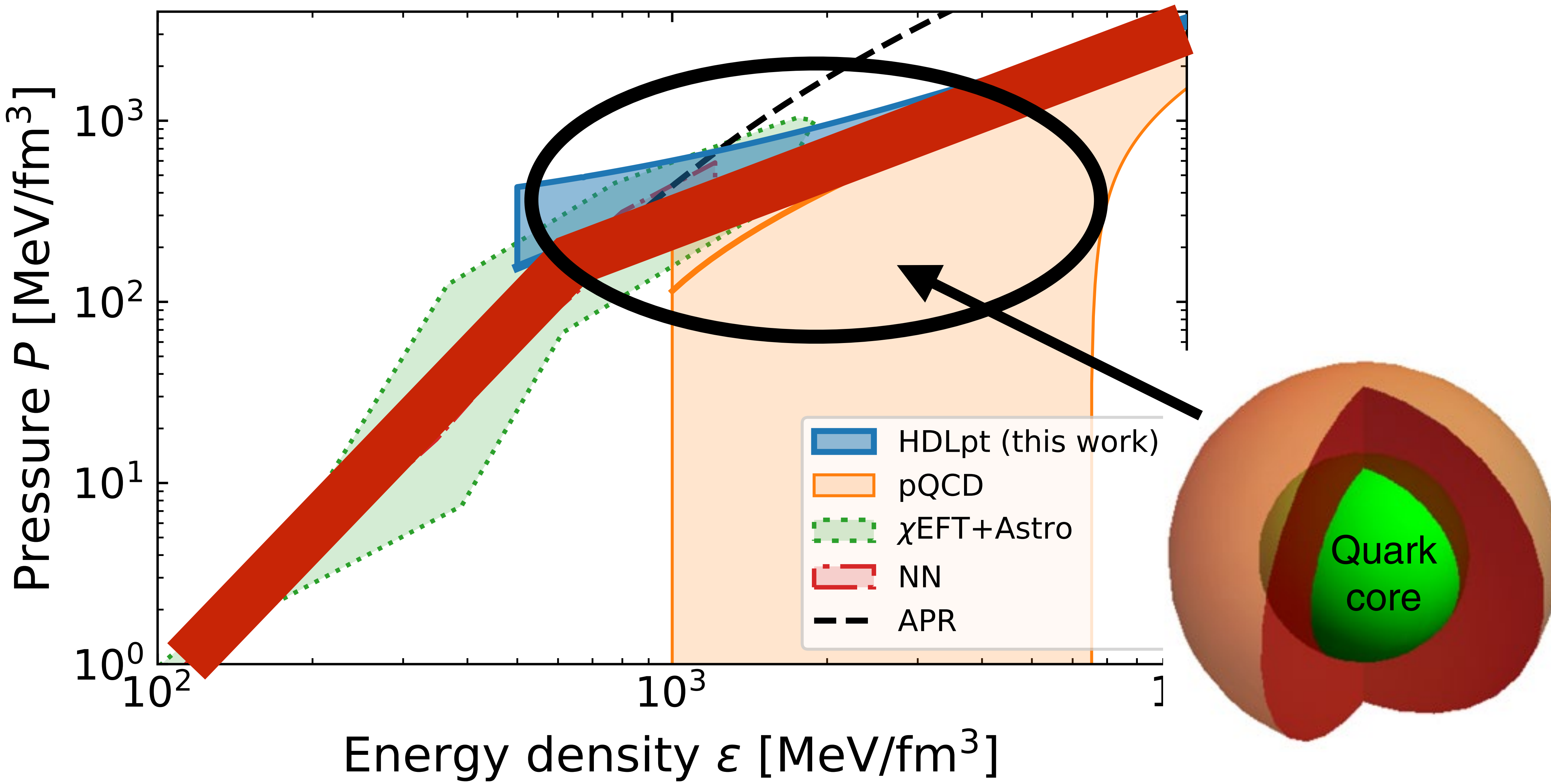
Clear separation!

Interpolated EoS (contains quark)

$$P/P_{\rm FD}$$

Quark core is not “exotic” anymore

Fujimoto, Fukushima: 2011.10891 (2020)



Summary

Developed **QCD-based & model-independent calculations** of the dense matter EoS:

1. Refined pQCD calculation of the EoS:

- Performed the novel calculation in perturbative QCD
- Result implies that the applicability is extended compared with the preceding calculation; **pQCD is not useless**

2. Observational constraints and quark core:

- Rapid slope change (likely a crossover) implied by the calculation
- This may be identified with the emergence of quark core; it is **not exotic anymore**